Amplify Desmos Math
NEW YORK

Teacher Edition Sampler
Grade 6

Inside you'll find:

- Program overview, scope and sequence, and correlations
- Complete sample lessons from Amplify Desmos Math
- Lesson plans from requested domains, partially designed

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About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify Desmos Math is based on the Illustrative Mathematics (IM) curriculum. IM 6–8 Math was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is © 2017–2019 Open Up Resources. Additional adaptations and updates to IM 6–8 Math are © 2019 Illustrative Mathematics. IM 9–12 Math is © 2019 Illustrative Mathematics. IM 6–8 Math and IM 9–12 are licensed under the Creative Commons Attribution 4.0 International license (CC BY 4.0). Additional modifications contained in Amplify Desmos Math are © 2020 Amplify Education, Inc. and its licensors. Amplify is not affiliated with the Illustrative Mathematics organization.

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55 Washington Street, Suite 800,
Brooklyn, NY 11201
www.amplify.com

Welcome reviewer

Welcome to your Amplify Desmos Math New York Teacher Edition sampler!

Amplify Desmos Math New York is the result of two groundbreaking research and development efforts in K–12 mathematics instruction led by the Amplify and Desmos Classroom teams. Merging the two teams in 2022 enabled us to build a new curriculum around the idea that all students deserve to engage in high-quality grade-level mathematics every day. Based on Illustrative Mathematics'® IM K–12 Math™, Amplify Desmos Math New York combines strong pedagogy, arresting design, and forward-looking collaborative technology to deliver a classroom experience that keeps students engaged and asking productive questions.

Every lesson in the Amplify Desmos Math digital platform has a corresponding lesson in the print teacher and student editions. While we are in the process of finalizing the print materials, we have provided exemplars highlighting the unique design and ease of use of the Amplify Desmos Math print resources. To provide content covering your specific domain requests, in this physical sampler we have included both robust Amplify Desmos Math lesson plans and partially designed lesson plans. However, all of the lessons can be reviewed in their complete forms online.

All Amplify Desmos Math lessons include:

- Easy-to-follow lesson plans, tested in classrooms across the country.
- Clear teaching suggestions and strategies, including math language routines.
- Recommended differentiation moves and practice sets.

Diagnostic, formative, and summative assessments are provided with each unit along with lesson-level checks for understanding.

Amplify and New York City have a long history of partnering to provide equitable, high-quality instruction to our next generation of leaders. We look forward to continuing this partnership with New York City Public Schools in middle school mathematics.

—Jason Zimba and the Amplify Desmos Math team

Amplify Desmos Math New York

Helping New York City teachers develop and celebrate student thinking

Deep and lasting learning occurs when students are able to make connections to prior thinking and experiences. This requires teachers to deliver math instruction that balances exploration and explanation, and that puts student thinking at the center of classroom instruction.

Amplify Desmos Math students are invited to explore the math that fills their everyday lives, while strengthening their knowledge of math facts, procedural skills, and conceptual knowledge. Using the Amplify Desmos Math print and digital lesson plans, teachers can confidently guide and instruct as they build on students' understandings to help them develop a better grasp of mathematics.

Amplify Desmos Math is **a truly student-centered program** built around three core tenets:

1

A strong foundation in problem-based learning is critical to developing deep conceptual understanding, procedural fluency, and application.

Students are introduced to interesting problems and leverage both their current understandings and problem-solving strategies to develop reasonable answers. The learning experience is an active one that leads students to explore, notice, question, solve, justify, explain, represent, and analyze. Teachers guide the process, supporting synthesis and sensemaking at the end of each lesson.

2

Technology can provide ongoing, enriched feedback that encourages students to persevere in problem solving.

Especially when new ideas are being introduced, Desmos Classroom technology shows students the meaning of their thinking in context, interpreting it mathematically rather than reducing it to a question of right or wrong. This creates a culture of going deep with mathematics and students as doers of mathematics, so that as learning progresses and correctness is the goal, incorrect answers become objects of curiosity rather than embarrassment. This information in response to student ideas is what we call "enriched feedback." Amplify Desmos Math New York offers more enriched feedback than any other math program.

3

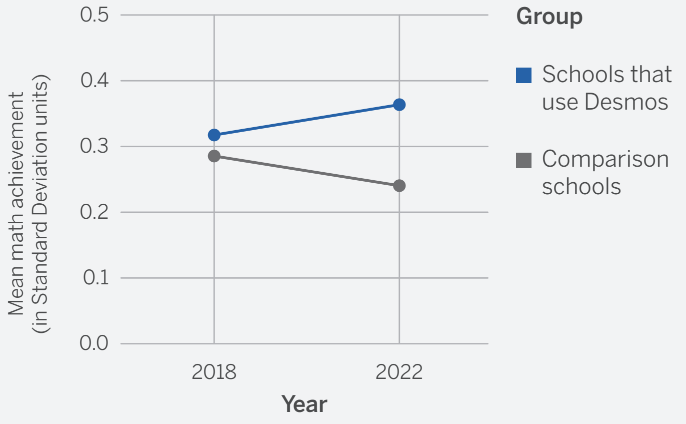
A commitment to access and equity should underpin every development decision.

All students can dive into problems on their own, and activities are designed to honor different approaches. Activities rely on collaboration and lots of hands-on, experiential learning.

And the program works.

Amplify Desmos Math New York expands on the Desmos Math 6–8 curriculum, which was recently proven to increase average math achievement in a study of more than 900 schools in nine states led by WestEd.

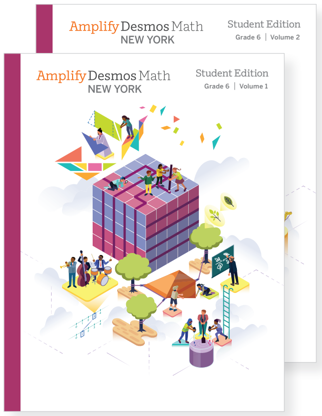
Mean Math Achievement for Desmos Schools and Matched Comparison Schools in 2018 and 2022



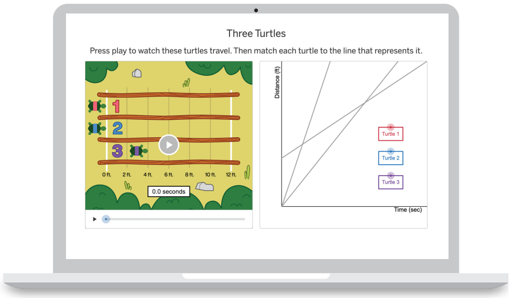
The Effect of Desmos Math Curriculum on Middle School Mathematics Achievement in Nine States. WestEd., (McKinney, D., Strother, S., Walters, K. & Schneider, S., 2023).

Amplify Desmos Math New York program resources

Student bundle includes:



NY Student Edition,
multivolume, consumable



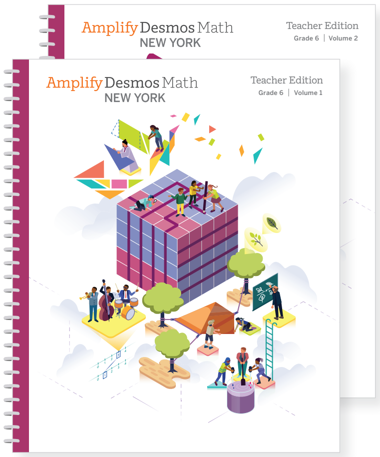
Three Turtles

Press play to watch these turtles travel. Then match each turtle to the line that represents it.

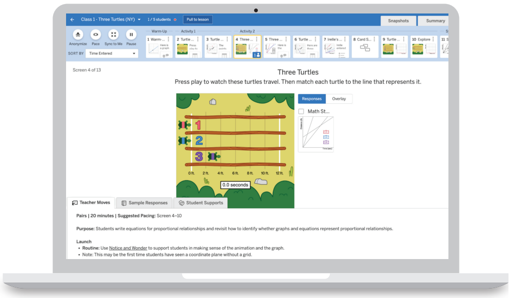
NY Digital Experience (English and Spanish), featuring:

- Interactive Student Activity Screens
- Enriched feedback
- Collaboration tools

Teacher bundle includes:



NY Teacher Edition,
multivolume, spiral-bound



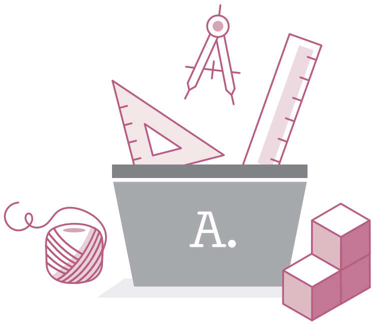
Three Turtles

Press play to watch these turtles travel. Then match each turtle to the line that represents it.

NY Digital Experience (English and Spanish), featuring:

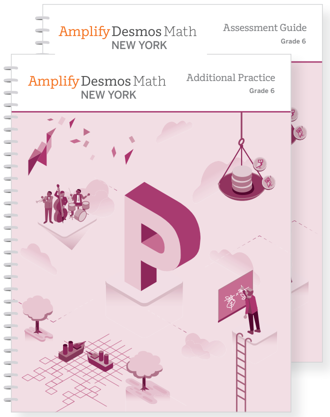
- Facilitation and progress monitoring tools
- Presentation Screens
- Instructional supports
- Assessment

Optional:



Middle School Manipulative Kit (Grades 6–8)

Extra Practice and Assessment Blackline Masters



Additional components and features may roll out over time.

Program architecture

Course

I mCLASS beginning-of-year diagnostic screener (before Unit 1)
I (before Unit 3)
I Interim Assesments (before Unit 4)
I (before Unit 6)
I (before Unit 8)

UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6	UNIT 7	UNIT 8
21 days	20 days	19 days	20 days	22 days	22 days	18 days	22 days

Unit

A Pre-Unit Check

A Sub-Unit Quiz

End-of-Unit Assessment A

Sub-Unit 1	Sub-Unit 2 (Practice Day)	Sub-Unit 3 (Practice Day)
1 2 3 4 5 6 7	8 9 10 11 12 PD	13 14 15 16 17 18 19 20 PD

Note: The number of sub-units and lessons vary from unit to unit; this depiction shows the general structure of a unit.

Lesson

Warm-up	>	Activity 1	>	Activity 2	>	Synthesis	>	Exit Ticket	>	Practice
🕘 5 min		🕘 15 min		🕘 15 min		🕘 5 min		🕘 5 min		🕘 timing varies
👤 👥 👪 👨‍👩‍👧‍👦		👤 👥 👪 👨‍👩‍👧‍👦		👤 👥 👪 👨‍👩‍👧‍👦		👨‍👩‍👧‍👦		👤		👤

Note: The number of activities and timing vary from lesson to lesson; this depiction shows the general structure of a lesson.

Key:

👤 Independent	👪 Small Groups
👥 Pairs	👨‍👩‍👧‍👦 Whole Class

Program Scope and Sequence

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 164 days total	Area and Surface Area 18 Instructional Days 3 Assessment Days 21 days total	Introducing Ratios 17 Instructional Days 3 Assessment Days 20 days total	Unit Rates and Percentages 16 Instructional Days 3 Assessment Days 19 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 158 days total	Scale Drawings 15 Instructional Days 3 Assessment Days 18 days total	Introducing Proportional Relationships 16 Instructional Days 3 Assessment Days 19 days total	Measuring Circles 13 Instructional Days 3 Assessment Days 16 days total	Proportional Relationships and Percentages 17 Instructional Days 3 Assessment Days 20 days total
Grade 8 150 days total	Rigid Transformation and Congruence 16 Instructional Days 4 Assessment Days 20 days total	Dilations, Similarity, and Introducing Slope 12 Instructional Days 4 Assessment Days 16 days total	Proportional and Linear Relationships 13 Instructional Days 3 Assessment Days 16 days total	Linear Equations and Linear Systems 17 Instructional Days 3 Assessment Days 20 days total

Unit 5	Unit 6	Unit 7	Unit 8
Decimal Arithmetic 18 Instructional Days 4 Assessment Days 22 days total	Expressions and Equations 19 Instructional Days 3 Assessment Days 22 days total	Positive and Negative Numbers 15 Instructional Days 3 Assessment Days 18 days total	Describing Data 19 Instructional Days 3 Assessment Days 22 days total
Operations With Positive and Negative Numbers 16 Instructional Days 4 Assessment Days 20 days total	Expressions, Equations, and Inequalities 21 Instructional Days 3 Assessment Days 24 days total	Angles, Triangles, and Prisms 16 Instructional Days 3 Assessment Days 19 days total	Probability and Sampling 19 Instructional Days 3 Assessment Days 22 days total
Functions and Volume 18 Instructional Days 4 Assessment Days 22 days total	Associations in Data 15 Instructional Days 2 Assessment Days 17 days total	Exponents and Scientific Notation 16 Instructional Days 3 Assessment Days 19 days total	The Pythagorean Theorem and Irrational Numbers 17 Instructional Days 3 Assessment Days 20 days total

Unit 1 Area and Surface Area

In this unit, students learn to calculate areas of polygons by decomposing, rearranging, and composing shapes. They also represent polyhedra with nets and calculate their surface areas.

Pre-Unit

Getting to Know Each Other

Pre-Unit Check

Sub-Unit 1 Area

1.01 Shapes on a Plane | Exploring Area

1.02 Letters | Area Strategies

1.03 Exploring Parallelograms | Parallelograms on a Grid

1.04 Off the Grid | Calculating Areas of Parallelograms

1.05 Exploring Triangles | Triangles on a Grid

1.06 Triangles and Parallelograms | Generalizing the Area of a Triangle

1.07 Off the Grid, Part 2 | Applying the Formula for the Area of a Triangle

1.08 Pile of Polygons | Investigating Polygons and Their Areas

Practice Day 1

Quiz

Sub-Unit 2 Surface Area

1.09 Renata's Stickers | Intro to Surface Area

1.10 Plenty of Polyhedra | Polyhedra and Their Faces

1.11 Nothing But Nets | Nets and Surface Area on a Grid

1.12 Face Value | Surface Area Off of a Grid

1.13 Take It To Go | Surface Area in Context

Practice Day 2

Quiz

End-Unit

End-of-Unit Assessment

Unit 2 Introducing Ratios

In this unit, students are introduced to the concept of ratios; represent ratios using double number line diagrams, tables, and tape diagrams; and use ratio reasoning to solve problems.

Pre-Unit

Pre-Unit Check

Sub-Unit 1 Introducing Ratios

2.01 Pizza Maker | Exploring Ratios

2.02 Ratio Rounds | Describing Ratios

2.03 Rice Ratios | Introduction to Equivalent Ratios

2.04 Fruit Lab | Creating Equivalent Ratios

2.05 Balancing Act | Introduction to Double Number Lines

2.06 Product Prices | Unit Prices

Practice Day 1

Quiz

Sub-Unit 2 Solving Problems with Ratios

2.07 Mixing Paint Part 1 | Comparing Ratios

2.08 World Records | Comparing Speeds

2.09 Disaster Preparation | Using Ratio Tables With Large Quantities

2.10 Balloons | Solving Multistep Ratio Problems

2.11 Community Life | Solving Equivalent Ratio Problems

2.12 Mixing Paint Part 2 | Part-Part-Whole Ratios

2.13 City Planning | Applying Part-Part-Whole Ratio Problems

2.14 Lunch Waste | Applying Ratio Strategies

Practice Day 2

End-Unit

End-of-Unit Assessment

Unit 3 Unit Rates and Percentages

In this unit, students apply the ratio reasoning they learned in Unit 2 to convert between units of measurement, solve problems with unit rates, and make sense of percentages.

Pre-Unit

Pre-Unit Check

Sub-Unit 1 Units and Measurement

3.01 Many Measurements | Everyday Measurements

3.02 Counting Classrooms | Measuring With Different Units

3.03 Pen Pals | Converting Units

Sub-Unit 2 Unit Rates

3.04 Model Trains | Comparing Rates

3.05 Soft Serve | Two Unit Rates

3.06 Welcome to the Robot Factory | Using Unit Rates

3.07 More Soft Serve | Solving Rate Problems

Practice Day 1

Quiz

Sub-Unit 3 Percentages

3.08 Lucky Duckies | Benchmark Percentages

3.09 Bicycle Goals | Friendly Percentages

3.10 What's Missing? | Solving Percentage Problems

3.11 Cost Breakdown | Any Percentage of a Number

3.12 More Bicycle Goals | Unknown Percentages

3.13 A Country as a Village | Applying Ratios, Rates, and Percentages

Practice Day 2

End-Unit

End-of-Unit Assessment

Unit 4 Dividing Fractions

In this unit, students develop multiple strategies for dividing fractions and apply those strategies to solve problems about areas and volumes with fractional dimensions.

Pre-Unit

Pre-Unit Check

Sub-Unit 1 Introduction to Dividing Fractions

4.01 Cookie Cutter | Estimating Quotients
4.02 Making Connections | Representing Division Situations
4.03 Flour Planner | How Many Groups? Part 1
4.04 Flower Planters | How Many in Each Group?

Sub-Unit 2 Dividing Fractions

4.05 Garden Bricks | How Many Groups? Part 2
4.06 Fill the Gap | More or Less Than One Group?
4.07 Break It Down | Using Common Denominators to Divide Fractions
4.08 Potting Soil | Dividing by Unit Fractions
4.09 Division Challenges | Two Strategies for Dividing Fractions
4.10 Swap Meet | Division of Fractions in Context

Practice Day 1

Quiz

Sub-Unit 3 Area and Volume With Fractions

4.11 Classroom Comparisons | Comparing Fractional Lengths
4.12 Puzzling Areas | Areas With Fractional Dimensions
4.13 Volume Challenges | Volumes With Fractional Dimensions
4.14 Planter Planner | Applying Fraction Division

Practice Day 1

End-Unit

End-of-Unit Assessment

Unit 5 Decimal Arithmetic

In this unit, students develop and use a variety of strategies for adding, subtracting, multiplying, and dividing decimals. They also explore the least common multiple and greatest common factor of two numbers.

Pre-Unit

Pre-Unit Check

Sub-Unit 1 Adding and Subtracting Decimals

5.01 Dishing Out Decimals | Reasoning With Decimals
5.02 Decimal Diagrams | Representing Decimals
5.03 Fruit by the Pound | Adding and Subtracting Decimals Part 1
5.04 Missing Digits | Adding and Subtracting Decimals Part 2
Quiz 1

Sub-Unit 2 Multiplying and Dividing Decimals

5.05 Decimal Multiplication | Introduction to Multiplying Decimals
5.06 Multiplying With Areas | Using Area Models to Multiply Decimals
5.07 Multiplication Methods | Multiplying Decimals Using Vertical Calculations
5.08 Division Diagrams | Making Sense of Decimal Division
5.09 Long Division Launch | Long Division and Decimals
5.10 Return of the Long Division | Long Division With Remainders
5.11 Movie Time | Dividing Decimals in Context
Practice Day 1
Quiz 2

Sub-Unit 3 Solving Problems With Decimals

5.12 Budget Vehicles | Operations With Decimals in Context
5.13 Grocery Prices | Percentages as Decimals

Sub-Unit 4 Least Common Multiple and Greatest Common Factor

5.14 Common Multiples | Investigating Least Common Multiples
5.15 Common Factors | Investigating Greatest Common Factor
Practice Day 2

End-Unit

End-of-Unit Assessment

Unit 6 Expressions and Equations

Students discover that the equal sign is more than a prompt, it's also a way to indicate balance — a critical understanding that allows them to move beyond the strictly numeric world and into the realm of algebra.

Pre-Unit

Pre-Unit Check

Sub-Unit 1 Solving Equations

6.01 Weight for It | Reasoning About Unknown Values
6.02 Five Equations | Tape Diagrams, Equations, Contexts
6.03 Hanging Around | Introduction to Balanced Hangers
6.04 Hanging It Up | Solving Equations
6.05 Swap and Solve | Solving Equations in Context

Sub-Unit 2 Equivalent Expressions

6.06 Vari-apples | Introduction to Variable Expressions
6.07 Border Tiles | Equivalent Expressions
6.08 Products and Sums | Distributive Property Part 1
6.09 Products, Sums, and Differences | Distributive Property Part 2

Practice Day 1

Quiz

Sub-Unit 3 Exponents

6.10 Powers | What Are Exponents?
6.11 Exponent Expressions | Exponents and Order of Operations
6.12 Squares and Cubes | Exponent Expressions With Variables

Sub-Unit 4 Introduction to Representing Relationships

6.13 Turtles All the Way | Stories and Tables and Variables, Oh My!
6.14 Representing Relationships | Interpreting Graphs of Relationships
6.15 Connecting Representations | Tables, Equations, and Graphs of Relationships
6.16 Subway Fares | Applying Relationships

Practice Day 2

End-Unit

End-of-Unit Assessment

Unit 7 Positive and Negative Numbers

In this unit, students explore positive and negative numbers in several contexts: on a number line, represented as inequalities, and in the coordinate plane.

Pre-Unit

Pre-Unit Check

Sub-Unit 1 Negative Numbers and Absolute Values

7.01 Can You Dig It? | Positive and Negative Numbers

7.02 Digging Deeper | Positions on the Number Line

7.03 Order in the Class | Comparing Numbers

7.04 Sub-Zero | Positive and Negative Numbers in Context

7.05 Distance on the Number Line | Introduction to Absolute Value

Practice Day 1

Quiz 2

Sub-Unit 2 Inequalities

7.06 Tunnel Travels | Graphing Inequalities

7.07 Comparing Weights | Writing Inequalities

7.08 Shira's Solutions | Solutions to Inequalities

Sub-Unit 3 The Coordinate Plane

7.09 Sand Dollar Search | Points in the Coordinate Plane

7.10 The A-maze-ing Coordinate Plane | Practice Plotting

7.11 Polygon Maker | Polygons in the Plane

7.12 Graph Telephone | The Coordinate Plane in Context

Practice Day 2

End-Unit

End-of-Unit Assessment

Unit 8 Describing Data

In this unit, students visualize data using dot plots, histograms, and box plots, as well as calculate measures of center and spread.

Pre-Unit

Pre-Unit Check

Sub-Unit 1 Visualizing Data

8.01 Screen Time | Asking Questions, Collecting Data, and Making Claims

8.02 Dot Plots | Visualizing Data With Dot Plots

8.03 Minimum Wage | Creating Dot Plots

8.04 Lots More Dots | Comparing Dot Plots

8.05 The Plot Thickens | Introduction to Histograms

8.06 DIY Histograms | Creating Histograms

Sub-Unit 2 Measuring Data: Mean and MAD

8.07 Snack Time | Introduction to the Mean

8.08 Pop It! | Sum of the Deviations

8.09 Hoops | Mean Absolute Deviation

8.10 Hollywood Part 1 | Using Mean and MAD to Analyze Actor Salaries

Practice Day 1

Quiz

Sub-Unit 3 Measuring Data: Median and IQR

8.11 Toy Cars | Introducing the Median

8.12 In the News | Comparing Measures of Center

8.13 Pumpkin Patch | Introduction to Quartiles

8.14 Car, Plane, Bus, or Train? | Box Plots, IQR, and Range

8.15 Hollywood Part 2 | Comparing Box Plots

8.16 Hollywood Part 3 | Using Statistics to Analyze Movies

Practice Day 2

End-Unit

End-of-Unit Assessment

New York State Next Generation Mathematics Learning Standards, Grade 6

The following shows the alignment of Amplify Desmos Math to the New York State Next Generation Mathematics Learning Standards for Grade 6 Mathematics.

NY-6.RP	**Ratios and Proportional Relationships**	**Lesson(s)**
Understand ratio concepts and use ratio reasoning to solve problems.		
NY-6.RP.1 *CCSS: 6.RP.A.1*	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	6.2.02, 6.2.03, 6.2.04, 6.2 Practice Day 1, 6.6.06, 6.6.15, 6.6.16
NY-6.RP.2 *CCSS: 6.RP.A.2*	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.	6.2.06, 6.2.08, 6.3.04, 6.3.05, 6.3.06, 6.3.07, 6.3 Practice Day 1, 6.3 Practice Day 2, 6.6.06, 6.6.15, 6.6.16
NY-6.RP.3 *CCSS: 6.RP.A.3*	Use ratio and rate reasoning to solve real-world and mathematical problems.	6.2.06, 6.2 Practice Day 1, 6.2.07, 6.2.08, 6.2.09, 6.2.10, 6.2.11, 6.2.12, 6.2.13, 6.2.14, 6.2 Practice Day 2, 6.3.05, 6.3.06, 6.3.07, 6.3 Practice Day 1, 6.3.08, 6.3.09, 6.3.10, 6.3.11, 6.3.12, 6.3.13, 6.3 Practice Day 2, 6.6.06, 6.6.15, 6.6.16, 6.6 Practice Day 2
NY-6.RP.3a *CCSS: 6.RP.A.3.A*	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	6.2.05, 6.2 Practice Day 1, 6.2.07, 6.2.08, 6.2.09, 6.2.10, 6.2.11, 6.2 Practice Day 2, 6.6.06, 6.6.15, 6.6.16, 6.6 Practice Day 2
NY-6.RP.3b *CCSS: 6.RP.A.3.B*	Solve unit rate problems.	6.2.06, 6.2 Practice Day 1, 6.2.08, 6.2 Practice Day 2, 6.3.04, 6.3.05, 6.3.06, 6.3.07, 6.3 Practice Day 1, 6.3 Practice Day 2, 6.5.12
NY-6.RP.3c *CCSS: 6.RP.A.3.C*	Find a percent of a quantity as a rate per 100. Solve problems that involve finding the whole given a part and the percent, and finding a part of a whole given the percent.	6.3.08, 6.3.09, 6.3.10, 6.3.11, 6.3.12, 6.3.13, 6.3 Practice Day 2, 6.5.13 **Alignment note:** NYS added finding a part of a whole given the percent. This concept is addressed in the indicated lessons.
NY-6.RP.3d *CCSS: 6.RP.A.3.D*	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	6.3.02, 6.3.03, 6.3.04, 6.3 Practice Day 1, 6.3 Practice Day 2

NY-6.NS	The Number System	Lesson(s)
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.		
NY-6.NS.1 *CCSS 6.NS.A.1*	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions.	6.4.03, 6.4.04, 6.4.05, 6.4.06, 6.4.07, 6.4.08, 6.4.09, 6.4.10, 6.4 Practice Day 1, 6.4.11, 6.4.12, 6.4.14, 6.4 Practice Day 2
Compute fluently with multi-digit numbers and find common factors and multiples.		
NY-6.NS.2 *CCSS 6.NS.B.2*	Fluently divide multi-digit numbers using a standard algorithm.	6.5.09, 6.5.10, 6.5.11, 6.5 Practice Day 1, 6.5 Practice Day 2
NY-6.NS.3 *CCSS 6.NS.B.3*	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	6.5.02, 6.5.03, 6.5.04, 6.5.05, 6.5.06, 6.5.07, 6.5.08, 6.5.10, 6.5.11, 6.5 Practice Day 1, 6.5.12, 6.5.13, 6.5 Practice Day 2, 6.6.04
NY-6.NS.4 *CCSS 6.NS.B.4*	Find the greatest common factor of two whole numbers less than or equal to 100. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor other than 1. Find the least common multiple of two whole numbers less than or equal to 12.	6.5.14, 6.5.15, 6.5 Practice Day 2
Apply and extend previous understandings of numbers to the system of rational numbers.		
NY-6.NS.5 *CCSS: 6.NS.C.5*	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	6.7.01, 6.7.04, 6.7 Practice Day 1, 6.7 Practice Day 2
NY-6.NS.6 *CCSS: 6.NS.C.6*	Understand a rational number as a point on the number line. Use number lines and coordinate axes to represent points on a number line and in the coordinate plane with negative number coordinates.	6.7.02, 6.7 Practice Day 1, 6.7 Practice Day 2
NY-6.NS.6a *CCSS: 6.NS.C.6.A*	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line. Recognize that the opposite of the opposite of a number is the number itself, and that 0 is its own opposite.	6.7.02, 6.7.03, 6.7 Practice Day 1, 6.7 Practice Day 2
NY-6.NS.6b *CCSS: 6.NS.C.6.B*	Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane. Recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	6.7.09, 6.7.10, 6.7 Practice Day 2
NY-6.NS.6c *CCSS: 6.NS.C.6.C*	Find and position integers and other rational numbers on a horizontal or vertical number line. Find and position pairs of integers and other rational numbers on a coordinate plane.	6.7.02, 6.7.03, 6.7 Practice Day 1, 6.7.09, 6.7.10, 6.7.12, 6.7 Practice Day 2

New York State Next Generation Mathematics Learning Standards, Grade 6

NY-6.NS.7 *CCSS: 6.NS.C.7*	Understand ordering and absolute value of rational numbers.	6.7.03, 6.7.05, 6.7 Practice Day 1
NY-6.NS.7a *CCSS: 6.NS.C.7.A*	Interpret statements of inequality as statements about the relative position of two numbers on a number line.	6.7.03, 6.7.05, 6.7 Practice Day 1
NY-6.NS.7b *CCSS: 6.NS.C.7.B*	Write, interpret, and explain statements of order for rational numbers in real-world contexts.	6.7.04, 6.7 Practice Day 1, 6.7.06, 6.7 Practice Day 2
NY-6.NS.7c *CCSS: 6.NS.C.7.C*	Understand the absolute value of a rational number as its distance from 0 on the number line. Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.	6.7.05, 6.7 Practice Day 1
NY-6.NS.7d *CCSS: 6.NS.C.7.D*	Distinguish comparisons of absolute value from statements about order.	6.7.05, 6.7 Practice Day 1
NY-6.NS.8 *CCSS: 6.NS.C.8*	Solve real-world and mathematical problems by graphing points on a coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	6.7.10, 6.7.11, 6.7.12, 6.7 Practice Day 2

NY-6.EE	**Expressions, Equations, and Inequalities**	**Lesson(s)**
Apply and extend previous understandings of arithmetic to algebraic expressions.		
NY-6.EE.1 *CCSS: 6.EE.A.1*	Write and evaluate numerical expressions involving whole number exponents.	6.6.10, 6.6.11, 6.6 Practice Day 2
NY-6.EE.2 *CCSS: 6.EE.A.2*	Write, read, and evaluate expressions in which letters stand for numbers.	6.6.06, 6.6.12
NY-6.EE.2a *CCSS: 6.EE.A.2.A*	Write expressions that record operations with numbers and with letters standing for numbers.	6.1.03, 6.1.06, 6.1 Practice Day 1, 6.1 Practice Day 2, 6.4.13, 6.6.06, 6.6.08, 6.6.09, 6.6 Practice Day 1, 6.6 Practice Day 2
NY-6.EE.2b *CCSS 6.EE.A.2.B*	Identify parts of an expression using mathematical terms (term, coefficient, sum, difference, product, factor, and quotient); view one or more parts of an expression as a single entity.	6.6.08, 6.6.09, 6.6 Practice Day 1 **Alignment note:** NYS added "difference," which is addressed in 6.6.09.
NY-6.EE.2c *CCSS: 6.EE.A.2.C*	Evaluate expressions given specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order (Order of Operations).	6.1.04, 6.1.07, 6.1 Practice Day 1, 6.1 Practice Day 2, 6.6.11, 6.6.12, 6.6 Practice Day 2
NY-6.EE.3 *CCSS: 6.EE.A.3*	Apply the properties of operations to generate equivalent expressions.	6.6.07, 6.6.08, 6.6.09, 6.6 Practice Day 1
NY-6.EE.4 *CCSS: 6.EE.A.4*	Identify when two expressions are equivalent.	6.6.07, 6.6.08, 6.6.09, 6.6 Practice Day 1, 6.6 Practice Day 2

Reason about and solve one-variable equations and inequalities.		
NY-6.EE.5 *CCSS: 6.EE.B.5*	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	6.6.02, 6.6.03, 6.6.04, 6.6.05, 6.6 Practice Day 1, 6.6 Practice Day 2, 6.7.08
NY-6.EE.6 *CCSS: 6.EE.B.6*	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. Understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	6.6.01, 6.6.03, 6.6.04, 6.6.05, 6.6.06, 6.6.16, 6.6 Practice Day 1, 6.6 Practice Day 2, 6.7.07, 6.7.08, 6.7 Practice Day 2
NY-6.EE.7 *CCSS: 6.EE.B.7*	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$; $x - p = q$; $px = q$; and $\frac{x}{p} = q$ for cases in which p, q, and x are all nonnegative rational numbers.	6.6.01, 6.6.02, 6.6.03, 6.6.04, 6.6.05, 6.6 Practice Day 1, 6.6 Practice Day 2 **Alignment note:** NYS added subtraction and division equations, which are present in 6.6.03, 6.6.05, and 6.6 Practice Day 2.
NY-6.EE.8 *CCSS 6.EE.B.8*	Write an inequality of the form $x > c$, $x \geq c$, $x \leq c$, or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of these forms have infinitely many solutions; represent solutions of such inequalities on a number line.	6.7.06, 6.7.07, 6.7.08, 6.7 Practice Day 2, 7.6.13 **Alignment note:** NYS added nonstrict inequalities, which are addressed in 7.6.13.
Represent and analyze quantitative relationships between dependent and independent variables.		
NY-6.EE.9 *CCSS 6.EE.C.9*	Use variables to represent two quantities in a real-world problem that change in relationship to one another. Given a verbal context and an equation, identify the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	6.6.13, 6.6.14, 6.6.15, 6.6.16, 6.6 Practice Day 2

New York State Next Generation Mathematics Learning Standards, Grade 6

NY-6.G	**Geometry**	**Lesson(s)**
Solve real-world and mathematical problems involving area, surface area, and volume.		
NY-6.G.1 *CCSS: 6.G.A.1*	Find area of triangles, trapezoids, and other polygons by composing into rectangles or decomposing into triangles and quadrilaterals. Apply these techniques in the context of solving real-world and mathematical problems.	6.1.01, 6.1.02, 6.1.03, 6.1.04, 6.1.05, 6.1.06, 6.1.07, 6.1.08, 6.1 Practice Day 1, 6.1 Practice Day 2, 6.6.11, 6.6.12, 6.6 Practice Day 2 **Alignment note:** NYS adds specific mention of trapezoids, which are addressed in 6.1.01, 6.1.02, and 6.1.08.
NY-6.G.2 *CCSS: 6.G.A.2*	Find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	6.4.13, 6.4.14, 6.4 Practice Day 2, 6.6.12
NY-6.G.3 *CCSS: 6.G.A.3*	Draw polygons in the coordinate plane given coordinates for the vertices. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	6.7.11
NY-6.G.4 *CCSS: 6.G.A.4*	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	6.1.11, 6.1.12, 6.1.13, 6.1 Practice Day 2
NY-6.G.5 *CCSS: part of 8.EE.A.2*	Use area and volume models to explain perfect squares and perfect cubes.	6.6.12, 8.8.01, 8.8.02, 8.8.03, 8.8.04, 8.8.05, 8.8 Practice Day 1, 8.8.10, 8.8 Practice Day 2, 8.8.14 **Alignment note:** NY adds area and volume models to the part of CCSS 8.EE.A.2 that mentions perfect squares and perfect cubes. Area and volume models are addressed in the indicated lessons.

NY-6.SP	Statistics and Probability	Lesson(s)
Develop understanding of statistical variability.		
NY-6.SP.1a *CCSS: 6.SP.A.1*	Recognize that a statistical question is one that anticipates variability in the data related to the question and accounts for it in the answers.	6.8.02, 6.8.06, 6.8.10, 6.8.16
NY-6.SP.1b *CCSS: 7.SP.A.1*	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population.	7.8.10, 7.8.11, 7.8.12, 7.8.15, 7.8 Practice Day 2
NY-6.SP.1c *CCSS: 7.SP.A.2*	Understand that the method and sample size used to collect data for a particular question is intended to reduce the difference between a population and a sample taken from the population so valid inferences can be drawn about the population. Generate multiple samples (or simulated samples) of the same size to recognize the variation in estimates or predictions.	7.8.12, 7.8.13, 7.8.15, 7.8 Practice Day 2
NY-6.SP.2 *CCSS: 6.SP.A.2*	Understand that a set of quantitative data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	6.8.04, 6.8.06, 6.8.09, 6.8 Practice Day 1, 6.8.11, 6.8.14, 6.8.15, 6.8 Practice Day 2
NY-6.SP.3 *CCSS: 6.SP.A.3*	Recognize that a measure of center for a quantitative data set summarizes all of its values with a single number while a measure of variation describes how its values vary with a single number.	6.8.07, 6.8.08, 6.8.09, 6.8.10, 6.8 Practice Day 1, 6.8.13, 6.8.14, 6.8.15, 6.8.16, 6.8 Practice Day 2
Summarize and describe distributions.		
NY-6.SP.4 *CCSS: 6.SP.B.4*	Display quantitative data in plots on a number line, including dot plots, and histograms.	6.8.02, 6.8.03, 6.8.04, 6.8.05, 6.8.06, 6.8 Practice Day 1, 6.8.11, 6.8.12, 6.8 Practice Day 2
NY-6.SP.5 *CCSS: 6.SP.B.5*	Summarize quantitative data sets in relation to their context.	6.8.10, 6.8 Practice Day 1, 6.8.12, 6.8.16, 6.8 Practice Day 2
NY-6.SP.5a *CCSS: 6.SP.B.5.A*	Report the number of observations.	6.8.02, 6.8.05, 6.8 Practice Day 1, 6.8.16
NY-6.SP.5b *CCSS: 6.SP.B.5.B*	Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.	6.8.01, 6.8.02, 6.8 Practice Day 1, 6.8.15, 6.8.16, 6.8 Practice Day 2
NY-6.SP.5c *CCSS: 6.SP.B.5.C*	Calculate range and measures of center, as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	6.8.07, 6.8 Practice Day 1, 6.8.11, 6.8.12, 6.8.13, 6.8.14, 6.8.16, 6.8 Practice Day 2
NY-6.SP.5d *CCSS: 6.SP.B.5.D*	Relate the range and the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.	6.8.12, 6.8.16

New York State Next Generation Mathematics Learning Standards, Grade 6

Investigate chance processes and develop, use, and evaluate probability models.		
NY-6.SP.6 *CCSS: 7.SP.C.5*	Understand that the probability of a chance event is a number between 0 and 1 inclusive, that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	7.8.02, 7.8.03, 7.8 Practice Day 1, 7.8 Practice Day 2
NY-6.SP.7 *CCSS: 7.SP.C.6*	Approximate the probability of a simple event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	7.8.02, 7.8.03, 7.8.04, 7.8.05, 7.8 Practice Day 1, 7.8 Practice Day 2 **Alignment note:** NYS changes "chance event" to "simple event." Simple events are addressed in the indicated lessons.
NY-6.SP.8 *CCSS: 7.SP.C.7*	Develop a probability model and use it to find probabilities of simple events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.	7.8.05, 7.8 Practice Day 1, 7.8 Practice Day 2
NY-6.SP.8a *CCSS: 7.SP.C.7.A*	Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of simple events.	7.8.02, 7.8 Practice Day 1, 7.8 Practice Day 2
NY-6.SP.8b *CCSS: 7.SP.C.7.B*	Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	7.8.04, 7.8.05, 7.8 Practice Day 1

The Standards for Mathematical Practice, New York State Next Generation Mathematics Learning Standards, Grade 6

The following shows the alignment of Amplify Desmos Math, Grade 6, to the Standards for Mathematical Practice for the New York State Next Generation Mathematics Learning Standards.

MP1 Make sense of problems and persevere in solving them. ***CCSS: MP1***	**Lesson(s)**
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	6.2.08, 6.2.11, 6.2.12, 6.3.08, 6.4.04, 6.4.10, 6.4.12, 6.6.02, 6.6.05, 6.7.05, 6.7.07, 6.7.11, 6.8.14
MP2 Reason abstractly and quantitatively. ***CCSS: MP2***	
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	6.2.07, 6.2.09, 6.3.04, 6.3.05, 6.3.09, 6.4.02, 6.4.08, 6.4.10, 6.6.02, 6.6.05, 6.6.06, 6.6.09, 6.6.11, 6.6.15, 6.6.16, 6.7.03, 6.7.04, 6.7.06, 6.7.07, 6.7.08, 6.7.12, 6.8.04, 6.8.08, 6.8.09, 6.8.11, 6.8.12
MP3 Construct viable arguments and critique the reasoning of others. ***CCSS: MP3***	
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	6.1.08, 6.1.10, 6.1.11, 6.2.01, 6.2.04, 6.2.07, 6.2.10, 6.2.11, 6.3.02, 6.3.05, 6.4.01, 6.4.06, 6.4.07, 6.5.01, 6.5.04, 6.5.06, 6.5.10, 6.5.14, 6.6.06, 6.6.07, 6.6.08, 6.6.10, 6.6.12, 6.6.13, 6.6.14, 6.6.15, 6.7.01, 6.7.04, 6.7.05, 6.7.08, 6.7.09, 6.7.10, 6.8.01, 6.8.02, 6.8.04, 6.8.05, 6.8.06, 6.8.07, 6.8.08, 6.8.10, 6.8.11, 6.8.12, 6.8.13, 6.8.15, 6.8.16

The Standards for Mathematical Practice, New York State Next Generation Mathematics Learning Standards, Grade 6

MP4 Model with mathematics. ***CCSS: MP4***	**Lesson(s)**
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	6.1.13, 6.2.09, 6.2.14, 6.4.14, 6.5.12, 6.6.16, 6.8.10, 6.8.14, 6.8.15, 6.8.16
MP5 Use appropriate tools strategically. ***CCSS: MP5***	
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	6.1.03, 6.1.05, 6.2.10, 6.2.11, 6.3.03, 6.3.10, 6.4.10, 6.5.10, 6.6.04
MP6 Attend to precision. ***CCSS: MP6***	
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	6.1.02, 6.1.08, 6.1.10, 6.1.11, 6.1.12, 6.2.02, 6.2.05, 6.2.08, 6.2.12, 6.3.01, 6.3.02, 6.3.03, 6.3.04, 6.3.09, 6.3.13, 6.4.05, 6.4.11, 6.5.03, 6.5.07, 6.5.15, 6.6.03, 6.6.12, 6.6.14, 6.7.01, 6.7.05, 6.7.08, 6.8.03, 6.8.06, 6.8.07, 6.8.08, 6.8.09, 6.8.13

MP7 Look for and make use of structure. ***CCSS: MP7***	**Lesson(s)**
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as *5 minus a positive number times a square* and use that to realize that its value cannot be more than 5 for any real numbers x and y.	6.1.01, 6.1.02, 6.1.09, 6.2.01, 6.2.03, 6.3.05, 6.3.06, 6.3.10, 6.4.01, 6.4.02, 6.4.06, 6.4.07, 6.5.02, 6.5.03, 6.5.05, 6.5.08, 6.6.03, 6.6.07, 6.6.08, 6.6.10, 6.6.11, 6.6.12, 6.7.02, 6.7.06, 6.7.09, 6.7.10, 6.7.11
MP8 Look for and express regularity in repeated reasoning. ***CCSS: MP8***	
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	6.1.03, 6.1.06, 6.1.07, 6.2.03, 6.2.04, 6.3.07, 6.3.08, 6.3.11, 6.4.08, 6.5.04, 6.5.05, 6.5.11, 6.6.09, 6.6.10, 6.8.07, 6.8.13

GRADE 6

Amplify Desmos Math
NEW YORK

Teacher Edition Sample Lessons

In this section, two lesson samples showcase the unique design and ease of use of lesson plans found in the Amplify Desmos Math New York Teacher Edition. All Teacher Edition lessons will be created following this structure and design for the 2024-2025 school year.

Contents of this lesson:

- **Teacher Edition Overview**
- **Lesson 3.04: Model Trains**
  Comparing Rates
- **Lesson 6.16: Subway Fares**
  Applying Relationships

Powerful learning experiences with the flexibility you need in the classroom.

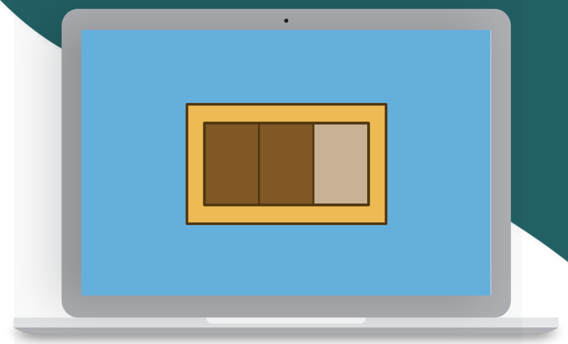
Every lesson in Amplify Desmos Math New York can be taught with students using print while the teacher projects digital Presentation Screens. For lessons that are best taught with students on devices, we make a clear recommendation and provide instructional guidance to support students using digital and on print. The robust collaboration tools, interactive visuals, and enriched feedback of Desmos technology are integral to daily learning as a whole class, pairs, or individuals.

Print and digital resources for every lesson!

Two types of lesson delivery

Digital Lesson RECOMMENDED

Assign Student Activity Screens for this lesson so students can explore visual potting soil models!



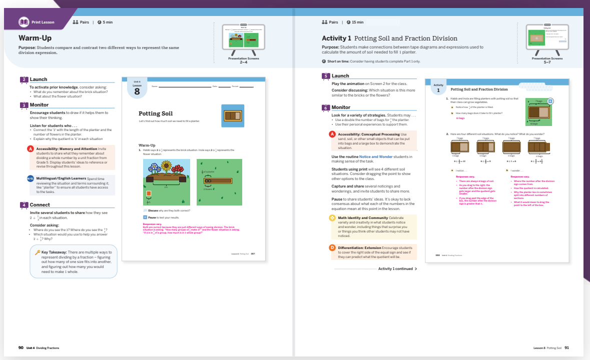
Digital recommended

Lesson goals best learned digitally

- Students use devices and interact with Student Activity Screens.
- Teachers present the Student Activity Screens to facilitate the lesson.
- Closely-aligned student print pages are available for off-line note taking or for students who may need to use print.
- About 75% of lessons

Print Lesson

Assign the Student Edition with Presentation Screens for this lesson.



Print

Lesson goals best learned with pencil-and-paper

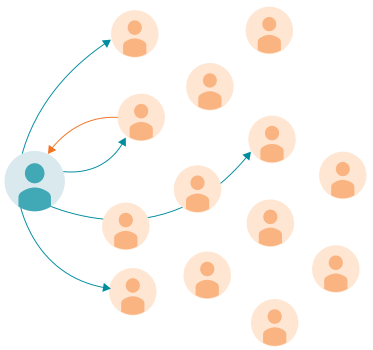
- Students interact with printed Student Edition pages and hands-on manipulatives (when applicable).
- Teachers present Presentation Screens to facilitate the lesson.
- Students must use consumable Student Edition.
- About 25% of lessons

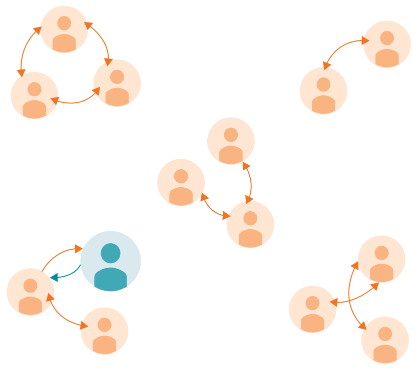
REVIEWER TIP

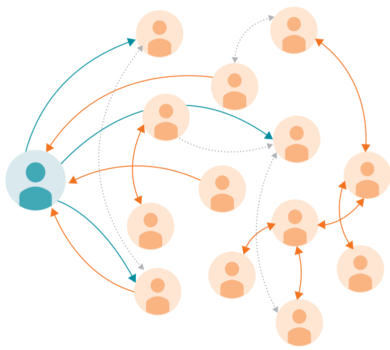
These print/digital flexibility enhancements can be found in Amplify Desmos Math New York lessons in this section and online, but are not yet available in the partially designed lesson plans.

Activities are designed to provide collaborative learning experiences.

Following a *Warm-Up*, a lesson includes two or more learning activities. All activities in Amplify Desmos Math New York utilize a *Launch, Monitor, Connect* framework to surface student thinking and spark interesting and productive discussions.







1

Launch

Teachers launch an activity and ensure students understand what's being asked. Launches are designed to ensure all students can access and engage with the problem.

2

Monitor

Students interact with each other to discuss and work out strategies for solving a problem. Teachers can better understand what their students are thinking so that they can choose their next move while students are working.

3

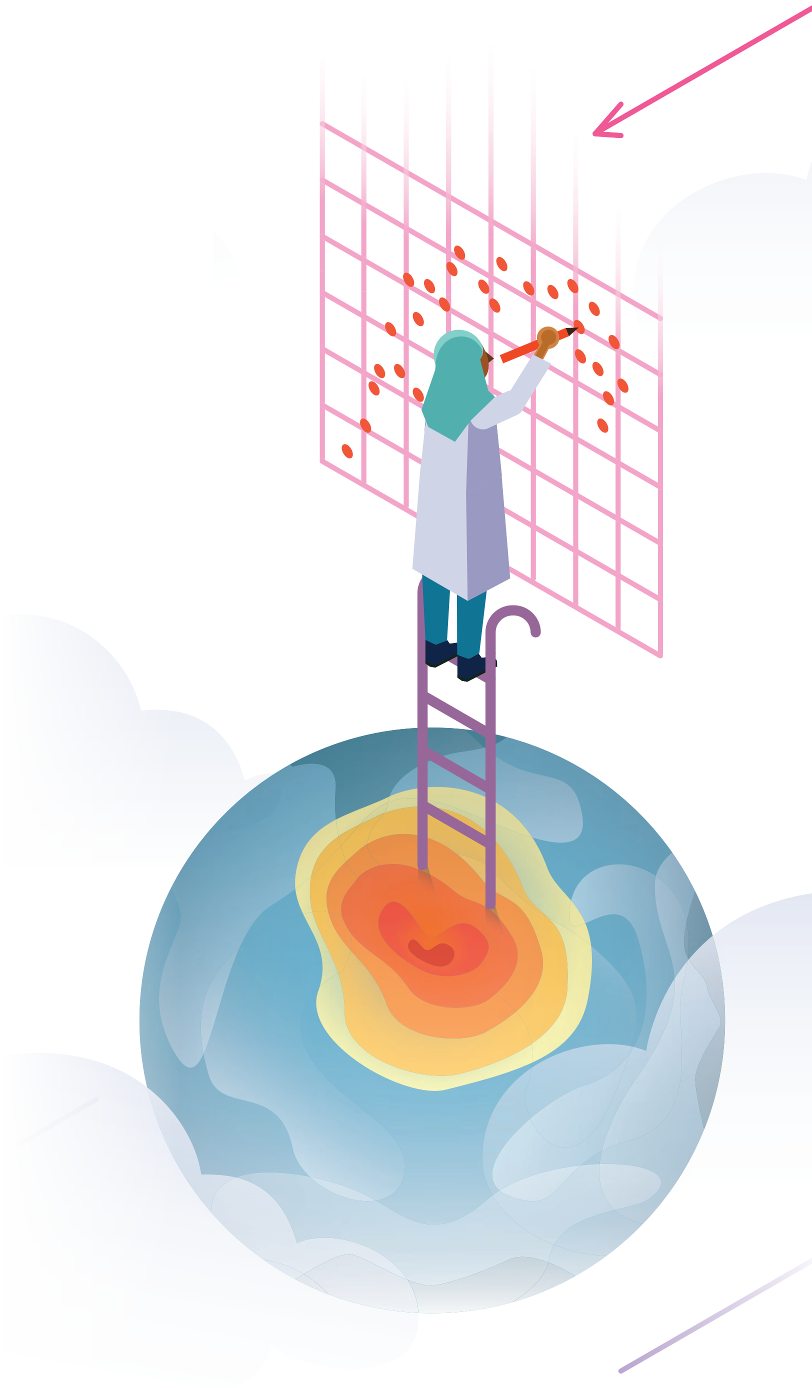
Connect

Students construct viable arguments and critique each other's reasoning.Then, at the end of the activity, they synthesize their learning with the teacher in a moment called the *Key Takeaway*.

Following all activities, each lesson wraps with *Synthesis and Summary* to consolidate thinking and refine strategies across activities. An *Exit Ticket* enables students to share how well they understood the math of the lesson and how they felt about learning that math.

Every moment in the classroom is valuable.

Teachers play an active role as discussion facilitators, monitoring student work in real time, choosing moments to share and discuss, and synthesizing learning.



At Amplify, we want teachers to spend their time focused on their students, rather than preparing instruction and managing materials. Our comprehensive Teacher Edition and intuitive technology is designed with busy educators in mind.

Inside the Amplify Desmos Math New York Teacher Edition, you'll find:

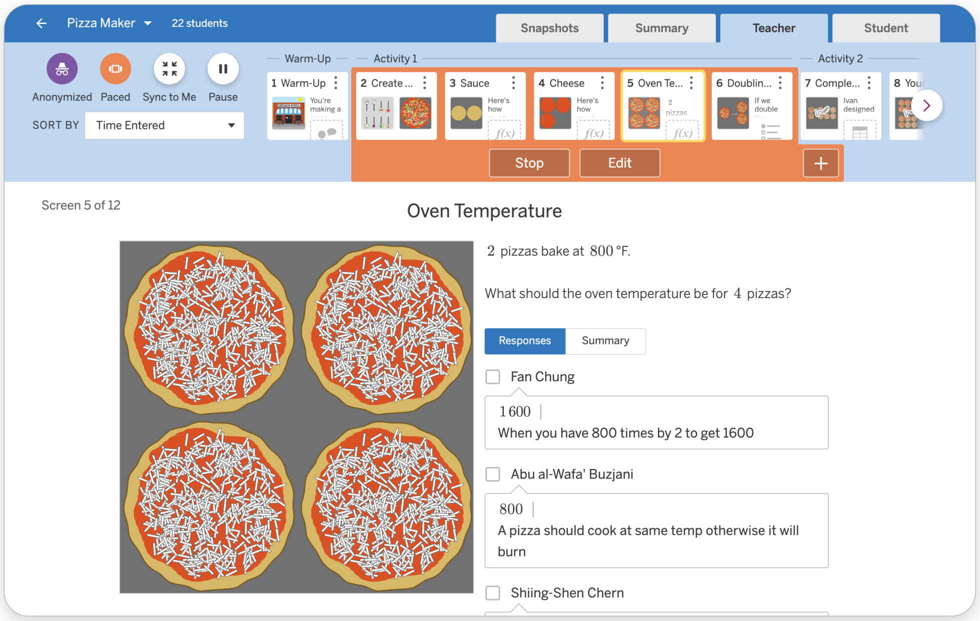
- ***Unit at a Glance*** and ***Lesson at a Glance*** sections to quickly understand what to expect from a unit or lesson.
- ***Focus and Coherence*** information to connect today's goals to prior and future learning.
- A ***Prep Checklist*** to prepare materials for the day's lesson.
- ***Suggested pacing*** to allot the appropriate amount of time for each activity.
- ***Visuals of student pages and screens*** to streamline lesson planning.
- ***Practice problem item analysis*** to easily map learning to Depth of Knowledge (DOK) levels.

REVIEWER TIP

These time-saving enhancements can be found in Amplify Desmos Math New York lessons in this section and online, but are not yet available in the partially designed lesson plans.

Teacher facilitation tools enable dynamic interactions.

The teacher dashboard gives you insight into student thinking in real time, meaning you can select student work to display and discuss quickly and easily, and ask better questions to guide more productive discussions.



Screen 5 of 12

Oven Temperature

2 pizzas bake at 800 °F.

What should the oven temperature be for 4 pizzas?

☐ Fan Chung

1600

When you have 800 times by 2 to get 1600

☐ Abu al-Wafa' Buzjani

800

A pizza should cook at same temp otherwise it will burn

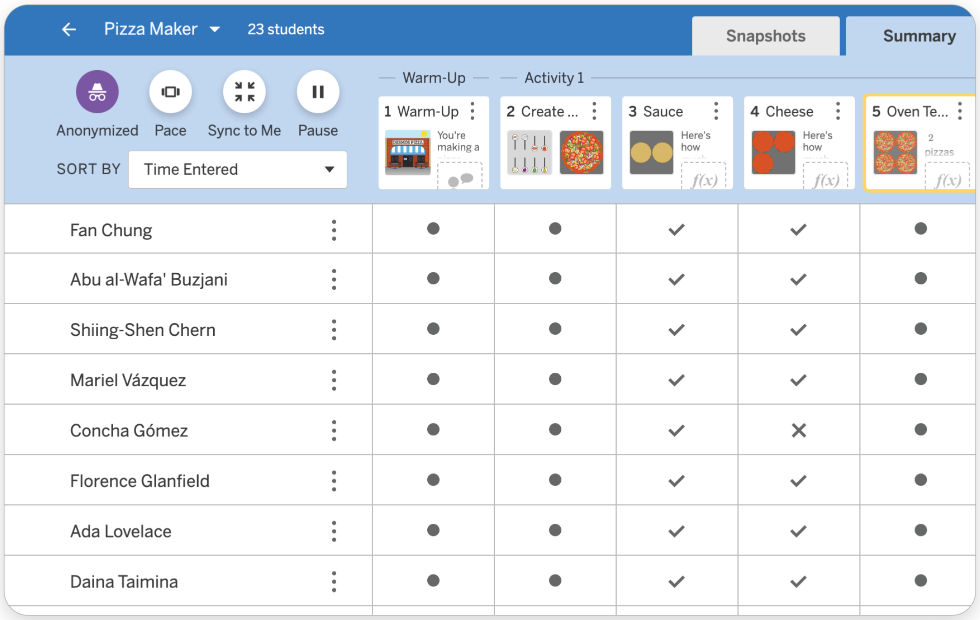
☐ Shiing-Shen Chern

Teacher view and pacing

The Teacher view gives you access to student responses, student-facing content, teacher moves, and sample responses, as well as the ability to pace screens.

Summary view

The Summary view shows you where students are working. If a question is auto-scored it shows how they are doing, and the ability to look at individual student work.



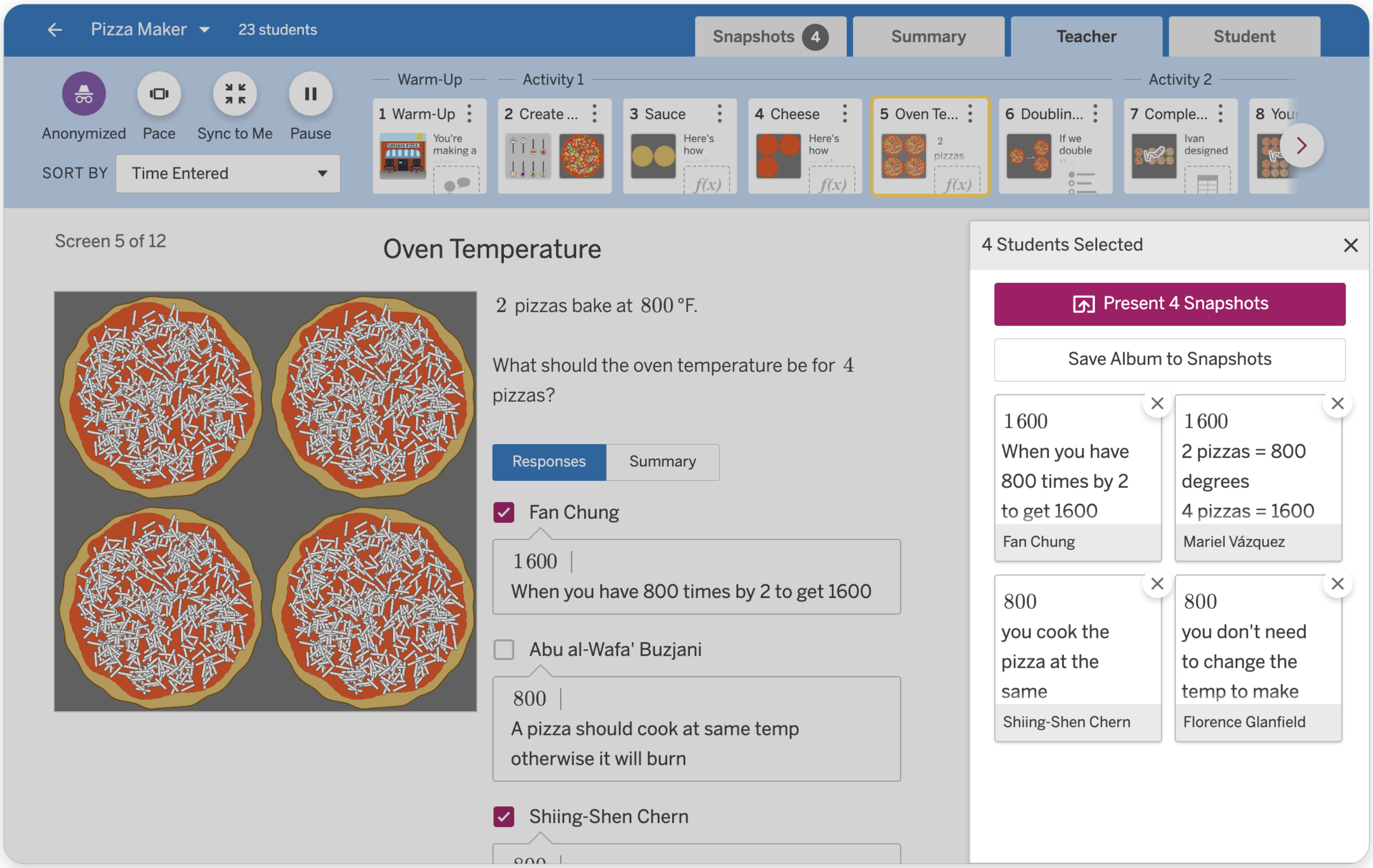
	1 Warm-Up	2 Create ...	3 Sauce	4 Cheese	5 Oven Te...
Fan Chung	●	●	✓	✓	●
Abu al-Wafa' Buzjani	●	●	✓	✓	●
Shiing-Shen Chern	●	●	✓	✓	●
Mariel Vázquez	●	●	✓	✓	●
Concha Gómez	●	●	✓	✗	●
Florence Glanfield	●	●	✓	✓	●
Ada Lovelace	●	●	✓	✓	●
Daina Taimina	●	●	✓	✓	●

Snapshots

When you find student work you want to share, you can collect it in your snapshots and then show individual or even groups of students' responses to move the conversation in the direction you want. Names can be anonymized to protect students' identity.

TRY IT OUT

Start your review at **amplify.com/math-review-nyc**



Screen 5 of 12

Oven Temperature

2 pizzas bake at 800 °F.

What should the oven temperature be for 4 pizzas?

☑ Fan Chung

1 600

When you have 800 times by 2 to get 1600

☐ Abu al-Wafa' Buzjani

800

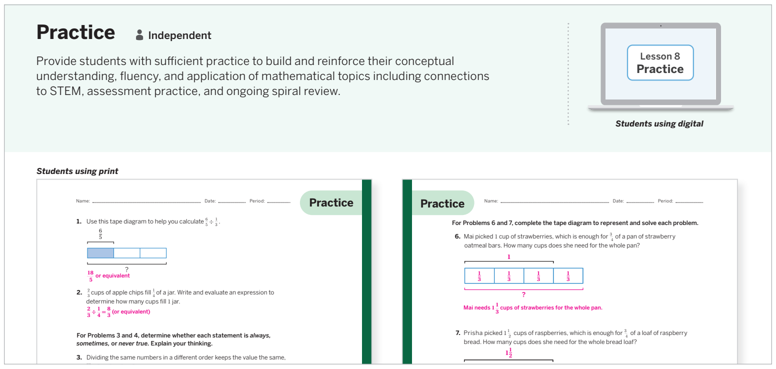
A pizza should cook at same temp otherwise it will burn

☑ Shiing-Shen Chern

1 600 When you have 800 times by 2 to get 1600 Fan Chung	1 600 2 pizzas = 800 degrees 4 pizzas = 1600 Mariel Vázquez
800 you cook the pizza at the same Shiing-Shen Chern	800 you don't need to change the temp to make Florence Glanfield

Practice makes progress.

When it comes to cementing new learning into long-term understanding, ample practice opportunities are key. Amplify Desmos Math New York builds practice opportunities into both daily instruction and independent practice.



Practice Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics including connections to STEM, assessment practice, and ongoing spiral review.

Lesson 8
Practice

Students using digital

Students using print

Practice

Name: Date: Period:

1. Use this tape diagram to help you calculate $\frac{6}{5} \div \frac{1}{3}$.

$\frac{18}{5}$ or equivalent

2. $\frac{2}{3}$ cups of apple chips fill $\frac{1}{4}$ of a jar. Write and evaluate an expression to determine how many cups fill 1 jar.

$\frac{2}{3} \div \frac{1}{4} = \frac{8}{3}$ (or equivalent)

For Problems 3 and 4, determine whether each statement is *always*, *sometimes*, or *never true*. Explain your thinking.

3. Dividing the same numbers in a different order keeps the value the same.

Practice

Name: Date: Period:

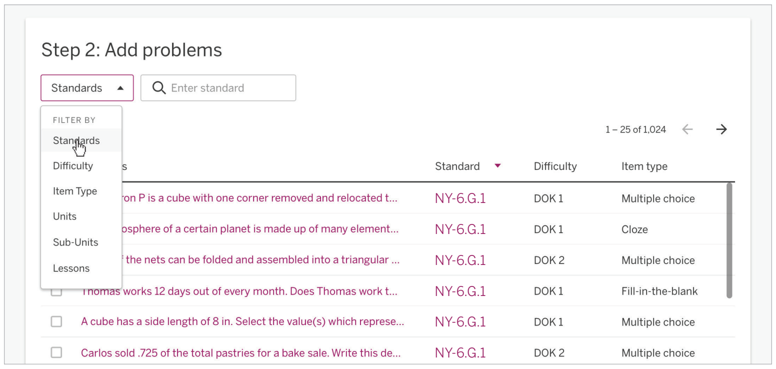
For Problems 6 and 7, complete the tape diagram to represent and solve each problem.

6. Mai picked 1 cup of strawberries, which is enough for $\frac{3}{4}$ of a pan of strawberry oatmeal bars. How many cups does she need for the whole pan?

Mai needs $1\frac{1}{3}$ cups of strawberries for the whole pan.

7. Prisha picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a loaf of raspberry bread. How many cups does she need for the whole bread loaf?

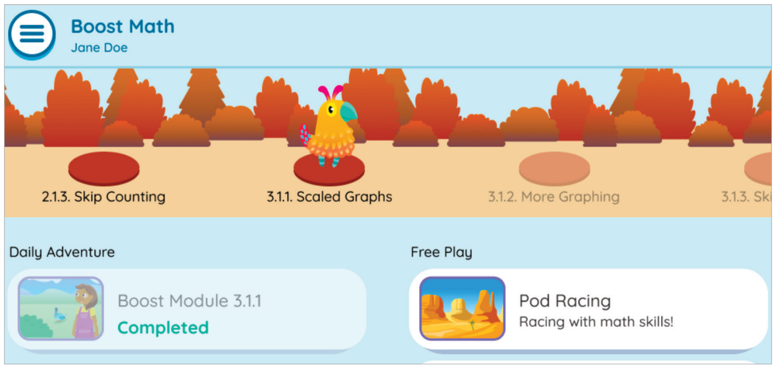
Daily practice problems for the day's lesson are included online and in the Student Edition, including fluency and test practice. This daily practice also includes *Spiral Review* to revisit formerly acquired math learning. A Depth of Knowledge (DOK) table is provided for practice problem item analysis and further insight into how students are doing conceptually.



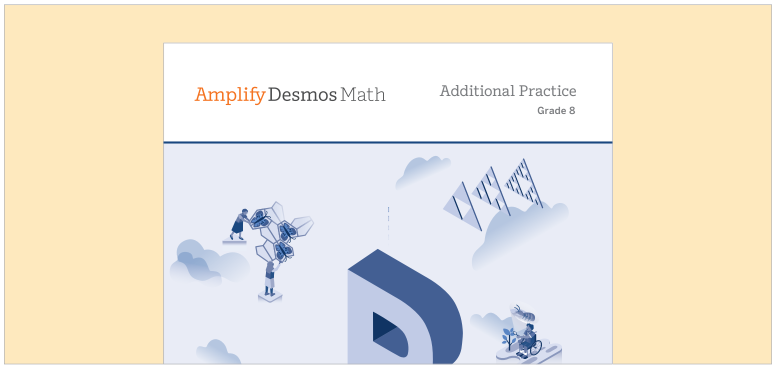
Step 2: Add problems

	s	Standard	Difficulty	Item type
	on P is a cube with one corner removed and relocated t...	NY-6.G.1	DOK 1	Multiple choice
	osphere of a certain planet is made up of many element...	NY-6.G.1	DOK 1	Cloze
	the nets can be folded and assembled into a triangular ...	NY-6.G.1	DOK 2	Multiple choice
☐	Thomas works 12 days out of every month. Does Thomas work t...	NY-6.G.1	DOK 1	Fill-in-the-blank
☐	A cube has a side length of 8 in. Select the value(s) which represe...	NY-6.G.1	DOK 1	Multiple choice
☐	Carlos sold .725 of the total pastries for a bake sale. Write this de...	NY-6.G.1	DOK 2	Multiple choice

An **online item bank** contains additional practice sets, or teachers can customize their own based on unit or sub-unit concepts and standards.



Boost Tutored Practice offers engaging, digital independent practice for students that provides access to grade-level math through personalized feedback that responds to student work to support their learning.



Additional Practice Blackline Masters contain additional practice problems to further address fluency, spiral review, and a variety of DOK questions in lesson learning, supporting differentiated practice based on the needs of students.

REVIEWER TIP

These practice enhancements can be found in Amplify Desmos Math New York lessons in this section, online, and in the Student Edition sampler, but are not yet available in the partially designed lesson plans.

In-the-moment instructional supports help teachers meet the needs of every learner.

Embedded instructional supports provide practical guidance for scaffolding or extending learning for all students using an asset-based approach.

D

Differentiation

Provides a lens with which to anticipate, view, and guide individual student work, including *Extensions* and *Differentiation Support* guidance. In addition, robust recommendations to *Support, Strengthen, and Stretch* are provided at the unit level.

