Amplify Math TENNESSEE

Grade 8

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Volume 1

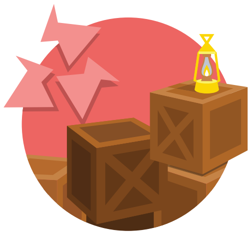
Unit 1 Rigid Transformations and Congruence

Unit Narrative:
The Art of Transformation

Students begin Grade 8 by joining talented architects, artists, and mathematicians in the study of two-dimensional figures. Equipped with their geometry toolkits, students manipulate familiar figures with new methods, and make key discoveries along the way.

Note: *Lessons in gray are recommended to be omitted.*

Sub-Unit Narrative:
How do you make a piece of cardboard come alive?
Pack your geometry toolkits for a transformational journey into the movement of figures.



Sub-Unit Narrative:
How can a crack make a piece of art priceless?
Something special happens when you perform rigid transformations on a figure.

Sub-Unit Narrative:
What's got 10 billion galaxies and goes great with maple syrup?
Construct a triangle from a straight angle and cut two parallel lines to see what angle relationships you notice.

Unit 2 Dilations and Similarity

Students explore a new type of transformation, dilations, and practice using dilations to create and recognize similar figures. Students' understanding of the characteristics of these similar figures, of similar triangles specifically, will serve as the foundation for their study of the slope of a line.

Unit Narrative:
More Than
Meets the Eye

Sub-Unit Narrative:
Would you put poison in your eye?

Shrink and stretch objects on and off the plane and study the characteristics of the figures you dilate.

Sub-Unit Narrative:
Do you really get what you pay for?

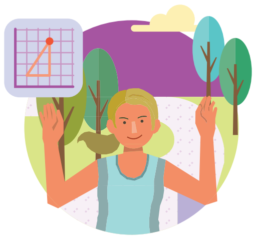
Learn how some companies use dilations to create similar, and slightly smaller, sized packaging, in a process called "shrinkflation."

Unit 3 Linear Relationships

Students make connections between the rate of change, slope, and the constant of proportionality, drawing on previous knowledge to explore an exciting new relationship: the linear relationship.

Unit Narrative:
A Straight Change

● = *Tennessee-specific lessons*



Sub-Unit Narrative:
How fast is a geography teacher?
On your mark, get set, go! Use your understanding of slope to show how a geography teacher shocked the world with her record setting speed.

Sub-Unit Narrative:
How did a coal mine help build America's most famous amusement park?
Use linear relationships to collect as many coins as you can at Honest Carl's Funtime World amusement park.

Sub-Unit Narrative:
How did a 16-year-old take down a Chicago Bull?
Create equations from linear relationships and find how a 16-year-old was able to beat Michael Jordan in a game of basketball.

Unit 4 Linear Equations and Systems of Linear Equations

Students begin this unit by developing algebraic methods for solving linear equations with variables on both sides of the equation. They then use these algebraic methods, along with graphs and tables, to solve systems of linear equations.

Unit Narrative:
The Path the Mind Takes

Note: *Lessons in gray are recommended to be omitted.*

Sub-Unit Narrative:
Who was the Father of Algebra?

When traders in 9th century Baghdad needed a better system for solving problems, a mathematician developed a new method he called "al-jabr" or algebra.

Sub-Unit Narrative:
How is anesthesia like buying live lobsters?

Now that you have practiced solving equations, take a closer look at how you can use linear equations to solve everyday problems.

Volume 2

Unit 5 Functions and Volume

By the end of this unit, students will have derived the formulas for the volumes of cylinders, cones, and spheres. But it all starts with a deep dive into the concept of what makes a relationship a function at the beginning of the unit.

Unit Narrative:
Pumping up the Volume on Functions

Unit 6 Exponents and Scientific Notation

This unit is about the numbers so large and so small that students must develop new ways of working with them. Students deepen their knowledge of exponents before exploring how powers of 10 and scientific notation can be used to write and work with numbers as small as the mass of a bacterium or as large as the number of atoms in the Universe.

Unit Narrative:
From Teeny-Tiny to Downright Titanic

Sub-Unit Narrative:
How many carbs are in a game of chess?

You probably already know a thing or two about exponents, but what happens when you multiply or divide expressions with exponents?

Sub-Unit Narrative:
Who should we call when we run out of numbers?

You'll work with numbers that are super small and incredibly large. But you won't waste your time writing pesky zeros!

Unit 7 Irrationals and the Pythagorean Theorem

Students study rational and irrational numbers using geometry and expressions before exploring a proof of the Pythagorean Theorem.

Unit Narrative:
The Mystery of the Pythagoreans

Sub-Unit 1 Rational and Irrational Numbers 727

Sub-Unit Narrative:
How rational were the Pythagoreans?
Find out if every number can be represented by a fraction.

Sub-Unit 2 The Pythagorean Theorem 773

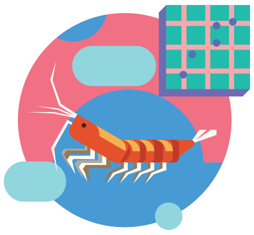
Sub-Unit Narrative:
What do the President of the United States and Albert Einstein have in common?
Uncover a special property of right triangles when you explore one of the nearly 500 proofs of the Pythagorean Theorem.

Unit 8 Associations in Data

What makes a cat logo consumer friendly? Is there a relationship between eye distance and height for a species of krill? Are adults just as likely to ride a bike as kids? Did the hole in the ozone layer have an association with skin cancer rates in Australia? In this unit, students will grapple with these questions and more, as they discover new ways to represent associations in data and build their data literacy.

Unit Narrative:
Data and the
Ozone Layer

Note: *Lessons in gray are recommended to be omitted.*
● = *Tennessee-specific lessons*



Sub-Unit 1 Associations in Data ... 833

Sub-Unit Narrative:
Who is the biggest mover and shaker in the Antarctic Ocean?
Explore the ozone hole using scatter plots, while learning about the different kinds of associations data can have.

Tennessee Mathematics Standards, Grade 8

8.NS	**The Number System**	**Lesson(s)**
8.NS.A	**Know that there are numbers that are not rational, and approximate them by rational numbers.**	
8.NS.A.1	Know that real numbers that are not rational are called irrational (e.g., π, $\sqrt{2}$, etc.). Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually or terminates, and convert a decimal expansion which repeats eventually or terminates into a rational number.	**Unit 7, Lessons** 6–8
8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers by locating them approximately on a number line diagram. Estimate the value of irrational expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.	**Unit 7, Lessons** 3–5
8.EE	**Expressions and Equations**	**Lesson(s)**
8.EE.A	**Work with radicals and integer exponents.**	
8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.	**Unit 6, Lessons** 2–8, 13–15
8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.	**Unit 7, Lessons** 2, 3, 5, 6
8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 and determine that the world population is more than 20 times larger.	**Unit 6, Lessons** 9–13, 15
8.EE.A.4	Using technology, solve real-world problems with numbers expressed in decimal and scientific notation. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).	**Unit 6, Lessons** 12–15
8.EE.B	**Understand the connections between proportional relationships, lines, and linear equations.**	
8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	**Unit 3, Lessons** 4–6
8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	**Unit 2, Lesson** 11 **Unit 3, Lessons** 9–15, 18, 19

Tennessee Mathematics Standards, Grade 8

8.EE.C	**Analyze and solve linear equations, linear inequalities, and systems of two linear equations.**	
8.EE.C.7	Solve linear equations in one variable.	**Unit 3, Lesson** 16 **Unit 4, Lessons** 4, 9, 10
8.EE.C.7a	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).	**Unit 4, Lessons** 7–9
8.EE.C.7b	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	**Unit 4, Lessons** 2, 5–9
8.EE.C.8	Analyze and solve systems of two linear equations graphically.	**Unit 4, Lessons** 10–12
8.EE.C.8a	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.	**Unit 3, Lesson** 17 **Unit 4, Lessons** 13, 14
8.EE.C.8b	Estimate solutions by graphing a system of two linear equations in two variables. Identify solutions by inspecting graphs of a system of linear equations in two variables.	**Unit 4, Lessons** 13, 14
8.EE.C.9	By graphing on the coordinate plane or by analyzing a given graph, determine the solution set of a linear inequality in one or two variables.	**Unit 3, Lessons** 17A
8.F	**Functions**	**Lesson(s)**
8.F.A	**Define, evaluate, and compare functions.**	
8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)	**Unit 5, Lessons** 2–5, 19, 20
8.F.A.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.	**Unit 5, Lessons** 3, 7, 8
8.F.A.3	Know and interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1, 1)$, $(2, 4)$ and $(3, 9)$, which are not on a straight line.	**Unit 3, Lessons** 8, 9 **Unit 5, Lessons** 7, 8, 19

8.F.B	**Use functions to model relationships between quantities.**	
8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**Unit 3, Lessons** 7–10, 13, 15 **Unit 5, Lessons** 4, 8–11
8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**Unit 5, Lessons** 5, 6, 10
8.G	**Geometry**	**Lesson(s)**
8.G.A	**Understand and describe the effects of transformations on two-dimensional figures and use informal arguments to establish facts about angles.**	
8.G.A.1	Describe the effect of translations, rotations, reflections, and dilations on two-dimensional figures using coordinates.	**Unit 1, Lessons** 1–8, 10, 15, 18 **Unit 2, Lessons** 4, 5, 12 **Unit 3, Lesson** 12
8.G.A.1a	Verify informally that lines are taken to lines, and determine when line segments are taken to line segments of the same length.	**Unit 1, Lessons** 9, 13, 14
8.G.A.1b	Verify informally that angles are taken to angles of the same measure.	**Unit 1, Lessons** 9, 14
8.G.A.1c	Verify informally that parallel lines are taken to parallel lines.	**Unit 1, Lessons** 13, 14
8.G.A.1d	Make connections between dilations and scale factors.	**Unit 2, Lessons** 3–5
8.G.A.2	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	**Unit 1, Lessons** 15–17 **Unit 2, Lessons** 8, 10
8.G.B	**Understand and apply the Pythagorean Theorem.**	
8.G.B.3	Explain a model of the Pythagorean Theorem and its converse.	**Unit 7, Lessons** 9, 10, 12
8.G.B.4	Know and apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	**Unit 7, Lessons** 11, 15, 16
8.G.B.5	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	**Unit 7, Lessons** 13, 14

Tennessee Mathematics Standards, Grade 8

8.G.C	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**	
8.G.C.6	Apply the formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems.	**Unit 5, Lessons** 12–21
8.SP	**Statistics and Probability**	**Lesson(s)**
8.SP.A	**Investigate patterns of association in bivariate data.**	
8.SP.A.1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	**Unit 8, Lessons** 1–3, 5–7
8.SP.A.2	Know that straight lines are widely used to model linear relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.	**Unit 8, Lessons** 4–7
8.SP.A.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting slopes and intercepts. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.	**Unit 8, Lessons** 5–7
8.SP.B	**Investigate chance processes and develop, use, and evaluate probability models.**	
8.SP.B.4	Find probabilities of and represent sample spaces for compound events using organized lists, tables, tree diagrams, and simulation.	**Unit 8, Lessons** 7C
8.SP.B.4a	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	**Unit 8, Lessons** 7B
8.SP.B.4b	Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.	**Unit 8, Lessons** 7A, 7B

Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Unit 1, Lessons 8, 17

Unit 3, Lessons 4, 5, 9, 12, 15, 18, 19

Unit 4, Lessons 2, 5, 11

Unit 5, Lessons 7, 21

Unit 6, Lessons 11, 13, 14

Unit 7, Lessons 10–12, 15, 16

Unit 8, Lesson 7C

MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

Unit 2, Lesson 1

Unit 3, Lessons 9, 11–13

Unit 4, Lessons 2–4, 10, 12–15

Unit 5, Lessons 3, 5, 10, 12, 16–20

Unit 6, Lessons 3, 6, 7, 11, 13, 15

Unit 7, Lessons 2, 3, 6, 8, 12

Unit 8, Lessons 1, 2, 5, 6

MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Unit 1, Lessons 1, 3, 4, 8, 10

Unit 2, Lessons 5–9, 11

Unit 3, Lessons 5, 10, 14, 16, 19

Unit 4, Lessons 2, 3, 5–9, 12

Unit 5, Lessons 6, 13, 15, 18

Unit 6, Lessons 8–10, 14

Unit 7, Lessons 5–7, 9

Unit 8, Lesson 4

Standards for Mathematical Practice

MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Unit 1, Lesson 18
Unit 2, Lessons 10, 12
Unit 3, Lessons 1, 2, 6, 10, 18
Unit 4, Lessons 1, 2, 10, 12
Unit 5, Lessons 1, 5, 6, 9, 11, 19–21
Unit 6, Lesson 11
Unit 8, Lessons 1, 5, 7, 7B, 7C

MP5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a compass, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Unit 1, Lessons 4, 9
Unit 2, Lesson 3
Unit 3, Lesson 19
Unit 4, Lessons 1
Unit 7, Lesson 3

MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

Unit 1, Lessons 2, 3, 5, 7, 8, 10
Unit 2, Lessons 3–7
Unit 3, Lessons 2, 3, 5, 8, 13–15, 18
Unit 4, Lesson 7
Unit 5, Lessons 4, 8
Unit 6, Lessons 9, 10, 12, 14
Unit 7, Lessons 1, 2, 4, 5, 11, 13
Unit 8, Lessons 3, 4

MP7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students see 7 × 8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Unit 1, Lessons 1, 3–9, 13–17

Unit 2, Lessons 2, 4, 8, 9, 11, 12

Unit 3, Lessons 6, 11, 13, 15, 17, 17A, 19

Unit 4, Lessons 4–6, 8, 9, 14

Unit 5, Lessons 1, 2, 4, 8, 13, 18

Unit 6, Lessons 1–3, 5, 6

Unit 7, Lessons 1, 7, 8, 11–114

Unit 8, Lessons 5, 7, 7A

MP8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $\frac{(y - 2)}{(x - 1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Unit 1, Lessons 6, 7

Unit 2, Lessons 5

Unit 3, Lessons 7, 11

Unit 5, Lessons 3, 7, 14

Unit 6, Lessons 3, 4, 6, 7

Unit 7, Lessons 9, 16

Unit 8, Lessons 7A

Practice Problem Analysis

Teachers may omit the following Practice Problems from the indicated lessons as they address topics beyond the scope of the Tennessee Mathematics Standards.

Unit 1: Rigid Transformations and Congruence

Lesson	Problem(s)
10	6

Unit 2: Dilations and Similarity

Lesson	Problem(s)
2	4

Unit 4: Linear Equations and Systems of Linear Equations

Lesson	Problem(s)
14	5

Unit 5: Functions and Volume

Lesson	Problem(s)
7	3
13	4

Unit 6: Exponents and Scientific Notation

Lesson	Problem(s)
6	5

Unit 7: Irrationals and the Pythagorean Theorem

Lesson	Problem(s)
1	5

Unit 8: Associations in Data

Lesson	Problem(s)
5	4
7	6

Solutions to Linear Inequalities

Let's use graphs to represent solutions of linear inequalities.

Focus

Goals

1. **Language Goal:** Given the graph of a related equation, determine the solution region to an inequality in two variables by testing the points on the line and on either side of the line. **(Speaking and Listening, Writing)**
2. **Language Goal:** Understand that the solutions to a linear inequality in two variables are represented graphically as a half-plane bounded by a line. **(Speaking and Listening, Writing)**

Coherence

Today

Students learn that solutions of two-variable inequalities involve pairs of values, which is similar to the solutions of two-variable equations. They graph solutions of inequalities, observing that solutions are not single points on a line but are composed of a region bounded by a line. Students determine whether the boundary line is included in the solution set. They write inequalities, given graphs that represent the solution regions **(MP7)**.

Previously

In Lesson 16 and 17, students explored linear equations with two variables and graphed an equation in the form of $Ax + By = C$. They determined whether points on the graph of the equation represent solutions to the equation.

Coming Soon

In Lesson 18, students will use multiple representations of real-world linear relationships, including equations, graphs, verbal descriptions, and tables.

Rigor

- Students develop **conceptual understanding** of graphical representations of solution sets of linear inequalities in two variables by making connections to graphs of two-variable linear equations.
- Students graph linear inequalities in two variables to build **procedural skills**.

Standards

Addressing

8.EE.C.9

By graphing on the coordinate plane or by analyzing a given graph, determine the solution set of a linear inequality in one or two variables.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	20 min	10 min	5 min	5 min
Independent	Small Groups	Pairs	Whole Class	Independent
	MP7			
8.EE.C.9	8.EE.C.9	8.EE.C.9	8.EE.C.9	8.EE.C.9

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, one per student (as needed)
- Activity 1 PDF (answers, for display)
- Anchor Chart PDF, *Graphing Linear Inequalities*
- Anchor Chart PDF, *Inequality Symbols and Key Phrases*
- calculators
- colored pencils

Math Language Development

New words

- ***boundary line***
- ***half-plane***

Review words

- *inequality*
- *boundary point*
- *solution set*

Amps | Featured Activity

Activity 2
Interactive Graphs

Students test ordered pairs in inequalities. The solutions and non-solutions are represented by different symbols on the graph. Students' points are then generated on a graph to reveal the boundary line and solution set to the inequality.

Amps
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Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel frustrated with their difficulty in looking for and making use of structure as they attempt to identify a clear boundary between the region of solutions and non-solutions **(MP7)**. Encourage students to persevere and continue plotting more points until the boundary becomes clearer. Encourage students to ask others to explain their strategy of the points they chose to plot.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

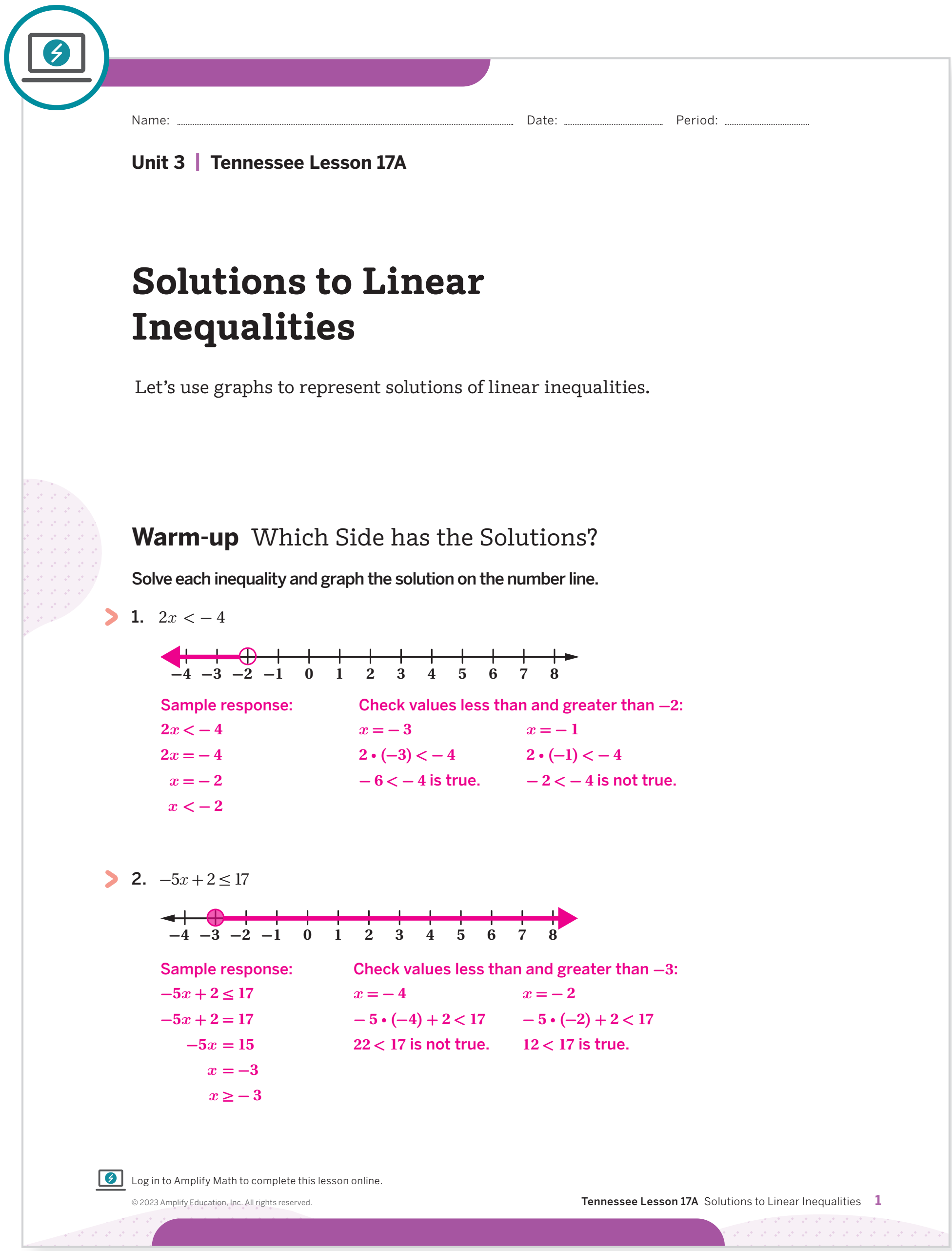
- In **Activity 2**, have students attempt only the first two problems. The third problem may be omitted or assigned as additional practice.

Independent | 5 min

Warm-up Which Side has the Solutions?

8.EE.C.9

Students solve inequalities by solving related equations, testing solutions, and graphing them on a number line.



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Unit 3 | Tennessee Lesson 17A

Solutions to Linear Inequalities

Let's use graphs to represent solutions of linear inequalities.

Warm-up Which Side has the Solutions?

Solve each inequality and graph the solution on the number line.

1. $2x < -4$

Sample response:

$2x < -4$

$2x = -4$

$x = -2$

$x < -2$

Check values less than and greater than −2:

$x = -3$

$2 \cdot (-3) < -4$

$-6 < -4$ is true.

$x = -1$

$2 \cdot (-1) < -4$

$-2 < -4$ is not true.

2. $-5x + 2 \leq 17$

Sample response:

$-5x + 2 \leq 17$

$-5x + 2 = 17$

$-5x = 15$

$x = -3$

$x \geq -3$

Check values less than and greater than −3:

$x = -4$

$-5 \cdot (-4) + 2 < 17$

$22 < 17$ is not true.

$x = -2$

$-5 \cdot (-2) + 2 < 17$

$12 < 17$ is true.

Log in to Amplify Math to complete this lesson online.

1 Launch

Give students individual work time followed by a whole class discussion about the steps of solving inequalities.

2 Monitor

Help students get started by asking:

- "What equation could you write to find the boundary value?"
- "How do you represent solutions to inequalities on the number line?"
- "How could you test whether a value is a solution to an inequality?"

Look for points of confusion:

- **Ignoring the negative sign on the variable when selecting values that are solutions.** Have students substitute the given values into the inequality and check whether it is still true.
- **Marking individual sets of points on the number line instead of showing the solution set by a line.** Have students try both integer and noninteger values to find the solution set.

Look for productive strategies:

- Using the sign of the values of x to reason about the answers.
- Testing numbers close to the boundary point to confirm its location.

3 Connect

Have individual students share how they graphed the solution to each inequality and wrote the solution in the form of an inequality, based on the graph.

Highlight the idea that there are an infinite number of values that make these inequalities true.

Ask whether it is possible to graph the inequalities in two variables on a number line. No, because inequalities in two variables are graphed on the coordinate plane like the equations in two variables.

Differentiated Support

Accessibility: *Memory; Organization*

To support working memory, provide students with sticky notes or mini whiteboards.

Accessibility: *Discussion Supports*

Display or provide students with the Anchor Chart PDF, *Inequality Symbols and Key Phrases* to support them when they explain their strategy. Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Power-up

To power up students' ability to write linear equations to represent scenarios, have students complete:

Tickets for a school play cost \$5 for adults and \$3 for students. At the end of the play, the school collected \$150 by selling n adult tickets and m student tickets.