A

Accessibility

Promotes main areas of cognitive functioning, including memory and attention, conceptual processing, visual-spatial processing, executive functioning, fine motor skills, and affective functioning.

ML/EL

Multilingual / English Learners

Provides math language development supports to help all students achieve the *Language Goal* of the lesson.

Math identity and Community

Highlights opportunities to recognize and celebrate the brilliance from all students.

Boost mini lessons

Offer teacher-led small group assistance to students who need more direct and explicit support to re-engage with grade-level math.

This just-in-time instruction is informed by assessment data such as pre-unit and sub-unit quizzes.

REVIEWER TIP

In-the-moment instructional supports are included in Amplify Desmos Math New York lessons in this section and online, but most are not yet available in the partially designed lesson plans.

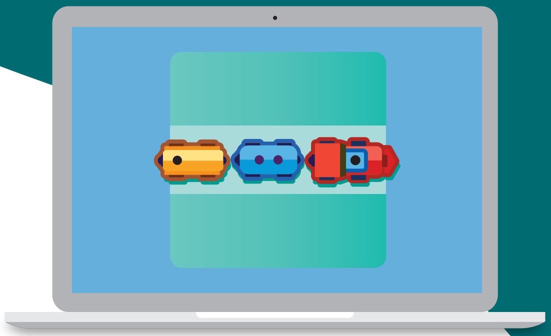
Unit 3

Lesson

4

Digital Lesson RECOMMENDED

This is a digital lesson. A print option is also available.



Model Trains

Comparing Ratios

Let's calculate unit rates and use them to compare speeds.

Focus and Coherence

Today's Goals

1. **Goal:** Calculate the speed of an object as a unit rate for the distance the object travels over time.
2. **Goal:** Use rate and ratio reasoning to compare rates expressed in different units.
3. **Language Goal:** Justify which of two objects is faster. **(Writing, Speaking, and Listening)**

Students compare the speeds of model trains and discuss unit rates using the term *per*. They consider what information is needed to determine which of two model trains is traveling faster and then compare speeds of model trains given in different units of measurement.

Prior Learning

In Lesson 3, students defined the terms *rate* and *unit rate* and used each to determine and compare speeds.

Future Learning

In Lesson 5, students will extend their understanding of unit rates by determining unit prices and identifying the two unit rates associated with any ratio.

Rigor and Balance

- Students continue to build **conceptual understanding** of equivalent ratios and unit rates by comparing speeds.
- Students **apply** their understanding of converting between measurement systems to comparing speeds.
- Students **reason adaptively** when they use multiple strategies to solve a problem.

Standards

Addressing

NY-6.RP.3b
Solve unit rate problems.

Also Addressing: **NY-6.RP.2**, **NY-6.RP.3d**

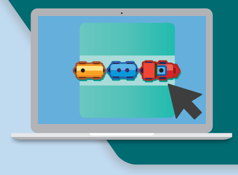
Mathematical Practices: MP2, MP6

Building On	Building Toward
NY-6.RP.3a	**NY-7.RP.1**
NY-5.MD.1	

Lesson at a Glance

~ 45 min

Standards: NY-6.RP.2, NY-6.RP.3b, NY-6.RP.3d



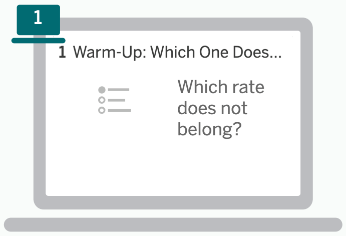
Why digital?

Students receive interpretive feedback as they calculate the unit rates of the model trains in real time.

Warm-Up

Independent | 7 min

Students compare and contrast four rates relating distance and time using the **Which One Doesn't Belong?** routine.



1

1 Warm-Up: Which One Does...

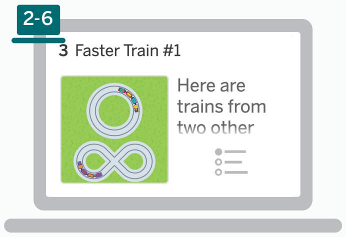
Which rate does not belong?

Pacing: Screen 1

Activity 1

Pairs | 15 min

Students calculate equivalent ratios and unit rates to help them compare speeds of model trains by analyzing two different strategies using the **MLR7: Compare and Connect** routine.



2-6

3 Faster Train #1

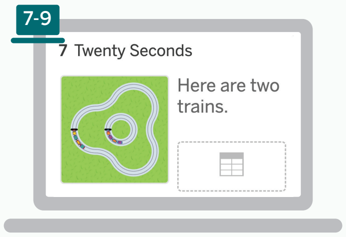
Here are trains from two other

Pacing: Screens 2–6

Activity 2

Pairs | 13 min

Students compare speeds given in different measurement systems.
Think-Pair-Share



7-9

7 Twenty Seconds

Here are two trains.

Pacing: Screens 7–9

Synthesis

Whole Class | 5 min

Students surface strategies for comparing speeds, including calculating a unit rate.

10

10 Synthesis

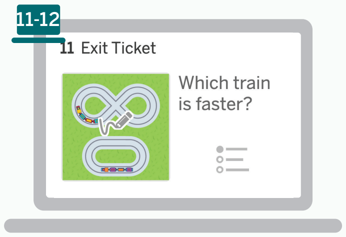
Describe two strategies

Pacing: Screen 10

Exit Ticket

Independent | 5 min

Students demonstrate their understanding by determining which train is traveling at a faster speed.



11-12

11 Exit Ticket

Which train is faster?

Pacing: Screens 11–12

Prep Checklist

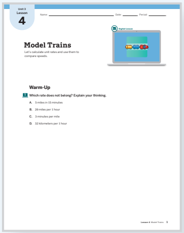
Assign the digital lesson. A print option is also available.

Students using digital:	**Students using print:**
Digital Lesson	Print Option in Student Edition
	Exit Ticket PDF

Digital Lesson | Independent | 7 min | Pacing: Screen 1

Warm-Up

Purpose: Students compare and contrast four rates relating distance and time using the Which One Doesn't Belong? routine.



Students using print

1 Launch

Support getting started by asking "How many minutes are in an hour? How many kilometers are in a mile?"

Use the Which One Doesn't Belong? routine to support students in noticing how the four rates are expressed, noting the different units of measurement and the order of those units.

1 Connect

Invite students to share which rate they decided doesn't belong and why. Encourage the use of mathematical language, such as *per*, *unit rate*, *for each*, or *for every*. As each option is identified, ask whether anyone else chose the same option but for a different reason. **(MP6)**

Consider asking, "Are any of these rates equivalent?"

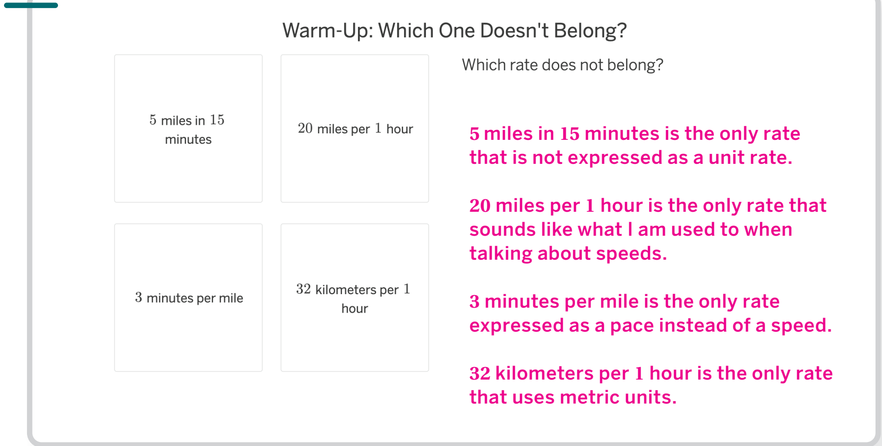
Emphasize that in the upcoming activities, the rates students will work with are distances over time, which are also known as speeds.

Math Identity and Community Consider celebrating variety and creativity in what students notice, including things that surprise you or you think other students may not have noticed.

M/EL **Multilingual/English Learners** Provide sentence frames to support students as they share their responses. For example, "Choice ______ doesn't belong because ______." or "Choice ______ is the only one that has/doesn't have ______." **(Reading and Speaking)**

Students using digital

1



Warm-Up: Which One Doesn't Belong?

5 miles in 15 minutes	20 miles per 1 hour
3 minutes per mile	32 kilometers per 1 hour

Which rate does not belong?

5 miles in 15 minutes is the only rate that is not expressed as a unit rate.

20 miles per 1 hour is the only rate that sounds like what I am used to when talking about speeds.

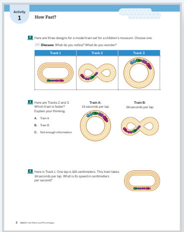
3 minutes per mile is the only rate expressed as a pace instead of a speed.

32 kilometers per 1 hour is the only rate that uses metric units.

Pairs | 15 min | Pacing: Screens 2–4

Activity 1 How Fast?

Purpose: Students calculate equivalent ratios and unit rates to help them compare speeds of model trains by analyzing two different strategies using the MLR7: Compare and Connect routine.



Students using print

2 Launch

Encourage connections by asking students if they have ever played with a model train set.

Support getting started by asking students what information they think is missing from the image they chose.

3-4 Monitor

Consider asking, "What information is needed to know which train is faster?"

D Differentiation

Look for students who:	Teacher Moves
Need help getting started. *(Screen 4)*	Ask, "What do you think it means to measure the speed of a train in centimeters per second?" **(MP6)**
Would benefit from a challenge during the activity.	**Extension:** Invite students to think about what a student who responded differently might have been thinking.

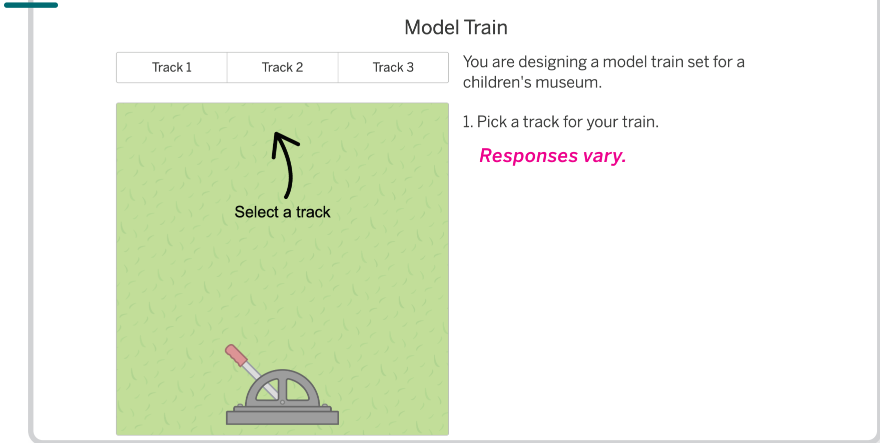
A Accessibility: Conceptual Processing

To assist students in recognizing the connections between new problems and prior work, consider displaying a double number line diagram or a table of equivalent ratios from an earlier lesson. Ask students how we might represent the ratio on this screen using a table or a double number line.

Activity 1 continued >

Students using digital

2



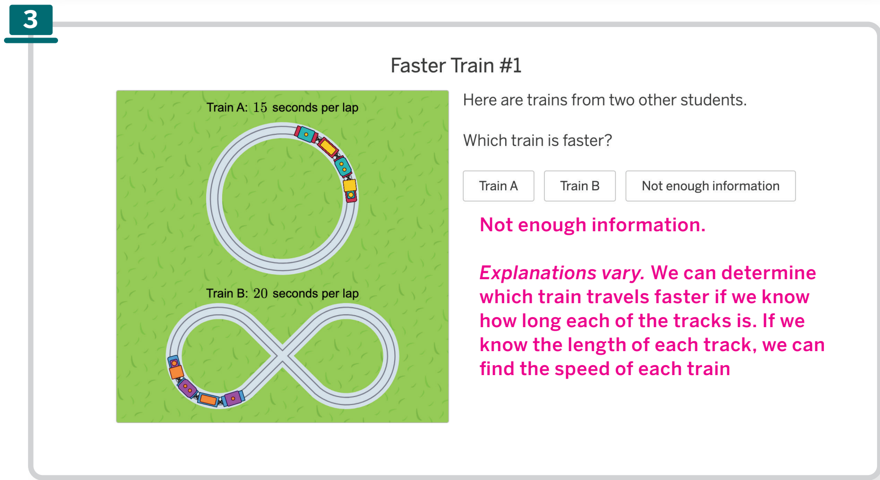
Model Train

You are designing a model train set for a children's museum.

1. Pick a track for your train.

Responses vary.

3



Faster Train #1

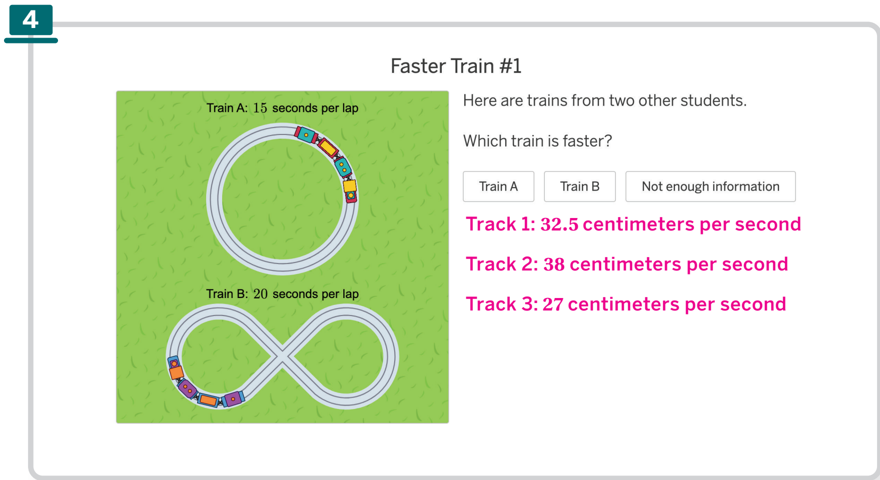
Here are trains from two other students.

Which train is faster?

Not enough information.

Explanations vary. We can determine which train travels faster if we know how long each of the tracks is. If we know the length of each track, we can find the speed of each train

4



Faster Train #1

Here are trains from two other students.

Which train is faster?

Track 1: 32.5 centimeters per second

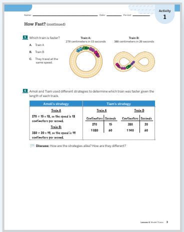
Track 2: 38 centimeters per second

Track 3: 27 centimeters per second

Digital Lesson | Pairs | 15 min | Pacing: Screens 2–6

Activity 1 How Fast? (continued)

Purpose: Students calculate equivalent ratios and unit rates to help them compare speeds of model trains by analyzing two different strategies using the MLR7: Compare and Connect routine.


Students using print

5-6 Monitor

Identify students who use similar strategies to Amoli's and Tiam's or other creative strategies. Plan to invite those students to share during the Connect. *(Screen 5)*

D Differentiation

Look for students who:	Teacher Moves
Recognize that Amoli's strategy involved using a *unit rate*. *(Screen 6)*	Invite them to share why Amoli divided by 15 for Train A.
Recognize that Tiam's strategy involved using *equivalent ratios*. *(Screen 6)*	Invite them to share how Tiam calculated that Train B travels 1 140 cm in 60 seconds.
Finish early. *(Screen 6)*	**Extension:** Invite students to determine how Amoli and Tiam would determine the speed of a train that traveled 300 cm in 40 seconds.

A Accessibility: Conceptual Processing To support students in expressing their thinking, encourage them to use the sketch tool to annotate or highlight each strategy. *(Screen 6)*

M/EL Multilingual/English Learners Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others. **(Speaking and Listening)** *(Screen 6)*

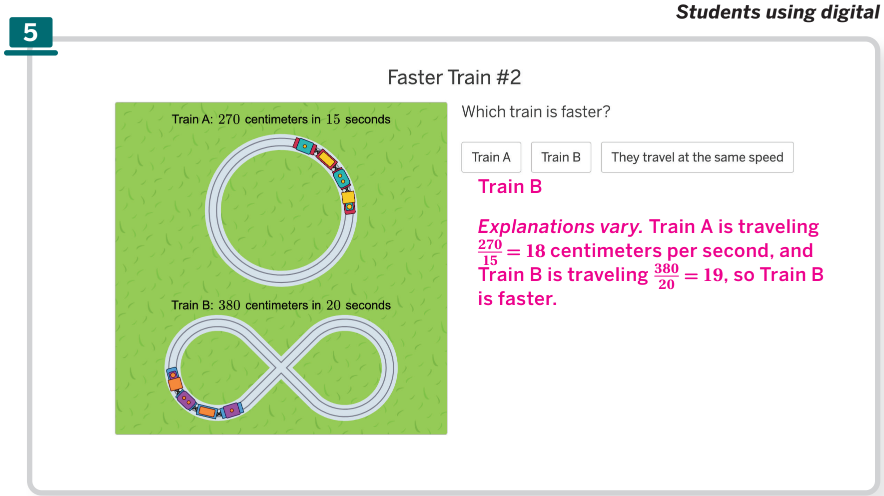
Math Identity and Community Consider renaming each strategy after a student in your class who used it.

6 Connect

Invite students to share how Amoli's and Tiam's strategies connect to their own.

MLR MLR7: Compare and Connect As students share their responses and reasoning, invite students to compare the two strategies and discuss with a partner. Consider asking, "Why did the two strategies lead to the same result of Train B being faster than Train A?"

Students using digital

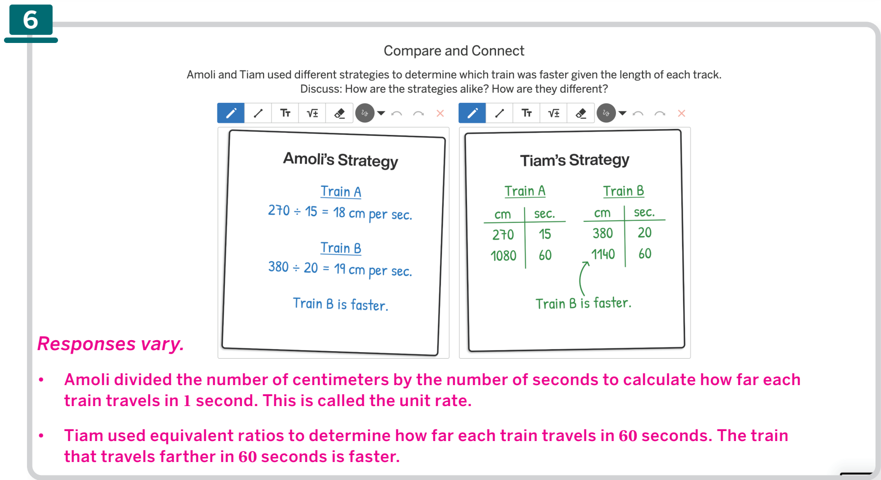


Faster Train #2

Which train is faster?

Train B

Explanations vary. **Train A is traveling $\frac{270}{15} = 18$ centimeters per second, and Train B is traveling $\frac{380}{20} = 19$, so Train B is faster.**



Compare and Connect

Amoli and Tiam used different strategies to determine which train was faster given the length of each track.
Discuss: How are the strategies alike? How are they different?

Amoli's Strategy

Train A
270 ÷ 15 = 18 cm per sec.

Train B
380 ÷ 20 = 19 cm per sec.

Train B is faster.

Tiam's Strategy

Train A

cm	sec.
270	15
1080	60

Train B

cm	sec.
380	20
1140	60

Train B is faster.

Responses vary.

- **Amoli divided the number of centimeters by the number of seconds to calculate how far each train travels in 1 second. This is called the unit rate.**
- **Tiam used equivalent ratios to determine how far each train travels in 60 seconds. The train that travels farther in 60 seconds is faster.**

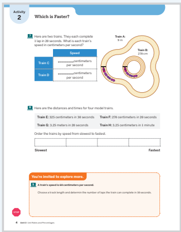
Key Takeaway: Speeds can be compared by determining the rate per 1, or the *unit rate*, which can be done by dividing one quantity by the other. Two speeds can also be compared by creating *equivalent ratios* such that either the distance or the time are the same.

Pairs | 13 min | Pacing: Screens 7–9

Activity 2 Which is Faster?

Purpose: Students compare speeds given in different measurement systems.

Short on time: Consider allowing students to order two trains of the four on Screen 8.



Students using print

7 Launch

Support getting started by asking, “How long would it take each train to complete a lap if it was traveling 1 centimeter per second?”

Use the Think-Pair-Share routine.

7-9 Monitor

Pause to ask students what they notice about the rates of the four model trains. *(Screen 8)*

D Differentiation

Look for students who:	**Teacher Moves**
Write the speed for Train C as $\frac{9}{20}$. *(Screen 7)*	**Support:** Invite students to state the units of the numerator and denominator separately. Then ask if they need to convert either measurement before doing division. **(MP6)**
Write the speed as 32 500 meters in 30 seconds for Train E. *(Screen 8)*	**Support:** Consider asking, “How many centimeters are in 1 meter?”
Use several different strategies for deciding which train is traveling the fastest. *(Screen 8)* **(MP2)**	Invite students to share during the Connect. **(MP6)**
Would benefit from a challenge during this activity. *(Screen 9)*	**Extension:** ***You're invited to explore more.*** Invite students who want to explore the relationship between distance, rate, and time to further discuss this task with a partner.

A Accessibility: Executive Functioning
To support organization in problem solving, consider chunking this activity by inviting students to select two trains, determine which is faster, and then compare the third train to the other two. Then have them continue this process until they have compared all four trains.

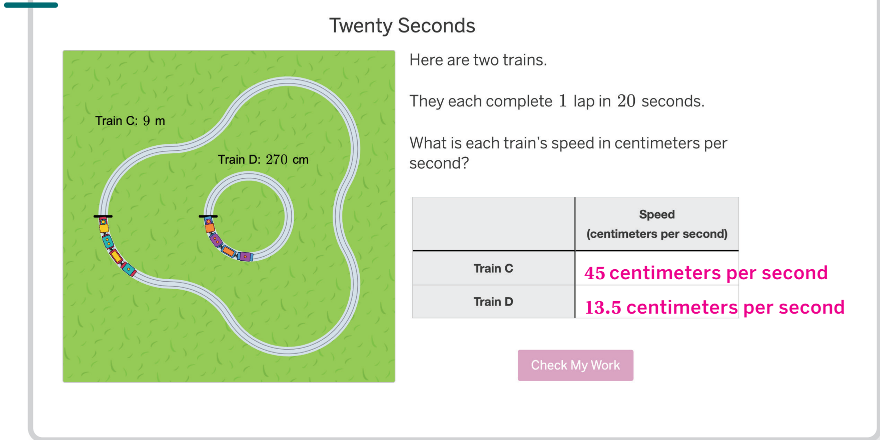
8 Connect

Invite students to share their strategies for comparing the speeds of the trains.

Math Identity and Community Consider asking the class why having more than one strategy might be useful.

Students using digital

7



Twenty Seconds

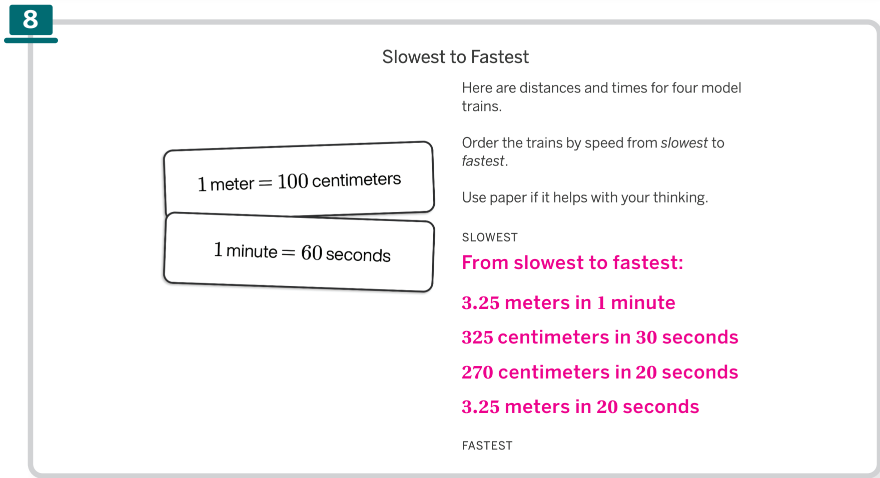
Here are two trains.

They each complete 1 lap in 20 seconds.

What is each train's speed in centimeters per second?

	Speed (centimeters per second)
Train C	45 centimeters per second
Train D	13.5 centimeters per second

8



Slowest to Fastest

Here are distances and times for four model trains.

Order the trains by speed from *slowest* to *fastest*.

Use paper if it helps with your thinking.

SLOWEST

From slowest to fastest:

3.25 meters in 1 minute

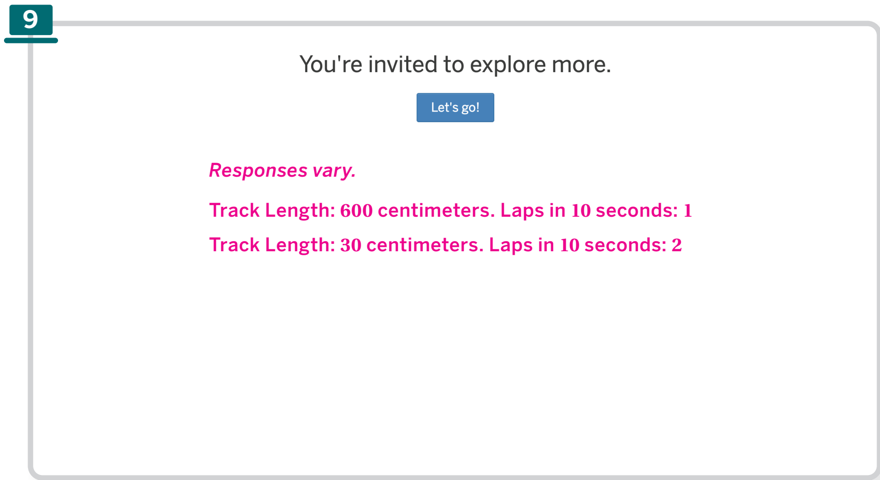
325 centimeters in 30 seconds

270 centimeters in 20 seconds

3.25 meters in 20 seconds

FASTEST

9



You're invited to explore more.

Responses vary.

Track Length: 600 centimeters. Laps in 10 seconds: 1

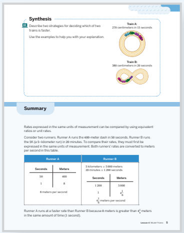
Track Length: 30 centimeters. Laps in 10 seconds: 2

Key Takeaway: To compare the speeds of different trains (or traveling objects), both the distance and the time must be measured in the same units of measurement.

Digital Lesson | Whole Class | 5 min | Pacing: Screen 10

Synthesis

Purpose: Students surface strategies for comparing speeds, including calculating a unit rate.



Students using print

10 Synthesis

Invite students to respond independently and then share their thinking with a partner.

Capture and share a variety of ideas, including:

- Calculating a unit rate for each train.
- Using equivalent ratios to see how far each train goes in the same amount of time.

Math Identity and Community If time allows, invite students to shout out students whose strategies they found most helpful.

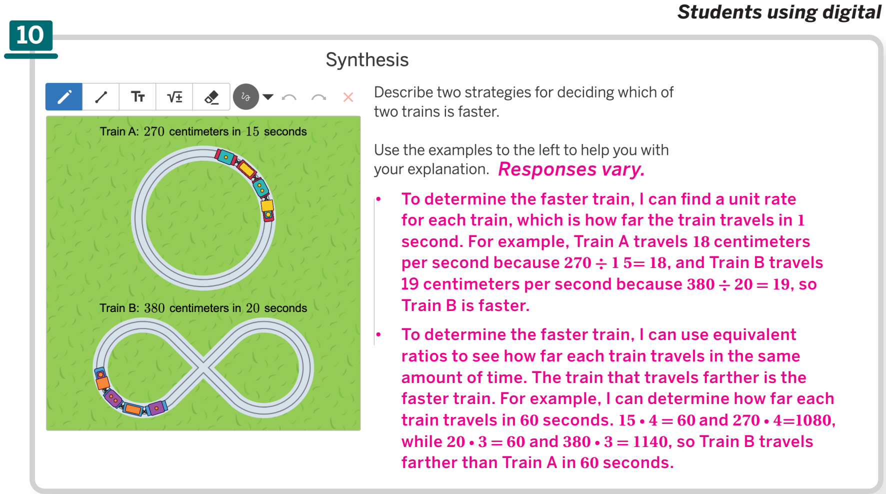
If time allows, invite students to share what makes sense to them about each strategy and what connections they see between their classmates' strategies.

Lesson Takeaway: There are multiple strategies that can be used to compare speeds.

Summary

Share the Summary. Students can refer back to this throughout the unit and course.

Students using digital



10

Synthesis

Train A: 270 centimeters in 15 seconds

Train B: 380 centimeters in 20 seconds

Describe two strategies for deciding which of two trains is faster.

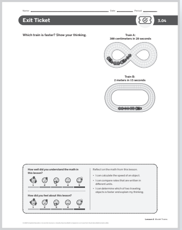
Use the examples to the left to help you with your explanation. *Responses vary.*

- To determine the faster train, I can find a unit rate for each train, which is how far the train travels in 1 second. For example, Train A travels 18 centimeters per second because $270 \div 1\ 5= 18$, and Train B travels 19 centimeters per second because $380 \div 20 = 19$, so Train B is faster.
- To determine the faster train, I can use equivalent ratios to see how far each train travels in the same amount of time. The train that travels farther is the faster train. For example, I can determine how far each train travels in 60 seconds. $15 \cdot 4 = 60$ and $270 \cdot 4=1080$, while $20 \cdot 3 = 60$ and $380 \cdot 3 = 1140$, so Train B travels farther than Train A in 60 seconds.

Independent | 5 min | Pacing: Screens 11–12

Exit Ticket

Purpose: Students demonstrate their understanding by determining which train is traveling at a faster speed.



Students using print

11-12 Today's Goals

Goal: Calculate the speed of an object as a unit rate for the distance the object travels over time.

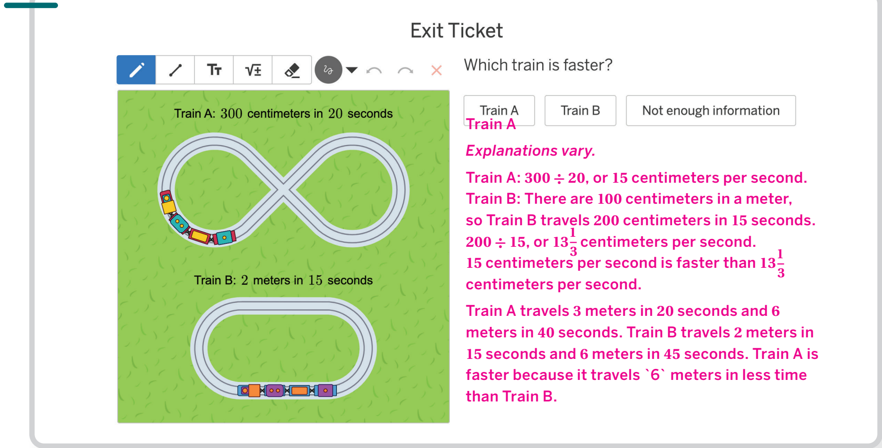
Goal: Use rate and ratio reasoning to compare rates expressed in different units.

Language Goal: Justify which of two objects is faster. **(Writing, Speaking, and Listening)**

Support for Future Learning: If students struggle with calculating unit rates, plan to emphasize this when opportunities arise over the next several lessons. For example, spend extra time in Lesson 5 discussing how to calculate and interpret the two unit rates for the same relationship.

Students using digital

11



Which train is faster?

Train A: 300 centimeters in 20 seconds

Train B: 2 meters in 15 seconds

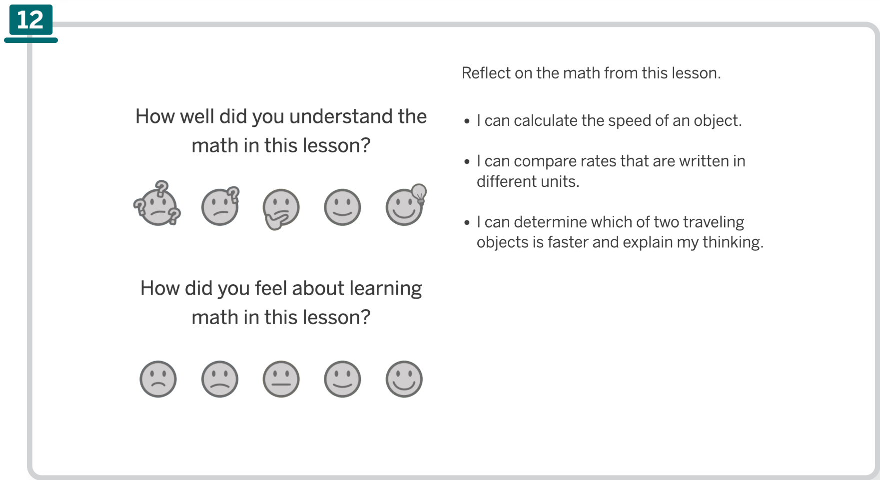
Train A

Explanations vary.

Train A: 300 ÷ 20, or 15 centimeters per second.
Train B: There are 100 centimeters in a meter, so Train B travels 200 centimeters in 15 seconds. 200 ÷ 15, or $13\frac{1}{3}$ centimeters per second.
15 centimeters per second is faster than $13\frac{1}{3}$ centimeters per second.

Train A travels 3 meters in 20 seconds and 6 meters in 40 seconds. Train B travels 2 meters in 15 seconds and 6 meters in 45 seconds. Train A is faster because it travels `6` meters in less time than Train B.

12



How well did you understand the math in this lesson?

How did you feel about learning math in this lesson?

Reflect on the math from this lesson.

- I can calculate the speed of an object.
- I can compare rates that are written in different units.
- I can determine which of two traveling objects is faster and explain my thinking.

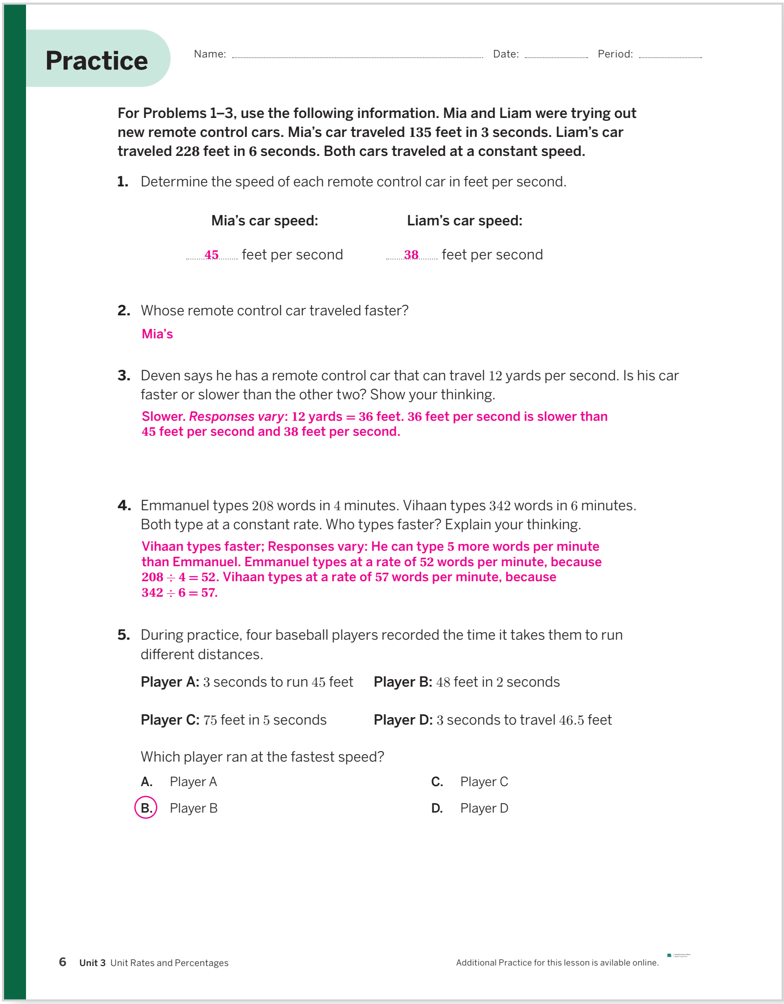
Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Students using digital

Students using print



Practice

Name: ______ Date: ______ Period: ______

For Problems 1–3, use the following information. Mia and Liam were trying out new remote control cars. Mia's car traveled 135 feet in 3 seconds. Liam's car traveled 228 feet in 6 seconds. Both cars traveled at a constant speed.

1. Determine the speed of each remote control car in feet per second.

Mia's car speed: 45 feet per second

Liam's car speed: 38 feet per second

2. Whose remote control car traveled faster?

Mia's

3. Deven says he has a remote control car that can travel 12 yards per second. Is his car faster or slower than the other two? Show your thinking.

Slower. *Responses vary*: 12 yards = 36 feet. 36 feet per second is slower than 45 feet per second and 38 feet per second.

4. Emmanuel types 208 words in 4 minutes. Vihaan types 342 words in 6 minutes. Both type at a constant rate. Who types faster? Explain your thinking.

Vihaan types faster; Responses vary: He can type 5 more words per minute than Emmanuel. Emmanuel types at a rate of 52 words per minute, because $208 \div 4 = 52$. Vihaan types at a rate of 57 words per minute, because $342 \div 6 = 57$.

5. During practice, four baseball players recorded the time it takes them to run different distances.

Player A: 3 seconds to run 45 feet

Player B: 48 feet in 2 seconds

Player C: 75 feet in 5 seconds

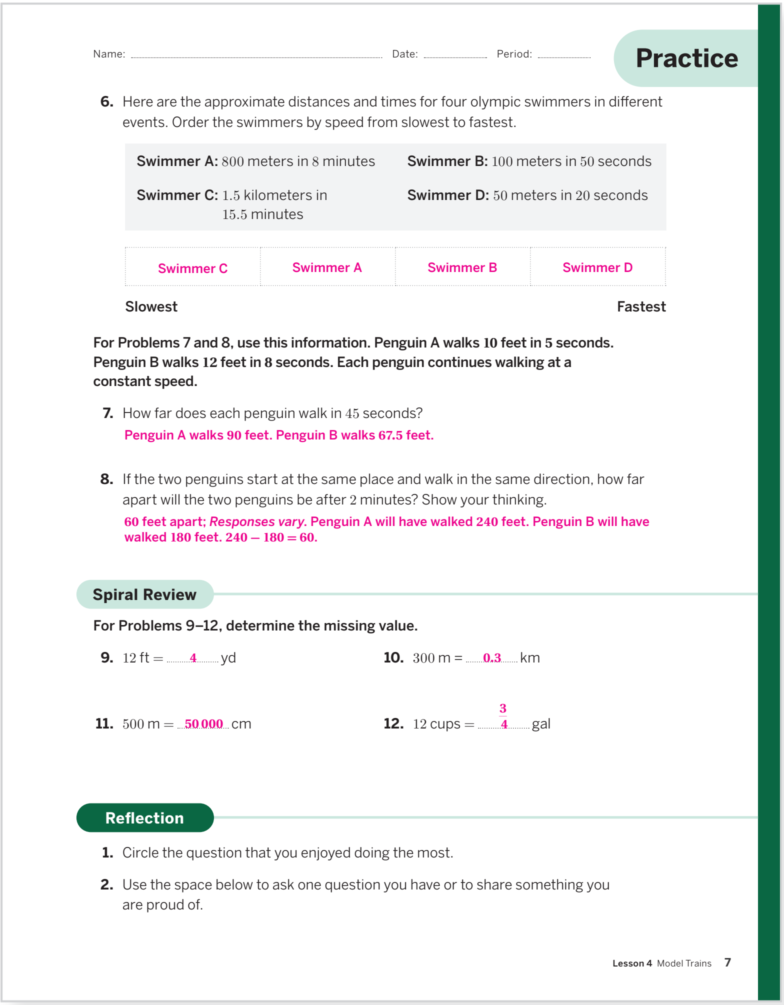
Player D: 3 seconds to travel 46.5 feet

Which player ran at the fastest speed?

A. Player A
[✓ B.] Player B
C. Player C
D. Player D

6 **Unit 3** Unit Rates and Percentages

Additional Practice for this lesson is avilable online.



Name: ______ Date: ______ Period: ______

Practice

6. Here are the approximate distances and times for four olympic swimmers in different events. Order the swimmers by speed from slowest to fastest.

Swimmer A: 800 meters in 8 minutes

Swimmer B: 100 meters in 50 seconds

Swimmer C: 1.5 kilometers in 15.5 minutes

Swimmer D: 50 meters in 20 seconds

Swimmer C	Swimmer A	Swimmer B	Swimmer D
Slowest			**Fastest**

For Problems 7 and 8, use this information. Penguin A walks 10 feet in 5 seconds. Penguin B walks 12 feet in 8 seconds. Each penguin continues walking at a constant speed.

7. How far does each penguin walk in 45 seconds?

Penguin A walks 90 feet. Penguin B walks 67.5 feet.

8. If the two penguins start at the same place and walk in the same direction, how far apart will the two penguins be after 2 minutes? Show your thinking.

60 feet apart; *Responses vary*. Penguin A will have walked 240 feet. Penguin B will have walked 180 feet. $240 - 180 = 60$.

Spiral Review

For Problems 9–12, determine the missing value.

9. 12 ft = 4 yd

10. 300 m = 0.3 km

11. 500 m = 50 000 cm

12. 12 cups = $\frac{3}{4}$ gal

Reflection

1. Circle the question that you enjoyed doing the most.
2. Use the space below to ask one question you have or to share something you are proud of.

Lesson 4 Model Trains 7

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1–4, 7	2	NY-6.RP.2, NY-6.RP.3b
Test Practice	5	2	NY-6.RP.2, NY-6.RP.3b
	6	2	NY-6.RP.3b, NY-6.RP.3d
	8	3	NY-6.RP.2, NY-6.RP.3b
Spiral Review			
Fluency	9–12	1	NY-5.MD.1

Unit 6

Lesson

16

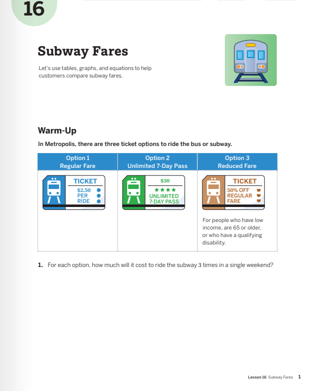
Subway Fares

Applying Relationships

Let's use tables, graphs, and equations to help customers compare subway fares.

Print Lesson

Assign the Student Edition with Presentation Screens for this lesson.



Unit 6

Lesson

16

Name: ________ Date: ________ Period: ________

Subway Fares

Let's use tables, graphs, and equations to help customers compare subway fares.

Warm-Up

In Metropolis, there are three ticket options to ride the bus or subway.

Option 1 Regular Fare	Option 2 Unlimited 7-Day Pass	Option 3 Reduced Fare
TICKET $2.50 PER RIDE	$30 ★★★★ UNLIMITED 7-DAY PASS	TICKET 50% OFF REGULAR FARE For people who have low income, are 65 or older, or who have a qualifying disability.

1. For each option, how much will it cost to ride the subway 3 times in a single weekend?

Lesson 16 Subway Fares 1

Focus and Coherence

Today's Goals

1. **Goal:** Create graphs, tables, and equations to represent situations.
2. **Language Goal:** Interpret representations to analyze an issue in society. **(Reading, Writing, Speaking, and Listening)**

Students use tables, graphs, and equations to interpret relationships and help customers make decisions about what type of transportation ticket to buy. Students are also invited to stand in the shoes of others, considering the potential impact of changing fares on customers with different needs. **(MP2)**

Prior Learning

In Lessons 13–15, students used tables, graphs, and equations to represent and compare proportional and nonproportional relationships.

Future Learning

In Grade 7, students will use multiple representations (tables and graphs) to decide whether two quantities are in a proportional relationship and use proportional relationships to solve multistep problems.

Rigor and Balance

- Students **apply** their understanding about ratios and rate to compare transportation tickets using tables, graphs, and equations.

Standards

Addressing

NY-6.EE.9

Use variables to represent two quantities in a real-world problem that change in relationship to one another.
Given a verbal context and an equation, identify the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Also Addressing: **NY-6.RP.1**, **NY-6.RP.2**, **NY-6.RP.3**, **NY-6.RP.3a**, **NY-6.EE.6**

Mathematical Practices: MP2, MP4

Building Toward

NY-7.RP.2

Amplify Desmos Math **NEW YORK**

Lesson Sample

Lesson at a Glance

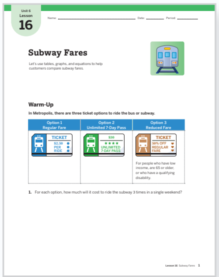
~ 45 min

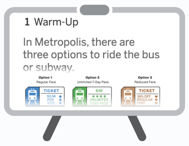
Standard(s): NY-6.EE.9, NY-6.RP.1, NY-6.RP.2, NY-6.RP.3, NY-6.RP.3a, NY-6.EE.6

Warm-Up

Pairs | 5 min

Students use the **Think-Pair-Share** routine to make sense of the context they will explore in this lesson: different fare options for riding the subway or bus.





1 Warm-Up

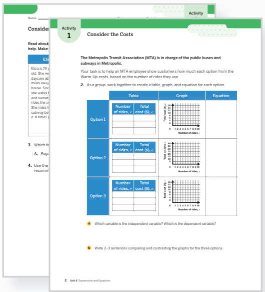
In Metropolis, there are three options to ride the bus or subway.

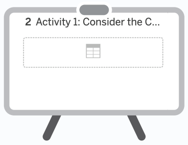
Presentation Screen 1

Activity 1

Small Groups | 15 min

Students create representations for the relationship between the number of rides and the total cost of each fare option and make decisions about which fare option to choose. **(MP2)**





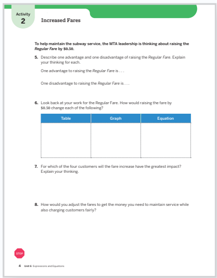
2 Activity 1: Consider the C...

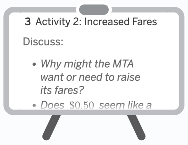
Presentation Screen 2

Activity 2

Small Groups | 15 min

Students use their representations from Activity 1 to analyze an issue in society: the impacts of raising transit fares. **(MP4)**





3 Activity 2: Increased Fares

Discuss:

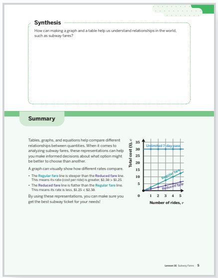
- *Why might the MTA want or need to raise its fares?*
- *Does $0.50 seem like a*

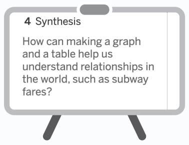
Presentation Screen 3

Synthesis

Whole Class | 5 min

Tables, graphs, and equations are all useful tools to analyze and compare relationships and make decisions. **MLR 1: Stronger and Clearer Each Time**





4 Synthesis

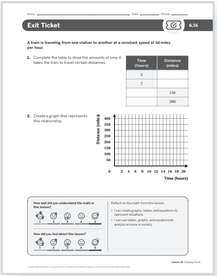
How can making a graph and a table help us understand relationships in the world, such as subway fares?

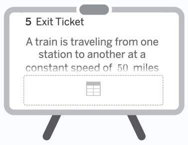
Presentation Screen 4

Exit Ticket

Independent | 5 min

Students create a table and a graph to represent the relationship between distance and time of a train traveling at a constant rate.





5 Exit Ticket

A train is traveling from one station to another at a constant speed of 50 miles

Presentation Screen 5

Prep Checklist

Students will use their Student Editions. Display the Teacher Presentation Screens. Go online to access the Teacher Presentation lesson screens.

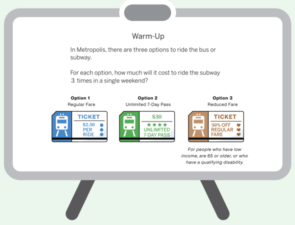
This lesson includes:

- Student Edition
- Exit Ticket PDF
- Large sheets of paper or graph paper (optional)

Pairs | 5 min

Warm-Up

Purpose: Students use the **Think-Pair-Share** routine to make sense of the context they will explore in this lesson: different fare options for riding the subway or bus.



Presentation Screen
1

1 Launch

To activate background knowledge, consider asking:

- "Does anyone have experience riding the subway or bus?"
- "What do you think an unlimited 7-day pass means?"

Use the Think-Pair-Share routine giving students one minute to think individually before sharing with their partner.

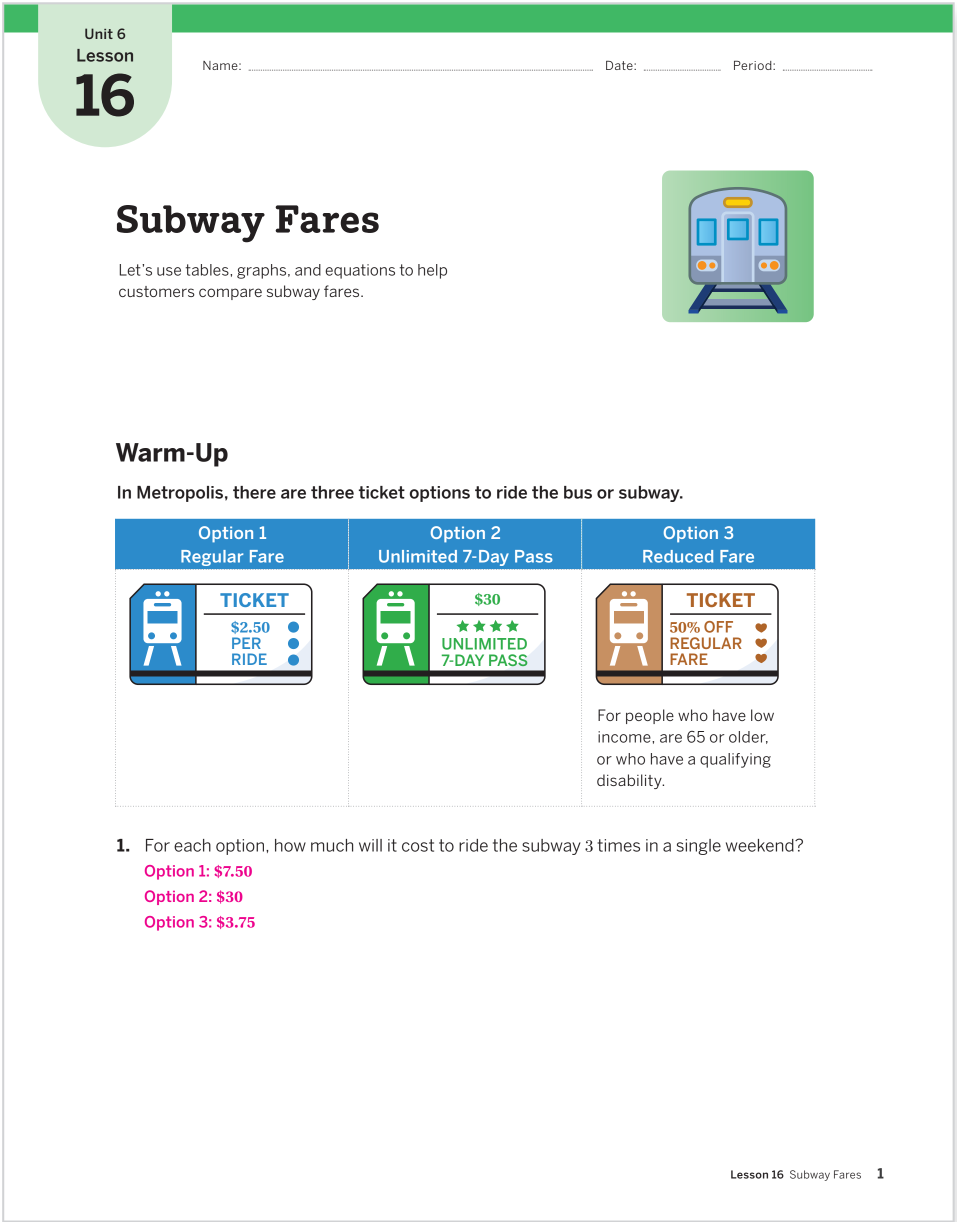
> **A Accessibility: Conceptual Processing**
> To support students getting started, encourage them to consider how much one trip would cost first.

1 Connect

Invite students to share strategies for calculating the cost of each option, particularly what "50% off the regular fare" means and why it costs $30 for only three rides with Option 2.

Consider asking:

- "When might Option 1 be a good choice? When might Option 2 be a good choice?"
- "Who might qualify for Option 3?"



Unit 6
Lesson
16

Name: ______ Date: ______ Period: ______

Subway Fares

Let's use tables, graphs, and equations to help customers compare subway fares.

Warm-Up

In Metropolis, there are three ticket options to ride the bus or subway.

Option 1 Regular Fare	Option 2 Unlimited 7-Day Pass	Option 3 Reduced Fare
TICKET $2.50 PER RIDE	$30 ★★★★ UNLIMITED 7-DAY PASS	TICKET 50% OFF REGULAR FARE For people who have low income, are 65 or older, or who have a qualifying disability.

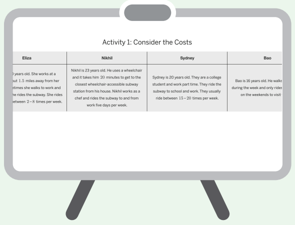
1. For each option, how much will it cost to ride the subway 3 times in a single weekend?
   Option 1: $7.50
   Option 2: $30
   Option 3: $3.75

Lesson 16 Subway Fares 1

Small Groups | 15 min

Activity 1 Considering the Cost

Purpose: Students create representations for the relationship between the number of rides and the total cost of each fare option and make decisions about which fare option to choose. **(MP2)**



Presentation Screen
2

2 Launch

To activate prior knowledge, consider asking "Why might it be helpful for an MTA employee to have a table, a graph, and an equation of the fare options? How might they be helpful to customers?"

Invite students to work as a group to create representations for each fare option. Consider providing large sheets of paper or graph paper to help group members see each others' thinking

Math Identity and Community Consider giving students time to discuss what they think they can contribute to their group, such as organization, asking good questions, creating graphs, making connections between representations, personal experience, etc.

2 Monitor

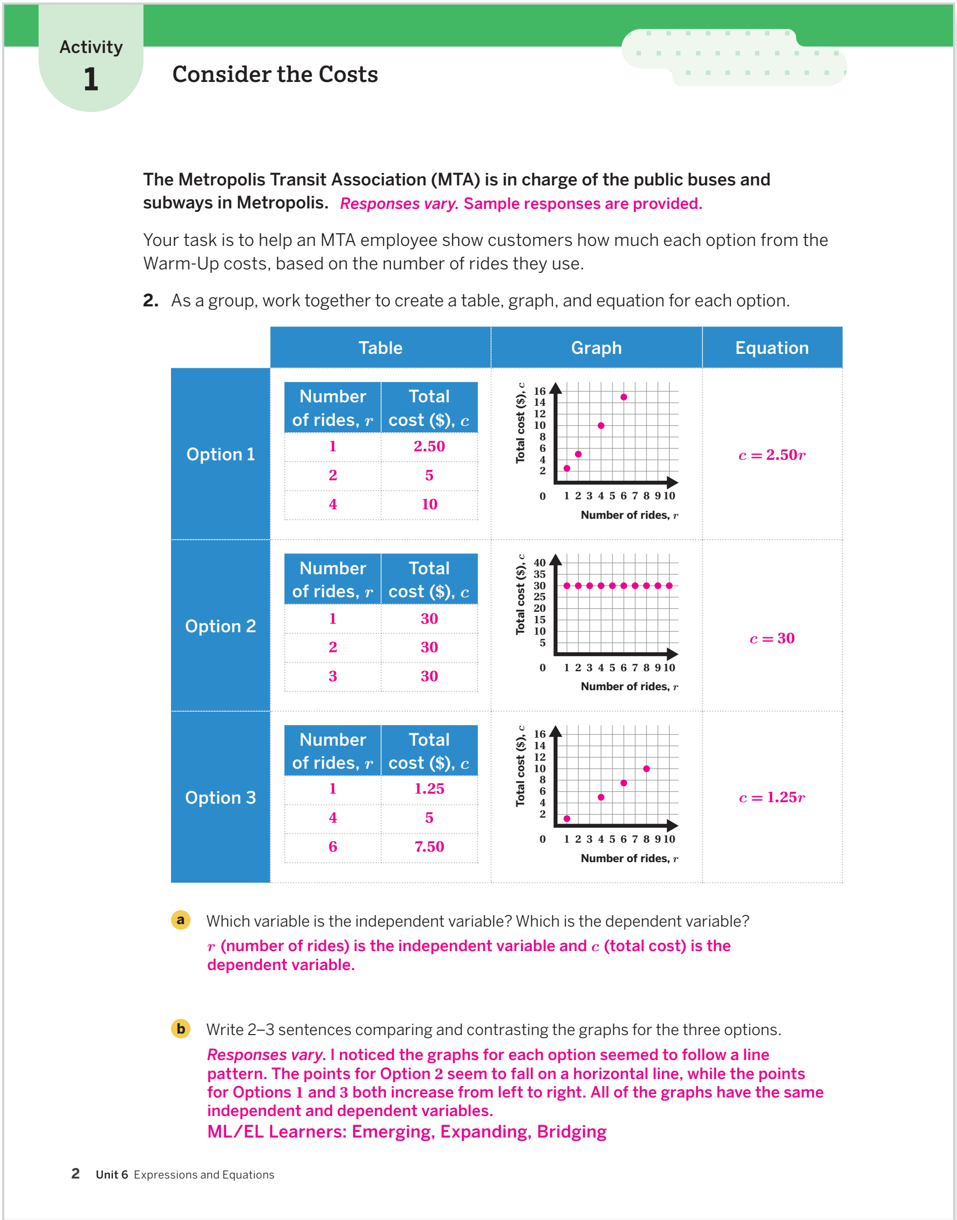
Encourage students to spend 7–10 minutes to create graphs, tables, and equations for each fare option. **(MP2)**

D Differentiation

Look for students who . . .	Teacher Moves
Wonder how to create a table and a graph for the unlimited fare.	**Support:** Consider asking, "How much would it cost to ride the subway once? Twice? 10 times?"
Swap the variables in their equations.	**Support:** Invite them to substitute 1 for the number of rides into their equation. Ask, "Does the cost match the values in your table and graph?"
Use unit rates to determine the cost of one ride in Options 1 and 3 and apply the unit rate to complete the table and write the equation.	Invite them to share why this strategy works for Options 1 and 3, but not Option 2

Pause to share students' ideas around creating their representations before students begin on Problem 3.

Activity 1 continued >



Activity
1

Consider the Costs

The Metropolis Transit Association (MTA) is in charge of the public buses and subways in Metropolis. *Responses vary.* Sample responses are provided.

Your task is to help an MTA employee show customers how much each option from the Warm-Up costs, based on the number of rides they use.

2. As a group, work together to create a table, graph, and equation for each option.

	Table		Graph	Equation
	Number of rides, r	Total cost ($), c		
Option 1	1	2.50		$c = 2.50r$
	2	5		
	4	10		
Option 2	1	30		$c = 30$
	2	30		
	3	30		
Option 3	1	1.25		$c = 1.25r$
	4	5		
	6	7.50		

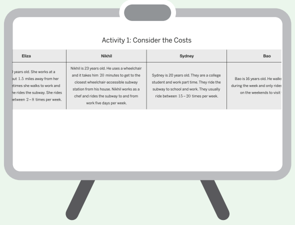
a Which variable is the independent variable? Which is the dependent variable?
r (number of rides) is the independent variable and c (total cost) is the dependent variable.

b Write 2–3 sentences comparing and contrasting the graphs for the three options.
Responses vary. I noticed the graphs for each option seemed to follow a line pattern. The points for Option 2 seem to fall on a horizontal line, while the points for Options 1 and 3 both increase from left to right. All of the graphs have the same independent and dependent variables.
ML/EL Learners: Emerging, Expanding, Bridging

2 Unit 6 Expressions and Equations

Activity 1 Considering the Cost (continued)

Purpose: Students create representations for the relationship between the number of rides and the total cost of each fare option and make decisions about which fare option to choose. **(MP2)**



Presentation Screen
2

2 Monitor

Encourage students to use any of the three representations their group created to answer Problem 3.

Consider asking, "Does your customer qualify for the reduced fare? How do you know?"

M/EL **Multilingual/English Learners** Invite students to describe or act out each customer's information in their own words to make sense of it.

2 Connect

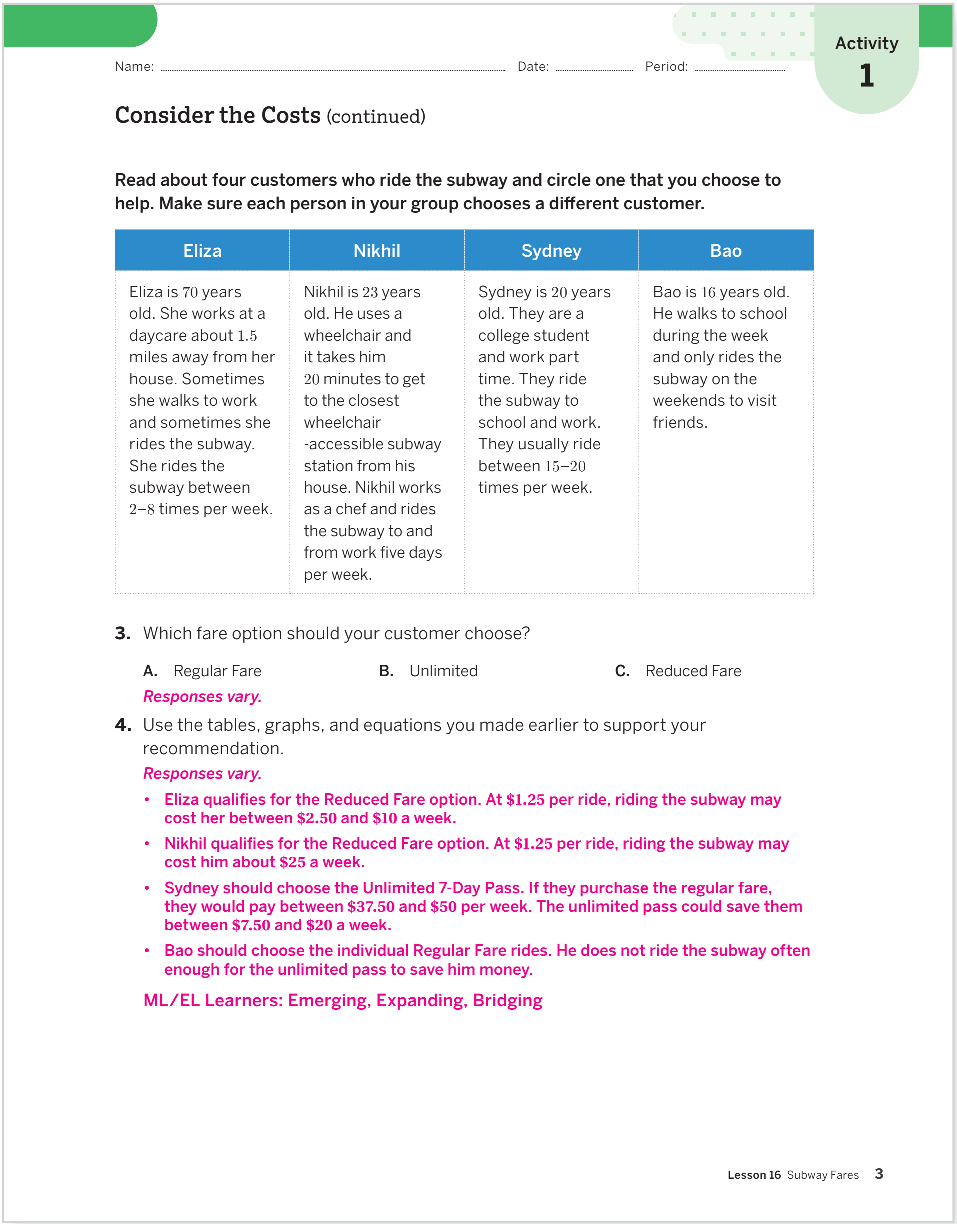
Invite students to share which option they selected for each customer and what evidence they used to support their claim.

Consider asking:

- "How are your tables for each option similar? How are they different?"
- "Which representation(s) helped you decide which option to choose for your customer? How did you use them?"

Math Identity and Community Consider asking, "Why do you think it's important to learn about different customers with different needs?"

Key Takeaway: Each representation (table, graph, and equation) can be used to compare fare options and to determine which fare option is the best choice depending on a person's individual needs.



Name: Date: Period:

Activity 1

Consider the Costs (continued)

Read about four customers who ride the subway and circle one that you choose to help. Make sure each person in your group chooses a different customer.

Eliza	Nikhil	Sydney	Bao
Eliza is 70 years old. She works at a daycare about 1.5 miles away from her house. Sometimes she walks to work and sometimes she rides the subway. She rides the subway between 2–8 times per week.	Nikhil is 23 years old. He uses a wheelchair and it takes him 20 minutes to get to the closest wheelchair -accessible subway station from his house. Nikhil works as a chef and rides the subway to and from work five days per week.	Sydney is 20 years old. They are a college student and work part time. They ride the subway to school and work. They usually ride between 15–20 times per week.	Bao is 16 years old. He walks to school during the week and only rides the subway on the weekends to visit friends.

3. Which fare option should your customer choose?

A. Regular Fare **B.** Unlimited **C.** Reduced Fare

Responses vary.

4. Use the tables, graphs, and equations you made earlier to support your recommendation.

Responses vary.

- **Eliza qualifies for the Reduced Fare option. At $1.25 per ride, riding the subway may cost her between $2.50 and $10 a week.**
- **Nikhil qualifies for the Reduced Fare option. At $1.25 per ride, riding the subway may cost him about $25 a week.**
- **Sydney should choose the Unlimited 7-Day Pass. If they purchase the regular fare, they would pay between $37.50 and $50 per week. The unlimited pass could save them between $7.50 and $20 a week.**
- **Bao should choose the individual Regular Fare rides. He does not ride the subway often enough for the unlimited pass to save him money.**

ML/EL Learners: Emerging, Expanding, Bridging

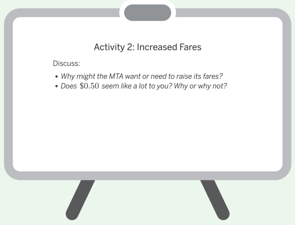
Lesson 16 Subway Fares 3

Small Groups | 15 min

Activity 2 Increased Fares

Purpose: Students use their representations from Activity 1 to analyze an issue in society: the impacts of raising transit fares. **(MP4)**

Short on time: Consider having students complete Problem 6, then discuss Problems 7 and 8 as a group.


Activity 2: Increased Fares

Discuss:

- *Why might the MTA want or need to raise its fares?*
- *Does $\$0.50$ seem like a lot to you? Why or why not?*

Presentation Screen
3

3 Launch

Consider discussing: "Why might the MTA want or need to raise its fares? Does $\$0.50$ seem like a lot to you? Why or why not?"

M/EL **Multilingual/English Learners** Group students with different strengths, including social, mathematical, and language strengths, to support each other in making sense of each of the questions and craft a response.

3 Monitor

Encourage students to use their work from Activity 1 to support their recommendations about who would be most impacted.

A **Accessibility: Conceptual Processing** To support students getting started, consider asking, "What do you think the new cost of each option would be?"

D **Differentiation**

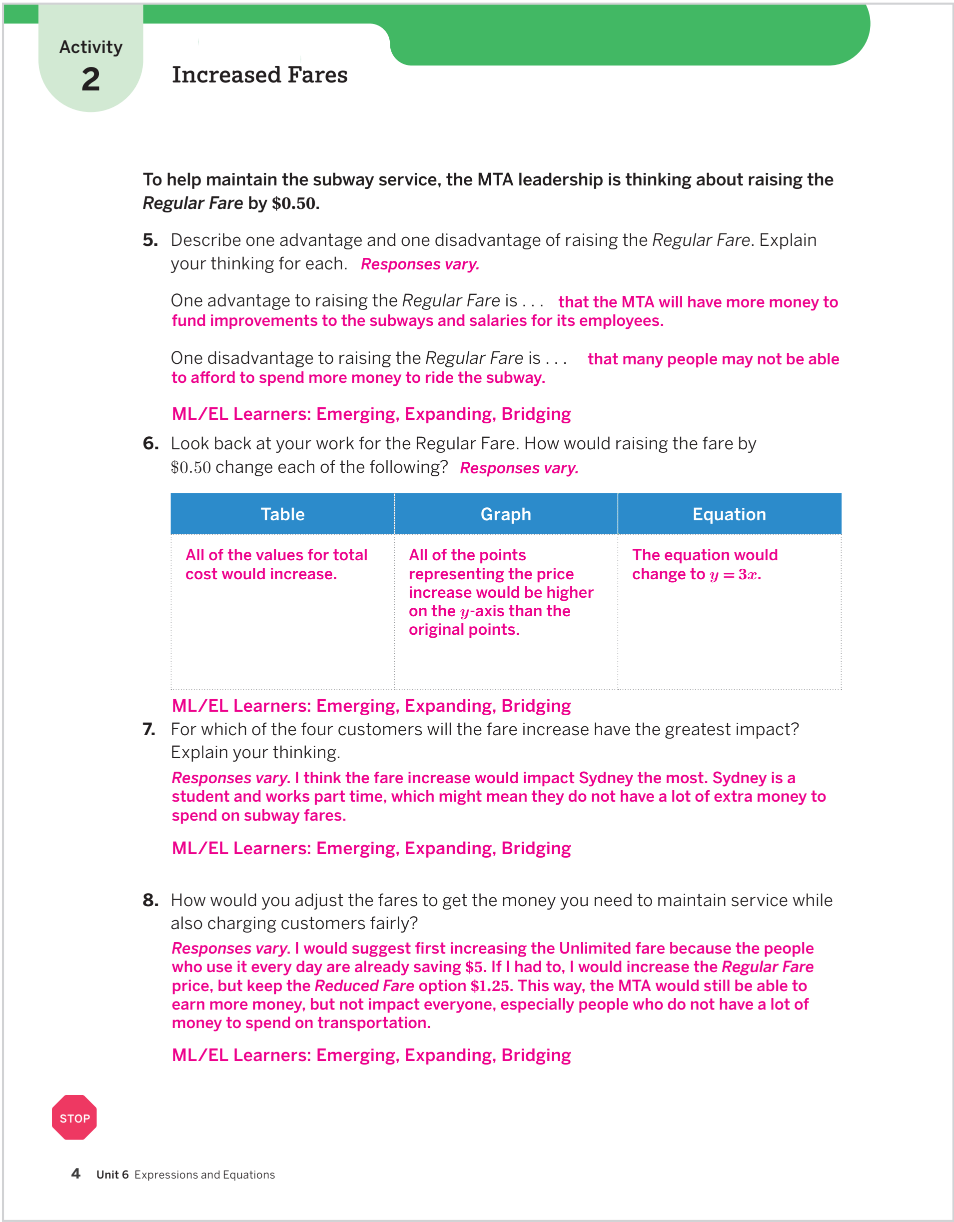
Look for students who . . .	Teacher Moves
Thinking the total cost would increase by $\$0.50$ total regardless of the number of rides.	**Support:** Consider asking, "How much would each ride cost after the increase? Two rides?"
Modeling the increase using a graph, table, or equation to reason about its impact.	**Extension:** Consider asking, "By what percent did the cost of one ride increase? What would be the new cost of the unlimited pass if it increased by the same amount?"

Pause to share students' ideas when most groups have responded to Problem 7.

3 Connect

Invite groups of students to share who they think would be most impacted by the increased fare and share the ideas students have for what the MTA should do.

Math Identity and Community Celebrate students who use information about each of the customers or their own sense of fairness to support them in their reasoning.


Activity
2

Increased Fares

To help maintain the subway service, the MTA leadership is thinking about raising the *Regular Fare* by $\$0.50$.

5. Describe one advantage and one disadvantage of raising the *Regular Fare*. Explain your thinking for each. ***Responses vary.***

   One advantage to raising the *Regular Fare* is . . . **that the MTA will have more money to fund improvements to the subways and salaries for its employees.**

   One disadvantage to raising the *Regular Fare* is . . . **that many people may not be able to afford to spend more money to ride the subway.**

   ML/EL Learners: Emerging, Expanding, Bridging

6. Look back at your work for the Regular Fare. How would raising the fare by $\$0.50$ change each of the following? ***Responses vary.***

Table	Graph	Equation
All of the values for total cost would increase.	All of the points representing the price increase would be higher on the y-axis than the original points.	The equation would change to $y = 3x$.

   ML/EL Learners: Emerging, Expanding, Bridging

7. For which of the four customers will the fare increase have the greatest impact? Explain your thinking.

   ***Responses vary.* I think the fare increase would impact Sydney the most. Sydney is a student and works part time, which might mean they do not have a lot of extra money to spend on subway fares.**

   ML/EL Learners: Emerging, Expanding, Bridging

8. How would you adjust the fares to get the money you need to maintain service while also charging customers fairly?

   ***Responses vary.* I would suggest first increasing the Unlimited fare because the people who use it every day are already saving $5. If I had to, I would increase the *Regular Fare* price, but keep the *Reduced Fare* option $1.25. This way, the MTA would still be able to earn more money, but not impact everyone, especially people who do not have a lot of money to spend on transportation.**

   ML/EL Learners: Emerging, Expanding, Bridging

STOP

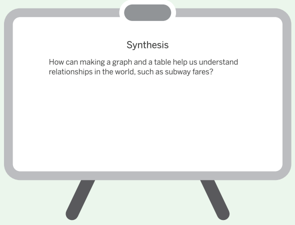
4 Unit 6 Expressions and Equations

Key Takeaway: An equation, table, or graph can model increasing the price of the Regular Fare ticket. In the equation, the coefficient will increase by $\$0.50$. In the table and graph, the y-values will increase proportionally to the number of tickets purchased.

Whole Class | 5 min

Synthesis

Key Takeaway: Tables, graphs, and equations are all useful representations to analyze and compare relationships and make decisions.



Synthesis

How can making a graph and a table help us understand relationships in the world, such as subway fares?

Presentation Screen
4

4 Synthesis

Encourage students to look back at Activity 1, to compare the tables and graphs for each option.

Use the routine MLR1: Stronger and Clearer Each Time to help students develop their ideas and language.

Look for a variety of ideas, including:

- Values in a table can be seen in a graph.
- A table helps organize thinking and graphs help visualize relationships.

Consider asking:

- "What other situations might graphs or tables be helpful for?"
- "When is a table more helpful? When is a graph more helpful?'
- "How can graphs and tables be used to help to compare relationships?"

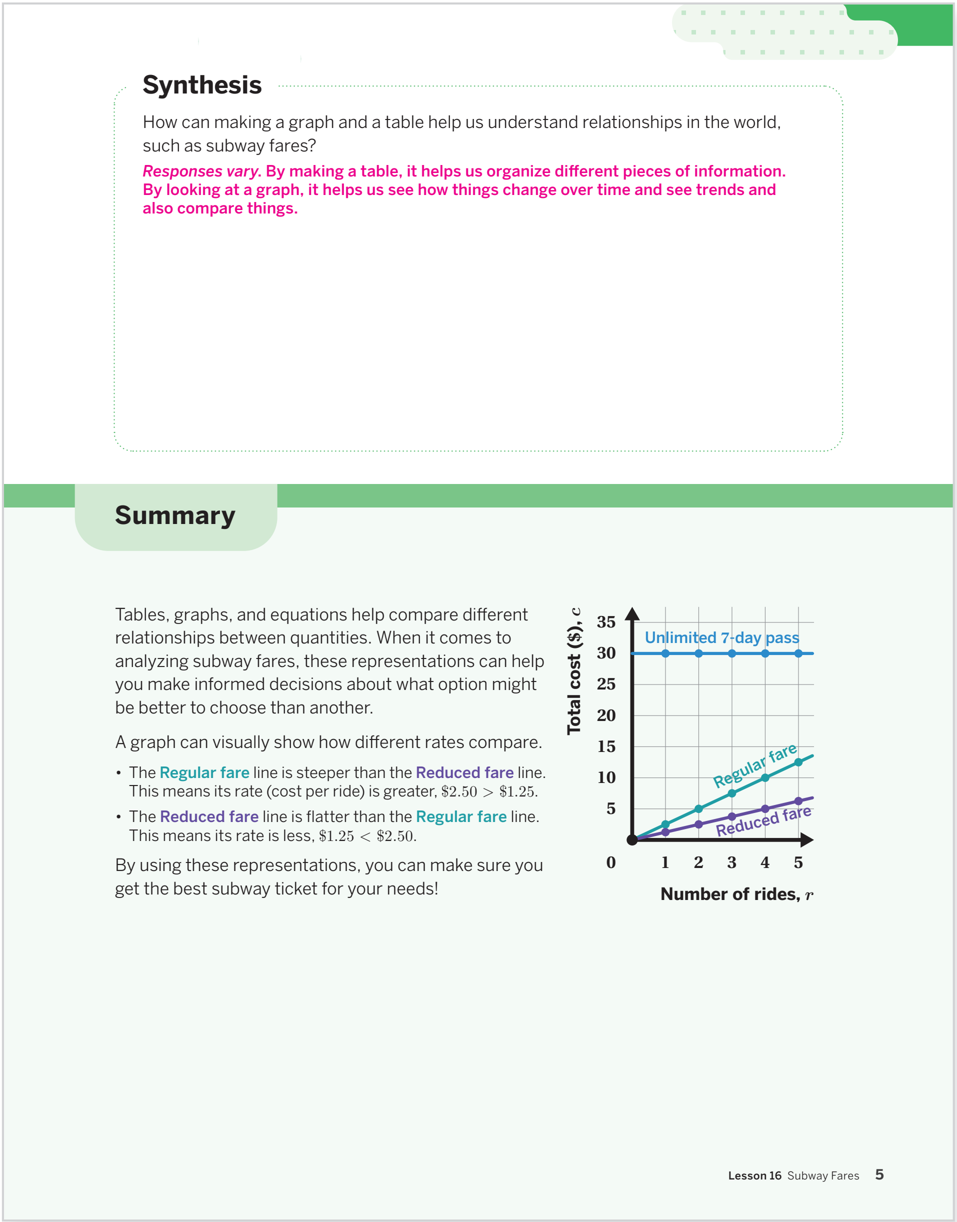
If time allows, invite students to make their response stronger and clearer.

M/EL **Multilingual/English Learners** Provide sentence frames to help them explain their thinking (e.g., Making a table and a graph can help us by ______.).

Lesson Takeaway: Tables, graphs, and equations are all useful representations of two-variable relationships. They can be used to analyze and compare relationships to make decisions.

Summary

Invite students to read the Summary. Share that students can refer back to the Summary throughout the unit and grade.



Synthesis

How can making a graph and a table help us understand relationships in the world, such as subway fares?

Responses vary. **By making a table, it helps us organize different pieces of information. By looking at a graph, it helps us see how things change over time and see trends and also compare things.**

Summary

Tables, graphs, and equations help compare different relationships between quantities. When it comes to analyzing subway fares, these representations can help you make informed decisions about what option might be better to choose than another.

A graph can visually show how different rates compare.

- The **Regular fare** line is steeper than the **Reduced fare** line. This means its rate (cost per ride) is greater, $\$2.50 > \$1.25$.
- The **Reduced fare** line is flatter than the **Regular fare** line. This means its rate is less, $\$1.25 < \$2.50$.

By using these representations, you can make sure you get the best subway ticket for your needs!

Lesson 16 Subway Fares 5

Independent | 5 min

Exit Ticket

Purpose: Students create a table and a graph to represent the relationship between distance and time of a train traveling at a constant rate.

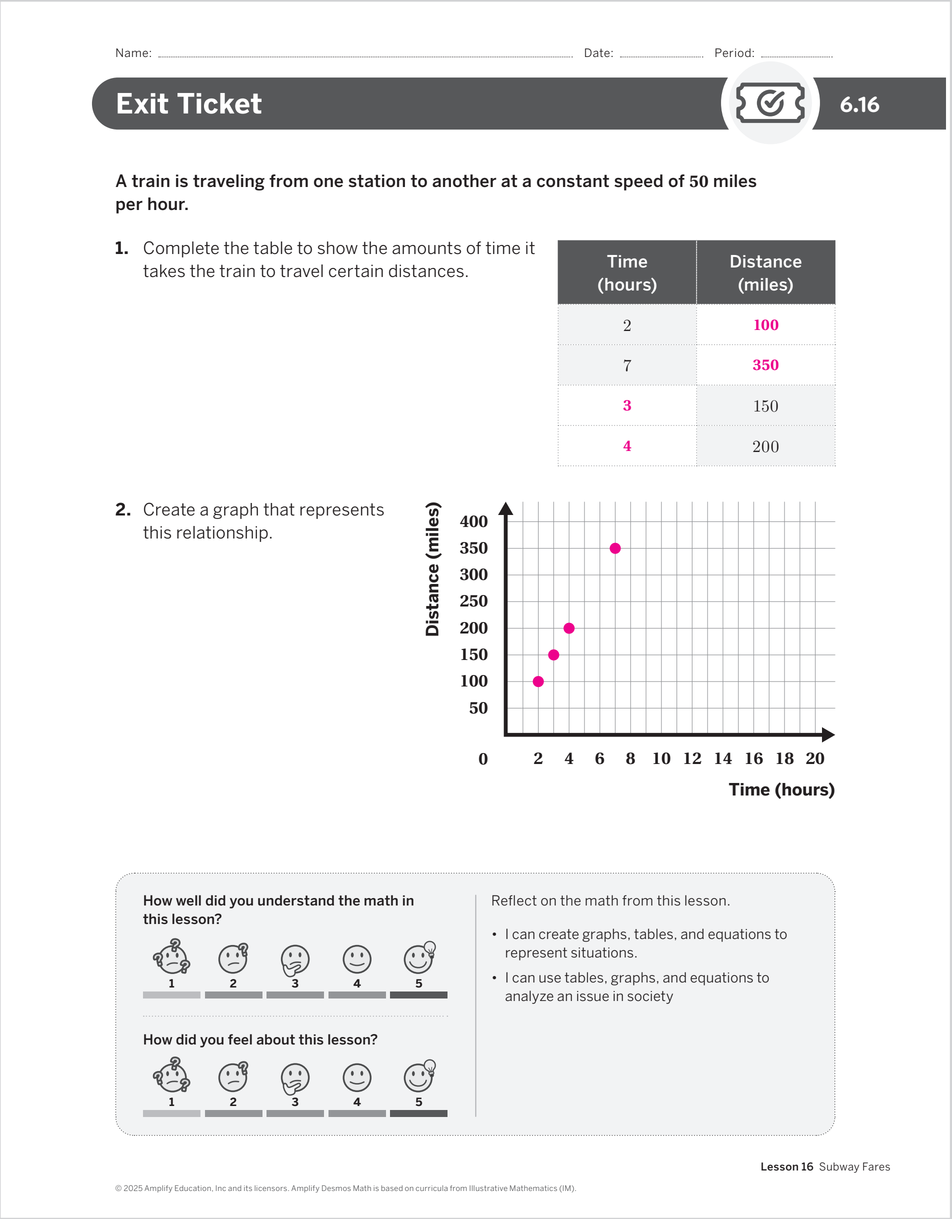
Presentation Screen
5

Learning Goals

Goal: Create graphs, tables, and equations to represent situations.

Language Goal: Interpret representations to analyze an issue in society. **(Reading, Writing, Speaking, and Listening)**

Support for Future Learning: If students struggle, consider reviewing this problem as a class before Practice Day 2 or offering individual support where needed during the practice day.



Name: ________ Date: ________ Period: ________

Exit Ticket 6.16

A train is traveling from one station to another at a constant speed of 50 miles per hour.

1. Complete the table to show the amounts of time it takes the train to travel certain distances.

Time (hours)	Distance (miles)
2	100
7	350
3	150
4	200

2. Create a graph that represents this relationship.

How well did you understand the math in this lesson?

1 2 3 4 5

How did you feel about this lesson?

1 2 3 4 5

Reflect on the math from this lesson.

- I can create graphs, tables, and equations to represent situations.
- I can use tables, graphs, and equations to analyze an issue in society

Lesson 16 Subway Fares

Practice

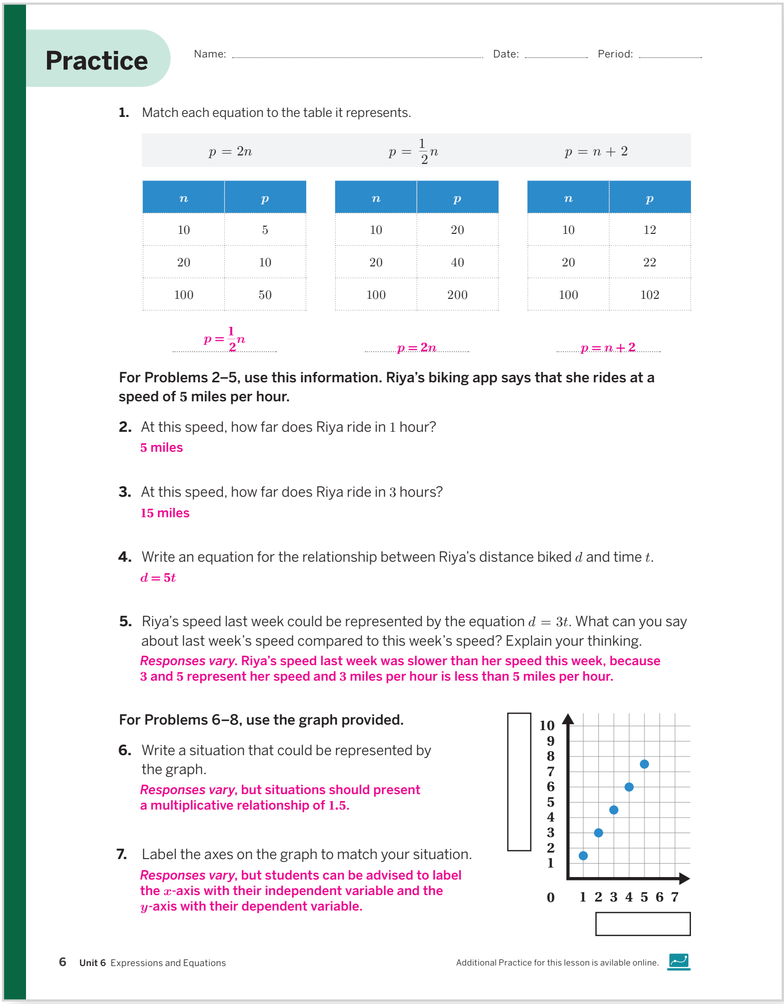
Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.


Lesson 16
Practice

Students using digital

Students using print



Practice

Name: ________ Date: ________ Period: ________

1. Match each equation to the table it represents.

$p = 2n$ $\quad p = \frac{1}{2}n \quad$ $p = n + 2$

n	p
10	5
20	10
100	50

$p = \frac{1}{2}n$

n	p
10	20
20	40
100	200

$p = 2n$

n	p
10	12
20	22
100	102

$p = n + 2$

For Problems 2–5, use this information. Riya's biking app says that she rides at a speed of 5 miles per hour.

2. At this speed, how far does Riya ride in 1 hour?
5 miles

3. At this speed, how far does Riya ride in 3 hours?
15 miles

4. Write an equation for the relationship between Riya's distance biked d and time t.
$d = 5t$

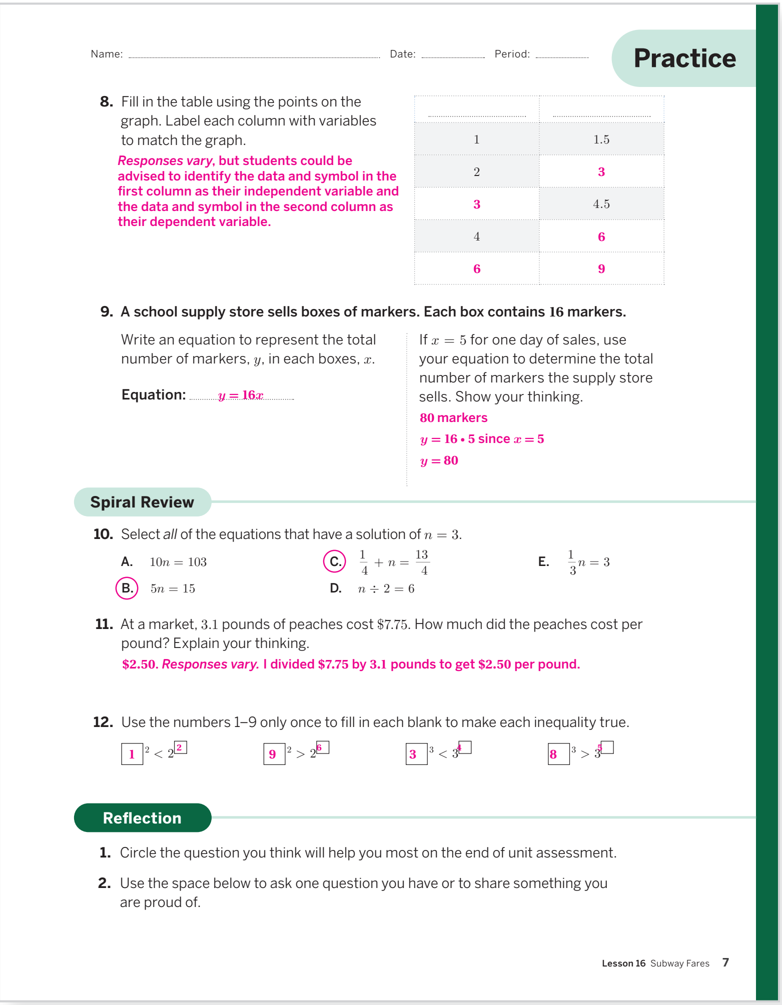
5. Riya's speed last week could be represented by the equation $d = 3t$. What can you say about last week's speed compared to this week's speed? Explain your thinking.
***Responses vary.* Riya's speed last week was slower than her speed this week, because 3 and 5 represent her speed and 3 miles per hour is less than 5 miles per hour.**

For Problems 6–8, use the graph provided.

6. Write a situation that could be represented by the graph.
***Responses vary*, but situations should present a multiplicative relationship of 1.5.**

7. Label the axes on the graph to match your situation.
***Responses vary*, but students can be advised to label the x-axis with their independent variable and the y-axis with their dependent variable.**

Additional Practice for this lesson is avilable online.



Name: ________ Date: ________ Period: ________

Practice

8. Fill in the table using the points on the graph. Label each column with variables to match the graph.
***Responses vary*, but students could be advised to identify the data and symbol in the first column as their independent variable and the data and symbol in the second column as their dependent variable.**

________	________
1	1.5
2	3
3	4.5
4	6
6	9

9. A school supply store sells boxes of markers. Each box contains 16 markers.

Write an equation to represent the total number of markers, y, in each boxes, x.

Equation: $y = 16x$

If $x = 5$ for one day of sales, use your equation to determine the total number of markers the supply store sells. Show your thinking.
80 markers
$y = 16 \cdot 5$ **since** $x = 5$
$y = 80$

Spiral Review

10. Select *all* of the equations that have a solution of $n = 3$.

A. $10n = 103$
[✓ B.] $5n = 15$
[✓ C.] $\frac{1}{4} + n = \frac{13}{4}$
D. $n \div 2 = 6$
E. $\frac{1}{3}n = 3$

11. At a market, 3.1 pounds of peaches cost \$7.75. How much did the peaches cost per pound? Explain your thinking.
\$2.50. *Responses vary.* I divided \$7.75 by 3.1 pounds to get \$2.50 per pound.

12. Use the numbers 1–9 only once to fill in each blank to make each inequality true.

$\boxed{1}^2 < 2^{\boxed{2}}$ $\quad \boxed{9}^2 > 2^{\boxed{6}}$ $\quad \boxed{3}^3 < 3^{\boxed{4}}$ $\quad \boxed{8}^3 > 3^{\boxed{5}}$

Reflection

1. Circle the question you think will help you most on the end of unit assessment.

2. Use the space below to ask one question you have or to share something you are proud of.

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1	1	NY-6.EE.9
	2, 3	1	NY-6.RP.3
	4, 5	2	NY-6.EE.9
	6–8	3	NY-6.RP.3
Test Practice	9	2	NY-6.EE.9, NY-6.EE.2c
Spiral Review			
Fluency	10	1	NY-6.EE.7
	11	1	NY-6.RP.3
	12	3	NY-6.EE.1

GRADE 6

Unit 2
Lesson Plans

Teacher lesson plans from Unit 2 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Ratios and Proportional Relationships** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 6 | Unit 2

Grade 6 Unit 2

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

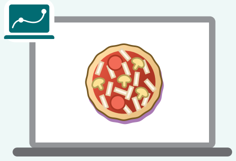
Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

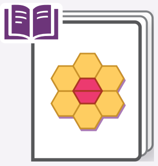
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



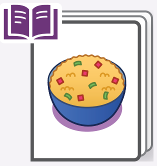
1 Pizza Maker

Informally explore ratios in context.



2 Ratio Rounds

Explain what a ratio is.



3 Rice Ratios

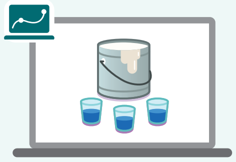
Explain what equivalent ratios are.

Assess and Respond

Quiz: Sub-Unit 1

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2

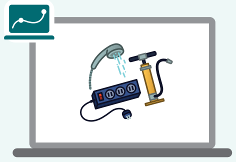


7 Mixing Paint, Part 1

Develop strategies for comparing ratios in context.

8 World Records

Calculate the speed of an object as a unit rate for the distance an object travels over time.



9 Disaster Preparation

Use tables of equivalent ratios to determine large unknown values in context.

14 Lunch Waste

Apply ratio reasoning to answer questions about a real-world situation.

Practice Day

Practice Day 2

Practice the concepts and skills developed during Lessons 7–14. Consider using this time to prepare for the upcoming Quiz.

Summative Assessment

End-of-Unit Assessment

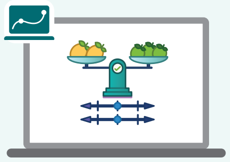
Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing: 19 days | **Short on time?** See pacing considerations below.

Pre-Unit Check: (Optional)
14 Lessons: 45 min each
2 Practice Days: 45 min each
1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min

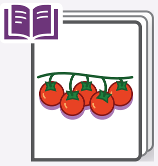
4 Fruit Lab

Generate equivalent ratios and justify that they are equivalent.



5 Balancing Act

Use double number line diagrams to solve problems.



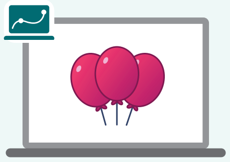
6 Product Prices

Use a double number line or table to calculate a unit price.

Practice Day

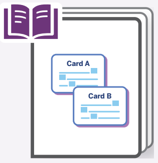
Practice Day 1

Practice the concepts and skills developed during Lessons 1–6. Consider using this time to prepare for the upcoming Quiz.



10 Balloons

Solve problems by reasoning about tables of equivalent ratios and double number line diagrams.



11 Community Life

Determine whether or not you can use equivalent ratios to solve a problem.

12 Mixing Paint, Part 2

Use and interpret tape diagrams to solve problems involving part-part-whole ratios.

13 City Planning

Create and use tape diagrams and tables to help solve problems involving part-part-whole ratios.

Pacing Considerations

Lesson 1: This lesson supports students in developing their intuition reasoning about ratios. If most students demonstrate a strong understanding of ratios in Problems 1, 2, and 5 of the Pre-Unit Check, this lesson may be omitted. If omitted, consider launching Lesson 2 using Screens 1–4 from Lesson 1.

Lesson 11: This lesson supports students in applying what they've learned about ratios to make sense of situations in context. If students show a strong understanding working with ratios in earlier lessons, this lesson may be omitted. If omitted, be sure to highlight different strategies for solving problems using equivalent ratios elsewhere in the unit.

Lessons 14: This lesson gives students an opportunity to use their personal experiences and different kinds of ratio reasoning to make sense of a common experience: school lunch and lunch waste. There is no new content introduced in this lesson.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Pizza Maker (NYC)

Lesson 1: Exploring Ratios

Overview

Students create their own pizzas and use them to explore ratio concepts.

Learning Goals

- Informally explore ratios in context.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students create their own pizzas and use them to explore ratio concepts. Students encounter concepts that *are* in ratio relationships (like number of toppings) and ones that *are not* in ratio relationships (like the temperature of the oven). This lesson is intended to draw on students' intuition and knowledge, and to surface strategies students will revisit throughout the unit.
Note: Students will be formally introduced to the term *ratio* in Lesson 2.

Lesson Summary

Warm-Up (5 minutes)
The purpose of the warm-up is to draw on student knowledge related to the context of the lesson: making pizzas.

Activity 1: Your Pizza (20 minutes)
The purpose of this activity is for students to use a pizza that they create to answer questions about how the number of pizzas affects parts of the pizza-making process (e.g., amount of sauce needed, oven temperature, etc.). Students use informal ratio thinking to determine needed amounts of cheese and sauce for different numbers of pizzas, and answer the question: *If we double the number of pizzas, what else does it make sense to double?*

Activity 2: Ivan's Pizza (10 minutes)
The purpose of this activity is for students students to begin informally developing strategies for solving ratio problems. Students closely examine one student's pizza and figure out how many toppings are needed for different numbers of pizzas. Students will be formally introduced to equivalent ratios in Lesson 3.

Lesson Synthesis (5 minutes)
The purpose of the synthesis is for students to begin describing strategies for solving ratio problems.

Cool-Down (5 minutes)

1 Warm-Up

You're making a

You're making a pizza. What are some things you need?

Teacher Moves

Overview: In this lesson, students create their own pizzas and use them to explore ratio concepts. In this warm-up, students draw on their knowledge related to the context of the lesson: making pizzas.

Launch
- Invite students to share any pizza-related stories they have or any experiences they have making pizza themselves.

Facilitation
- If students struggle, consider asking: *What kind of toppings would you need? What do you need other than toppings?*
- Consider using the snapshot tool or dashboard's teacher view to highlight several responses. Monitor for students who mention ovens or other cooking items, and return to these responses on Screen 5, where students examine oven temperature.

Math Community
- Consider highlighting unique or creative responses.

Suggested Pacing: Screen 1

Sample Responses

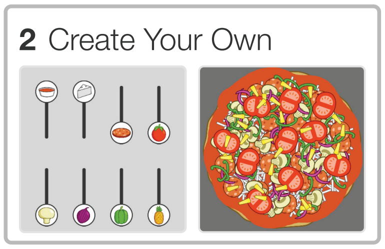
Responses vary.

- Pizza dough or flour and water
- An oven
- Pepperoni or other toppings
- Tomatoes or tomato sauce
- Cheese

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Direct student attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.



Teacher Moves

Overview: Students use a pizza that they make to answer questions about how the number of pizzas affects parts of the pizza-making process (e.g., amount of sauce needed, oven temperature, etc.).

Launch
- Invite students to work *individually.*
- Consider pausing the activity and asking students: *If you could choose anything, what toppings would you include?*

Facilitation
- If students ask, consider acknowledging that this pizza maker has constraints and asking students to make the pizza they would *most like* even if it's not their ideal pizza.
- Consider using the snapshot tool or the dashboard's teacher view to highlight several student pizzas. Ask students to share about their inspiration.

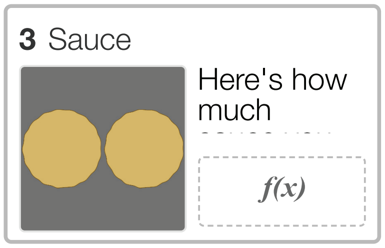
Math Community
- Consider asking a question like: *Pineapple—yay or nay?*

Note: Because of the constraints of the lesson, every pizza requires some cheese and sauce. Consider inviting students to think of alternate cheeses and sauces they could use.

Suggested Pacing: Screen 2

Sample Responses

Pizzas vary.



Here's how much sauce you used for 1 pizza.

How much sauce do you need for 2 pizzas?

Teacher Moves

Launch
- Share with students that they will be making more copies of the pizzas they designed.

Facilitation
- If students are struggling, consider inviting them to enter any number and use the feedback to revise their thinking.

Early Student Thinking

- Students may think that one more pizza requires one more ounce of sauce. Encourage them to reflect on why this is not enough sauce.

Suggested Pacing: Screen 3–6

Sample Responses

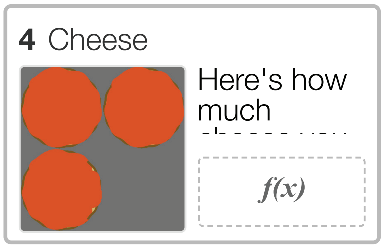
Responses vary. The amount of sauce for two pizzas should be double what students used on one pizza.

Student Supports

Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Direct student attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.



4 Cheese

Here's how much

Here's how much cheese you used for 2 pizzas.

How much cheese do you need for 3 pizzas?

Teacher Moves

Facilitation

- If students are struggling, consider asking: *How much cheese would you need for* 1 *pizza?*

Sample Responses

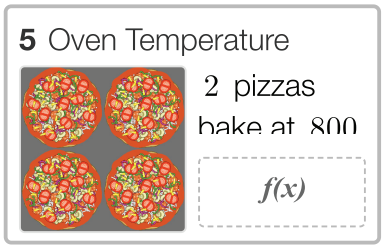
Responses vary. The amount of cheese should be $\frac{3}{2}$ the amount of cheese for 2 pizzas.

Student Supports

Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Invite students to draw out the problem (e.g., circles to represent the pizzas and lines to represent the cheese on the pizzas) to support them in their thinking.



5 Oven Temperature

2 pizzas bake at 800

2 pizzas bake at 800 °F.

What should the oven temperature be for 4 pizzas?

Teacher Moves

Facilitation

- Emphasize the range of reasonable responses on this screen. It's okay—even desirable—to lack consensus. Students are not expected to have previous knowledge about oven temperatures.
- Consider snapshotting different responses and inviting students to share their reasoning (MP3).

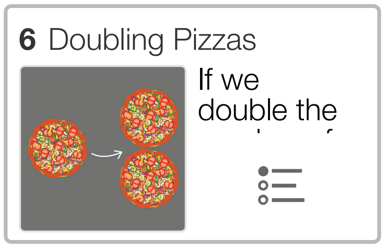
Discussion Questions

- *What would happen if we tried to cook* 10 *pizzas?* 100 *pizzas?*

Note: 800 °F is a typical pizza oven temperature, which may differ from home ovens.

Sample Responses

Responses and explanations vary.



6 Doubling Pizzas

If we double the

If we double the number of pizzas, it makes sense to double the amount of sauce.

What else do you think it makes sense to double?

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface features of real-world situations that are and are not in ratio relationships.

Facilitation

- When most students have responded, discuss each item one at a time as a class, inviting students to justify whether or not it makes sense to double that item when you double the number of pizzas (MP3).

Discussion Questions

- *What else makes sense to double when you double the number of pizzas? What else doesn't make sense?*

Early Finishers

- Encourage students to write down two more things that do make sense to double and two more things that don't make sense to double.

Math Community

- Consider asking if any students revised their thinking and to give credit to the student who helped them change their mind.

Sample Responses

Responses vary.

Student Supports

Students With Disabilities

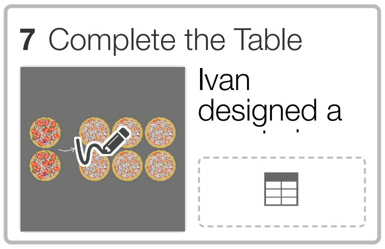
- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners

- *Receptive Language: Eliminate Barriers*

Discuss what each choice means before students respond, including pictures if appropriate.



7 Complete the Table

Ivan designed a

Ivan designed a vegetarian pizza.

The table shows what is needed to make 2 pizzas.

What is needed to make 6 pizzas?

Teacher Moves

Overview: Students begin informally developing strategies for solving ratio problems.

Launch

- Share that we will be making lots of a pizza someone else designed.
- Direct students' attention to the calculator button. Encourage them to use it whenever they find it helpful.

Facilitation

- Monitor for students who use the sketch tool to highlight structure in the image on the left. Consider sharing this with the class to support students later in the lesson (MP7).

Readiness Check (Problem 1)

- If students struggled, consider spending extra time on this screen. Invite students to share the strategies they used to determine the ingredients needed to make 6 pizzas.

Suggested Pacing: Screen 7–9

Sample Responses

Tomato: 60 slices

Onion: 90 slices

Pepper: 72 slices

Student Supports

Students With Disabilities

• *Conceptual Processing: Eliminate Barriers*

Invite students to use the sketch tool to draw out the problem to support them in their thinking.

8 Your Favorite Mistake

It takes 20 tomato

It takes 20 tomato slices to make 2 of Ivan's pizzas.

Two students made a mistake when making 6 pizzas.

Select your favorite mistake.

Teacher Moves

Facilitation

- When most students have responded, facilitate a class discussion.
- Consider monitoring for students who discuss what each fictional student might have been thinking. Invite these students to share.

Discussion Questions

- *What do you think Ivan was thinking? How might he have gotten* 120 *?*
- *What do you think Jada was thinking? How might she have gotten* 24 *?*

Math Community

- Consider inviting students to share what they think we can learn from looking at incorrect thinking.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language.

Sample Responses

Responses vary.

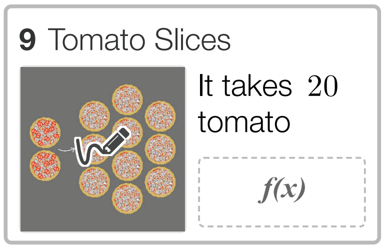
- **Ivan:** Make sure to pay attention to how many pizzas you already have.
- **Jada:** It may be helpful to think about how many tomato slices an individual pizza has on it.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



9 Tomato Slices

It takes 20 tomato

It takes 20 tomato slices to make 2 of Ivan's pizzas.

How many does it take to make 10 pizzas?

Teacher Moves

Facilitation

- Invite students to compare their strategy with a classmate. If they solved the problem the same way, encourage students to think of a different strategy (MP7).

Discussion Questions

- *There is often more than one way to solve a problem. Can you think of a different way to get the same answer?*

Early Finishers

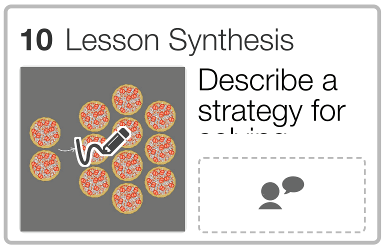
- Encourage students to think about how many pizzas they could make with 165 tomato slices, and how they would distribute the tomatoes.

Math Community

- Consider inviting students to share the value of having more than one strategy for the same problem.

Sample Responses

100 slices



10 Lesson Synthesis

Describe a strategy for

Describe a strategy for solving problems like the one on the previous screen.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to begin describing strategies for solving ratio problems.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Math Community

- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Suggested Pacing: Screen 10

Sample Responses

Responses vary.

- Figure out how many you need for each pizza and then multiply by the number of pizzas you have.
- Figure out how many groups you have and multiply by that number. There are 5 groups of 2 pizzas, so you just need to multiply the number of tomato slices by 5.

Student Supports

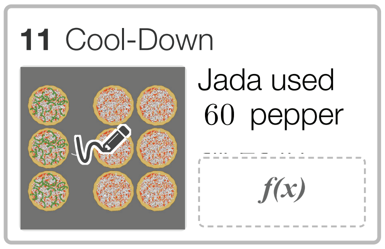
Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., First, ________. Then, _______.).

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



11 Cool-Down

Jada used 60 pepper

Jada used 60 pepper slices to make 3 pizzas.

How many does it take to make 6 of Jada's pizzas?

Teacher Moves

Support for Future Learning

- Students will have more chances to develop their understanding of ratios throughout Unit 2.

Suggested Pacing: Screens 11–12

Sample Responses

120 pepper slices

12

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can determine which quantities make sense to double in context and which do not.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Ratio Rounds (NYC)

Lesson 2: Describing Ratios

Purpose

The purpose of this lesson is for students to learn what ratios are and several ways to describe them. Students are given cards that have different numbers of pizzas and toppings, and they find partners using instructions related to the ratios of toppings. In pairs, students practice describing ratios (_____ of *these* for every ______ of *those*; the ratio of *these* to *those* is ____ to ____) and using ____ : ____ notation. Students also analyze descriptions of ratios.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Ratio Cards

- Print and cut out one card for each student.
- For classes with fewer than 24 students, distribute the cards in the order they appear. For example, for a class of 20 students use the first 20 cards.
- For classes with more than 24 students, print additional copies of the original cards. For example, for a class of 30 students use all of the cards and a second copy of the first 6 cards.

Warm-Up (10 minutes)

Overview: Students engage in the Number Talk routine to surface strategies for multiplying whole numbers. This is part of a series of warm-ups to strengthen strategies around multiplication.

Launch

- Display Sheet 1 of the Teacher Projection Sheets.
- Remind students that the · is a symbol for multiplication.
- Consider sharing that the purpose is to hear as many different strategies as possible.

How Number Talk Works

- Give students one minute to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own or one of the possible strategies in the sample responses.
- Select several students to share different strategies. Use the sample responses as examples of possible student strategies. Record strategies for all to see, along with the name of the student who shared each one.
- Display the next expression and repeat. If it makes sense, encourage students to use a strategy described by a classmate in the previous round.

Activity 1: Ratio Rounds (10 minutes)

Overview: Students learn three ways to describe ratios. The structure of this activity supports student collaboration with many different partners and allows for movement around the classroom.

Launch

- Distribute one ratio card and a Student Worksheet to each student.
- Give students one minute to figure out the number of each topping on their pizza. Share that the possible toppings are mushrooms, pepperoni, pineapple, and tomatoes.

Facilitation

- This activity is composed of three rounds.
- For each round, display the prompt to help students find their partner (Sheets 5, 7, and 9).
- Give students one minute to find a new partner for the round and introduce themselves.
- Display the example and clarify the example as a class (Sheets 6, 8, and 10).
- Give partners two minutes to describe both their and their partner's ratios on their worksheet.

There are many possible teacher moves when facilitating this activity. Consider selecting or modifying these suggestions to meet the culture and needs of your class:

- Set a timer or play music while students look for a partner. When the timer goes off, students pair up with the person closest to them who meets the requirements.
- Consider inviting students to sit with their partner to have a surface for writing and to allow students still looking for a partner to easily identify unpaired classmates.

Note: If students are unable to find a partner in a given round (i.e., there is an odd number of students, one or more students think none of their classmates' figures meet the requirement, etc.), invite them to join any pair of students.

Math Community

- Consider adding get-to-know-you questions for partners to answer after introducing themselves and before sharing their ratios.

Intermission (5 minutes)

Overview: Students practice writing their own descriptions of ratios.

Launch

- Display Sheet 11 of the Teacher Projection Sheets.

Facilitation

- Give students 1–2 minutes to respond quietly on their worksheet.
- Invite them to share their descriptions with a partner and add any they didn't have.
- Facilitate a whole-class conversation where students share ratios they noticed and how they described them.

Discussion Questions

- *What other ways could we describe the same ratio?*
- *What other ratios do you see in the image?*

Math Community

- Consider highlighting unique ratios (e.g., For every 8 slices, there are 8 mushrooms.).

Activity 2: Two Truths and a Lie (10 minutes)

Overview: Students analyze descriptions of ratios and write their own (MP6).

Launch

- Invite students to work *in pairs.*
- Display Sheet 12 of the Teacher Projection Sheets.
- This may be students' first time engaging in the game Two Truths and a Lie. Consider asking any students who have played the game to share how it works (e.g., you are given three sentences, two of which are true and one of which is false. You need to figure out which sentence is false).

Facilitation

- Discuss the question on Sheet 12 of the Teacher Projection Sheets so that students understand the structure of the activity.
- Give students about five minutes to work through the activity. Encourage students to share their reasoning with a partner and work to reach an agreement about which statement is false.

Math Community

- If time allows, consider including a personal Two Truths and a Lie about yourself.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

- The purpose of this discussion is to surface the three different ways of describing ratios.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their response based on the discussion.

Discussion Questions

- *What other ratios do you see in this diagram?*
- *What is helpful about using the colon? What is helpful about saying "for every"?*

Cool-Down (5 minutes)

Support for Future Learning

- Students will have more chances to describe ratios and think about equivalent ratios in Lesson 3.

Rice Ratios (NYC)

Lesson 3: Introduction to Equivalent Ratios

Purpose

This lesson introduces students to the term *equivalent ratios*, meaning two ratios that have the same relationship but different quantities. Students use diagrams and their intuition to generate equivalent ratios. Students will formalize strategies for creating equivalent ratios in Lesson 4.

Preparation

Worksheet

- *Activity 1–2:* Print one sheet for each student. If facilitating Activity 2 in stations or as Level Up, cut the recipes on Pages 2–3.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Warm-Up (10 minutes)

Overview: Surface students' strategies for multiplying whole numbers. This is part of a series of warm-ups to strengthen students' strategies around multiplication.

Launch

- Invite students to work *individually.*
- Display Sheet 1 of the Teacher Projection Sheets.

Facilitation

- Display each of the five Teacher Project Sheets one at a time.
- Use the instructional routine Number Talk to help students look for and make use of the structure of the expressions in order to develop and name strategies (MP7).

How Number Talk Works

- Give students one minute to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own or one of the possible strategies in the sample responses.
- Select several students to share different strategies. Use the sample responses as examples of possible student strategies. Record strategies for all to see, along with the name of the student who shared each one.
- Display the next expression and repeat. If it makes sense, encourage students to use a strategy described by a classmate in the previous round.

Math Community

- Consider inviting students to share if any of the strategies from the last number talk were helpful to them here and to name students whose ideas they found helpful.

Activity 1: Rice Advice (10 minutes)

Overview: Students are introduced to the concept of *equivalent ratios* in the context of making rice.

Set Up and Launch

- Invite students to work *in pairs.*
- Display Sheet 6 of the Teacher Projection Sheets.
- Consider asking: *Does anyone in your family make rice? Have you ever made rice?*

Facilitation

- Give students one minute to discuss with a partner how the recipes are the same and different.
- Invite several students to share. Consider recording students' ideas on the board to refer back to throughout the lesson and unit.
- If it does not come up naturally, consider asking: *Do you think the rice from each bag would taste the same or different? Why?*
- Distribute one worksheet to each student and have students write their responses on paper.
- Introduce the term *equivalent ratio*, then invite students to answer Problems 2–4.
- When most students have completed Problem 3, hold a discussion about equivalent ratios.
- **Routine (optional):** Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their definition of equivalent ratios.

Key Discussion 🔑

How would you explain to someone what equivalent ratios are?

Math Community

- Invite students to use what they know about cooking or recipes as they work.

Readiness Check (Problems 2 and 3)

- If students struggled with Problem 2, consider reviewing it between Activities 1 and 2. If possible, make connections between the strategies students used on Problem 2 and the language of equivalent ratios.
- If students struggled with Problem 3, consider asking students: *How are equivalent fractions similar to and different from equivalent ratios?*

Activity 2: Rice Around the World (15 minutes)

Overview: Students use equivalent ratios to adapt rice recipes from around the world (MP8).

Set Up and Launch

- Consider asking: *What is your favorite rice dish? When or where do you eat it?*
- Display Sheet 7 of the Teacher Projection Sheets and share that these are four rice dishes that we'll examine in today's lesson.

Facilitation

There are several ways to facilitate this activity:

1. Invite students in each pair to complete the task for the first recipe independently, then to come to an agreement about how much of each ingredient is needed. When students agree, continue to the next recipe.
2. Distribute one set of recipes per pair. Invite one student in each pair to share their thinking for the first recipe aloud while their partner writes and asks clarifying questions. After both students agree on the first recipe, invite students to switch who writes and who talks.
3. **Level Up:** Cut up the recipes. Make a pile for each recipe at the front of the room for students to pick up. Distribute one copy of "Jollof Rice" to each pair. Once they complete "Jollof Rice," review their thinking and highlight any incorrect solutions for students to try again. Once students have successfully completed "Jollof Rice," invite them to pick up a copy of "Arroz Con Leche" to complete. Continue this process until students have completed all four tasks.

When most students have completed Arroz Con Leche, consider discussing Valeria's thinking as a class.

Discussion Questions

- *What might Valeria have been thinking? How could we show her that her idea is incorrect?*

Math Community

- Invite students to share any experiences they may have with the dishes listed here, or other rice dishes their families make.

Early Finishers

- Encourage students to choose one of the recipes and determine the ingredients needed to make the dish for the whole class.

Note: Click the links below to view the complete recipes used in this activity. Note that some details of the recipes have been modified to fit this lesson. This is especially true of Arroz con Leche, which requires about twice as much liquid as listed in the student-facing ingredients in order to have the proper milky consistency.

Jollof Rice, Arroz Con Leche, Champorado, Risotto

Lesson Synthesis (5 minutes)

Key Discussion 🔑

The purpose of this discussion is to solidify students' understanding of equivalent ratios.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their response based on the discussion.

Discussion Questions

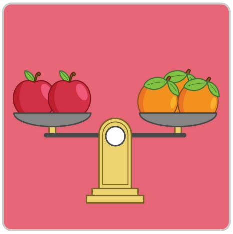
- *How would you say what ________ said in your own words?*
- *How do you know the ratio you created is equivalent to Bags A and B?*

Cool-Down (5 minutes)

Support for Future Learning

Students will have more chances to explore equivalent ratios in Lesson 4.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Fruit Lab (NYC)

Lesson 4: Creating Equivalent Ratios

Overview

Students explore how to generate equivalent ratios in the context of balancing fruit on scales.

Learning Goals

- Explain that multiplying each amount by the same number yields an equivalent ratio.
- Decide whether two ratios are equivalent.
- Generate equivalent ratios and justify that they are equivalent.

Vocabulary

- table

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to explore how to generate equivalent ratios in the context of balancing fruit on scales. This builds on what students learned in Lesson 3 about what equivalent ratios are. By the end of this lesson, students should be able to explain that multiplying each amount by the same number yields an equivalent ratio.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce the context of balancing fruits and for students to begin to generate equivalent ratios. Students adjust the numbers of apples and oranges on a scale to create several ways to balance the scale.

Activity 1: Comparing Apples to Oranges (5 minutes)

The purpose of this activity is for students to analyze a set of equivalent ratios and generate equivalent ratios for one relationship. This activity prepares students to explore several different ratios of fruits in Activity 2.

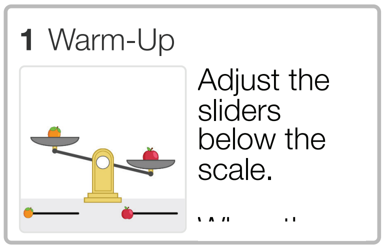
Activity 2: Fruit Lab (25 minutes)

The purpose of this activity is for students to explore strategies for generating equivalent ratios and determining whether two ratios are equivalent or not. Students first explore in the Fruit Lab, then analyze several different fictional students' strategies for creating equivalent ratios. Students should leave this activity recognizing which operations do and do not create equivalent ratios.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to describe how to determine equivalent ratios that balance the scale when they know a ratio that does.

Cool-Down (5 minutes)



1 Warm-Up

Adjust the sliders below the scale.

Adjust the sliders below the scale.

When the scale balances, press "Record."

Find as many ways as you can to balance the scale.

Teacher Moves

Overview: In this lesson, students explore how to generate equivalent ratios in the context of balancing fruit on scales. This warm-up introduces the context of balancing fruits and invites students to generate equivalent ratios.

Launch

- Consider using the dashboard's student view to drag each slider and invite students to notice what happens to the scale.

Facilitation

- Give students 2–3 minutes to find as many ways as they can to balance the scale.

Early Finishers

- Encourage students to think of other combinations of apples and oranges that would balance the scale.

Note: In this lesson, we assume that all fruits of the same type have the same weight. For example, all apples weigh the same.

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

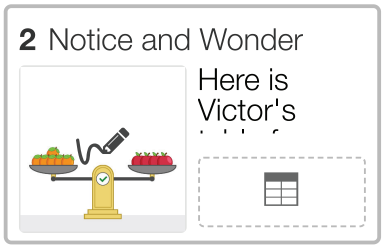
3 oranges and 2 apples
6 oranges and 4 apples
9 oranges and 6 apples
12 oranges and 8 apples
15 oranges and 10 apples
18 oranges and 12 apples

Student Supports

Students With Disabilities

- *Fine Motor Skills: Strategic Pairing*

Allow students who struggle with fine motor skills to dictate use of the graph and sketch tool as needed throughout the lesson.



2 Notice and Wonder

Here is Victor's

Here is Victor's table from the previous screen.

Teacher Moves

Overview: In this short activity (Screens 2–3), students analyze a set of equivalent ratios and generate equivalent ratios for one relationship.

Launch
- Invite students to work *individually.*

Facilitation
- Select several student responses using the snapshot tool.
- Monitor for students who make connections to Lesson 3 about equivalent ratios.
- When most students have responded, facilitate a discussion of student responses to both this screen and the following screen.

Math Community
- Highlight both informal and formal noticings. Invite students to share if they had noticings or wonderings that are similar to the ones shared.

Suggested Pacing: Screens 2–3

Sample Responses

Responses vary.

- I notice that there are always more oranges than apples.
- I notice that the number of apples is always even.
- I notice that the top row is 5 times the middle row and the last row is 2 times the middle row.
- I notice that the rows are all equivalent ratios.

- I wonder how many apples you could fit on the scale.
- I wonder if you could balance with 3 apples or another odd number.
- I wonder if you can always make a new combination that balances.

3 Balance Another

Here is Victor's

Here is Victor's table from the previous screen.

Enter another equivalent ratio in the last row.

Try to find one that you think no one else will think of.

Teacher Moves

Facilitation

- Encourage students to use the feedback on the screen to help them revise their thinking.
- Select several equivalent and not equivalent ratios. Facilitate a discussion around students' ratios and their responses from the previous screen.

Discussion Questions

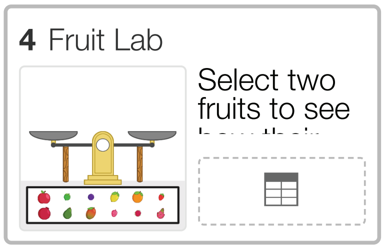
- *What are equivalent ratios? Where do we see equivalent ratios here?*
- *How is balancing apples and oranges similar to the recipes from last lesson?*
- *Is this ratio of apples to oranges equivalent to the original? How do you know?*
- *What do you think would happen if we wanted to balance* 400 *apples?*

Early Finishers

- Encourage students to try several different ratios of apples to oranges, then to describe a strategy for checking whether a ratio will balance the scale without pressing "Try It."

Sample Responses

Responses vary. Any ratio of oranges to apples equivalent to $3:2$.



4 Fruit Lab

Select two fruits to see

Select two fruits to see how their weights compare.

On paper, write several equivalent ratios of those fruits. Check your work below.

Teacher Moves

Overview: In Activity 2 (Screens 4–7), students explore strategies for generating equivalent ratios and determining whether two ratios are equivalent or not. Students should leave this activity recognizing that adding the same number to both values in a ratio does not create an equivalent ratio but multiplying both values by the same number does.

Launch

- Invite students to work *in pairs.*
- Consider demonstrating how the Fruit Lab works using the dashboard's student view.
- Consider inviting students to share their favorite fruits or any stories they have about the fruits in the Fruit Lab. There may be fruits that are new for students.
- Distribute one supplement to each student.

Facilitation

- Give students 5–10 minutes to select pairs of fruits, then to generate several equivalent ratios of those fruits.
- Encourage students to record their equivalent ratios in tables to help them notice patterns and develop strategies (MP8).
- Encourage students to share the strategies they are using with their partner, then to test those strategies on new pairs of fruits.

Early Student Thinking

- Students might always double the given amounts to create equivalent ratios.
- Consider encouraging these students to try out other strategies for creating equivalent ratios, then to use the "Check My Work" button to test their strategies.

Suggested Pacing: Screen 4

Sample Responses

Responses vary based on choice of fruits.

Are You Ready for More? (on paper)

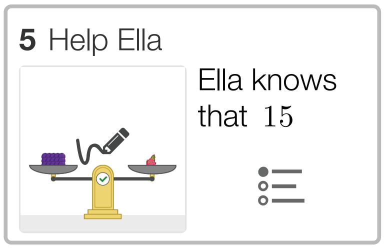
Responses and explanations vary. A ratio of 11 kiwis to 10 pears would balance. I can use equivalent ratios to determine that 12 peaches balance with both 33 kiwis and 30 pears, so those amounts of kiwis and pears will balance with one another. $33 : 30$ is equivalent to $11 : 10$.

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

Check in with individual students to assess for understanding as needed at each step of the activity.



5 Help Ella

Ella knows that 15

Ella knows that 15 grapes balance with 1 dragon fruit.

She says 16 grapes will balance with 2 dragon fruits.

Will this $16 : 2$ ratio balance the scale?

Teacher Moves

Facilitation

- When most students have responded, consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.
- If there is conflict, consider inviting students to share their reasoning for each choice (MP3). Students can go back to the previous screen to test out Ella's claim.

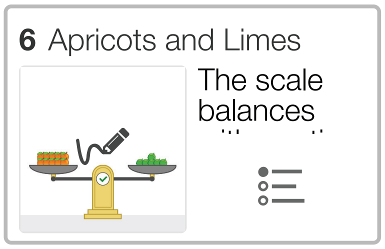
Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify to help students with communicating their ideas.

Suggested Pacing: Screen 4–7

Sample Responses

Will not balance

Explanations vary. Ella doubled the amount of dragon fruits but didn't double the amount of grapes, so the dragon fruit side will be heavier than the grape side.



6 Apricots and Limes

The scale balances

The scale balances with a ratio of 10 apricots to 6 limes.

Select **all** of the equivalent ratios.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to ensure students have strategies for determining whether two ratios are equivalent, and to generalize that multiplication and division create equivalent ratios while addition and subtraction do not.

Facilitation

- When most students have responded, discuss each ratio one by one as a class.
- Invite students to share their reasoning, then to build on the reasoning of others (MP3).
- If it is helpful, students can check their thinking using the Fruit Lab screen.

Discussion Questions

- *What might a person who wrote this ratio be thinking?*
- *Is this ratio equivalent to* $10:6$*? How do you know?*
- *In general, what strategies work for creating equivalent ratios? Why does that make sense to you?*

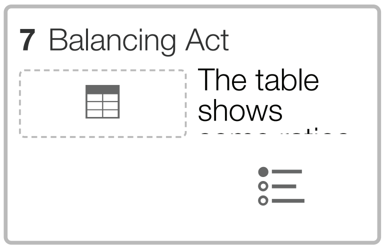
Math Community

- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their definition of how to create equivalent ratios.

Sample Responses

- 50 apricots to 30 limes
- 5 apricots to 3 limes


7 Balancing Act

The table shows

The table shows some ratios of limes to lychees that balance the scale.

Dyani says 22 limes will balance with 55 lychees.

Will the $22:55$ ratio balance?

Teacher Moves

Facilitation

- If time allows, consider inviting students to share their thinking.

Early Student Thinking

- Students may say that this ratio will not balance because it involves adding $20+2$ and $50+5$.
- Consider asking these students: *Could we use multiplication to get the ratio* $22:55$*?* Invite students to return to the Fruit Lab screen to try 22 limes and 55 lychees.

Early Finishers
- Encourage students to create their own ratio, then to trade with a classmate and invite them to write equivalent ratios.

Sample Responses

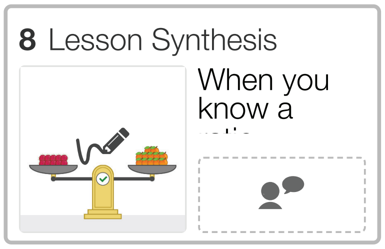
Will balance

Explanations vary. $2 \cdot 11 = 22$ and $5 \cdot 11 = 55$, so it will balance because you just have 11 groups of fruits that balance.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



8 Lesson Synthesis

When you know a

When you know a ratio balances a scale, how can you create equivalent ratios that also balance the scale?

Use the example if it helps you show your thinking.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to surface strategies for generating equivalent ratios.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions
- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Math Community
- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Suggested Pacing: Screen 8

Routine (optional): Consider using the mathematical language routine Collect and Display to collect strategies for generating equivalent ratios. Consider adding to any visual display related to equivalent ratios from the previous lesson.

Sample Responses

Responses vary. You can create equivalent ratios by multiplying or dividing both numbers in the ratio by the same number. For example, the ratio of rambutans to apricots is $10:12$. This means that $20:24$ and $5:6$ will also balance because they are multiplied or divided by 2. Also, $100:120$ would balance because that is just multiplying both numbers by 10.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., When you know a ratio balances a scale, you can create equivalent ratios by ________.).

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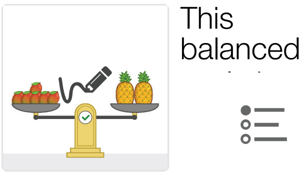
9 Cool-Down



This balanced

This balanced scale has 6 mangos and 2 pineapples.

Select **all** of the combinations that will balance the scale.

Teacher Moves

Support for Future Learning

- If students struggle, consider reviewing this screen as a class before Lesson 5. Understanding how to create equivalent ratios will help students as they begin determining missing quantities in Lessons 5 and 6.

Suggested Pacing: Screens 9–10

Sample Responses

- 3 mangos and 1 pineapple
- 24 mangos and 8 pineapples
- 60 mangos and 20 pineapples

10

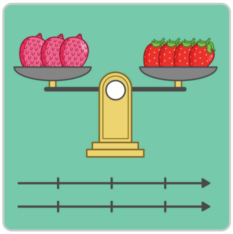
This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain which operations create equivalent ratios.
- I can decide whether two ratios are equivalent.
- I can create equivalent ratios and explain how I know they are equivalent.

Digital Lesson

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Balancing Act (NYC)

Lesson 5: Introduction to Double Number Lines

Overview

Students are introduced to the double number line as a tool for solving problems involving equivalent ratios.

Learning Goals

- Explain how to use a double number line to generate equivalent ratios.
- Use double number line diagrams to solve problems.

Vocabulary

- double number line

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Students are introduced to the double number line as a tool for solving problems involving equivalent ratios. This lesson builds on what students learned about equivalent ratios in Lessons 3 and 4. Students use fully and partially labeled double number lines to answer questions about balancing different fruits.

Lesson Summary

Warm-Up (10 minutes)

In this warm-up, students engage in the Number Talk routine to surface strategies for multiplying whole numbers. This is part of a series of warm-ups to strengthen strategies around multiplication.

Activity 1: Introducing Double Number Lines (15 minutes)

The purpose of this activity is to introduce students to double number lines. Students make connections between the scale representation from Lesson 4 and the double number line.
In this activity, the double number lines are fully labeled and students interpret them in order to solve problems about equivalent ratios.

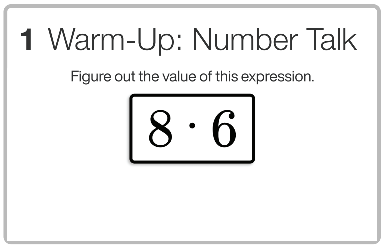
Activity 2: Using Double Number Lines (10 minutes)

The purpose of this activity is for students to use partially labeled double number lines to solve problems. This invites students to make sense of how to construct double number lines on their own in order to solve problems.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to summarize how to use a double number line diagram to solve problems with ratios.

Cool-Down (5 minutes)



1 Warm-Up: Number Talk

Figure out the value of this expression.

$\boxed{8 \cdot 6}$

Teacher Moves

Overview: In this lesson, students are introduced to the double number line as a tool for solving problems involving equivalent ratios. In this warm-up, students engage in the Number Talk routine to surface strategies for multiplying whole numbers.

Launch

- Use the dashboard's student view to display Screen 1.
- Invite students to share any strategies they remember from previous number talks in this unit.

How Number Talk Works

- Give students one minute to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own or one of the possible strategies in the sample responses.
- Select several students to share different strategies. Use the sample responses as examples of possible student strategies. Record strategies for all to see, along with the name of the student who shared each one.
- Display the next expression and repeat. If it makes sense, encourage students to use a strategy described by a classmate in the previous round.

Suggested Pacing: Screens 1–4, one screen at a time

Sample Responses

$8 \cdot 6 = 48$

Strategies students might use in the Number Talk

Student Supports

Students With Disabilities

- *Fine Motor Skills: Strategic Pairing*

Allow students who struggle with fine motor skills to dictate use of the graph and sketch tool as needed throughout the lesson.

2 Warm-Up: Number Talk

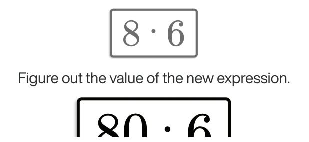

8 · 6

Figure out the value of the new expression.

80 · 6

Sample Responses

$80 \cdot 6 = 480$

Strategies students might use in the Number Talk

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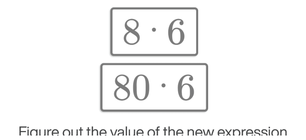
3 Warm-Up: Number Talk


8 · 6

80 · 6

Figure out the value of the new expression

Sample Responses

$79 \cdot 6 = 474$

Strategies students might use in the Number Talk

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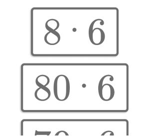
4 Warm-Up: Number Talk


8 · 6

80 · 6

Sample Responses

$48 \cdot 6 = 288$

Strategies students might use in the Number Talk

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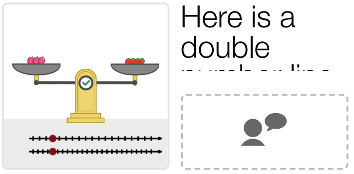
5 Double Number Lines


Here is a double

Here is a double number line.

Drag the movable points to explore how it works.

What do you notice? What do you wonder?

Teacher Moves

Overview: In Activity 1 (Screens 5–9), students are introduced to double number lines.

Launch

- Arrange students into *pairs.*
- Invite students to share why they think this is called a double number line.
- Consider using the dashboard's student view to demonstrate how to move the movable points on the double number line.

Facilitation

- Encourage students to read others’ responses and decide if others' noticings and wonderings were similar to or different from their own.

Math Community
- Consider referring back to students’ noticings and wonderings throughout the lesson where applicable.

Readiness Check (Problem 4)
- If students struggled, consider writing an incorrect solution to this problem and reviewing it as a class.

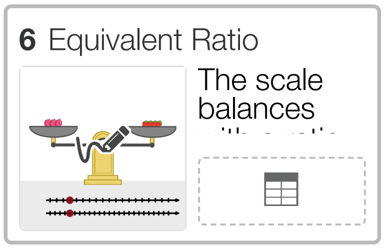
Suggested Pacing: Screens 5–6

Sample Responses

Responses vary.

- I notice that moving the top point changes the number of lychees and moving the bottom point changes the number of strawberries.
- I notice that the top is going by 3 s and the bottom is going by 4 s.
- I notice that if the points line up, then the scale balances.
- I notice that if you make a ratio of the top and bottom numbers that line up, they are all equivalent to $3:4$.

- I wonder if the lines go on forever.
- I wonder if you can make the scale balance any other way.
- I wonder how to balance the scale with 6 strawberries.



6 Equivalent Ratio

The scale balances

The scale balances with a ratio of 3 lychees to 4 strawberries.

1. Enter several equivalent ratios in the table. Try to find one that none of your classmates will.

Teacher Moves

Facilitation
- Encourage students to use the sketch tool to show where their ratios appear on the double number line.
- If time allows, use the snapshot tool to select and display several ratios. Monitor for students who use creative ratios such as $4.5:6$. Make connections between the double number line and the scale.

Discussion Questions
- *How can you use the double number line to decide if the scale will balance?*

- *Someone said that the ratio of* 8 *lychees to* 11 *strawberries balances. What advice would you give them?*

Early Finishers

- Encourage students to enter ratios whose number do not directly appear on the double number line.

Sample Responses

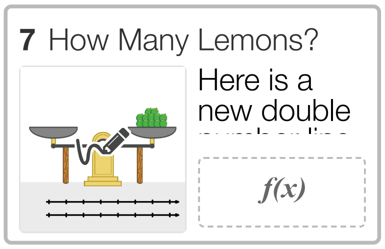
Responses vary. Any ratio equivalent to 3 lychees $:$ 4 strawberries.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time. This may include reading the information in the table and in the graph.



7 How Many Lemons?

Here is a new double

Here is a new double number line.

The scale balances with a ratio of 4 lemons to 6 limes.

How many lemons will balance with 12 limes?

Teacher Moves

Launch

- Consider asking: *Where do you see the* 4 *lemons to* 6 *limes ratio in the double number line?*
- Invite students to use the sketch tool to show their thinking on the double number line.

Facilitation

- Encourage students to use the feedback on the screen to help them revise their thinking.

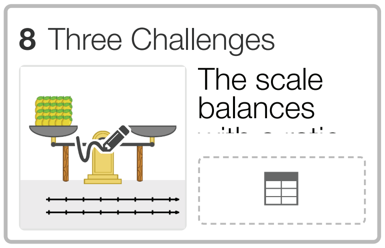
Early Student Thinking

- Students might say that you need 18 lemons because they noticed that the ratio $12 : 18$ is on the double number line. Consider asking students what the 12 on each number line represents.

Suggested Pacing: Screens 7–9

Sample Responses

8 lemons



8 Three Challenges

The scale balances

The scale balances with a ratio of 4 lemons to 6 limes.

Use the double number line to help solve each challenge.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface how students used the double number line to solve problems involving equivalent ratios.

Facilitation

- Encourage students to draw their own number line on blank paper if it is helpful.
- When most students have responded, discuss each question one by one, inviting students to share how they used the double number line or other strategies to support their thinking.

Discussion Questions

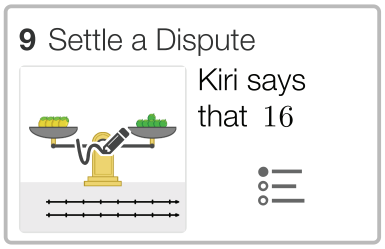
- *What strategy did you use? How was the double number line helpful?*
- *Which challenge was the most challenging for you? Why?*
- *Are there any questions that would be hard to answer using a double number line?*

Math Community

- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.
- Consider asking the class why having more than one strategy might be useful.

Sample Responses

- 30 limes
- 28 lemons
- 48 limes



9 Settle a Dispute

Kiri says that 16

Kiri says that 16 lemons will balance with 24 limes.
Lola says that 36 lemons will balance with 24 limes.

Who is correct?

Teacher Moves

Facilitation

- Consider pausing to discuss how to use the double number line if there is a lack of consensus on this screen.

Discussion Questions

- *What might Lola have been thinking?*
- *What advice would you give to Lola?*
- *What does each* 24 *in the diagram represent?*

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

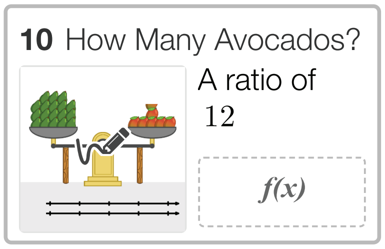
Early Finishers

- Encourage students to write their own questions that could be answered using this double number line, then to trade with a classmate.

Sample Responses

Kiri

Explanations vary. Since we have 24 limes, we need to look at the bottom row of the double number line and find the 24 there.



10 How Many Avocados?

A ratio of 12

A ratio of 12 avocados : 10 mangos balances the scale.