Write an equation that shows the total amount of money collected from student and adult tickets. $5n + 3m = 150$

Use: Before the Warm-up

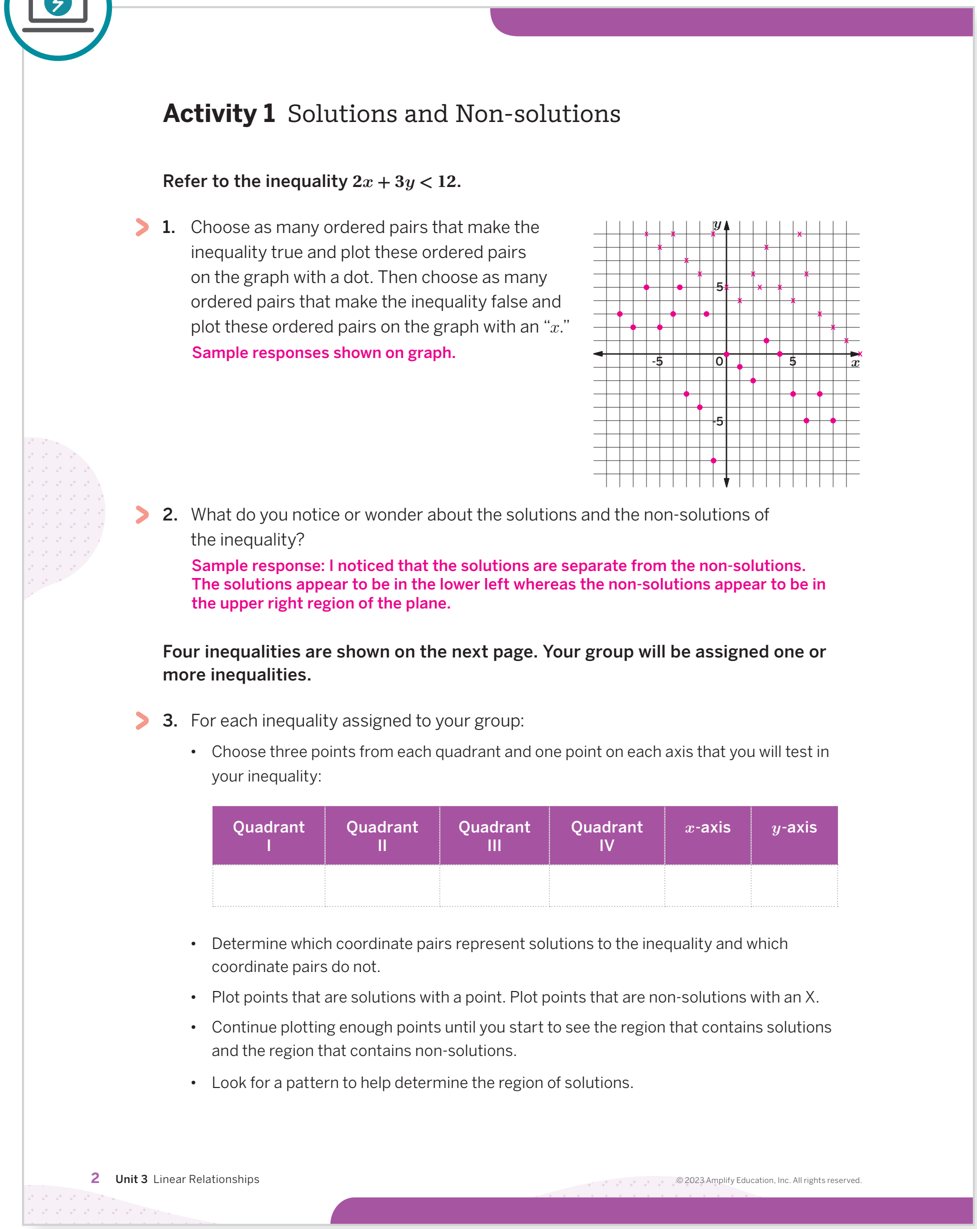
Informed by: Performance on Lesson 17, Practice Problem 6

Small Groups | 20 min

Activity 1 Solutions and Non-solutions

MP7
8.EE.C.9

Students graph linear inequalities to explore graphical representations of their solution sets.



Activity 1 Solutions and Non-solutions

Refer to the inequality $2x + 3y < 12$.

1. Choose as many ordered pairs that make the inequality true and plot these ordered pairs on the graph with a dot. Then choose as many ordered pairs that make the inequality false and plot these ordered pairs on the graph with an "x."
   Sample responses shown on graph.

2. What do you notice or wonder about the solutions and the non-solutions of the inequality?
   Sample response: I noticed that the solutions are separate from the non-solutions. The solutions appear to be in the lower left whereas the non-solutions appear to be in the upper right region of the plane.

Four inequalities are shown on the next page. Your group will be assigned one or more inequalities.

3. For each inequality assigned to your group:
   - Choose three points from each quadrant and one point on each axis that you will test in your inequality:

Quadrant I	Quadrant II	Quadrant III	Quadrant IV	x-axis	y-axis

   - Determine which coordinate pairs represent solutions to the inequality and which coordinate pairs do not.
   - Plot points that are solutions with a point. Plot points that are non-solutions with an X.
   - Continue plotting enough points until you start to see the region that contains solutions and the region that contains non-solutions.
   - Look for a pattern to help determine the region of solutions.

2 **Unit 3** Linear Relationships

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1 Launch

Ask, "Do the values in the ordered pair $(-1, 1)$ make the inequality $2x + 3y < 12$ true? What about $(0, 5)$?

Have students complete Problems 1—2 independently. Then conduct the ***Notice and Wonder*** routine.

Display Problem 3, and assign each group a different inequality. Give each group time to determine and graph points. Collect ordered pairs from each group, using differing symbols to plot their points.

2 Monitor

Help students get started by providing the general form of an ordered pair for each quadrant and axis, such as $(-x, y)$ and $(x, 0)$. Prompt students to choose ordered pairs using these forms.

Look for points of confusion:

- **Having difficulty distinguishing between solutions and non-solutions.** Provide students with two colored pencils, assigning different colors for solutions and non-solutions.
- **Choosing a limited number of points and not seeing the boundary line.** Have students try to plot non-solution and solution points that are closer and closer together.

Look for productive strategies:

- Continuing to choose points from each quadrant and each axis until a boundary line becomes clearer.
- Testing more points close to the apparent boundary line to confirm its location.
- Changing the inequality symbol to an equal sign and graphing the equation as the boundary line.

Activity 1 continued ›

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with the Activity 1 PDF table. Have them use the given ordered pairs for each graph as a starting point. Then ask them to generate six more of their own ordered pairs to test, recording their ordered pairs in the table.

Extension: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which the solutions and non-solutions are represented by different symbols on the graph. Students' points are generated on the graph to reveal the boundary line and solution set to the inequality.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students respond to the Ask questions, highlight the mathematical terms and phrases they use, such as *half-plane, boundary line, solution, solid line, dashed line, inequality symbol, etc.*

English Learners

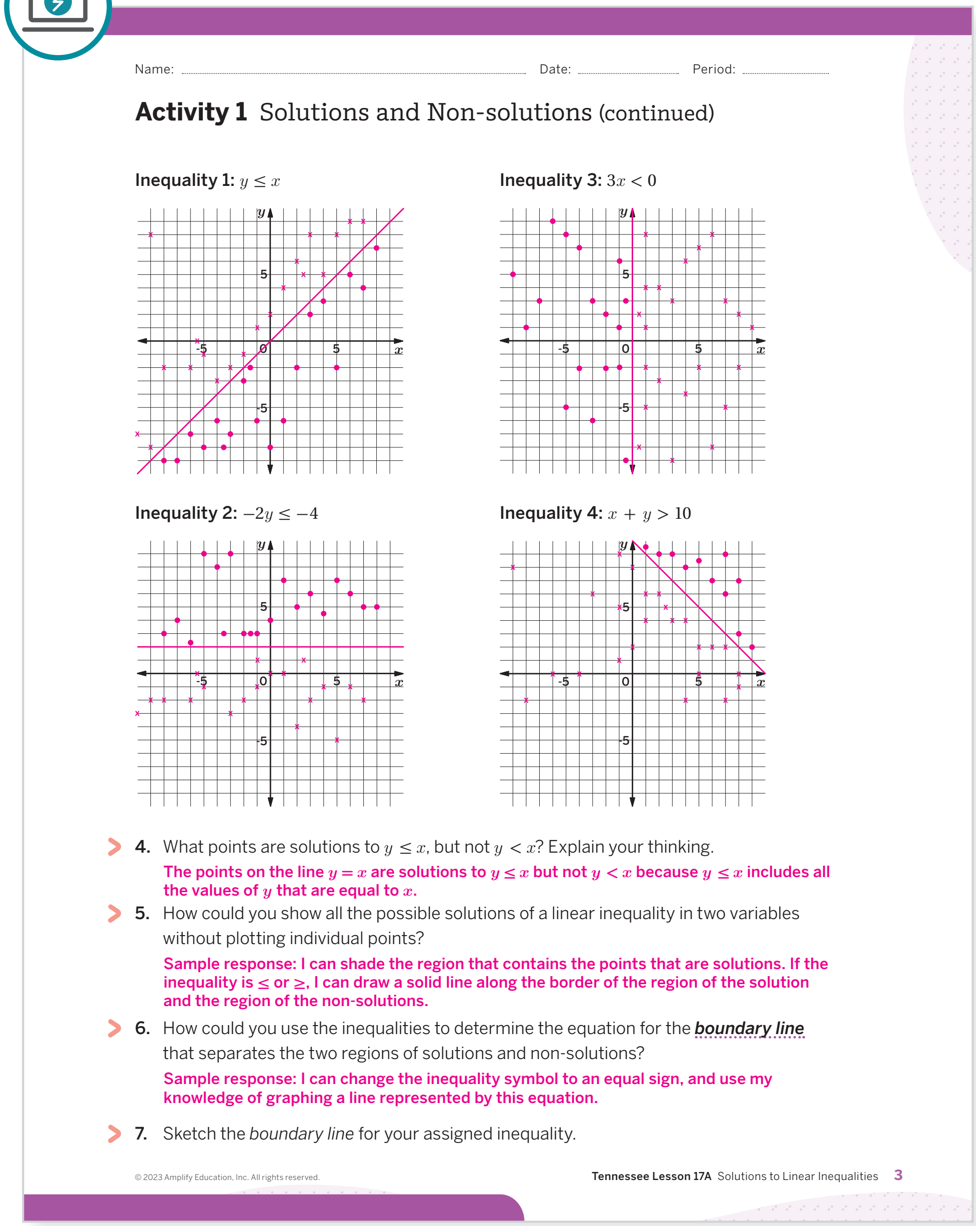
Annotate the graph of Inequality 1 to illustrate the boundary line and the two half-planes.

Small Groups | 20 min

MP7
8.EE.C.9

Activity 1 Solutions and Non-solutions (continued)

Students graph linear inequalities to explore graphical representations of their solution sets.



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Activity 1 Solutions and Non-solutions (continued)

Inequality 1: $y \leq x$

Inequality 3: $3x < 0$

Inequality 2: $-2y \leq -4$

Inequality 4: $x + y > 10$

4. What points are solutions to $y \leq x$, but not $y < x$? Explain your thinking.

   The points on the line $y = x$ are solutions to $y \leq x$ but not $y < x$ because $y \leq x$ includes all the values of y that are equal to x.

5. How could you show all the possible solutions of a linear inequality in two variables without plotting individual points?

   Sample response: I can shade the region that contains the points that are solutions. If the inequality is ≤ or ≥, I can draw a solid line along the border of the region of the solution and the region of the non-solutions.

6. How could you use the inequalities to determine the equation for the ***boundary line*** that separates the two regions of solutions and non-solutions?

   Sample response: I can change the inequality symbol to an equal sign, and use my knowledge of graphing a line represented by this equation.

7. Sketch the *boundary line* for your assigned inequality.

3 Connect

Have groups of students share their graphs showing where a clear boundary can be seen between two regions.

Highlight the importance of creating an organized list so that ordered pairs are neither missed nor repeated. **(MP7)**

Highlight that the solution and non-solution regions are separated by a clear boundary, called a boundary line. The boundary line separates the coordinate plane into two half-planes.

Define the terms ***boundary line*** and ***half plane***.

Ask:

- "How could you account for all solutions on these graphs?" I can shade the half-plane that contains the points that are solutions. (Point out that if the boundary line is part of the solution, a solid line should be used. If it is not, a dashed line should be used.)
- "How does the inequality symbol affect this boundary?" There are only solutions along this boundary line if the inequality symbol is ≥ or ≤. If the sign is < or >, then the points on the boundary line are not part of the solution of the inequality.

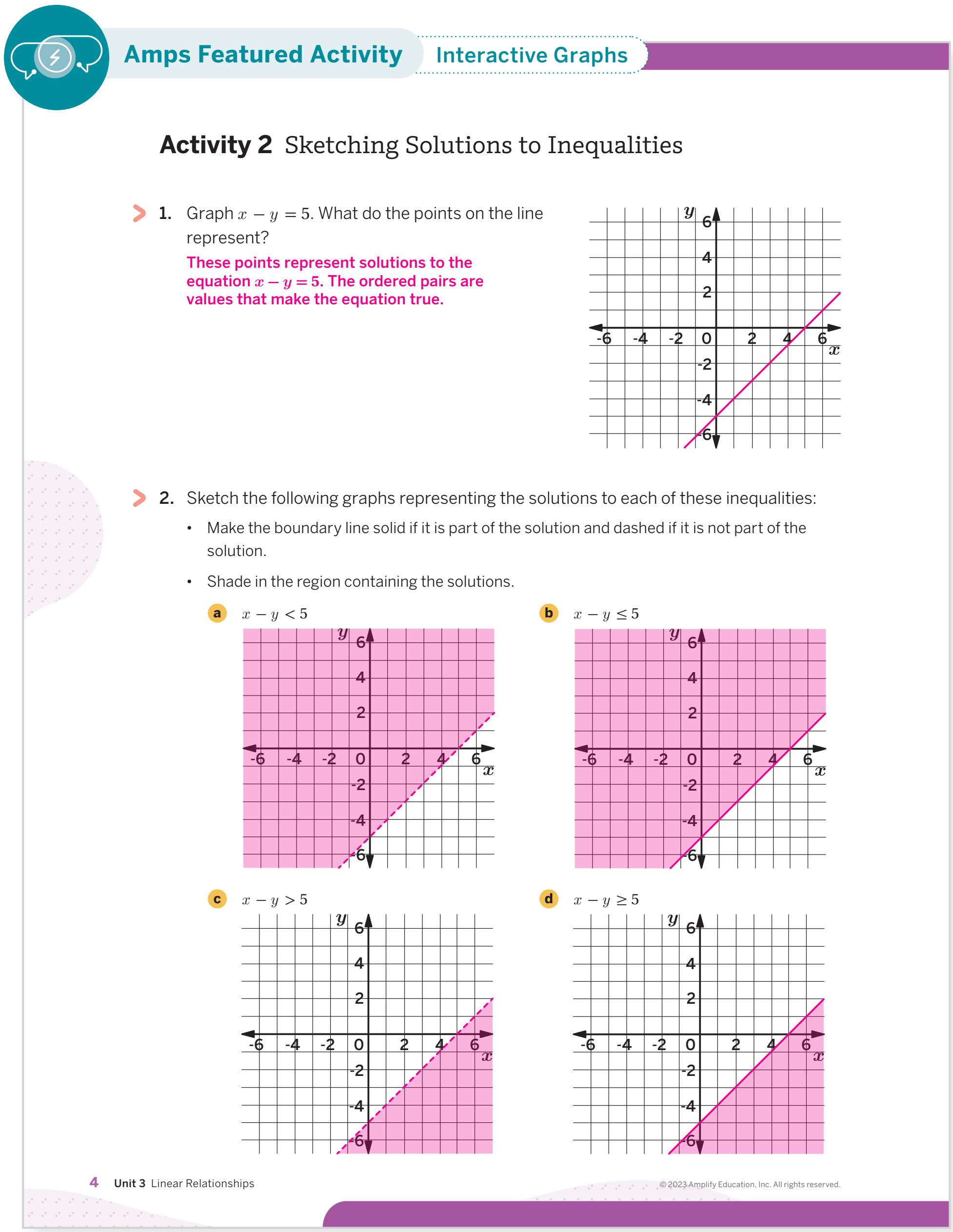
Pairs | 10 min

Activity 2 Sketching Solutions to Inequalities

8.EE.C.9

Students graph the solutions of inequalities and write inequalities whose solutions could be represented by given graphs.

Amps Featured Activity Interactive Graphs



Activity 2 Sketching Solutions to Inequalities

1. Graph $x - y = 5$. What do the points on the line represent?

   These points represent solutions to the equation $x - y = 5$. The ordered pairs are values that make the equation true.

2. Sketch the following graphs representing the solutions to each of these inequalities:
   - Make the boundary line solid if it is part of the solution and dashed if it is not part of the solution.
   - Shade in the region containing the solutions.

   a $x - y < 5$

   b $x - y \leq 5$

   c $x - y > 5$

   d $x - y \geq 5$

4 Unit 3 Linear Relationships

1 Launch

Have partners discuss the question, "How do we know if the boundary line is included in the solutions?" If the inequality symbol includes "equal to," then the points along the line are included.

Set an expectation for the amount of time students have to work individually on the activity.

2 Monitor

Help students get started by saying, "Use ordered pairs of points on either side of the boundary line to determine which region contains the solutions."

Look for points of confusion:

- **Assuming the original inequality symbol can be used to determine whether to shade "above" (for $>$ or $\geq$) or "below" (for $<$ or $\leq$) the boundary line.** Have students check this reasoning by testing ordered pairs on either side of the boundary line.
- **Struggling to write an equation for a vertical line or a horizontal line.** Have students determine and plot the coordinates of several points on the line and look for a pattern.

Look for productive strategies:

- Determining intercepts and the slope of the boundary line in Problem 3 to determine the equation of the boundary line.
- Testing ordered pairs on either side of the boundary line to determine the inequality symbol.

Activity 2 continued >

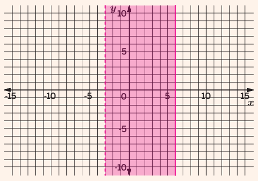
Differentiated Support

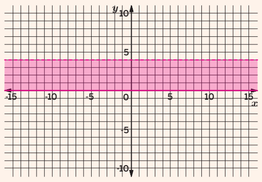
Accessibility: *Guide Processing and Visualization*

For Problem 3, encourage students to first think of the graph as a straight line, without an inequality or any shading. Then have them determine the inequality symbol.

Extension: *Math Enrichment*

Have students graph the inequalities $-3 < x \leq 6$ and $0 \leq y < 4$.





Math Language Development

MLR5: Critique, Correct, Clarify

During the Connect, display an incorrect inequality and incorrect reasoning, such as "The inequality in Problem 3a is $y > 3$ because all the points to the right of 3 are shaded." Ask these questions:

- **Critique:** "Why is this statement incorrect?"
- **Correct:** "How would you correct this statement?"
- **Clarify:** "How do you know your statement is correct?"

English Learners

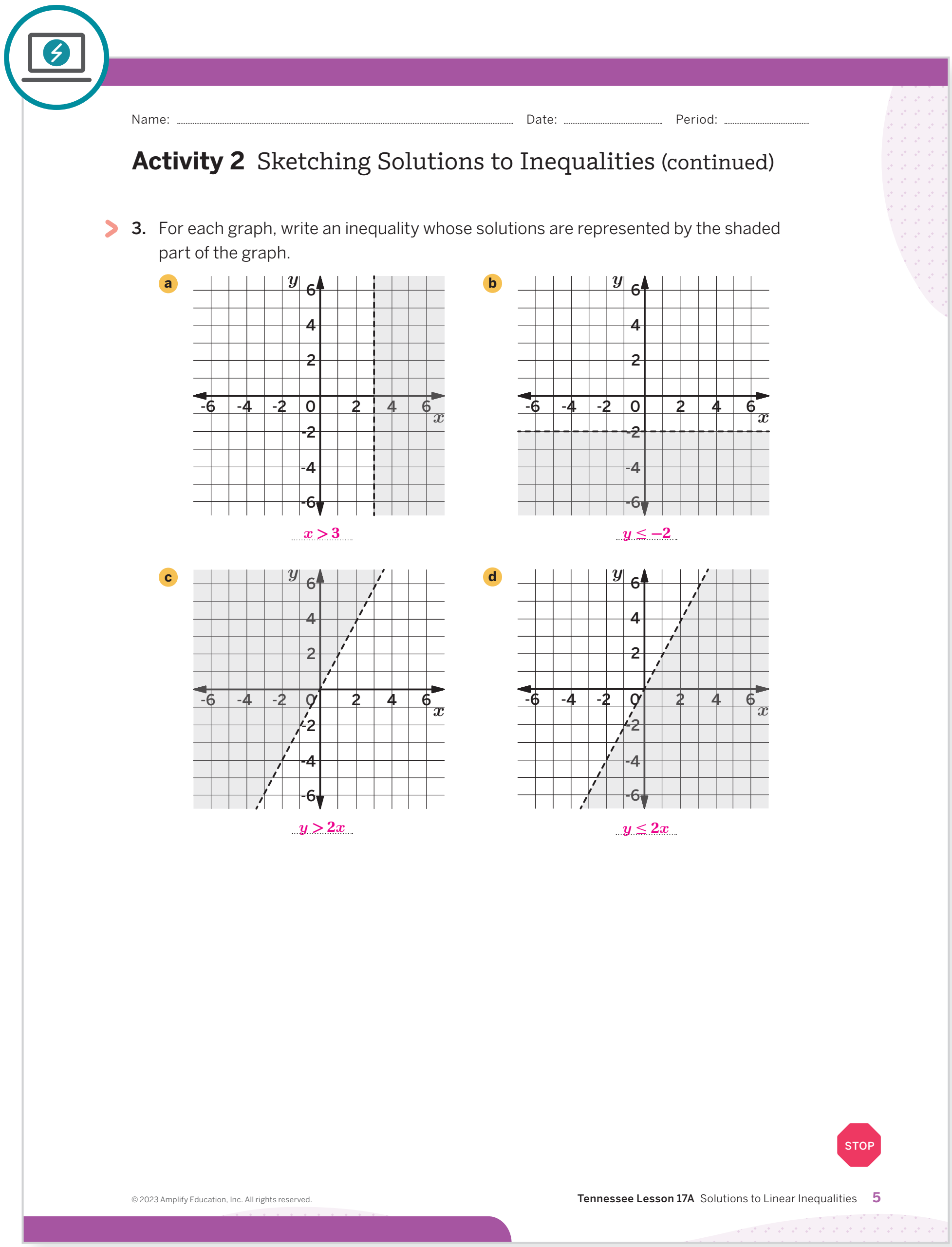
After the discussion, clearly annotate the incorrect part of the statement.

Pairs | 10 min

Activity 2 Sketching Solutions to Inequalities (continued)

8.EE.C.9

Students graph the solutions of inequalities and write inequalities whose solutions could be represented by given graphs.



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Activity 2 Sketching Solutions to Inequalities (continued)

3. For each graph, write an inequality whose solutions are represented by the shaded part of the graph.

a $x > 3$

b $y \leq -2$

c $y > 2x$

d $y \leq 2x$

Tennessee Lesson 17A Solutions to Linear Inequalities 5

3 Connect

Have individual students share their graphs for Problem 2 and their strategies for writing their inequalities in Problem 3.

Ask:

- What methods did you use to graph the boundary line?" Sample response: I determined the x- and y-intercepts of the equation that go along with each inequality, then connected these two points with a line.
- "Does checking one ordered pair in the solution region confirm that you graphed the inequality accurately?" As long as the boundary line is graphed correctly, checking the coordinates of a single point in the solution region confirms you graphed the inequality accurately.

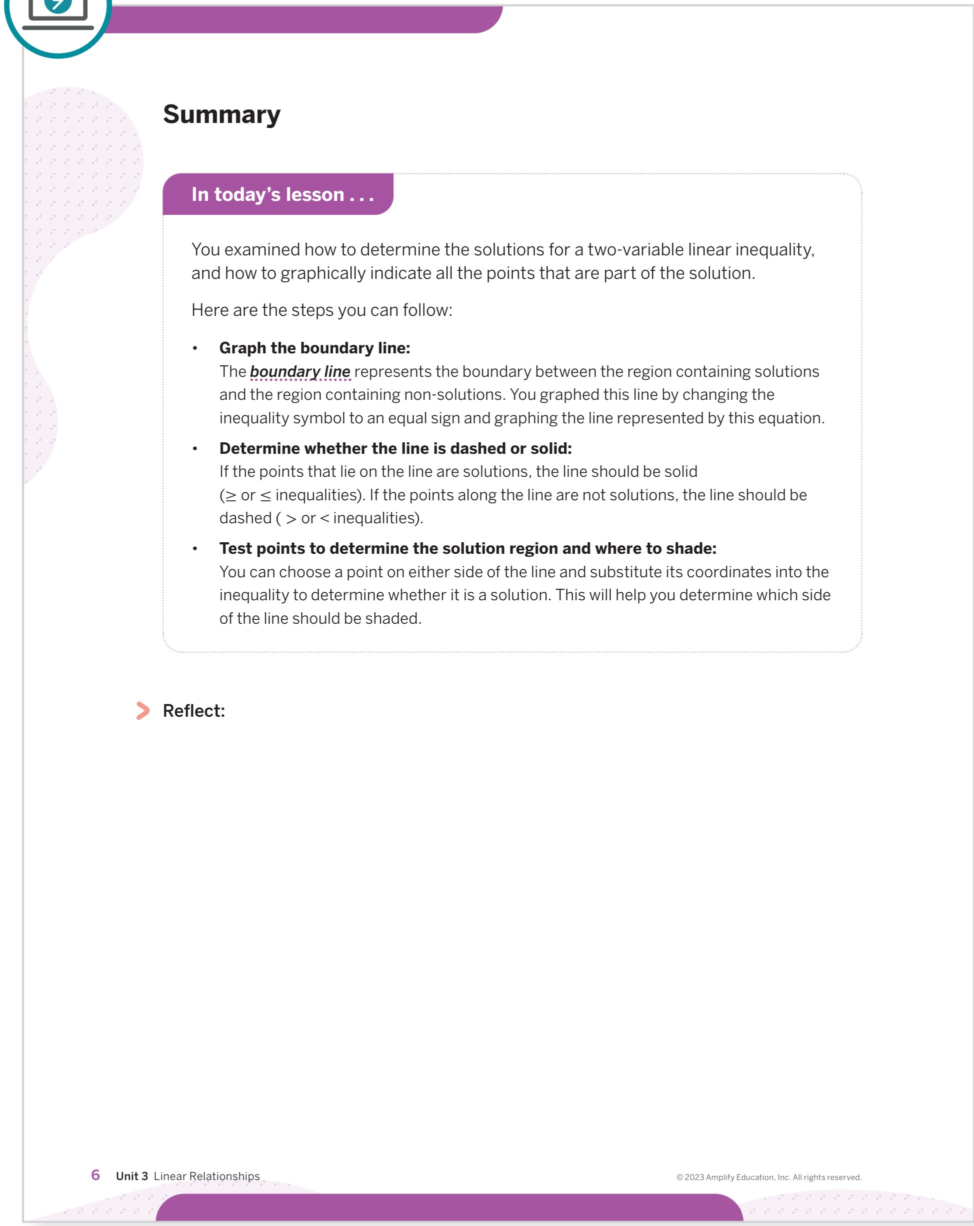
Highlight that graphing an accurate boundary line is critical to start plotting an inequality. It is important to check the accuracy of the boundary line before moving on to the next steps. Once students graph the boundary line, they can change it to a dashed or solid line depending on the inequality symbol. For inequality symbols $<$ and $>$, the line should be dashed. For $\leq$ and $\geq$, the line should be solid. Then, they can check which region they need to shade by choosing a point on either side of the line. If the inequality is true for the chosen point, shade the region that contains the chosen point. If the inequality is not true, shade the other region.

Whole Class | 5 min

Summary

8.EE.C.9

Review and synthesize how to graph the solution to a linear inequality in two variables.



Summary

In today's lesson . . .

You examined how to determine the solutions for a two-variable linear inequality, and how to graphically indicate all the points that are part of the solution.

Here are the steps you can follow:

- **Graph the boundary line:**
  The ***boundary line*** represents the boundary between the region containing solutions and the region containing non-solutions. You graphed this line by changing the inequality symbol to an equal sign and graphing the line represented by this equation.
- **Determine whether the line is dashed or solid:**
  If the points that lie on the line are solutions, the line should be solid (≥ or ≤ inequalities). If the points along the line are not solutions, the line should be dashed (> or < inequalities).
- **Test points to determine the solution region and where to shade:**
  You can choose a point on either side of the line and substitute its coordinates into the inequality to determine whether it is a solution. This will help you determine which side of the line should be shaded.

> Reflect:

Synthesize

Display Anchor Chart PDF, Graphing Linear Inequalities, to graph the inequality $y - x < 0$.

Have students share their method for graphing the boundary line.

Highlight that graphing the boundary line is a critical first step in determining where to shade to represent the solutions of linear inequalities in two variables.

Formalize vocabulary:

- ***boundary line***
- ***half-plane***

Ask:

- "What would you do next after graphing the boundary line?" Sample response: I would look at the inequality symbol to determine if the line is solid or dashed, pick a point on either side of the boundary, and then substitute the values of the ordered pair into the inequality to determine where to shade.
- "What methods can you use to check your graph?" Sample response: I could test points from both sides of the boundary line, as well as on the boundary line, to confirm the shading and whether the line should be dashed or solid.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What strategies did you find helpful today when graphing a boundary line and determining which region contains the solutions?"
- "Were any strategies or tools not helpful? Why?"

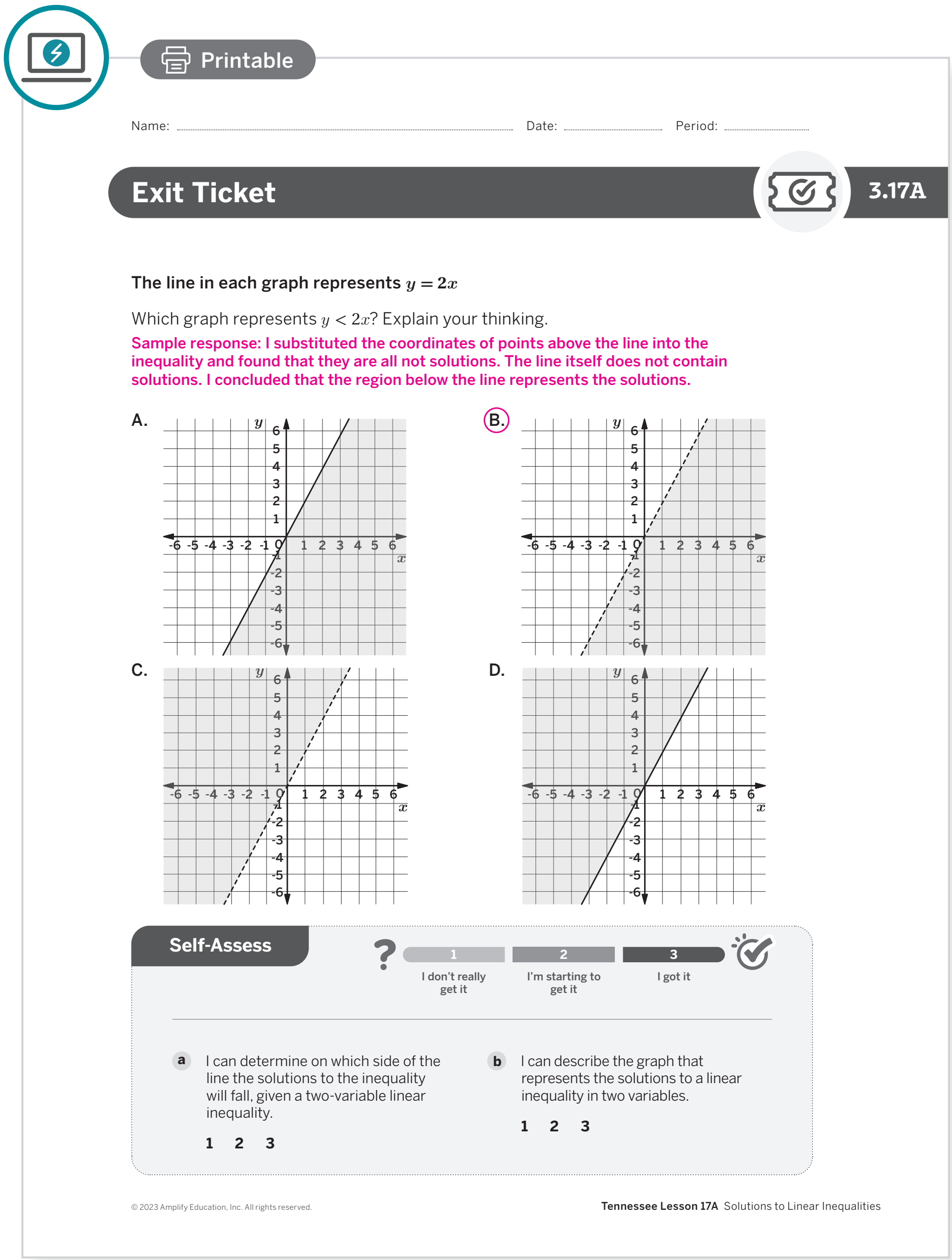
Independent | 5 min

Exit Ticket

8.EE.C.9

Students demonstrate their understanding by identifying the graph of the solutions to a linear inequality in two variables.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

3.17A

The line in each graph represents $y = 2x$

Which graph represents $y < 2x$? Explain your thinking.

Sample response: I substituted the coordinates of points above the line into the inequality and found that they are all not solutions. The line itself does not contain solutions. I concluded that the region below the line represents the solutions.

Self-Assess

1 I don't really get it

2 I'm starting to get it

3 I got it

a I can determine on which side of the line the solutions to the inequality will fall, given a two-variable linear inequality.

1 2 3

b I can describe the graph that represents the solutions to a linear inequality in two variables.

1 2 3

Success looks like . . .

- **Language Goal:** Given the graph of a related equation, determining the solution region to an inequality in two variables by testing the points on the line and on either side of the line. **(Speaking and Listening, Writing)**
    - » Identifying the boundary line and the solution region of $y < 2x$ as solid or dashed.
- **Language Goal:** Understanding that the solutions to a linear inequality in two variables are represented graphically as a half-plane bounded by a line. **(Speaking and Listening, Writing)**
    - » Identifying the line itself does not contain the solutions, but the region below the line contains the solutions of $y < 2x$

Suggested next steps

If students choose Graph A, consider:

- Reviewing how to determine if the boundary line is included in the solution, from Activity 1.
- Assigning Practice Problem 2.

If students choose the Graphs C or D, consider:

- Reviewing how to determine which region to shade from Activity 1.
- Assigning Practice Problem 2.
- Asking, "How can you use a point from either side of the boundary line to determine where to shade?"

Professional Learning

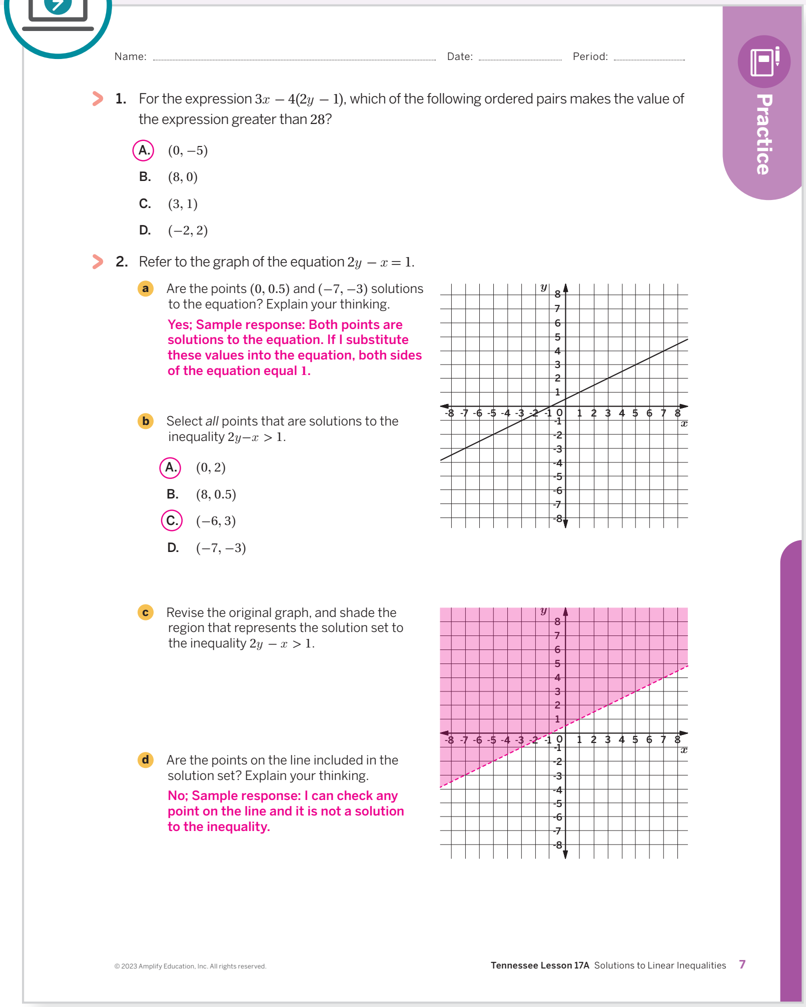
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students graphed linear inequalities in two variables. How did that build on the earlier work students did with graphing linear equations? What different ways did students approach graphing the boundary line? What might you change for the next time you teach this lesson?

Practice

Independent



Practice

Name: ________ Date: ________ Period: ________

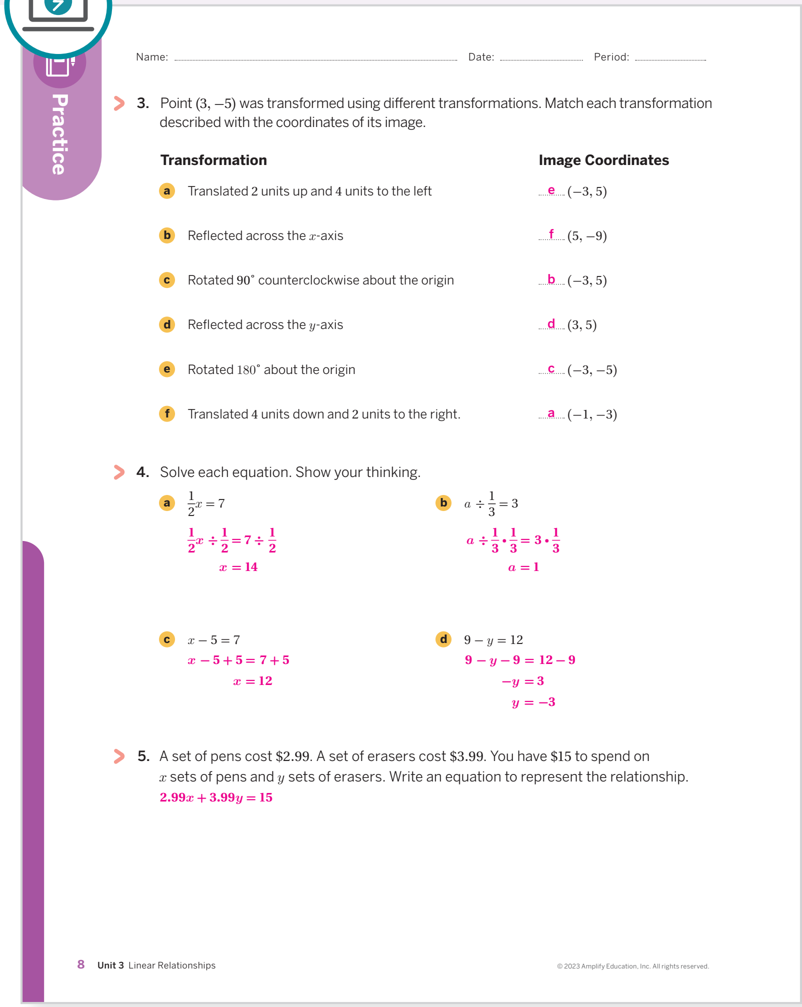
1. For the expression $3x - 4(2y - 1)$, which of the following ordered pairs makes the value of the expression greater than 28?
   - [✓ A.] $(0, -5)$
   - B. $(8, 0)$
   - C. $(3, 1)$
   - D. $(-2, 2)$

2. Refer to the graph of the equation $2y - x = 1$.
   - **a** Are the points $(0, 0.5)$ and $(-7, -3)$ solutions to the equation? Explain your thinking.

     Yes; Sample response: Both points are solutions to the equation. If I substitute these values into the equation, both sides of the equation equal 1.
   - **b** Select *all* points that are solutions to the inequality $2y - x > 1$.
     - [✓ A.] $(0, 2)$
     - B. $(8, 0.5)$
     - [✓ C.] $(-6, 3)$
     - D. $(-7, -3)$
   - **c** Revise the original graph, and shade the region that represents the solution set to the inequality $2y - x > 1$.
   - **d** Are the points on the line included in the solution set? Explain your thinking.

     No; Sample response: I can check any point on the line and it is not a solution to the inequality.

Tennessee Lesson 17A Solutions to Linear Inequalities 7



Practice

Name: ________ Date: ________ Period: ________

3. Point $(3, -5)$ was transformed using different transformations. Match each transformation described with the coordinates of its image.

Transformation	Image Coordinates
a Translated 2 units up and 4 units to the left	**e** $(-3, 5)$
b Reflected across the x-axis	**f** $(5, -9)$
c Rotated 90° counterclockwise about the origin	**b** $(-3, 5)$
d Reflected across the y-axis	**d** $(3, 5)$
e Rotated 180° about the origin	**c** $(-3, -5)$
f Translated 4 units down and 2 units to the right.	**a** $(-1, -3)$

4. Solve each equation. Show your thinking.
   - **a** $\frac{1}{2}x = 7$

     $\frac{1}{2}x \div \frac{1}{2} = 7 \div \frac{1}{2}$

     $x = 14$
   - **b** $a \div \frac{1}{3} = 3$

     $a \div \frac{1}{3} \cdot \frac{1}{3} = 3 \cdot \frac{1}{3}$

     $a = 1$
   - **c** $x - 5 = 7$

     $x - 5 + 5 = 7 + 5$

     $x = 12$
   - **d** $9 - y = 12$

     $9 - y - 9 = 12 - 9$

     $-y = 3$

     $y = -3$

5. A set of pens cost \$2.99. A set of erasers cost \$3.99. You have \$15 to spend on x sets of pens and y sets of erasers. Write an equation to represent the relationship.

   $2.99x + 3.99y = 15$

8 **Unit 3** Linear Relationships

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 1	8.EE.C.9	2
	2	Activity 2	8.EE.C.9	2
Spiral	3	Unit 1 Lesson 7	8.G.A.1	2
	4	Grade 7	7.EE.B.4a	2
Formative ⚡	5	Unit 3 Lesson 18	8.EE.B	2

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice.**

Keeping Track of All Possible Outcomes

Let's represent sample spaces using organized lists, tables, and tree diagrams.

Focus

Goals

1. **Language Goal:** Compare and contrast different methods for representing the sample space of a multi-step event, and evaluate their usefulness. **(Speaking and Listening, Writing)**
2. Create an organized list, table, or tree diagram to represent the sample space of a multi-step event, and determine the total number of possible outcomes.

Coherence

Today

Students make use of the structure of organized lists, tables, and tree diagrams as methods for listing the sample space for *multi-step events* **(MP7)**. For an experiment with multiple events, they notice that the total number of outcomes in the sample space can be found by multiplying the number of outcomes for each event **(MP8)**.

Previously

In Lesson 7, students analyzed and interpreted bivariate data. They examined data in a scatter plot, created a linear model, and studied data values predicted by the linear model.

Coming Soon

In Tennessee Lesson 7B, students will determine the probabilities of multi-step events.

Rigor

- Students build **conceptual understanding of** multi-step events.

Standards

Addressing

8.SP.B.4b

Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, identify the outcomes in the sample space which compose the event.

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3 (Optional)	Summary	Exit Ticket
5 min	15 min	15 min	15 min	5 min	5 min
Independent	Pairs	Small groups	Pairs	Whole Class	Independent
MP7	MP7	MP8			
8.SP.B.4b	8.SP.B.4b	8.SP.B.4b	8.SP.B.4b	8.SP.B.4b	8.SP.B.4b

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice

Math Language Development

New words

- *multi-step event*
- *tree diagram*

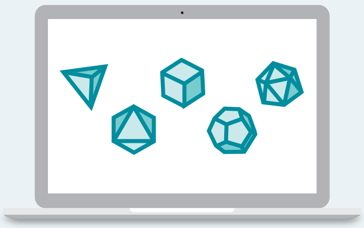
Review words

- *event*
- *outcome*
- *sample space*

Amps | **Featured Activity**

Activity 2
Student Choice

Students choose one of four experiments for which they determine the sample space.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel disorganized in Activity 2 as they try to list the sample spaces of multi-step events. They may repeat or forget outcomes as they try to make use of the structures of organized lists, tables, or tree diagrams. **(MP7)**. Help them grow their organizational skills as they list and pair the outcomes of one event with the outcomes of the other event, ensuring that all possible outcomes are listed and not repeated.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

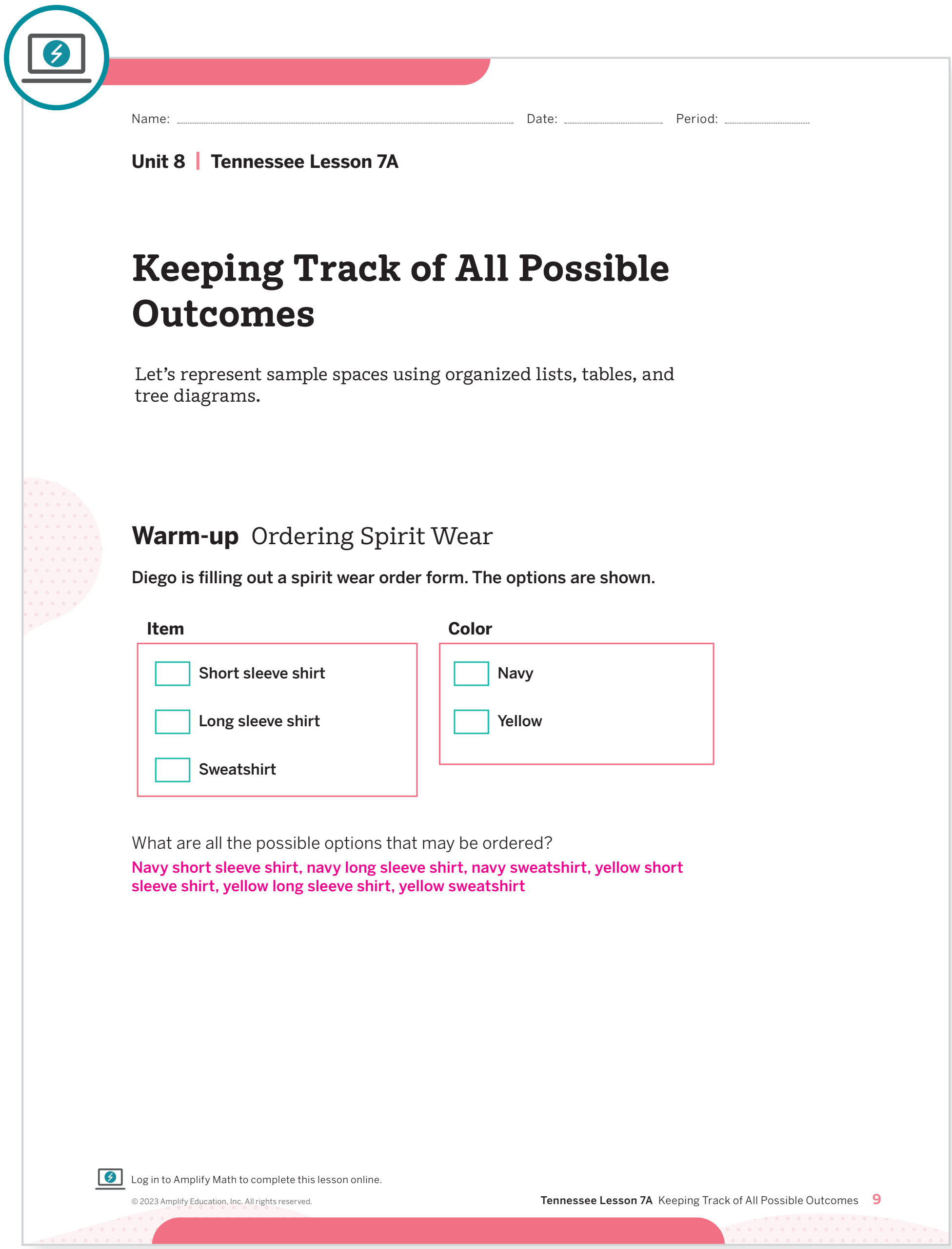
- Optional **Activity 3** may be omitted or assigned as additional practice.

Independent | 5 min

Warm-up Ordering Spirit Wear

MP7
8.SP.B.4b

Students use their own methods to organize different outcomes that prepare them for keeping track of all possible outcomes in a multi-step probability event.



Name: ____________ Date: ________ Period: ________

Unit 8 | Tennessee Lesson 7A

Keeping Track of All Possible Outcomes

Let's represent sample spaces using organized lists, tables, and tree diagrams.

Warm-up Ordering Spirit Wear

Diego is filling out a spirit wear order form. The options are shown.

Item	**Color**
☐ Short sleeve shirt	☐ Navy
☐ Long sleeve shirt	☐ Yellow
☐ Sweatshirt	

What are all the possible options that may be ordered?

Navy short sleeve shirt, navy long sleeve shirt, navy sweatshirt, yellow short sleeve shirt, yellow long sleeve shirt, yellow sweatshirt

Log in to Amplify Math to complete this lesson online.

1 Launch

Activate students' background knowledge by asking whether they have ever ordered clothing items with different options.

2 Monitor

Help students get started by asking them what color short sleeve shirts they could order.

Look for points of confusion:

- **Not including a possible option.** Encourage students to organize the possible options by creating a list or table.

Look for productive strategies:

- Organizing the possible options using a list or table.

3 Connect

Display the choices from the Warm-up.

Have students share their methods for organizing the possible options.

Ask:

- "How do you know that you listed all of the possible options?"
- "How do you know that you did not repeat an option?"

Highlight the importance of creating an organized list so that possible outcomes are neither missed nor repeated (MP7).

Define the term ***multi-step event*** as an event that consists of two or more events. Tell students that, because Diego is selecting an item and a color, the order is considered a multi-step event. This may also be referred to as compound events.

Power-up

To power up students' ability to interpret data, display the following table and have students complete:

The table shows the number of animals in a pet store. The number of fish is twice the amount of the number of birds. How many fish are in the pet store?

Animal	Rabbit	Birds	Cats	Fish
Number	7	10	2	20

Use: Before the Warm-up

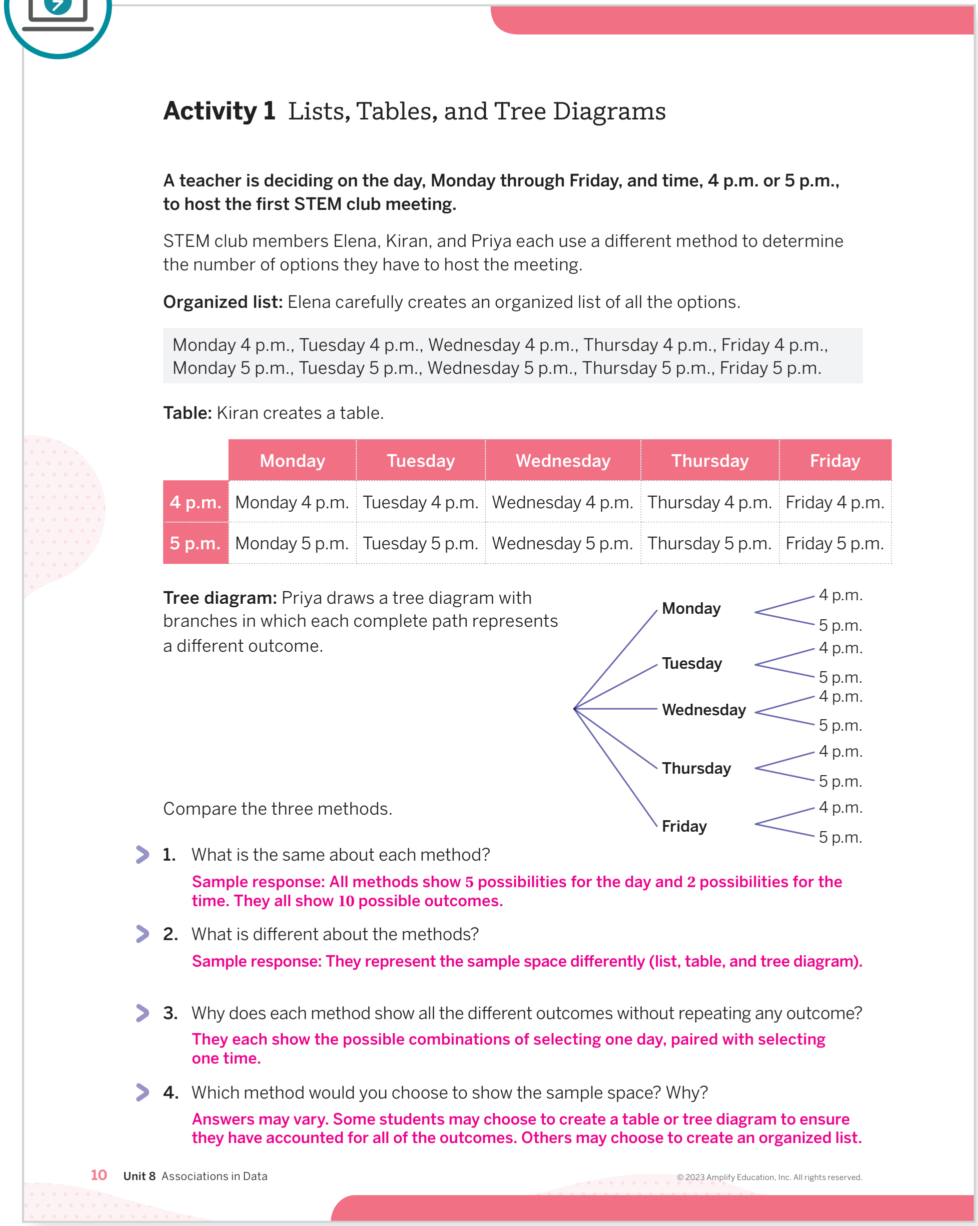
Informed by: Performance on Lesson 7, Practice Problem 6

Pairs | 15 min

MP7
8.SP.B.4b

Activity 1 Organized Lists, Tables, and Tree Diagrams

Students are shown three methods for writing sample spaces to understand the possible outcomes of a *multi-step event*.