How many mangos will balance with 18 avocados?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Overview: In Activity 2 (Screens 10–13), students use partially labeled double number lines to solve problems.

Launch

- Invite students to share how this double number line is similar to and different from the ones they have seen so far in this lesson.

Facilitation
- If students are struggling to get started, encourage them to label the rest of the tick marks in the double number line.
- Encourage students to use the sketch tool or blank paper to show their thinking.

Suggested Pacing: Screens 10–13

Sample Responses

15 mangos

11 Your Favorite Mistake

Omari and Pilar both

Omari and Pilar both labeled the rest of the diagram on the previous screen. They each made a mistake.

Select one student. Describe something they did well and what advice you would give them.

Teacher Moves

Facilitation
- When most students have responded, consider pausing and discussing this screen as a class.

Discussion Questions
- *What might Omari be thinking? What might Pilar be thinking?*
- *What is important to remember when making double number lines?*

Math Community
- Consider inviting students to share what they think we can learn from looking at incorrect thinking.

Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify to help students communicate their ideas.

Sample Responses

Responses vary.

- **Something Omari did well:** He counted by a consistent amount.
- **Advice for Omari:** Look at the size of the jump between the tick marks. Think about how to make every single jump the same.

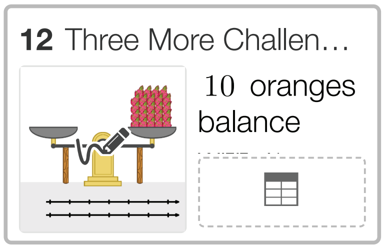
- **Something Pilar did well:** She knew that the top number line goes by 6 s.
- **Advice for Pilar:** Both number lines do not need to go by the same number. Fix the bottom number line so that the bottom goes by 5 s.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



12 Three More Challen…

10 oranges balance

10 oranges balance with 8 dragon fruits.

Use the double number line to help solve each challenge.

Teacher Moves

Facilitation

- If students are struggling, invite them to label all of the tick marks on the double number line.
- Consider reminding students that they can add numbers that are in between tick marks if they want to.
- Note: Students will need to determine values that are in between the given tick marks on these challenges (MP6).

Progress Check

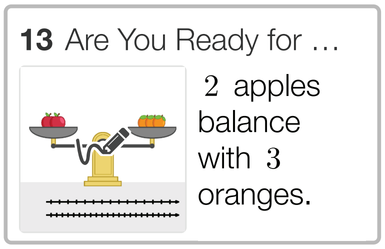
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Math Community

- Monitor for unique or creative uses of the double number line.

Sample Responses

- 30 oranges
- 4 dragon fruits
- 25 oranges


13 Are You Ready for …

2 apples balance with 3 oranges.

2 apples balance with 3 oranges.

On paper, answer the following:

1. How many oranges will balance with 101 apples?

2. How many apples will balance with $25\frac{1}{2}$ oranges?

3. Create your own problem involving equivalent ratios of apples and oranges. Then trade with a classmate.

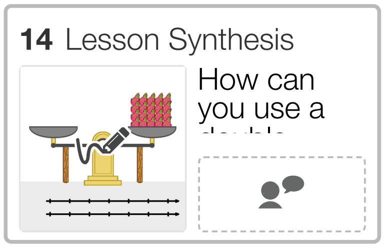
Teacher Moves

Facilitation

- Invite students who finish Screens 10–12 early to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion.

Sample Responses

1. $151\frac{1}{2}$ oranges
2. 17 apples
3. *Questions vary.*


14 Lesson Synthesis

How can you use a

How can you use a double number line diagram to solve problems with ratios?

Use the sketch tool if it helps you show your thinking.

Teacher Moves

Key Discussion Screen

- The purpose of this discussion is to summarize how to use a double number line diagram to solve problems with ratios.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their response with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *When is a double number line useful? When might it be less useful?*

Suggested Pacing: Screen 14

Sample Responses

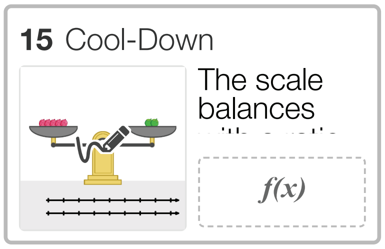
Responses vary. Set up two double number lines so that the ratio you know lines up vertically. Fill in several tick marks on the double number line. Find the value you know on one number line and look at the value that lines up on the other number line. If you don't have that value, make a new tick mark with the value you know and figure out the value that lines up on the other number line.

Student Supports

Multilingual Learners

- *Routine:* *Collect and Display*

Circulate and listen to students talk as they describe how to use double number line diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.



15 Cool-Down

The scale balances

The scale balances with a ratio of 5 lychees to 2 limes.

How many lychees will balance with 10 limes?

Teacher Moves

Support for Future Learning

- If students struggle, plan to emphasize this when opportunities arise in Lesson 6 and beyond. It may be helpful to review this screen as a class before Lesson 6.

Suggested Pacing: Screens 15–16

Sample Responses

25 lychees

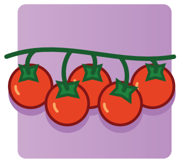
16

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain how to use a double number line diagram to find equivalent ratios.
- I can use double number line diagrams to solve problems.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Product Prices (NYC)

Lesson 6: Unit Prices

Purpose

Students calculate unit prices and use unit prices to solve problems. This lesson builds on what students learned about equivalent ratios in Lessons 3–5 and introduces a new idea: unit price. The work with unit price in this lesson supports students in using unit rates to compare ratios in Lessons 7 and 8.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Cards

- Print and cut one set of single-sided cards for each group of 2–4 students.

Warm-Up (5 minutes)

Overview: Students are introduced to the idea of partial prices and use both mathematics and their outside knowledge to reason about what would be a fair price.

Launch

- Display Sheet 1 of the Teacher Projection Sheets.
- Consider asking a question like: *When might you only want to buy one tomato?*

Facilitation

- Give students 1–2 minutes to think independently and then share their thoughts with a partner.
- Invite several students to share their thinking. It's okay to lack consensus. This warm-up is intentionally ambiguous: there are valid arguments for why the price is and is not fair.
- If it does not come up naturally, consider asking: *A student said that* 40 *cents would be more fair than 50 cents. What might this student have been thinking? Do you agree?*

Discussion Questions

- *Why might someone think this price per tomato is not fair?*
- *Why might someone think this price per tomato is fair?*

Math Community

- Invite students to take another perspective by asking: *Why might someone else have answered differently?*

Activity 1: How Much for One? (10 minutes)

Overview: Students are introduced to the idea of *unit price* as the price per item.

Launch

- Invite students to work *in groups of 2–4.*
- Distribute one worksheet to each student.
- Distribute one set of cards to each group. Give students one minute to share with a group what they notice and wonder about the cards.
- Consider sharing that for the rest of the lesson, we will calculate prices by using equivalent ratios. For example, if 5 tomatoes cost $\$2$, 1 tomato costs $\$0.40$.

Facilitation

- Give students 3–5 minutes to sort the cards and discuss their reasoning as a group.
- When most students have sorted cards A, B, and C, consider pausing for a brief discussion.
- If it does not come up naturally, introduce the term *unit price* as the price per item.

Discussion Questions

- *How did you decide which item is most expensive?*
- *If someone says the lightbulbs or the deodorants are the most expensive, what might they be thinking?*
- *What strategy could we use to figure out the price per item?*

Math Community

- Consider communicating with students the value of different strategies for sorting the cards.

Readiness Check (Problem 5)

- If students struggled with Problem 5, consider spending extra time during this activity sharing strategies for calculating unit prices.

Early Finishers

- Encourage students to continue to Activity 2.

Activity 2: How Much for Many? (20 minutes)

Overview: Students use strategies, including calculating the unit price, to calculate partial prices.

Facilitation

- Give students 10–15 minutes to work together as they answer each question.
- Consider circulating and choosing one student from each group at random to share their thinking on one card as a check for understanding throughout the activity.
- When most students have answered Problem 3, consider pausing and inviting students to share which card Mariana was working on, and how they know.

- Consider monitoring for students who wrote different advice on Problem 6. Invite these students to share their thinking during a whole-class discussion.

Discussion Questions

- *Who made a mistake similar to Mariana's or Naoki's? What did that help you learn?*
- *How can we offer advice that is helpful and supportive?*
- *What is important to remember when solving problems like these?*

Math Community

- Consider asking students to share how it can be helpful to look at their and others' mistakes.

Early Finishers

- Encourage students to answer the "Are You Ready for More?" question. Remind students that they can purchase different numbers of each item if they want to.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

- The purpose of this discussion is to invite students to use their creativity and to surface strategies for using unit prices to solve problems.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share the cards they created.

Discussion Questions

- *What is similar about these cards? What is different?*
- *Is there a strategy that could help us answer each of these cards?*

Math Community

- Consider highlighting unique or creative cards. Ask the author to speak about their inspiration.

Cool-Down (5 minutes)

Support for Future Learning

- Students will have more chances to think about unit rates in Lessons 7 and 8. Plan to emphasize strategies that involve calculating a unit rate during Activity 1 of Lesson 7.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

6.2 Practice Day 1 (NYC)

Preparation

Student Worksheet

- Print one double-sided sheet for each student.

Task Cards

- *Option 1 (Stations)*: Print two single-sided sets of task cards for the entire class (10 cards total).
- *Option 2 (Level Up)*: Print one single-sided set of task cards for each group of 2–3 students. Make a pile for each task card in a central location for students to drop off and pick up.

Instructions

Option 1: Stations

- Arrange students into *groups of 2–4.*
- Distribute one Student Worksheet to each student.

Options for student movement:

- As students finish a station, instruct them to move on to a new station.
- After a set amount of time, instruct students to move as a group from station to station.
- After a set amount of time, instruct students to move to a new station such that no one from their previous group is in their new group.

Option 2: Level Up

- Arrange students into *groups of 2–3.*
- Distribute one Student Worksheet to each student.
- Share with students that there are five tasks.
- Distribute one copy of "Rainy Roof" to each group.

- Once the group completes "Rainy Roof," review their thinking. Consider choosing one student at random in the group to share the group's ideas.
- Once you have reviewed their work on "Rainy Roof," invite them to pick up a copy of "Potato Price" to complete together.

- Continue this process until students have completed all five tasks.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Mixing Paint, Part 1 (NYC)

Lesson 7: Comparing Ratios

Overview

Students develop and share strategies for comparing ratios.

Learning Goals

- Develop strategies for comparing ratios in context.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop and share strategies for comparing ratios. Students use paint mixtures as a context for thinking about how ratios compare to one another. This lesson is designed to elicit several different strategies, including but not limited to comparing unit rates and creating equal amounts of one quantity.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to continue to strengthen their strategies around multiplication of whole numbers.

Activity 1: Developing Strategies for Comparing (15 minutes)

The purpose of this activity is for students to develop and name several strategies for comparing two ratios. Students create their own paint color by mixing colored tint into white paint, then create arguments to determine which of the two paint colors will be a darker blue. The strategies surfaced in this activity should support students in Activity 2.

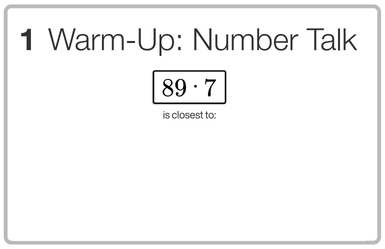
Activity 2: Using Strategies for Comparing (15 minutes)

The purpose of this activity is for students to practice comparing ratios using strategies from Activity 1 and new strategies they develop. Students order several shades of the same color from darkest to lightest, and compare ratios in a series of repeated challenges.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to generalize a strategy for comparing two ratios.

Cool-Down (5 minutes)



1 Warm-Up: Number Talk

$89 \cdot 7$

is closest to:

Teacher Moves

Overview: In this lesson, students develop and share strategies for comparing ratios. In this warm-up, students continue to strengthen their strategies around multiplication of whole numbers.

Launch

- Consider starting with the lesson paused and sharing that the purpose is to hear as many different ways of knowing as possible.

Facilitation

- Consider using the dashboard's student view to display the prompt.
- Give students one minute to think quietly. Invite them to signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- Select several students to share different responses and how they know. Record their thinking for all to see, along with their names.
- Use the instructional routine Number Talk to help students surface and name strategies for estimating the value of $89 \cdot 7$.

Discussion Questions

- *How would you convince someone ______ is not correct?*

Math Community

- Consider inviting students to think about why estimation can be helpful even if you never get an exact answer.

Suggested Pacing: Screen 1

Sample Responses

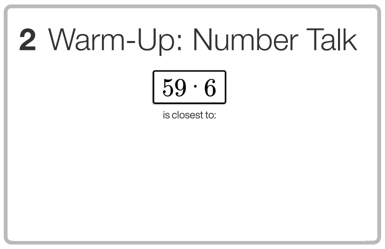
600

Student Supports

Students With Disabilities

- *Fine Motor Skills: Strategic Pairing*

Allow students who struggle with fine motor skills to dictate use of the graph and sketch tool as needed throughout the lesson.



2 Warm-Up: Number Talk

$59 \cdot 6$

is closest to:

Teacher Moves

Facilitation

- Consider using the dashboard's student view to display the prompt.
- Give students one minute to think quietly. Invite them to signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- Select several students to share different responses and how they know. Record their thinking for all to see, along with their names.
- Use the instructional routine Number Talk to help students surface and name strategies for estimating the value of $59 \cdot 6$.

Discussion Questions

- *How would you convince someone* ______ *is not correct?*

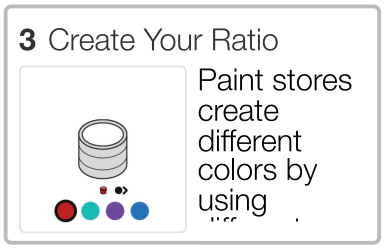
Math Community

- Consider inviting students to share if anyone's strategy from the previous screen helped them on this screen.

Suggested Pacing: Screen 2

Sample Responses

300



3 Create Your Ratio

Paint stores create different colors by using

Paint stores create different colors by using different ratios of white paint to tint.

1. Click to choose a tint color.

2. Drag the point to adjust the amount. Press "Mix It."

Teacher Moves

Overview: In Activity 1 (Screens 3–7), students develop and name several strategies for comparing two ratios.

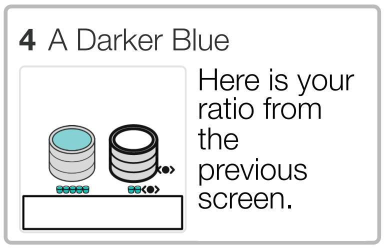
Launch

- Demonstrate how to change tint colors and amount of tint using the dashboard's student view.
- Consider inviting students to share any experience they have with mixing paint.

Math Community

- Consider asking: *If you could choose any color, what color would you choose and why?*

Suggested Pacing: Screens 3–7



4 A Darker Blue

Here is your ratio from the previous screen.

Here is your ratio from the previous screen.

Drag the points to create a new ratio.

Can you find two different ways to make a darker color? A lighter color?

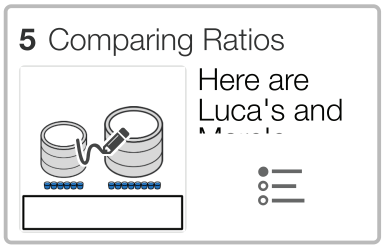
Teacher Moves

Facilitation

- Invite students to spend 2–3 minutes adjusting the amount of white paint and colored tint, and sharing what they notice with a partner.
- Consider pausing the class to discuss multiple strategies for creating a darker color.
- If it doesn't come up naturally, consider asking what happens if you keep the tint the same and change the amount of white paint.

Discussion Questions

- *Is there a different way we could create a darker color? A lighter color?*
- *How does changing the amount of white paint affect the color?*



5 Comparing Ratios

Here are Luca's and

Here are Luca's and Marc's ratios.

Which will make a darker blue?

Use paper to support your thinking.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface multiple strategies for comparing ratios that students can refer back to throughout the lesson and unit.

Facilitation

- Encourage students to share their reasoning with a partner and to work together to make their reasoning as strong and clear as possible (MP3).
- Monitor for students who show their thinking in different ways or use different strategies, including but not limited to calculating a

unit rate, creating equal amounts of blue tint, or creating equal amounts of white paint.

- When most students have responded, display several student responses and sketches and facilitate a conversation.
- Consider recording students' strategies in a public place for students to refer to later in the lesson.

Discussion Questions

- *How did you decide which ratio makes a darker blue?*
- *What is a different way to explain why Luca's ratio makes a darker blue?*

Early Student Thinking

- Students might notice that Marc's ratio has 2 more ounces of blue and 2 more gallons of white pain, and conclude that this make the same blue. Consider inviting them to continue to Screen 6 and reflect on why Luca made a darker blue.

Math Community

- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Luca's ratio

Explanations vary.

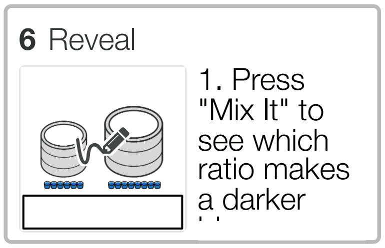
- Luca uses 3 ounces of blue for every gallon of white, and Marc only uses 2 ounces of blue for every gallon of white.
- If you double Luca's ratio, both ratios would have 4 gallons of white paint. Luca's would have 12 ounces of blue and Marc's would have 8 ounces of blue.
- If you multiply Luca's ratio by 4 and Marc's ratio by 3 then they would have the same amount of blue tint. Luca's ratio would have 24 ounces of blue and 8 gallons of white. Marc's ratio would have 24 ounces of blue and 12 gallons of white. Luca uses less white paint, so his would make a darker blue.

Student Supports

Multilingual Learners

- *Routine:* *Stronger and Clearer Each Time*

To support students in describing which ratio will be a darker blue, use this routine with successive pair shares to give students a structured opportunity to revise their initial ideas. After writing an initial explanation, have pairs share what they wrote to get feedback from a partner. Have each student meet with a second partner to provide extra verbal practice, idea sharing, and feedback. Have students return to their seats and write down their revised transformation description.



6 Reveal

1. Press "Mix It" to see which ratio makes a darker

1. Press "Mix It" to see which ratio makes a darker blue.

2. Discuss with a partner a different strategy you could use to compare the ratios.

Teacher Moves

Facilitation

- After students have pressed “Mix It," invite them to go back to the previous screen and either revise their thinking or make their thinking stronger and clearer.
- Invite students to use the sketch tool to show their thinking if it is helpful.

Sample Responses

Responses vary.

7 Select All

Here is Marc's ratio:

Here is Marc's ratio:

8 ounces purple : 4 gallons white

Which of the following will result in a darker purple?

Teacher Moves

Facilitation

- Consider displaying the distribution of responses using the dashboard’s teacher view, calling attention to any conflict or consensus you see.

Discussion Questions

- *What do you think will happen if we add the same amount of purple tint and white paint? Why?*

Early Student Thinking

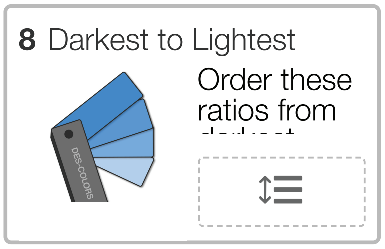
- Students may think that adding 2 ounces of purple and 2 gallons of white paint will keep the paint the same color. Consider asking: *If I added 2 ounces of purple and 2 gallons of white paint, how much purple would there be per gallon of white?*

Early Finishers

- Encourage students to think of an amount of purple tint and white paint they could be added to create the *same* purple.

Sample Responses

- Using less white paint
- Adding purple tint



8 Darkest to Lightest

Order these ratios from

Order these ratios from darkest blue to lightest blue.

Use paper to support your thinking.

Teacher Moves

Overview: In Activity 2 (Screens 8–10), students practice comparing ratios using strategies from Activity 1 and new strategies they develop.

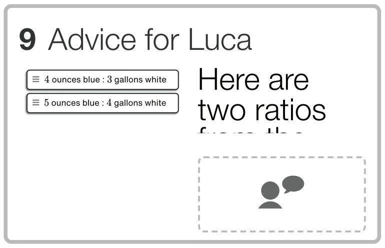
Launch
- Consider sharing that students can order the paint colors using any strategies they've heard or new ones that they come up with (MP2).

Facilitation
- Give students several minutes to order the colors. Encourage students to use paper to support their thinking.
- If students are struggling to get started, invite them to select any two paint colors and compare those, then to compare a third color to the ones they've already ordered.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 8–10

Sample Responses

From darkest blue to lightest blue:
- 10 ounces blue : 6 gallons white
- 9 ounces blue : 6 gallons white
- 4 ounces blue : 3 gallons white
- 5 ounces blue : 4 gallons white



9 Advice for Luca

4 ounces blue : 3 gallons white

5 ounces blue : 4 gallons white

Here are two ratios

Here are two ratios from the previous screen.

Luca says they make the same shade of blue.

What advice would you give to Luca?

Teacher Moves

Facilitation
- Select and sequence several student responses using the snapshot tool. Monitor for students who reference how you can determine which color is a darker blue.

Discussion Questions

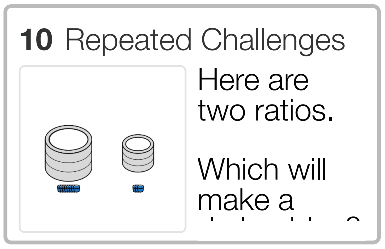
- *Why might Luca think they make the same shade of blue?*
- *What ratios would be the same color as* 4 *ounces blue* : 3 *gallons white?*
- *Which color makes a darker blue? How can you convince someone else?*

Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify to help students communicate their ideas.

Sample Responses

Responses vary.

- It's helpful to compare the colors when you have the same number of gallons of white or ounces of blue to compare them. You could make both colors have 12 gallons of white by multiplying the first color by 4 and the second color by 3.
- You can figure out if the colors are the same shade by calculating how much blue for every gallon of white. The first color is $\frac{4}{3}$ or about 1.333 ounces of blue for every gallon of white. The second color is $\frac{5}{4}$ or 1.25 ounces of blue for every gallon of white.



10 Repeated Challenges

Here are two ratios.

Which will make a

total correct: 0

current streak: 0
longest streak: 0

Teacher Moves

How Repeated Challenges Work

- Students are presented with a variety of challenges one at a time, in this case determining which ratio makes a darker blue.
- The challenges typically increase in difficulty as they continue.

Facilitation

- Give students 5–7 minutes to complete as many challenges as they can.
- Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

Math Community
- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

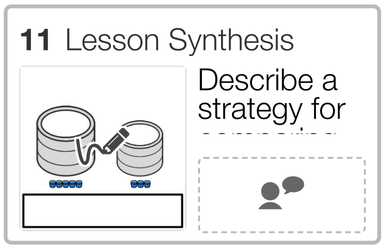
Sample Responses

This screen contains an unlimited number of randomized challenges. The challenges progress in difficulty.

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Begin with a demonstration of the first problem to provide access to students who benefit from clear and explicit instructions. Check in with individual students, as needed, to assess for comprehension during each step of the activity.



11 Lesson Synthesis

Describe a strategy for

Describe a strategy for comparing two ratios, such as these recipes for paint mixes.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to generalize a strategy for comparing two ratios.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- **Note:** This ratio is the third challenge from the Repeated Challenges screen.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions
- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates’ strategies?*

Math Community
- Invite students to share strategies they’ve found most helpful and attribute them to the students who shared them.

Suggested Pacing: Screen 11

Sample Responses

Responses vary.

- You can compare two ratios by making both ratios have the same number of one quantity. In this example, you could multiply Ratio B by 2 so that both gallons of white paint are the same and then compare the ounces of blue tint.
- You can compare ratios by calculating how much each ratio is per 1. Here, Ratio A has $\frac{5}{4} = 1.25$ ounces of blue per gallon of white and Ratio B has $\frac{3}{2} = 1.5$ ounces of blue per gallon of white.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

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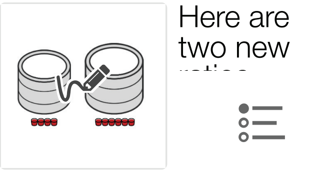
12 Cool-Down



Here are two new

Here are two new ratios:

Ratio A
4 ounces red : 3 gallons white

Ratio B
6 ounces red : 4 gallons white

Which will make a darker red?

Teacher Moves

Support for Future Learning
- Students will have more chances to develop their understanding of comparing ratios in Lesson 8.

Suggested Pacing: Screens 12–13

Sample Responses

Ratio B

Explanations vary.

- Ratio B has 1.5 ounces of red per gallon of white and Ratio A has about 1.33 ounces of red per gallon of white. More red tint means a darker red.
- If you make both ratios use 12 ounces of red, Ratio A uses $3 \cdot 3 = 9$ gallons of white paint, but Ratio B only uses $4 \cdot 2 = 8$ gallons of white. Less white paint means a darker red.

13

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use strategies to compare ratios in context.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

World Records (NYC)

Lesson 8: Comparing Speeds

Purpose

Students calculate unit rates and use them to compare speeds. This builds on the work students did with unit prices in Lesson 6 and with comparing paint mixes in Lesson 7. This lesson is also the first time where students use rates and ratios involving abstract quantities like time.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Setup

- Before class, set up four or more paths with a 2-meter warm-up zone and a 10-meter measuring zone like the image on the Student Worksheet.

Materials

- Stopwatch or other timer

Note: The videos on the Teacher Presentation Screens contain sound. This can be helpful for creating context and generating interest, but sound is not *necessary* to facilitate the lesson.

Warm-Up (5 minutes)

Overview: Students compare two vehicles that travel the same distance in different amounts of time. Students are also introduced to ratios in the context of distance and time.

Launch

- Invite students to work *individually.*
- Display Screen 1 of the Teacher Presentation Screens.

Facilitation

- Give students one minute to think quietly, then another minute to share with a partner.
- Consider polling the class to see the distribution of responses.
- Invite students to share reasons why each vehicle might be faster.

Discussion Questions

- *Why might someone have said the car? The bus? The same speed?*

Math Community

- Consider highlighting the value of changing one’s mind by asking if any students revised their thinking from what they originally thought.

Activity 1: Moving 10 Meters (10 minutes)

Overview: Students calculate their own walking speed from data they collect.

Launch

- Invite students to work *in groups of 3–4.*
- Distribute one Student Worksheet to each student.
- Display Screen 2 of the Teacher Presentation Screens.
- Demonstrate one round of data collection to review the steps on the screen. Consider inviting a student to act as the mover as you act as the timer.

Facilitation

- Invite students to use one of the set up paths to collect their data, taking turns as the mover and timer until all group members have their own data.
- To optimize class time, consider setting a time limit of 5–7 minutes for this activity.
- When students have collected their data, consider choosing a random student to share their data. Then answer Problems 3 and 4 as a class using that students' data.
- Consider drawing a double number line on the board and annotating the unit rate for students to refer back to throughout the lesson.

Discussion Questions

- *How was the double number line helpful? What other strategies could we use to determine the speed?*

Early Finishers

- Encourage students to answer questions like: *At this rate, how far can you walk in a minute? How long would it take you to walk a kilometer?*

Activity 2: World Records (20 minutes)

Overview: Students use unit rates to compare the speeds of world-record holders in different sports and justify their reasoning.

Launch

- Invite students to work in *groups of 3–4.*
- Display Screen 3 of the Teacher Presentation Screens.
- Invite students to share any experiences they have with climbing, hurdles, or swimming. Spend adequate time here to ensure students understand what these three sports are.

Facilitation

- Give students one minute to predict the order individually, then another two minutes to come to consensus as a group.
- Invite several students to share their ideas and reasoning. It's okay—even desirable—to lack consensus at this stage. The activity will build toward consensus throughout the activity.

- Display Screen 4 and play the 30-second video.
- Give students one minute to revise their thinking about the order.

- Display Screen 5. Give students one minute to make sense of the table as a group (MP1) and record the information on their worksheet.
- Consider a question like: *What do you think "approximate times" means? Why do you think we are using approximate times?*

- Give students several minutes to determine the order using the approximate times.
  Note: Students may notice that both the swimmer and the climber moved at same speed using the approximate times. Consider asking them: *What do you need to determine who was faster?*
- Consider giving students a shared workspace, like a whiteboard or large blank paper, to write their thinking as they work.

- When most students have determined an order using the approximate times, consider pausing and watching the videos on Screen 6 as a class.
- Invite students to share what they notice and wonder about the actual times compared to the approximate times. (MP6)
- Display Screen 7 for students to reference after they finish Problem 4.

Discussion Questions

- *What strategies were most helpful for comparing speeds?*
- *For what other problems might this strategy be helpful?*

Math Community

- Consider inviting students to reflect on qualities of productive group work before beginning this activity. If it makes sense, invite students to share something they bring to a group or display a list of important qualities in productive group work for students to select from.

Early Finishers

- Distribute the "Are You Ready for More?" page to groups who have finished Problem 4 before the class watches the full videos.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

- The purpose of this discussion is to surface strategies for comparing speeds, including calculating unit rate.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If it does not come up naturally, consider asking students how calculating a unit rate might be helpful.

Discussion Questions

- *What is a unit rate? How does it help us compare speeds?*
- *When is calculating the unit rate helpful? When might a different strategy be helpful?*

Cool-Down (5 minutes)

Support for Future Learning

- Students will continue to use double number lines and other strategies to solve problems with ratios. If students struggle to compare ratios, consider reviewing this question before Practice Day 2 or offering individual support where needed during the practice day.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Disaster Preparation (NYC)

Lesson 9: Using Ratio Tables With Large Quantities

Overview

Students use tables of equivalent ratios to determine large unknown values in context.

Note: Activity 2 includes a paper supplement.

Learning Goals

- Use tables of equivalent ratios to determine large unknown values in context.

Materials

- Tools for creating a visual display

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to use tables of equivalent ratios to determine large unknown values in context. Students analyze recommendations from the Federal Emergency Management Association (FEMA) about various items to have on hand in case of a disaster. Students calculate the recommended amount of each item and decide how they might alter those recommendations based on their own intuition or lived experience.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce the context of cities storing items in case of a disaster. Some of these items may appear throughout the rest of the lesson.

Activity 1: Shower, Power, and Air (10 minutes)

The purpose of this activity is for students to use what they have learned about equivalent ratios to calculate the number of shower stalls, power strips, and air pumps needed for cities of different sizes in case of a disaster. This activity is facilitated as a slow reveal, where students have an opportunity to gradually make sense of each piece of information and then compare their thoughts with FEMA’s recommendations.

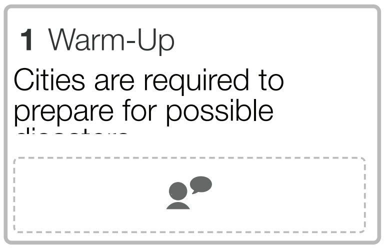
Activity 2: FEMA Poster (20 minutes)

The purpose of this activity is for students to use tables of equivalent ratios to determine large unknown values in context. Students calculate the recommended number of other items, such as paper towels, for new cities. Then they create a poster describing the number of each item needed for a city of their choice. Students will see recommendations that create equivalent ratios and ones that do not (e.g., having 6 pairs of crutches).

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to explain how to use a table of equivalent ratios to determine unknown values.

Cool-Down (5 minutes)



1 Warm-Up

Cities are required to prepare for possible

Cities are required to prepare for possible disasters.

What are three things a city should have for its people in case of a disaster?

Teacher Moves

Overview: In this lesson, students use ratio tables to determine large unknown quantities in context. In this warm-up, students are introduced to the context of cities storing items in case of a disaster.

Launch
- Ask students to share why they think cities should have items in case of a disaster.
- The topic of disasters may affect students differently based on their and their community's experiences. Consider discussing disasters generally rather than in specific detail.

Facilitation
- Give students 1–2 minutes to respond and share their responses with a partner.
- Consider highlighting the variety of items students have described using the dashboard's teacher view or snapshot tool.

Early Finishers
- Encourage students to predict how cities figure out how many of these items they will need.

Math Community
- Celebrate the diversity of thoughts within your class.

Suggested Pacing: Screen 1

Sample Responses

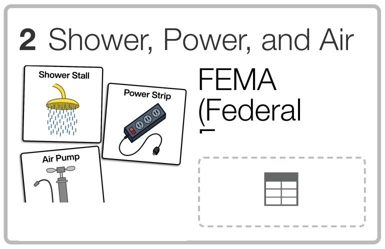
Responses vary.

- Toilets
- Beds
- Food
- Blankets
- Water
- Extra clothes

Student Supports

Students With Disabilities
- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time.



2 Shower, Power, and Air

FEMA (Federal

FEMA (Federal Emergency Management Agency) has a list of items that cities should prepare in case of a disaster. Three items are shown.

Atka, Alaska, is a very small city.

How many of each item do you think Atka should have?

Teacher Moves

Overview: In Activity 1 (Screens 2–5), students use what they have learned about equivalent ratios to calculate the number of shower stalls, power strips, and air pumps needed for cities of different sizes in case of a disaster.

This activity is facilitated as a slow reveal, where students have an opportunity to make sense of each piece of information one at a time and then compare their thoughts with FEMA's recommendations. Here is a video of Desmos's Dan Meyer facilitating a similar lesson using this slow-reveal format.

Launch

- Invite students to work *in pairs.*
- Ask students to define each item to ensure that the class has shared context.
- Consider inviting students to consider why shower stalls, power strips, and air pumps might be useful to people in a disaster. For example, air pumps are particularly useful for wheelchair wheels.

Facilitation

- Give students one minute to respond and think of an explanation for their estimates.
- Invite several students to share their estimates and the reasons for them.
- Emphasize the range of student responses. It's okay—even desirable—to lack consensus at this stage.

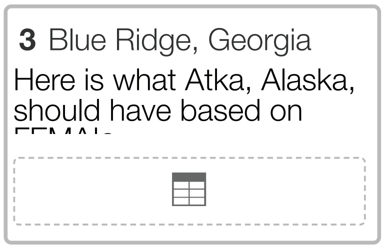
Discussion Questions

- *Why do you think FEMA has this list of items?*
- *Why is having enough of each item important? Why is it useful not to have too many?*

Suggested Pacing: Screen 2

Sample Responses

Responses vary.



3 Blue Ridge, Georgia

Here is what Atka, Alaska, should have based on

Here is what Atka, Alaska, should have based on FEMA's recommendations.

How many of each item would you recommend for Blue Ridge, Georgia?

Teacher Moves

Launch

- Consider inviting students to discuss with a partner how FEMA's recommendations compare to theirs and to what extent they agree with what FEMA recommended.

Facilitation

- Give students one minute to determine how many of each item they would recommend for Blue Ridge and to be ready to explain their reasoning (MP2).
- Consider using the snapshot tool to select and sequence several student responses.
- Invite students to share how they determined their recommendations. It's okay—even desirable—to lack consensus.

Discussion Questions

- *How is ______'s strategy similar to _____'s strategy?*
- *What might ________ have done to get the numbers in their recommendation?*

Early Student Thinking

- Students may guess rather than use reason to determine their estimate. Consider asking these students: *How would you know if your estimate is enough? Too much?*

Math Community

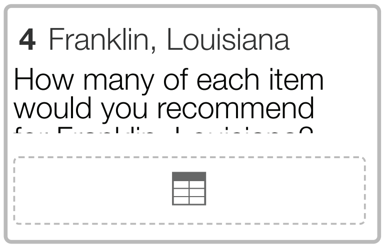
- Consider highlighting student work that does and does not use equivalent ratios.

Note: Here is FEMA's full list of recommendations. Some details have been modified to fit this lesson.

Suggested Pacing: Screens 2–3 (so students can go back to check their responses)

Sample Responses

Responses vary. If students create equivalent ratios, it would be 24 shower stalls, 30 power strips, and 12 air pumps.



4 Franklin, Louisiana

How many of each item would you recommend

How many of each item would you recommend for Franklin, Louisiana?

Use paper if it helps you with your thinking.

Teacher Moves

Facilitation

- Give students one minute to compare their responses with FEMA's guidelines.
- Ask a question like: *How do you think FEMA's recommendations work for these items?*

- Then give students one minute to determine how many of each item they would recommend for Franklin and to be ready to explain their reasoning (MP2).
- Consider using the snapshot tool to select and sequence several student responses. Invite students to justify their recommendations.

Discussion Questions

- *How is ______'s strategy similar to _____'s strategy?*
- *Why do you think air pumps might be different than shower stalls or power strips?*

Suggested Pacing: Screen 3–4 (so students can go back to check their responses)

Sample Responses

Responses vary. If students use the same strategy as was used for Blue Ridge, it would be 200 shower stalls, 250 power strips, and 2 air pumps.

Note: FEMA's guidelines for shower stalls and power strips follow a ratio relationship, but students don't need to assume this at this stage in the lesson as long as they can defend their answer.

5 Here is what FEMA re…

Teacher Moves

Key Discussion Screen

- The purpose of this discussion is to surface strategies for using equivalent ratios to determine large values.

Facilitation

- Give students 1–2 minutes to discuss with a partner.
- Consider recording students' descriptions of strategies and calculations on the board so students can refer back to them during Activity 2.

Discussion Questions

- *We predicted that FEMA's recommendations for shower stalls and power strips would change based on the population, and air pumps wouldn't. What do you think now?*
- *What would you do to calculate the number of shower stalls in a different city?*
- *Do any of these numbers seem not quite right to you?*

Math Community

- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their strategy for determining large unknown values.

Suggested Pacing: Screen 5

Sample Responses

Responses vary.

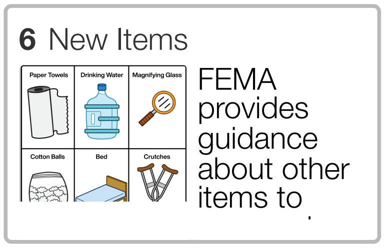
- They figured out how many times bigger the population was than Atka, then multiplied that number by 4 for shower stalls and 5 for power strips. There are always 2 air pumps.
- I think 250 power strips feels like a lot. You need outlets for all of them to plug into and finding 250 outlets seems tough.
- I think "2 air pumps always" is a weird rule. It might make sense for small cities, but what about a city with hundreds of thousands of people.

Student Supports

Multilingual Learners

- *Routine:* Collect and Display

Circulate and listen to students talk as they describe FEMA's strategy. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.



6 New Items

FEMA provides guidance about other items to .

FEMA provides guidance about other items to prepare in case of disaster.

Here are a few more.

Follow the directions on your worksheet.

Teacher Moves

Overview: In Activity 2, students calculate the recommended number of other items, such as paper towels, for new cities. Then they create a poster describing the number of each item needed for a city of their choice.

Launch

- Invite students to work in *groups of 3–4.*
- Distribute one paper supplement to each student.
- Invite students to clarify any objects from the image or list that they are not sure about.

Facilitation

- Give students several minutes to read FEMA's recommendations aloud as a group and ensure everyone understands them. **Note:** Students can either count the number of cotton balls or the number of *bags* of cotton balls. This is left intentionally ambiguous.
- Once a group has confirmed they understand the recommendations, give them several minutes to make recommendations and to analyze FEMA's guidelines.
- When most students have finished Problems 1 and 2, consider facilitating a brief discussion or sharing an answer key and inviting groups to revise their recommendations and discuss the reasoning.
- Give students 5–10 minutes to choose a city and create a poster with their recommendations (MP4).
- If time allows, invite students who have completed their posters to do a gallery walk to compare the strategies they used with those of their classmates. Consider using the mathematical language routine Compare and Connect.

Materials

- Tools for creating a visual display

Sample Responses

1.

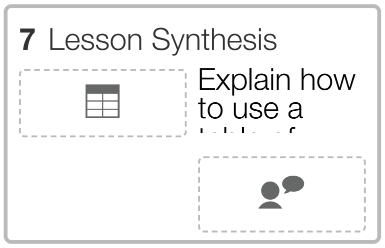
Charlestown: 60 rolls of paper towels, 6 magnifying glasses, 6 pairs of crutches, and 450 cotton balls (or 9 bags)
Whitney, Texas: 400 rolls of paper towels, 40 magnifying glasses, 6 pairs of crutches, and 3 000 cotton balls (or 60 bags)
Burlington, Vermont: 10 000 rolls of paper towels, 1 000 magnifying glasses, 6 pairs of crutches, and 75 000 cotton balls (or 1 500 bags)

2. *Responses vary*.
- I don't think Burlington really needs 1 000 magnifying glasses. I can understand having a few magnifying glasses on hand in every city, but I'm not sure that cities need 1 for every 50 people.
- FEMA's guidance for crutches seems like a mistake. There should be a ratio between the number of crutches and the number of people, like 6 pairs of crutches for every 100 people.

Student Supports

Students With Disabilities
- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., presenting one problem at a time), which will aid students who benefit from support with organizational skills in problem-solving.



7 Lesson Synthesis

Explain how to use a

Explain how to use a table of equivalent ratios to determine unknown values.

Use the table if it helps you explain your thinking.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to explain how to use a table of equivalent ratios to determine unknown values.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions
- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Suggested Pacing: Screen 7

Sample Responses

Responses vary.

- I can make an equivalent ratio by multiplying or dividing the numbers as needed. For example, if there are 4 shower stalls for every 100 people, I can multiply both quantities by 50 to determine that we need $50 \cdot 4 = 200$ shower stalls for 5000 people.
- I can find a unit rate and then use it to figure out any new number. In the example, there are 25 people for every shower stall. For a town with 5000 people, $5000 \div 25 = 200$ shower stalls.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.


8 Cool-Down

FEMA recommends 20 rolls of duct tape and 4

FEMA recommends 20 rolls of duct tape and 4 pairs of scissors for every 100 people.

Complete the table according to FEMA's recommendations.

Teacher Moves

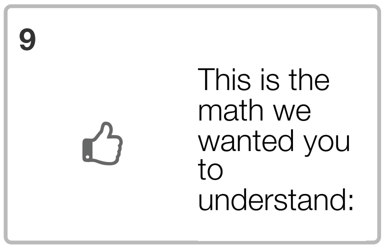
Support for Future Learning

- If students struggle, consider reviewing this screen as a class before Lesson 10.

Suggested Pacing: Screens 8–9

Sample Responses

Population 300**:** 60 rolls of duct tape and 12 scissors
Population 4000**:** 800 rolls of duct tape and 160 scissors

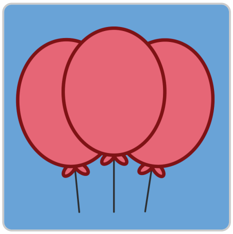

9

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use tables to determine missing values in a situation that involves large numbers.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Balloons (NYC)

Lesson 10: Solving Multistep Ratio Problems

Overview

Students develop and use tools to solve multistep problems involving equivalent ratios.

Learning Goals

- Solve problems by reasoning about tables of equivalent ratios and double number line diagrams.
- Compare tools for determining missing values involving equivalent ratios.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop and use tools to solve multistep problems involving equivalent ratios. Students determine missing values in equivalent ratios using double number lines, tables, and other tools. This lesson surfaces strategies such as calculating a unit rate or multiplying up to a friendly number and then dividing. Students will have more opportunities to practice these strategies in Lesson 11.

Lesson Summary

Warm-Up (10 minutes)

The purpose of the warm-up is for students engage in the Number Talk routine to surface strategies for multiplying whole numbers. This is the last in a series of warm-ups to strengthen strategies around multiplication.

Activity 1: Developing Strategies (15 minutes)

The purpose of this activity is for students to surface and analyze tools for solving ratio problems where it is helpful to calculate an intermediate equivalent ratio. Students consider strategies such as calculating a unit rate or using a friendly equivalent ratio. Students can use any representations to approach each problem, including but not limited to tables and double number lines.

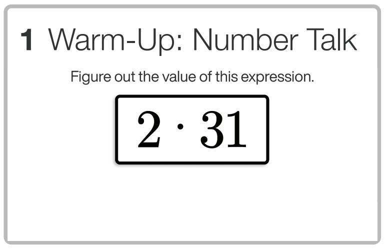
Activity 2: Using Your Strategies (10 minutes)

The purpose of this activity is for students to practice solving multistep ratio problems. This activity includes repeated challenges where students have multiple opportunities to calculate a missing number of balloons or marbles in different ratios.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to describe a strategy for determining missing values in equivalent ratios, like an unknown number of balloons or marbles.

Cool-Down (5 minutes)



1 Warm-Up: Number Talk

Figure out the value of this expression.

$\boxed{2 \cdot 31}$

Teacher Moves

Overview: In this lesson, students develop and use strategies to solve multistep problems involving equivalent ratios. In this warm-up, students engage in the Number Talk routine to strengthen strategies around multiplication.

Launch

- Use the dashboard's student view to display Screen 1.
- Invite students to share any strategies they remember from previous number talks in this unit.

How Number Talk Works

- Give students one minute to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own or one of the possible strategies in the sample responses.
- Select several students to share different strategies. Use the sample responses as examples of possible student strategies. Record strategies for all to see, along with the name of the student who shared each one.
- Display the next expression and repeat. If it makes sense, encourage students to use a strategy described by a classmate in the previous round.

Readiness Check (Problem 6)

- If students struggled, consider reviewing Problem 6 before beginning this lesson. Consider asking a question like: *How are the second and third equations related?*

Suggested Pacing: Screens 1–4, one screen at a time

Sample Responses

$2 \cdot 31 = 62$

Strategies students might use in the Number Talk

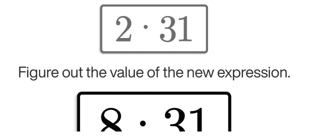
Student Supports

Multilingual Learners

- *Receptive/Expressive Language: Strategic Pairing*

Pair students to aid them in comprehension and expression of understanding.

2 Warm-Up: Number Talk



$2 \cdot 31$

Figure out the value of the new expression.

$8 \cdot 31$

Sample Responses

$8 \cdot 31 = 248$

Strategies students might use in the Number Talk

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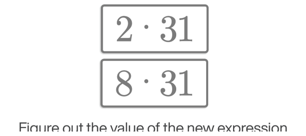
3 Warm-Up: Number Talk



$2 \cdot 31$

$8 \cdot 31$

Figure out the value of the new expression.

Sample Responses

$9 \cdot 31 = 279$

Strategies students might use in the Number Talk

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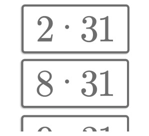
4 Warm-Up: Number Talk



$2 \cdot 31$

$8 \cdot 31$

Sample Responses

$11 \cdot 31 = 341$

Strategies students might use in the Number Talk

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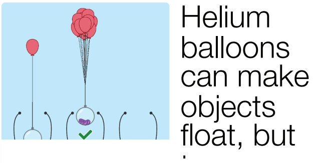
5 Balloon Float



Helium balloons can make objects float, but

Helium balloons can make objects float, but too many balloons will make objects fly away!

Teacher Moves

Overview: In Activity 1 (Screens 5–9), students surface and analyze tools for solving ratio problems where it is helpful to calculate an intermediate equivalent ratio.

Launch

- Invite students to work *in pairs.*
- Share that in this lesson, we will be floating groups of marbles with balloons. Consider inviting students to share balloon- or marble-related story.

Suggested Pacing: Screens 5–9

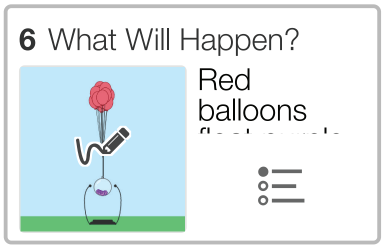
Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*
Check in with individual students, as needed, to assess for comprehension during each step of the activity.

- *Memory: Processing Time*
Provide sticky notes or mini whiteboards to aid students with working memory challenges.

- *Visual-Spatial Processing: Visual Aids*
Provide printed copies of the representations for students to draw on or highlight.



6 What Will Happen?

Red balloons

Red balloons float purple marbles at a ratio of $12 : 4$.

What will happen to the marbles if we add 1 balloon and 1 marble?

Teacher Moves

Facilitation

- Display the distribution of responses. Invite students to justify their responses (MP3).
- If it does not come up naturally, consider asking: *How many balloons would you need to float* 1 *marble?*

Early Student Thinking

- Students may say that it will float in place because you are adding the same amount of balloons and marbles. Consider asking: *What would happen if you just had* 1 *balloon and* 1 *marble?*

Math Community

- Consider inviting students to think about what a student who responded differently might have been thinking.

Note: This is the first time students have solved problems where the ratio $12 : 4$ is unitless. Students may need time to determine which number represents balloons and which represents marbles.

Sample Responses

Sink down

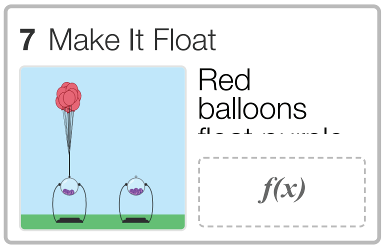
Explanations vary. 12 balloons float 4 marbles, so you need 3 balloons for every marble. If you add 1 marble, you must add 3 balloons in order to keep it afloat. 1 balloon isn't enough to float 1 marble, so it would sink.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



7 Make It Float

Red balloons

Red balloons float purple marbles at a ratio of $12 : 4$.

How many red balloons will float 6 purple marbles?

Use paper if it helps you with your thinking.

Teacher Moves

Facilitation

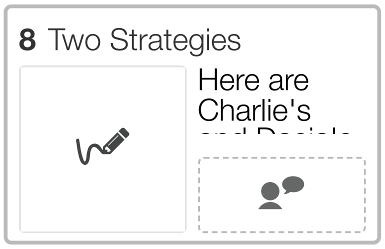
- Encourage students to use any tools they want throughout the lesson to help them with their thinking. It may be helpful to brainstorm tools that students may want find helpful, such as a double number line, table, or calculator. (MP5)
- Encourage students to use the feedback on the screen to help them revise their thinking.
- Monitor for students who use strategies similar to Charlie's and Daeja's thinking on the next screen or other creative strategies. Invite these students to share their thinking during the discussion on that screen.

Early Student Thinking

- Students may recognize that 6 is half of 12 and write 2 balloons because half of 4 is 2. Consider asking these students: *What information is missing: balloons or marbles?*

Sample Responses

18 red balloons



8 Two Strategies

Here are Charlie's

Here are Charlie's and Daeja's strategies for determining how many red balloons will float 6 purple marbles.

1. Select each student's name to see their strategy.

2. Explain how you think one student could finish their strategy to solve the problem.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to explore two tools for solving multistep ratio problems. Students should leave this discussion with at least one tool that makes sense to them (MP5).

Facilitation

- Encourage students to use the sketch tool to show their thinking.
- Select and sequence several sketches and responses using the snapshot tool.
- Invite students to summarize each student's strategy and build on each other's thinking.
- Spend adequate time here clarifying each student's strategy.

Discussion Questions

- *How would you describe Charlie's strategy in your own words? Daeja's?*
- *What are the advantages of Daeja's strategy? Charlie's?*
- *Could you use Charlie's strategy on a double number line? Could you use Daeja's strategy in a table?*

Math Community

- Consider asking the class why having more than one strategy might be useful.
- Consider renaming these strategies after the students in your class who used them.

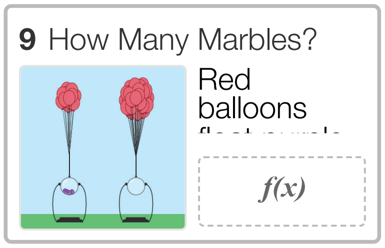
Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary.

- Charlie could finish by multiplying both the 3 and the 1 by 6. He would get 18 balloons.

- Daeja could finish by multiplying both the 6 balloons and the 2 marbles by 3. She would get 18 balloons. She could also just continue counting on the double number line and get to 18 balloons for 6 marbles.



9 How Many Marbles?

Red balloons

Red balloons float purple marbles at a ratio of $12 : 4$.

How many purple marbles will 30 red balloons float?

Use paper if it helps you with your thinking.

Teacher Moves

Progress Check

- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking

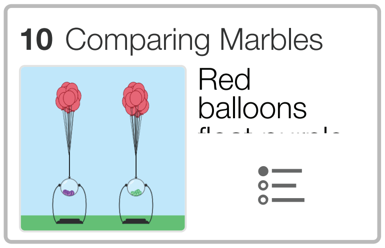
- Students may calculate the number of balloons needed to float 30 marbles (90 balloons).
- Consider asking these students: *What information is missing: balloons or marbles?*

Early Finishers

- Encourage students to write their own problem with a missing number of balloons or marbles. Trade problems with a partner and invite them to solve it or explain why the problem is not possible.

Sample Responses

10 purple marbles



10 Comparing Marbles

Red balloons

Red balloons float purple marbles at a ratio of $12 : 4$.

Red balloons float green marbles at a ratio of $15 : 6$.

Which is heavier: a purple marble or a green marble?

Teacher Moves

Overview: In Activity 2 (Screens 10–13), students practice solving multistep ratio problems.

Launch

- Consider inviting students to think about which past lessons have involved comparing two ratios. Invite students to generate strategies that we could use to compare two ratios.

Facilitation

- Consider pausing for a discussion if there is no consensus on this screen, or offer individual support where needed.

Math Community

- Consider inviting students to think about what a student who responded differently might have been thinking.

Sample Responses

Purple

Explanations vary.

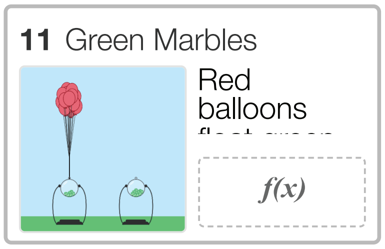
- You need $\frac{12}{4} = 3$ balloons to float every purple marble but only $\frac{15}{6} = 2.5$ balloons to float every green marble.
- You would need $12 \cdot 3 = 36$ balloons to float 12 purple marbles and only $15 \cdot 2 = 30$ balloons to float 12 green marbles.

Student Supports

Multilingual Learners

- *Routine:* *Compare and Connect*

After students share their approaches for deciding which marble is heavier, ask groups to discuss: *What is similar and what is different between the approaches?* Ask students to describe what worked well with their approach and what might make an approach more complete or easier to understand.



11 Green Marbles

Red balloons

Red balloons float green marbles at a ratio of $15 : 6$.

How many red balloons will float 10 green marbles?

Use paper if it helps you with your thinking.

Teacher Moves

Math Community

- Consider highlighting the value of reflecting on feedback. Ask students who entered an incorrect answer to share how they used the feedback to help them revise their answer.

Early Student Thinking

- Students may notice that 10 is $\frac{2}{3}$ of 15 and calculate $\frac{2}{3}$ of 6 marbles.
- Consider asking: *Is the* 10 *related to the* 15 *balloons or the* 6 *marbles?*

Sample Responses

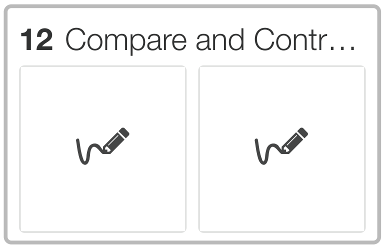
25 red balloons

Student Supports

Multilingual Learners

- *Routine: Collect and Display*

Circulate and listen to students talk as they describe how to use ratios to determine the missing number of balloons. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.



12 Compare and Contr…

Teacher Moves

Facilitation

- Invite students to spend 2–3 minutes discussing this screen with a partner.
- When most students have discussed, facilitate a whole-class discussion to describe each strategy.
- Invite students to share other strategies they used that are not pictured here.
- If it does not come up naturally, consider asking: *Why do you think Daeja figured out how many balloons she needed for* 30 *marbles?*

Discussion Questions

- *How would you summarize Daeja's strategy? Charlie's?*
- *Whose strategy was more similar to yours?*
- *What are the advantages of Charlie's strategy? Daeja's?*

Math Community

- Consider revisiting the names of strategies surfaced earlier in the lesson and asking if either of the strategies on this screen matches with one of those.

Sample Responses

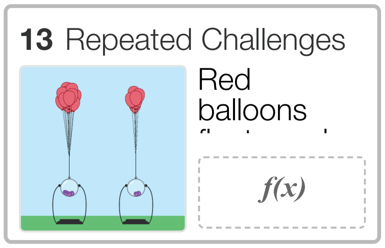
Responses vary.

Similarities

- Both students used a table and both students found a different ratio before they found the ratio they needed to solve the problem.
- Both students used multiplication.

Differences

- Charlie only used multiplication, while Daeja used multiplication *and* division.
- Charlie calculated how many balloons for 1 marble and then multiplied to get 6.
- Daeja found a number that works well with both 6 marbles and 10 marbles. She figured out how many balloons she would need for 30 marbles, then figured out the amount for 10 marbles.



13 Repeated Challenges

Red balloons

Red balloons float purple marbles at a ratio of $6 : 2$.

How many purple marbles will 12 red balloons float? (Challenges vary.)

Use paper if it helps you with your thinking.

Teacher Moves

How Repeated Challenges Work

- Students are presented with a variety of challenges one at a time, in this case, calculating the missing number of balloons or marbles.
- The challenges typically increase in difficulty as they continue.

Facilitation

- Give students 5–7 minutes to complete as many challenges as they can.
- Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

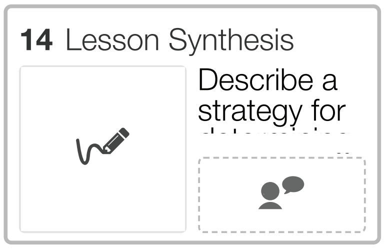
Math Community

- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Sample Responses

This screen contains an unlimited number of randomized challenges. The challenges progress in difficulty.

The solution to the first challenge is 4 purple marbles.



14 Lesson Synthesis

Describe a strategy for

Describe a strategy for determining missing values in equivalent ratios, like an unknown number of balloons or marbles.

Use the sketch tool if it helps you show your thinking.

Teacher Moves

Key Discussion Screen

- The purpose of this discussion is to compare strategies for determining missing values in equivalent ratios.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Suggested Pacing: Screen 14

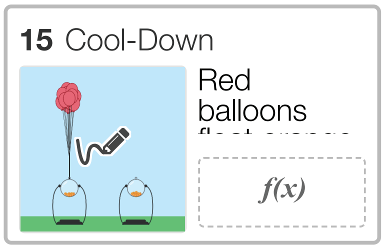
Sample Responses

Responses vary.

One strategy is to figure out a unit rate, so how many balloons for every marble or how many marbles for every balloon. Then you can multiply your unit rate by the number of marbles or balloons you know to figure out the missing one.

Another strategy is to find a different equivalent ratio that would be helpful as a middle step.

You can use a double number line or a table to help you with your thinking.



15 Cool-Down

Red balloons float orange

Red balloons float orange marbles at a ratio of $12 : 8$.

How many red balloons will float 10 orange marbles?

Use paper if it helps you with your thinking.

Teacher Moves

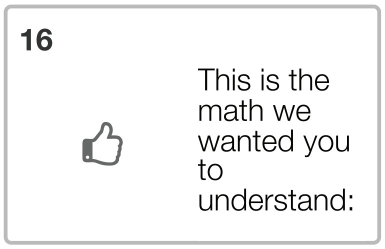
Support for Future Learning

- Students will have more chances to solve multistep ratio problems in Lesson 11. If many students struggle, consider reviewing this screen as a class before beginning Lesson 11.

Suggested Pacing: Screens 15–16

Sample Responses

15 red balloons



16

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can solve problems using tables and double number line diagrams.
- I can compare different strategies for determining missing values.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Community Life (NYC)

Lesson 11: Solving Equivalent Ratio Problems

Purpose

In this lesson, students apply everything they've learned about ratios to make sense of situations in context. Students use a combination of ratio thinking and their own knowledge to answer a series of questions related to community life. Students are invited to pause and make sense of each situation before solving.

Preparation

Worksheet

- *Activity 1–3:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Cards

- Print one set of double-sided cards for every group of 2–3 students.

Materials

- Calculator

Warm-Up (5 minutes)

Overview: Students use the routine Three Reads to make sense of a situation described in words (MP1). This reasoning is an example of what students will do on their own in the rest of the lesson.

Launch

- Invite students to work *in pairs.*
- Display Sheet 1 of the Teacher Projection Sheets.

Facilitation

- The purpose of this routine is to invite students to slow down and focus on a situation before calculating. This may be the first time students experience a Three Reads routine. Here is a brief summary:
    1. Display Sheet 1 and give students one minute to read quietly and then discuss with their partner what the situation is about. Then invite them to share with the class.
    2. Display Sheet 2. Consider asking a student to read the situation aloud or invite students to read in pairs. Then give students 1–2 minutes to draw a visual to represent the situation.
    3. Display Sheet 3. Read the situation once more and invite students to brainstorm a question we might ask. If it does not come up naturally, consider inviting students to answer the question: *How much will the rice cost?*

Math Community

- Consider inviting students to share why reading a question multiple times might be helpful in order to normalize slowing down when solving a complex problem.

Support for Multilingual Learners

Receptive/Expressive Language: Strategic Pairing

Pair students to aid them in comprehension and expression of understanding.

Activity 1: Sort 'em (10 minutes)

Overview: Students determine which questions from a variety of situations could be solved using equivalent ratios. Students will answer these questions in Activities 2 and 3.

Launch

- Invite students to work in *groups of 2–3.*
- Distribute one worksheet to each student and one set of cards to each group.
- Have students display the cards with the numberless side facing up.
- Invite each group to select one card and make sense of it together. Encourage students to ask themselves questions similar to the warm-up (MP1).

Facilitation

- Give students several minutes to read each card and to reach an agreement together about how to sort the cards.
- Consider inviting students to check with a teacher or another group and justify their choices.
- During the check in, consider choosing one student at random from each group to share the group's thinking for one card (e.g., Card B).

Early Finishers

- Encourage students who finish early to begin Activity 2.

Math Community

- Before beginning this activity, consider inviting students to brainstorm how they will work together to hear from everyone in their group.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Support for Students With Disabilities

Conceptual Processing: Processing Time

Begin with a demonstration of the first problem to provide access to students who benefit from clear and explicit instructions. Check in with individual students, as needed, to assess for comprehension during each step of the activity.

Activity 2: Closer Look (10 minutes)

Overview: Students analyze tools for answering the question on one card (Card D) before answering questions on their own in Activity 3.

Facilitation

- Invite students to turn over Card D to reveal the values for that problem.
- Circulate to check students' progress on Problem 1. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- When most students have answered Problem 4, consider facilitating a whole-class discussion.
- Consider polling the class to see the distribution of responses. Invite students who said it was reasonable and students who said it was not reasonable to share their thinking.
- Encourage students to use a calculator whenever they find it helpful throughout the lesson.

Discussion Questions

- *How did you decide whether 6 hospitals is reasonable?*
- *How could we use the table to decide if 6 hospitals is reasonable? The double number line?*

Early Finishers

- Encourage students who finish early to begin Activity 3.

Math Community

- It's okay—even desirable—to lack consensus at this stage. Consider focusing on the reasoning students used to justify their answer (MP3) instead of coming to a definitive conclusion.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Support for Students With Disabilities
Conceptual Processing: Eliminate Barriers
Assist students in recognizing the connections between new problems and prior work. Students may benefit from a review of different representations to activate prior knowledge.

Activity 3: Solve 'em (10 minutes)

Overview: Students answer questions about situations involving equivalent ratios.

Facilitation

- Give students several minutes to select cards and answer the question on them.
- Encourage students to select the tool that they think will most help them answer each question or to try multiple tools and compare the results. (MP5)
- Consider posting an answer key for students to compare their thinking to.

- Consider inviting students to find someone who selected the same card(s) and to compare their thinking.
- Consider focusing on one card as a class, like Card C, and inviting several students to share the strategies they used to answer the question.

Discussion Questions

- *What tools did you find most helpful in answering these questions?* (MP5)
- *How are __________'s strategy and ____________'s strategy similar? How are they different?*
- *Where else in our communities might we see equivalent ratios?*

Math Community

- Highlight different successful ways of answering the same question on each card. Invite students to reflect on which strategy they prefer and why.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

The purpose of this discussion is to surface strategies for solving ratio problems in context.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their response based on the discussion.

Discussion Questions

- *How are ___________'s and __________'s strategies similar? How are they different?*
- *How would you say what ________ said in your own words?*

Math Community

- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Cool-Down (5 minutes)

Support for Future Learning

Consider reviewing this screen as a class before Practice Day 2 or offering individual support where needed during the practice day.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Mixing Paint, Part 2 (NYC)

Lesson 12: Part-Part-Whole Ratios

Overview

Students extend what they've learned about ratios to explore ratios where both quantities are given in the same unit.

Learning Goals

- Use and interpret tape diagrams to solve problems involving part-part-whole ratios (e.g., green paint is made from 5 parts blue and 3 parts yellow paint).

Vocabulary

- tape diagram

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to extend what they've learned about ratios to explore ratios where both quantities are given in the same unit. These relationships, called part-part-whole ratios, enable students to answer new questions like “If I want 10 cups of paint in total, how many cups of red paint and blue paint will I need?”

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to focus students’ attention on how the amount of each quantity in a ratio affects the whole. This may be the first time students see ratios where it makes sense to consider the total amount.

Activity 1: Intro to Tape Diagrams (15 minutes)

The purpose of this activity is to introduce students to a new tool for approaching problems involving part-part-whole ratios: the tape diagram. Students are given the opportunity to solve a problem given a total amount of paint, then see how a tape diagram might be useful in solving it.

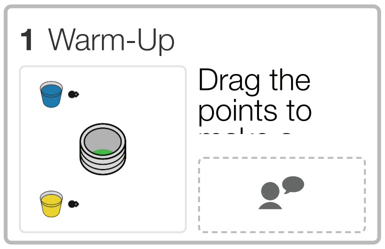
Activity 2: Solving More Problems (15 minutes)

The purpose of this activity is for students to apply what they’ve learned in Activity 1 to new paint ratios. Students can use tape diagrams to support their thinking. This activity also invites students to pay close attention to units of measurement (e.g., cups vs. gallons) as they examine ratios of paint.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to make connections between tape diagrams and part-part-whole ratio relationships.

Cool-Down (5 minutes)



1 Warm-Up

Drag the points to

Drag the points to make a new color.

What do you notice? What do you wonder?

Teacher Moves

Overview: In this lesson, students extend what they've learned about ratios to explore ratios where both quantities are given in the same unit. The purpose of the warm-up is to focus students' attention on how the amount of each quantity in a ratio affects the whole.

Launch

- Consider inviting students to share differences between this method of mixing paint (pouring two cups into a bucket) and the method of mixing paint from Lesson 7 (pouring small amounts of tint into white paint).

Facilitation

- Give students 2–3 minutes to create different shades and amounts of paint.
- Use the routine Notice and Wonder to surface what students noticed and wondered.
- Consider monitoring for different ways of creating a full can and using the snapshot tool to capture them. Highlight these during the discussion.

Discussion Questions

- *When is the can filled exactly? Is there more than one ratio that fills the can exactly?*
- *What happens when you create equivalent ratios of blue to yellow?*

Math Community

- Consider inviting students to signal if they also noticed or wondered something shared by a classmate.

Readiness Check (Problem 7)

- If students struggled with this problem, consider spending extra time on this warm-up. Invite students to determine what fraction of the total paint is made of blue paint and yellow paint for different mixes.

Early Finishers

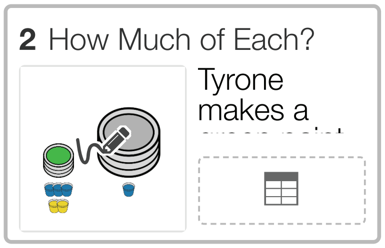
- Encourage students to determine how many different ratios fill the can exactly and to justify their response.

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

- I notice that the more paint you add, the more the bucket fills up.
- I notice that it takes 8 cups of paint to fill the bucket.
- I notice that the more yellow you add, the lighter the green gets.
- I wonder what happens if you make equivalent ratios of yellow and blue paint.
- I wonder if the bucket of paint is always the same size.
- I wonder what would happen if you mixed half a cup of yellow with half a cup of blue.



2 How Much of Each?

Tyrone makes a

Tyrone makes a green paint by mixing 3 cups of blue with 2 cups of yellow.

He now needs 20 more cups of green paint to finish painting a mural.

How much of each color should he mix?

Teacher Moves

Overview: In Activity 1 (Screens 2–4), students are introduced to a new tool for approaching problems involving part-part-whole ratios: the tape diagram.

Launch

- Read the problem as a class. Share that we will spend a few minutes on this problem to hear many different ways to approach it.
- Consider demonstrating by entering an incorrect number of cups of blue and yellow paint. Invite students to explain how they know it doesn't work.

Facilitation

- If students are having trouble getting started, encourage them to enter values and ask: *Does this make a blue to yellow ratio of* $3:2$*? Does this make* 20 *cups total?*
- Monitor for different strategies, both correct and incorrect. Some students may pay more attention to 20 cups total, while others will pay more attention to the $3:2$ ratio (MP1).
- Spend adequate time discussing students' approaches and ensure everyone understands why the answer is 12 cups of blue and 8 cups of yellow.

Discussion Questions

- *What are some things you tried that didn't work? How did you know they did not work?*

- *What did you start thinking about: the 20 cups or the $3 : 2$ ratio? How did you think about both at once?*

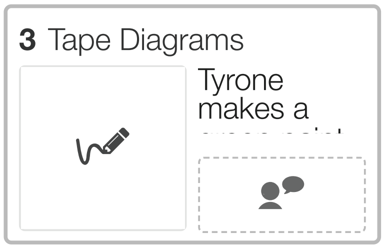
Early Finishers
- Encourage students to show their reasoning using the sketch tool.

Math Community
- Intentionally select and sequence correct and incorrect strategies. Consider inviting students to share what they think we can learn from looking at both correct and incorrect thinking.

Suggested Pacing: Screen 2

Sample Responses

12 blue cups and 8 yellow cups



3 Tape Diagrams

Tyrone makes a

Tyrone makes a green paint by mixing 3 cups of blue with 2 cups of yellow.

Tyrone drew a tape diagram to help him figure out that he needs 12 cups of blue and 8 cups of yellow to make 20 cups of green paint.