Activity 1 Lists, Tables, and Tree Diagrams

A teacher is deciding on the day, Monday through Friday, and time, 4 p.m. or 5 p.m., to host the first STEM club meeting.

STEM club members Elena, Kiran, and Priya each use a different method to determine the number of options they have to host the meeting.

Organized list: Elena carefully creates an organized list of all the options.

Monday 4 p.m., Tuesday 4 p.m., Wednesday 4 p.m., Thursday 4 p.m., Friday 4 p.m.,
Monday 5 p.m., Tuesday 5 p.m., Wednesday 5 p.m., Thursday 5 p.m., Friday 5 p.m.

Table: Kiran creates a table.

	Monday	Tuesday	Wednesday	Thursday	Friday
4 p.m.	Monday 4 p.m.	Tuesday 4 p.m.	Wednesday 4 p.m.	Thursday 4 p.m.	Friday 4 p.m.
5 p.m.	Monday 5 p.m.	Tuesday 5 p.m.	Wednesday 5 p.m.	Thursday 5 p.m.	Friday 5 p.m.

Tree diagram: Priya draws a tree diagram with branches in which each complete path represents a different outcome.

Compare the three methods.

1. What is the same about each method?
   Sample response: All methods show 5 possibilities for the day and 2 possibilities for the time. They all show 10 possible outcomes.
2. What is different about the methods?
   Sample response: They represent the sample space differently (list, table, and tree diagram).
3. Why does each method show all the different outcomes without repeating any outcome?
   They each show the possible combinations of selecting one day, paired with selecting one time.
4. Which method would you choose to show the sample space? Why?
   Answers may vary. Some students may choose to create a table or tree diagram to ensure they have accounted for all of the outcomes. Others may choose to create an organized list.

10 **Unit 8** Associations in Data

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1 Launch

Review the prompt, and then tell students there are multiple methods for organizing the number of options for the STEM club meeting. One method is called a ***tree diagram***. Each branch represents an outcome, and the end of the branches can be counted to determine the total number of outcomes.

2 Monitor

Help students get started by asking them to carefully examine the organized list, table, and tree diagram.

Look for points of confusion:

- **Misinterpreting the tree diagram.** Help students see that a single outcome is represented by following one complete path of branches. Have students highlight the paths using different colors.

Look for productive strategies:

- Checking one sample space against another to ensure each method shows all of the outcomes.

3 Connect

Display the three sample space representations.

Have students share their preferred method. Use the ***Poll the Class*** routine, and then ask one student for each method to explain why they prefer it.

Highlight that all three methods show the same possible outcomes but organize the information differently. **(MP7)**.

Ask, "In what situations do you think each method would be useful?"
Sample response: A tree diagram is useful for providing a visual of the outcomes. A table can be useful for organizing two events, each with a large number of outcomes. An organized list can be useful for quickly determining the sample space.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Annotate the tree diagram by writing Day above to the first column and *Time* above the second column to help students make sense of the two events occurring. Encourage students to use their finger to trace the possible outcomes.

Math Language Development

MLR7: Compare and Connect

During the Connect, have students compare the three methods used to represent the sample space. Connect how the three different representations all show the same information. Ask:

- "How many total possible outcomes are there? How do you see this in the organized list? Table? Tree diagram?"
- "How does the tree diagram show this is a multi-step event?"

English Learners

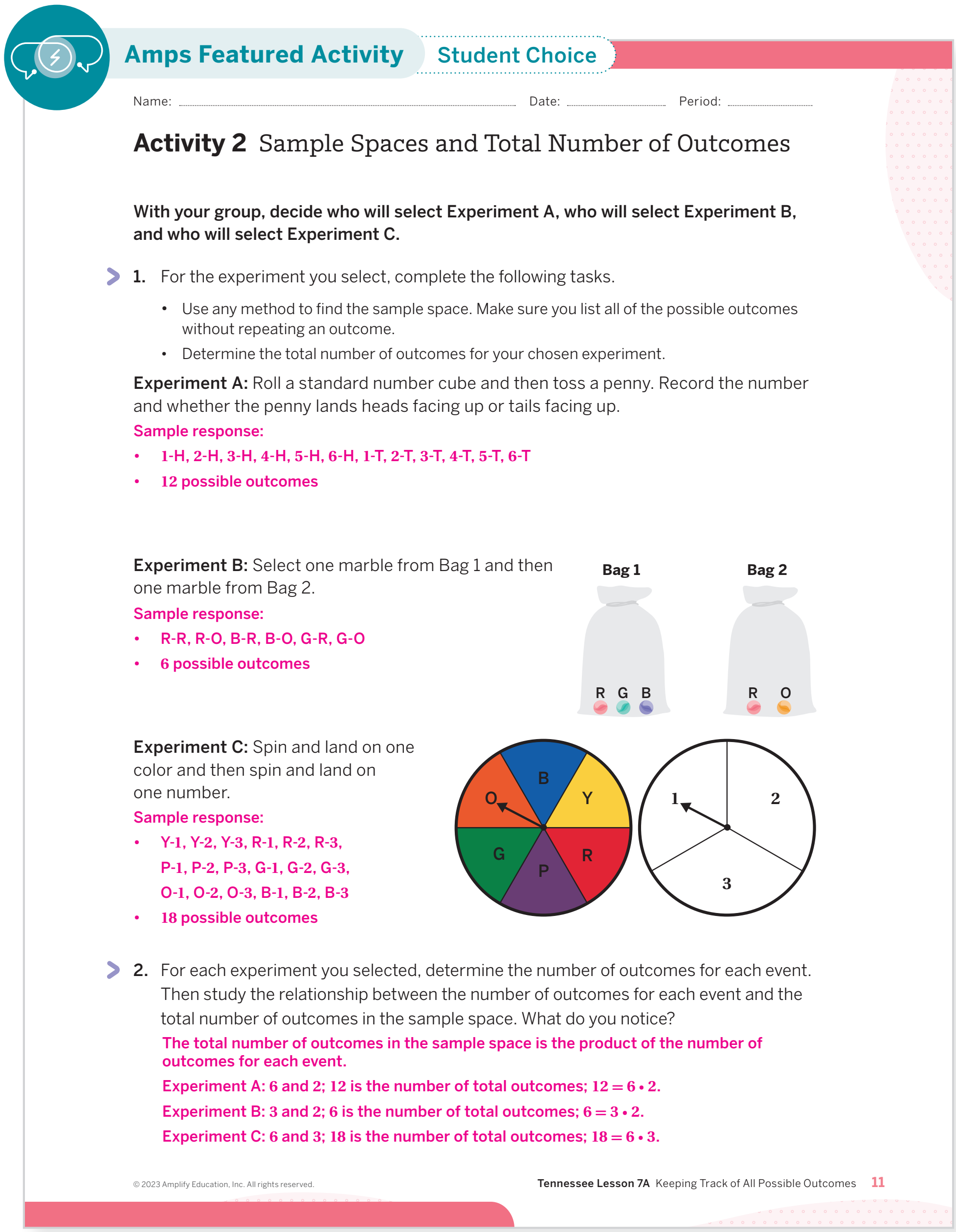
Color code one of the outcomes in each representation to illustrate how they are each displayed.

Small groups | 15 min

MP8
8.SP.B.4b

Activity 2 Sample Spaces and Total Number of Outcomes

Students record the sample spaces to learn that the total number of outcomes can be found by multiplying the number of outcomes of each experiment.



Amps Featured Activity Student Choice

Name: ______ Date: ______ Period: ______

Activity 2 Sample Spaces and Total Number of Outcomes

With your group, decide who will select Experiment A, who will select Experiment B, and who will select Experiment C.

1. For the experiment you select, complete the following tasks.
    - Use any method to find the sample space. Make sure you list all of the possible outcomes without repeating an outcome.
    - Determine the total number of outcomes for your chosen experiment.

Experiment A: Roll a standard number cube and then toss a penny. Record the number and whether the penny lands heads facing up or tails facing up.

Sample response:

- 1-H, 2-H, 3-H, 4-H, 5-H, 6-H, 1-T, 2-T, 3-T, 4-T, 5-T, 6-T
- 12 possible outcomes

Experiment B: Select one marble from Bag 1 and then one marble from Bag 2.

Sample response:

- R-R, R-O, B-R, B-O, G-R, G-O
- 6 possible outcomes

Experiment C: Spin and land on one color and then spin and land on one number.

Sample response:

- Y-1, Y-2, Y-3, R-1, R-2, R-3, P-1, P-2, P-3, G-1, G-2, G-3, O-1, O-2, O-3, B-1, B-2, B-3
- 18 possible outcomes

2. For each experiment you selected, determine the number of outcomes for each event. Then study the relationship between the number of outcomes for each event and the total number of outcomes in the sample space. What do you notice?

The total number of outcomes in the sample space is the product of the number of outcomes for each event.

Experiment A: 6 and 2; 12 is the number of total outcomes; $12 = 6 \cdot 2$.

Experiment B: 3 and 2; 6 is the number of total outcomes; $6 = 3 \cdot 2$.

Experiment C: 6 and 3; 18 is the number of total outcomes; $18 = 6 \cdot 3$.

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Tennessee Lesson 7A Keeping Track of All Possible Outcomes 11

1 Launch

In groups of three, have each student choose a different experiment and use any method to create a sample space.

2 Monitor

Help students get started by having them decide which method they will use to organize the sample space.

Look for points of confusion:

- **Repeating outcomes or missing outcomes.** Point out any repeated or missing outcomes. Ask students how they can check to see whether they have listed all the possible outcomes. Consider having students show the sample space using another method to check their work.

3 Connect

Have students share their sample spaces. Include displays of each method.

Ask, "What structure or method did you use for each experiment to make sure all outcomes were included, without duplicates?"

Highlight students using different methods for the different experiments. Connect the total possible outcomes for each experiment to the product of the number of outcomes for each event in the experiment. Some students might see this connection better by creating a table or a tree diagram **(MP8)**.

Differentiated Support

Extension: ***Math Enrichment***

Have students use the relationship they noticed in this activity to determine the total number of outcomes for the following multi-step events.

1. Toss a coin 5 times and record each outcome.
   32 total outcomes; $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
2. From Experiment C, spin the color spinner 3 times and then spin the number spinner 3 times and record each outcome.
   5,832 total outcomes; $6 \cdot 6 \cdot 6 \cdot 3 \cdot 3 \cdot 3 = 5832$

Math Language Development

MLR7: Compare and Connect

During the Connect, as students share their observations, provide the following sentence frames to help organize their thinking.

- "The number of outcomes in each event are ___ to give the total number of outcomes in the sample space"
- "The total number of outcomes in the sample space is the ___ of the number of outcomes in each event."

English Learners

All students, including English Learners, may benefit from organizing the pattern in a table to visualize the relationship more clearly.

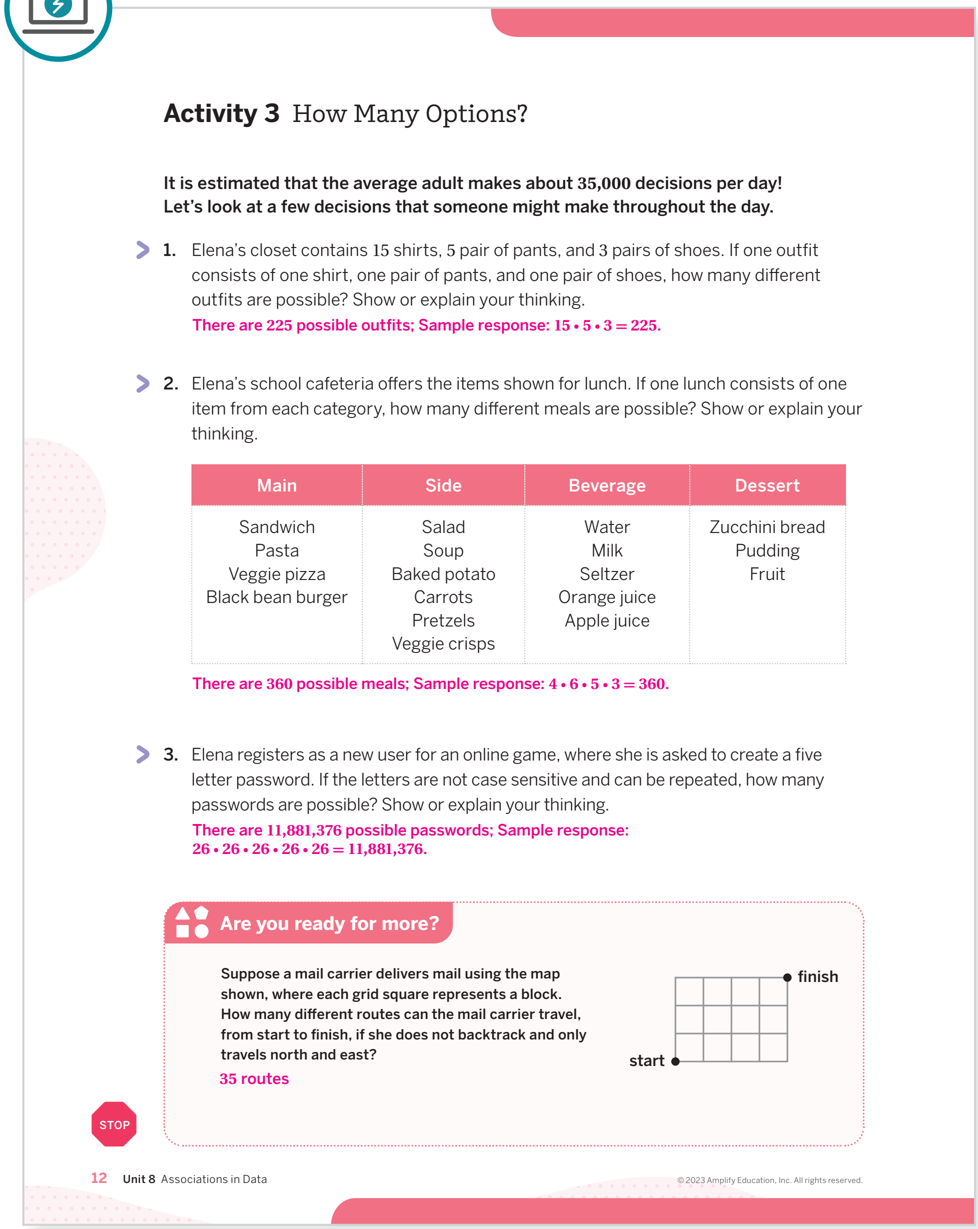
Optional

Pairs | 15 min

Activity 3 How Many Options?

8.SP.B.4b

Students apply their understanding of calculating the total number of possible outcomes in a multi-step event without listing all of the possible outcomes in the sample space.



Activity 3 How Many Options?

It is estimated that the average adult makes about 35,000 decisions per day! Let's look at a few decisions that someone might make throughout the day.

1. Elena's closet contains 15 shirts, 5 pair of pants, and 3 pairs of shoes. If one outfit consists of one shirt, one pair of pants, and one pair of shoes, how many different outfits are possible? Show or explain your thinking.
   There are 225 possible outfits; Sample response: $15 \cdot 5 \cdot 3 = 225$.

2. Elena's school cafeteria offers the items shown for lunch. If one lunch consists of one item from each category, how many different meals are possible? Show or explain your thinking.

Main	Side	Beverage	Dessert
Sandwich	Salad	Water	Zucchini bread
Pasta	Soup	Milk	Pudding
Veggie pizza	Baked potato	Seltzer	Fruit
Black bean burger	Carrots	Orange juice	
	Pretzels	Apple juice	
	Veggie crisps		

   There are 360 possible meals; Sample response: $4 \cdot 6 \cdot 5 \cdot 3 = 360$.

3. Elena registers as a new user for an online game, where she is asked to create a five letter password. If the letters are not case sensitive and can be repeated, how many passwords are possible? Show or explain your thinking.
   There are 11,881,376 possible passwords; Sample response: $26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 11,881,376$.

Are you ready for more?

Suppose a mail carrier delivers mail using the map shown, where each grid square represents a block. How many different routes can the mail carrier travel, from start to finish, if she does not backtrack and only travels north and east?

35 routes

STOP

12 **Unit 8** Associations in Data

1 Launch

Ask, "What types of decisions does a person make each day?"

2 Monitor

Help students get started by having them underline the possible outcomes for each event.

Look for points of confusion:

- **Creating a sample space to determine the possible outcomes.** Tell students that, while this is a valid strategy, it may not be the most efficient because it might take a lot of time and space.
- **Struggling to answer Problem 3.** Explain a password that is not case sensitive means it does not matter whether the password includes capital or lowercase letters. Then remind students that there are 26 letters in the alphabet. Allow students write their answer as an expression using exponents.

Look for productive strategies:

- Noticing the order in which they multiply the outcomes does not matter.

3 Connect

Have students share their responses. For Problem 3, select students who wrote different expressions, such as $26 \cdot 26 \cdot 26 \cdot 26 \cdot 26$ or 26^5

Highlight that some situations have very large sample spaces and recording every possible outcome is not an efficient method. Instead, the total number of possible outcomes can be determined by multiplying the number of outcomes for each event.

Ask, "For Problem 3, how would the number of total possible outcomes change if the letters were case sensitive?" There would be 52^5, or 380,204,032, possible passwords.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Suggest that students create an expression using blanks and then fill each blank with the number of outcomes of each event. For example, for Problem 3, students may write ___ • ___ • ___ • ___ • ___, and then fill in the blanks with the number of outcomes for each event.

Extension: *Math Enrichment*

For Problem 3, have students determine the number of total possible outcomes if the letters cannot be repeated. $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22$, or 7,893,600 possible passwords.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, display the incorrect statement for Problem 3, "There are 130 possible passwords because $26 \cdot 5 = 130$."

- *Critique:* "Do you agree or disagree with this statement? Why or why not?"
- *Correct:* "Write a corrected statement that is now true."
- *Clarify:* "What was the most likely misunderstanding of the person who wrote this incorrect statement?" The person multiplied the number of events by the number of possible outcomes for one event.

English Learners

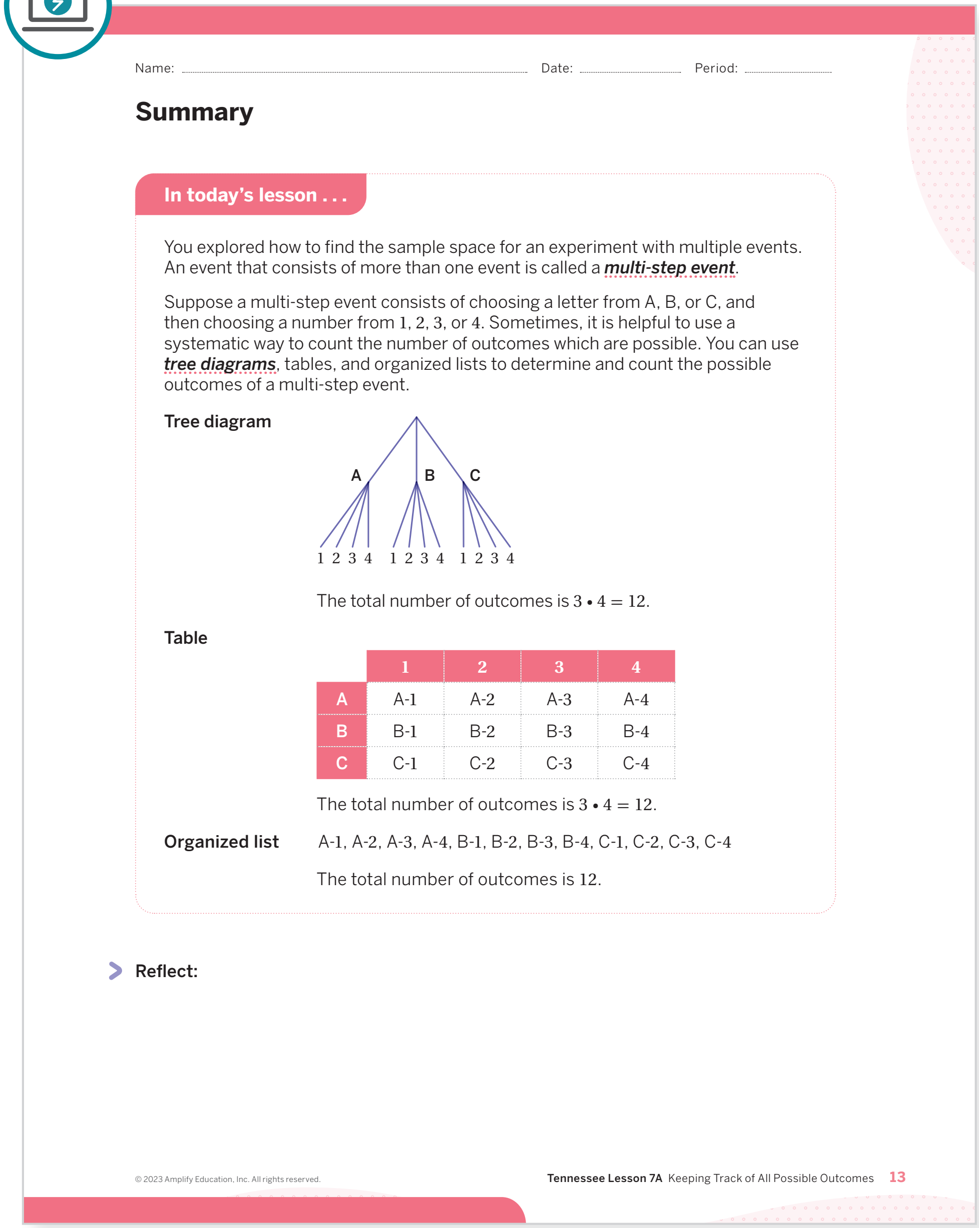
Allow students time to rehearse what they will say with a partner before sharing with the whole class.

Whole Class | 5 min

Summary

8.SP.B.4b

Review and synthesize how to determine the total number of outcomes using organized lists, tables, and tree diagrams.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how to find the sample space for an experiment with multiple events. An event that consists of more than one event is called a ***multi-step event***.

Suppose a multi-step event consists of choosing a letter from A, B, or C, and then choosing a number from 1, 2, 3, or 4. Sometimes, it is helpful to use a systematic way to count the number of outcomes which are possible. You can use ***tree diagrams***, tables, and organized lists to determine and count the possible outcomes of a multi-step event.

Tree diagram

The total number of outcomes is $3 \cdot 4 = 12$.

Table

	1	2	3	4
A	A-1	A-2	A-3	A-4
B	B-1	B-2	B-3	B-4
C	C-1	C-2	C-3	C-4

The total number of outcomes is $3 \cdot 4 = 12$.

Organized list A-1, A-2, A-3, A-4, B-1, B-2, B-3, B-4, C-1, C-2, C-3, C-4

The total number of outcomes is 12.

Reflect:

Synthesize

Display the Summary.

Have students share which method they would choose to determine the sample space of a multi-step event.

Formalize vocabulary:

- *multi-step event*
- *tree diagram*

Highlight that an organized list, table, or tree diagram are ways to represent a sample space.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How is determining the sample space of a multi-step event similar to how you determined the sample space of a single-step event? How is it different?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the terms *multi-step event* and *tree diagram* that were added to the display during the lesson.

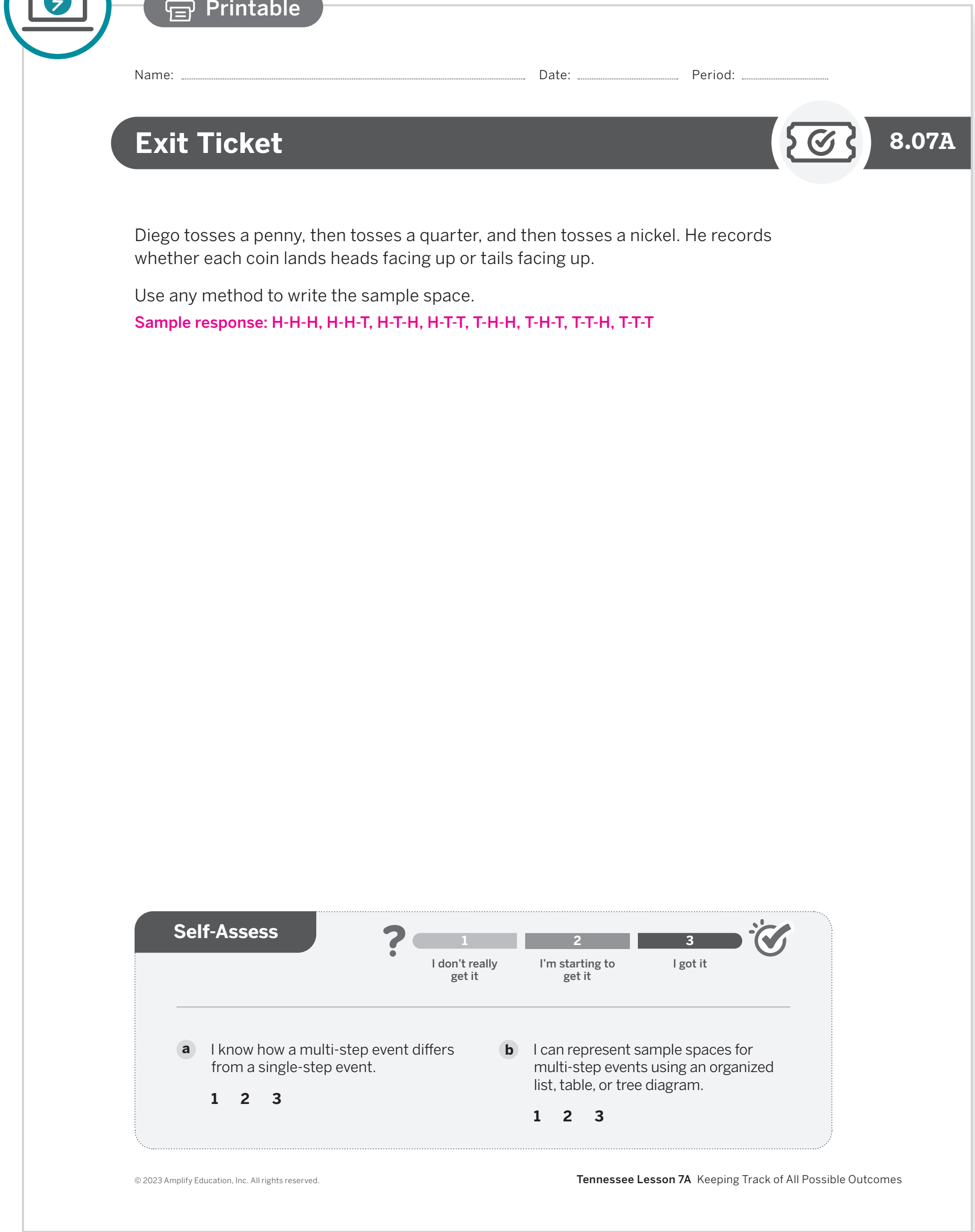
Independent | 5 min

Exit Ticket

8.SP.B.4b

Students demonstrate their understanding by creating a sample space and calculating the total number of possible outcomes for a multi-step event.

Printable



Name: ________ Date: ________ Period: ________

Exit Ticket

8.07A

Diego tosses a penny, then tosses a quarter, and then tosses a nickel. He records whether each coin lands heads facing up or tails facing up.

Use any method to write the sample space.

Sample response: H-H-H, H-H-T, H-T-H, H-T-T, T-H-H, T-H-T, T-T-H, T-T-T

Self-Assess

1	2	3
I don't really get it	I'm starting to get it	I got it

a I know how a multi-step event differs from a single-step event.

1 2 3

b I can represent sample spaces for multi-step events using an organized list, table, or tree diagram.

1 2 3

Success looks like . . .

- **Language Goal:** Comparing and contrasting different methods for representing the sample space of a multi-step event, and evaluating their usefulness. **(Speaking and Listening, Writing)**
- **Goal:** Creating an organized list, table, or tree diagram to represent the sample space of a multi-step event, and determining the total number of possible outcomes.
  - » Listing the sample space using an organized list, table, or tree diagram.

Suggested next steps

If students do not correctly write the sample space, consider:

- Providing a table for students to complete.
- Reviewing Activity 1.

Professional Learning

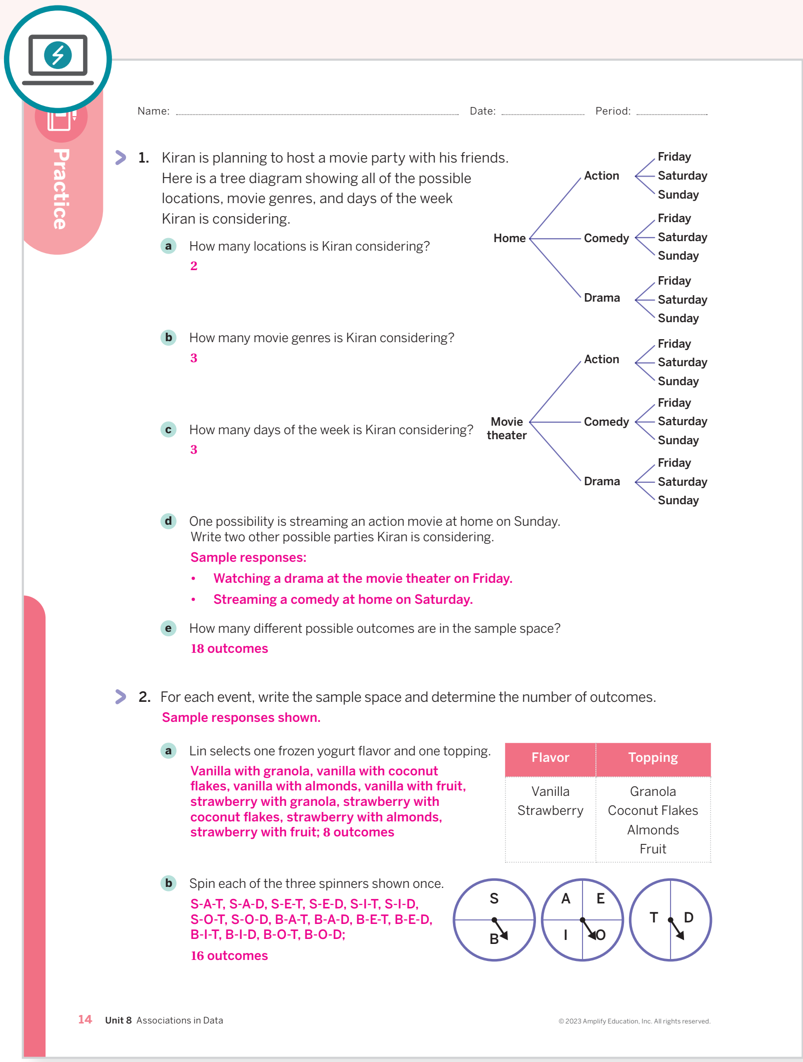
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In this lesson, students used tree diagrams and tables to determine the probability of multi-step events. How did this build on the early work that students did with sample space and simple events?
- What routines enabled all students to do math in today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Name: ________ Date: ________ Period: ________

1. Kiran is planning to host a movie party with his friends. Here is a tree diagram showing all of the possible locations, movie genres, and days of the week Kiran is considering.

a How many locations is Kiran considering?

2

b How many movie genres is Kiran considering?

3

c How many days of the week is Kiran considering?

3

d One possibility is streaming an action movie at home on Sunday. Write two other possible parties Kiran is considering.

Sample responses:

- **Watching a drama at the movie theater on Friday.**
- **Streaming a comedy at home on Saturday.**

e How many different possible outcomes are in the sample space?

18 outcomes

2. For each event, write the sample space and determine the number of outcomes.

Sample responses shown.

a Lin selects one frozen yogurt flavor and one topping.

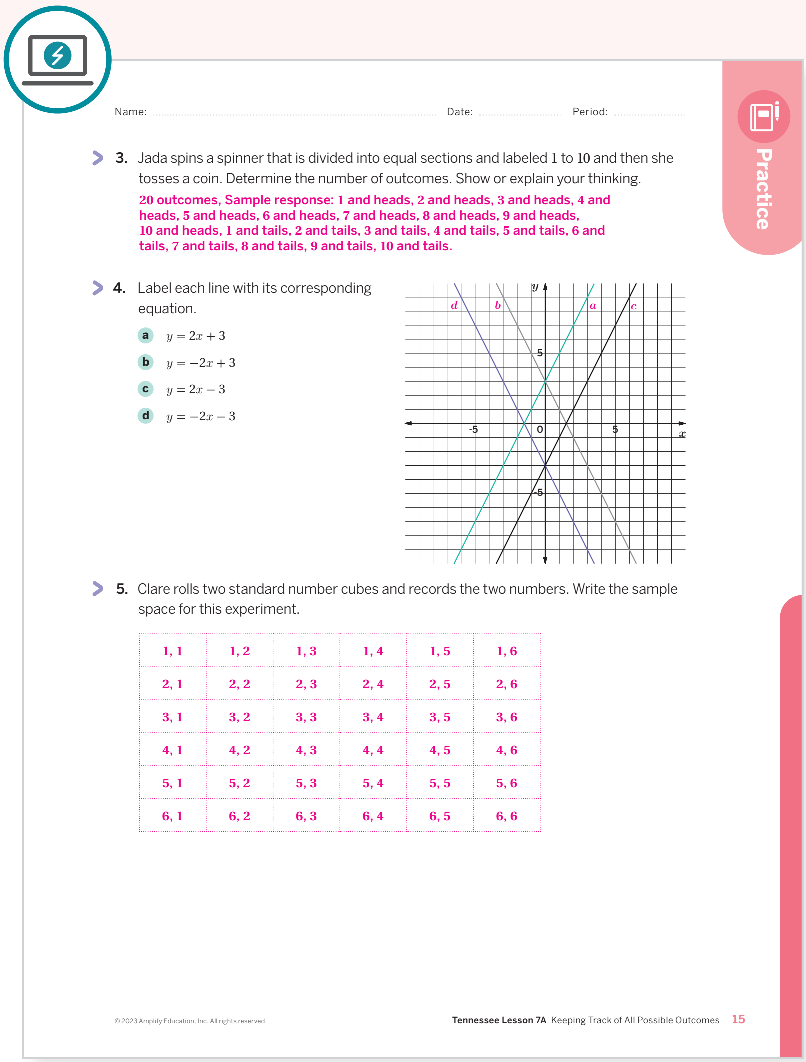
Vanilla with granola, vanilla with coconut flakes, vanilla with almonds, vanilla with fruit, strawberry with granola, strawberry with coconut flakes, strawberry with almonds, strawberry with fruit; 8 outcomes

Flavor	Topping
Vanilla Strawberry	Granola Coconut Flakes Almonds Fruit

b Spin each of the three spinners shown once.

S-A-T, S-A-D, S-E-T, S-E-D, S-I-T, S-I-D, S-O-T, S-O-D, B-A-T, B-A-D, B-E-T, B-E-D, B-I-T, B-I-D, B-O-T, B-O-D;

16 outcomes



Name: ________ Date: ________ Period: ________

3. Jada spins a spinner that is divided into equal sections and labeled 1 to 10 and then she tosses a coin. Determine the number of outcomes. Show or explain your thinking.

20 outcomes, Sample response: 1 and heads, 2 and heads, 3 and heads, 4 and heads, 5 and heads, 6 and heads, 7 and heads, 8 and heads, 9 and heads, 10 and heads, 1 and tails, 2 and tails, 3 and tails, 4 and tails, 5 and tails, 6 and tails, 7 and tails, 8 and tails, 9 and tails, 10 and tails.

4. Label each line with its corresponding equation.

a $y = 2x + 3$

b $y = -2x + 3$

c $y = 2x - 3$

d $y = -2x - 3$

5. Clare rolls two standard number cubes and records the two numbers. Write the sample space for this experiment.

1, 1	**1, 2**	**1, 3**	**1, 4**	**1, 5**	**1, 6**
2, 1	**2, 2**	**2, 3**	**2, 4**	**2, 5**	**2, 6**
3, 1	**3, 2**	**3, 3**	**3, 4**	**3, 5**	**3, 6**
4, 1	**4, 2**	**4, 3**	**4, 4**	**4, 5**	**4, 6**
5, 1	**5, 2**	**5, 3**	**5, 4**	**5, 5**	**5, 6**
6, 1	**6, 2**	**6, 3**	**6, 4**	**6, 5**	**6, 6**

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 1	8.SP.B.4b	2
	2	Activity 2	8.SP.B.4b	2
	3	Activities 2 and 3	8.SP.B.4b	1
Spiral	4	Unit 3 Lesson 12	8.EE.B.6	2
Formative ⚡	5	Unit 8 Tennessee Lesson 7B	8.SP.B.4b	1

⚡ **Power-up:** If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice.**

Probabilities of Multi-step Events

Let's determine the probabilities of multi-step events.

Focus

Goals

1. Use any method to determine the probability of a multi-step event.
2. **Language Goal:** Compare the likelihoods of events by computing the probabilities of the events, and explain the reasoning. **(Speaking and Listening)**

Coherence

Today

Students continue to write sample spaces for multi-step events and begin using the sample spaces to calculate the probabilities of events. Students apply their understanding of probabilities to order the likelihood of multi-step events **(MP4)**.

Previously

In Tennessee Lesson 7A, students determined the sample space and reasoned that multiplying the number of outcomes for each event gave the total number of possible outcomes in an experiment.

Coming Soon

In Tennessee Lesson 7C, students will build conceptual understanding of how chance events can be used to simulate real-world situations.

Rigor

- Students build **conceptual understanding** of determining the probabilities for a multi-step event.
- Students **apply** their understanding of sample spaces to determine the probabilities of multi-step events.

Standards

Addressing

8.SP.B.4a
Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Also Addressing: **8.SP.B.4b**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
5 min	10 min	20 min	5 min	5 min
Independent	Pairs	Pairs	Whole Class	Independent
		MP4		
7.SP.C.6*	8.SP.4a	8.SP.4a, 8.SP.4b	8.SP.4a, 8.SP.4b	8.SP.4a, 8.SP.4b

**In this activity, students build on their understanding of probabilities of single-step events from Grade 7.*

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 2 PDF (as needed)
- Activity 2 PDF (sample space answers)
- colored pencils (as needed)

Math Language Development

Review words

- *multi-step event*
- *outcome*
- *probability*
- *sample space*

Amps | **Featured Activity**

Activity 2
Formative Feedback for Students

Students order the likelihood of events and are given immediate feedback as to whether their ordering is correct.

Amps powered by desmos

Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel lost as they determine the likelihood of events in Activity 2 **(MP4)**. Encourage students to list the different methods they can use to represent the sample space. Listing the methods will provide options for students and allow them to take control over their learning.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

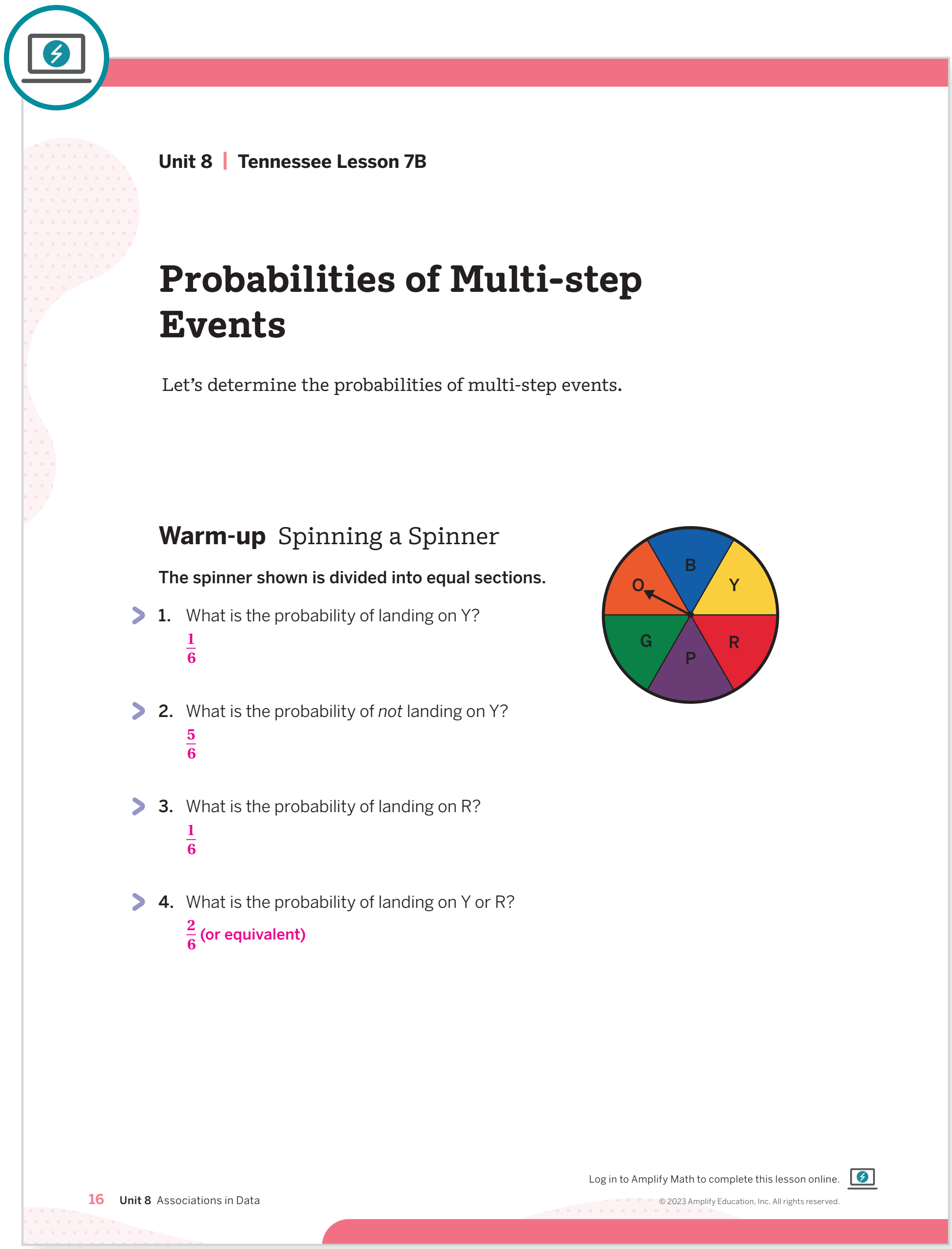
- The **Warm-up** may be omitted.
- In **Activity 2**, have students determine the most likely and least likely events.

Independent | 5 min

Warm-up Spinning a Spinner

7.SP.C.6

Students review how to determine the probability of single-step events to prepare them for determining the probability of multi-step events.



Unit 8 | Tennessee Lesson 7B

Probabilities of Multi-step Events

Let's determine the probabilities of multi-step events.

Warm-up Spinning a Spinner

The spinner shown is divided into equal sections.

1. What is the probability of landing on Y? $\frac{1}{6}$
2. What is the probability of *not* landing on Y? $\frac{5}{6}$
3. What is the probability of landing on R? $\frac{1}{6}$
4. What is the probability of landing on Y or R? $\frac{2}{6}$ (or equivalent)

Log in to Amplify Math to complete this lesson online.

16 **Unit 8** Associations in Data

1 Launch

Activate prior knowledge by asking students how they can determine the probability of an event.

2 Monitor

Help students get started by asking what fraction of the circle represents each color.

Look for productive strategies:

- Determining the probability for Problem 2 by subtracting the probability of landing on Y from 1.
- Writing a ratio, fraction, or decimal for the probability.

3 Connect

Have students share their responses and strategies for determining their responses.

Ask:

- "Suppose you spin the spinner two times. How many total outcomes are there?" 36 Sample response: $6 \cdot 6 = 36$.
- "Suppose you spin the spinner two times. How many favorable outcomes are there for landing on a Y and then an R?" 1
- "How do you think you can determine the probability of a multi-step event?"

Highlight that the probability of a multi-step event can be found the same way as the probability of a simple event. The ratio of the favorable outcomes to the total number of possible outcomes is determined.

Power-up

To power up students' ability to write the sample space of multi-step events, have students complete:

Noah is planning his birthday party. He is considering two locations (Carnival, Park), and three days of the week (Friday, Saturday, Sunday). Use any method to determine the sample space.

Carnival and Friday, Carnival and Saturday, Carnival and Sunday, Park and Friday, Park and Saturday, Park and Sunday.

Use: After the Warm-up.

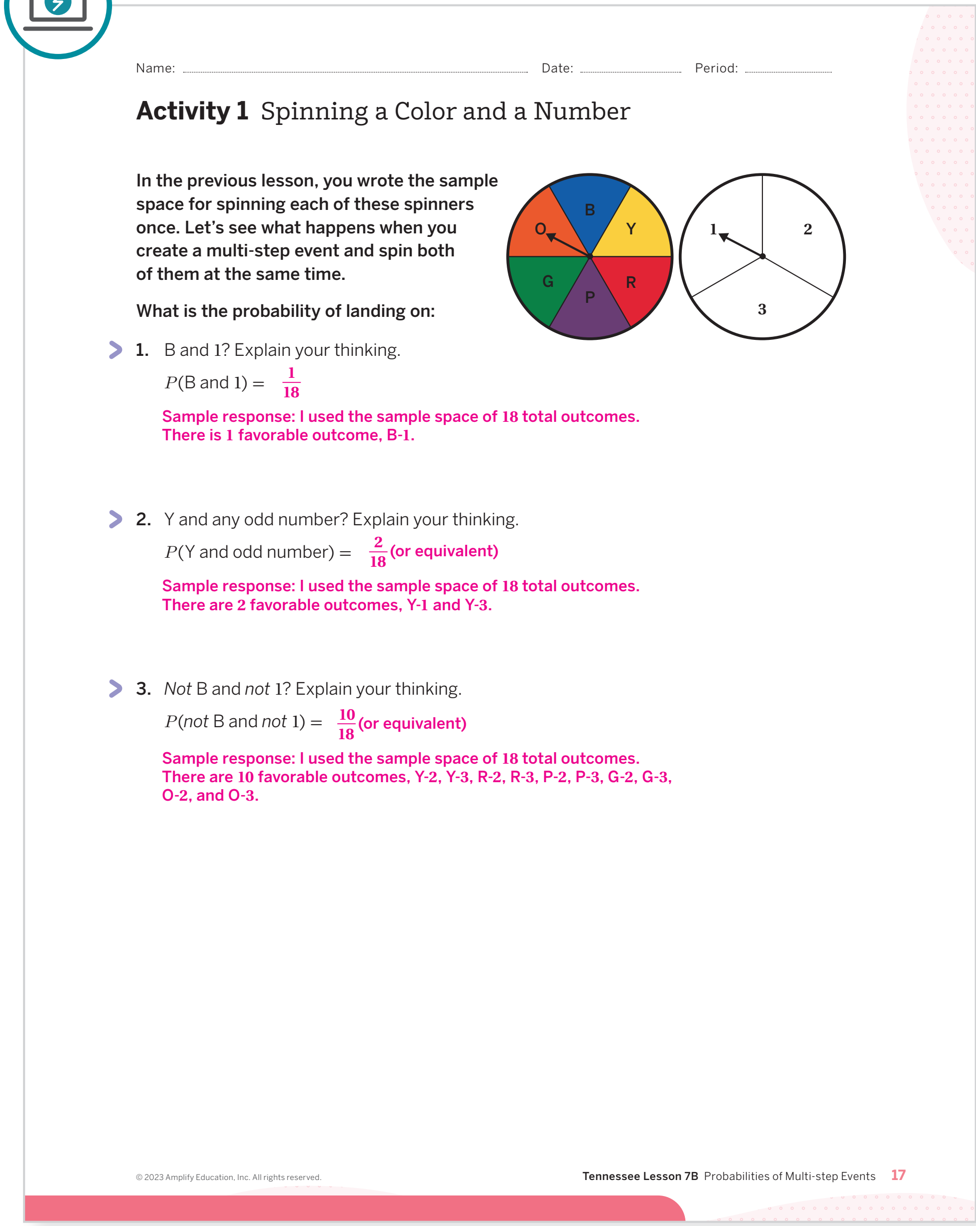
Informed by: Performance on Tennessee 7A, Practice Problem 5

Pairs | 10 min

Activity 1 Spinning a Color and a Number

8.SP.4a

Students use the sample space of a multi-step experiment to calculate the probability of multi-step events.



Name: ______ Date: ______ Period: ______

Activity 1 Spinning a Color and a Number

In the previous lesson, you wrote the sample space for spinning each of these spinners once. Let's see what happens when you create a multi-step event and spin both of them at the same time.

What is the probability of landing on:

1. B and 1? Explain your thinking.

   $P(\text{B and }1) =$ $\frac{1}{18}$

   Sample response: I used the sample space of 18 total outcomes. There is 1 favorable outcome, B-1.

2. Y and any odd number? Explain your thinking.

   $P(\text{Y and odd number}) =$ $\frac{2}{18}$ (or equivalent)

   Sample response: I used the sample space of 18 total outcomes. There are 2 favorable outcomes, Y-1 and Y-3.

3. *Not* B and *not* 1? Explain your thinking.

   $P(\textit{not}\text{ B and }\textit{not}\ 1) =$ $\frac{10}{18}$ (or equivalent)

   Sample response: I used the sample space of 18 total outcomes. There are 10 favorable outcomes, Y-2, Y-3, R-2, R-3, P-2, P-3, G-2, G-3, O-2, and O-3.