Where do you see the $3 : 2$ ratio, 20, and 12 represented in Tyrone's diagram?

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is for students to describe how the tape diagram represents the ratio and how it can be used to help solve problems.

Launch
- Consider starting with the screen paused and inviting students to share what they notice and wonder about the tape diagram.

Facilitation
- Encourage students to use the sketch tool if it helps them show their thinking.
- Select and sequence sketches and text responses. Monitor for students who explain their thinking in different ways.
- Spend adequate time here to ensure students understand how to construct a tape diagram for a situation.

Discussion Questions

- *How could Tyrone's tape diagram help him solve the problem?*
- *How would the tape diagram be different if the ratio of blue to yellow was* $4 : 1$*?*
- *How would the tape diagram be different if it were* 10 *total cups of paint?*

Suggested Pacing: Screen 3–4

Sample Responses

Responses vary.

- The $3 : 2$ ratio is shown by three blue boxes and two yellow boxes.
- The 20 total cups is shown by the total amount in the diagram. $4 + 4 + 4 + 4 + 4 = 20$ total cups.
- The 12 blue cups are shown by the total amount in the blue boxes.

Student Supports

Students With Disabilities

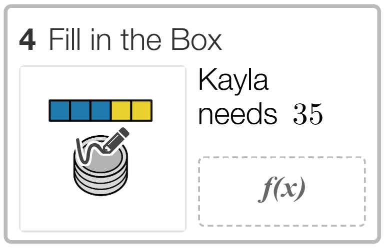
• *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

• *Expressive Language: Visual Aids*

Create or review an anchor chart that publicly displays a tape diagram to aid in explanations and reasoning.



4 Fill in the Box

Kayla needs 35

$f(x)$

Kayla needs 35 gallons of the same green paint.

She used this tape diagram to determine how much of each paint color she needs.

What number of gallons would go into each box in the tape diagram?

Teacher Moves

Progress Check

- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking

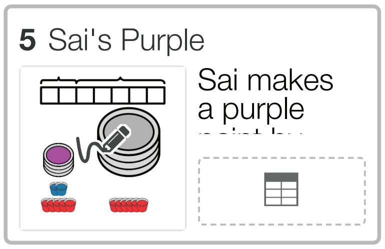
- Students might assume that there should be 35 gallons or 5 gallons in each box of the tape diagram. Encourage students to use the feedback to adjust their thinking, then to explain to a partner why 7 gallons makes sense in this situation.

Early Finishers

- Encourage students to write a general strategy for how to figure out what value goes in each box of a tape diagram.

Sample Responses

7 gallons



5 Sai's Purple

Sai makes a purple

Sai makes a purple paint by mixing 2 cups of blue and 5 cups of red.

How much of each color should they mix if they need 28 cups of purple paint?

Use the tape diagram if it helps you with your thinking.

Teacher Moves

Overview: In Activity 2 (Screens 5–10), students apply what they've learned in Activity 1 to new paint ratios.

Facilitation

- Consider reading the problem as a class.
- Encourage students to use the sketch tool if it helps them with their thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 5–10

Sample Responses

8 blue cups and 20 red cups


6 Same Color?

Sai makes a purple

Sai makes a purple paint by mixing 2 cups of blue and 5 cups of red.

Which of the following would make the same color?

Teacher Moves

Facilitation

- The purpose of this screen is for students to attend to the units of measure in ratios.
- When most students have responded, display the distribution of responses using the dashboard’s teacher view.
- Spend adequate time discussing why 2 quarts blue and 5 quarts red will create the same color but 2 cups blue and 5 gallons red will not (MP6).

Discussion Questions

- *Why do you think using quarts instead of cups would make the same color?*

Math Community

- Consider inviting students to think of reasons someone might have selected 2 cups blue and 5 gallons red.

Sample Responses

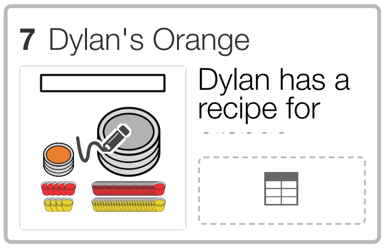
- 1 cup blue and 2.5 cups red
- 2 quarts blue and 5 quarts red

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.


7 Dylan's Orange

Dylan has a recipe for

Dylan has a recipe for orange paint that mixes 5 parts red paint and 4 parts yellow paint.

How much of each color should he mix if he needs 54 cups of orange paint?

Draw your own tape diagram if it helps you with your thinking.

Teacher Moves

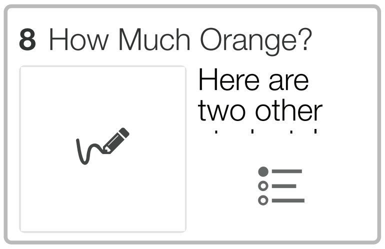
Facilitation

- Screens 7–9 are intended for students to practice using tape diagrams to solve part-part-whole problems. Consider selecting which screens you think best match the needs of your students.
- Consider using the snapshot tool to capture how students used the tape diagram.

Note: This is the first time students see the word “parts” to describe a ratio.

Sample Responses

30 red cups and 24 yellow cups



8 How Much Orange?

Here are two other

Here are two other students' recipes for orange paint.

Which student makes more paint?

Teacher Moves

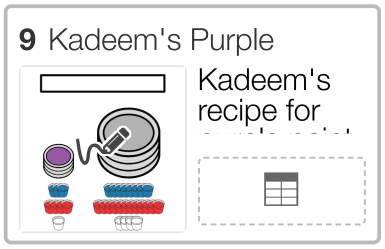
Discussion Questions

- *How did you figure out how much orange paint Ethan used?*

Sample Responses

Ethan

Explanations vary. Ethan made $7 \cdot 6 = 42$ cups of paint. Zion only made $5 \cdot 7 = 35$ cups of paint.



9 Kadeem's Purple

Kadeem's recipe for

Kadeem's recipe for purple paint calls for 3 parts blue, 4 parts red, and 1 part white paint.

He needs 24 liters of purple paint to paint his mural.

How much of each color will he need?

Teacher Moves

Facilitation

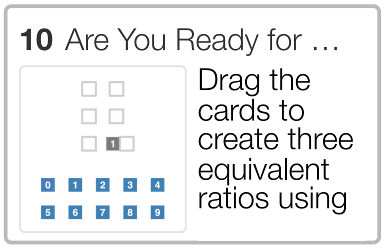
- Consider using the snapshot tool to capture how students used the tape diagram.

Discussion Questions
- *How is solving problems with* 3 *colors of paint similar to solving problems with* 2 *colors of paint? How is it different?*

Math Community
- Consider highlighting unique or creative uses of the tape diagram to solve the problem.

Sample Responses

9 blue L, 12 red L, 3 white L



10 Are You Ready for …

Drag the cards to create three equivalent ratios using

Drag the cards to create three equivalent ratios using the digits 0 through 9.

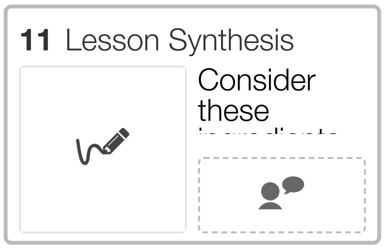
Teacher Moves

Facilitation
- Invite students who finish Screens 5–9 early to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion.

Sample Responses

Responses vary.

- $1:2$, $3:6$, and $9:18$
- $2:4$, $3:6$, and $5:10$



11 Lesson Synthesis

Consider these

Consider these ingredients for a mango lassi drink.

Explain how this tape diagram represents this situation.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to surface connections between tape diagrams and part-part-whole ratio relationships.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *What is important to think about when making a tape diagram for a situation?*
- *How does changing the value in each box affect the total amount of mango lassi?*

Routine (optional): Consider using the mathematical language routine Collect and Display to capture students' insights about tape diagrams for reference later.

Suggested Pacing: Screen 11

Sample Responses

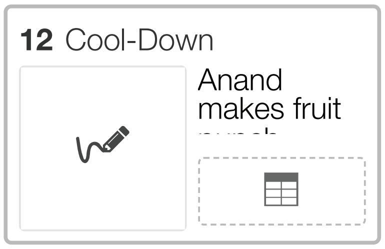
Responses vary. The mango lassi has 12 cups of liquid and there are 6 boxes in the tape diagram, so every box represents 2 cups. The 6 cups of mango are shown by the 3 yellow boxes, because $3 \cdot 2 = 6$. The 4 cups of yogurt are the 2 blue boxes, and the 2 cups of milk are the 1 white box.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., Two things I know about positive and negative numbers are ______.).


12 Cool-Down

Anand makes fruit

Anand makes fruit punch by mixing 4 liters of cranberry juice and 3 liters of ginger ale.

How much of each ingredient would Anand need to make 35 liters of fruit punch for a party?

Teacher Moves

Support for Future Learning

- Students will have more chances to develop their understanding of part-part-whole relationships in Lesson 13. Consider reviewing this screen as a class before Lesson 13 or offering individual support where needed.

Suggested Pacing: Screens 12–13

Sample Responses

20 cranberry L and 15 ginger ale L

13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use and interpret tape diagrams to solve problems involving part-part-whole ratios.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

City Planning (NYC)

Lesson 13: Applying Part-Part-Whole Ratio Problems

Overview

Students apply the strategies they learned in Lesson 12 to solve problems involving part-part-whole ratios in the context of housing and green space in neighborhoods.

Learning Goals

- Create and use tape diagrams and tables to help solve problems involving part-part-whole ratios.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to apply the strategies they learned in Lesson 12 to solve problems involving part-to-part ratios. Students use these strategies to act as city planners and think about ratios of market-rate to affordable housing and building space to green space.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to begin thinking about the characteristics of neighborhoods and what makes a neighborhood a place they might want to live.

Activity 1: Market-Rate and Affordable Housing (15 minutes)

The purpose of this activity is for students to solve problems involving ratios of affordable housing to market-rate housing. Students calculate missing parts given the whole in part-to-part ratio relationships. They also determine whether or not neighborhoods meet city requirements for the ratio of affordable housing to market-rate housing.

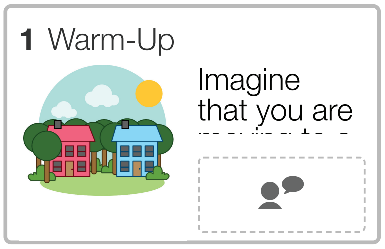
Activity 2: Green Space (15 minutes)

The purpose of this activity is for students to use what they've learned and their personal knowledge to analyze different neighborhoods. Students consider the ratio of building space to green space, and the combined ratio of market-rate housing to affordable housing to green space. In this activity, students have the opportunity to design their own neighborhood that meets the city's requirements.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to describe and refine strategies for determining the parts given a whole in a part-to-part ratio situation.

Cool-Down (5 minutes)



1 Warm-Up

Imagine that you are

Imagine that you are moving to a new city.

What would be important to you when looking for a place to live?

Teacher Moves

Overview: In this lesson, students apply the strategies they learned in Lesson 12 to solve problems involving part-to-part ratios in the context of housing and green space in neighborhoods. In this warm-up, students begin thinking about the characteristics of neighborhoods and what makes a neighborhood a place they might want to live.

Launch

- Consider asking students: *If you could move anywhere, where would you move?*

Facilitation

- Give students 1–2 minutes to work independently, then share their responses with a partner.
- Consider generating a long list of characteristics that are important to students.
- If it doesn't come up naturally, consider asking about resources like grocery stores or about affordability or about green spaces like parks, etc.

Math Community

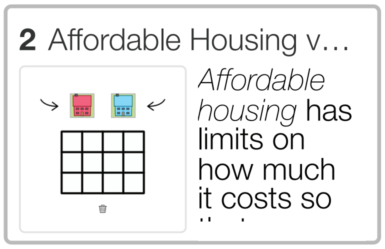
- Conversations about fair housing can be very different for different students, in different classrooms, and in different communities. As students share their perspectives of fairness, consider a) the culture of your classroom b) how you will facilitate a discussion where students who may bring perspectives different from their peers and from your own, and c) how you will check-in with students throughout and after the discussion.

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

- Close to work and school
- Affordable
- Near my friends and family
- Has a grocery store and a park nearby



2 Affordable Housing v…

Affordable housing has limits on how much it costs so

Affordable housing has limits on how much it costs so that households have money left over for other necessities, like food and healthcare.

Market-rate housing has no limits on how much it costs.

Many cities have a shortage of affordable housing.

Create a neighborhood that you think has a fair balance of housing.

Teacher Moves

Overview: In Activity 1 (Screens 2–6), students solve problems involving ratios of affordable housing to market-rate housing.

Launch

- Invite students to work *in pairs.*
- Start with the activity paused and review what *affordable housing* and *market-rate housing* mean.
- Consider asking: *Why is it important to have both market-rate and affordable housing in a neighborhood?*
- Demonstrate how to create a neighborhood using the dashboard's student view.

Suggested Pacing: Screens 2–3

Sample Responses

Responses vary.

Student Supports

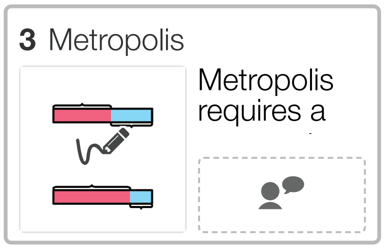
Students With Disabilities

- *Fine Motor Skills: Strategic Pairing*

Allow students who struggle with fine motor skills to dictate use of the graphs and sketch tools as needed throughout the lesson.

- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.



Metropolis requires a $7 : 2$ ratio of market-rate housing to affordable housing units.

Here's the neighborhood you created and a neighborhood in Metropolis.

How does your neighborhood compare to Metropolis's requirement?

Teacher Moves

Facilitation

- Encourage students to examine both the tape diagrams and the neighborhoods, and to make connections between them.
- Select several student responses and tape diagrams using the dashboard's snapshot tool.
- Facilitate a discussion to make connections between Metropolis's requirements, the tape diagram, and the neighborhood, and to raise questions of fairness.

Discussion Questions

- *Which neighborhood has more affordable housing units? Which has a larger portion of affordable housing units?*
- *Where can you see the* $7 : 2$ *in Metropolis's tape diagram?*
- *Do you think Metropolis's requirements are fair? Why or why not?*

Early Finishers

- Encourage students to determine the ratio of market-rate housing to affordable housing units for the neighborhood they designed and to calculate how many of each type of unit they would have if their neighborhood was the size of Metropolis's.

Sample Responses

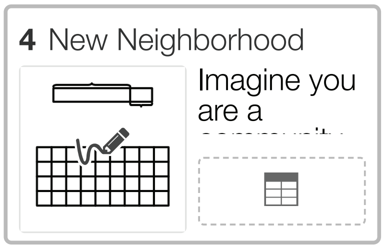
Responses vary. The Metropolis neighborhood has more affordable housing units than mine does, but it also has more units overall. My tape diagram has a larger portion of affordable housing, so my neighborhood has more of a balance between affordable and market-rate housing.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.



4 New Neighborhood

Imagine you are a

Imagine you are a community planner.

Your task is to make sure each neighborhood has a $7:2$ ratio of market-rate units to affordable units.

A neighborhood is developing 36 units of land.

How many of each type of housing should the city plan for in order to meet the requirement?

Teacher Moves

Launch

- Consider sharing with students that in this lesson, they will be community planners in Metropolis. Their task is to make sure each neighborhood has the correct ratio of different types of housing units.
- Encourage students to use paper to help them with their thinking.

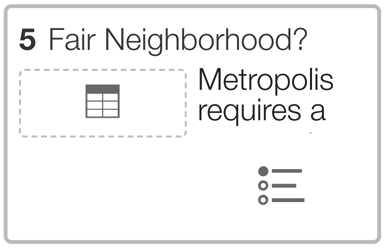
Progress Check

- Encourage students to use the feedback on the screen to help them revise their thinking.

Suggested Pacing: Screens 4–6

Sample Responses

28 market-rate units and 8 affordable units



5 Fair Neighborhood?

Metropolis requires a

Metropolis requires a $7:2$ ratio of market-rate units to affordable units.

Does this neighborhood meet the requirement?

Teacher Moves

Facilitation

- Consider discussing the distribution of responses during the discussion on Screen 6.

Early Student Thinking

- Students may think that the neighborhood does meet the requirements because 10 is a multiple of 2 and because $62 + 10 = 72$.

- Consider asking: *If there were* 10 *affordable housing units, how many market-rate units would there be to create a* $7:2$ *ratio?*

Sample Responses

No

Explanations vary. If there were 10 affordable units, then there should be 35 market-rate housing units because $2 \cdot 5 = 10$ and $7 \cdot 5 = 35$. This neighborhood has more than 35 market-rate housing units, so it does not meet the requirement.

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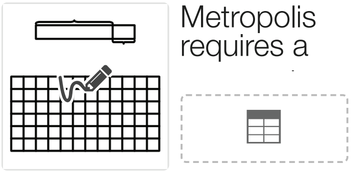
6 Fix It!



Metropolis requires a

Metropolis requires a $7:2$ ratio of market-rate units to affordable units.

Adjust the number of units of each type of housing so that this neighborhood meets Metropolis's requirement.

Teacher Moves

Progress Check

- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- If students need more practice solving problems like these, invite them to calculate the number of market-rate and affordable housing units for the neighborhood they designed on Screen 2, if their neighborhoods were 36 or 72 units large.

Early Finishers

- Encourage students to determine the number of market-rate and affordable housing units the neighborhood they designed would have if it were the size of this neighborhood.

Math Community

- Invite students to share strategies they’ve found most helpful for solving this problem and attribute them to the students who shared them.

Sample Responses

56 market-rate units and 16 affordable units

7 New City, New Neigh…

Urban green

Urban green spaces, such as parks and gardens, give people space for physical activity, relaxation, peace, and an escape from heat.

Here are two neighborhoods in Evergreen City.

Where would you prefer to live?

Teacher Moves

Overview: In Activity 2 (Screens 7–10), students use what they've learned and their personal knowledge to analyze different neighborhoods.

Launch
- Consider asking: *What could green space be used for?*

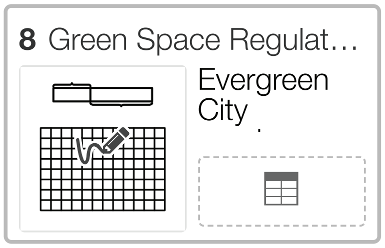
Facilitation
- Use the dashboard's teacher view to display the distribution of responses.
- Invite students to share why someone might have chosen each neighborhood.

Suggested Pacing: Screen 7

Sample Responses

Responses and explanations vary.

- I would rather live in Neighborhood A because you can do more things with a big green space. I would love to have a big park where I can run around.
- I would prefer to live in Neighborhood B because then every house is close to at least one bit of green space.



8 Green Space Regulat…

Evergreen City .

Evergreen City requires $3 : 5$ units of green space to building space.

The city is developing 96 units of land for a new neighborhood.

How many of each type of space should the city plan for?

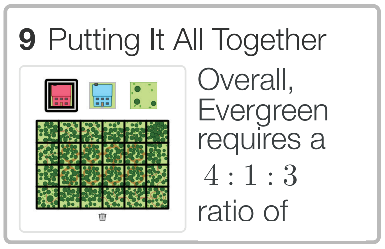
Teacher Moves

Progress Check
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 8–10

Sample Responses

36 units of green space and 60 units of building space



9 Putting It All Together

Overall, Evergreen requires a $4:1:3$ ratio of

Overall, Evergreen requires a $4:1:3$ ratio of market-rate housing to affordable housing to green space.

Here are 24 units of land.

Design a neighborhood that meets Evergreen City's requirements.

Teacher Moves

Facilitation

- If students are struggling to start, invite them to determine how many of each type of unit they will need.
- Use the dashboard's snapshot tool or teacher view to display several students' neighborhoods. Include ones that do and do not meet the requirements.
- Invite students to notice and wonder about the different neighborhoods.
- Invite several students to share their strategies both for determining how many of each type of unit there would be and how they should be distributed.

Discussion Questions

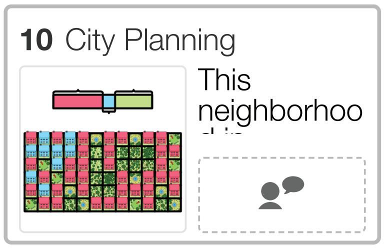
- *Does this neighborhood meet the requirements? How do you know?*
- *What is similar about each neighborhood? What is different?*
- *Why do you think _______ chose to design their neighborhood like that?*

Math Community

- Consider highlighting unique or creative neighborhoods using the snapshot tool or the dashboard's teacher view. Ask the author to speak about their inspiration.

Sample Responses

Responses vary. All neighborhoods should have 12 units of market-rate housing, 3 units of affordable housing, and 9 units of green space.



10 City Planning

This neighborhoo

This neighborhood in Evergreen City meets the requirement of a $4:1:3$ ratio of market-rate housing to affordable housing to green space.

However, residents claim this neighborhood is not fair.

1. Why might the residents feel it is not fair?

2. What changes do you think should be made?

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is for students to consider how city regulations may support fairness and how they may also fall short.

Facilitation
- When most students have responded, facilitate a whole-class discussion.

Discussion Questions
- *Why might some people say this is fair? Why might others say it is not fair? What do you think?*
- *What might be more impactful, changing the ratios or changing the layout of the neighborhood?*

Early Finishers
- Encourage students to answer a question like: *Which ratio would you change if you could change one (market-rate to affordable units or building space to green space)?*

Math Community
- Invite students to bring their own sense of fairness into this conversation, and to leave the discussion without a final conclusion about whether or not the neighborhood is designed fairly.

Sample Responses

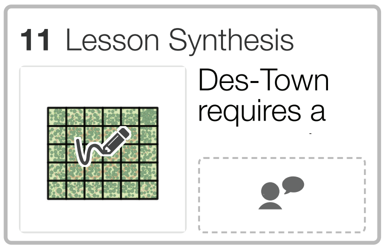
Responses vary.

- The residents might think this neighborhood is not fair because almost all of the green space is by the market-rate housing. Also, there is much more market-rate housing than affordable housing.
- I would change how the affordable housing is distributed so that everyone benefits from the green space. I would also increase the units of affordable housing.

Student Supports

Students With Disabilities
- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.



11 Lesson Synthesis

Des-Town requires a

Des-Town requires a $3 : 2$ ratio of building space to green space.

Explain how a city planner can determine how many units of building space can be developed in this neighborhood.

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to surface and refine strategies for determining the parts given a whole in a part-to-part ratio situation.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.

Discussion Questions
- *What makes sense to you about this strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Suggested Pacing: Screen 11

Routine (optional): Consider using the mathematical language routine Collect and Display to capture students' strategies.

Sample Responses

Responses vary. The total units in the ratio is $3 + 2 = 5$, so you can divide the number of units you have by 5 and then multiply by either 3 or 2 to figure out how many of each type of housing unit you need. In this example, there are 30 total units, so there would be $\frac{30}{5} \cdot 3 = 18$ units of building space and $\frac{30}{5} \cdot 2 = 12$ units of green space.

Student Supports

Multilingual Learners

- *Routine: Collect and Display*

Circulate and listen to students talk as they describe their strategy for determining the number of units for each type of land. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.

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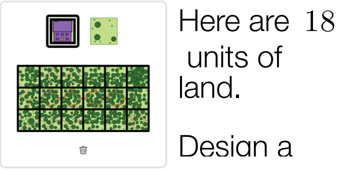
12 Cool-Down



Here are 18 units of land.

Design a

Here are 18 units of land.

Design a neighborhood that has a $5 : 4$ ratio of building space to green space.

Teacher Moves

Support for Future Learning

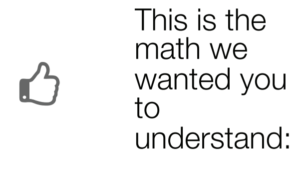
- Consider reviewing this screen as a class before Practice Day 2 or offering individual support where needed during the practice day. A strong understanding of how to determine the parts given the whole will support students in the End-Unit Assessment.

Suggested Pacing: Screens 12–13

Sample Responses

10 building units and 8 green spaces

13



This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can create and use tape diagrams to solve problems involving part-part-whole ratios.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Lunch Waste (NYC)

Lesson 14: Applying Ratio Strategies

Purpose

Students use their personal experiences and the different kinds of ratio reasoning they've learned to make sense of a common experience: school lunch and lunch waste. This lesson surfaces ideas of unit rates, equivalent ratios, and part-part-whole ratios, giving students the flexibility to choose their own strategies as they make sense of how much waste is generated and analyze solutions for reducing the amount of lunch waste.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Warm-Up (5 minutes)

Overview: Students are introduced to thinking about how much gets thrown away at lunch time.

Launch

- Display Sheet 1 of the Teacher Projection Sheets.

Facilitation

- Give students 1–2 minutes to think independently and share their responses with a partner.
- If students are struggling to create an estimate, consider inviting them to think of a number they think would be too high and a number they think would be too low.
- Consider writing a list of lunch trash and a range of student estimates on the board to refer back to during Activities 1 and 2.

Discussion Questions

- *How do you feel about the amount of things we throw away at lunch time?*
- *Do you think this is more or less than the amount of trash created at home? At other schools?*

Math Community

- Invite students to use their personal experiences to inform their estimates.

Activity 1: How Much Waste? (10 minutes)

Overview: Students use ratio relationships to determine how much trash is created at a school.

Launch

- Invite students to work *in groups of 2–3.*
- Display Sheet 2 of the Teacher Projection Sheets.
- Read the situation aloud. Consider asking: *Why might Maria and Hoang have studied their class instead of the whole school?*
- Spend adequate time here to ensure students understand what the three types of trash were.
- Distribute one student worksheet to each student.

Facilitation

- Give students 5–7 minutes to answer Problems 1–3.
- When most students have responded, consider reviewing each question as a class.
- **Note:** Consider changing the number of school days in a year to match your school.

Discussion Questions

- *What strategies were helpful in answering each question?*
- *Do you think our answer for the total amount of waste at this school is accurate? Even if it isn't accurate, why might this number be useful?*

Math Community

- Encourage students to name strategies that were helpful.
- Invite students to think about the waste that their school generates.

Early Finishers

- Encourage students to try "Are You Ready for More?" on Page 2 of the Student Worksheet.

Activity 2: Cutting Waste (20 minutes)

Overview: Use ratio strategies to analyze ways to reduce lunch waste.

Launch

- Invite students to work in *groups of 3–4.*
- Display Sheet 3 of the Teacher Projection Sheets.
- Invite students to generate ideas for how to reduce waste at this school (or their school).
- If it does not come up naturally, consider discussing what composting, recycling, and reusable trays are.

Facilitation

- Give students 7–10 minutes to answer Problems 1–3.2.
- If students are struggling with Problem 1, consider asking: *How much waste did we create with styrofoam trays?*
- When most students have responded to Problem 2, consider sharing strategies for figuring out how many students would need to compost to meet their goal.
- When students have created a plan in Problem 3.2, invite them to compare plans with a different group, then revise their plan based on what they learned from that group.

Discussion Questions

- *What do you think of the strategies proposed in this lesson (i.e., switching to reusable trays and composting)? What other strategies might be effective?*
- *How much waste do you think is possible to remove? Is* 5000 *pounds reasonable?*
- *How did you decide on your plan to reduce lunch waste? What was important to you?*

Math Community

- Consider highlighting different strategies used to approach the same problem.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

The purpose of this discussion is for students to start to consider lunch waste in their own lives and what a first action step to reduce waste might look like.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their plans and who they would present it to.

Discussion Questions

- *How did you decide who to share your recommendation with?*
- *What action steps could we take now?*
- *How reasonable do you think your reduction plan is? What would need to be true to make this happen? Are you reducing by a lot? A little?*

Cool-Down (5 minutes)

Support for Future Learning

Consider reviewing this question as a class before Practice Day 2 or offering individual support where needed during the practice day. Several questions on the practice day invite students to compare ratios.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

6.2 Practice Day 2 (NYC)

Preparation

Student Workspace

- Print one double-sided copy for each student.

Cards

- *Group Questions:* Print and cut one set of cards for each group of students.
- *Solve and Swap:* Print and cut enough sets of cards so that there is one card for each student (e.g., two sets for a class of 24; three sets for a class of 36).

Instructions

Option 1: Group Questions

This structure supports student collaboration and focuses students' attention on one problem at a time.

- Arrange students into *groups of 2–3.* Print and cut out one set of 12 cards for each group.
- Invite students to select one card to work on at a time as a group.
- Give each student the Student Workspace Sheet to complete as they work together. Encourage students to justify their reasoning as they discuss their strategies.
- If time allows, invite students to order the cards from what they think will be more challenging to what they think will be less challenging. This helps them prioritize if they are not able to answer all 12 questions.
- Consider posting the answer key, or walking around with it and providing feedback to students as they work.

Option 2: Solve and Swap

This structure supports student collaboration with many different partners and allows for movement around the classroom. Students are positioned as experts as they discuss each problem and support one another.

- Print and cut out enough sets of cards so that there is one card for each student.
- Give each student a card and invite them to answer the question on their worksheet.
- Invite students to circulate the class with their card and pair up with a classmate. In the pair, each student should solve the problem on their partner's card, collaborating as needed.
- If a pair of students end up with a problem they've already solved, invite them to compare strategies and solutions with their partner.
- When both students have completed their problems, invite them to swap cards, stand up with a hand up, and find another classmate to pair up with.
- Repeat the process.

GRADE 6

Unit 3
Lesson Plans

Teacher lesson plans from Unit 3 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Ratios and Proportional Relationships** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 6 Unit 3

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

Digital Lessons

Assess and Respond

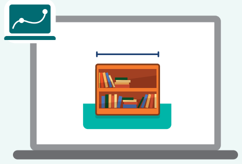
Pre-Unit Check
(Optional)

Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1

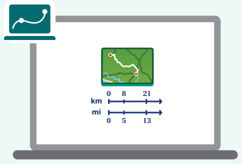
1 Many Measurements

Determine whether a certain unit measures length, area, time, mass, or volume.



2 Counting Classrooms

Convert measurements from one unit to another in the same system of measurement.



3 Pen Pals

Convert measurements from one unit to another in different measurement systems.

Practice Day

Practice Day 1

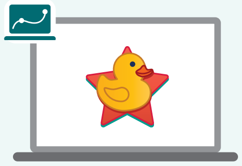
Practice the concepts and skills developed during Lessons 1–7. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

Quiz: Sub-Unit 1

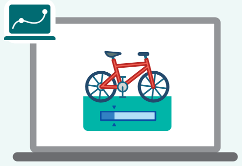
Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2



8 Lucky Duckies

Understand the word percent and that the symbol % means "for every 100."



9 Bicycle Goals

Use a double number line, tape diagram, or table to determine unknown parts or wholes given friendly percentages.

Practice Day

Practice Day 2

Practice the concepts and skills developed during Lessons 1–13. Consider using this time to prepare for the upcoming Quiz.

Summative Assessment

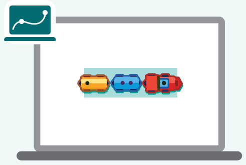
End-of-Unit Assessment

Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing: 17 days | **Short on time?** See pacing considerations below.

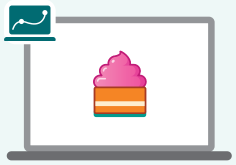
Pre-Unit Check: (Optional)
13 Lessons: 45 min each
2 Practice Days: 45 min each

1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



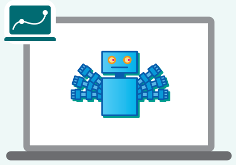
4 Model Trains

Use rate and ratio reasoning to compare rates expressed in different units.



5 Soft Serv

Calculate and interpret the two unit rates for the same relationship.

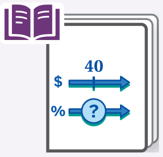


6 Welcome to the Robot Factory

Explain how to multiply by a unit rate to go from one column to another in a table of equivalent ratios.

7 More Soft Serve

Use unit rates to make comparisons and calculate unknown quantities.

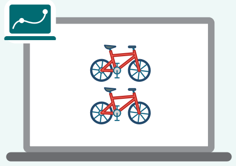


10 What's Missing?

Create tape diagrams, double number line diagrams, or tables to determine unknown parts, percentages, or wholes.

11 Cost Breakdown

Calculate any percentage of a number (e.g. 32% of 40).



12 More Bicycle Goals

Calculate an unknown percentage (e.g., 32 is what % of 40?)

13 A Country as a Village

Use rates and percentages to analyze characteristics of a country's population.

Pacing Considerations

Lesson 1: The purpose of this lesson is for students to practice working with the relative sizes of different units of measure. If students show a strong understanding of units of measure in Problem 1 of the Pre-Unit Check, this lesson may be omitted.

Lesson 2-3: These lessons support students in working with different units of measure and converting between units. They could be consolidated into one class period if students show a strong proficiency applying ratio reasoning to convert units.

Lesson 7: This lesson gives students an opportunity to practice using what they've learned about unit rates to make comparisons and calculate unknowns. There is no new content introduced in this lesson.

Lesson 13: This lesson gives students an opportunity to use what they've learned about rates and percentages to analyze characteristics of countries' populations. There is no new content introduced in this lesson.

Many Measurements (NYC)

Lesson 1: Everyday Measurements

Purpose

In this lesson, students recall what they've learned in previous grades about different units, what those units measure, and their relative sizes. Students describe different units of measurement and connect them with measurements of everyday objects, such as the width of a finger or the weight of a slice of bread. A strong understanding of what different units measure and their sizes will be helpful when students convert units in Lessons 2 and 3.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Cards

- Print and cut one set of single-sided cards for each pair of students.

Warm-Up (5 minutes)

Overview: Students discuss informal and formal ways of making comparisons based on measurements such as length, volume, and mass and weight.

Launch

- Invite students to work *in pairs.*
- Display Sheet 1 of the Teacher Projection Sheets.

Facilitation

- Give students 1–2 minutes to think quietly, then share their reasoning with a partner.
- Poll the class to see the distribution of responses. Invite students who selected each option to share how they know and what they could do to be more certain.
- **Note:** The third option is purposefully ambiguous. If it does not come up naturally, ask: *How could we be more certain?*

Discussion Questions

- *What tools could we use to be more certain?*
- *Which question is asking about length? Which is asking about mass or weight? Volume?* (If it does not come up naturally, consider defining these words to help students be successful in Activity 2.)

Math Community

- Celebrate students who use their personal experiences to support them in their reasoning.

Support for Multilingual Learners

Receptive/Expressive Language: Strategic Pairing

Consider bringing in images of physical representations of each object to support students in accessing this activity.

Activity 1: Describe It (10 minutes)

Overview: Students describe units of measurement in whatever ways make sense to them.

Launch

- Invite students to work *in pairs.*
- Display Sheet 2 of the Teacher Projection Sheets.
- Consider sharing that we will be using many measurement words in this lesson and ask students to name some measurement words that they already know.
- Consider waiting to distribute the Student Worksheet until after the launch.

Facilitation

- Give partners 1–2 minutes to describe one foot in as many different ways as they can.
- Invite several students to share their descriptions. Students may use their hands or draw on a piece of paper or make connections to familiar objects.
- Distribute one student worksheet to each student.
- Give students 5–7 minutes to describe as many measurements as they can.

Discussion Questions

- *What do you think makes some units easier or harder to describe?*
- *Why do you think we have different units that measure the same thing (like feet or miles)?*
- *Where in the world do you see* 1 *square foot or* 1 *square inch?*

Math Community

- Consider highlighting the variety of ways students describe measurements, along with any unique or creative ways.

Activity 2: Sort It (20 minutes)

Overview: Students sort units of measurement by what these units measure (attribute), then sort them by relative size, and then make connections between these units of measurement and everyday objects.

Launch

- Display Sheet 3 of the Teacher Projection Sheets. Give students one minute to decide what attribute kilograms and miles measure, then discuss.

Facilitation

- Distribute one set of measurement cards to each pair, or one set of all the cards to each pair and invite them to put the picture cards aside for now.
- Review each of the steps of the activity. Invite students to ask their classmate if there is a unit of measurement they do not know.
- **Note:** An ounce can measure either volume or weight. Consider sharing with students that there will be four words for each attribute to help them decide how to sort "ounce."
- Consider including a checkpoint after students have completed Steps 1 and 2 of the activity or posting a key for students to compare their thinking to before beginning Step 3.
- Once students have successfully completed Steps 1 and 2, invite them to match each image card with a measurement card.
- Spend adequate time to ensure students have a shared understanding of what attribute each unit measures and how they compare in size. This will be helpful in Lessons 2 and 3 (MP6).

- **Note on mass and weight:** Formally mass and weight are defined as separate attributes, where weight depends on gravity, while mass is constant. You would weigh fewer pounds on the moon, but your mass would be the same number of kilograms. Students will learn this distinction in science classes, especially in later grades. For the purposes of this lesson, mass and weight are both attributes that help us describe how heavy an object is in our everyday worlds on Earth.

Discussion Questions

- *What unit is larger: a pound or an ounce? How did you decide?*
- *What strategies did you find most helpful for determining which unit was larger?*
- *What else could we measure about the dime? The slice of bread?*

Early Finishers

- Encourage students to find objects in the classroom that measure about 1 centimeter, 1 liter, 1 pound, etc.

Routine (optional): Consider using the mathematical language routine Collect and Display to create a reference for students to refer to throughout the unit.

Are You Ready for More?

Encourage students to add any other conversions that they have heard of or have researched on the internet. Their worksheet may be used as a resource in Lessons 2–4. Encourage students to add additional conversions that they find helpful along the way.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

The purpose of this discussion is to surface many possible measurements of the same object.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.

Discussion Questions

- *Would you use ounces or pounds to measure the weight of the can? Why?*
- *Would you use inches or feet to measure the length of the can? Why?*

Support for Students With Disabilities

Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Support for Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., We could measure ______. To measure ______, I would use ______.).

Cool-Down (5 minutes)

Support for Future Learning

If some students struggle, consider reviewing this question as a class at the beginning of Lesson 2. Students will need to understand which units represent larger and smaller quantities in Lessons 2–4.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Counting Classrooms

Lesson 2: Measuring With Different Units

Overview

Students convert measurements from one unit to another.

Learning Goals

- Explain why it takes more of a smaller unit or fewer of a larger unit to measure the same quantity.
- Convert measurements from one unit to another in the same system of measurement.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to begin thinking about converting measurements from one unit to another. Students convert between measurements in the same measurement system (e.g., feet and yards or centimeters and meters) and explain why it takes more of a smaller unit or fewer of a larger unit to measure the same quantity.

Lesson Summary

Warm-Up (10 minutes)

In this warm-up, students engage in the Number Talk routine to surface strategies for multiplying whole numbers by fractions. This the first in a series of warm-ups to strengthen strategies around multiplication of fractions.

Activity 1: Pencil Length (10 minutes)

The purpose of this activity is for students to consider using different units to measure the same object. They consider the relationship between the size of the unit and how many of them are needed to measure an object. For example, you will need more inches than feet to measure the same object because 1 foot is larger than 1 inch.

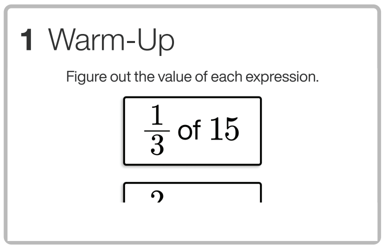
Activity 2: Same Length, Different Units (15 minutes)

The purpose of this activity is for students to convert lengths given in one unit of measurement to a different unit in the same system of measurement. Students consider which unit of measurement they would find most helpful to measure a distance and then convert that distance from one unit to another. This activity includes repeated challenges where students are given information in one unit and are asked to convert it to a different unit.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate their thinking around the idea that when you measure one object using different units, you will need more of the smaller unit. This reasoning will support them in determining whether their answers make sense throughout the rest of the unit.

Cool-Down (5 minutes)



1 Warm-Up

Figure out the value of each expression.

$\frac{1}{3}$ of 15

Teacher Moves

Overview: In this lesson, students begin thinking about converting measurements from one unit to another. In this warm-up, students engage in the Number Talk routine to surface strategies for multiplying whole numbers by fractions. This is the first in a series of warm-ups to strengthen strategies around multiplication of fractions.

Launch

- Use the dashboard's student view to display Screen 1.
- Invite students to share what they remember about how Number Talk work.

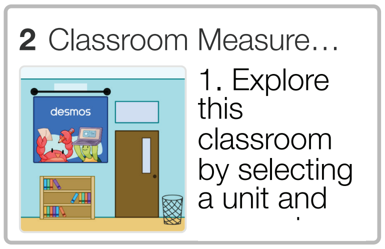
How Number Talk Works

- Give students two minutes to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own or one of the possible strategies in the sample responses.
- Select several students to share different strategies. Use the sample responses as examples of possible student strategies. Record strategies for all to see, along with the name of the student who shared each one.

Suggested Pacing: Screen 1

Sample Responses

Strategies students might use in the Number Talk



2 Classroom Measure…

1. Explore this classroom by selecting a unit and

1. Explore this classroom by selecting a unit and measuring several objects.

2. Discuss with a partner: *Which units were most useful for measuring each object?*

Teacher Moves

Overview: In Activity 1 (Screens 2–5), students consider using different units to measure the same object. They consider the relationship between the size of the unit and how many of them are needed to measure an object.

Launch

- Demonstrate how to change the units and drag the points using the dashboard's student view.

- Invite students to predict how long an object will be before measuring it.

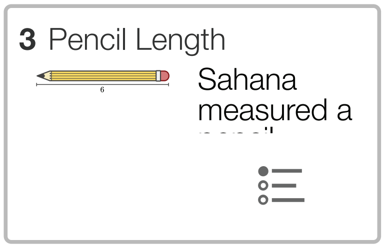
Facilitation
- Give students 2–3 minutes to explore, then to discuss the question with a partner.
- Circulate to listen to pair discussions and observe patterns.
- If time allows, invite a few students to share when they used each of the units.

Suggested Pacing: Screen 2

Student Supports

Students with Disabilities
- *Fine Motor Skills: Strategic Pairing*

Allow students who struggle with fine motor skills to dictate use of the sketch tool and movable points as needed throughout the lesson and unit.


3 Pencil Length

Sahana measured a

Sahana measured a pencil.

She said its length is 6.

Which unit do you think she used to measure the pencil?

Teacher Moves

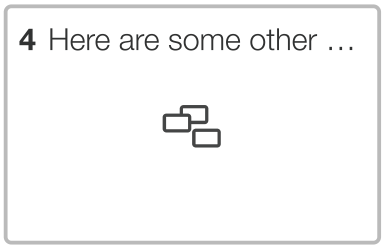
Launch
- Invite students to work *in pairs.*
- Ask students: *Why does it matter what unit she used?* (MP6)

Facilitation
- Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.

Suggested Pacing: Screens 3–5

Sample Responses

Students who select inches or centimeters will be marked correct on this screen.



4 Here are some other …

Teacher Moves

Facilitation

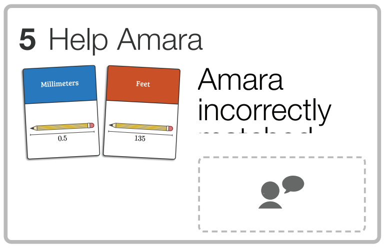
- Encourage students to share their reasoning with a partner and work to reach an agreement together about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.
- Monitor for students who make the same error that is shown on the next screen. Invite these students to reflect on their thinking after the discussion on the following screen.

Math Community

- Celebrate students who use their personal experiences to support them in their reasoning.

Sample Responses

Image solution



5 Help Amara

Amara incorrectly

Amara incorrectly matched these two pairs of cards.

What would you say to help her understand her mistake?

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface strategies for reasoning about the relationship between the size of a unit and how many of them you need to measure an object. This reasoning will support students throughout the lesson and unit.

Facilitation

- Select and sequence several student responses using the snapshot tool.
- Facilitate a whole-class discussion around students' responses.

Discussion Questions

- *About how big is a millimeter? A foot?*
- *Would you need more millimeters or feet to measure a pencil? What about a car? A worm?*
- *Could we measure a pencil in miles? Would we need more or fewer than* 0.5 *?*

Early Finishers

- Encourage students to estimate how many centimeters or yards long the pencil would be.

Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify to help students communicate their ideas.

Sample Responses

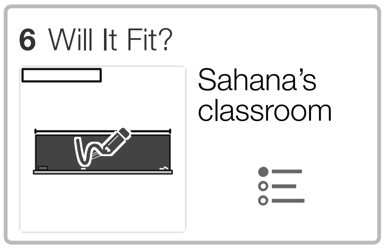
Responses vary. I know that a pencil is less than 1 foot long, so the measurement in feet will be less than 1. Also, think about which unit is bigger: millimeters or feet. You need more of the smaller unit to measure the same distance.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.


6 Will It Fit?

Sahana's classroom

Sahana's classroom is 5 meters wide.

Her teacher wants to buy a new chalkboard and finds one that is 400 centimeters wide.

Will the new chalkboard fit on the wall?

Teacher Moves

Overview: In Activity 2 (Screens 6–9), students convert lengths given in one unit of measurement to a different unit in the same system of measurement.

Launch

- Share with students that we will be converting from one unit to another to measure the same object.

Facilitation

- Give students 1–2 minutes to respond. Then consider sharing strategies as a class.
- Use the dashboard's teacher view to monitor students' responses. Facilitate a whole-class discussion if there is not consensus.
- If it does not come up naturally, highlight students who converted 5 meters into 500 centimeters and students who converted 400 centimeters into 4 meters.

Early Student Thinking
- Students may say it will not fit because 400 is larger than 5. Consider asking: *What units are each number in? Why might that matter?*

Suggested Pacing: Screens 6–9

Sample Responses

Yes

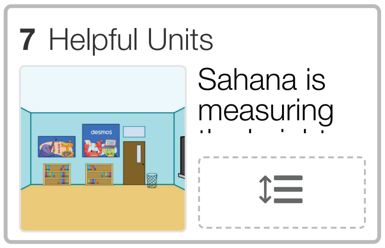
Explanations vary.

- 5 meters is equal to 500 centimeters since 1 meter equals 100 centimeters. 500 centimeters is longer than the width of the chalkboard.
- 400 centimeters is the same as 4 meters because every 100 centimeters is 1 meter. This means that the chalkboard is shorter than the wall.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., The new chalkboard will/will not fit on the wall because ________.).



7 Helpful Units

Sahana is measuring

Sahana is measuring the height of the classroom.

1. Order these units from most to least helpful for measuring the height of the classroom.

2. Discuss your thinking with a partner.

Teacher Moves

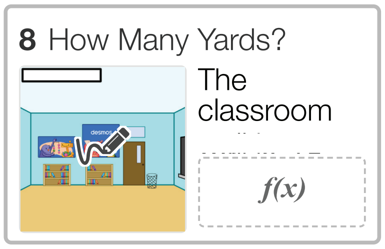
Facilitation
- Consider asking: *Why might Sahana want to know the height of her classroom?*
- Circulate to listen to student conversations. Monitor for students who use language from the key discussion screen (MP3).
- If time allows, display the most popular orderings using the dashboard's teacher view.

Math Community

- Consider celebrating the range of student responses on this screen. It's okay—even desirable—to lack consensus.

Sample Responses

Responses vary.



8 How Many Yards?

The classroom

The classroom wall is 12 feet tall.

How many yards is that?

Teacher Moves

Progress Check

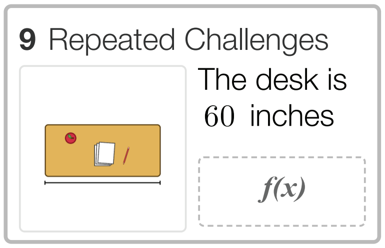
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Math Community

- Consider inviting students who entered 36 yards to share what they learned from the feedback.

Sample Responses

4 yards



9 Repeated Challenges

The desk is 60 inches

The desk is 60 inches wide.

How many feet is that? (Challenges vary.)

Teacher Moves

How Repeated Challenges Work

- Students are presented with a variety of challenges one at a time, in this case converting the measurement of an object from one unit to another.
- The challenges typically increase in difficulty as they continue.

Facilitation

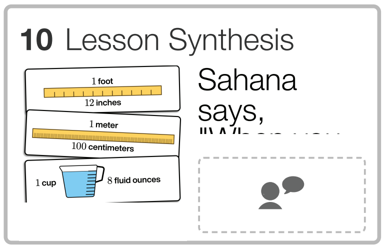
- Give students 5–7 minutes to complete as many challenges as they can.
- Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

Math Community
- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Sample Responses

There are 10 total challenges.

The first few responses are:
- 5 feet
- 42 feet
- 220 millimeters



10 Lesson Synthesis

Sahana says,

Sahana says, "When you measure an object using different units, you need more of the smaller unit."

What do you think about Sahana’s claim? Explain your reasoning.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to revisit the idea that when you measure one object using different units, you need more of the smaller unit. This reasoning will support students in determining whether their answers make sense throughout the rest of the unit.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions
- *Do you think Sahana’s idea is true all the time?*
- *Why might Sahana’s idea be useful to you as you are solving problems?*

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Suggested Pacing: Screen 10

Sample Responses

Responses vary. Sahana's claim is true because if a unit is smaller, then you would need more of them to get to the same measurement. If you have a really big unit like miles, then you don't need as many of them. Sometimes you even need less than 1.

Student Supports

Students With Disabilities

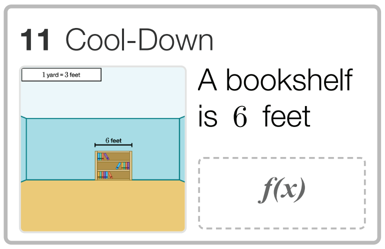
- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., I think Sahana's claim is true/not true because ______.).


11 Cool-Down

A bookshelf is 6 feet

$f(x)$

A bookshelf is 6 feet wide.

How many yards is that?

Teacher Moves

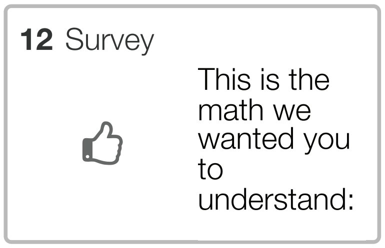
Support for Future Learning

- If students struggle, plan to emphasize this when opportunities arise in Lesson 3. It may be helpful to review this screen as a class before Lesson 3 and invite students to share whether the number of yards should be larger or smaller than the number of feet.

Suggested Pacing: Screens 11–12

Sample Responses

2 yards


12 Survey

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain why it takes more of a smaller unit to measure the same quantity.

- I can convert measurements from one unit to another in the same system of measurement.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Pen Pals

Lesson 3: Converting Units

Overview

Students convert measurements based on information from pen pals around the world.

Learning Goals

- Convert measurements from one unit to another in different measurement systems.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Students convert measurements based on information from pen pals around the world. They convert measurements from one unit to another in different measurement systems, recalling what they've learned in previous lessons about converting between units of measure and using ratio tools from Unit 6.2.

Lesson Summary

Warm-Up (5 minutes)

The purpose of this warm-up is to introduce students to the context of the lesson: pen pals from around the world. Students learn about the pen pals' favorite things, some of which will be included throughout the lesson.

Activity 1: Favorite Things (20 minutes)

The purpose of this activity is for students to apply their knowledge of ratios to convert measurements from one unit of measurement to another. Students begin by considering distances from school given in different units of measurement, and order these measurements from least to greatest by estimating. Later, they consider different tools that can be used with more complex conversions.

Activity 2: Recipes (10 minutes)

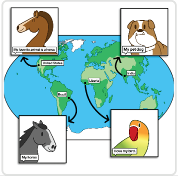
The purpose of this activity is for students to practice converting measurements from one unit of measurement to another using the strategies developed during the previous activity.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to make connections between ratio reasoning strategies and converting between units of measurement in different systems.

Cool-Down (5 minutes)

1 Warm-Up



Teacher Moves

Overview: In this lesson, students convert measurements based on information from pen pals around the world. In this warm-up, students are introduced to the pen pals and some of their favorite things.

Launch

- Invite students to work *in pairs.*
- Consider asking students what they know about pen pals, and if anyone in the class has ever had a pen pal.
- Invite students to share their own favorite foods, animals, or sports.

Facilitation

- Give students 1–2 minutes to discuss with a partner.
- Invite several students to share what they notice and wonder with the class.

Readiness Check (Problem 2)

- If most students struggle, consider reviewing this problem as a class before beginning this lesson, or spending extra time reviewing the Practice Problem warm-ups for Lessons 1–3.

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

- I notice some of the pen pals live in rural areas. I wonder if their hobbies are different from people who live in busier areas.
- I notice that most of the pen pals like football. I wonder why football is different in different countries?
- I notice that some of the pen pals like to make food. I wonder if I would be able to make those recipes using the things I have in my kitchen.

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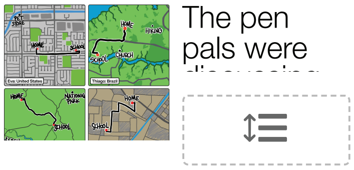
2 Distance From School



The pen pals were

The pen pals were discussing how far they each live from school.

Use your best estimates to order the pen pals from closest to farthest from school.

Teacher Moves

Overview: In Activity 1 (Screens 2–6), students apply their knowledge of ratios to convert measurements from one unit of measurement to another.

Launch
- Invite students to work *in pairs.*
- Share that people in different countries may use different measurement systems and ask students what they know about that.

Facilitation
- Give students 1–2 minutes to think independently and then work together to order the distances.
- Emphasize the range of student responses on this screen. It's okay—even desirable—to lack consensus at this stage. The activity will build toward consensus on later screens when students are given a ratio to use to convert between different units of measurement.

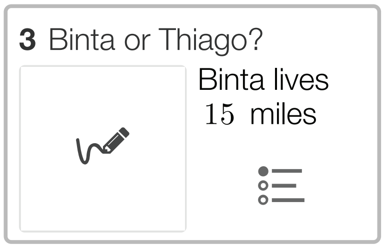
Early Finishers
- Invite students to research how far they live from school and from other places that are important to them.

Math Community
- Celebrate students who use their personal experiences to support them in their reasoning.

Suggested Pacing: Screen 2

Sample Responses

Responses vary. There is no correctness on this screen. Students should use their intuition about the sizes of the given units to help them estimate the closest and farthest distance from school.



3 Binta or Thiago?

Binta lives 15 miles

Binta lives 15 miles from her school in Liberia.

Thiago lives 20 kilometers from his school in Brazil.

Who lives closer to their school?

Teacher Moves

Launch
- Tell students that we'll be helping the pen pals convert distances from school from one unit of measurement to another.

Facilitation
- Direct students' attention to the calculator button near the top of the screen.

- Encourage students to use paper to help them with their thinking throughout this lesson.
- Highlight students who use representations such as tables or double number lines.

Possible Discussion Question
- *Why might it be valuable to have different units of measurement?*

Note: This may be the first time students see the ≈ symbol used to represent an approximate equivalence.

Suggested Pacing: Screens 3–6

Sample Responses

Thiago

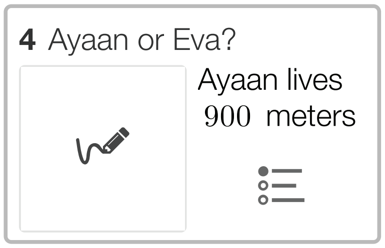
Explanations vary.

- Binta lives 15 miles or 24 kilometers from school. Thiago lives 20 kilometers from his school, so he lives closer.
- Thiago lives 20 kilometers or 12.5 miles from school. Binta lives 15 miles from school, so she lives farther from school.

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

To assist students in recognizing the connections between new problems and prior work, consider displaying a double number line or a table of equivalent ratios from Unit 6.2. Ask students how we might represent the ratio on this screen using a table or a double number line.



4 Ayaan or Eva?

Ayaan lives 900 meters

Ayaan lives 900 meters from his school in India.

Eva lives 2000 feet from her school in the United States.

Who lives closer to their school?

Teacher Moves

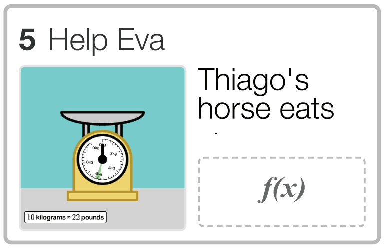
Progress Check
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Eva

Explanations vary.

- Ayaan lives 900 meters from school, which is about 3000 feet. Eva lives 2000 feet from her school, so she lives closer.
- Eva lives 2000 feet from school, which is about 600 meters. Ayaan lives 900 meters from his school, so he lives farther.



5 Help Eva

Thiago's horse eats

10 kilograms ≈ 22 pounds

Thiago's horse eats about 6 kilograms of hay per day.

Eva wants to know about how many pounds this is.

About how many pounds is the same as 6 kilograms?

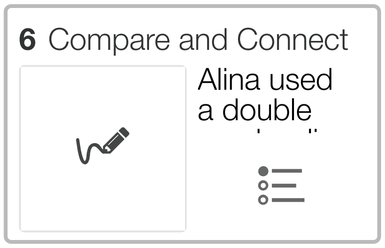
Teacher Moves

Facilitation

- To support students getting started, consider asking: *How are* 10 *kilograms and* 22 *pounds related? How many pounds do you think is about the same as* 20 *kilograms? How can we organize these pairs of numbers to help us convert between units?*
- Encourage students to use the feedback on the screen to help them revise their thinking.
- Consider monitoring for students who use tools similar to Alina's or Maia's on the next screen or other creative strategies. Invite these students to share their thinking during the discussion on the next screen. (MP5)

Sample Responses

13.2 pounds



6 Compare and Connect

Alina used a double

Alina used a double number line and Maia used a table to decide the pounds of hay Thiago's horse eats.

Pick a student and explain their thinking.

Teacher Moves

Key Discussion Screen

- The purpose of this discussion is to surface different tools and strategies for converting between units of measurement in different systems, including using equivalent ratios and using a unit rate.

Facilitation

- Select and sequence several student responses using the snapshot tool.
- When most students have responded, consider pausing the class to discuss these strategies.
- Spend adequate time discussing how Alina and Maia each used equivalent ratios to determine the number of pounds (MP6).

Discussion Questions

- *How would you describe Alina's strategy in your own words? Maia's?*
- *Why do you think Maia used a* 1 *in her ratio table?*
- *What are the advantages of Alina's strategy? Maia's?*

Early Finishers

- Invite students to research other units used to measure hay and calculate how much hay Thiago's horse eats in those units.

Math Community

- Consider renaming Alina's and Maia's strategies after the students in your class who used them.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary.

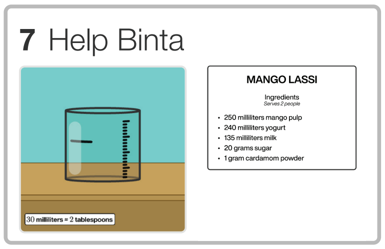
- Alina multiplied each of the numbers by 6 to get 60 kilograms ≈ 132 pounds. Then she divided each of these numbers by 10 to get 6 kilograms ≈ 13.2 pounds.
- Maia divided each of the numbers by 10 to determine the number of pounds per kilogram. Then she multiplied by 6 because there are 6 kilograms.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



7 Help Binta

30 milliliters ≈ 2 tablespoons

MANGO LASSI

Ingredients

Serves 2 people

- 250 milliliters mango pulp
- 240 milliliters yogurt
- 135 milliliters milk
- 20 grams sugar
- 1 gram cardamom powder

Binta decides to make Ayaan's recipe.

The recipe calls for 135 milliliters of milk.

About how many tablespoons of milk should Binta use?

Teacher Moves

Overview: In Activity 2 (Screens 7–9), students practice converting measurements from one unit of measurement to another using the strategies developed during the previous activity.

Launch

- Invite students to work *individually.*
- Consider asking students if they have ever used a recipe, and what sorts of units they remember using.
- Share that different countries may use different measurement systems, and that we'll be helping the pen pals convert recipe ingredients from one unit of measurement to another.

Facilitation

- To support students getting started, consider inviting them to make a table using the given relationship between milliliters and tablespoons and ask: *Where would* 135 *go?*
- Encourage students to use any tool they find helpful to solve the problem. (MP5)
- If you notice students are guessing and checking, consider providing these students with strategies by highlighting their classmates' thinking.

Pacing: Screens 7–9

Sample Responses

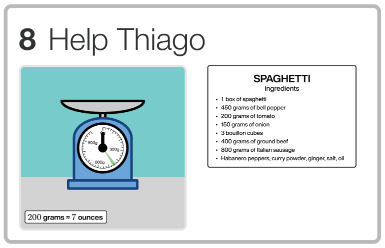
9 tablespoons

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting students to share what they notice and wonder about the recipe before responding. Consider clarifying as a class which part of the recipe we will focus on in this question and inviting students who struggle with visual-spatial processing to record the 135 milliliters of milk on a sheet of blank paper.



8 Help Thiago

SPAGHETTI
Ingredients

- 1 box of spaghetti
- 450 grams of bell pepper
- 200 grams of tomato
- 150 grams of onion
- 3 bouillon cubes
- 400 grams of ground beef
- 800 grams of Italian sausage
- Habanero peppers, curry powder, ginger, salt, oil

200 grams ≈ 7 ounces

Thiago decides to make Binta's recipe.

The recipe calls for 450 grams of bell pepper.

About how many ounces of bell peppers should Thiago use?

Teacher Moves

Progress Check

- To support students getting started, consider inviting them to make a table using the ratio of grams to ounces and ask: *Where would* 450 *go?*
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Math Community

- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Sample Responses

15.75 ounces

9 Are You Ready for M…

People
have known

People have known that Earth is round for over 2000 years, but at first, they didn’t know how big Earth was.

Eratosthenes was the first recorded person to calculate the distance around Earth's equator in about 240 BCE. He used an estimated distance from Alexandria to Syene along with lengths of shadows to calculate the distance around Earth's equator to be about $250\,000$ stadia.

The actual distance is about $24\,901$ miles.

How close was Eratosthenes calculation to the actual distance? Explain your thinking.

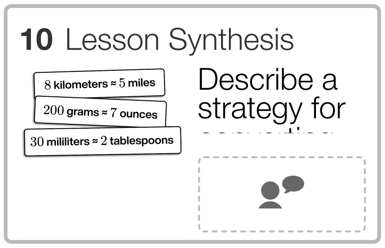
Teacher Moves

Facilitation

- Invite students who finish Screens 7–8 early to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion.

Sample Responses

$1\,299$ miles. $250\,000$ stadia is about $524 \cdot 50 = 26\,200$ miles. This is $24\,901 - 26\,200 = 1\,299$ miles different from the actual distance around Earth.



10 Lesson Synthesis

8 kilometers ≈ 5 miles

200 grams ≈ 7 ounces

30 mililiters ≈ 2 tablespoons

Describe a strategy for

Describe a strategy for converting a measurement from one unit to another.

Use the examples to the left if they help you with your thinking.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is for students to make connections between ratio reasoning strategies and converting between units of measurement in different systems.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Suggested Pacing: Screen 10

Sample Responses

Responses vary.

- To convert measurements from one unit to another, you can use a table of values or a double number line. You need a ratio to start with, and then you can multiply and divide to get to the number you are trying to convert to.

- You can find how many per one, and then multiply to get your answer. For example, if I want to convert kilometers to miles, I know there is $\frac{5}{8}$ of a mile for every 1 kilometer. Then I multiply $\frac{5}{8}$ by the number of kilometers I have to find the number of miles.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

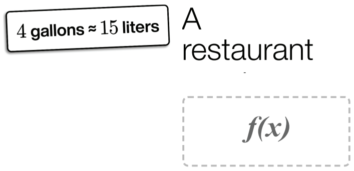
Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., A strategy for converting a measurement from one unit to another is ______.).

11 Cool-Down



A restaurant needs 5 gallons of ice cream for dessert one night.

About how many liters is this?

Teacher Moves

Support for Future Learning

- If students struggle, consider making time to explicitly revisit these ideas. A strong understanding of the strategies that can be used to convert between units in different systems of measurement will support students as they learn about unit rates in Section 2.

Sample Responses

18.75 liters

12

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

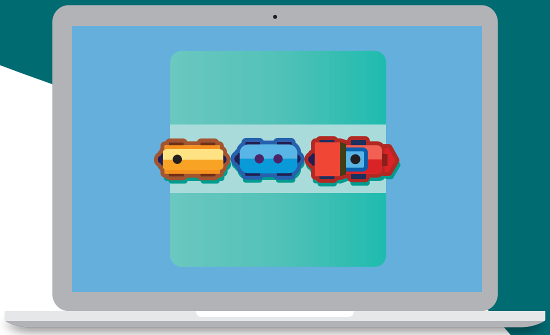
- I can convert measurements from one unit to another in different measurement systems.

Unit 3
Lesson
4

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Digital Lesson RECOMMENDED

This is a digital lesson. A print option is also available.



Model Trains

Comparing Ratios

Let's calculate unit rates and use them to compare speeds.

Focus and Coherence

Today's Goals

1. **Goal:** Calculate the speed of an object as a unit rate for the distance the object travels over time.
2. **Goal:** Use rate and ratio reasoning to compare rates expressed in different units.
3. **Language Goal:** Justify which of two objects is faster. **(Writing, Speaking, and Listening)**

Students compare the speeds of model trains and discuss unit rates using the term *per*. They consider what information is needed to determine which of two model trains is traveling faster and then compare speeds of model trains given in different units of measurement.

Prior Learning

In Lesson 3, students defined the terms *rate* and *unit rate* and used each to determine and compare speeds.

Future Learning

In Lesson 5, students will extend their understanding of unit rates by determining unit prices and identifying the two unit rates associated with any ratio.

Rigor and Balance

- Students continue to build **conceptual understanding** of equivalent ratios and unit rates by comparing speeds.
- Students **apply** their understanding of converting between measurement systems to comparing speeds.
- Students **reason adaptively** when they use multiple strategies to solve a problem.

Standards

Addressing

NY-6.RP.3b
Solve unit rate problems.

Also Addressing: **NY-6.RP.2**, **NY-6.RP.3d**

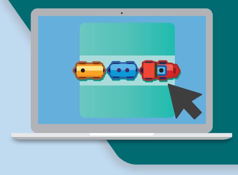
Mathematical Practices: MP2, MP6

Building On	Building Toward
NY-6.RP.3a	NY-7.RP.1
NY-5.MD.1	

Lesson at a Glance

~ 45 min

Standards: NY-6.RP.2, NY-6.RP.3b, NY-6.RP.3d



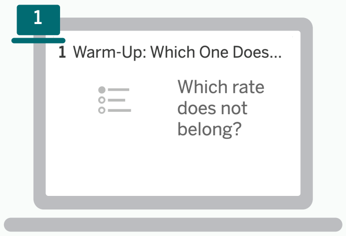
Why digital?

Students receive interpretive feedback as they calculate the unit rates of the model trains in real time.

Warm-Up

Independent | 7 min

Students compare and contrast four rates relating distance and time using the **Which One Doesn't Belong?** routine.



1

1 Warm-Up: Which One Does...

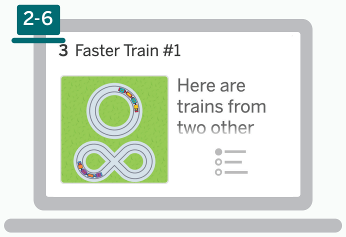
Which rate does not belong?

Pacing: Screen 1

Activity 1

Pairs | 15 min

Students calculate equivalent ratios and unit rates to help them compare speeds of model trains by analyzing two different strategies using the **MLR7: Compare and Connect** routine.



2-6

3 Faster Train #1

Here are trains from two other

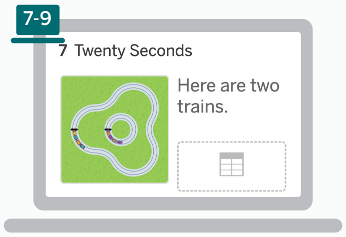
Pacing: Screens 2–6

Activity 2

Pairs | 13 min

Students compare speeds given in different measurement systems.

Think-Pair-Share



7-9

7 Twenty Seconds

Here are two trains.

Pacing: Screens 7–9

Synthesis

Whole Class | 5 min

Students surface strategies for comparing speeds, including calculating a unit rate.

10

10 Synthesis

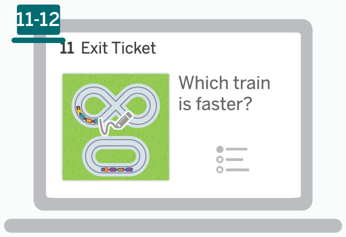
Describe two strategies

Pacing: Screen 10

Exit Ticket

Independent | 5 min

Students demonstrate their understanding by determining which train is traveling at a faster speed.



11-12

11 Exit Ticket

Which train is faster?

Pacing: Screens 11–12

Prep Checklist

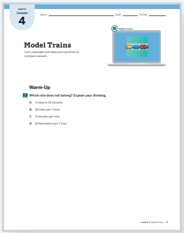
Assign the digital lesson. A print option is also available.

Students using digital:	**Students using print:**
Digital Lesson	Print Option in Student Edition Exit Ticket PDF

Digital Lesson Independent | 7 min | Pacing: Screen 1

Warm-Up

Purpose: Students compare and contrast four rates relating distance and time using the Which One Doesn't Belong? routine.



Students using print

1 Launch

Support getting started by asking "How many minutes are in an hour? How many kilometers are in a mile?"

Use the Which One Doesn't Belong? routine to support students in noticing how the four rates are expressed, noting the different units of measurement and the order of those units.

1 Connect

Invite students to share which rate they decided doesn't belong and why. Encourage the use of mathematical language, such as *per*, *unit rate*, *for each*, or *for every*. As each option is identified, ask whether anyone else chose the same option but for a different reason. **(MP6)**

Consider asking, "Are any of these rates equivalent?"