Tennessee Lesson 7B Probabilities of Multi-step Events 17

1 Launch

Have students refer to the sample space they wrote in Activity 2 from the previous lesson.

2 Monitor

Help students get started by determining the ratio of the favorable outcomes to the total number of possible outcomes for each problem.

Look for points of confusion:

- **Miscounting any outcomes.** Have students use another method to record their sample space and check their answers.

Look for productive strategies:

- Circling or highlighting favorable outcomes.

3 Connect

Have students share their reasoning for determining the number of favorable outcomes and the probabilities for each event.

Highlight students who determined the probabilities using an organized list, table, or tree diagram. Be sure to show student work of each method.

Ask, "If you know the sample space and the number of favorable events, how can you determine the probability of a multi-step event?" Just as with a simple event, the total number of possible outcomes is the divisor and the number of favorable events is the dividend of the probability ratio.

Differentiated Support

Accessibility: ***Vary Demands to Optimize Challenge***

If students need more processing time, have them focus on completing Problems 1 and 2 and only work on Problem 3 if they have time available.

Math Language Development

MLR5: Co-craft Questions

During the Launch, display both spinners and have students work with a partner to write 2–3 mathematical questions they could ask about this multi-step event. Sample questions shown.

- How many total possible outcomes are there?
- How does the likelihood of landing on Yellow and 1 relate to the likelihood of landing on Red and 3?
- What is the probability of landing on Blue and an odd number?

English Learners

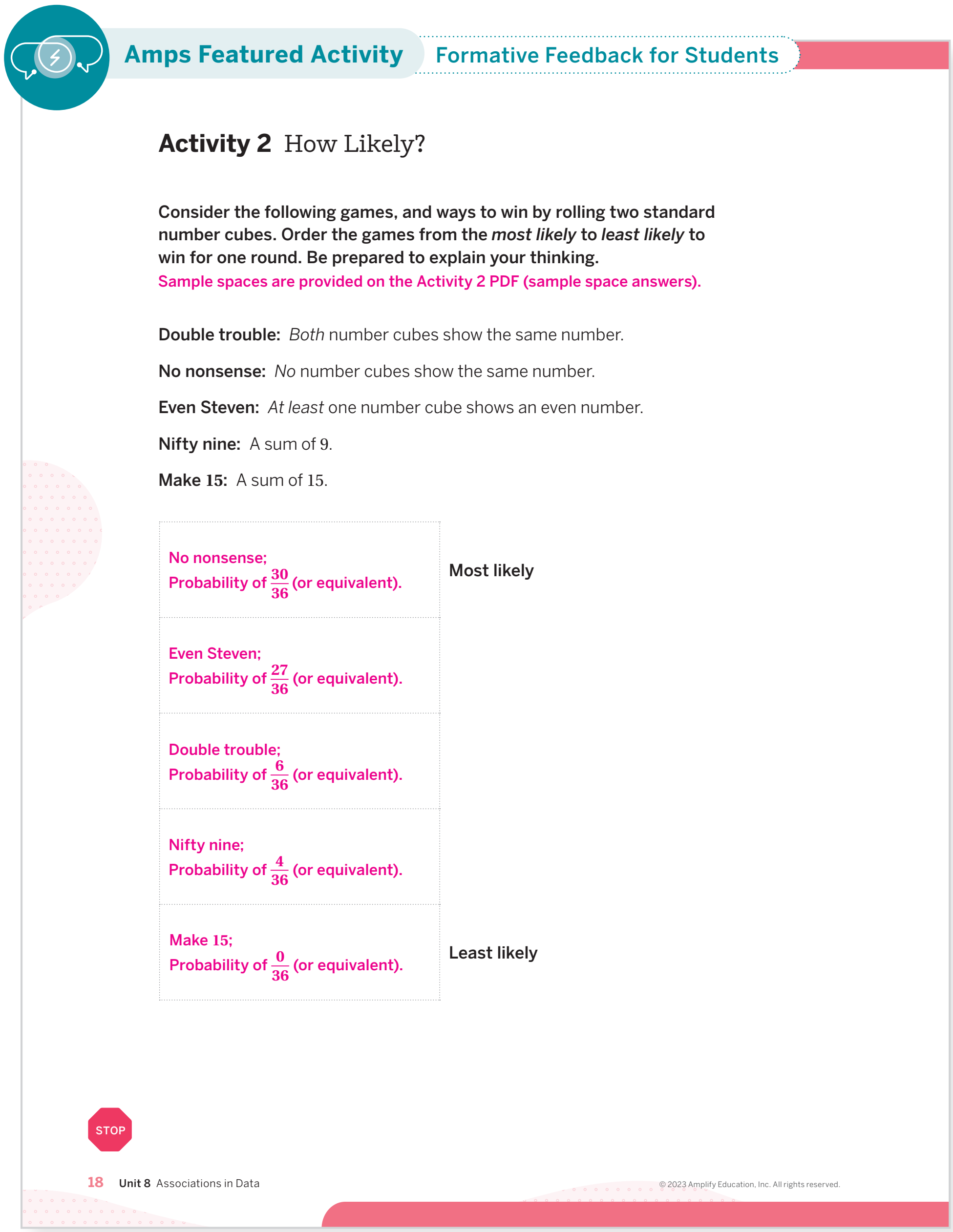
To support students in developing metalinguistic awareness, model how to craft a mathematical question. Consider displaying one of the sample questions.

Pairs | 20 min

MP4
8.SP.4a, 8.SP.4b

Activity 2 How Likely?

Students apply their understanding of probabilities to order the likelihood of multi-step events **(MP4).**



Amps Featured Activity Formative Feedback for Students

Activity 2 How Likely?

Consider the following games, and ways to win by rolling two standard number cubes. Order the games from the *most likely* to *least likely* to win for one round. Be prepared to explain your thinking.
Sample spaces are provided on the Activity 2 PDF (sample space answers).

Double trouble: *Both* number cubes show the same number.

No nonsense: *No* number cubes show the same number.

Even Steven: *At least* one number cube shows an even number.

Nifty nine: A sum of 9.

Make 15: A sum of 15.

No nonsense; Probability of $\frac{30}{36}$ (or equivalent).	**Most likely**
Even Steven; Probability of $\frac{27}{36}$ (or equivalent).	
Double trouble; Probability of $\frac{6}{36}$ (or equivalent).	
Nifty nine; Probability of $\frac{4}{36}$ (or equivalent).	
Make 15; Probability of $\frac{0}{36}$ (or equivalent).	**Least likely**

STOP

18 **Unit 8** Associations in Data

1 Launch

Review the prompt and games listed with the class. Answer any questions students may have.

2 Monitor

Help students get started by having them create a sample space for rolling two number cubes.

Look for points of confusion:

- **Struggling to create a sample space.** Provide students with the Activity 2 PDF. Have them complete Table A with the sample space for the numbers shown on each number cube. Have students complete Table B with the sample space for the sum of rolling the two number cubes.
- **Confusing the order of the events after students determine the probabilities.** Encourage students to write fractions that have the same denominator.
- **Struggling with the probability for *Make* 15.** Remind students that the probability of an event can be a value from 0 to 1.

Look for productive strategies:

- Creating two different sample spaces – one for the number shown on each number cube, and another for the sum of the numbers shown on the number cubes.
- Determining the probability of *No nonsense* by subtracting the probability of *Double Trouble* from 1.

3 Connect

Have students share their strategies for ordering the games. Select students who used different methods to determine the probabilities of the games.

Ask, "Why is *Make* 15 the least likely to win?"
There are no two numbers on a number cube that make a sum of 15.

Highlight that, just as with the probabilities of single-step events, the probabilities of multi-step events are expressed using numbers from 0 to 1, where 0 represents an impossible event and 1 represents a certain event.

Differentiated Support

Accessibility: *Guide Processing and Visualization*

Provide students with a copy of the Activity 2 PDF to help them make sense of the sample space for the multi-step event. Suggest that students use colored pencils to mark the favorable events in the sample space.

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on determining the games that are the most likely and least likely to win. If they have time available, have them determine the likelihoods of winning the remaining games.

Extension: *Math Extension*

Have students determine the rules for a game in which the probability of winning is more likely than *Even Steven,* but less likely than *No Nonsense*.
Sample response: Not rolling a sum of 5 or not rolling a sum of 9.

Math Language Development

MLR8: Discussion Supports

During the Connect, as students share the strategies they used to compare the games, display the sentence frames to help them organize their thinking, such as:

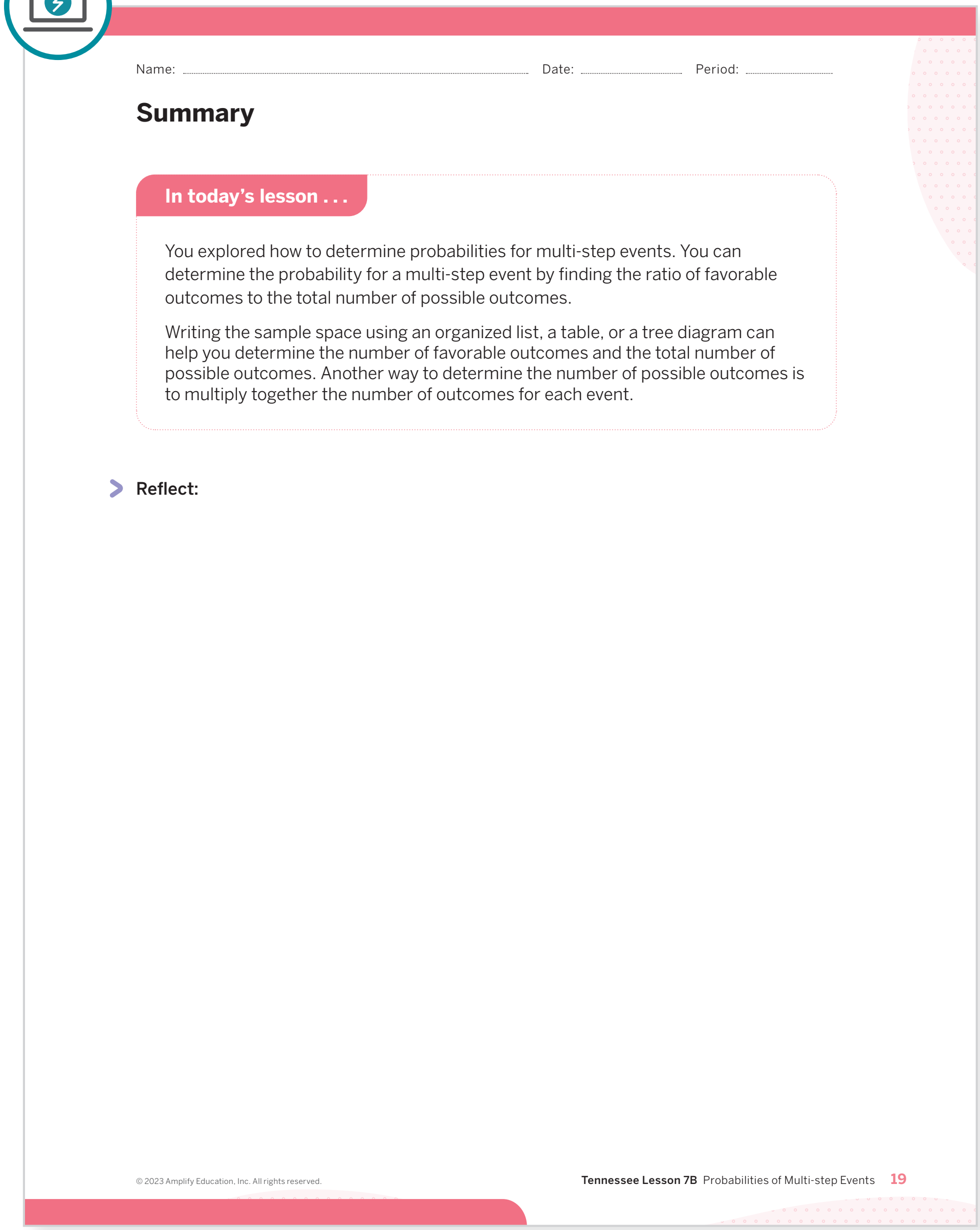
- "First I ___, then I . . ."
- "I noticed that ___, so I . . ."

Whole Class | 5 min

Summary

8.SP.4a, 8.SP.4b

Review Review and synthesize how to determine the probability of multi-step events.



Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored how to determine probabilities for multi-step events. You can determine the probability for a multi-step event by finding the ratio of favorable outcomes to the total number of possible outcomes.

Writing the sample space using an organized list, a table, or a tree diagram can help you determine the number of favorable outcomes and the total number of possible outcomes. Another way to determine the number of possible outcomes is to multiply together the number of outcomes for each event.

Reflect:

Synthesize

Have students share the strategies they used for determining the probability of multi-step events.

Ask, "If the probability of a multi-step event is 0, what does that tell you about the outcome of the event? What about the probability of 1?"
A probability of 0 means that the event will not happen, while a probability of 1 means that the event will definitely happen.

Highlight that determining the probability of a multi-step event is found by calculating the ratio of the number of favorable events to the total number of possible events.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "How did creating a sample space help you in determining the probability of multi-step events?"

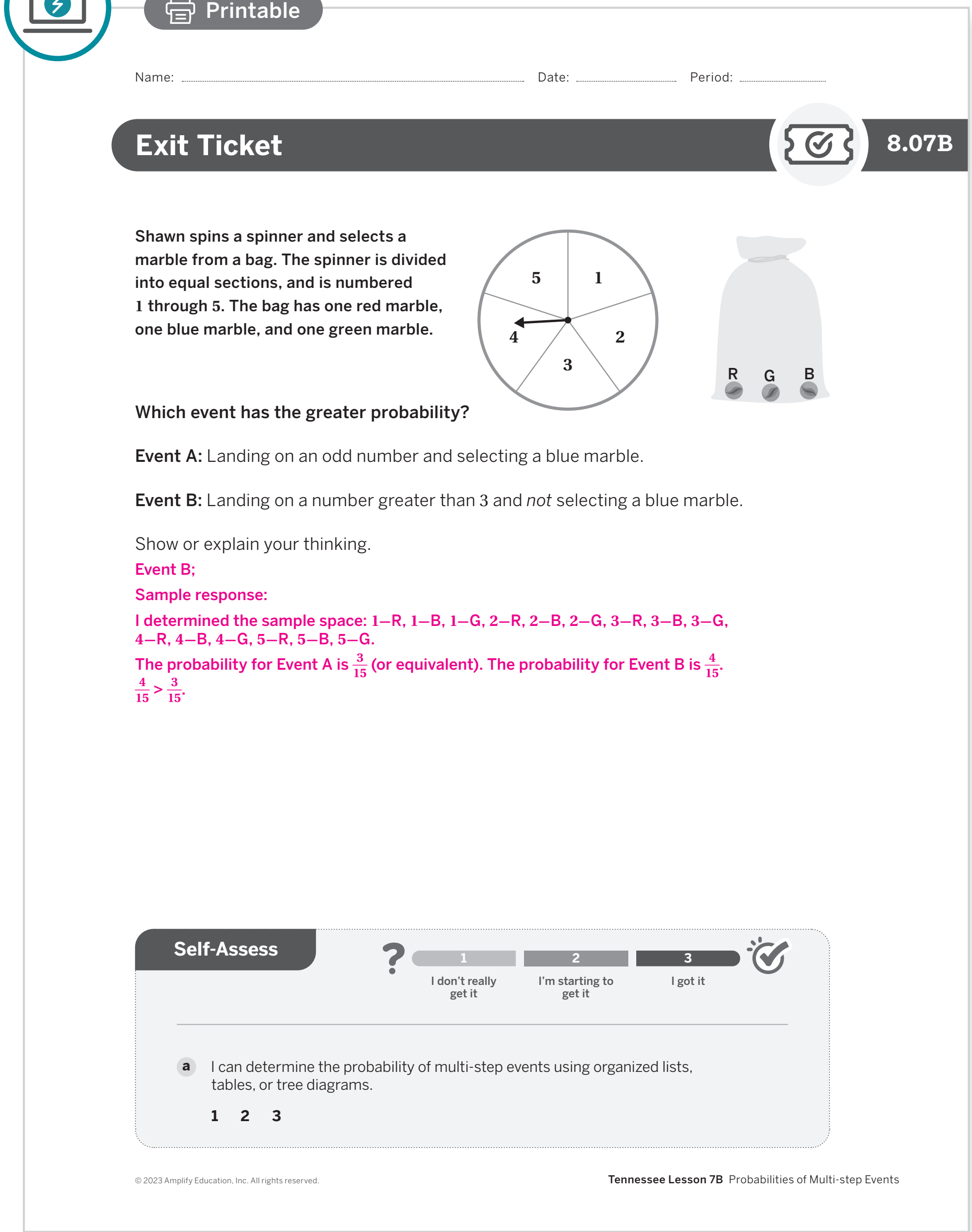
Independent | 5 min

Exit Ticket

8.SP.4a, 8.SP.4b

Students demonstrate their understanding by determining the probabilities of multi-step events.

Printable



Name: ______ Date: ______ Period: ______

Exit Ticket

8.07B

Shawn spins a spinner and selects a marble from a bag. The spinner is divided into equal sections, and is numbered 1 through 5. The bag has one red marble, one blue marble, and one green marble.

Which event has the greater probability?

Event A: Landing on an odd number and selecting a blue marble.

Event B: Landing on a number greater than 3 and *not* selecting a blue marble.

Show or explain your thinking.

Event B;

Sample response:

I determined the sample space: 1—R, 1—B, 1—G, 2—R, 2—B, 2—G, 3—R, 3—B, 3—G, 4—R, 4—B, 4—G, 5—R, 5—B, 5—G.

The probability for Event A is $\frac{3}{15}$ (or equivalent). The probability for Event B is $\frac{4}{15}$. $\frac{4}{15} > \frac{3}{15}$.

Self-Assess

? 1	2	3
I don't really get it	I'm starting to get it	I got it

a I can determine the probability of multi-step events using organized lists, tables, or tree diagrams.

1 2 3

Tennessee Lesson 7B Probabilities of Multi-step Events

Success looks like . . .

- **Goal:** Using any method to determine the probability of a multi-step event.
  - » Writing the sample space to determine probabilities.
  - » Multiplying the number of outcomes for each event to determine probabilities.
- **Language Goal:** Comparing the likelihoods of events by computing the probabilities of the events, and explain the reasoning. **(Speaking and Listening)**
  - » Computing the probabilities for each event, and determining the greater fraction as belonging to the event with the greater probability.

Suggested next steps

If students do not compute the correct probabilities, consider:

- Having them write the sample space using any method.
- Reviewing Activity 1.

If students do not select Event B, consider:

- Reviewing how to write coordinates from Having them write fractions with the same denominator.
- Providing them with a calculator.
- Reviewing Activity 2.

Professional Learning

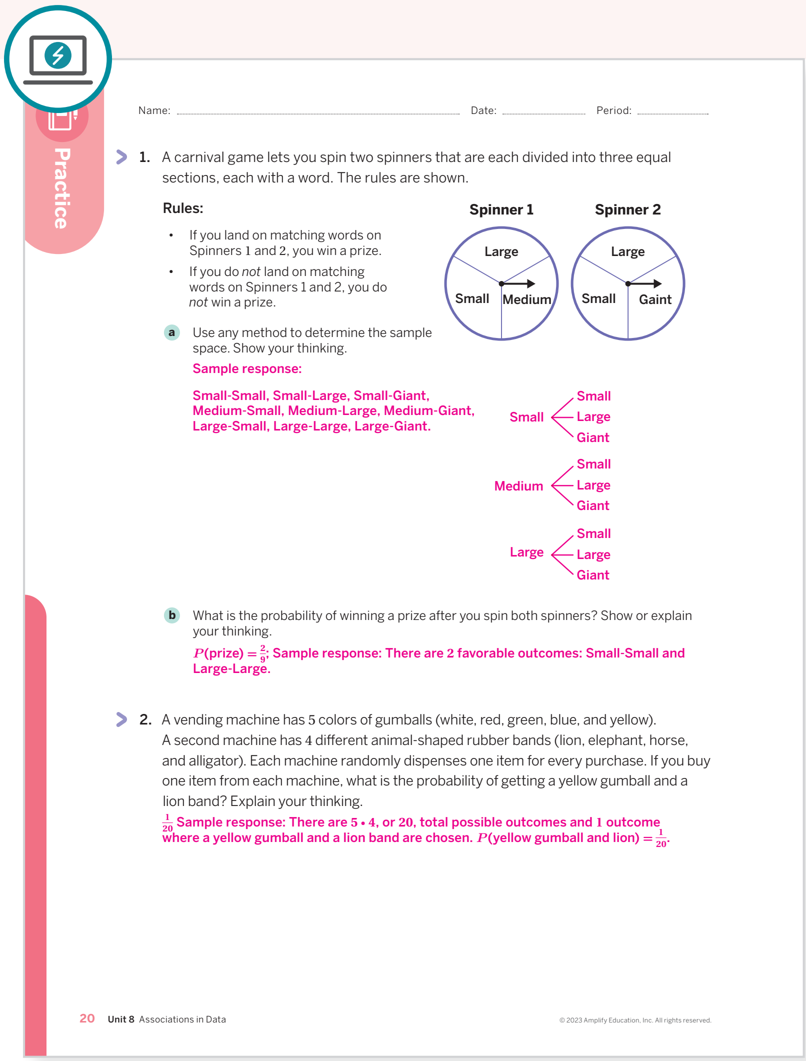
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? In what ways in Activity 2 did things happen that you did not expect?
- What did the use of sample spaces to determine probabilities of multi-step events reveal about your students as learners? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. A carnival game lets you spin two spinners that are each divided into three equal sections, each with a word. The rules are shown.

Rules:

- If you land on matching words on Spinners 1 and 2, you win a prize.
- If you do *not* land on matching words on Spinners 1 and 2, you do *not* win a prize.

a. Use any method to determine the sample space. Show your thinking.

Sample response:

Small-Small, Small-Large, Small-Giant, Medium-Small, Medium-Large, Medium-Giant, Large-Small, Large-Large, Large-Giant.

b. What is the probability of winning a prize after you spin both spinners? Show or explain your thinking.

P(prize) $= \frac{2}{9}$; Sample response: There are 2 favorable outcomes: Small-Small and Large-Large.

2. A vending machine has 5 colors of gumballs (white, red, green, blue, and yellow). A second machine has 4 different animal-shaped rubber bands (lion, elephant, horse, and alligator). Each machine randomly dispenses one item for every purchase. If you buy one item from each machine, what is the probability of getting a yellow gumball and a lion band? Explain your thinking.

$\frac{1}{20}$ Sample response: There are $5 \cdot 4$, or 20, total possible outcomes and 1 outcome where a yellow gumball and a lion band are chosen. P(yellow gumball and lion) $= \frac{1}{20}$.

20 **Unit 8** Associations in Data

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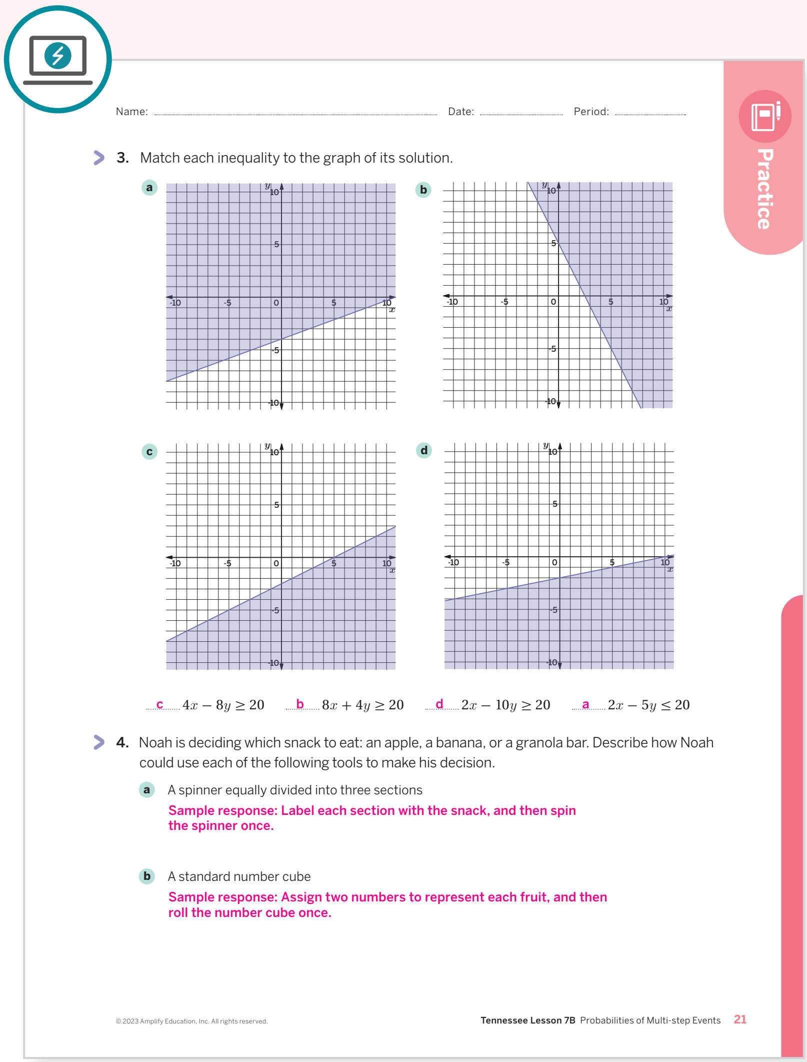

Name: Date: Period:

3. Match each inequality to the graph of its solution.

c $4x - 8y \geq 20$ b $8x + 4y \geq 20$ d $2x - 10y \geq 20$ a $2x - 5y \leq 20$

4. Noah is deciding which snack to eat: an apple, a banana, or a granola bar. Describe how Noah could use each of the following tools to make his decision.

a. A spinner equally divided into three sections

Sample response: Label each section with the snack, and then spin the spinner once.

b. A standard number cube

Sample response: Assign two numbers to represent each fruit, and then roll the number cube once.

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Tennessee Lesson 7B Probabilities of Multi-step Events 21

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activities 1-2	8.SP.B.4a, 8.SP.B.4b	2
	2	Activities 1-2	8.SP.B.4a	2
Spiral	3	Unit 3 Tennessee Lesson 17A	8.EE.C.9	2
Formative	4	Unit 8 Tennessee Lesson 7C	8.SP.B.4	2

Power-up: If students need additional support with the key prerequisite concept or skill this problem addresses, consider assigning the Power-up in the next lesson.

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice.**

Simulating Multi-step Events

Let's simulate multi-step events.

Focus

Goals

1. **Language Goal:** Coordinate a real-world situation and a chance event which could be used to simulate that situation. **(Speaking and Listening)**
2. **Language Goal:** Perform a multi-step simulation, and use the results to estimate the probability of a multi-step event in a real-world situation (using other representations). **(Speaking and Listening, Writing)**

Coherence

Today

Students see that multi-step events can be simulated by using multiple chance experiments. Students use simulations to determine probabilities and consider how real-world situations could be represented using simulations **(MP1, MP4)**.

Previously

In Tennessee Lesson 7B, students determined probabilities of multi-step events.

Coming Soon

In future grades, students will deepen their knowledge of probabilities of independent and dependent events.

Rigor

- Students build **conceptual understanding** of how chance events can be used to simulate real-world situations.

Standards

Addressing

8.SP.B.4

Find probabilities of and represent sample spaces for compound events using organized lists, tables, tree diagrams, and **simulation.**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	**Activity 1**	**Activity 2** (optional)	**Summary**	**Exit Ticket**
10 min	20 min	10 min	5 min	10 min
Pairs	Small Groups	Small Groups	Whole Class	Independent
MP4	MP4	MP1		
8.SP.B.4	8.SP.B.4	8.SP.B.4	8.SP.B.4	8.SP.B.4

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at **learning.amplify.com**.

Practice Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, *Graphington Slopes*, pre-cut spinner, one per group
- Activity 1 PDF, *Graphington Slopes: Theoretical Probability* (mathematical information for the teacher)
- bags with slips of paper
- paper clips
- number cubes

Math Language Development

New words

- ***simulation***

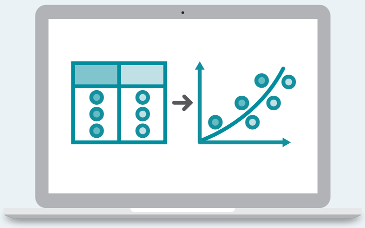
Review words

- *chance experiment*
- *multi-step event*
- *outcome*
- *probability*
- *sample space*

Amps | **Featured Activity**

Activity 1
Aggregate Class Data

Students perform their individual simulations followed by the entire class's data aggregating to create a greater number of trials. Students compare their simulation probability to the larger class's probability.



Building Math Identity and Community

Connecting to Mathematical Practices

Students may grow disinterested with the repetitive nature of a simulation; they may want to be impulsive and make assumptions instead of modeling the event and completing the simulation **(MP4)**. Encourage students to persist and work as a group to finish all trials of the simulation because accurate results are needed for the entire class to learn.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

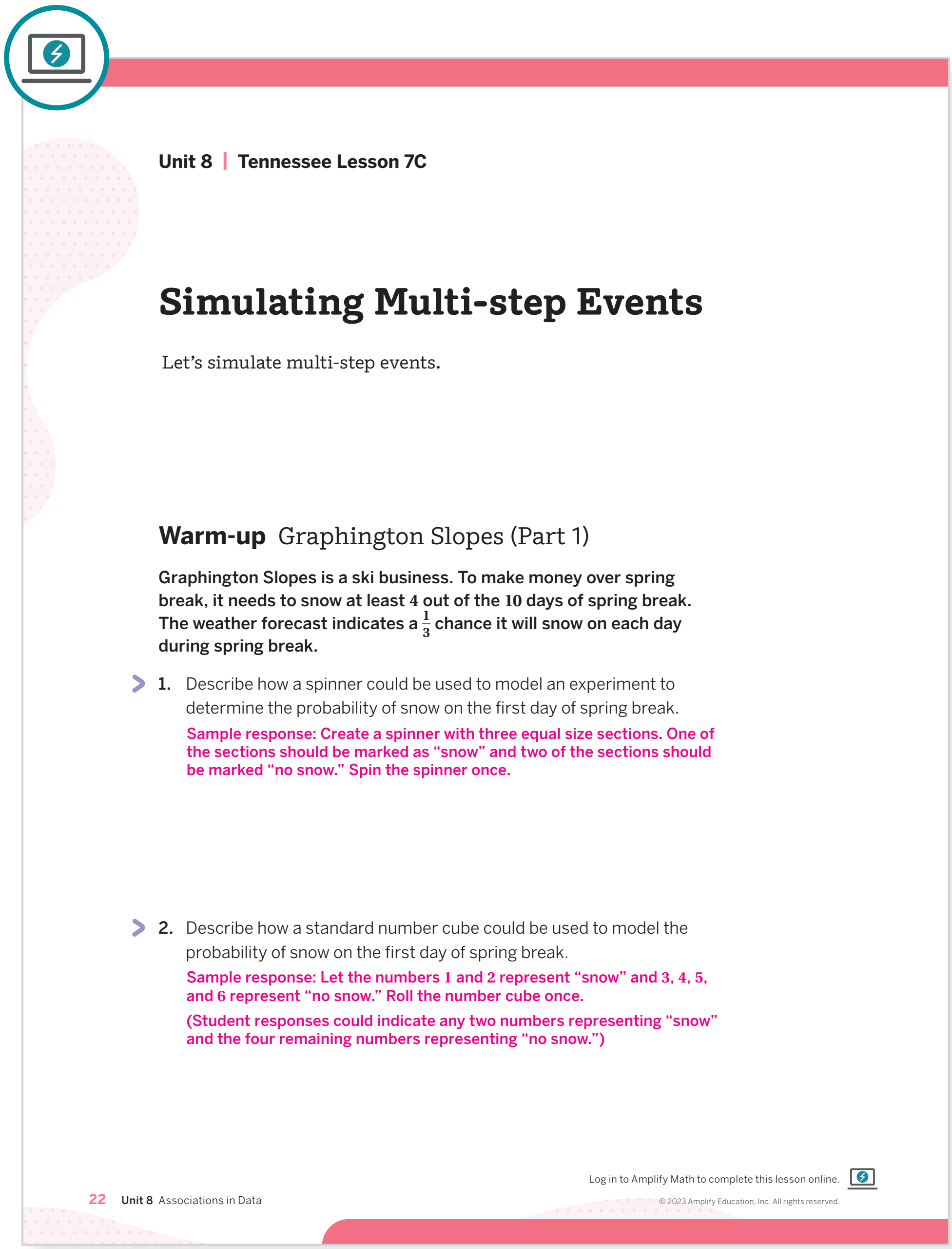
- Complete the **Warm-up** as a whole class.
- Reduce the number of simulations in **Activity 1** to three instead of five.
- Optional **Activity 2** may be omitted.

Pairs | 10 min

MP4
8.SP.B.4

Warm-up Graphington Slopes (Part 1)

Students design simulations of a real-world event to use in Activity 1.



Unit 8 | Tennessee Lesson 7C

Simulating Multi-step Events

Let's simulate multi-step events.

Warm-up Graphington Slopes (Part 1)

Graphington Slopes is a ski business. To make money over spring break, it needs to snow at least 4 out of the 10 days of spring break. The weather forecast indicates a $\frac{1}{3}$ chance it will snow on each day during spring break.

1. Describe how a spinner could be used to model an experiment to determine the probability of snow on the first day of spring break.
   Sample response: Create a spinner with three equal size sections. One of the sections should be marked as "snow" and two of the sections should be marked "no snow." Spin the spinner once.

2. Describe how a standard number cube could be used to model the probability of snow on the first day of spring break.
   Sample response: Let the numbers 1 and 2 represent "snow" and 3, 4, 5, and 6 represent "no snow." Roll the number cube once.
   (Student responses could indicate any two numbers representing "snow" and the four remaining numbers representing "no snow.")

Log in to Amplify Math to complete this lesson online.

22 Unit 8 Associations in Data

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1 Launch

Read the scenario to the class and ask, "What is the probability it will snow on any given day?"

2 Monitor

Help students get started by describing a spinner that is divided into thirds. Ask, "How many sections represent snow and how many sections represent no snow?"

Look for points of confusion:

- **Having difficulty describing a spinner showing a third as snow.** Let students draw a spinner instead of describing it.
- **Not understanding how the number cube could be used.** Have students write the sample space of rolling a number cube then circle one third of it.

3 Connect

Define the term ***simulation*** as an experiment used to estimate or predict the probability of a real-world event. The chance experiments designed in Problems 1 and 2 are examples of simulations **(MP4).**

Highlight why simulations are used to model a compound event using everyday objects like number cubes, marbles, coins, or spinners to help estimate probabilities.

Ask:

- "How can we adjust the simulations from Problems 1 or 2 to determine the probability of Graphington Slopes making money?" (Take this conversation and lead into the Launch of Activity 1).
- "Which simulation would you like to perform to help determine the probability?" (Based on group responses, provide those materials to the groups for Activity 1.)

Differentiated Support

Accessibility: *Activate Background Knowledge*

Students are likely familiar with weather forecasts. Ask, "When a weather forecast indicates a probability of snow (or other weather), what does that mean to you? Do you think a $\frac{1}{3}$ chance of snow means that snow is likely or unlikely?"

Accessibility: *Optimize Access to Tools*

Provide students with a blank spinner and a number cube that they can physically manipulate and hold to help them visualize how they could use each one to model the probability.

Power-up

To power up students' ability to describe how a tool could be used to determine a possible outcome, have students complete:

Mai is deciding which she should complete first, her math homework or science homework. Describe how flipping a coin could help her with making a decision.
Sample response: Mai can assign landing on heads to math homework and landing on tails to science homework, and then flip the coin once.

Use: Before the Warm-up

Informed by: Performance on Tennessee Lesson 7B, Practice Problem 5

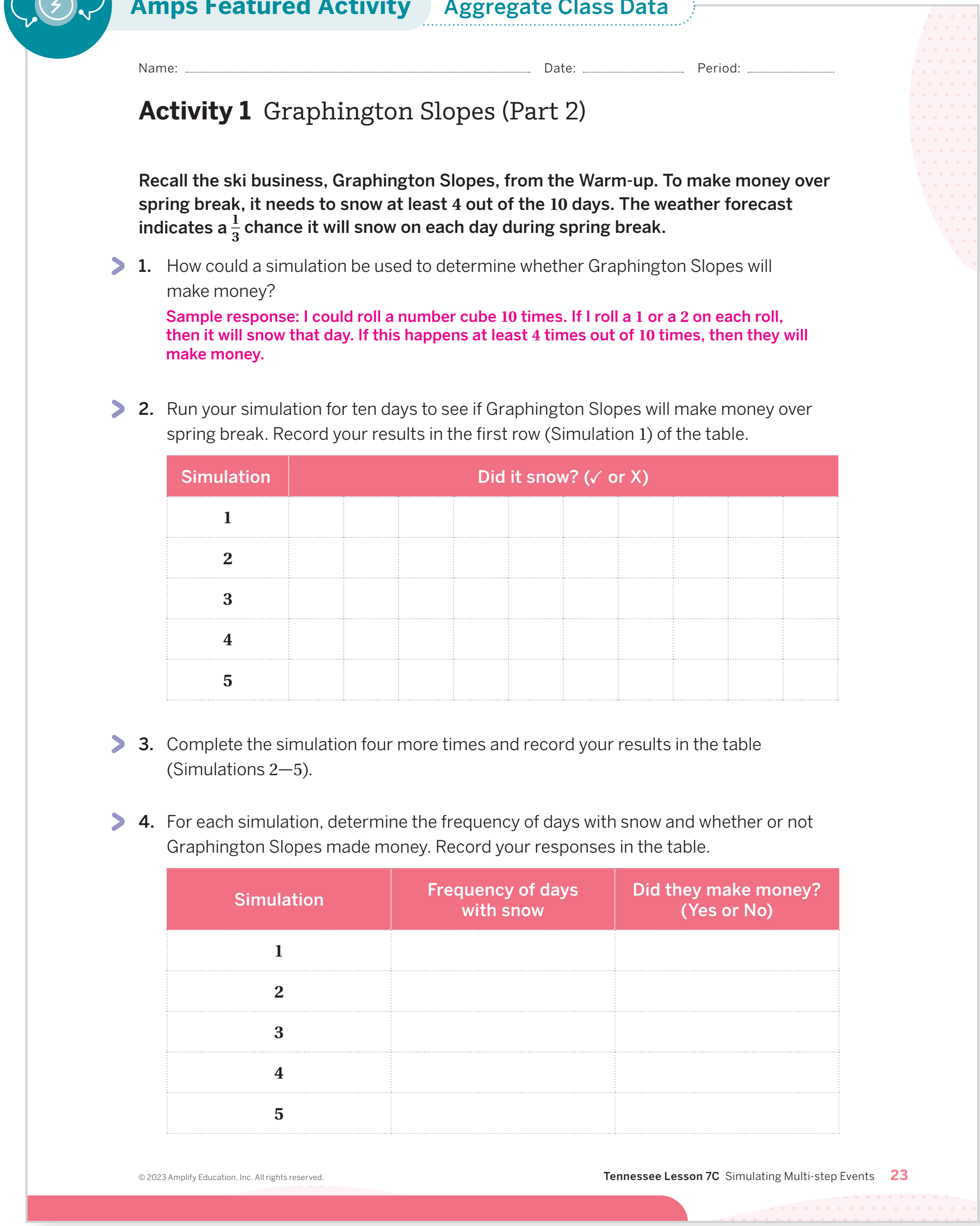
Small Groups | 20 min

MP4
8.SP.B.4

Activity 1 Graphington Slopes (Part 2)

Students perform the simulation created in the Warm-up to determine the relative frequency of a real-world event **(MP4).**

Amps Featured Activity Aggregate Class Data



Name: Date: Period:

Activity 1 Graphington Slopes (Part 2)

Recall the ski business, Graphington Slopes, from the Warm-up. To make money over spring break, it needs to snow at least 4 out of the 10 days. The weather forecast indicates a $\frac{1}{3}$ chance it will snow on each day during spring break.

1. How could a simulation be used to determine whether Graphington Slopes will make money?
   Sample response: I could roll a number cube 10 times. If I roll a 1 or a 2 on each roll, then it will snow that day. If this happens at least 4 times out of 10 times, then they will make money.

2. Run your simulation for ten days to see if Graphington Slopes will make money over spring break. Record your results in the first row (Simulation 1) of the table.

Simulation	Did it snow? (✓ or X)									
1										
2										
3										
4										
5										

3. Complete the simulation four more times and record your results in the table (Simulations 2—5).

4. For each simulation, determine the frequency of days with snow and whether or not Graphington Slopes made money. Record your responses in the table.

Simulation	Frequency of days with snow	Did they make money? (Yes or No)
1		
2		
3		
4		
5		

Tennessee Lesson 7C Simulating Multi-step Events 23

1 Launch

Provide groups with the Activity 1 PDF, pre-cut spinner, paper clips, number cubes, and bags with slips of paper. Give students 2 minutes to answer Problem 1 and have a whole class discussion regarding any questions they have before starting the simulation.

2 Monitor

Help students get started by helping them analyze and understand their simulation results. Once the simulation is complete in Problem 2, ask, "How do we know if Graphington Slopes will make money?"

Look for points of confusion:

- **Struggling with how to find frequency.** Remind students that *frequency* means the number of occurrences, for example, how many times it snowed.
- **Confusing the phrase "at least 4"**. Remind students it means 4 or more days of snow.
- **Totaling the number of snow days across all simulations.** Remind students one complete simulation represents 10 days, not any more or less.

Activity 1 continued ›

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides, in which they can perform their own simulations and view the class data aggregated. Students can then compare their individual simulation probability to the class's probability.

Extension: *Math Enrichment*

Ask students how the design of their simulation would change if the weather forecast indicated a 10% chance of snow on each day.
Answers may vary.

Math Language Development

MLR1: Stronger and Clearer Each Time

After students complete Problem 1, have groups meet with another group to share and receive feedback on the design of their simulations. Have reviewers provide feedback using these questions:

- Does the response include a description of the tool to use and how to use it (spinner, number cube, slips of paper, etc.)?
- Does the response include a description of what the favorable outcome represents when using the tool?

Have groups revise their designs, based on the feedback, and proceed with the remainder of the activity.

English Learners

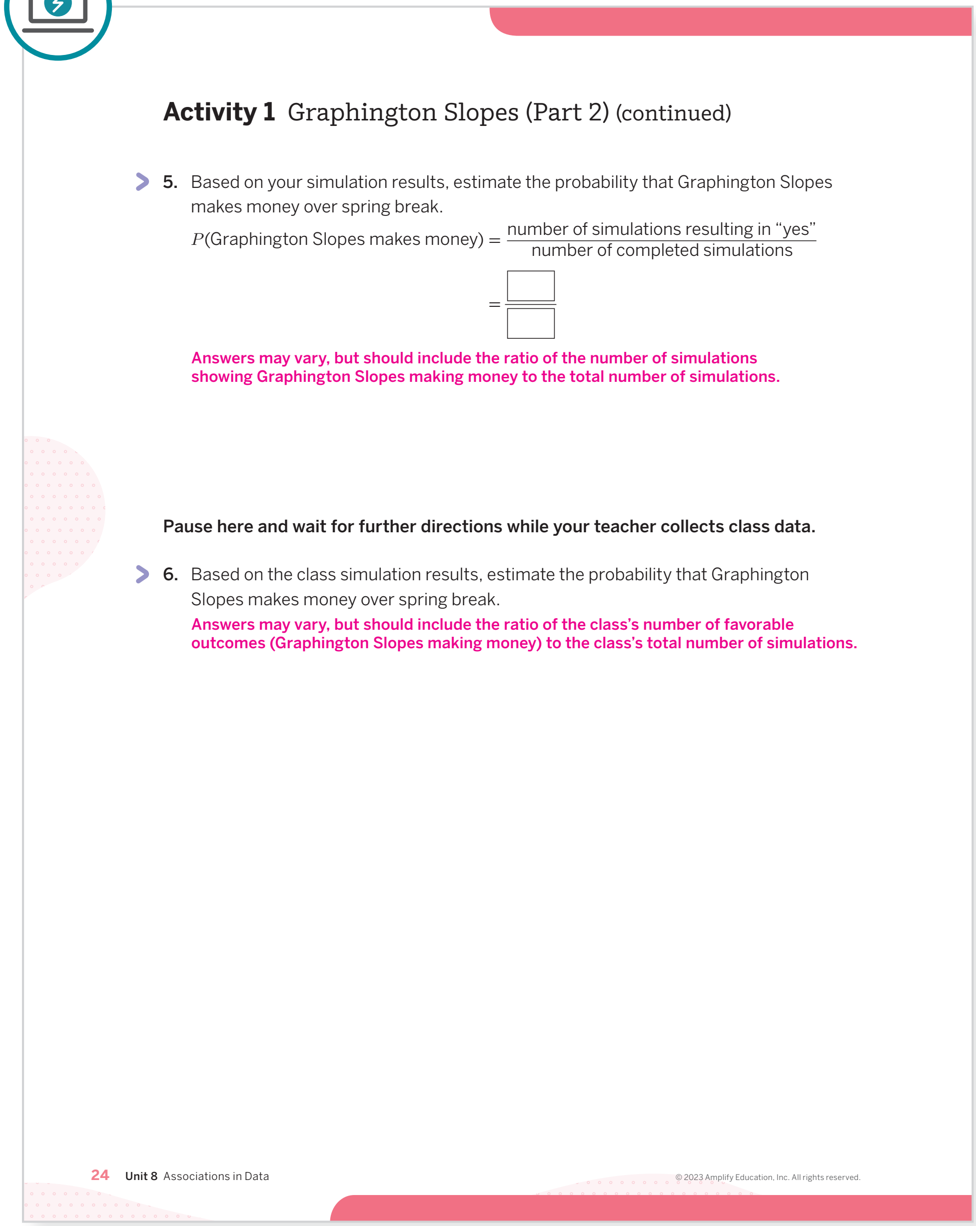
Suggest that students draw diagrams or pictures to include in their descriptions.

Small Groups | 20 min

Activity 1 Graphington Slopes (Part 2) (continued)

MP4
8.SP.B.4

Students perform the simulation created in the Warm-up to determine the relative frequency of a real-world event **(MP4).**



Activity 1 Graphington Slopes (Part 2) (continued)

5. Based on your simulation results, estimate the probability that Graphington Slopes makes money over spring break.

$P(\text{Graphington Slopes makes money}) = \frac{\text{number of simulations resulting in "yes"}}{\text{number of completed simulations}}$

$= \frac{\square}{\square}$

Answers may vary, but should include the ratio of the number of simulations showing Graphington Slopes making money to the total number of simulations.

Pause here and wait for further directions while your teacher collects class data.

6. Based on the class simulation results, estimate the probability that Graphington Slopes makes money over spring break.

Answers may vary, but should include the ratio of the class's number of favorable outcomes (Graphington Slopes making money) to the class's total number of simulations.

3 Connect

Have groups of students share their simulation results and display them for the class to see. Have students complete Problem 5 using the class data.

Highlight that the experimental (observed) probability approaches the theoretical (expected) probability when many trials are observed.

Note: At this level, make the assumption that the class's simulation result is the theoretical probability of the event. In later grades, students will learn to calculate the precise theoretical probability of compound events. The theoretical probability of it snowing at least 4 days out of the 10 days is 0.44 Refer to the Activity 1 PDF, *Graphington Slopes: Theoretical Probability* for an explanation.

Ask:

- "How close was your group's estimated probability to the class's probability?"
- "Is your group's probability a good representation of the class probability?"
- "The class performed ___ simulations and calculated the probability of Graphington Slopes making money to be ___. What do you think you could do to make the estimated probability the same as the expected probability?" Perform more simulations.
- "Do you anticipate Graphington Slopes will make money this year?" Answers may vary depending on the class's experimental (observed) probability. If probability is less than 0.5, perhaps students would not expect to make money.

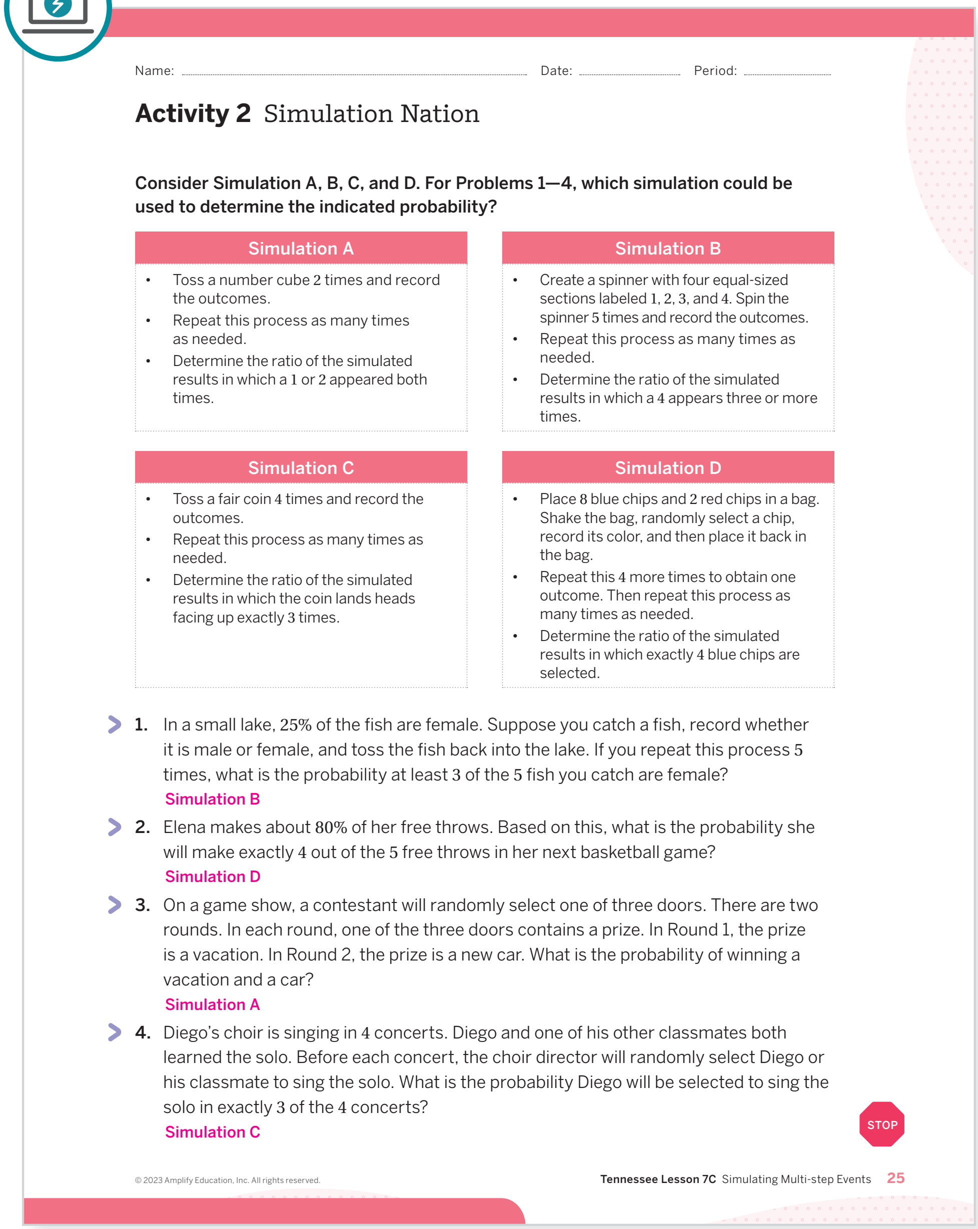
Optional

Small Groups | 10 min

MP1
8.SP.B.4

Activity 2 Simulation Nation

Students decide which simulations could be used to determine indicated probabilities to make sense of real-world situations and simulations.