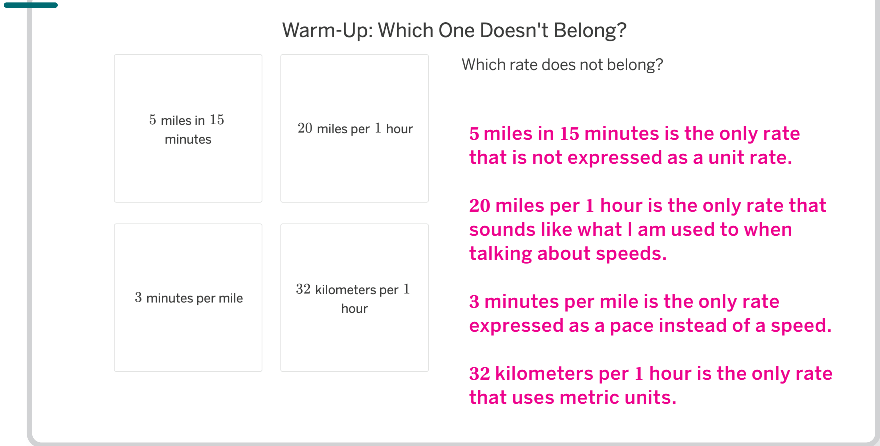
Emphasize that in the upcoming activities, the rates students will work with are distances over time, which are also known as speeds.

Math Identity and Community Consider celebrating variety and creativity in what students notice, including things that surprise you or you think other students may not have noticed.

M/EL **Multilingual/English Learners** Provide sentence frames to support students as they share their responses. For example, "Choice ______ doesn't belong because ______." or "Choice ______ is the only one that has/doesn't have ______." **(Reading and Speaking)**

Students using digital

1



Warm-Up: Which One Doesn't Belong?

5 miles in 15 minutes	20 miles per 1 hour
3 minutes per mile	32 kilometers per 1 hour

Which rate does not belong?

5 miles in 15 minutes is the only rate that is not expressed as a unit rate.

20 miles per 1 hour is the only rate that sounds like what I am used to when talking about speeds.

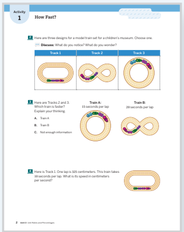
3 minutes per mile is the only rate expressed as a pace instead of a speed.

32 kilometers per 1 hour is the only rate that uses metric units.

Pairs | 15 min | Pacing: Screens 2–4

Activity 1 How Fast?

Purpose: Students calculate equivalent ratios and unit rates to help them compare speeds of model trains by analyzing two different strategies using the MLR7: Compare and Connect routine.



Students using print

2 Launch

Encourage connections by asking students if they have ever played with a model train set.

Support getting started by asking students what information they think is missing from the image they chose.

3-4 Monitor

Consider asking, "What information is needed to know which train is faster?"

D Differentiation

Look for students who:	Teacher Moves
Need help getting started. *(Screen 4)*	Ask, "What do you think it means to measure the speed of a train in centimeters per second?" **(MP6)**
Would benefit from a challenge during the activity.	**Extension:** Invite students to think about what a student who responded differently might have been thinking.

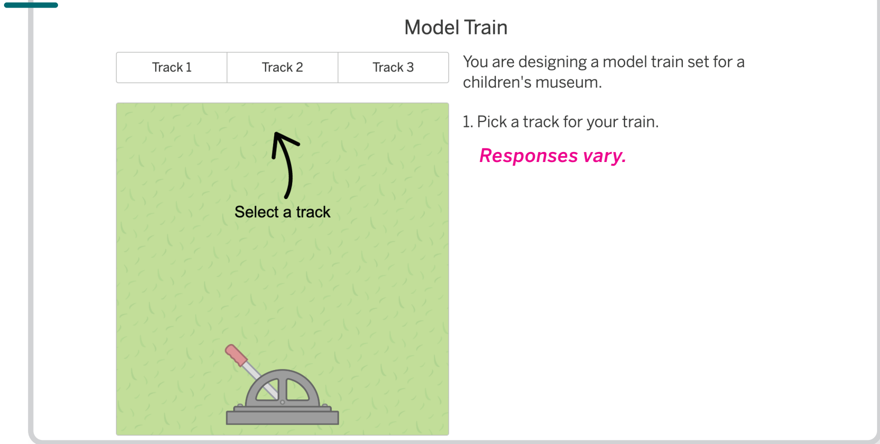
A Accessibility: Conceptual Processing

To assist students in recognizing the connections between new problems and prior work, consider displaying a double number line diagram or a table of equivalent ratios from an earlier lesson. Ask students how we might represent the ratio on this screen using a table or a double number line.

Activity 1 continued >

Students using digital

2



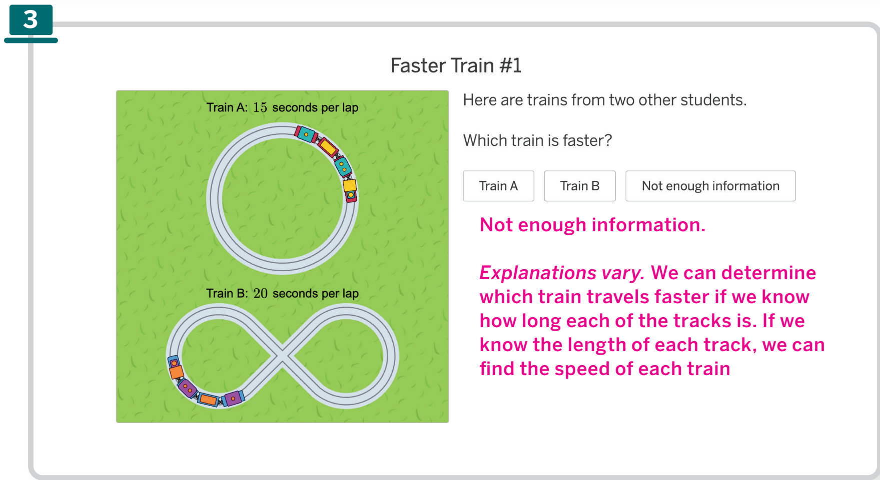
Model Train

You are designing a model train set for a children's museum.

1. Pick a track for your train.

Responses vary.

3



Faster Train #1

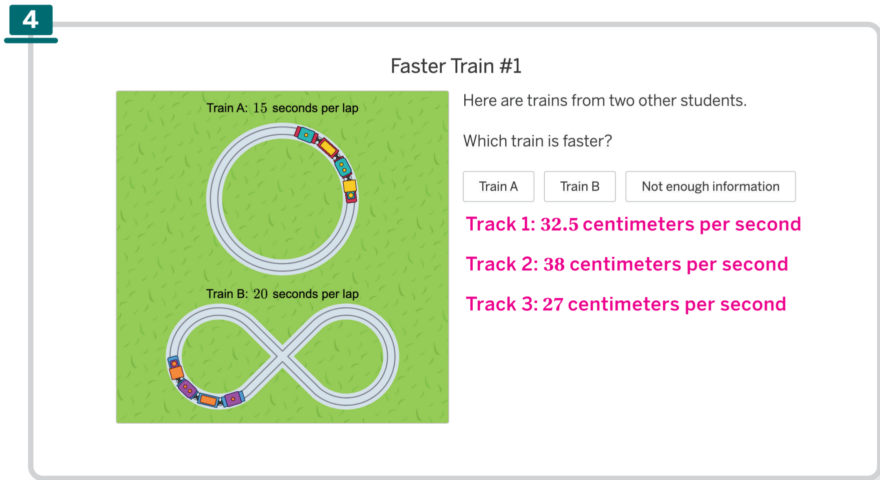
Here are trains from two other students.

Which train is faster?

Not enough information.

Explanations vary. **We can determine which train travels faster if we know how long each of the tracks is. If we know the length of each track, we can find the speed of each train**

4



Faster Train #1

Here are trains from two other students.

Which train is faster?

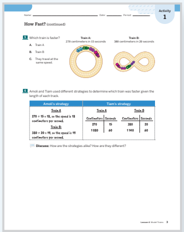
Track 1: 32.5 centimeters per second

Track 2: 38 centimeters per second

Track 3: 27 centimeters per second

Activity 1 How Fast? (continued)

Purpose: Students calculate equivalent ratios and unit rates to help them compare speeds of model trains by analyzing two different strategies using the MLR7: Compare and Connect routine.



Students using print

5-6 Monitor

Identify students who use similar strategies to Amoli's and Tiam's or other creative strategies. Plan to invite those students to share during the Connect. *(Screen 5)*

D Differentiation

Look for students who:	**Teacher Moves**
Recognize that Amoli's strategy involved using a *unit rate*. *(Screen 6)*	Invite them to share why Amoli divided by 15 for Train A.
Recognize that Tiam's strategy involved using *equivalent ratios*. *(Screen 6)*	Invite them to share how Tiam calculated that Train B travels 1 140 cm in 60 seconds.
Finish early. *(Screen 6)*	**Extension:** Invite students to determine how Amoli and Tiam would determine the speed of a train that traveled 300 cm in 40 seconds.

A Accessibility: Conceptual Processing
To support students in expressing their thinking, encourage them to use the sketch tool to annotate or highlight each strategy. *(Screen 6)*

M/EL Multilingual/English Learners Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others. **(Speaking and Listening)** *(Screen 6)*

Math Identity and Community Consider renaming each strategy after a student in your class who used it.

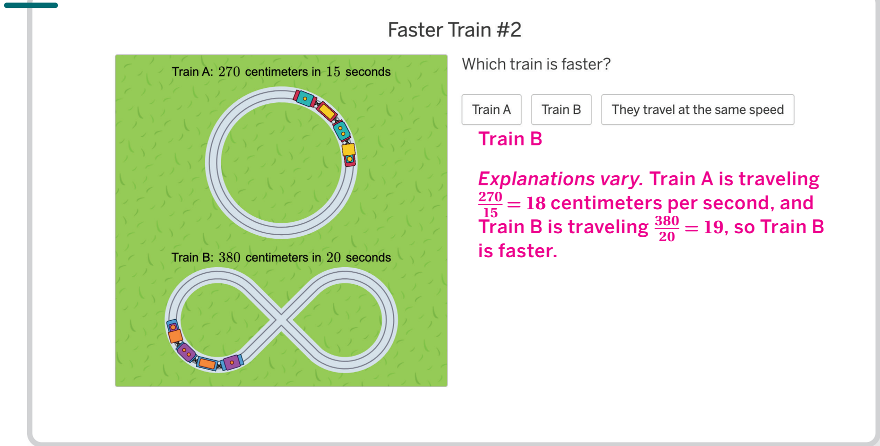
6 Connect

Invite students to share how Amoli's and Tiam's strategies connect to their own.

MLR MLR7: Compare and Connect As students share their responses and reasoning, invite students to compare the two strategies and discuss with a partner. Consider asking, "Why did the two strategies lead to the same result of Train B being faster than Train A?"

Students using digital

5



Faster Train #2

Train A: 270 centimeters in 15 seconds

Train B: 380 centimeters in 20 seconds

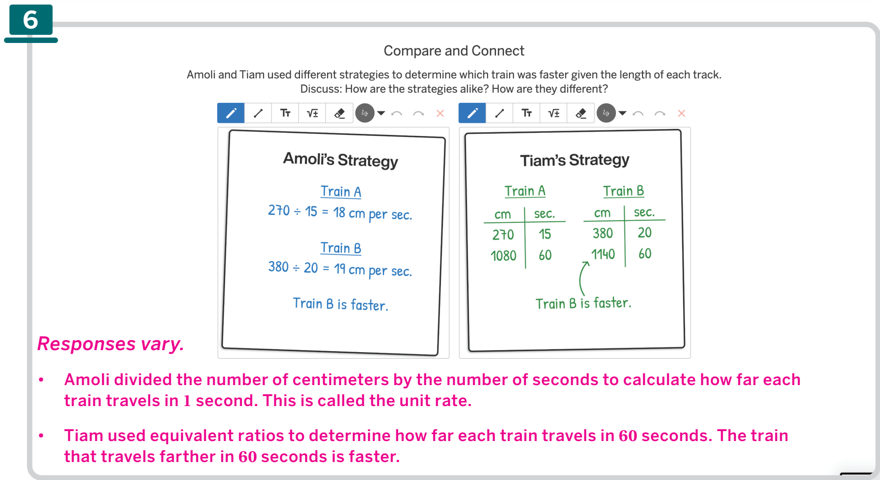
Which train is faster?

☐ Train A ☐ Train B ☐ They travel at the same speed

Train B

Explanations vary. Train A is traveling $\frac{270}{15} = 18$ centimeters per second, and Train B is traveling $\frac{380}{20} = 19$, so Train B is faster.

6



Compare and Connect

Amoli and Tiam used different strategies to determine which train was faster given the length of each track.
Discuss: How are the strategies alike? How are they different?

Amoli's Strategy

Train A
270 ÷ 15 = 18 cm per sec.

Train B
380 ÷ 20 = 19 cm per sec.

Train B is faster.

Tiam's Strategy

Train A

cm	sec.
270	15
1080	60

Train B

cm	sec.
380	20
1140	60

Train B is faster.

Responses vary.

- Amoli divided the number of centimeters by the number of seconds to calculate how far each train travels in 1 second. This is called the unit rate.
- Tiam used equivalent ratios to determine how far each train travels in 60 seconds. The train that travels farther in 60 seconds is faster.

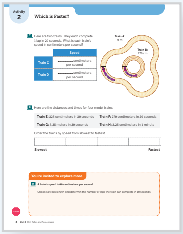
Key Takeaway: Speeds can be compared by determining the rate per 1, or the *unit rate*, which can be done by dividing one quantity by the other. Two speeds can also be compared by creating *equivalent ratios* such that either the distance or the time are the same.

Pairs | 13 min | Pacing: Screens 7–9

Activity 2 Which is Faster?

Purpose: Students compare speeds given in different measurement systems.

Short on time: Consider allowing students to order two trains of the four on Screen 8.



Students using print

7 Launch

Support getting started by asking, “How long would it take each train to complete a lap if it was traveling 1 centimeter per second?”

Use the Think-Pair-Share routine.

7-9 Monitor

Pause to ask students what they notice about the rates of the four model trains. *(Screen 8)*

D Differentiation

Look for students who:	Teacher Moves
Write the speed for Train C as $\frac{9}{20}$. *(Screen 7)*	**Support:** Invite students to state the units of the numerator and denominator separately. Then ask if they need to convert either measurement before doing division. **(MP6)**
Write the speed as 32 500 meters in 30 seconds for Train E. *(Screen 8)*	**Support:** Consider asking, “How many centimeters are in 1 meter?”
Use several different strategies for deciding which train is traveling the fastest. *(Screen 8)* **(MP2)**	Invite students to share during the Connect. **(MP6)**
Would benefit from a challenge during this activity. *(Screen 9)*	**Extension:** ***You’re invited to explore more.*** Invite students who want to explore the relationship between distance, rate, and time to further discuss this task with a partner.

A Accessibility: Executive Functioning
To support organization in problem solving, consider chunking this activity by inviting students to select two trains, determine which is faster, and then compare the third train to the other two. Then have them continue this process until they have compared all four trains.

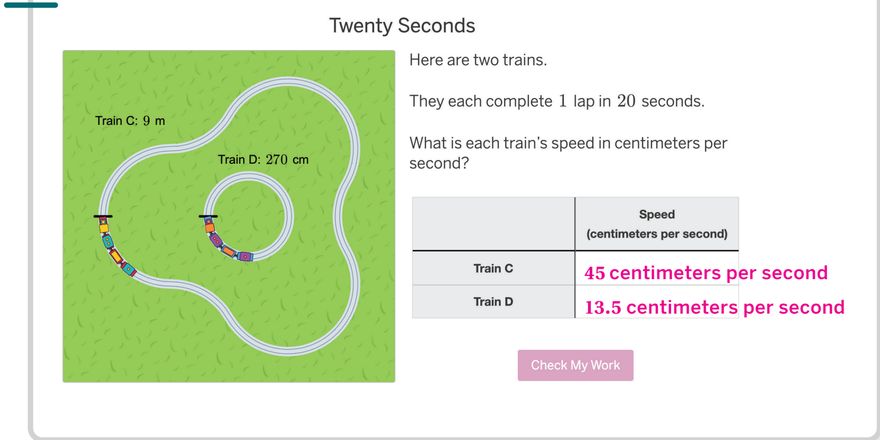
8 Connect

Invite students to share their strategies for comparing the speeds of the trains.

Math Identity and Community Consider asking the class why having more than one strategy might be useful.

Students using digital

7



Twenty Seconds

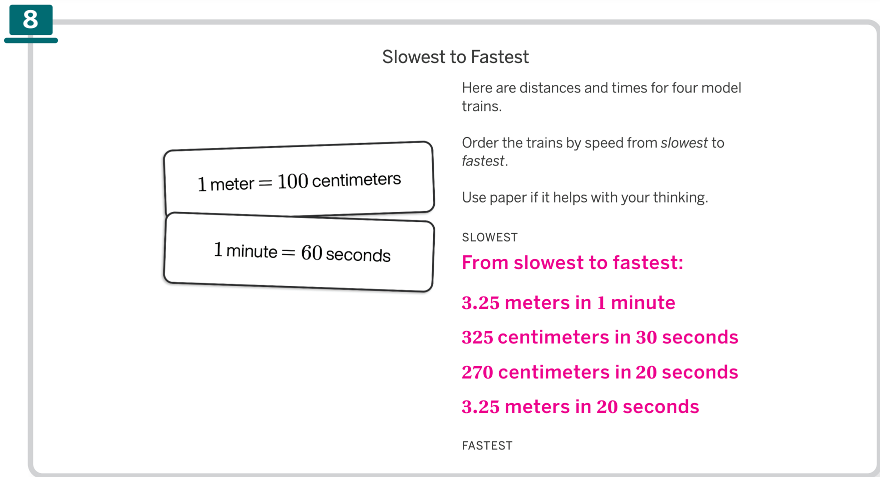
Here are two trains.

They each complete 1 lap in 20 seconds.

What is each train’s speed in centimeters per second?

	Speed (centimeters per second)
Train C	45 centimeters per second
Train D	13.5 centimeters per second

8



Slowest to Fastest

Here are distances and times for four model trains.

Order the trains by speed from *slowest* to *fastest*.

Use paper if it helps with your thinking.

1 meter = 100 centimeters

1 minute = 60 seconds

SLOWEST

From slowest to fastest:

3.25 meters in 1 minute

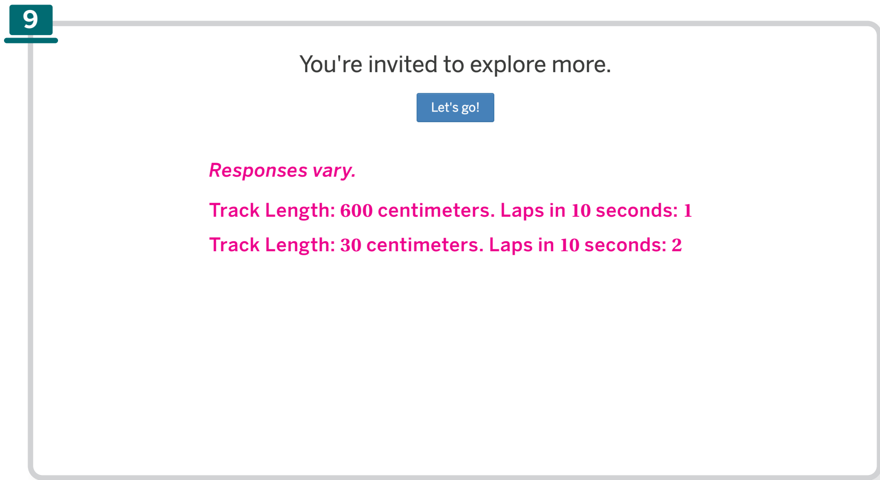
325 centimeters in 30 seconds

270 centimeters in 20 seconds

3.25 meters in 20 seconds

FASTEST

9



You're invited to explore more.

Responses vary.

Track Length: 600 centimeters. Laps in 10 seconds: 1

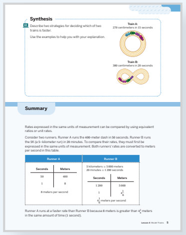
Track Length: 30 centimeters. Laps in 10 seconds: 2

Key Takeaway: To compare the speeds of different trains (or traveling objects), both the distance and the time must be measured in the same units of measurement.

Digital Lesson | Whole Class | 5 min | Pacing: Screen 10

Synthesis

Purpose: Students surface strategies for comparing speeds, including calculating a unit rate.



Students using print

10 Synthesis

Invite students to respond independently and then share their thinking with a partner.

Capture and share a variety of ideas, including:

- Calculating a unit rate for each train.
- Using equivalent ratios to see how far each train goes in the same amount of time.

Math Identity and Community If time allows, invite students to shout out students whose strategies they found most helpful.

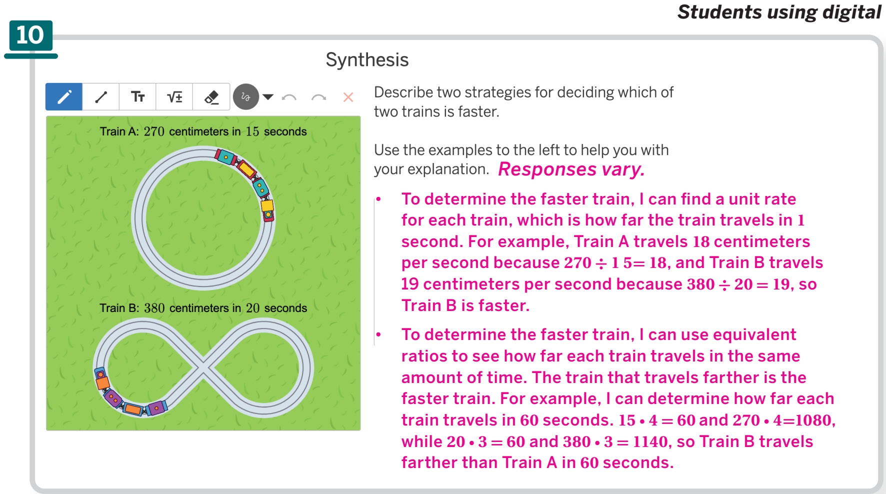
If time allows, invite students to share what makes sense to them about each strategy and what connections they see between their classmates' strategies.

Lesson Takeaway: There are multiple strategies that can be used to compare speeds.

Summary

Share the Summary. Students can refer back to this throughout the unit and course.

Students using digital



10

Synthesis

Train A: 270 centimeters in 15 seconds

Train B: 380 centimeters in 20 seconds

Describe two strategies for deciding which of two trains is faster.

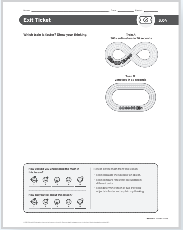
Use the examples to the left to help you with your explanation. ***Responses vary.***

- **To determine the faster train, I can find a unit rate for each train, which is how far the train travels in 1 second. For example, Train A travels 18 centimeters per second because $270 \div 1\,5= 18$, and Train B travels 19 centimeters per second because $380 \div 20 = 19$, so Train B is faster.**
- **To determine the faster train, I can use equivalent ratios to see how far each train travels in the same amount of time. The train that travels farther is the faster train. For example, I can determine how far each train travels in 60 seconds. $15 \cdot 4 = 60$ and $270 \cdot 4=1080$, while $20 \cdot 3 = 60$ and $380 \cdot 3 = 1140$, so Train B travels farther than Train A in 60 seconds.**

Independent | 5 min | Pacing: Screens 11–12

Exit Ticket

Purpose: Students demonstrate their understanding by determining which train is traveling at a faster speed.



Students using print

11-12 Today's Goals

Goal: Calculate the speed of an object as a unit rate for the distance the object travels over time.

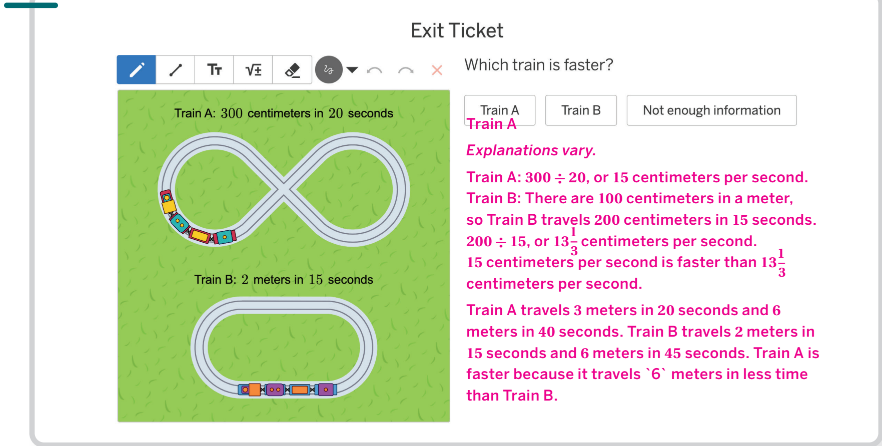
Goal: Use rate and ratio reasoning to compare rates expressed in different units.

Language Goal: Justify which of two objects is faster. **(Writing, Speaking, and Listening)**

Support for Future Learning: If students struggle with calculating unit rates, plan to emphasize this when opportunities arise over the next several lessons. For example, spend extra time in Lesson 5 discussing how to calculate and interpret the two unit rates for the same relationship.

Students using digital

11



Exit Ticket

Which train is faster?

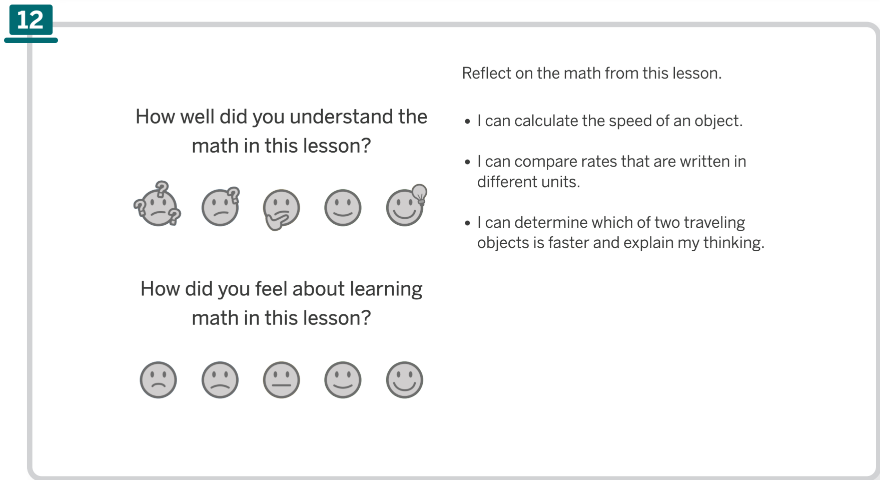
Train A

Explanations vary.

Train A: 300 ÷ 20, or 15 centimeters per second.
Train B: There are 100 centimeters in a meter, so Train B travels 200 centimeters in 15 seconds. 200 ÷ 15, or $13\frac{1}{3}$ centimeters per second. 15 centimeters per second is faster than $13\frac{1}{3}$ centimeters per second.

Train A travels 3 meters in 20 seconds and 6 meters in 40 seconds. Train B travels 2 meters in 15 seconds and 6 meters in 45 seconds. Train A is faster because it travels `6` meters in less time than Train B.

12



How well did you understand the math in this lesson?

How did you feel about learning math in this lesson?

Reflect on the math from this lesson.

- I can calculate the speed of an object.
- I can compare rates that are written in different units.
- I can determine which of two traveling objects is faster and explain my thinking.

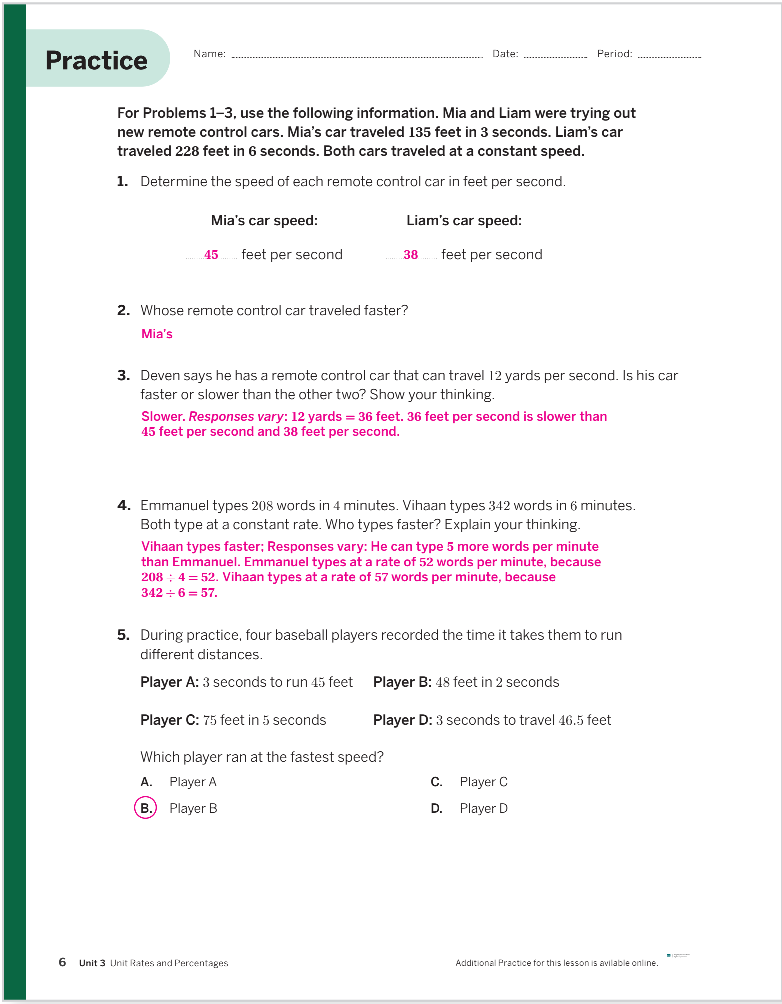
Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Students using digital

Students using print



Practice

Name: .... Date: .... Period: ....

For Problems 1–3, use the following information. Mia and Liam were trying out new remote control cars. Mia's car traveled 135 feet in 3 seconds. Liam's car traveled 228 feet in 6 seconds. Both cars traveled at a constant speed.

1. Determine the speed of each remote control car in feet per second.

Mia's car speed:	**Liam's car speed:**
45 feet per second	38 feet per second

2. Whose remote control car traveled faster?

   Mia's

3. Deven says he has a remote control car that can travel 12 yards per second. Is his car faster or slower than the other two? Show your thinking.

   Slower. *Responses vary*: 12 yards $= 36$ feet. 36 feet per second is slower than 45 feet per second and 38 feet per second.

4. Emmanuel types 208 words in 4 minutes. Vihaan types 342 words in 6 minutes. Both type at a constant rate. Who types faster? Explain your thinking.

   Vihaan types faster; Responses vary: He can type 5 more words per minute than Emmanuel. Emmanuel types at a rate of 52 words per minute, because $208 \div 4 = 52$. Vihaan types at a rate of 57 words per minute, because $342 \div 6 = 57$.

5. During practice, four baseball players recorded the time it takes them to run different distances.

   Player A: 3 seconds to run 45 feet

   Player B: 48 feet in 2 seconds

   Player C: 75 feet in 5 seconds

   Player D: 3 seconds to travel 46.5 feet

   Which player ran at the fastest speed?

   A. Player A

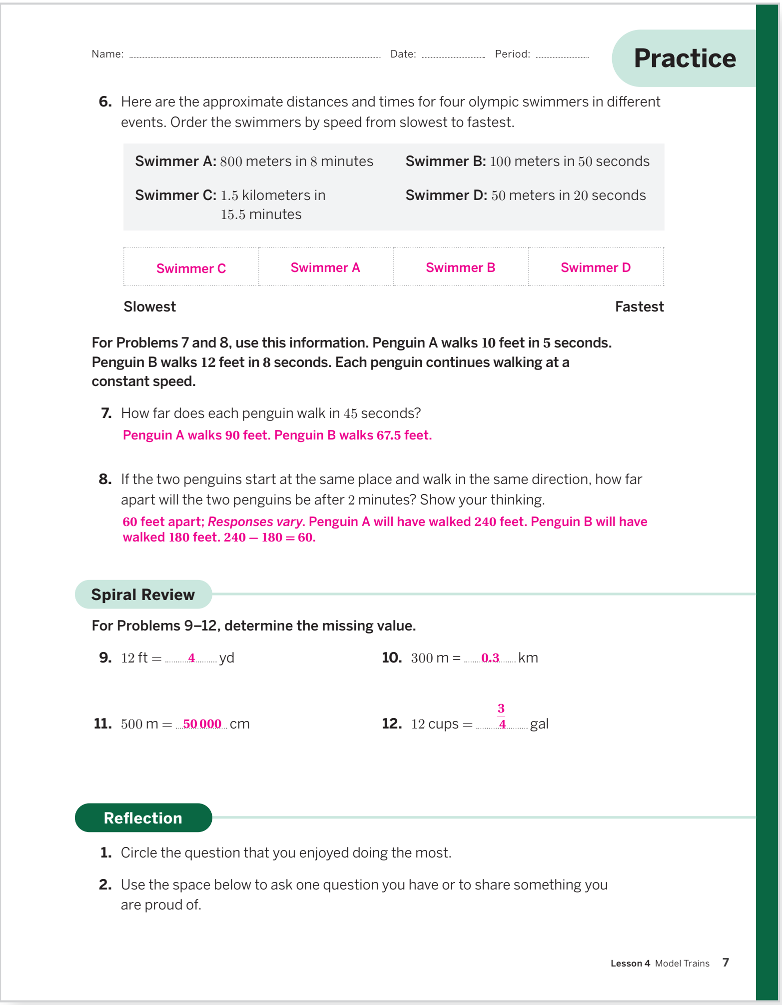
   [✓ B.] Player B

   C. Player C

   D. Player D

6 **Unit 3** Unit Rates and Percentages

Additional Practice for this lesson is avilable online.



Name: .... Date: .... Period: ....

Practice

6. Here are the approximate distances and times for four olympic swimmers in different events. Order the swimmers by speed from slowest to fastest.

   Swimmer A: 800 meters in 8 minutes

   Swimmer B: 100 meters in 50 seconds

   Swimmer C: 1.5 kilometers in 15.5 minutes

   Swimmer D: 50 meters in 20 seconds

Swimmer C	Swimmer A	Swimmer B	Swimmer D
Slowest			**Fastest**

For Problems 7 and 8, use this information. Penguin A walks 10 feet in 5 seconds. Penguin B walks 12 feet in 8 seconds. Each penguin continues walking at a constant speed.

7. How far does each penguin walk in 45 seconds?

   Penguin A walks 90 feet. Penguin B walks 67.5 feet.

8. If the two penguins start at the same place and walk in the same direction, how far apart will the two penguins be after 2 minutes? Show your thinking.

   60 feet apart; *Responses vary*. Penguin A will have walked 240 feet. Penguin B will have walked 180 feet. $240 - 180 = 60$.

Spiral Review

For Problems 9–12, determine the missing value.

9. 12 ft $=$ **4** yd

10. 300 m $=$ **0.3** km

11. 500 m $=$ **50 000** cm

12. 12 cups $=$ $\mathbf{\frac{3}{4}}$ gal

Reflection

1. Circle the question that you enjoyed doing the most.
2. Use the space below to ask one question you have or to share something you are proud of.

Lesson 4 Model Trains 7

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1–4, 7	2	NY-6.RP.2, NY-6.RP.3b
Test Practice	5	2	NY-6.RP.2, NY-6.RP.3b
	6	2	NY-6.RP.3b, NY-6.RP.3d
	8	3	NY-6.RP.2, NY-6.RP.3b
Spiral Review			
Fluency	9–12	1	NY-5.MD.1

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Soft Serve

Lesson 5: Two Unit Rates

Overview

Students learn that every ratio relationship has two associated unit rates and that each unit rate is useful for solving different problems.

Learning Goals

- Calculate and interpret the two unit rates for the same relationship.
- Choose which unit rate to use to solve a given problem and explain the choice.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to learn that every ratio relationship has two associated unit rates and that each unit rate can be useful for solving different problems. Students have used unit rates in both the previous lesson and in Unit 2. This is the first lesson that focuses on determining both unit rates for a ratio. Students consider strategies for calculating each unit rate and choosing which unit rate to use to solve a given problem.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce the context of ordering soft serve and for students to consider unit rate in order to compare the prices of different size orders.

Activity 1: Two Unit Rates (20 minutes)

The purpose of this activity is for students to calculate and interpret the two unit rates for the same relationship and use them to solve problems. Students are asked to complete custom orders where a customer requests either a certain number of ounces of soft serve or a certain cost of soft serve. Students consider how to determine a unit rate to help fulfill each order and also which unit rate might be most useful.

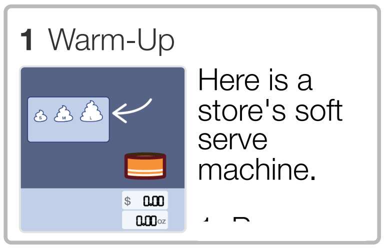
Activity 2: New Flavors (10 minutes)

The purpose of this activity is for students to apply what they learned in Activity 1 to new flavors. Students calculate two unit rates for a different type of soft serve, use those unit rates to solve a problem, and complete a card sort connecting ratios and unit rates.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to interpret unit rates in a different context. Students calculate both unit rates and explain what each means.

Cool-Down (5 minutes)


1 Warm-Up

Here is a store's soft serve machine.

Here is a store's soft serve machine.

1. Press each size to see the prices.

2. Discuss with a classmate: *Which size offers the best deal?*

Teacher Moves

Overview: In this lesson, students learn that every ratio relationship has two associated unit rates and that each unit rate can be useful for solving different problems. This warm-up introduces the context of ordering soft serve and invites students to consider unit rate in order to compare the prices of different size orders.

Launch

- Consider asking students to share any personal experiences they have had with soft serve machines or stores.
- Demonstrate how to press each size to reveal its price using the dashboard's student view. Ask students what they think the S, M, and L stand for.

Facilitation

- Give students 1–2 minutes to think independently and then discuss their responses with a partner.
- Poll the class to see the distribution of thoughts about which size offers the best deal. Invite several students to share their reasoning.
- If it does not come up naturally, consider sharing that someone said they are all the same and inviting students to discuss what that person's argument might have been. (MP3)

Discussion Questions

- *What strategy did you use to decide which was the best deal? How else could you know?*

Math Community

- Celebrate strategies that include both mathematical calculations and personal experience (e.g., Since the rate is the same for all three sizes, the large is the best deal because I get the most ounces of soft serve. More soft serve is better than less.).

Suggested Pacing: Screen 1

Sample Responses

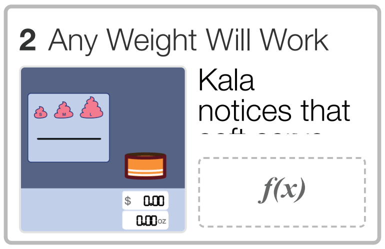
Responses vary. Every size is 40 cents per ounce, so the rate is the same for all sizes and there is no best deal.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.



2 Any Weight Will Work

Kala notices that

Kala notices that soft serve costs the same per ounce no matter what size you get.

She suggests that the store put the rate on the menu.

How much does soft serve cost per ounce?

Teacher Moves

Overview: In Activity 1 (Screens 2–7), students calculate and interpret the two unit rates for the same relationship and use them to solve problems.

Launch

- Invite students to work *in pairs.*
- Invite students to share experiences where the price of something is dependent on its weight (e.g., the deli counter, bulk bins).

Facilitation

- Encourage students to use the feedback on the screen to help them revise their thinking.

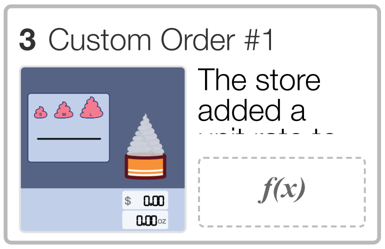
Early Student Thinking

- Students may write 40 dollars per ounce. Encourage students to look at the relationship between the cost they calculated and what they wanted.

Suggested Pacing: Screens 2–5

Sample Responses

0.40 dollars per ounce



3 Custom Order #1

The store added a

The store added a unit rate to the menu.

A customer asks for 8 ounces of soft serve.

How much will it cost?

Teacher Moves

Progress Check

- Encourage students to use paper if it helps them with their thinking throughout this lesson.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

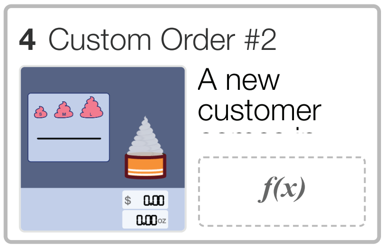
$\$3.20$

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

To support students getting started, consider asking: *What representation can we use for this information? Where would the* 8 *go in our representation?*



4 Custom Order #2

A new customer

A new customer comes in with $\$3$ and wants to spend it all on soft serve.

How many ounces can they get for $\$3$?

Teacher Moves

Facilitation

- To support students getting started, consider inviting them to look at the menu and estimate how much soft serve they think they can get for $\$3$.
- Consider monitoring for students who use strategies similar to Neena's on the next screen or other creative strategies. Invite these students to share their thinking during the discussion on that screen.

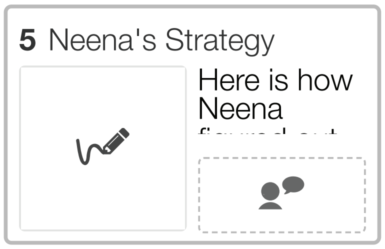
Early Student Thinking

- Students may use the unit rate they calculated earlier and write $3 \cdot 0.40 = 1.2$ ounces. Encourage these students to make a

table and to record all of the information they know.
- Ask questions like: *Where would* $\$3$ *go on your table? How much do you think* 1.2 *ounces would cost?* (MP2)

Sample Responses

7.5 ounces


5 Neena's Strategy

Here is how Neena

Here is how Neena figured out how much soft serve you can get for $\$3$.

1. Discuss Neena's strategy with a classmate.

2. Explain or show where you can see **ounces per dollar** in Neena's work.

Teacher Moves

Facilitation
- Select and sequence several student responses and sketches using the snapshot tool.

Discussion Questions
- *Why do you think Neena divided by* 2 *as her first step?*
- *What does the* 2.5 *mean about the soft serve?*
- *If the cost were* $\$2$ *for* 6 *ounces, what would the ounces per dollar be?* (MP7)

Early Finishers
- Ask students: *What do you think Neena would do if she had* $\$5$*?* $\$6.75$*?*

Math Community
- Rename Neena's strategy after a student in your class who used a similar strategy.

Sample Responses

Responses vary. It is 2.5 ounces per dollar. It is the number of ounces for 1 dollar, which means that every dollar gets you 2.5 ounces of soft serve.

Student Supports

Multilingual Learners
- *Receptive/Expressive Language: Strategic Pairing*

Pair students to aid them in expressing their ideas about Neena’s work.

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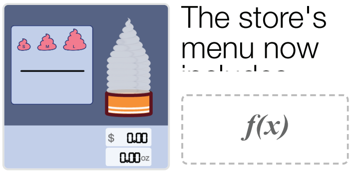
6 Custom Order #3



The store's menu now includes **both** unit rates.

A new customer comes in with $\$7$ and wants to spend it all on soft serve.

How many ounces of soft serve can they get for $\$7$?

Teacher Moves

Launch

- Consider sharing that now that we have two unit rates (dollars per ounce and ounces per dollar), we can consider which one would be useful in different situations.

Progress Check

- Encourage students to use the feedback on the screen to help them revise their thinking.
- This problem will be discussed in more detail on the next screen.

Suggested Pacing: Screens 6–7

Sample Responses

17.5 ounces

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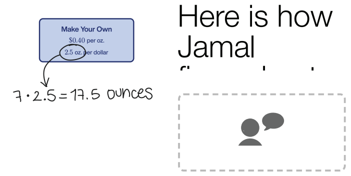
7 Jamal's Strategy



Here is how Jamal figured out how much soft serve you can get for $\$7$.

How do you think Jamal knew which unit rate to use?

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to consider which unit rate to use in solving a given problem and to explain the choice.

Facilitation

- Select and sequence several student responses using the snapshot tool or the dashboard’s teacher view.
- When most students have responded, discuss Jamal’s strategy and generalize about when each unit rate might be useful.

Discussion Questions

- *Why do you think Jamal multiplied by* 2.5 *ounces per dollar and not* 0.40 *dollars per ounce?*
- *When would multiplying by* 0.40 *make sense?*
- *How might this apply to other situations not about soft serve?* (MP7)

Early Finishers

- Encourage students to write each unit rate as a fraction and answer: *How are these two unit rates related?*

Math Community

- Consider naming powerful strategies you hear about determining when to use each unit rate after the students who use them and using those names throughout the rest of the lesson and unit.

Sample Responses

Responses vary. The soft serve is 2.5 ounces per dollar, which means for every dollar, you can buy 2.5 ounces. There is $\$7$ to spend, so I think that's why Jamal multiplied 7 by 2.5.

Student Supports

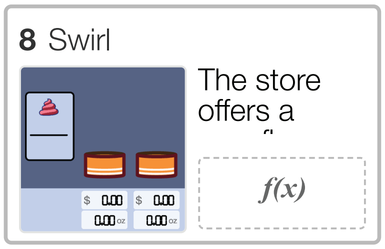
Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., I think Jamal knew to multiply by the unit rate of ___ per ___ because ______.).

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



The store offers a new flavor, Swirl, with this pricing:
$\$5$ for every 4 ounces.

How much does Swirl cost **per ounce**?

Teacher Moves

Overview: In Activity 2 (Screens 8–10), students apply what they have learned in Activity 1 to new flavors.

Launch

- Invite students to think about what flavors they think are in the swirl.

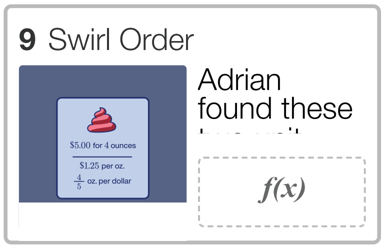
Progress Check

- Encourage students to use the feedback on the screen to help them revise their thinking.

Suggested Pacing: Screens 8–10

Sample Responses

- $\$1.25$ per ounce
- 0.8 or $\frac{4}{5}$ ounces per dollar



9 Swirl Order

Adrian found these

Adrian found these two unit rates for Swirl soft serve.

How much does 7 ounces of Swirl cost?

Teacher Moves

Facilitation

- Encourage students to read others’ responses and decide if others' strategies were similar to or different from their own.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

$\$8.75$

Explanations vary. Since I know the amount of ounces, I can multiply by the dollars per ounce. It is $\$1.25$ per ounce, so $7 \cdot 1.25 = 8.75$.

---

10 Chocolate or Vanilla?

Teacher Moves

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Math Community

- Consider polling the class about their preference for chocolate or vanilla soft serve.

Sample Responses

Image solution

Student Supports

Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts by inviting students to choose one card at a time and decide if it represents chocolate or vanilla soft serve.

---

11 Lesson Synthesis

A model train goes 8

A model train goes 8 inches in 2 seconds.

1. Calculate two unit rates for the model train and enter them in the table.

2. Explain the meaning of each of the numbers you found.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to interpret unit rates in a different context.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *How did you calculate each unit rate? How was the process similar and different for each one?*
- *How did you know what the first unit rate meant? How is it different from the second unit rate?*
- *In general, how do you calculate unit rates?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Suggested Pacing: Screen 11

Sample Responses

- 4 inches
- $\frac{1}{4}$ seconds

Responses vary. The first number means that the train moves 4 inches every second. The second number means that it takes $\frac{1}{4}$ of a second to travel every inch.

Student Supports

Students With Disabilities

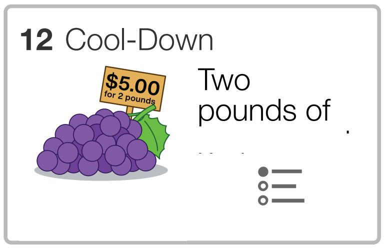
- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., The first number means ______. The second number means ______.).



12 Cool-Down

Two pounds of .

Two pounds of grapes cost $\$5$.

Jordan says that's 2.5 pounds per dollar.
Emika says it's 0.4 pounds per dollar.

Which rate is correct?

Teacher Moves

Support for Future Learning

- If students struggle, plan to emphasize this when opportunities arise in the following lesson.

Suggested Pacing: Screens 12–13

Sample Responses

0.4 pounds per dollar

Explanations vary. Pounds per dollar means how many pounds for 1 dollar. Since $\$5$ gets 2 pounds, dividing both numbers by 5 will get the pounds for 1 dollar: $\frac{2}{5} = 0.4$.

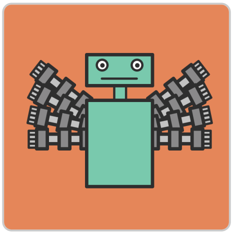
13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can calculate and interpret the two unit rates for the same relationship.
- I can choose which unit rate to use to solve a problem and explain my choice.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Welcome to the Robot Factory

Lesson 6: Using Unit Rates

Overview

Students recognize that in a table of equivalent ratios, they can multiply by a unit rate to go from one column to another (MP7).

Learning Goals

- Explain how to multiply by a unit rate to go from one column to another in a table of equivalent ratios.
- Use unit rates to complete a table of equivalent ratios.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to recognize that in a table of equivalent ratios, they can multiply by a unit rate to go from one column to another (MP7). This lesson invites students to think about unit rates not only as a number across from 1 in a table, but also as the relationship between the numbers in each row of a table. Students use unit rates to help run a robot factory, first examining the relationship between arms and fingers on each robot, then between the number of robots and the amount of paint needed.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to surface what students already know about tables of equivalent ratios using a familiar context: unit conversion.

Activity 1: Arms and Fingers (15 minutes)

The purpose of this activity is for students to use a unit rate between the number of fingers and arms on a robot to determine unknown values. Students describe strategies for using the number of fingers on the robot to determine the number of arms and vice versa.

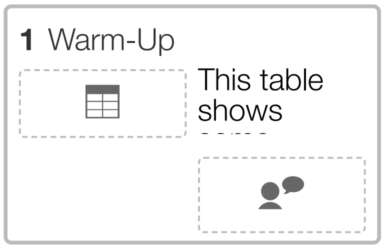
Activity 2: Painting Robots (15 minutes)

The purpose of this activity is for students to continue exploring how multiplying by a unit rate can help determine unknown values in a table of equivalent ratios. They examine relationships between the number of robots and the amount of paint needed to color them.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to explain how you can determine unknown values in a table of equivalent ratios.

Cool-Down (5 minutes)



1 Warm-Up

This table shows

This table shows some lengths in both inches and feet.

What are three things you notice about the table?

Teacher Moves

Overview: In this lesson, students recognize that in a table of equivalent ratios, they can multiply by a unit rate to go from one column to another. This warm-up surfaces what students already know about tables of equivalent ratios in a familiar context: unit conversion.

Launch

- Invite students to share what they remember about measuring in inches and feet.

Facilitation

- Give students 1–2 minutes to think independently and then to share their responses with a partner.
- Consider sharing student thinking using either the dashboard's teacher view or the snapshot tool.
- If it does not come up naturally, consider asking: *Is there a relationship between the numbers in each row?*

Discussion Questions

- *What did you notice? What do you wonder?*
- *Do you think these patterns are true for other tables of equivalent ratios?*

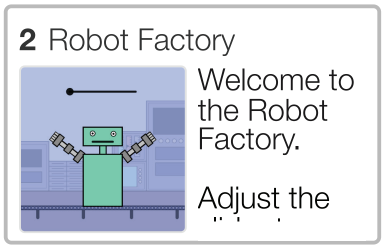
Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

- I notice that every length is different.
- I notice all of the lengths in inches are even numbers.
- I notice that the number on the right is the number on the left times 12.



2 Robot Factory

Welcome to the Robot Factory.

Adjust the

Welcome to the Robot Factory.

Adjust the slider to control how many arms the robot has.

Teacher Moves

Overview: In Activity 1 (Screens 2–5), students use a unit rate between the number of fingers and arms on a robot to determine unknown values.

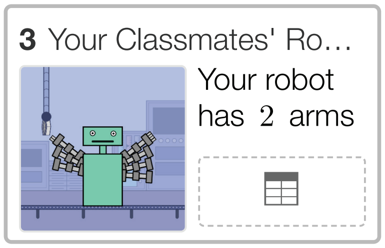
Launch

- Invite students to work *in pairs.*
- Encourage students to adjust the slide to see how it changes the robot.

Facilitation

- Give students one minute to select the number of arms on their robot and then continue to the next screen.

Suggested Pacing: Screens 2–5



3 Your Classmates' Ro…

Your robot has 2 arms

Your robot has 2 arms and 8 fingers.

Your classmates chose different numbers of arms.

Complete the table with the number of fingers on their robots.

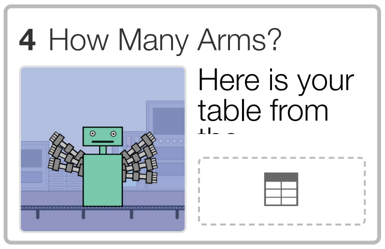
Teacher Moves

Facilitation

- To support students struggling to get started, consider asking: *What information would be helpful to figure out how many fingers this robot needs? What is the relationship between the number of fingers and the number of arms?*
- Consider monitoring for students who discuss multiplying by 4 to determine the number of fingers. Invite these students to share their thinking during the discussion on Screen 5.

Sample Responses

Responses vary. All numbers of fingers are 4 times the number of arms.



4 How Many Arms?

Here is your table from

Here is your table from the previous screen.

A new row has been added.

How many arms go with this many fingers?

Teacher Moves

Progress Check

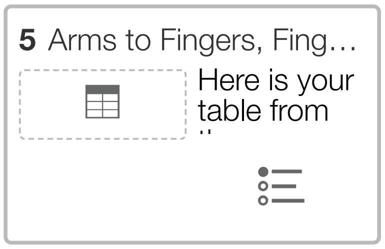
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling before the conversation on Screen 5.

Early Student Thinking

- Students may notice that 44 is four more than 40 and write 14 arms.
- Consider asking these students to adjust their response based on the feedback. Then ask: *What is the relationship between* 11 *and* 44*? How is this like the relationship between* 10 *and* 40*?*

Sample Responses

11 arms



5 Arms to Fingers, Fing…

Here is your table from

Here is your table from the previous screen.

Choose one question to answer:

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface that you can multiply by a unit rate to determine an unknown number of arms or fingers.

Facilitation

- While students are working, select and sequence several student responses using the snapshot tool. Monitor for students who mention unit rates, and for students who describe different strategies (e.g., divide by 4 vs. multiply by $\frac{1}{4}$).

Discussion Questions

- *How would you say ____________'s strategy in your own words?*

- *Do you think this strategy will work for any number of fingers and arms? Do you think it will work for feet and inches?*

Early Finishers

- Invite students to decide if it is possible for this robot to have 50 fingers.

Math Community

- Invite students to share what they found helpful about each of the shared responses.

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Sample Responses

Responses vary.

I can determine the number of arms by dividing by 4.

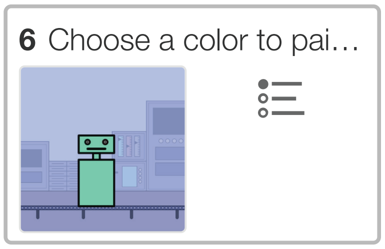
I can determine the number of fingers by multiplying by 4.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Consider reading each question aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.



6 Choose a color to pai…

Teacher Moves

Overview: In Activity 2 (Screens 6–10), students continue exploring how multiplying by a unit rate can help determine unknown values in a table of equivalent ratios.

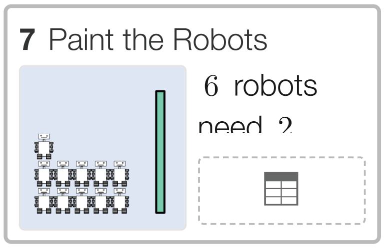
Launch

- Consider sharing with students that they will be determining how much paint they need to paint different-size robots.

Facilitation

- Give students one minute to select the color of their robot and then continue to the next screen.

Suggested Pacing: Screens 6–10



7 Paint the Robots

6 robots need 2

6 robots need 2 gallons of paint.

Complete the table.

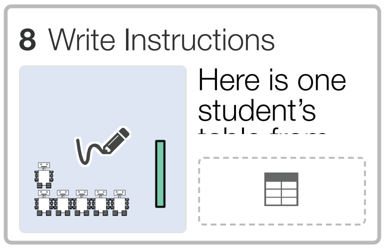
Teacher Moves

Progress Check

- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- Students may struggle to calculate the amount of paint needed for 11 robots if they are only thinking about 3 as a unit rate. Consider asking: *What is the other unit rate for this ratio? How many gallons of paint are needed per robot?*

Sample Responses

- 5 gallons
- 7 gallons
- $\frac{11}{3}$ gallons



8 Write Instructions

Here is one student's

Here is one student's table from the previous screen.

Teacher Moves

Facilitation

- Select and sequence several student responses using the snapshot tool. Share several responses and invite students to compare and contrast them.

Discussion Questions

- *What do these instructions have in common?*
- *Where do we see unit rates in each of these instructions?*
- *What is the advantage of having a set of instructions that work for any number of robots?*

Math Community

- Consider snapshotting imprecise or unfinished explanations. During the discussion, highlight the strengths of these explanations by asking students to identify what parts of each explanation they found valuable and to make them stronger and clearer as a class.

Sample Responses

Responses vary. Take the number of robots and multiply by the number of gallons per robot, which is $\frac{1}{3}$.

Student Supports

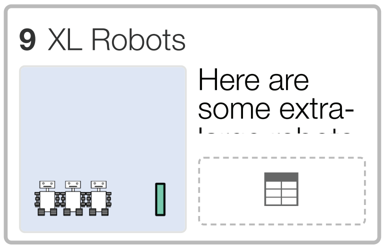
Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Invite students to describe their thinking aloud with a partner before writing it.

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., First, ________. Then, _______.).



9 XL Robots

Here are some extra-

Here are some extra-large robots.

4 robots need 10 gallons of paint.

Complete the table.

Teacher Moves

Progress Check
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- Monitor for students who describe their strategies using language similar to that shared on the previous screen.

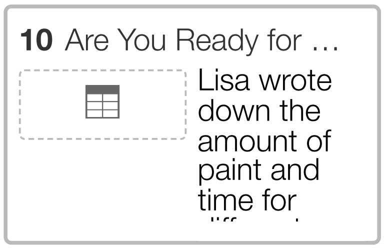
Sample Responses

- 17.5 gallons
- 22.5 gallons
- 32.5 gallons

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

To support students getting started, consider asking: *How are* 4 *robots and* 10 *gallons of paint related?* If students say to add 6, invite them to try $7 + 6 = 13$ and to use the feedback to revise their thinking.



10 Are You Ready for …

Lisa wrote down the amount of paint and time for

Lisa wrote down the amount of paint and time for different numbers of robots.

Some of the numbers are missing.

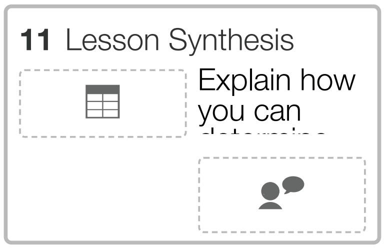
Complete the table. Then press "Check My Work."

Teacher Moves

Facilitation
- Invite students who finish Screens 6–9 early to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion, or if enough students approach the problem, consider inviting students to share aloud.

Sample Responses

Row 1: 5, 2, 4
Row 2: 12.5, 5, 10
Row 3: 15, 6, 12
Row 4: 2.5, 1, 2



11 Lesson Synthesis

Explain how you can

Explain how you can determine unknown values, like the amount of paint for robots, in a table of equivalent ratios.

Use this table if it helps you explain your thinking.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is for students to explain how you can determine unknown values in a table of equivalent ratios

Facilitation

- Give students 1–2 minutes to respond and one minute to share their response with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *What is helpful about this strategy? What questions do you have?*
- *What connections do you see between your classmates' strategies?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Suggested Pacing: Screen 11

Sample Responses

Responses vary. In a table of equivalent ratios, you can multiply by a unit rate to go from one column to another.

Student Supports

Students With Disabilities

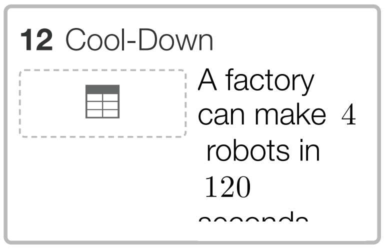
- *Executive Functioning: Visual Aids*

Consider using the routine Collect and Display to capture students' explanations for the class to refer back to throughout the lesson and unit.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., You can determine unknown values in a table of equivalent ratios by ______.).



12 Cool-Down

A factory can make 4 robots in 120

A factory can make 4 robots in 120 seconds.

Complete the table.

Teacher Moves

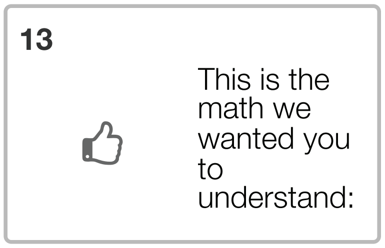
Support for Future Learning

- If students struggle, plan to emphasize this when opportunities arise in Lesson 7, particularly on Screens 6–8.

Suggested Pacing: Screens 12–13

Sample Responses

- 750 seconds
- 9 robots
- 30 seconds



13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I know that you can multiply by a unit rate to go from one column to another in a table of equivalent ratios.
- I can use unit rates to complete a table of equivalent ratios.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

More Soft Serve

Lesson 7: Solving Rate Problems

Overview

Students practice using what they've learned about unit rates to make comparisons and calculate unknowns.

Learning Goals

- Use unit rates to make comparisons and calculate unknown quantities.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to practice using what they've learned about unit rates to make comparisons and calculate unknowns. This lesson returns to the context of soft serve and invites students to use what they learned in the previous two lessons to engage with problems that include more complex decimals and fractions. This lesson also offers students an opportunity to create their own soft serve challenge for their classmates.

Lesson Summary

Warm-Up (5 minutes)

In the warm-up, students engage in the Number Talk routine to surface strategies for multiplying whole numbers by fractions. This the third in a series of warm-ups to strengthen strategies around multiplication of fractions.

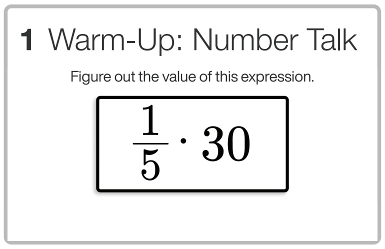
Activity 1: More Soft Serve (30 minutes)

The purpose of this activity is for students to continue to practice using unit rates to solve problems. Students compare soft serve shops, help those shops fulfill orders, then create rates at their own shop and challenge their classmates. Students should leave this activity with a strong understanding of how to calculate both unit rates in a ratio relationship and use them to determine unknown quantities.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to surface strategies for using unit rates to calculate unknown values.

Cool-Down (5 minutes)



1 Warm-Up: Number Talk

Figure out the value of this expression.

$$\frac{1}{5} \cdot 30$$

Teacher Moves

Overview: In this lesson, students practice using what they've learned about unit rates to make comparisons and calculate unknowns. In this warm-up, students engage in the Number Talk routine to surface strategies for multiplying whole numbers by fractions. This is the third in a series of warm-ups to strengthen strategies around multiplication of fractions.

Launch

- Use the dashboard's student view to display Screen 1.
- Invite students to share any strategies they remember from previous number talks in this unit.

How Number Talk Works

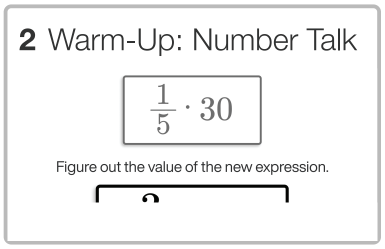
- Give students one minute to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own or one of the possible strategies in the sample responses.
- Select several students to share different strategies. Use the sample responses as examples of possible student strategies. Record strategies for all to see, along with the name of the student who shared each one.
- Display the next expression and repeat. If it makes sense, encourage students to use a strategy described by a classmate in the previous round.

Suggested Pacing: Screens 1–4, one screen at a time

Sample Responses

- $\frac{1}{5} \cdot 30 = 6$

Strategies students might use in the Number Talk



2 Warm-Up: Number Talk

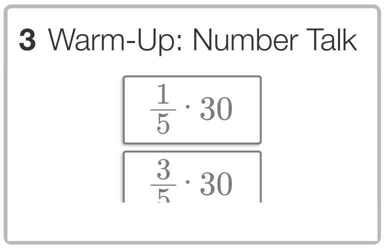
$\frac{1}{5} \cdot 30$

Figure out the value of the new expression.

Sample Responses

$\frac{3}{5} \cdot 30 = 18$

Strategies students might use in the Number Talk



3 Warm-Up: Number Talk

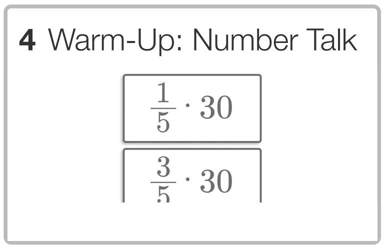
$\frac{1}{5} \cdot 30$

$\frac{3}{5} \cdot 30$

Sample Responses

$\frac{3}{5} \cdot 15 = 9$

Strategies students might use in the Number Talk



4 Warm-Up: Number Talk

$\frac{1}{5} \cdot 30$

$\frac{3}{5} \cdot 30$

Sample Responses

$\frac{3}{5} \cdot 3 = 1.8$

Strategies students might use in the Number Talk

5 The Best Deal

Which soft serve shop

Which soft serve shop has the best price per ounce?

Teacher Moves

Overview: In Activity 1 (Screens 5–9), students continue to practice using unit rates to solve problems.

Launch
- Invite students to work *individually.*
- Invite students to share what they remember from the last lesson about soft serve.

Facilitation
- Give students 1–2 minutes to work independently and then read their classmates' responses.
- Consider displaying the distribution of responses using the dashboard’s teacher view, calling attention to any conflict or consensus you see.

- If there is not consensus, invite students to explain why someone might have selected each of the most popular responses.

Math Community
- Celebrate students who used different strategies to determine which soft serve shop has the best price.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Suggested Pacing: Screens 5–7

Sample Responses

Shop B

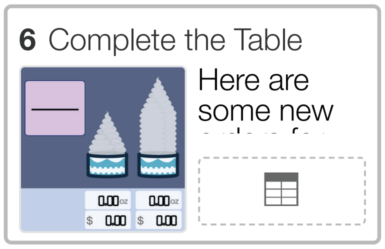
Explanations vary.

- Shop B is a better deal than Shop A because you get more soft serve for less money. Shop B is a better deal than Shop C because it would cost $\$1.20 \cdot 1.5 = \$1.80$ at Shop C for 6 ounces.
- Shop B is the best deal because it has the lowest price per ounce. Shop A is $\$0.40$ per ounce, Shop B is $\$0.25$ per ounce, and Shop C is $\$0.30$ per ounce.

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

To assist students in recognizing the connections between new problems and prior work, consider inviting them to notice and wonder about the three shops before responding. Ask a question like: *What strategies might help us decide which shop has the best price per ounce?*



Here are some new orders for Shop B.

Enter the missing values.

Teacher Moves

Progress Check
- Encourage students to use a calculator and paper if it helps them with their thinking.
- To support students getting started, consider asking: *Which unit rate would be helpful here?*

- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

- $\$2$
- 14 ounces

7 Riya's Strategy

On the previous

On the previous screen, Riya multiplied across the table by the unit rates.

Explain how you could use Riya's strategy to calculate the cost of a 10.5-ounce soft serve.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to review how to select which unit rate would be useful to solve different problems (MP8).

Facilitation
- Monitor for students who show their thinking in different ways. Display several student responses and sketches.
- Invite several students to share how they think Riya knew which number to multiply by in each case.

Discussion Questions
- *Why do you think Riya multiplied by different numbers for each question?*
- *What other strategies could Riya have used to answer these questions?*

Early Finishers
- Encourage students to make up their own amount or cost of soft serve and challenge a classmate to determine the unknown cost or number of ounces.

Math Community
- Consider renaming Riya's strategy after the students in your class who used them.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Sample Responses

Responses vary. You could multiply 10.5 by 0.25 because you know the number of ounces and 0.25 is the dollars per ounce.

Student Supports

Students With Disabilities

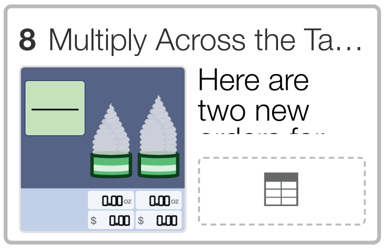
- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



8 Multiply Across the Ta…

Here are two new

Here are two new orders for Shop C.

Use Riya's strategy to calculate the missing values.

Teacher Moves

Launch

- Consider drawing students' attention to the fact that this is a different soft serve shop and sharing that they will be creating their own shop shortly.

Progress Check

- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 8–9

Sample Responses

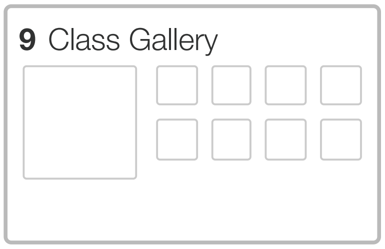
- 9.5 ounces
- $\$3.30$

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

To support students in using unit rates to calculate unknown quantities on this and other screens, consider providing blank copies of double number lines or tables.



9 Class Gallery

Teacher Moves

How Challenge Creator Works

- Students are prompted to "Make My Challenge," in this case, setting a cost and weight for a small soft serve, calculating both unit rates, and setting the weight of a medium and the cost of a large soft serve.
- When students have created their soft serve shop, they submit their challenge to the Class Gallery. Students should then select classmates' challenges to solve, which involves calculating the unknown cost of a medium and the weight of a large.

Facilitation

- Give students several minutes to create their own challenge and more time to solve their classmates' challenges.

- Encourage students to go back and review their classmates' responses to the challenge they created.
- Consider using the teacher dashboard to display the variety of the unit rates students created.
- Ask a question like: *What is the same about solving different people's challenges? What is different?*

Note: This Challenge Creator may take 15 minutes or more.

Math Community
- Consider inviting students to share challenges they found particularly fun or creative.

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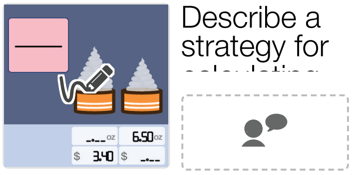
10 Lesson Synthesis



Describe a strategy for

Describe a strategy for calculating the unknown weights or costs of different soft serve orders.

Use the example to the left if it helps you with your thinking.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to surface strategies for using unit rates to calculate unknown values.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their response with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions
- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Math Community
- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Suggested Pacing: Screen 10

Sample Responses

Responses vary. The most important thing is to choose the correct unit rate. In the problem on the left, you can figure out the unknown weight by multiplying the cost $(\$3.40)$ by the ounces per dollar (2.5). In the problem on the right, you can figure out the unknown cost by multiplying the weight (6.5 ounces) by the dollars per ounce ($\$0.40$).

Student Supports

Students With Disabilities

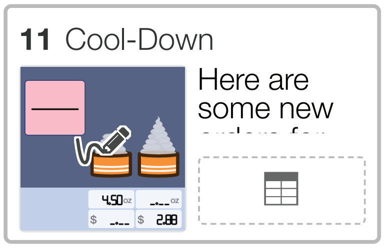
- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., You can calculate the cost of a soft serve order by ______. You can calculate the weight of a soft serve order by ______.).



11 Cool-Down

Here are some new

Here are some new orders for Shop A.

Enter the missing values.

Teacher Moves

Support for Future Learning

- If students struggle, consider reviewing this screen as a class before Practice Day 1 or offering individual support where needed during the Practice Day. Students will need a strong understanding of how to use unit rates to determine unknown values.

Suggested Pacing: Screens 11–12

Sample Responses

- $\$1.80$
- 7.2 ounces

12

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can make comparisons and calculate unknown quantities using unit rates.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

6.3 Practice Day 1 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided sheet for each student.

Task Cards

- *Option 1 (Stations)*: Print two single-sided sets of task cards for the entire class (8 cards total).
- *Option 2 (Task Cards):* Print one single-sided set of task cards for each group of 2–3 students.

Instructions

Option 1: Stations

- Arrange students into *groups of 2–4.*
- Distribute one Student Workspace Sheet to each student.
- Place one task card at each station. It may be helpful to have two copies of the same task card at one station.

- Invite students to work together as they solve the problems on each task card.
- If students have extra time at a station, invite them to try the “Are You Ready for More?” task.

Options for student movement:

- As students finish a station, instruct them to move on to a new station.
- After a set amount of time, instruct students to move as a group from station to station.
- After a set amount of time, instruct students to move to a new station such that no one from their previous group is in their new group.

Option 2: Task Cards

- Arrange students into *groups of 2–3.*
- Distribute one Student Workspace Sheet to each student and one set of task cards to each group.

- Invite groups to choose one task card to start with. Encourage students to share their reasoning as a group and work to reach an agreement together about how to answer each question.
- When groups have completed all four tasks, invite them to select 1–2 “Are You Ready for More?” tasks to think about.

- Consider posting the answer key, or walking around with it and providing feedback to students as they work.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Lucky Duckies

Lesson 8: Benchmark Percentages

Overview

Students build on their experiences with percentages in order to reason about these benchmark percentages: 10%, 25%, 50%, and 75%.

Learning Goals

- Understand the word *percent* and that the symbol $\%$ means "for every 100."
- Calculate $A\%$ of B where A is 10, 25, 50, or 75.

Vocabulary

- percent

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to build on personal experiences they have with percentages in order to reason about these benchmark percentages: 10%, 25%, 50%, and 75%. This lesson also informally introduces tape diagrams as a model for representing percentages in the context of a ducky carnival game.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce the context of a ducky carnival game and to surface what students already know about the concept of 50%.

Activity 1: Ducky Game Design (20 minutes)

The purpose of this activity is for students to reason about the relationship between the percentage of winners, the number of winners, and the number of total duckies. Students explore how the number of winners changes as the number of total duckies changes and calculate the number of winners for a specific game.

Activity 2: Ducky Challenges (10 minutes)

The purpose of this activity is for students to practice solving problems involving calculating the number of winners given a benchmark percentage and a total number of duckies. Students use tape diagrams to connect percentages and fractions. The repetition of the benchmark percentages allows students to notice regularity and engage in MP8.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to explain what they have learned about percentages, and what a percent of a number means.

Cool-Down (5 minutes)

1 Warm-Up

In this carnival game, players win a prize if

In this carnival game, players win a prize if they catch a rubber ducky with a star on the bottom.

Try catching a ducky with a star.

Teacher Moves

Overview: In this lesson, students build on personal experiences they have with percentages in order to reason about these benchmark percentages: 10%, 25%, 50%, and 75%. This lesson also informally introduces the tape diagram as a model for representing percentages. This warm-up introduces the context of a ducky carnival game and surfaces what students already know about the concept of 50%.

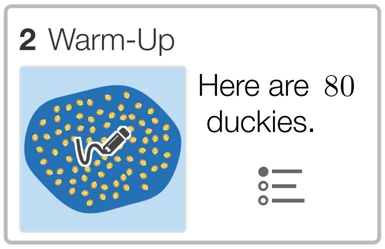
Launch

- Use the dashboard's student view to demonstrate how to click a duck to "catch" it.
- Consider asking students to share experiences they have had with similar games, carnivals, or school fairs.

Facilitation

- Give students 1–2 minutes to try to catch a ducky with a star, then invite them to continue to the next screen.

Suggested Pacing: Screens 1–2



2 Warm-Up

Here are 80 duckies.

Here are 80 duckies.

Which game is easier to win?

Teacher Moves

Facilitation

- The purpose of this screen is to make sense of how 50 is different from 50% (MP1).
- When most students have responded, consider showing the distribution of responses using the dashboard's teacher view.
- Invite students to share their reasoning for their responses.
- Spend adequate time here to ensure that all students understand what "50% of the duckies have stars" means.

Discussion Questions

- *How is* 50 *different from* 50%?

- *What does* 50% *mean?*
- *Would your answer change if there were* 100 *duckies?* 150 *duckies?*

Early Finishers

- Invite students to describe a way to change the ducky game so that each multiple choice option would be the correct answer.

Math Community

- Consider inviting students to think about what a student who responded differently might have been thinking.

Sample Responses

50 duckies have stars

Explanations vary. There are 80 total duckies and 50% is half, so 40 duckies have stars. The game with 50 duckies with stars has more stars, so it is easier to win.

Student Supports

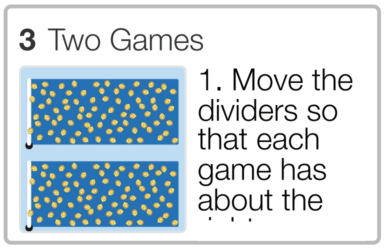
Students With Disabilities

- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

- *Conceptual Processing: Processing Time*

To support students in attending to the differences in the choices, consider asking: *How are these two games similar? How are they different?*



3 Two Games

1. Move the dividers so that each game has about the

1. Move the dividers so that each game has about the right number of duckies with stars.

Teacher Moves

Overview: In Activity 1 (Screens 3–7), students reason about the relationship between the percentage of winners, the number of winners, and the number of total duckies.

Launch

- Share with students that they are going to analyze several different ducky games.

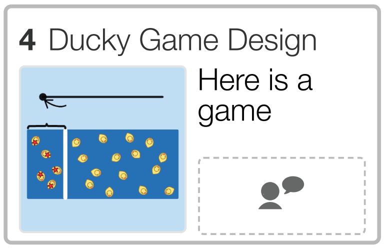
- Consider demonstrating how to move the dividers using the dashboard's student view.

Facilitation
- Give students 1–2 minutes to place each line according to the game instructions.
- Circulate to listen to how students describe their strategy for placing each line. Monitor for students who connect percentages and fractions (e.g. 25% as $\frac{1}{4}$, or 25% as $\frac{1}{2}$ of 50%). Invite these students to share their thinking.
- **Note:** Students may wonder what the other white lines that appear are. These are their classmates' responses.

Suggested Pacing: Screens 3–4

Sample Responses

Image solution



4 Ducky Game Design

Here is a game

Here is a game where 25% of the duckies have stars.

1. Drag the point to adjust the total number of duckies.
2. Describe what 25% of a number means.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to come to a shared understanding of the relationship between the total number of duckies, the percentage of duckies with stars, and the number of duckies with stars.

Facilitation
- Encourage students to change the total number of duckies and to share anything they notice and wonder with a partner (MP1).
- Select and sequence several student responses using the snapshot tool.
- Facilitate a whole-class discussion. Consider adjusting the number of duckies to 100 as a starting point.

Discussion Questions
- *Why do you think there are* 25 *duckies with stars when there are* 100 *duckies total?*

- *Do you think there will always be* 25 *duckies with stars?*
- *Why do you think* 25% *takes up* $\frac{1}{4}$ *of the pool?*
- *What changes when you change the number of total duckies? What stays the same?*

Early Finishers

- Encourage students to write a strategy for calculating 25% of any number.

Math Community

- Invite students to share if they found one of the explanations discussed particularly helpful.

Sample Responses

Responses vary.

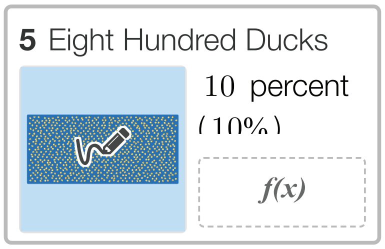
- 25% means 25 out of every 100. If you have 300 duckies, $25 \cdot 3 = 75$ of them would have stars.
- 25% means $\frac{1}{4}$ of something. If you have 25% of the duckies, then you have $\frac{1}{4}$ of all the duckies.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



5 Eight Hundred Ducks

10 percent (10%)

10 percent (10%) means 10 for every 100.

This game has 800 duckies. 10% of them have stars.

How many of these duckies have stars?

Teacher Moves

Facilitation

- Read the first sentence aloud.
- Encourage students to use the feedback on the screen to help them revise their thinking.

Suggested Pacing: Screens 5–7

Sample Responses

80 duckies

Student Supports

Multilingual Learners

- *Receptive/Expressive Language: Physical Models*

Consider bringing in images or physical representations of 10% to support students in accessing the language on this screen.

6 Santiago's Strategy

Here is how Santiago

Here is how Santiago figured out the number of duckies that are winners when 10% out of 800 duckies win.

Explain or show what he may have been thinking.

Teacher Moves

Facilitation

- Encourage students to read others’ responses and decide if others' descriptions were similar to or different from their own.

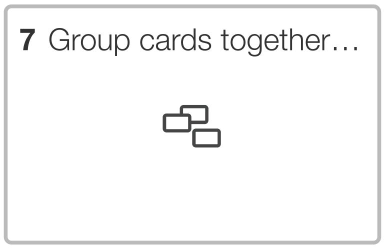
Early Student Thinking

- Students may say that Santiago divided by 10 because he was looking for 10%. Consider asking these students if that strategy would work for 25% as well.

Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify to help students communicate their ideas.

Sample Responses

Responses vary. Santiago knew that there are ten 10s in 100, so he split the tape diagram into 10 pieces and put an equal number in each piece so that the total was 800.



7 Group cards together…

Teacher Moves

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Discussion Questions

- *What would a tape diagram with* 75% *shaded green look like?*

Early Finishers

- Encourage students to create their own cards so that each group has three cards: a fraction, a percentage, and a tape diagram.

Sample Responses

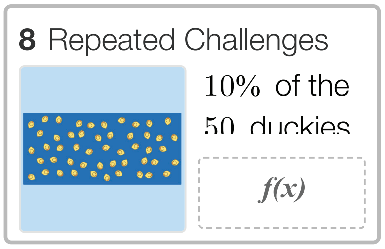
Image solution

Student Supports

Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts by inviting students to start by choosing one of the tape diagram cards and deciding what percent of the diagram is shaded green.



8 Repeated Challenges

10% of the 50 duckies have stars. (Challenges vary)

How many duckies have stars?

Teacher Moves

Overview: In Activity 2 (Screens 8–9), students practice solving problems involving calculating the number of winners given a benchmark percentage and a total number of duckies.

How Repeated Challenges Work

- Students are presented with a variety of challenges one at a time, in this case determining how many duckies will have stars.
- The challenges typically increase in difficulty as they continue.

Facilitation

- Give students 5–7 minutes to complete as many challenges as they can.
- Encourage students to use the sketch tool or paper to help them with their thinking.

Math Community

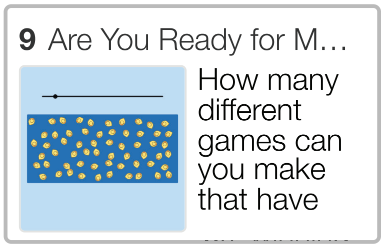
- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Suggested Pacing: Screen 8, opening to Screen 9 after several minutes

Sample Responses

The solutions to the first few challenges are:

- 5 duckies
- 50 duckies
- 150 duckies
- 12 duckies



9 Are You Ready for M…

How many different games can you make that have

How many different games can you make that have 30 winning duckies?

1. Select a percentage.

2. Drag the point to select the total number of duckies.

Teacher Moves

Facilitation

- After several challenges, encourage students to explore this screen.
- If time allows, consider inviting students to share different ways they made the game have 30 winning duckies.

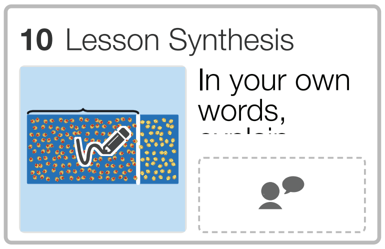
Sample Responses

10% of 300 duckies

25% of 120 duckies

50% of 60 duckies

75% of 40 duckies



10 Lesson Synthesis

In your own words,

In your own words, explain what 75% of a number means.

Teacher Moves

Key Discussion Screen
- The purpose of this discussion is to surface what 75% of a number means.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.

Discussion Questions
- *How is* 75% *similar to and different from* 25% *or* 50%*?*
- *How would* 70% *be different from* 75%*? What about* 90%*?*

Suggested Pacing: Screen 10

Sample Responses

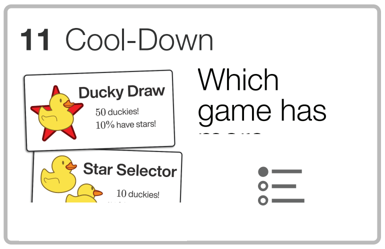
Responses vary.

- 75% means 75 out of every 100. If you have 200 duckies, $75 \cdot 2 = 150$ of them would have stars.
- 75% means $\frac{3}{4}$ of something. If you have done 75% of your homework, then you have done $\frac{3}{4}$ of your homework.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., 75% of a number means ______.).



11 Cool-Down

Which game has

Which game has more duckies with stars?

Teacher Moves

Support for Future Learning

- Students will have more chances to develop their understanding of reasoning about and calculating percentages in Lessons 9 and 10.

Readiness Check (Problem 5)

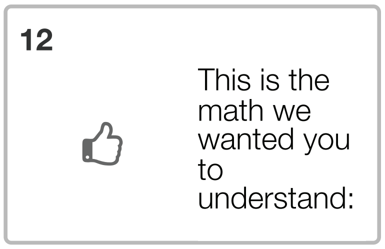
- If most students struggled on Problem 5, give them an opportunity to revise their responses after this lesson.

Suggested Pacing: Screens 11–12

Sample Responses

They have the same number of duckies with stars.

Explanations vary. 10% of 50 is 5, and 50% of 10 is also 5.



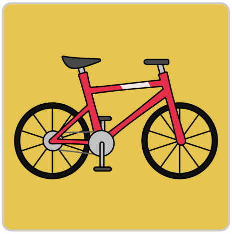
12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use the word *percent* and the symbol $\%$ to mean "for every 100 ."
- I can calculate 10%, 25%, 50%, or 75% of a number.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Bicycle Goals

Lesson 9: Friendly Percentages

Overview

Students make connections between tape diagrams and double number lines that represent percentages, and then use double number lines and tables to solve problems involving friendly percentages (multiples of benchmark percentages).

Learning Goals

- Make connections between percentages and ratios.
- Use a double number line, tape diagram, or table to determine unknown parts or wholes given friendly percentages.

Vocabulary

- percentage

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Students make connections between tape diagrams and double number lines that represent percentages, and then use double number lines and tables to solve problems involving friendly percentages (multiples of benchmark percentages). This builds on Unit 2 where students used double number lines to solve problems with ratios. This is also the first lesson in which students consider percentages greater than 100%.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to see the connection between the bicycle interaction and the progress bar, and make sense of what it means to ride a percentage of a goal. Students explore how the biker's distance affects the progress bar and then answer a question using a benchmark percentage.

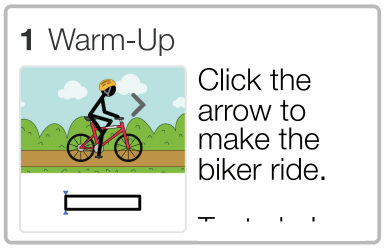
Activity 1: Bicycle Goals (30 minutes)

The purpose of this activity is for students to use double number lines, tables, and other strategies to solve problems involving friendly percentages. Students determine unknown parts and unknown wholes in a variety of challenges within the bicycle goal context. Students will calculate unknown percentages for the first time in the next lesson.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to make connections between solving problems with percentages and solving problems with ratios.

Cool-Down (5 minutes)



1 Warm-Up

Click the arrow to make the biker ride.

Click the arrow to make the biker ride.

Try to help them beat their goal.

Teacher Moves

Overview: In this lesson, students make connections between tape diagrams and double number lines that represent percentages, and then use double number lines and tables to solve problems involving friendly percentages (multiples of benchmark percentages). In this warm-up, students explore the connection between the bicycle interaction and the progress bar, and make sense of what it means to ride a percentage of a goal.

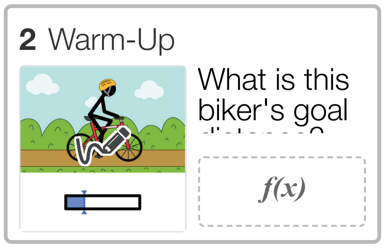
Launch

- Invite students to share where they have seen progress bars like this one before or what the longest distance they have ever biked is.

Facilitation

- Allow students one minute to play. Then invite them to continue to the next screen.

Suggested Pacing: Screens 1–2



2 Warm-Up

What is this biker's goal

What is this biker's goal distance?

Teacher Moves

Facilitation

- Select and sequence several student sketches using the snapshot tool. Monitor for students who used different types of reasoning to get to 60 km.
- Spend adequate time here to ensure that students understand what 100% means in this situation and how it is represented on the progress bar.

Discussion Questions

- *How is this situation similar to the duckies lesson? How is it different?*
- *What does* 100% *mean in this situation? Where do we see it on the progress bar?*

Early Finishers

- Encourage students to determine how far the biker would have gone if they rode 75% or 125% of their goal.

Math Community

- Celebrate students who use their personal experiences to support them in their reasoning.

Sample Responses

60 kilometers

Explanations vary.

- 25% is like $\frac{1}{4}$ so the whole goal would be 100% or $15 \cdot 4 = 60$ kilometers.
- There are four 25 percents in 100%, so you need to multiply by 4 to get the goal distance.

3 Challenge #1

Alejandro's goal was to

Alejandro's goal was to ride 36 kilometers.

His app says he rode 75% of his goal.

How far did he ride?

Teacher Moves

Overview: In Activity 1 (Screens 3–10), students use double number lines, tables, and other strategies to solve problems involving friendly percentages.

Launch

- Invite students to work *in pairs.*
- Read the question aloud. Ask: *Where would the* 36 *go on the progress bar?*
- Invite students to estimate the answer by asking: *Should it be more or less than* 36 *kilometers? More or less than half of* 36*?*

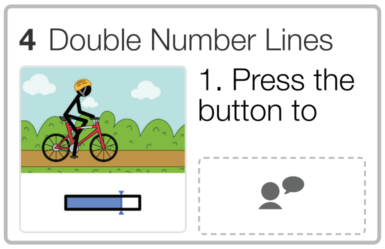
Progress Check

- Encourage students to use paper or the sketch tool to help them with their thinking.

Suggested Pacing: Screens 3–4

Sample Responses

27 kilometers



4 Double Number Lines

1. Press the button to

1. Press the button to see a double number line.

2. Describe how you can tell the goal distance is 36 kilometers.

Teacher Moves

Facilitation

- Encourage students to read others’ responses and decide if others' descriptions were similar to or different from their own.
- When most students have responded, invite them to share what they notice or wonder about the double number line.

Discussion Questions

- *Where do you see Alejandro’s goal on the double number line?*
- *What does the* 150 *on the double number line mean? What would* 150% *of a goal mean?*
- *How could a progress bar be helpful? How could a double number line be helpful?*

Early Finishers

- Invite students to use the double number line to calculate 40% of the goal and 120% of the goal.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

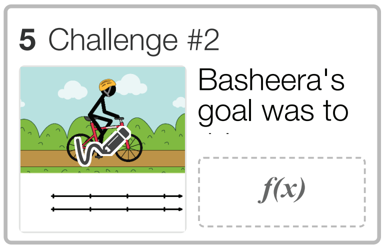
Responses vary. The goal is always 100% and 36 km on the top number line lines up with 100% on the bottom number line.

Student Supports

Multilingual Learners

- *Expressive Language: Strategic Pairing*

Invite students to describe their thinking aloud with a partner before writing it.


5 Challenge #2

Basheera's goal was to

Basheera's goal was to ride 12 kilometers.

Her app says she rode 150% of her goal.

How far did she ride?

Teacher Moves

Launch

- Consider inviting students to compare and contrast this double number line with the one on the previous screen.

Facilitation

- To support students getting started, ask: *How could we determine* 50% *of the goal?*
- **Note:** This is the first time students are asked to reason with percentages over 100%.

Suggested Pacing: Screens 5–7

Sample Responses

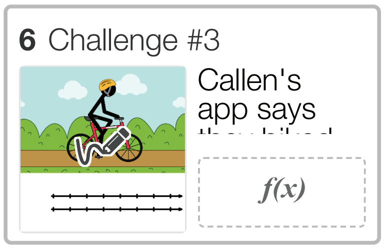
18 kilometers

Student Supports

Students With Disabilities

- *Executive Functioning: Visual Aids*

Consider using the routine Collect and Display to capture students' words and sketches for the class to refer back to throughout the lesson and unit.


6 Challenge #3

Callen's app says

Callen's app says they biked 8 kilometers, which is 20% of their goal.

What was their goal distance?

Teacher Moves

Facilitation

- Encourage students to use the sketch tool or paper to show their thinking. Consider using the snapshot tool to select student sketches for the discussion on the next screen.
- Consider monitoring for students who use strategies similar to those on the next screen or other creative strategies. Invite these

students to share their thinking during the discussion on that screen.

Early Student Thinking

- Students may calculate 20% of 8, which is 1.6 kilometers. Invite them to use the feedback to revise their answer.
- Consider asking: *What does the 8 represent about the biker?* (MP2)

Sample Responses

40 kilometers

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7 Two Strategies

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface how to use a double number line diagram and table to determine an unknown whole given a part and a percentage.

Facilitation

- To encourage student movement, consider displaying this screen using the dashboard's student view and inviting students to stand with a new partner and discuss what each ratio means about the biker's goal. Then, ask students to find a different partner across the room and discuss again. (MP2)

Discussion Questions

- *Where do we see the goal distance in the table? Where do we see it on the double number line?*
- *How can you use a table to figure out a goal? How can you use a double number line to figure out a goal?*
- *How are these strategies similar? How are they different?*

Early Finishers

- Invite students to figure out what the goal was if 8 kilometers was 40% of the goal instead. What about 5% of the goal?

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary. The student on the left made a table. They multiplied by 5 so that the percent in the right column was 100%. The student on the right filled in the double number line until they found the number that lined up with 100%.

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

To support students in expressing their thinking, encourage them to use the sketch tool to draw on or highlight each strategy.

---

8 Match cards that repr…

Teacher Moves

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Math Community

- Consider celebrating students who revised their thinking as they worked on the card sort.

Suggested Pacing: Screens 8–10

Sample Responses

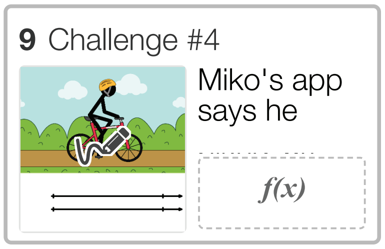
Image solution

Student Supports

Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts by inviting students to choose one card at a time and decide what number represents the whole.



9 Challenge #4

Miko's app says he

Miko's app says he biked 30 kilometers, which is 120% of his goal.

What was his goal distance?

Teacher Moves

Progress Check

- To help students get started, consider asking: *Do you think Miko's goal was more or less than* 30 *kilometers?*
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- **Note:** This is the first double number line where the number line is not subdivided for students. (MP6)

Sample Responses

25 kilometers

10 Are You Ready for …

How many ways can

How many ways can you make the biker ride 10 kilometers?

Enter a goal and a percentage for the biker to ride.

Teacher Moves

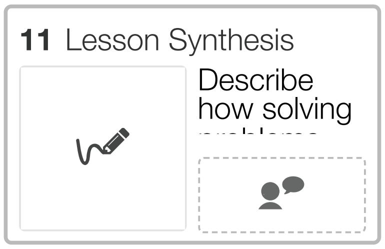
Facilitation

- Invite students who finish Screens 8–9 early to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion.

Sample Responses

Responses vary.

- 50 kilometers, 20%
- 100 kilometers, 10%



11 Lesson Synthesis

Describe how solving

Describe how solving problems with percentages is like solving problems with ratios.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface connections between solving problems with percentages and solving problems with ratios.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.

- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions
- *How are percentages and ratios similar? How are they different?*
- *When might a double number line be helpful for solving problems with percentages? What about a table? A tape diagram?*

Suggested Pacing: Screen 11

Sample Responses

Responses vary. A percentage is like a ratio but always with the number 100 somewhere. You can make a double number line for a ratio and also for percentages. Tables and tape diagrams can help with ratios and percentages—the whole tape diagram is another way to say 100%.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., Solving problems with percentages is like solving problems with ratios because ______.).

---

12 Cool-Down

Callen bought new sneakers for $\$60$.

Miko bought sneakers that cost 80% of that price.

How much did Miko pay for his sneakers?

Teacher Moves

Support for Future Learning
- Students will continue to develop their understanding of calculating unknowns involving percentages in Lessons 10 and 11.

Suggested Pacing: Screens 11–12

Sample Responses

$\$48$

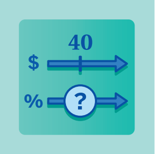
13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can make connections between percentages and ratios.
- I can use a double number line, tape diagram, or table to determine unknown parts or wholes.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



What's Missing? (NYC)

Lesson 10: Solving Percentage Problems

Purpose

This lesson is a continuation of students' work with percentages in Lessons 8 and 9. In this lesson, students create tape diagrams, double number lines, and tables to solve problems with percentages. The focus of this lesson is on determining which quantity is missing (the part, percentage, or whole), and then choosing a representation to make sense of the situation and answer the question. (MP5)

Note: This is the first lesson where students are asked to calculate an unknown percentage.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Cards

- Print and cut one set of cards (half sheet) for each student.

Materials

- Tape or glue (for attaching cards to the Student Worksheet)

Warm-Up (10 minutes)

Overview: In this warm-up, students engage in the Number Talk routine to strengthen strategies around multiplication involving fractions.

Launch

- Display Sheet 1 of the Teacher Projection Sheets.
- Invite students to share any strategies they remember from previous number talks in this unit.

Facilitation

- Use the instructional routine Number Talk to help students look for and make use of the structure of the expressions in order to develop and name strategies. (MP7)

How Number Talk Works

- Give students one minute to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own or one of the possible strategies in the sample responses.
- Select several students to share different strategies. Record strategies for all to see, along with the name of the student who shared each one.
- Display the next expression and repeat.

Math Community

- Consider inviting students to share if any of the strategies from the last number talk were helpful to them here and to name students whose ideas they found helpful.

Activity 1: What's Missing? (15 minutes)

Overview: Students sort cards that represent the same percentage situation, then create any missing representations or solutions. The purpose of this activity is for students to make connections between situations, representations, and solutions involving percentages.

Launch

- Invite students to work *in pairs.*
- Display Sheet 5 of the Teacher Projection Sheets.
- Read the two situations aloud. Consider inviting students to paraphrase each situation in their own words, or to act each situation out. Then give students one minute to think about how the situations are similar and different, and then to discuss with a partner and the class.

Note: These two situations appear in the second part of this activity and students will have an opportunity to answer each question then.

Facilitation

- Display Sheet 6 of the Teacher Projection Sheets. Consider modeling what it looks like to place a card on the worksheet.
- Distribute one set of cards to each student.
- Give students several minutes to decide which representations and solutions match each situation. Note: Students will need to write one of the situations. Consider inviting students to use color or arrows to make connections between the situation, representation, and solution.
- If students are struggling, invite them to select one of the double number lines and decide which situation it matches.
- Consider checking in with pairs in between steps 2 and 3, inviting pairs to check in with another pair, or posting an answer key.

Discussion Questions

- *How did you know which representation matched Question 1? 2? 3?*
- *How did you know whether you were looking for the percentage, sale price, or regular price?*
- *Where did you look on the double number line to figure out the regular price?*

Math Community

- Consider asking students why having more than one representation is helpful and why someone might prefer a double number line, table, or tape diagram.
- Consider highlighting unique or creative questions that students wrote.

Early Finishers

- Encourage students to begin Activity 2.

Support for Students With Disabilities

Receptive Language: Processing Time

Consider reading the prompt on Sheet 5 of the Teacher Projection Sheets aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Activity 2: Sale Price and Regular Price (10 minutes)

Overview: Students practice choosing and creating representations and solving problems with unknown parts, percentages, and wholes.

Launch

- Invite students to work *in pairs.*
- Consider sharing with students that in this activity, they will be thinking about the sale price and regular price of different items. (Note: In Activity 1, they thought about the amount saved.)

Facilitation

- Consider inviting students to make connections between these problems and the problems in Activity 1. It may be helpful for students to answer: *Which one of the questions from Activity 1 is this most like?*
- Monitor for students who use different representations to solve each problem. Invite students to compare which they chose and their solutions with their classmates. (MP5)
- As time allows, choose one problem to select and sequence several student responses. Invite these students to share their thinking for that problem. Support students in making connections between these responses.

Discussion Questions

- *How did you know whether you were looking for the percentage, sale price, or regular price?*
- *Where can you see the solution on the double number line? Where can you see it on the table?*

Math Community

- Consider sharing incomplete thinking and inviting the class to help finish it in order to highlight the value of draft thinking.

Routine (optional): Consider using the mathematical language routine Collect and Display.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

The purpose of this discussion is to surface how to use a double number line diagram to determine a missing whole.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their response based on the discussion.

Discussion Questions

- *How could we use this image to determine what percent Eliza had walked to school when she walked for 9 minutes?*
- *When is this strategy useful? When might this strategy not be useful?*

Cool-Down (5 minutes)

Support for Future Learning

If students struggle, consider reviewing the cool-down as a class before Practice Day 2 or offering individual support where needed during the practice day. This is the last lesson that focuses on calculating a whole given a part and a percentage.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Cost Breakdown

Lesson 11: Any Percentage of a Number

Overview

Students develop and use strategies for calculating any percentage of a number.

Learning Goals

- Calculate any percentage of a number (e.g., 32% of 40).
- Explain why $A\%$ of B can be calculated as $\frac{A}{100} \cdot B$ or $\frac{B}{100} \cdot A$.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop and use strategies for calculating any percentage of a number. Students investigate the breakdown of money spent on clothing and calculate the amount of money that goes toward expenses, profit, and inventory for various items. As they make repeated calculations, students look for and express regularity in their work (MP8). They will make connections between strategies for calculating unknown parts and expressions such as $\frac{A}{100} \cdot B$ and $\frac{B}{100} \cdot A$.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to reason about non-friendly percentages (ones that are not multiples of benchmark percentages). This is the first time that students will see percentages like 32% in this unit. Students complete a series of repeated challenges in which they estimate an unknown percentage shown on a tape diagram.

Activity 1: One Strategy (10 minutes)

The purpose of this activity is for students to learn one strategy for calculating any percentage of a number: calculating the amount for 1% and then multiplying by the percentage ($\frac{A}{100} \cdot B$). Students explore the breakdown of where the money goes when two store owners Ada and Bao sell a product, then calculate this breakdown for a T-shirt and a pair of shoes.

Activity 2: A Second Strategy (20 minutes)

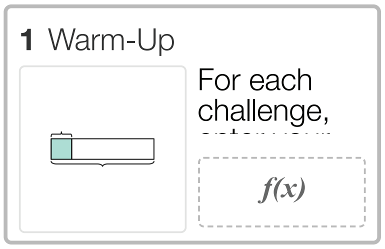
The purpose of this activity is for students to learn a second strategy for calculating any percentage of a number: thinking of the percentage as that many cents per dollar, then multiplying by the whole ($\frac{B}{100} \cdot A$).

This activity ends with a set of repeated challenges where students can use any strategy to calculate an unknown part given a percentage and a whole.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to surface strategies for calculating any percentage of a number.

Cool-Down (5 minutes)


1 Warm-Up

For each challenge,

For each challenge, enter your best estimate of the missing percent.

Complete as many challenges as you want!

Teacher Moves

Overview: In this lesson, students develop and use strategies for calculating any percentage of a number. In this warm-up, students reason about non-friendly percentages (e.g., 32%). This is the first time that students will see non-friendly percentages.

Launch
- Consider displaying the dashboard's teacher view and doing the first estimation together as a class.

Facilitation
- Give students 2–3 minutes to complete as many challenges as they can.
- Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

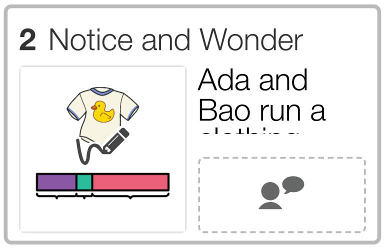
Math Community
- Consider celebrating students who persisted through struggle by inviting them to share what helped them persist and what strategies they found helpful.

Readiness Check (Problems 6 and 7)
- If most students struggle with Problem 6, consider reviewing it either before or after the warm-up. Invite students to make connections between "of" language and multiplication. Consider making an anchor chart for students to refer back to for the rest of the unit.
- If students struggle with Problem 7, consider reviewing it either before or after the warm-up. Invite students to make connections between multiplying by $\frac{1}{100}$ and dividing by 100.

Suggested Pacing: Screen 1

Sample Responses

This screen contains an unlimited number of randomized challenges. The challenges progress in difficulty.



2 Notice and Wonder

Ada and Bao run a

Ada and Bao run a clothing store.

The diagram shows where the money goes when Ada and Bao sell a T-shirt.

What do you notice? What do you wonder?

Teacher Moves

Overview: In Activity 1 (Screens 2–6), students learn one strategy for calculating any percentage of a number: calculating the amount for 1% and then multiplying by the percentage ($\frac{A}{100} \cdot B$).

Launch

- Invite students to work *in pairs.*
- Consider inviting students into the situation by asking: *Have you ever wondered where your money goes when you buy something? Do the store owners get to keep all of it? Where else does it go?*

Facilitation

- Give students 1–2 minutes to think independently and then share their responses with a partner.
- If it doesn't come up naturally, discuss each category and what it includes. Cost to run the store includes rent, salary for employees, furniture for the store, electricity, etc. Ada and Bao's profit includes the amount of money that Ada and Bao get to take home. Inventory cost includes the cost to buy the item for the store, which includes the cost of the material, the labor to make the shirt, shipping it to the store, etc.

Discussion Questions

- *What do you think the cost to run the store includes? Inventory cost?*
- *What do these percentages add up to? Why does that make sense?*

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Suggested Pacing: Screen 2

Sample Responses

Responses vary.

- I notice that the profit is the smallest percentage.
- I notice that they all add up to 100%.
- I notice that the inventory cost is the biggest percentage.

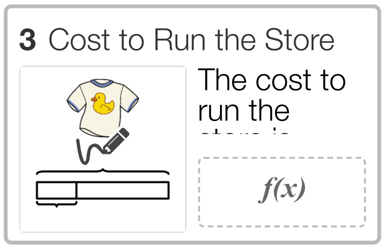
- I wonder how much of the cost to run the store goes to paying the people who work at the store.
- I wonder if Ada and Bao share the profit equally.

Student Supports

Multilingual Learners

- *Receptive/Expressive Language: Strategic Pairing*

Consider defining each of the words in the tape diagram as a class. Consider specifically addressing words that have multiple meanings, like “run."



3 Cost to Run the Store

The cost to run the

The cost to run the store is 30% of the total cost.

How much of a $\$40$ shirt goes to running the store?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Launch
- Consider sharing that in this lesson, we are going to calculate how much money goes to each of the categories from the previous screen based on the cost of the item.

Progress Check
- To support students getting started, consider asking: *What would the cost to run the store be if they were* 10% *of the total cost of the shirt? What representations have helped us make sense of situations with percentages?*
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 3–6

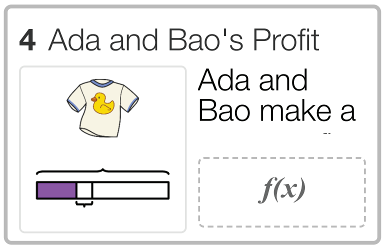
Sample Responses

$\$12$

Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

To support students in completing the problems in this activity, consider providing blank copies of double number lines or tables.



4 Ada and Bao's Profit

Ada and Bao make a

Ada and Bao make a 12% profit of the total cost.

What is their profit on a $\$40$ shirt?

Teacher Moves

Facilitation
- To support students getting started, consider asking: *What is a number that you know is too low? Too high?*

- Consider monitoring for students who use strategies similar to Bao's on the next screen or other creative strategies. Invite these students to share their thinking during the discussion on the next screen.
- **Note:** On the next screen, students will be presented with one strategy for solving problems like these.

Sample Responses

$\$4.80$

Student Supports

Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

To assist students in recognizing the connections between new problems and prior work, consider asking: *How much would Ada and Bao make if their profit was* 1%*?*

5 Bao's Strategy

Here is how Bao

Here is how Bao calculated 12% of $\$40$.

Explain how you could use Bao's strategy to calculate the inventory cost (58% of $\$40$).

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to make sense of one strategy for calculating any percentage of a number.

Facilitation

- Select and sequence several student sketches and responses using the snapshot tool.
- When most students have responded, invite several students to explain Bao’s strategy and how they would use it to calculate 58% of 40.
- Consider writing the expression for 12% of 40 on the board, then following it with the expression for 58% of 40. Invite students to explain what each part of the expression means.

Discussion Questions

- *Why do you think Bao divided by* 100 *as a first step? Why did he then multiply by* 12*?*
- *What does the* $\frac{40}{100}$ *mean in the situation?*

- *Which parts of the expression will be the same for calculating* 58% *of* 40*? Which parts will be different?*

Math Community

- Consider naming powerful explanations you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary. You would still start by dividing by 100 and get $\frac{40}{100}$ but then you would multiply by 58 instead of 12. The expression would be $\frac{40}{100} \cdot 58$.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

6 Ada and Bao's Profit

Here is a $\$75$ pair of

Here is a $\$75$ pair of shoes. Calculate each value.

Teacher Moves

Facilitation

- After the discussion on the previous screen, complete the first category together using Bao's strategy. Demonstrate that they can enter expressions like $\frac{75}{100} \cdot 30$ and the computer will calculate the total for them.
- Give students several minutes to calculate each unknown value.

Early Finishers

- Encourage students to think of an item they would be interested in selling and how much profit they would want to make from that item. Invite them to calculate how much the total cost of the item would need to be in order to earn that profit.

Sample Responses

Cost to Run the Store: $\$22.50$

Ada and Bao's Profit: $\$9$

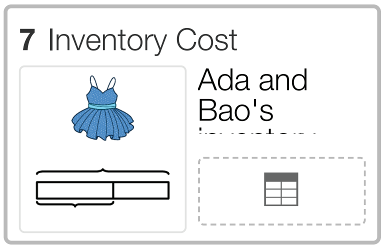
Inventory Cost: $\$43.50$

Student Supports

Students With Disabilities

- *Executive Functioning: Visual Aids*

Consider using the routine Collect and Display to capture students' expressions for the class to refer back to throughout the lesson.



7 Inventory Cost

Ada and Bao's

Ada and Bao's inventory costs are 58% of the total cost.

What is their inventory cost for each item in the table?

Teacher Moves

Overview: In Activity 2 (Screens 7–10), students learn a second strategy for calculating any percentage of a number: thinking of the percentage as that many cents per dollar, then multiplying by the whole ($\frac{B}{100} \cdot A$).

Launch

- Consider sharing with students that we are going to focus just on inventory cost for several different items. Invite students to use any patterns they notice to help them.

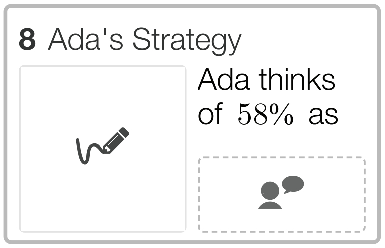
Facilitation

- Consider highlighting the fact that the first inventory cost is $\$58$ because 58% means 58 for every 100.
- Monitor for students who recognize that you can use the cost of the sticker to help determine the cost of the jeans and the hat.
- To support students getting started on the jeans, consider asking: *What if the item were* $\$2$*?* $\$5$*?*

Suggested Pacing: Screens 7–10

Sample Responses

Dress: $\$58$
Sticker: $\$0.58$
Jeans: $\$37.12$
Hat: $\$15.66$



8 Ada's Strategy

Ada thinks of 58% as

Ada thinks of 58% as 58 cents for every dollar, so he writes $\frac{58}{100} \cdot 64$ to calculate 58% of $\$64$.

Describe how Ada might calculate 36% of $\$15$.

Teacher Moves

Key Discussion Screen
- The purpose of this discussion is to make sense of a second strategy for calculating any percentage of a number.

Facilitation
- Select and sequence several student sketches and responses using the snapshot tool.
- When most students have responded, invite several students to explain Ada's strategy and how they would use it to calculate 36% of 15.
- Consider writing the expression for 58% of 64 on the board, then following it with the expression for 36% of 15. Invite students to explain what each part of the expression means.

Discussion Questions
- *Why do you think Ada divided by* 100 *as a first step? Why did he then multiply by* 64*?*
- *What does the* $\frac{58}{100}$ *mean in the situation?*
- *Which parts of the expression will be the same for calculating* 36% *of* 15*? Which parts will be different?*

Math Community
- Consider asking the class why having more than one strategy might be useful.

Sample Responses

Responses vary. You would still start by dividing by 100 and get $\frac{36}{100}$ but then you would multiply by 15 instead of 64. The expression would be $\frac{36}{100} \cdot 15$.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

---

9 Card Sort

Teacher Moves

Facilitation

- Consider inviting students to return to Ada's and Bao’s strategies if they are stuck.
- Encourage students to share their reasoning with a partner and work to reach an agreement together about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Early Student Thinking

- Students may sort $\frac{100}{36} \cdot 48$ with “What is 36% of $48?$”
- Consider asking: *What is the value of* $\frac{100}{36} \cdot 48$*? Does it have the same value as* $\frac{36}{100} \cdot 48$*?*

Sample Responses

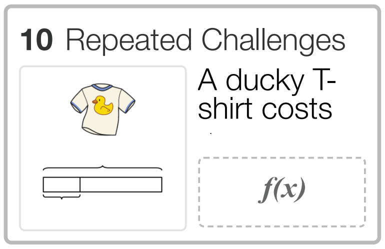
Image solution

Student Supports

Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts by inviting students to choose one question card at a time and ask: *How might Ada solve this problem? What about Bao?*



10 Repeated Challenges

A ducky T-shirt costs

A ducky T-shirt costs $\$22$.

The material cost is 16% of the total cost.

Calculate the material cost. [Challenges vary.]

Teacher Moves

How Repeated Challenges Work

- Students are presented with a variety of challenges one at a time—in this case, calculating an unknown partial cost of an item.
- The challenges typically increase in difficulty as they continue.

Facilitation

- Give students 5–7 minutes to complete as many challenges as they can.
- Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

Math Community

- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

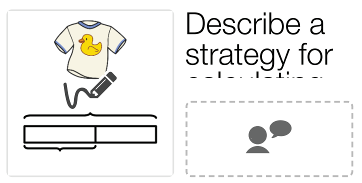
Sample Responses

This screen contains an unlimited number of randomized challenges. The challenges progress in difficulty.

The first few responses are:

- $16\% \text{ of } \$22 = \$3.52$
- $9\% \text{ of } \$30 = \$2.70$
- $8\% \text{ of } \$48 = \$3.84$

11 Lesson Synthesis



Describe a strategy for

Describe a strategy for calculating a percentage of a number.

Use the example if it helps you explain your thinking.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface strategies for calculating any percentage of a number.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Suggested Pacing: Screen 11

Sample Responses

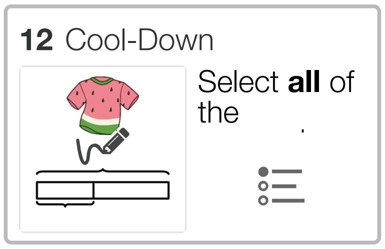
Responses vary. One strategy is to figure out how much 1% would be, then multiply that number by the percent that you actually have. Another strategy is to think about the percent as a fraction out of 100 and then multiply by the total number you have.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., One strategy for calculating a percent of a number is to ______.).



12 Cool-Down

Select **all** of the .

Select **all** of the expressions that can be used to calculate 43% of $\$26$.

Teacher Moves

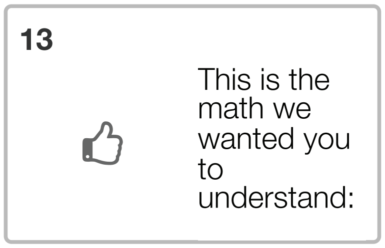
Support for Future Learning

- If students struggle, consider reviewing this screen as a class before Lesson 13 or offering individual support where needed during Lesson 13 and Practice Day 2.

Suggested Pacing: Screens 12–13

Sample Responses

- $\frac{26}{100} \cdot 43$
- $\frac{43}{100} \cdot 26$



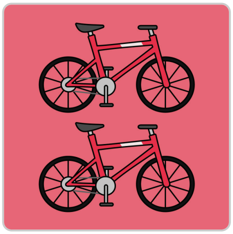
13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can calculate any percentage of a number.
- I can explain two different expressions for calculating a percentage of a number.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



More Bicycle Goals

Lesson 12: Unknown Percentages

Overview

Students develop and use one or more strategies to calculate unknown percentages.

Note: Includes an optional double number line and table template for students to use throughout the lesson.

Learning Goals

- Calculate an unknown percentage (e.g., 32 is what $\%$ of 40).
- Explain why an unknown percentage can be calculated as $\frac{100}{B} \cdot C$ or $\frac{C}{B} \cdot 100$.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop and use one or more strategies to calculate any unknown percentage. This lesson is similar to Lesson 11 in that students make connections between strategies and expressions. In this lesson, students connect strategies for calculating unknown percentages and expressions such as $\frac{100}{B} \cdot C$ and $\frac{C}{B} \cdot 100$.

Lesson Summary

Warm-Up (10 minutes)

In this warm-up, students engage in the Number Talk routine to surface strategies for multiplying fractions by fractions. This the last in a series of warm-ups to strengthen strategies around multiplication of fractions.

Activity 1: One Strategy (15 minutes)

The purpose of this activity is for students to reason about a strategy for calculating any unknown percentage: calculating what percent 1 would be, and then multiplying to determine the percent ($\frac{100}{B} \cdot C$). Students return to the context of bicycle goals from Lesson 9.

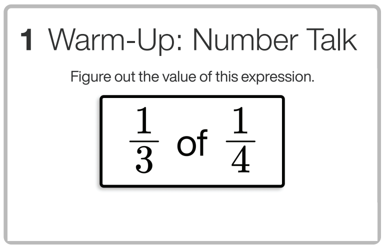
Activity 2: Practice With Calculating Unknown Percentages (10 minutes)

The purpose of this activity is for students to continue to make sense of strategies for calculating an unknown percentage. Students are exposed to the expression $\frac{C}{B} \cdot 100$ as they reason about Alejandro's, Basheera's, and Callen’s goals for themselves.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to make connections between an expression used to calculate an unknown percentage and a situation in context.

Cool-Down (5 minutes)



1 Warm-Up: Number Talk

Figure out the value of this expression.

$\frac{1}{3}$ of $\frac{1}{4}$

Teacher Moves

Overview: In this lesson, students develop and use one or more strategies for calculating unknown percentages. In this warm-up, students engage in the Number Talk routine to surface strategies for multiplying fractions by fractions. This the last in a series of warm-ups to strengthen strategies around multiplication of fractions.

Launch

- Use the dashboard's student view to display Screen 1.
- Invite students to share any strategies they remember from previous number talks in this unit.

How Number Talk Works

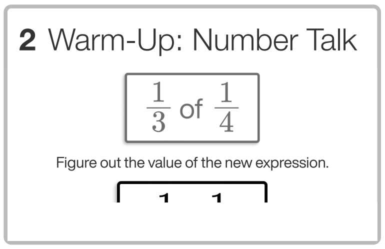
- Give students one minute to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own or one of the possible strategies in the sample responses.
- Select several students to share different strategies. Use the sample responses as examples of possible student strategies. Record strategies for all to see, along with the name of the student who shared each one.
- Display the next expression and repeat. If it makes sense, encourage students to use a strategy described by a classmate in the previous round.

Suggested Pacing: Screens 1–4, one screen at a time

Sample Responses

$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$

Strategies students might use in the Number Talk



2 Warm-Up: Number Talk

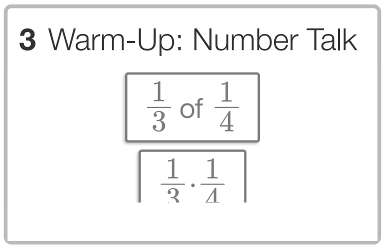
$\frac{1}{3}$ of $\frac{1}{4}$

Figure out the value of the new expression.

Sample Responses

$$\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

Strategies students might use in the Number Talk



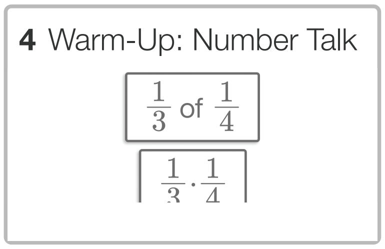
3 Warm-Up: Number Talk

$\frac{1}{3}$ of $\frac{1}{4}$

Sample Responses

$$\frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12}$$

Strategies students might use in the Number Talk



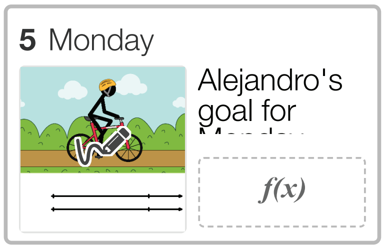
4 Warm-Up: Number Talk

$\frac{1}{3}$ of $\frac{1}{4}$

Sample Responses

$$\frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12}$$

Strategies students might use in the Number Talk



5 Monday

Alejandro's goal for

Alejandro's goal for Monday was to ride 20 kilometers.

His app says he rode 40% of his goal.

How far did he ride?

Teacher Moves

Overview: In Activity 1 (Screens 5–9), students reason about a strategy for calculating any unknown percentage: calculating what percent 1 would be, and then multiplying to determine the percentage ($\frac{100}{B} \cdot C$).

Students return to the context of bicycle goals from Lesson 9.

Launch

- Invite students to work *in pairs.*
- Invite students to share what they remember about bicycle goals from the previous lesson.

Progress Check

- Encourage students to use the sketch tool and/or paper to show their thinking.
- Students may make a double number line or table, like in Lessons 9 and 10, or use one of the strategies from Lesson 11 to support them in their thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Math Community

- Consider celebrating all of the different strategies students use to approach this problem.

Suggested Pacing: Screens 5–9

Sample Responses

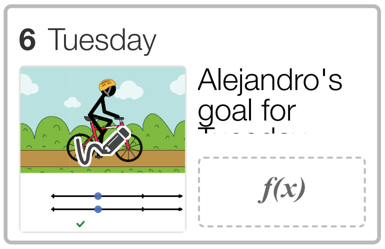
8 kilometers

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the double number line and table template to support students with organizing their thinking and making connections throughout this lesson.



6 Tuesday

Alejandro's goal for

Alejandro's goal for Tuesday was to ride 20 kilometers.

His app says he rode 10 kilometers.

What percent of his goal did he ride?

Teacher Moves

Facilitation

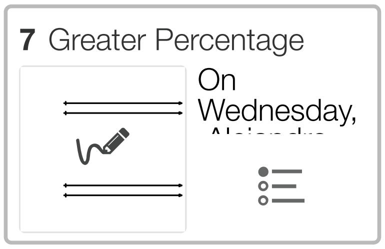
- To support students getting started, consider asking: *What fraction of Alejandro's goal did he ride?*

Early Student Thinking

- Students might calculate 10% of 20 and write 2.
- Consider sharing with students the question that they *did* answer and ask: *Am I trying to figure out a percent or a distance?*

Sample Responses

50%



7 Greater Percentage

On Wednesday,

On Wednesday, Alejandro and Basheera rode 17 kilometers.

Alejandro's goal was 20 kilometers.
Basheera's goal was 50 kilometers.

Who rode a greater percentage of their goal?

Teacher Moves

Facilitation

- If students are wondering if they need to calculate Alejandro's and Basheera's percentages, consider asking: *Do you need to know the exact percentage to decide whose was greater?* Encourage students to use reasoning to help them answer the question.
- If students are wondering how the double number line is helpful, consider asking: *Where would you put the goal on each double number line? What percent would it be? Where would the* 17 *go on each line?*
- Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Invite several students to share their reasoning.

Discussion Questions

- *How did you decide who rode a greater percentage of their goal?*

Math Community

- Invite students to think about why someone might have chosen differently.

Sample Responses

Alejandro

Explanations vary.

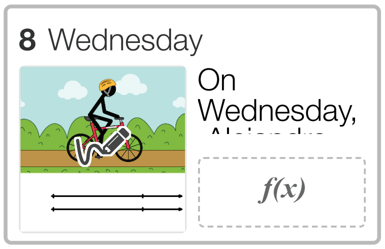
- Alejandro had a smaller goal, so even though Alejandro and Basheera biked the same distance, Alejandro biked a bigger portion of his goal.
- Alejandro biked over 50% of his goal and Basheera biked less than 50% of her goal.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.


8 Wednesday

On Wednesday,

On Wednesday, Alejandro and Basheera rode 17 kilometers.

Alejandro's goal was 20 kilometers.

What percent of his goal did he ride?

Teacher Moves

Facilitation

- To support students getting started, consider asking: *What is a number that you know is too low? Too high?*
- **Note:** On the next screen, students will be presented with one strategy for solving problems like these. It is okay if students use reasoning or guess-and-check strategies here.
- Consider monitoring for students who use strategies similar to Alejandro's on the next screen or other creative strategies. Invite these students to share their thinking during the discussion on the next screen.

Sample Responses

85%


9 Alejandro's Strategy

Here is how Alejandro

Here is how Alejandro calculated 17 out of 20 as a percentage.

Explain how you could use Alejandro's strategy to calculate 13 out of 20 as a percentage.

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to make sense of one strategy for calculating any unknown percentage.

Facilitation

- Select and sequence several student sketches and responses using the snapshot tool.
- When most students have responded, invite several students to explain Alejandro’s strategy and how they would use it to calculate 13 out of 20 as a percentage.

- Consider writing the expression for 17 out of 20 on the board, then following it with the expression for 13 out of 20. Invite students to explain what each part of the expression means.

Discussion Questions

- *Why do you think Alejandro multiplied by* $\frac{1}{20}$ *as a first step? Why did he then multiply by* 17*?*
- *What does the* $\frac{100}{20}$ *mean in the situation?*
- *Which parts of the expression will be the same for calculating* 13 *out of* 20*? Which parts will be different?*

Math Community

- Consider naming powerful explanations you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Early Finishers

- Encourage students to calculate 13 out of 20 as a percentage in one or two other ways.

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Sample Responses

Responses vary. You would still start by dividing 100 by 20 to get the percentage of the goal for 1 kilometer. Since you want to know the percentage of the goal for 13 kilometers, you would multiply by 13 instead of 17. The expression would be $\frac{100}{20} \cdot 13$.

Student Supports

Students With Disabilities

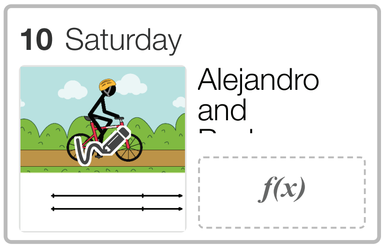
- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners

- *Routine: Collect and Display*

Circulate and listen to students talk as they describe how to calculate 13 out of 20 as a percentage. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.


10 Saturday

Alejandro and

Alejandro and Basheera set a new goal for Saturday: 30 kilometers.

They rode 36 kilometers.

What percent of their goal did they ride?

Teacher Moves

Overview: In Activity 2 (Screens 10–14), students continue to make sense of strategies for calculating an unknown percentage.

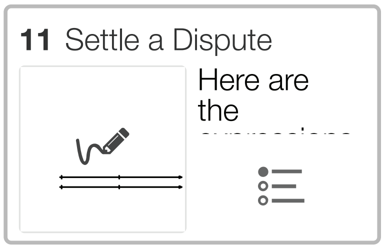
Launch
- Read the prompt as a class. Consider inviting students to share how this situation is similar to and different from the previous one.
- Consider reminding students that they can enter expressions like $2+2$ and the computer will calculate the total for them.

Progress Check
- To support students getting started, consider asking: *Do you think they biked less or more than* 100% *of their goal? How do you know?*
- **Note:** This is the first place that students calculate a percentage over 100%.

Suggested Pacing: Screens 10–14

Sample Responses

120%


11 Settle a Dispute

Here are the

Here are the expressions Alejandro and Basheera used to calculate 36 out of 30 as a percentage.

Who wrote a correct expression?

Teacher Moves

Facilitation
- Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or

consensus you see.
- If it does not come up naturally, consider inviting students to use the built-in calculator to determine the value of each expression.
- Spend adequate time to ensure students understand that both expressions are correct and have at least one reason why.

Discussion Questions
- *How are Alejandro's and Basheera's expressions similar? How are they different?*
- *Which expression do you prefer? Why?*
- *How could you convince someone that the expression* $\frac{30}{36} \cdot 100$ *is incorrect?*

Early Student Thinking
- Students may not recognize that Basheera's expression is correct because it does not follow the same strategy of calculating what percent 1 kilometer is. Invite these students to calculate the value of Basheera's expression or to test Basheera's expression on the previous screen.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Both

Explanations vary. Even though their expressions look different, they have the same value (120). Both expressions multiply 36 and 100 and divide by 30.

Student Supports

Multilingual Learners
- *Receptive/Expressive Language: Strategic Pairing*

Pair students to aid them in comprehension and expression of understanding.

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., Alejandro's/Basheera's expression is correct because ________.).


12 Three Friends, Thre…

On Sunday, Alejandro,

On Sunday, Alejandro, Basheera, and Callen rode 40 kilometers.

They each had different goals, as shown in the table.

Calculate each person's percentage of their goal.

Teacher Moves

Facilitation

- To support students getting started, invite them to mark where 40 kilometers is on the three number lines.

Progress Check

- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling. Students may need support connecting each row of the table to each rider.
- If time allows, consider inviting students to share the strategies or expressions they used to calculate each unknown percentage.

Sample Responses

- 80%
- 160%
- 62.5%

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13 Card Sort

Teacher Moves

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Sample Responses

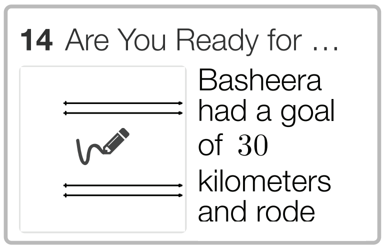
Image solution

Student Supports

Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts by inviting students to choose one question card at a time and decide what number represents the whole.



14 Are You Ready for …

Basheera had a goal of 30 kilometers and rode

Basheera had a goal of 30 kilometers and rode 51 kilometers.

Callen had a goal of 20 kilometers and rode 41 kilometers.

On paper, determine who rode a greater percentage of their goal in as many different ways as you can.

Teacher Moves

Facilitation

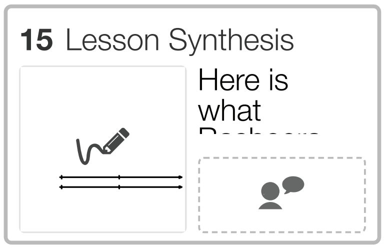
- Invite students who finish Screens 10–13 early to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion.

Sample Responses

Callen

Explanations vary.

- Callen rode more than twice their goal. Basheera rode less than twice her goal.
- Callen rode $\frac{41}{20} \cdot 100 = 205\%$ of their goal. Basheera rode $\frac{51}{30} \cdot 100 = 170\%$ of her goal.



15 Lesson Synthesis

Here is what

Here is what Basheera wrote to solve a bicycle challenge.

What do 31, 25, and 124 represent in this scenario?

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to make connections between an expression used to calculate an unknown percentage and a situation in context.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their response with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

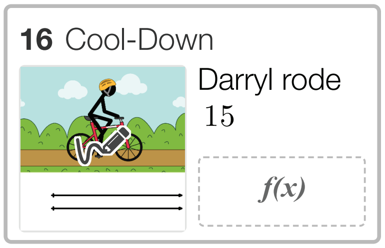
- *Why do you think it is important to know what each number represents?*
- *What is important to remember about calculating unknown percentages?*

Suggested Pacing: Screen 15

Sample Responses

Responses vary.

- 31 represents the distance that Basheera biked.
- 25 represents the distance of Basheera's goal.
- 124 represents the percentage of Basheera's goal that she biked.



16 Cool-Down

Darryl rode 15

Darryl rode 15 kilometers of his 12-kilometer goal.

What percent of his goal did he ride?

Teacher Moves

Support for Future Learning

- If students struggle, consider reviewing this screen as a class before Lesson 13 or offering individual support where needed as students are making their posters. This is the last lesson that focuses explicitly on calculating any unknown percentage.

Suggested Pacing: Screens 16–17

Sample Responses

125%

17

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can calculate an unknown percentage.
- I can explain different expressions for calculating an unknown percentage.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

A Country as a Village

Lesson 13: Applying Ratios, Rates, and Percentages

Overview

Students use what they've learned about rates and percentages to analyze characteristics of countries' populations.

Learning Goals

- Use rates and percentages to analyze characteristics of a country's population.

Materials

- Tools for creating a visual display

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to use what they've learned about rates and percentages to analyze characteristics of countries' populations. Students imagine countries as villages of 100 people and consider how many people in that village would have each of several different characteristics, including but not limited to the number of people who own a car or the number of people who are under 15 years old.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to engage with the types of facts they will be analyzing throughout the lesson and consider the different ways in which facts are presented.

Activity 1: Brazil as a Village (10 minutes)

The purpose of this activity is for students to engage in the type of thinking they will use to make a poster in Activity 2. Students use facts about Brazil to determine how many people and what percentage of people in Brazil have certain characteristics. Percentages are represented by the number of people in a 100-person village with that characteristic.

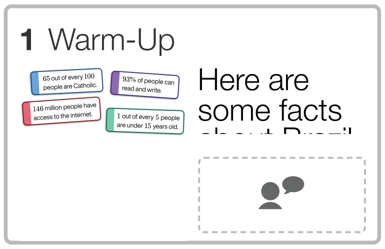
Activity 2: Make a Poster (20 minutes)

The purpose of this activity is for students to apply their learning from Units 2 and 3 to imagine a country as a village of 100 people. Students calculate unknown percentages and use ratio thinking to determine how many people have each of the characteristics in a country of students' choosing.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consider how working with percentages is like working with a village of 100 people.

Cool-Down (5 minutes)



1 Warm-Up

65 out of every 100 people are Catholic.

93% of people can read and write.

146 million people have access to the internet.

1 out of every 5 people are under 15 years old.

Here are some facts

Here are some facts about Brazil.

List at least two things you find surprising about these facts.

Teacher Moves

Overview: In this lesson, students use what they've learned about rates and percentages to analyze characteristics of countries' populations. In this warm-up, students engage with the types of facts they will be analyzing throughout the lesson and consider the different ways in which facts are presented.

Launch
- Read each of the facts aloud as a class. Invite students to paraphrase each fact to ensure they understand what each fact means.
- Consider inviting students to share what they know about Brazil: *Where is it on a map? What else do you know about Brazil?*

Facilitation
- Use the teacher dashboard to highlight several student responses.

Discussion Questions
- *What did you find surprising?*
- *How do you think this is similar to or different from where we live?*

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

- I was surprised that Brazil has 146 million people.
- 7% of people in Brazil cannot read and write.
- More than half of people in Brazil are Catholic.

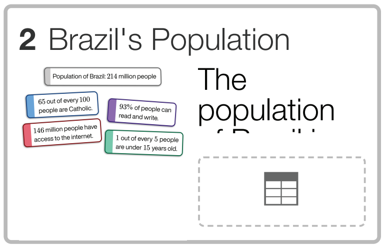
Student Supports

Multilingual Learners
- *Receptive/Expressive Language: Strategic Pairing*

Consider reviewing each fact as a class and clarifying any unknown words. Consider spending extra time on phrases such as "access to the internet."

- *Routine: Co-Craft Questions and Problems*

Before answering the questions in this activity, invite students to write possible mathematical questions about the activity. In pairs, students can compare their questions. Then invite pairs to share their questions with the class.



2 Brazil's Population

Population of Brazil: 214 million people

65 out of every 100 people are Catholic.

93% of people can read and write.

146 million people have access to the internet.

1 out of every 5 people are under 15 years old.

The population

The population of Brazil is about 214 million people.

How many people in Brazil have each of these characteristics?

Teacher Moves

Overview: In Activity 1 (Screens 2–3), students engage in the type of thinking they will use to make a poster in Activity 2.

Launch

- Invite students to work *in groups of 2–3.*
- Read the task aloud. Consider sharing that these are the same facts as the warm-up.
- Consider asking: *What strategies can help us figure out an unknown amount of people?*

Facilitation

- Encourage students to use paper to help them with their thinking throughout the lesson.
- Give students 3–5 minutes to determine the number of people in Brazil with each characteristic.
- After 2–3 minutes, consider reviewing how students calculated the number of people who were Catholic. All students should have a strategy for moving between number of people and percentages in order to support them in Activity 2.
- After students show that they understand how to calculate the total given a ratio or percentage, invite them to continue to Screen 3.

Discussion Questions

- *How did you determine the number of people who are Catholic in Brazil?*
- *How might a table be helpful? A double number line?*

Early Student Thinking

- Students may be stuck on the number of people who have internet access because there are no calculations needed to arrive at the answer. Consider asking: *What information are you trying to figure out? What do you know?*
- If students enter the full number instead of the number in millions (e.g., $146\,000\,000$ vs. 146), remind them to consider the units for the column in the table (MP6).

Suggested Pacing: Screens 2–3

Sample Responses

- **Catholic:** 139.1 million
- **Can read and write:** 199.02 million
- **Have internet access:** 146 million
- **Under 15 years old:** 42.8 million

Student Supports

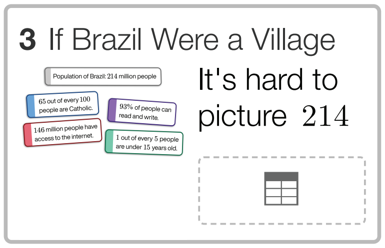
Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

To support organization in problem solving, consider chunking this activity by inviting students to select one fact at a time and then calculate the number of people in Brazil with that characteristic.

- *Conceptual Processing: Eliminate Barriers*

Students may benefit from a review of different representations from the unit to activate prior knowledge.



3 If Brazil Were a Village

It's hard to picture 214

It's hard to picture 214 million people.

Imagine Brazil were a village with just 100 people. How many people would have each of these characteristics?

Teacher Moves

Launch

- Consider clarifying the prompt as a class. Ask: *What would it mean for a country to be a village of* 100 *people?*

Facilitation

- When most students have determined the number of people with internet access, invite them to share their strategies.
- If it does not come up naturally, discuss what to do when you calculate a number of people that is not a whole number (MP6). Consider using the snapshot tool to capture students who wrote 68, 68.22, and 69 people.

Discussion Questions

- *Which of these facts was easier to figure out? More challenging? Why do you think so?*
- *What does the answer* 68.22 *mean in the third row? Should we round that? If so, how?*

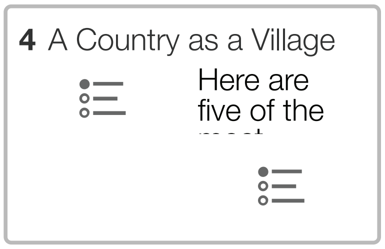
Early Finishers

- Invite students to support their classmates. A strong understanding of this task will support students in making their own posters in Activity 2.

Math Community
- Consider revisiting the names of students whose ideas throughout the unit were helpful in solving this task.

Sample Responses

- **Catholic:** 65 people
- **Can read and write:** 93 people
- **Have internet access:** 68 people
- **Under** 15 **years old:** 20 people



4 A Country as a Village

Here are five of the

Here are five of the most populated countries in the world.

Pick a country. Make a poster describing what this country would look like as a 100-person village.

Be sure to include these items in your poster:

Teacher Moves

Overview: In Activity 2 (Screen 4), students use everything they have learned in Units 2 and 3 to imagine a country as a village of 100 people.

Launch
- Invite students to work *in groups of 2–3.*
- Share with students that they will select a country and make a poster about that country as a village. The poster should clearly show their reasoning and each of the items on the checklist.