Name: ______ Date: ______ Period: ______

Activity 2 Simulation Nation

Consider Simulation A, B, C, and D. For Problems 1—4, which simulation could be used to determine the indicated probability?

Simulation A

- Toss a number cube 2 times and record the outcomes.
- Repeat this process as many times as needed.
- Determine the ratio of the simulated results in which a 1 or 2 appeared both times.

Simulation B

- Create a spinner with four equal-sized sections labeled 1, 2, 3, and 4. Spin the spinner 5 times and record the outcomes.
- Repeat this process as many times as needed.
- Determine the ratio of the simulated results in which a 4 appears three or more times.

Simulation C

- Toss a fair coin 4 times and record the outcomes.
- Repeat this process as many times as needed.
- Determine the ratio of the simulated results in which the coin lands heads facing up exactly 3 times.

Simulation D

- Place 8 blue chips and 2 red chips in a bag. Shake the bag, randomly select a chip, record its color, and then place it back in the bag.
- Repeat this 4 more times to obtain one outcome. Then repeat this process as many times as needed.
- Determine the ratio of the simulated results in which exactly 4 blue chips are selected.

1. In a small lake, 25% of the fish are female. Suppose you catch a fish, record whether it is male or female, and toss the fish back into the lake. If you repeat this process 5 times, what is the probability at least 3 of the 5 fish you catch are female?
   Simulation B
2. Elena makes about 80% of her free throws. Based on this, what is the probability she will make exactly 4 out of the 5 free throws in her next basketball game?
   Simulation D
3. On a game show, a contestant will randomly select one of three doors. There are two rounds. In each round, one of the three doors contains a prize. In Round 1, the prize is a vacation. In Round 2, the prize is a new car. What is the probability of winning a vacation and a car?
   Simulation A
4. Diego's choir is singing in 4 concerts. Diego and one of his other classmates both learned the solo. Before each concert, the choir director will randomly select Diego or his classmate to sing the solo. What is the probability Diego will be selected to sing the solo in exactly 3 of the 4 concerts?
   Simulation C

STOP

Tennessee Lesson 7C Simulating Multi-step Events 25

1 Launch

Read the directions and explain to students that each simulation will be used exactly one time.

2 Monitor

Help students get started by having them determine the probabilities represented in each simulation, and then reading each Problem to see which match.

Look for points of confusion:

- **Struggling to make the connection between Problem 3 and the number cube.** Have students write the sample space of a cube and circle the numbers 1 and 2. Ask them what the probability is of rolling a 1 or 2.

Look for productive strategies:

- Explaining their reasoning with precise mathematical language.

3 Connect

Display the Problems and simulations.

Have students share their matches and how the strategies they used to make sense of each problem **(MP1)**.

Highlight how the simulations model the problems using the explanations from the students.

Ask:

- "For Problem 1, why is the spinner spun five times?" Five fish were caught.
- "For Problem 1, why does the same number need to be spun 3 or more times?" We are trying to determine the probability of at least 3 being female.
- "For any given problem, could part of it be changed and still result in the simulation working?" Answers may vary, but should include maintaining the probability of the experiment.

Differentiated Support

Accessibility: *Optimize Access to Tools*

Provide students with blank spinners, number cubes, coins, slips of paper, and a bag that they can physically manipulate to help them visualize how they could use each one to simulate the situations.

Accessibility: *Vary Demands to Optimize Challenge*

Consider having students complete only Problems 1—3 and remove Simulation C from the list of simulations to choose.

Math Language Development

MLR7: Compare and Connect

During the Connect, as students explain how they determined their matches, listen for and amplify the connection between the numerical quantities in the simulation and the situation. Display sentence frames for students to use, such as:

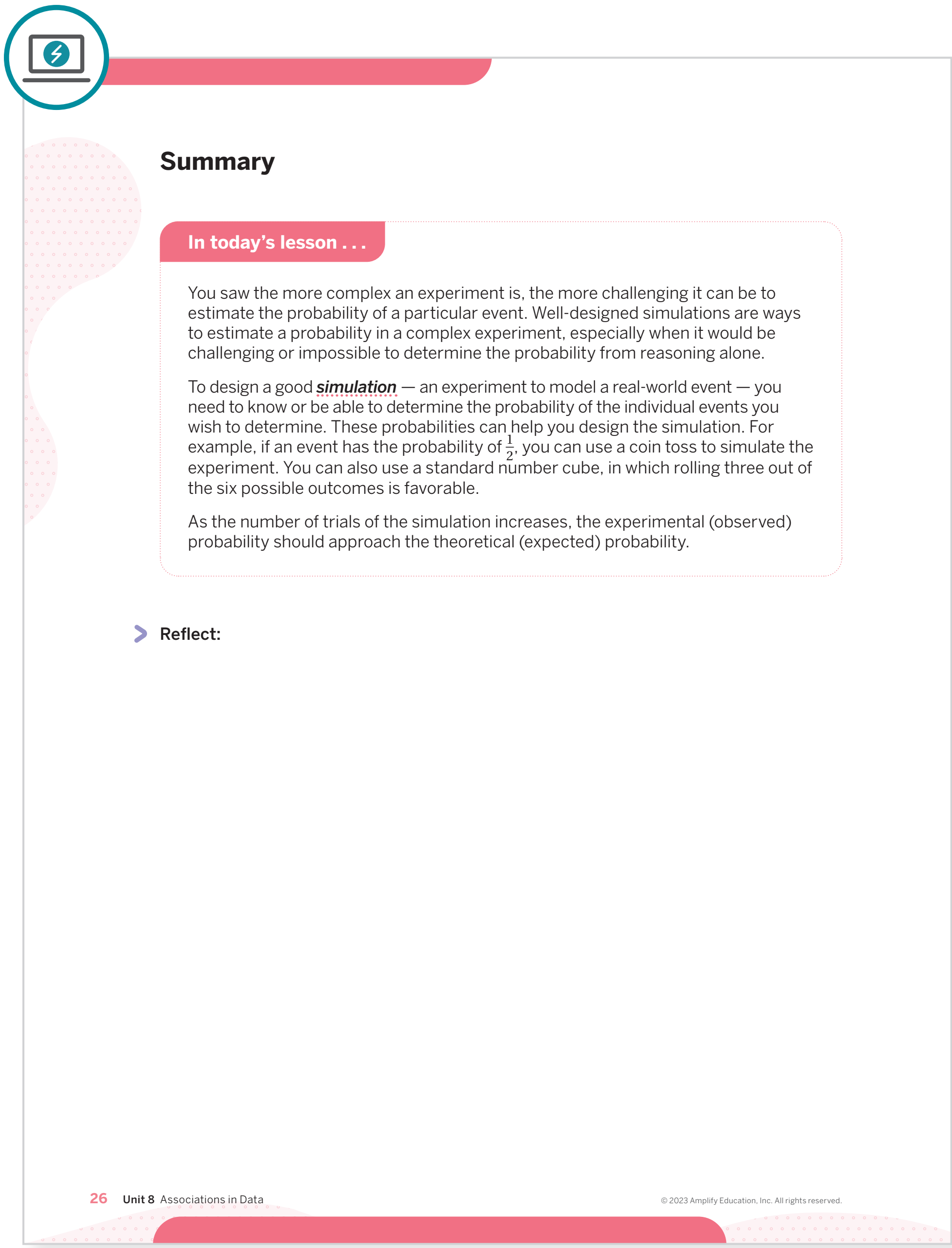
- "Simulation____ matches with Problem ____ because . . . "
- "I saw that the text mentions____, so I . . . "
- "I know that the spinner matches ____, because . . ."

Whole Class | 5 min

Summary

8.SP.B.4

Review and synthesize how simulations are used to estimate probabilities.



Summary

In today's lesson . . .

You saw the more complex an experiment is, the more challenging it can be to estimate the probability of a particular event. Well-designed simulations are ways to estimate a probability in a complex experiment, especially when it would be challenging or impossible to determine the probability from reasoning alone.

To design a good ***simulation*** — an experiment to model a real-world event — you need to know or be able to determine the probability of the individual events you wish to determine. These probabilities can help you design the simulation. For example, if an event has the probability of $\frac{1}{2}$, you can use a coin toss to simulate the experiment. You can also use a standard number cube, in which rolling three out of the six possible outcomes is favorable.

As the number of trials of the simulation increases, the experimental (observed) probability should approach the theoretical (expected) probability.

Reflect:

26 **Unit 8** Associations in Data

Synthesize

Have students share what they understand about simulations and how they are used to estimate probabilities.

Highlight experimental (observed) probability is calculated as the ratio of the number of observed favorable cases to the number of completed simulations. Performing more simulations should result in an observed probability which is closer to the expected probability. For instance, a simulation using 10,000 trials should have a better estimated probability than one using only 100 trials.

Formalize vocabulary: ***simulation***

Ask, "Each day, a student randomly reaches into a bowl of fruit and picks one for his lunch. To simulate the situation, he creates a spinner with four equal sections labeled: *apple*, *orange*, *pear*, and *peach*. Why might this simulation not represent the situation very well?" This simulation assumes each fruit is equally likely to be chosen. We do not know if there are the same number of each fruit. Also, as the week progresses, the remaining fruit might not have the same ratio as it did at the beginning of the week.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflect* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Our world is really complicated — how can we simulate parts of it to make better predictions?"

Math Language Development

MLR2: Collect and Display

As students formalize the new vocabulary for this lesson, ask them to refer to the class display that you started in this unit. Ask them to review and reflect on any terms and phrases related to the term *simulation* that were added to the display during the lesson.

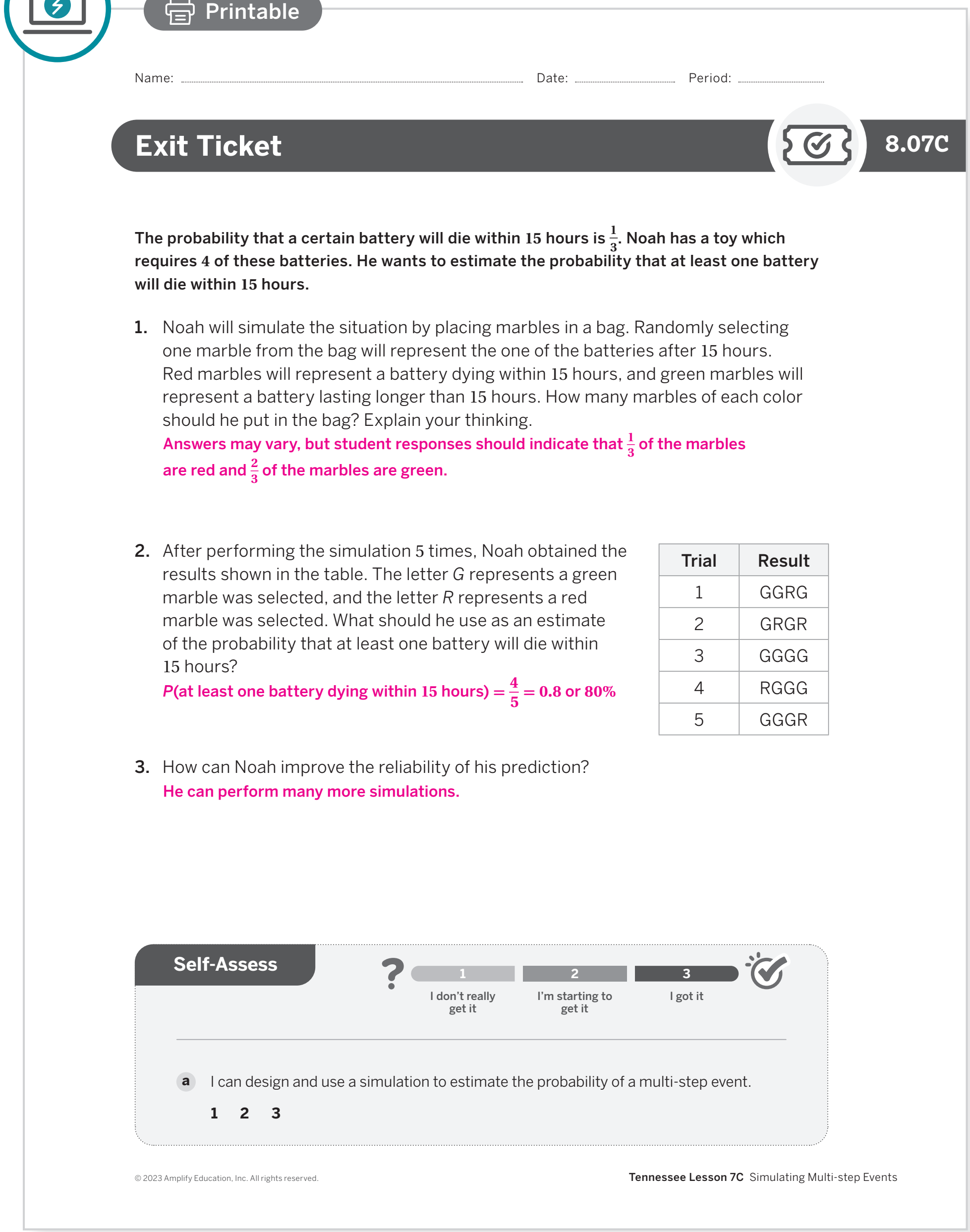
Independent | 10 min

Exit Ticket

8.SP.B.4

Students demonstrate their understanding by designing a simulation and using it to estimate probabilities.

Printable



Name: ______________________ Date: __________ Period: __________

Exit Ticket

8.07C

The probability that a certain battery will die within 15 hours is $\frac{1}{3}$. Noah has a toy which requires 4 of these batteries. He wants to estimate the probability that at least one battery will die within 15 hours.

1. Noah will simulate the situation by placing marbles in a bag. Randomly selecting one marble from the bag will represent the one of the batteries after 15 hours. Red marbles will represent a battery dying within 15 hours, and green marbles will represent a battery lasting longer than 15 hours. How many marbles of each color should he put in the bag? Explain your thinking.
   Answers may vary, but student responses should indicate that $\frac{1}{3}$ of the marbles are red and $\frac{2}{3}$ of the marbles are green.

2. After performing the simulation 5 times, Noah obtained the results shown in the table. The letter *G* represents a green marble was selected, and the letter *R* represents a red marble was selected. What should he use as an estimate of the probability that at least one battery will die within 15 hours?
   P(at least one battery dying within 15 hours) $= \frac{4}{5} = 0.8$ or 80%

Trial	Result
1	GGRG
2	GRGR
3	GGGG
4	RGGG
5	GGGR

3. How can Noah improve the reliability of his prediction?
   He can perform many more simulations.

Self-Assess

? 1 I don't really get it | 2 I'm starting to get it | 3 I got it

a I can design and use a simulation to estimate the probability of a multi-step event.

1 2 3

Success looks like . . .

- **Language Goal:** Coordinating a real-world situation and a chance event which could be used to simulate that situation. **(Speaking and Listening)**
  - » Responses for Problem 1 indicate that $\frac{1}{3}$ of the marbles are red and $\frac{2}{3}$ of the marbles are green.
- **Language Goal:** Performing a multi-step simulation, and use the results to estimate the probability of a multi-step event in a real-world situation (using other representations). **(Speaking and Listening, Writing)**
  - » Probability written in Problem 2 is equivalent to $\frac{4}{5}$.

Suggested next steps

If students struggle to solve Problem 1, consider:

- Providing red and green marbles to help them visualize how they could simulate the situation.
- Reviewing ratios and equivalent fractions.
- Assigning Practice Problem 1.

If students write an incorrect probability for Problem 2, consider:

- Reviewing Activity 1, Problem 4.
- Assigning Practice Problem 2.

Professional Learning

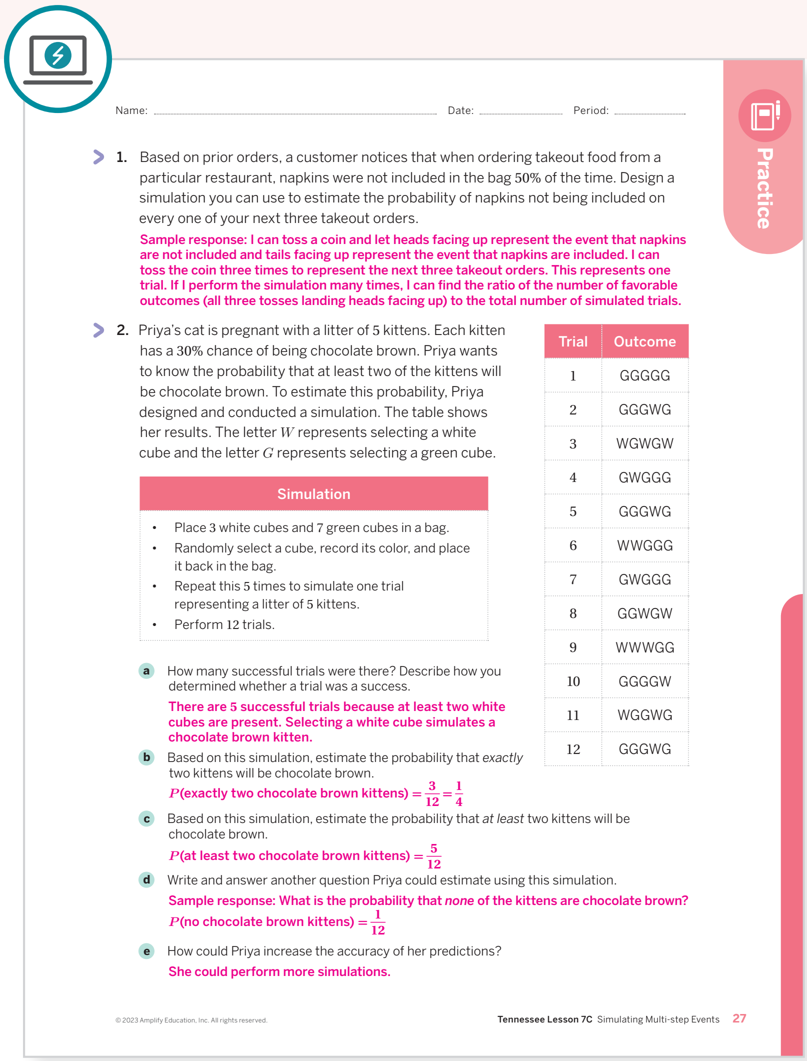
This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What surprised you as your students worked on Activity 1?
- Have you changed any ideas you used to have about simulations as a result of today's lesson? What might you change for the next time you teach this lesson?

Practice

Independent



Name: Date: Period:

1. Based on prior orders, a customer notices that when ordering takeout food from a particular restaurant, napkins were not included in the bag 50% of the time. Design a simulation you can use to estimate the probability of napkins not being included on every one of your next three takeout orders.

   Sample response: I can toss a coin and let heads facing up represent the event that napkins are not included and tails facing up represent the event that napkins are included. I can toss the coin three times to represent the next three takeout orders. This represents one trial. If I perform the simulation many times, I can find the ratio of the number of favorable outcomes (all three tosses landing heads facing up) to the total number of simulated trials.

2. Priya's cat is pregnant with a litter of 5 kittens. Each kitten has a 30% chance of being chocolate brown. Priya wants to know the probability that at least two of the kittens will be chocolate brown. To estimate this probability, Priya designed and conducted a simulation. The table shows her results. The letter W represents selecting a white cube and the letter G represents selecting a green cube.

Trial	Outcome
1	GGGGG
2	GGGWG
3	WGWGW
4	GWGGG
5	GGGWG
6	WWGGG
7	GWGGG
8	GGWGW
9	WWWGG
10	GGGGW
11	WGGWG
12	GGGWG

Simulation

- Place 3 white cubes and 7 green cubes in a bag.
- Randomly select a cube, record its color, and place it back in the bag.
- Repeat this 5 times to simulate one trial representing a litter of 5 kittens.
- Perform 12 trials.

a. How many successful trials were there? Describe how you determined whether a trial was a success.

There are 5 successful trials because at least two white cubes are present. Selecting a white cube simulates a chocolate brown kitten.

b. Based on this simulation, estimate the probability that *exactly* two kittens will be chocolate brown.

P(exactly two chocolate brown kittens) $= \frac{3}{12} = \frac{1}{4}$

c. Based on this simulation, estimate the probability that *at least* two kittens will be chocolate brown.

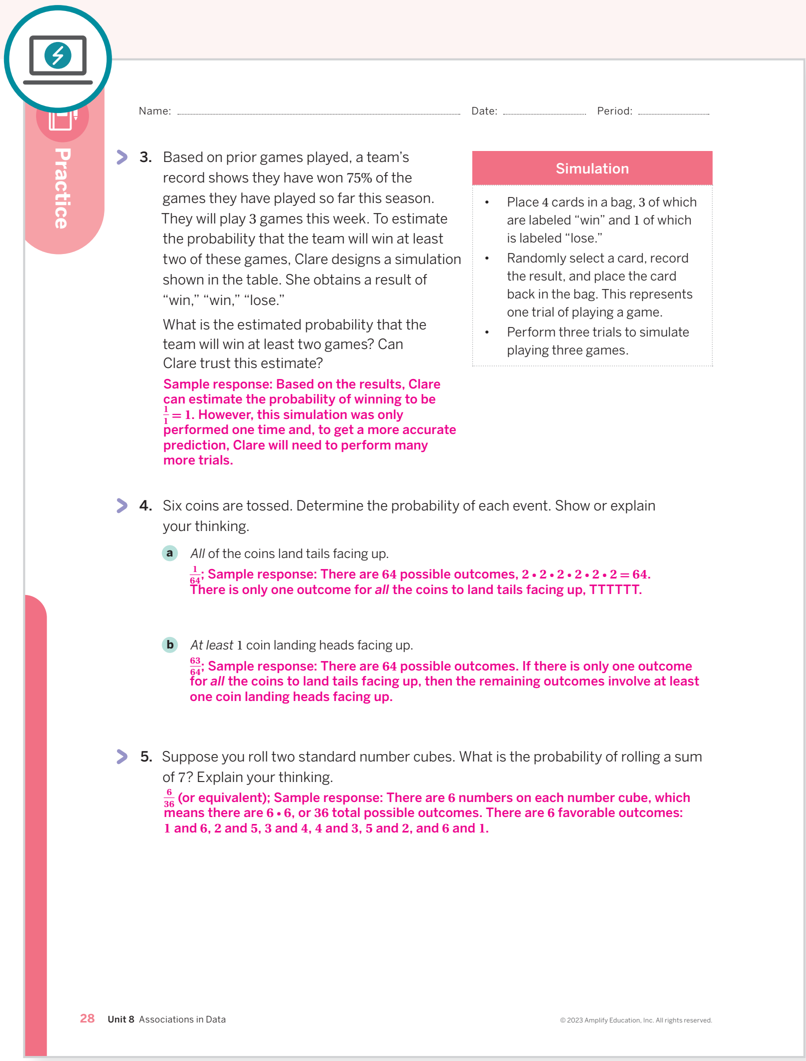
P(at least two chocolate brown kittens) $= \frac{5}{12}$

d. Write and answer another question Priya could estimate using this simulation.

Sample response: What is the probability that *none* of the kittens are chocolate brown?
P(no chocolate brown kittens) $= \frac{1}{12}$

e. How could Priya increase the accuracy of her predictions?

She could perform more simulations.



Name: Date: Period:

3. Based on prior games played, a team's record shows they have won 75% of the games they have played so far this season. They will play 3 games this week. To estimate the probability that the team will win at least two of these games, Clare designs a simulation shown in the table. She obtains a result of "win," "win," "lose."

   What is the estimated probability that the team will win at least two games? Can Clare trust this estimate?

   Sample response: Based on the results, Clare can estimate the probability of winning to be $\frac{1}{1} = 1$. However, this simulation was only performed one time and, to get a more accurate prediction, Clare will need to perform many more trials.

Simulation

- Place 4 cards in a bag, 3 of which are labeled "win" and 1 of which is labeled "lose."
- Randomly select a card, record the result, and place the card back in the bag. This represents one trial of playing a game.
- Perform three trials to simulate playing three games.

4. Six coins are tossed. Determine the probability of each event. Show or explain your thinking.

   a. *All* of the coins land tails facing up.

   $\frac{1}{64}$; Sample response: There are 64 possible outcomes, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$. There is only one outcome for *all* the coins to land tails facing up, TTTTTT.

   b. *At least* 1 coin landing heads facing up.

   $\frac{63}{64}$; Sample response: There are 64 possible outcomes. If there is only one outcome for *all* the coins to land tails facing up, then the remaining outcomes involve at least one coin landing heads facing up.

5. Suppose you roll two standard number cubes. What is the probability of rolling a sum of 7? Explain your thinking.

   $\frac{6}{36}$ (or equivalent); Sample response: There are 6 numbers on each number cube, which means there are $6 \cdot 6$, or 36 total possible outcomes. There are 6 favorable outcomes: 1 and 6, 2 and 5, 3 and 4, 4 and 3, 5 and 2, and 6 and 1.

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 2	8.SP.B.4	2
	2	Activity 1	8.SP.B.4	2
	3	Activity 2	8.SP.B.4	2
Spiral	4	Unit 8 Tennessee Lesson 7B	8.SP.B.4a	2
	5	Unit 8 Tennessee Lesson 7B	8.SP.B.4a	2

Additional Practice Available

For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice.**