Facilitation
- Give students one minute to explore this screen and read each of the facts.
- Invite students to select a country or assign countries at random.
- Give students 10–15 minutes to answer the questions and create a poster.
- After 7–10 minutes, encourage students who selected different countries to compare their draft thinking and give each other feedback before returning to finish their posters.
- If time allows, encourage students to do a gallery walk to see how other groups created their posters and how the countries compare.

Discussion Questions

- *What strategies were most helpful in determining each characteristic?*
- *How are your countries similar? How are they different?*

Early Finishers

- Encourage students to either research other facts about the country they chose, or to research these facts about a country meaningful to them.

Math Community

- Encourage students to share what they found helpful or informative about each other's posters.

Routine (optional): Consider using the mathematical language routine Compare and Connect.

Sample Responses

Responses vary.

China: 20 people own a car, 18 people are under 15 years old, there are 6 cats, 70 people have access to the internet, 52 people do not identify with any particular religion.

India: 7 people own a car, 29 people are under 15 years old, there are 17 cats, 45 people have access to the internet, 80 people practice Hinduism.

USA: 82 people own a car, 20 people are under 15 years old, there are 33 cats, 85 people have access to the internet, 78 people practice Christianity.

Indonesia: 8 people own a car, 27 people are under 15 years old, there are 50 cats, 73 people have access to the internet, 87 people practice Islam.

Nigeria: 6 people own a car, 41 people are under 15 years old, there are 3 cats, 50 people have access to the internet, 54 people are Muslim.

Student Supports

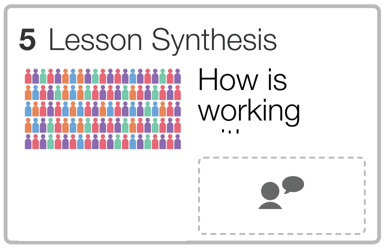
Students With Disabilities

- *Receptive Language: Processing Time*

Read statements and problems aloud for students who benefit from extra processing time.

- *Conceptual Processing: Processing Time*

To check for understanding, consider giving students time to discuss with a partner all of the things that should be included on the poster.



5 Lesson Synthesis

How is working ...

How is working with percentages like working with a village of 100 people?

Teacher Moves

Key Discussion Screen 🔑

- The purpose of this discussion is to surface how working with percentages is like working with a village of 100 people.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions

- *What did you find new or surprising about the country you chose?*
- *What do you think is important to remember about this lesson?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Suggested Pacing: Screen 5

Sample Responses

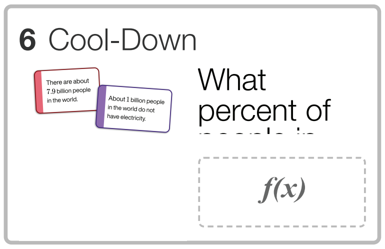
Responses vary. Percentages are a number out of 100, so a percentage is kind of the same as how many people out of a village of 100. For example, if 62 out of 100 people wear glasses, then 62% of people wear glasses.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., Working with percentages is like working with a village of 100 people because ______.).



6 Cool-Down

There are about 7.9 billion people in the world.

About 1 billion people in the world do not have electricity.

What percent of

What percent of people in the world do not have electricity?

Teacher Moves

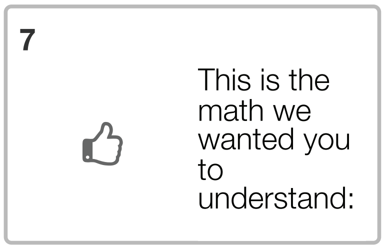
Support for Future Learning

- If some students struggle, consider offering individual support where needed during Practice Day 2. Students will not be directly assessed on situations involving a village of 100 people but should know how to calculate an unknown percentage.

Suggested Pacing: Screens 6–7

Sample Responses

- About 12.7%
- 12 or 13 people



7

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use rates and percentages to analyze characteristics of a country's population.

6.3 Practice Day 2 (NYC)

Preparation

Student Worksheet

- Print one double-sided sheet for each student.

Task Cards

- *Option 1 (Level Up)*: Print one single-sided set of task cards for each group of 2–3 students. Make a pile for each task card in a central location for students to drop off and pick up.
- *Option 2 (Stations)*: Print two single-sided sets of task cards for the entire class (8 cards total).

Note: There are two tasks on Page 1. Cut the page in half to separate the two tasks.

Instructions

Option 1: Level Up

- Arrange students into *groups of 2–3.*
- Distribute one Student Workspace Sheet to each student.
- Share with students that there are four tasks.
- Distribute one copy of “Running Teams” to each group.

- Once a group completes “Running Teams,” review their thinking. Consider choosing one student at random in the group to share the group’s ideas.
- Once you have reviewed their work on “Running Teams,” invite them to pick up a copy of “Lap Predictions” to complete together.

- Continue this process until students have completed all four tasks. Invite students who finish early to try the “Are You Ready for More?”

Option 2: Stations

- Arrange students into *groups of 2–4.*
- Distribute one Student Workspace Sheet to each student.
- Place one task card at each station. It may be helpful to have two copies of the same task card at one station.
- Invite students to work together as they solve the problems on each task card.

Options for student movement:

- As students finish a station, instruct them to move on to a new station.
- After a set amount of time, instruct students to move as a group from station to station.
- After a set amount of time, instruct students to move to a new station such that no one from their previous group is in their new group.

GRADE 6

Unit 6
Lesson Plans

Teacher lesson plans from Unit 6 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 6 Unit 6

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

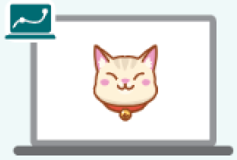
Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

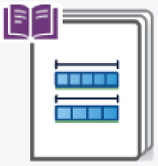
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



1 Weight for It

Connect tape diagrams and equations of the form x+p=q and px=q.

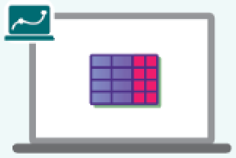


2 Five Equations

Connect tape diagrams, equations, and verbal descriptions in context.

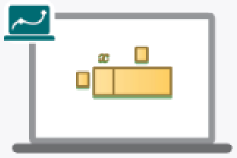
3 Hanging Around

Use balanced hangers to explain how to solve equations of the form x+p=q and px=q.



8 Products and Sums

Use an area model to write different equivalent expressions.



9 Products, Sums, and Differences

Use the distributive property to write equivalent algebraic expressions, including expressions involving subtraction.

Practice Day

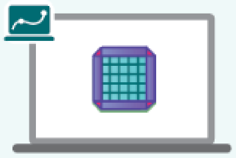
Practice Day 1

Practice the concepts and skills developed during Lessons 1–9. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

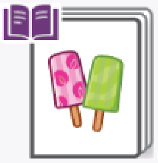
Quiz: Sub-Unit 1

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.



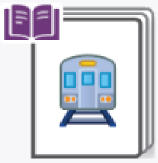
14 Representing Relationships

Represent relationships using tables and graphs.



15 Connecting Representations

Connect graphs, tables, and equations that represent the same relationship.



16 Subway Fares

Use tables, graphs, and equations to analyze an issue in society.

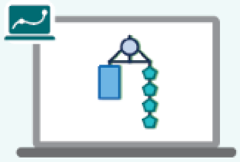
Practice Day

Practice Day 2

Practice the concepts and skills developed during Lessons 1–16. Consider using this time to prepare for the upcoming Quiz.

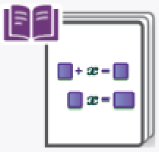
Pacing: 20 days | **Short on time?** See pacing considerations below.

Pre-Unit Check: (Optional)
16 Lessons: 45 min each
2 Practice Days: 45 min each
1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



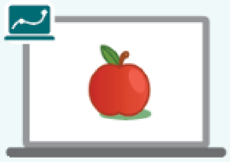
4 Hanging It Up

Solve equations of the form x+p=q and px=q that include whole numbers, decimals, and fractions.



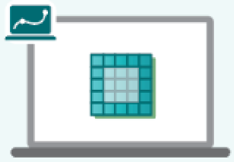
5 Swap and Solve

Use equations of the form x+p=q or px=q to solve problems in context and interpret the meaning of the solution.



6 Vari-apples

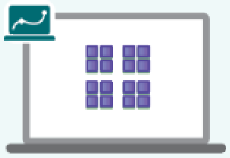
Write expressions involving variables to represent situations.



7 Border Tiles

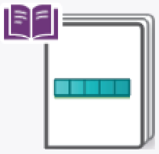
Explain what it means for two expressions to be equivalent.

Sub-Unit 2



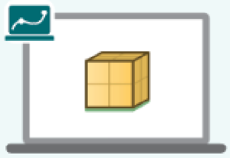
10 Powers

Explain the meaning of an expression with an exponent, such as 3^5.



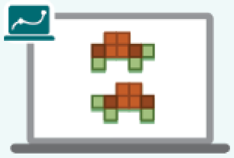
11 Exponent Expressions

Evaluate numerical expressions that have an exponent and one other operation (addition, subtraction, multiplication, or division)



12 Squares and Cubes

Evaluate expressions that have a variable, an exponent, and one other operation for a given value of the variable.



13 Turtles All the Way

Use rates and percentages to analyze characteristics of a country's population.

Summative Assessment

End-of-Unit Assessment

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing Considerations

Lesson 1: The purpose of this lesson is to lay the foundation for solving equations, including revisiting tape diagrams and introducing equations. If students show a strong understanding of determining unknown values and connecting tape diagrams to equations in Problems 1 and 2 of the Readiness Check, this lesson may be omitted.

Lesson 5: This lesson invites students to use what they have learned to write their own situations based on equations, then to interpret the meaning of those solutions in the situation. It can be skipped if students show a strong proficiency in moving fluidly between situations, equations, and solutions in earlier lessons.

Lesson 14: This lesson invites students to make connections between tables and graphs of relationships. This is the first time that students revisit graphs in the coordinate plane since Grade 5. If students show a strong understanding of plotting and interpreting points in Problem 8 of the Readiness Check, this lesson may be omitted.

Lesson 16: This lesson gives students an opportunity to apply their understanding of representing relationships to make sense of an issue in society: how much to charge for transportation fares. There is no new content introduced in this lesson.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Weight for It

Lesson 1: Reasoning About Unknown Values

Overview

Students use reasoning, equations, and tape diagrams to determine unknown weights on a see-saw.

Learning Goals

- Connect tape diagrams and equations of the form $x + p = q$ and $px = q$.
- Use reasoning and tape diagrams to determine unknown values.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to use reasoning, equations, and tape diagrams to determine unknown weights on a see-saw. This lesson introduces variables as letters that can represent different quantities, one of which will make an equation true (or in this case, balance the see-saw). Students make connections between the visual of animals and weights on a see-saw and equations and tape diagrams.

Lesson Summary

Warm-Up (5 minutes)

The purpose of this warm-up is for students to explore how changing the weight of an object affects a see-saw. Students will use the see-saw context to determine unknown weights throughout the lesson.

Activity 1: Equations and Tape Diagrams (15 minutes)

The purpose of this activity is to introduce students to equations and tape diagrams as representations for determining unknown values. Students solve several problems about determining the weight of an animal on a see-saw, then examine a representation that a fictional student used to make sense of the problem.

Activity 2: Determining Unknown Weights (15 minutes)

The purpose of this activity is for students to practice using reasoning, equations, and tape diagrams to determine unknown values. Students connect tape diagrams, equations, see-saws, and unknown weights, as well as make their own see-saw problem for other students to solve. This activity also introduces unknown weights that are decimals and fractions.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate what they have learned about equations and tape diagrams to represent situations with unknowns.

Cool-Down (5 minutes)

1 Warm-Up

Here are some weights on a seesaw.

Here are some weights on a seesaw.

1. Drag the movable point to adjust one of the weights.

2. Discuss what you notice and wonder.

Teacher Moves

Overview: In this lesson, students use reasoning, equations, and tape diagrams to determine unknown weights on a see-saw. In this warm-up, students explore how changing the weight of an object affects a see-saw using the Notice and Wonder routine.

Launch
- Invite students to work *individually.*
- Consider asking students to share their favorite item in a playground.
- If needed, consider reviewing that "lb." is an abbreviation of the word “pounds.”

Facilitation
- Give students 1–2 minutes to think independently and then share their responses with a partner.
- Use the dashboard’s teacher view or snapshot tool to highlight several students' noticings and wonderings.
- If it does not come up naturally, consider asking the discussion questions below.

Discussion Questions
- *What happens when the weight is less than* 4 *pounds? Greater than* 4 *pounds?*
- *What happens when the weight is exactly* 4 *pounds? Why might that make sense?*

Math Community
- Consider celebrating variety and creativity in what students notice and wonder, including things that surprise you or things you think other students may not have noticed.

Readiness Check (Problem 1)
If most students struggled, consider revisiting this problem before beginning Lesson 1 and drawing a tape diagram to represent each choice. Consider leaving the tape diagrams and equations up for students to reference as they work through Lesson 1.

Suggested Pacing: Screen 1

Sample Responses

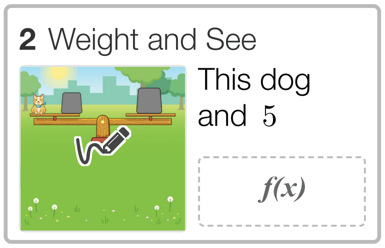
Responses vary.

- I notice that the weight can be any number between 1 and 10 pounds.
- I notice when the weight is 4 pounds, the see-saw balances.
- I notice that as you get further away from 4 pounds, the see-saw tips more.
- I wonder what is special about the number 4.
- I wonder how many pounds the see-saw can hold.
- I wonder if I would fall off if a giant weight fell on the other end of a see-saw.

Student Supports

Students With Disabilities
Executive Functioning: Graphic Organizers
Provide students a T-chart to record what they notice and wonder before they are expected to share their ideas with others.

Fine Motor Skills: Strategic Pairing
Allow students who struggle with fine motor skills to dictate use of the sketch tool and clickable buttons as needed throughout the lesson and unit.



2 Weight and See

This dog and 5

This dog and 5 pounds balance with a 17 lb. weight.

How much does the dog weigh?

Teacher Moves

Overview: In Activity 1 (Screens 2–5), students are introduced to equations and tape diagrams as representations for determining unknowns.

Launch
- Invite students to work *in pairs.*
- Consider asking: *How is this situation similar to the warm-up? How is it different?*

Facilitation
- Encourage students to come to an agreement about the weight of the dog before pressing “Try It.”

Suggested Pacing: Screen 2–5

Sample Responses

12 pounds

3 "d" Is for Dog

Tariq called the dog's

Tariq called the dog's weight d and wrote an equation.

Explain how the equation is like balancing the dog and weights.

Teacher Moves

Facilitation

- Encourage students to read others' responses and decide if others' explanations are similar to or different from their own.
- **Note:** This is the first time students see a letter used to represent an unknown quantity. If time allows, consider pausing and discussing how each piece of the equation is represented on the see-saw.

Discussion Questions

- *Where do you see each piece of the see-saw represented in the equation?*
- *Why do you think Tariq used the letter* d*? Could he have used a different letter?*

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

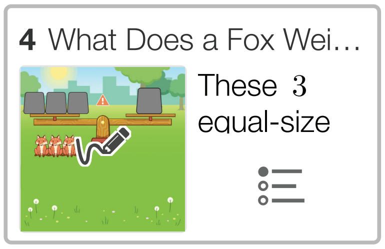
Responses vary. The left side of the equation is like the left side of the see-saw. It is the weight of the dog plus 5 pounds, so the left side of the equation is $d + 5$. The right side of the equation is like the right side of the see-saw.

Student Supports

Multilingual Learners

Routine: Stronger and Clearer Each Time

To support students in describing connections between the equation and the see-saw, use this routine with successive pair shares to give students a structured opportunity to revise their initial ideas. After writing an initial explanation, have pairs share what they wrote to get feedback from a partner. Have each student meet with a second partner to provide extra verbal practice, idea sharing, and feedback. Have students return to their seats and write down their revised transformation description.



4 What Does a Fox Wei…

These 3 equal-size

These 3 equal-size foxes balance with an 18 lb. weight.

Pick an equation that represents this situation.

Teacher Moves

Facilitation

- Encourage students to use the feedback on the screen to help them revise their thinking.
- Monitor for students who select each of the correct responses and for students who debate between $3 \cdot x = 18$ and $x + x + x = 18$.

Early Student Thinking

- Students may select $3 + x = 18$. Ask these students to compare this see-saw to the situation on the previous screen. Then invite them to describe a see-saw where $3 + x = 18$ would be the correct equation. Discuss how the situation on this screen is different from the previous screen and how that would affect the related equation.

Discussion Questions

- *Is it possible for more than one equation to be correct? How do you know?*

Sample Responses

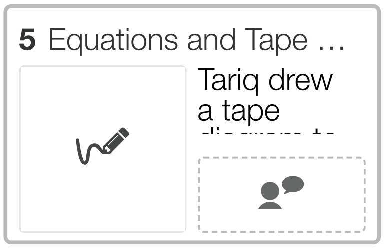
- $3 \cdot x = 18$
- $x + x + x = 18$

Student Supports

Students With Disabilities

Conceptual Processing: Checks for Understanding

Invite students to talk aloud about their thinking to help verbalize which concepts make sense and which do not.



5 Equations and Tape …

Tariq drew
a tape

Tariq drew a tape diagram to help figure out the weight of each fox.

Explain how the tape diagram is like the equations.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to connect tape diagrams to equations to help determine unknown weights.

Facilitation

- Encourage students to use the sketch tool if it helps them explain their thinking.
- Monitor for students who use different ways to explain how the tape diagram is like the equations, including students who make connections to the foxes on the see-saw (MP2).
- Select and sequence several student responses and sketches to help facilitate a whole-class discussion.

Discussion Questions

- *Where do you see the* $3 \cdot x$ *in the tape diagram? What about the* 18*? What about the* $x + x + x$*?*
- *What are the advantages of an equation? What are the advantages of a tape diagram?*

Early Finishers

- Encourage students to draw a tape diagram to represent each of the choices they did not select on Screen 4.

Math Community

- Consider snapshotting imprecise or unfinished explanations. During the discussion, highlight the strengths of these explanations by asking students to identify the parts of each explanation they found valuable and to make them stronger and clearer as a class.

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Sample Responses

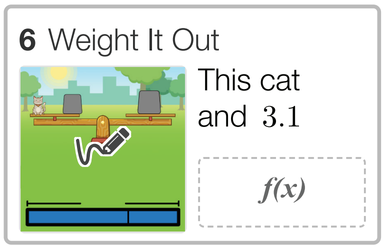
Responses vary. The tape diagram is like the left side of the equation. There are 3 x's, so the tape diagram has 3 parts labeled x. The total in the tape diagram is like the right side of the equation.

Student Supports

Students With Disabilities

Receptive Language: Processing Time
Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners
Expressive Language: Eliminate Barriers
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



6 Weight It Out

This cat and 3.1

This cat and 3.1 pounds balance with a 9.3 lb. weight.

How much does the cat weigh?

Use the tape diagram if it helps you with your thinking.

Teacher Moves

Overview: In Activity 2 (Screens 6–9), students practice using reasoning, equations, and tape diagrams to determine unknown values.

Launch

- Consider starting with the activity paused and inviting students to discuss with a partner how the tape diagram and the equation represent the see-saw.
- Then unpause and invite students to determine how much the cat weighs.

Suggested Pacing: Screen 6–9

Sample Responses

6.2 pounds

Student Supports

Students With Disabilities
Memory: Processing Time
Consider inviting students with working memory challenges to record a strategy for finding the weight of an animal on a piece of blank paper to refer back to throughout the remainder of the lesson.

7 Group the cards that …

Teacher Moves

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- Use the dashboard's summary view to monitor students' thinking. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Sample Responses

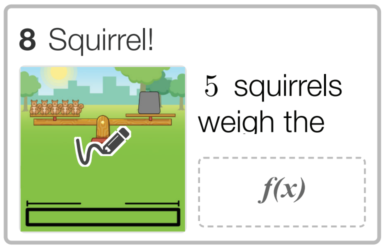
Image solution

Student Supports

Students With Disabilities

Executive Functioning: Eliminate Barriers

Chunk this activity into more manageable parts by inviting students to choose one equation or tape diagram card at a time and to match it with the corresponding see-saw.



8 Squirrel!

5 squirrels weigh the

5 squirrels weigh the same as a 7 lb. weight.

How much does each squirrel weigh?

Draw a tape diagram if it's helpful.

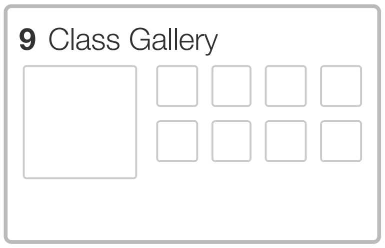
Teacher Moves

Facilitation

- To support students getting started, consider asking students to estimate how much a squirrel weighs before calculating.
- Encourage students to use the feedback on the screen to help them revise their thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

1.4 pounds (or equivalent)



9 Class Gallery

Teacher Moves

In this Challenge Creator, students set up their own see-saw and define the weights. Students then challenge themselves to determine the unknown weight of the animal on their and their classmates' see-saws.

Facilitation

- Give students several minutes to create their own challenge and more time to solve their classmates' challenges.
- Encourage students to go back and review their classmates' responses to the challenge they created.
- While students are working, monitor for and highlight creative challenges and solutions.
- **Note:** We anticipate this Challenge Creator may take 10 minutes or more.

Math Community

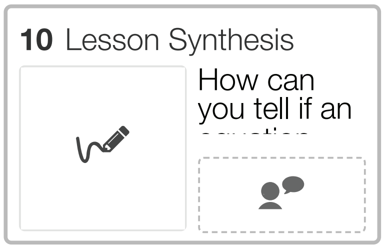
- Consider inviting students to share challenges they found particularly fun or creative.

Student Supports

Students With Disabilities

Executive Functioning: Eliminate Barriers

Consider helping students that are struggling to get started by identifying one or two challenges other students have created that might be a good place to start.



10 Lesson Synthesis

How can you tell if an

How can you tell if an equation and a tape diagram match?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is for students to consolidate what they have learned about equations and tape diagrams to represent situations with unknowns.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their responses stronger and clearer based on the discussion.

Discussion Questions

- *When might making a drawing be helpful?*
- *When might an equation be helpful?*
- *When might a tape diagram be helpful?*

Suggested Pacing: Screen 10

Sample Responses

Responses vary.

- I can tell if an equation and a tape diagram match when all of the pieces in the tape diagram are the same as one side of an equation and the total length of the tape diagram is the same as the other side of the equation.
- I can tell if an equation and a tape diagram match by seeing if I can draw the same picture for both of them, like a see saw or another drawing.
- I can tell if an equation and a tape diagram match by plugging the answer into the equation and the tape diagram and seeing if they are both correct.

Student Supports

Students With Disabilities
Receptive Language: Processing Time
Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from

both reading and listening.

---

11 Cool-Down

Teacher Moves

Support for Future Learning
Students will have more chances to develop their understanding of representing equations with tape diagrams in Lesson 2.

Readiness Check (Problem 2)
If most students struggled, consider revisiting these problems as a class after Lesson 1 and asking: *What would this look like if it were represented with a see-saw? What about a tape diagram?*

Suggested Pacing: Screens 11–12

Sample Responses

Image solution

12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can make connections between tape diagrams and equations.
- I can use reasoning and tape diagrams to figure out unknown values.

Five Equations (NYC)

Lesson 2: Tape Diagrams, Equations, Contexts

Purpose

In Lesson 1, students saw equations and tape diagrams as ways to represent balancing weights on a see-saw. This lesson introduces situations described in words. Students connect tape diagrams, equations, and situations, and use them as tools to answer questions in context. This lesson also introduces the terms *variable* and *solution*.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Warm-Up (10 minutes)

Overview: Students use the routine Three Reads to make sense of a situation described in words (MP1). This reasoning is an example of what students will do on their own in the rest of the lesson.

Launch

- Invite students to work *in pairs.*
- Display Sheet 1 of the Teacher Projection Sheets.

Facilitation

- This routine invites students to slow down and make sense before calculating.

    1. Display Sheet 1 and give students one minute to read quietly and then discuss with their partner what the situation is about. Then invite them to share with the class.

    2. Display Sheet 2. Consider asking a student to read the situation aloud, or invite students to read in pairs. Give students one minute to discuss what they think a *variable* is. Create a class definition and record it for students to refer to throughout the lesson and unit. Then give students 1–2 minutes to draw a visual to represent the situation.

    3. Display Sheet 3. Read the situation once more and invite students to brainstorm a question we might ask about the situation. If it does not come up naturally, consider inviting students to answer the question: *How much does each cat weigh?*

Math Community

- Consider inviting students to share why reading a question multiple times might be helpful in order to normalize slowing down when solving a complex problem.

Activity 1: Equations and Tape Diagrams (10 minutes)

Overview: Students make connections between equations and tape diagrams.

Launch

- Invite students to work *individually with the support of a partner.*
- Give students one minute to consider each of the equations.
- **Note:** This may be the first time students see a variable and a constant next to each other, such as $5x$. To support students in sense-making, ask: *What do you think the term $5x$ means in $5x = 20$?*

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement as they match.
- Circulate to monitor student progress and look for creative tape diagrams in Problem 5.
- Facilitate a discussion around any tape diagrams where there is not consensus and highlight student creativity in their own tape diagrams in Problem 5.

Discussion Questions

- *How are the equations that match the first tape diagram and second tape diagram similar? How are they different? How is that reflected in the tape diagrams?*
- *Here is _______'s tape diagram. What equation do you think they were representing?*

Math Community

- Consider highlighting the value of changing one's mind by asking if any students revised their thinking throughout the activity. If applicable, encourage students to celebrate classmates whose ideas helped them understand connections between equations and tape diagrams.

Support for Students With Disabilities

Visual-Spatial Processing: Eliminate Barriers

Consider inviting students to use objects like cube, chips, or pennies to model each of the tape diagrams.

Memory: Visual Aids

To support students with working memory challenges, consider writing each equation on a slip of paper, then inviting students to physically place the equations next to the tape diagram they think represents each equation, similar to a card sort.

Activity 2: Which Equation? (15 minutes)

Overview: Students connect the features of a situation and an equation, and then determine a solution and interpret the meaning of the solution in the situation (MP2).

Launch

- Invite students to work *in pairs.*
- Display Sheet 4 of the Teacher Projection Sheets.
- Give students one minute to discuss what they think a *solution* is. [A solution is a value of a variable that makes the equation true.] Then, create a class definition and record it for students to refer to throughout the lesson and unit.

Facilitation

- Give students 5–10 minutes to analyze each of the three situations.
- Here are two options for facilitating this activity:
    - Ask students to match equations and situations individually, and then compare and work together to write the solution and the meaning of the solution.
    - Invite students to take turns describing their thinking on a situation out loud as the other student records their thinking on paper, asking questions if they disagree.
- Consider posting the answer key, or walking around with it and providing feedback to students as they work.
- Facilitate a whole-class discussion to surface strategies students used to decide which diagram matched each situation and the meaning of the solution.
- Consider sharing the correct solutions with the class in order to focus the conversation on students' reasoning.

Discussion Questions

- *Here is _______'s situation. What equation do you think they were representing?*
- *How did you decide which equation matched Mohamed's situation? What does the solution mean?*

Early Finishers

- Invite students to trade their situation with another student and ask them to figure out which diagram they selected.

Math Community

- Consider highlighting students who used connections to their own lives when deciding their matches.

Support for Multilingual Learners

Receptive Language: Eliminate Barriers

Invite students to read each situation aloud and paraphrase what it means, clarifying any unknown words or concepts before answering each question.

Support for Students With Disabilities

Visual-Spatial Processing: Visual Aids

To support students in making connections between representations, provide printed copies of Sheet 4 of the Teacher Projection Sheets for students to draw on or highlight.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

The purpose of this discussion is to consolidate connections between equations and situations.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their responses based on the discussion.

Discussion Questions

- *Which equation matches Kwasi’s situation? How do you know?*
- *In general, what strategies do you use to decide if an equation and a situation match?*

Support for Students With Disabilities

Memory: Processing Time

To support students with working memory challenges, consider inviting them to refer back to Activity 2 to help them with their response.

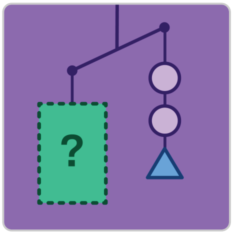
Cool-Down (5 minutes)

Support for Future Learning

Students will have more chances to connect situations and equations in Lesson 5.

Digital Lesson

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Hanging Around

Lesson 3: Introduction to Balanced Hangers

Overview

Students use hangers to represent equations and then determine unknown values that balance each hanger.

Learning Goals

- Make connections between balanced hangers and true equations.
- Use balanced hangers to explain how to solve equations of the form $x + p = q$ and $px = q$.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to use hangers to represent equations, then determine unknown values that balance each hanger. This lesson is slightly more abstract than the previous lessons, which prepares students to solve equations without an explicit visual in Lesson 4. Students make connections between equations and hangers, and then use either representation to help them determine unknown values.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to make sense of how a hanger works and to make connections between hangers and the balancing work of Lesson 1.

Activity 1: Connect It (15 minutes)

The purpose of this activity is for students to make connections between balanced hangers and equations. Students compare and contrast hangers that represent equations with addition and multiplication, and determine the value of the variable that makes the hanger balance.

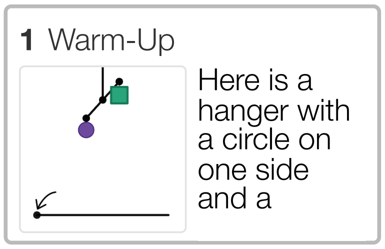
Activity 2: Make It, Solve It (15 minutes)

The purpose of this activity is for students to develop fluency determining unknown values in equations and hangers. Students create their own hanger and then determine the value of the variable that makes the hanger balance for their own hangers and and their classmates'.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to summarize how a hanger can be helpful in determining the solution to an equation.

Cool-Down (5 minutes)



1 Warm-Up

Here is a hanger with a circle on one side and a

Here is a hanger with a circle on one side and a square on the other.

1. Drag the movable point to adjust the weight of the square.

2. Discuss what you think the weight of the circle is.

Teacher Moves

Overview: In this lesson, students use hangers to represent equations, then determine unknown values that balance each hanger. In this warm-up, students make sense of how a hanger works and make connections between hangers and the balancing work of Lesson 1.

Launch
- Invite students to work *in pairs.*
- Consider asking: *Where have you seen something like this in real life?*
- Introduce the word “hanger” for those who might not be familiar with it.

Facilitation
- Give students 1–2 minutes to work independently and then to discuss with a partner.

Discussion Questions
- *How did you decide what the weight of the circle was?*
- *What does it mean when the hanger is tilted to the left? To the right?*

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

- A little more than 6
- About 6

Student Supports

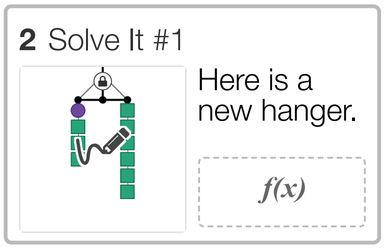
Students With Disabilities

Fine Motor Skills: Strategic Pairing

Allow students who struggle with fine motor skills to dictate use of the sketch tool and clickable buttons as needed throughout the lesson and unit.

Conceptual Processing: Eliminate Barriers

Use objects like clothes on hangers to demonstrate how a hanger works in real life.



2 Solve It #1

Here is a new hanger.

Here is a new hanger.

What value of x balances the hanger?

Teacher Moves

Overview: In Activity 1 (Screens 2–7), students make connections between balanced hangers and equations.

Launch

- Invite students to work *in pairs.*
- Consider starting with the activity paused and asking students to determine a value of x that they know *does not* balance the hanger.

Facilitation

- Invite students who are struggling to enter any value of x and use the feedback to help them revise their thinking.
- Encourage students to use the sketch tool to show their thinking.

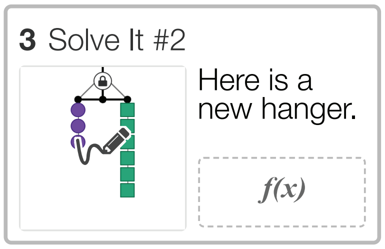
Early Student Thinking

- Students may notice that there are 3 squares on the left side and use this to explain why the value of x is 3.
- Consider asking these students: *How would the value of* x *change if there were only* 2 *squares on the left?*

Suggested Pacing: Screens 2–4

Sample Responses

$x = 3$



3 Solve It #2

Here is a new hanger.

Here is a new hanger.

What value of x balances the hanger?

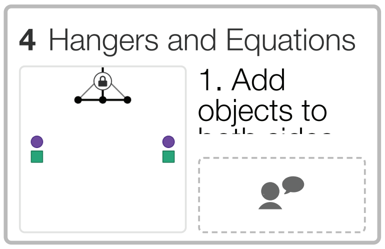
Teacher Moves

Progress Check

- Use the dashboard's summary view to monitor students' thinking.
- Monitor for students who compare this challenge to the challenge on the previous screen, then invite them to share during the discussion on Screen 7.

Sample Responses

$x = 2$



4 Hangers and Equations

1. Add objects to

1. Add objects to both sides of this hanger to make an equation.

2. Explain how the equation is like the hanger.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to surface connections between equations and hangers. Students can make hangers to help them solve equations throughout the unit.

Facilitation

- Monitor for students who create equations that include addition, multiplication, or both.
- Select and sequence several students' hangers and equations using the snapshot tool. Then invite students to discuss how you can see the same information in each hanger and equation (MP7).
- Consider sharing out equations that students made and challenging the rest of the class to recreate that equation using their own hanger.

Discussion Questions

- *How do we write the equation when there is more than one x on a side? Why does that make sense?*
- *What do you think would happen if you swapped the weights on each side of the hanger?*

Early Finishers

- Encourage students to challenge each other to make a hanger that represents an equation they wrote, then to determine the value of x that would make that hanger balance.

Math Community

- Consider celebrating student creativity in both their hangers and their explanations.

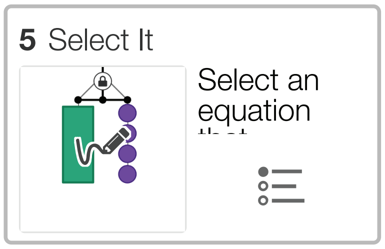
Sample Responses

Responses vary. You can see the same information in the equation and in the hanger. Each side of the equation is like one side of the hanger. If you have 5 x's and a 3 on one side, the left side of the equation would be $5x + 3$. The equal sign is like the hanger being balanced.

Student Supports

Students With Disabilities
Receptive Language: Processing Time
Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners
Expressive Language: Eliminate Barriers
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



5 Select It

Select an equation

Select an equation that represents this hanger.

Teacher Moves

Progress Check

- Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.
- Monitor for students who select each of the correct responses and for students who debate between $11 = 4x$ and $11 = x + x + x + x$.

Suggested Pacing: Screens 5–7

Sample Responses

- $11 = 4x$
- $11 = x + x + x + x$

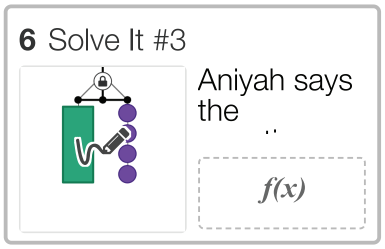
Explanations vary. There is an 11 on the left side of the hanger and $4x$'s on the right side of the hanger.

Student Supports

Students With Disabilities

Visual-Spatial Processing: Eliminate Barriers

To support students in recognizing differences between each equation, consider inviting them to compare and contrast each equation before responding or inviting a partner to read each equation aloud.



6 Solve It #3

Aniyah says the

Aniyah says the equation $11 = 4x$ represents this hanger.

Use the hanger or equation to figure out the solution.

Teacher Moves

Facilitation

- To support students getting started, consider asking: *Will the scale balance if $x = 4$? How do you know?*
- Monitor for students who use different strategies, including guessing and checking, reasoning using the hanger, and using the equation. Support students in making connections between these strategies, and discuss the strengths and weaknesses of each strategy as time allows.

Discussion Questions

- *How are ________'s and _______'s strategies similar? How are they different?*

Early Student Thinking

- Students may notice that $11 = 4 + 7$ and write 7 as the value of x.
- Invite these students to use the sketch tool to replace each x with a 7 and ask: *Will this balance? How do you know?*

Sample Responses

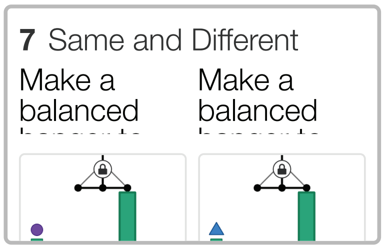
$x = 2\frac{3}{4}$ (or equivalent)

Student Supports

Students With Disabilities

Memory: Processing Time

Consider inviting students with working memory challenges to record a strategy for determining the value of x that balances this hanger on a piece of blank paper in order to refer back to it throughout the remainder of the lesson.



7 Same and Different

Make a balanced

Make a balanced

Make a balanced hanger to represent $5x = 8$.

Teacher Moves

Facilitation

- Encourage pairs to compare their hangers to their classmates' and discuss any discrepancies.
- Consider selecting both correct and incorrect hangers using the snapshot tool and inviting students to decide which equation each hanger matches, or to write a new equation if it matches neither (MP7).

Discussion Questions

- *How are your hangers similar? How are they different?*
- *Which equation do you think is easier to solve? Why?*
- *What value makes each equation true? How do you know?*

Early Finishers

- Encourage students to determine the solution to each equation and convince a classmate their solution is correct.

Math Community

- Consider inviting students to share what they can learn from looking at both correct and incorrect thinking.

Routine (optional): Consider using the mathematical language routine Compare and Connect to support students in making connections between the equations and hangers.

Sample Responses

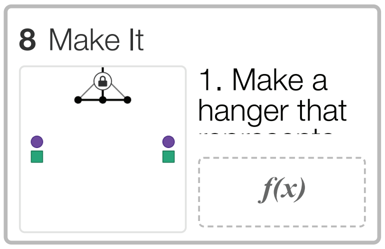
Image solution

Student Supports

Students With Disabilities

Memory: Processing Time

Consider inviting students with working memory challenges to look for similarities and differences between their task here and the hanger on the previous screen.



8 Make It

1. Make a hanger that

1. Make a hanger that represents $6 = x + 2$.

Teacher Moves

Overview: In Activity 2 (Screens 8–9), students develop fluency connecting equations and hangers and determining unknown values.

Launch

- Consider sharing that now we will put all of these ideas together, connecting equations to hangers and determining unknown values.

Progress Check

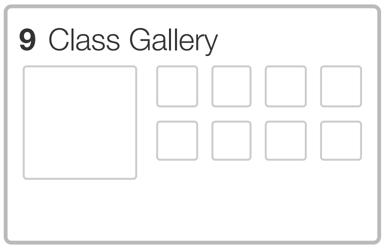
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling. These skills will be helpful before students begin the Challenge Creator on the next screen.

Suggested Pacing: Screens 8–9

Sample Responses

1. Image solution

2. $x = 4$



9 Class Gallery

Teacher Moves

Overview: In this Challenge Creator, students create their own hanger. Then they challenge their classmates to make a hanger that represents their equation and determine the value of x that balances the hanger (MP6).

Facilitation

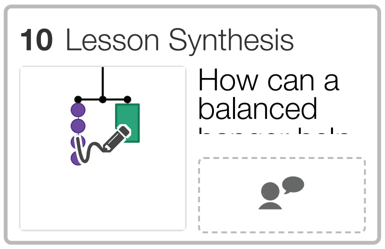
- Give students several minutes to create their own challenge and more time to solve their classmates' challenges.
- Encourage students to go back and review their classmates' responses to the challenge they created.
- While students are working, monitor for and highlight creative challenges and solutions.
- **Note:** We anticipate this Challenge Creator may take 10 minutes or more.

Math Community

- Consider inviting students to share challenges they found particularly fun or creative.

Student Supports

Students With Disabilities
Executive Functioning: Eliminate Barriers
Consider helping students that are struggling to get started by identifying one or two challenges other students have created that might be a good place to start.


10 Lesson Synthesis

How can a balanced

How can a balanced hanger help us figure out the solution to an equation?

Use the hanger and equation if that helps you with your thinking.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to summarize how a hanger can be helpful in determining the solution to an equation.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their responses stronger and clearer based on the discussion.

Discussion Questions

- *How is a balanced hanger similar to a see-saw or a tape diagram? How is it different?*
- *What are the advantages of a hanger? What are the advantages of an equation?*

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Suggested Pacing: Screen 10

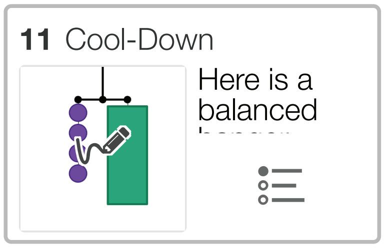
Sample Responses

Responses vary. Balanced hangers help us figure out the solution to an equation because they help us see what the value of x must be to make both sides have the same total.

Student Supports

Students With Disabilities
Receptive Language: Processing Time
Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners
Expressive Language: Eliminate Barriers
Provide sentence frames to help students explain their thinking (e.g., You can use a balanced hanger to help figure out the solution to an equation by ______.).



11 Cool-Down

Here is a balanced

Here is a balanced hanger.

1. Which equation does this hanger represent?

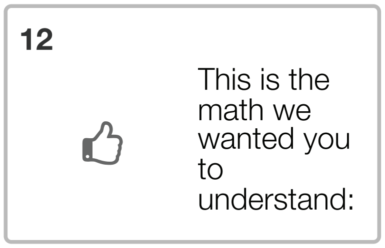
Teacher Moves

Support for Future Learning
If students struggle to connect equations and hangers, consider reviewing this screen before beginning Lesson 4. Students will have more chances to develop their understanding of solving equations in Lesson 4.

Suggested Pacing: Screens 11–12

Sample Responses

1. $4x = 24$
2. $x = 6$



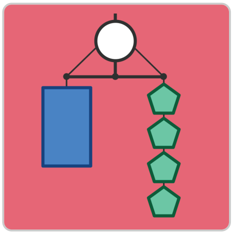
12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can make connections between balanced hangers and true equations.
- I can use balanced hangers to solve equations.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Hanging It Up

Lesson 4: Solving Equations

Overview

Students develop fluency with solving equations, particularly with equations that include decimals and fractions.

Learning Goals

- Solve equations of the form $x + p = q$ and $px = q$ that include whole numbers, decimals, and fractions.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop fluency with solving equations, particularly equations that include decimals and fractions. Students revisit connections between equations, balanced hangers, and tape diagrams, and then consider solutions to equations. Students can use a variety of tools to help them determine solutions to equations.

Lesson Summary

Warm-Up (5 minutes)

In this warm-up, students engage in the Number Talk routine to revisit strategies for adding and subtracting with decimals from Unit 5. Students will use these strategies throughout the lesson as they solve equations that involve decimals.

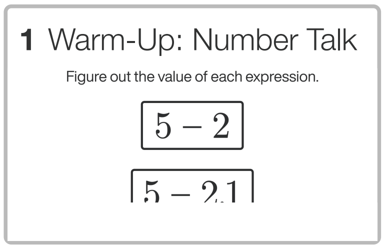
Activity 1: Solving and Solutions (30 minutes)

The purpose of this activity is for students to use what they have learned in previous lessons to help them develop fluency in solving equations. Students revisit both balanced hangers and tape diagrams, then transition away from visual representations as they continue to develop and solidify strategies for solving equations. This activity ends with a set of Repeated Challenges where students can develop fluency in solving. Equations in this lesson include whole numbers, decimals, and fractions.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to solidify their understanding of how to tell if a value is a solution to an equation.

Cool-Down (5 minutes)



1 Warm-Up: Number Talk

Figure out the value of each expression.

$\boxed{5 - 2}$

$5 - 2.1$

Teacher Moves

Overview: In this lesson, students develop fluency in solving equations, particularly equations that include decimals and fractions. In this warm-up, students engage in the Number Talk routine to revisit strategies from Unit 5 for adding and subtracting with decimals. Students will use these strategies throughout the lesson as they solve equations that involve decimals.

Launch

- Consider sharing with students that in this Number Talk, they will have time to think about all four expressions at once and then share their strategies.

Facilitation

- Give students 1–2 minutes to think quietly and then signal when they have an answer and a strategy. Encourage students to think of more than one strategy.
- If students are struggling, consider sharing a strategy of your own.
- Select several students to share different strategies. Use the sample responses as examples of possible student strategies. Record strategies for all to see, along with the name of the student who shared each one.

Early Finishers

- Encourage students who finish early to write their own expression that has the same value as the fourth expression.

Suggested Pacing: Screen 1

Sample Responses

3

2.9

2.83

2.983

Student Supports

Students With Disabilities

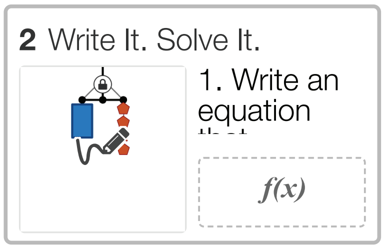
Conceptual Processing: Checks for Understanding

Invite students to talk aloud about their thinking to help them verbalize how each calculation is similar to and different from the previous one.

Fine Motor Skills: Strategic Pairing

Allow students who struggle with fine motor skills to dictate use of the sketch tool and clickable buttons as needed throughout the lesson and unit.

Memory: Eliminate Barriers
To support students with working memory challenges, consider inviting them to record their work for each calculation as they are working on a piece of blank paper.



2 Write It. Solve It.

1. Write an equation

1. Write an equation that matches the hanger.

Teacher Moves

Overview: In this activity (Screens 2–8), students use what they have learned in the previous three lessons to help them develop fluency in solving equations.

Launch
- Consider sharing with students that they will use all of the skills they have developed in this unit so far to help them, and then ask: *What do you think will be helpful to remember from what we have learned so far?*

Progress Check
- This is a review of the learning from the previous lesson.
- Use the dashboard's summary view to monitor students' thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Math Community
- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Readiness Check (Problems 3 and 4)
- If most students struggled, consider spending extra time on this screen surfacing strategies for determining the solution to this equation.
- Consider creating or referencing an anchor chart from Unit 5 with common strategies for adding, subtracting, multiplying, and dividing decimals.

Suggested Pacing: Screens 2–3

Sample Responses

1. $7 = 4x$

2. 1.75 (or equivalent)

Student Supports

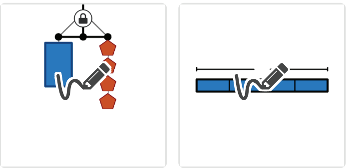
Students With Disabilities

Conceptual Processing: Eliminate Barriers

Use objects like chips or pennies to represent known and unknown weights described on the screen and throughout the lesson.

---

3 Two Representations



Teacher Moves

Facilitation

- When most students have discussed this screen, invite several of them to share how a hanger or a tape diagram might help us determine a solution.
- If it does not come up naturally, consider monitoring for and highlighting students who thought of the solution as $\frac{7}{4}$ and as 1.75.

Discussion Questions

- *What are the advantages of the hanger for solving equations? What about the tape diagram?*
- *Where can we check* 1.75 *or* $\frac{7}{4}$ *using the hanger? Using the tape diagram?*

Early Finishers

- Encourage students to write another equation that has the same solution and represent it on paper using a tape diagram or a balanced hanger.

Sample Responses

Responses vary.

- There are 4 sections in the tape diagram that total to 7, so each one has to be $\frac{7}{4}$.
- If you put in 1.75 for each of the weights on the hanger, it becomes 7, which will balance with the 7 on the left.

Student Supports

Multilingual Learners
Expressive Language: Visual Aids
Consider directing students to the visual displays created in earlier lessons that show connections between tape diagrams, equations, and images.

---

4 Match each equation …

Teacher Moves

Launch

- Consider sharing with students that they can use any strategy they want, including drawing diagrams or using reasoning, to help them match equations to values.

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- Use the dashboard’s summary view to monitor students’ thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 4–8

Sample Responses

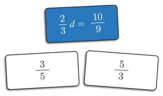
Image solution

Student Supports

Students With Disabilities
Executive Functioning: Eliminate Barriers
Chunk this activity into more manageable parts by inviting students to choose one equation card at a time and match it to its solution card(s). It may be helpful for students to group cards they think could possibly be the solution at first so that they have a smaller number of choices to consider.

---

5 Settle a Dispute



Imani and Deiondre

Imani and Deiondre solved this equation.

Imani says the solution is $d = \frac{3}{5}$.

Deiondre says that the solution is $d = \frac{5}{3}$.

Who is correct?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to surface strategies for determining the solution to an equation, especially one that involves fractions.

Facilitation

- Consider displaying the distribution of responses using the dashboard's teacher view.
- Monitor for students who use different reasoning in their responses. Then select and sequence several responses to make sense of as a class.
- Facilitate a discussion to review how to check if a value is a solution to an equation or to review fraction multiplication.

Discussion Questions

- *How can we check if* $\frac{3}{5}$ *is a solution?* $\frac{5}{3}$ *?*

Math Community

- Consider inviting students to think about what a student who responded differently might have been thinking.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Deiondre

Explanations vary. We are trying to figure out what number times $\frac{2}{3}$ equals $\frac{10}{9}$ and $\frac{2}{3} \cdot \frac{5}{3} = \frac{10}{9}$.

I know it can't be $d = \frac{3}{5}$ because the denominator would be 15 instead of 9 .

Student Supports

Students With Disabilities
Receptive Language: Processing Time
Read the prompt aloud for students who benefit from extra processing time. Invite students to paraphrase what each student's argument is before responding.

Multilingual Learners

Expressive Language: Eliminate Barriers

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

6 Solutions

Here is a new

Here is a new equation: $x + 2.01 = 3.5$.

Describe how you could figure out what value of x makes this equation true.

Teacher Moves

Facilitation

- Students may wonder if they need to solve the equation on this screen. Consider sharing that the purpose of this screen is to share strategies for figuring out unknown values rather than focusing on what the solution is.
- To support students getting started, consider asking: *What would you change about this equation to make it friendlier to figure out the solution? How is this equation similar to and different from that one?*
- Monitor for students who describe a variety of tools and strategies they would use, including but not limited to creating tape diagrams or hangers, using undoing steps, or reasoning about the solution. (MP5)
- Select and sequence several student strategies, then invite those students to share with the class.
- Consider recording students' strategies for others to reference as they work on the Repeated Challenges on the next screen.

Discussion Questions

- *How are these strategies similar? How are they different?*
- *How would you describe ________'s strategy in your own words?*

Math Community

- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Sample Responses

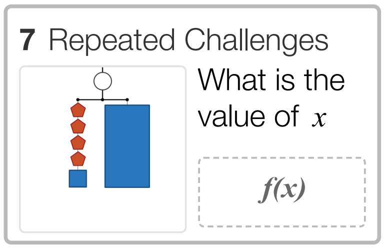
Responses vary. I would figure out what number I need to add to 2.01 to get 3.5. I know it has to be less than 1.5 because $2 + 1.5 = 3.5$.

Student Supports

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their strategy (e.g., To figure out the value of x that makes this equation true, ________.).



7 Repeated Challenges

What is the value of x

What is the value of x that makes this equation true?

$$5x = 30$$

[Equations will vary]

Teacher Moves

How Repeated Challenges Work

- Students are presented with a variety of challenges one at a time (in this case, solving equations).
- The challenges typically increase in difficulty as they continue (in this case, they will include more decimals and fractions).

Facilitation

- Give students 5–7 minutes to complete as many challenges as they can.
- Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.
- Consider encouraging students who have solved more than 10 correct equations to explore the “Are You Ready for More” on the next screen.

Math Community

- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Sample Responses

This screen contains an unlimited number of challenges. The first few challenges are the same for each student; additional challenges are randomized.

The first few responses are:

9

8

$\frac{1}{6}$

2.83

Student Supports

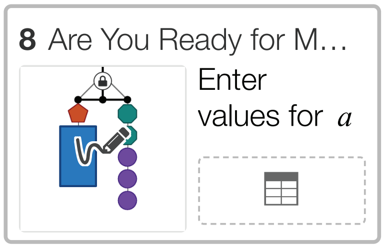
Students With Disabilities

Conceptual Processing: Checks for Understanding

Invite students to talk aloud about their thinking to help them verbalize which concepts make sense and which do not.

Memory: Visual Aids

Consider inviting students with working memory challenges to record their work for each problem on a piece of blank paper to refer back to throughout the remainder of the lesson and unit.



8 Are You Ready for M…

Enter values for a

Enter values for a, b, and c to make the hanger balance.

Try to figure out as many different values as you can.

Teacher Moves

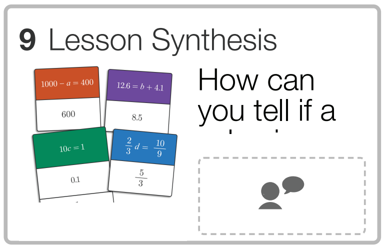
Facilitation

- Invite students who finish several of the repeated challenges on the previous screen to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion.
- If time allows, invite all students to explore this screen and share what they notice and wonder.

Sample Responses

Responses vary.

- $a = 2, b = 1.5, c = 3$
- $a = 7, b = 1, c = 5$
- $a = 90, b = 20, c = 20$



9 Lesson Synthesis

How can you tell if a

How can you tell if a value is a solution to an equation?

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to revisit and consolidate strategies for determining if a value is a solution to an equation.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their responses stronger and clearer based on the discussion.

Discussion Questions

- *What makes sense to you about each strategy? What does not make sense?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Suggested Pacing: Screen 9

Sample Responses

Responses vary. You can plug the number into where the variable is in the equation and see if the equation is true. For example, $1000 - 600 = 400$, so 600 is a solution.

Student Supports

Students With Disabilities

Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

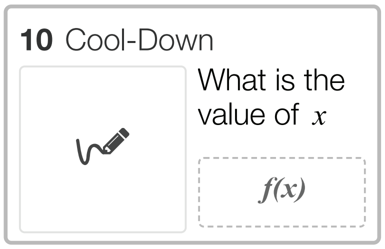
Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., You can tell a value is a solution to an equation by ______________.).

---

10 Cool-Down



What is the value of x

What is the value of x that makes this equation true?

$$2.18 + x = 6$$

Teacher Moves

Support for Future Learning

If students struggle, consider reviewing this screen as a class before Lesson 5 or offering individual support where needed during Lesson 5 or Practice Day 1.

Suggested Pacing: Screens 10–11

Sample Responses

3.82

11

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can solve equations that include whole numbers, decimals, and fractions.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Swap and Solve (NYC)

Lesson 5: Solving Equations in Context

Purpose

In this lesson, students connect equations to situations by writing their own situations to match equations, and then trade situations with classmates. Students will determine the solution to a situation and interpret the solution in the context of the situation (MP2).

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Materials

- Index cards or slips of colored paper (one per student)

Warm-Up (10 minutes)

Overview: Students use the routine Three Reads to make sense of a situation described in words (MP1). This reasoning is an example of what students will do on their own in the rest of the lesson.

Launch

- Invite students to work *in pairs.*
- Display Sheet 1 of the Teacher Projection Sheets.

Facilitation

- This routine invites students to slow down and make sense before calculating.
    1. Display Sheet 1 and give students one minute to read quietly and then discuss with their partner what the situation is about. Then invite them to share with the class. Consider asking: *What experiences do you have with situations like this?*
    2. Display Sheet 2. Consider asking a student to read the situation aloud or invite students to read in pairs. Invite students to discuss what the variable represents.
    3. Display Sheet 3. Read the situation once more and discuss which equation represents the situation. Encourage students to justify their arguments.

Math Community

- Consider inviting students to share why reading a question multiple times might be helpful in order to normalize slowing down when solving a complex problem.

Activity 1: Stronger and Clearer Each Time (10 minutes)

Overview: Students write situations that match equations and then use the routine Stronger and Clearer Each Time to refine their ideas and language.

Launch

- Invite students to work *individually.*
- Display Sheet 4 of the Teacher Projection Sheets.

Facilitation

- Consider sharing the purpose of the activity with students, specifically that they will be writing a situation to match one of the equations, then getting feedback on their situation before they trade their situation with other students.
- Circulate to ensure that students have solved the equation they selected correctly before they write a situation. Consider encouraging students to convince a partner that their solution is correct before moving on to Step 2.
- When all students have either started or written a situation to match their equation, facilitate two or more rounds of the routine Stronger and Clearer Each Time.

Math Community

- It may be vulnerable for students to share their situations with others for a number of reasons. Consider spending time normalizing this experience.
- Encourage students to shout out a partner or a classmate for something that student did that helped them during this activity.

Support for Students With Disabilities

Executive Functioning: Eliminate Barriers

Consider checking in with students after each step of the activity to prevent students from being overwhelmed.

Conceptual Processing: Processing Time

To check for understanding, consider asking students: *How do you know that your situation matches your equation?*

Support for Multilingual Learners

Expressive Language: Eliminate Barriers

Consider reviewing the word problems in the warm-up and in Lesson 2 to give students examples of the structures of these types of problems.

Activity 2: Trade and Solve (15 minutes)

Overview: Students trade their situation with several classmates, and then write an equation for each classmate's situation and solve it. This structure supports student collaboration with many different partners and allows for movement around the classroom.

Launch

- Invite students to work *in pairs.*
- Display Sheet 5 of the Teacher Projection Sheets.
- Distribute one index card or slip of colored paper to each student. Invite students to write the second draft of their situation on the slip of paper.

Facilitation

- Arrange students into pairs and invite students to write their partner's name on the back of the student worksheet.
- Invite each student to write an equation to represent their partner's situation, determine a solution, and write the meaning of the solution. (MP2)
- When both students have done all of the above for each other's situations, invite them to stand up with a hand up and find another classmate to pair up with.
- Repeat the process as many times as time allows.

Discussion Questions

- *What clues helped you figure out what equation to write for each situation?*

Math Community

- Consider inviting students to share situations they found particularly fun or challenging.

Support for Students With Disabilities

Conceptual Processing: Eliminate Barriers

Demonstrate the steps of the activity using the word problem from the warm-up and a student volunteer. Ask students to paraphrase the steps of the activity in their own words before beginning.

Support for Multilingual Learners

Receptive Language: Eliminate Barriers

Encourage students to read each situation aloud and paraphrase what it means, clarifying any unknown words or concepts before answering each question.

Lesson Synthesis (5 minutes)

Key Discussion 🔑
The purpose of this discussion is to summarize what students learned about writing equations to represent situations.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their response based on the discussion.

Discussion Questions

- *Why do you think this is important to remember?*
- *What mistakes might someone make when trying to write an equation from a situation?*

Cool-Down (5 minutes)

Support for Future Learning
Consider reviewing this prompt as a class before Practice Day 1 or offering individual support where needed during the practice day. Students will have opportunities to write expressions from situations in Part 2 of this unit, but this is the last lesson explicitly focused on solving equations.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Vari-apples

Lesson 6: Introduction to Variable Expressions

Overview

Students use expressions with variables to represent situations (in this case, the cost of some number of pounds of fruit).

Learning Goals

- Write expressions involving variables to represent situations.

Vocabulary

- coefficient
- term

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to use expressions with variables to represent situations (in this case, the cost of some number of pounds of fruit). Students come to recognize variable expressions as a way to express answers to questions like: *How can you describe the cost of any number of pounds of apples?* Students also practice writing these types of expressions. They will extend their thinking about expressions with one variable to equations with two variables in Lesson 13 of this unit.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to surface different ways of calculating the area of rectangles made up of two parts. They will work with areas of rectangles like these in Lessons 8 and 9.

Activity 1: Intro to Variable Expressions (20 minutes)

The purpose of this activity is to introduce students to using variables to represent any number of some quantity (in this case, any number of pounds of various fruits). Students calculate the cost to order several different numbers of pounds of apples, then generalize using an expression. This activity ends by inviting students to write their own expression to calculate the total cost of any number of pounds of limes.

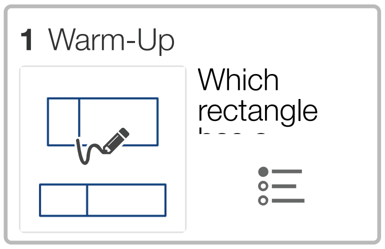
Activity 2: Comparing Variable Expressions (10 minutes)

The purpose of this activity is for students to explore variable expressions that involve addition, and to compare and contrast expressions involving multiplication and addition. Students explore the cost of delivering fruit for a fixed fee per delivery and compare this to the cost per pound from Activity 1.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consider the advantages of expressions with variables and expressions with numbers.

Cool-Down (5 minutes)



1 Warm-Up

Which rectangle

Which rectangle has a greater area?

Teacher Moves

Overview: In this lesson, students use expressions with variables to represent situations (in this case, the cost of some number of pounds of fruit). This warm-up is intended to surface different ways of calculating the area of rectangles made up of two parts. Students will work with areas of rectangles like these in Lessons 8 and 9.

Launch

- Consider briefly reviewing how to calculate the area of a rectangle and sharing that we will be studying rectangles that look like this later in the unit.

Facilitation

- Give students one minute to respond and to share their thinking with a partner.
- Monitor for students who calculate the area of each small rectangle and for students who add the two measurements on the width and calculate the area of the whole rectangle at once.
- Whether or not there is consensus, invite students to justify how they calculated the total area of the rectangle (MP3).

Early Finishers

- Encourage students to think of a different way to calculate the area of each rectangle, or another way to convince someone that their choice is correct.

Suggested Pacing: Screen 1

Sample Responses

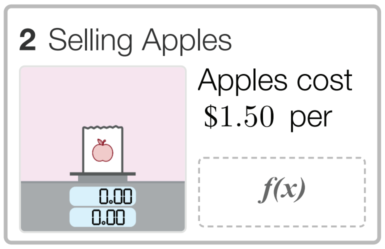
A

Explanations vary.

- I added up the areas of the small rectangles. Rectangle A's area is $6+15=21$ square units. Rectangle B's area is $6+10=16$ square units.
- Rectangle A is a 3-by-7 rectangle and the bottom rectangle is a 2-by-8 rectangle. $3\cdot 7=21$ and $2\cdot 8=16$.
- The first small rectangle is the same size just turned, so we only have to compare the rectangle on the right. A 3-by-5 rectangle is bigger than a 2-by-5 rectangle.

Student Supports

Students With Disabilities
Conceptual Processing: Eliminate Barriers
To assist students in recognizing the connections between new problems and prior work, consider asking: *What other rectangles have we seen like this before?* Consider making explicit connections to the area models in Unit 5.



2 Selling Apples

Apples cost $\$1.50$ per

Apples cost $\$1.50$ per pound.

A customer wants to buy 3 pounds of apples.

How much should you charge them?

Teacher Moves

Overview: In Activity 1 (Screens 2–6), students are introduced to using variables to represent any number of some quantity (in this case, any number of pounds of various fruits).

Launch

- Consider asking students: *What is your favorite type of apple? How many apples do you think there are in* 3 *pounds of apples?*

Facilitation

- To support students' getting started, consider asking: *How might a double number line or a table help you with your thinking?*
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Math Community

- Celebrate students who use their personal experiences to support them in their reasoning.

Suggested Pacing: Screens 2–6

Sample Responses

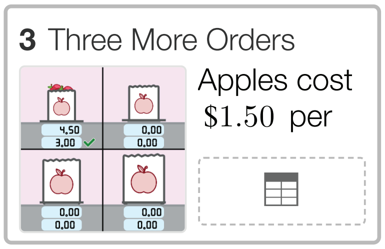
4.50 dollars

Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers

Use objects like blocks or chips to demonstrate the situation described on this screen, where each object represents one pound of apples.



3 Three More Orders

Apples cost $\$1.50$ per

Apples cost $\$1.50$ per pound.

How much should you charge for each order?

Teacher Moves

Facilitation

- Use the dashboard's summary view to monitor students' thinking.
- Monitor for students who use ratio strategies, such as making a double line or determining a unit rate, to help them with their thinking. Consider inviting students to share these strategies during the discussion on Screen 5.

Early Student Thinking

- Students might continue to add 1.50 for each row of the table.
- Consider asking these students: *How many pounds does the order in the third row want?*

Sample Responses

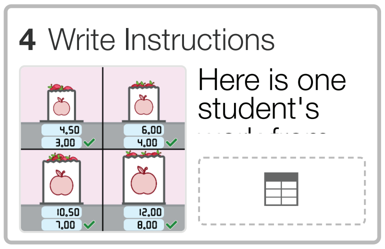
6.00
10.50
12.00

Student Supports

Students With Disabilities

Memory: Processing Time

To support students with working memory challenges, consider inviting them to record each calculation on a piece of blank paper as they are working.



4 Write Instructions

Here is one student's

Here is one student's work from the previous screen.

Teacher Moves

Facilitation

- Consider monitoring for students who describe the cost in different ways, including adding 1.50 until they get to the number of pounds they want or multiplying the number of pounds by 1.50.
- If possible, make connections between students' descriptions on this screen and expressions on Screen 5 during the key discussion on that screen.

Math Community
- Consider celebrating descriptions that use different levels of formality and precision.

Sample Responses

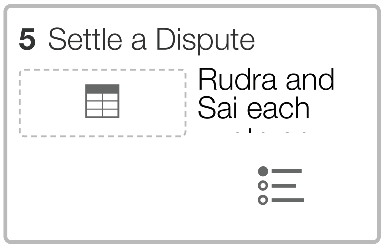
Responses vary.

- Multiply the number of pounds you want by 1.50 to get the cost.
- Keep adding 1.50 until you get to the number of pounds you want.

Student Supports

Students With Disabilities
Conceptual Processing: Eliminate Barriers
To assist students in recognizing the connections between this task and their work on the previous screen, consider inviting them to explain how they calculated one row of the table, such as when there were 7 pounds of apples.

Multilingual Learners
Expressive Language: Eliminate Barriers
Provide sentence frames to help students explain their thinking (e.g., To determine the cost of any number of apples, first ______. Then, _______.).



5 Settle a Dispute

Rudra and Sai each

Rudra and Sai each wrote an expression to describe the cost of p pounds of apples.

Rudra: $p + 1.50$
Sai: $1.50p$

Who is correct?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to make connections between an expression with variables and a situation (MP2).

Facilitation
- Select and sequence several student explanations using the snapshot tool.
- When most students have responded, display the distribution of responses using the dashboard's teacher view.

- Facilitate a conversation around how to know which expression is correct based on the situation and the table.
- Spend adequate time here to ensure that students understand why $1.50p$ is correct and $p + 1.50$ is incorrect.

Discussion Questions

- *How can the table help us check which expression is correct?*
- *How can the situation help us know which expression is correct?*

Early Student Thinking

- Students may select Rudra because $3 + 1.50 = 4.50$ or because they used adding on previous screens.
- Consider sharing that if p represented the number of pounds before the number they wanted, then their expression would be correct. Invite these students to substitute 4 for p in the expression to see if they are correct.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Sai $1.50p$

Responses vary.

- Sai's expression works for every pair of numbers in the table. For example, $1.50(8) = 12$, but $8 + 1.50$ does not equal 12.
- Apples are 1.50 per apple, which means that you have to multiply to get the total number of apples.

Student Supports

Students With Disabilities

Receptive Language: Processing Time

Read the prompt aloud for students who benefit from extra processing time.

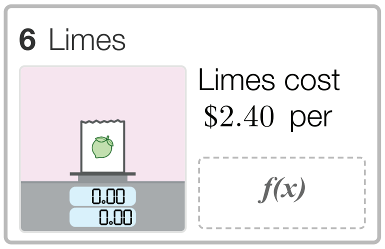
Visual-Spatial Processing: Eliminate Barriers

To support students in recognizing the differences between each equation, consider inviting students to compare and contrast each equation before responding or inviting a partner to read each equation aloud.

Multilingual Learners

Expressive Language: Eliminate Barriers

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



6 Limes

Limes cost $\$2.40$ per

Limes cost $\$2.40$ per pound.

How much should you charge for p pounds of limes?

Teacher Moves

Progress Check

- Use the dashboard's summary view to monitor students' thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Note: Consider sharing that in this example the *coefficient* of p is 2.4 because 2.4 is multiplied by p to determine the cost of the limes.

Early Finishers

Encourage students to answer these questions: *How many limes could you buy for* $\$60$*?* [$25$ limes] *What about* $\$100$*?* [$41$ limes]

Sample Responses

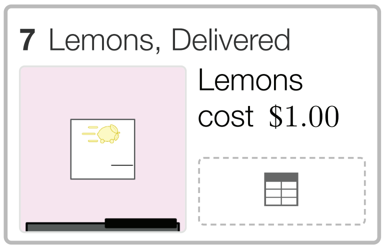
$2.40p$

Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers

To assist students in recognizing the connections between new problems and prior work, consider asking: *How is buying limes similar to and different from buying apples? How should our equation change to show how limes are different*?



7 Lemons, Delivered

Lemons cost $\$1.00$

Lemons cost $\$1.00$ per pound, and for an extra $\$5.00$, you can have the entire order delivered!

What is the total cost for each of these deliveries?

Teacher Moves

Overview: In Activity 2 (Screens 7–11), students explore variable expressions that involve addition, and compare and contrast expressions involving multiplication and addition.

Launch

- Consider starting with the activity paused and asking students to explain to a partner why it would cost $\$8.00$ to buy 3 lemons.

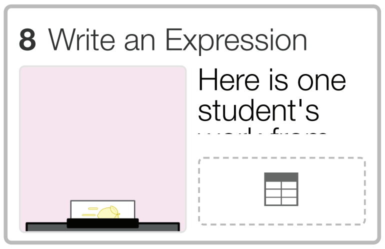
Progress Check

- Use the dashboard's summary view to monitor students' thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 7–11

Sample Responses

10.00
15.00
17.50



8 Write an Expression

Here is one student's

Here is one student's work from the previous screen.

Teacher Moves

Facilitation

- To support students getting started, consider asking students to describe in words how they figured out the cost of each pound of lemons and to generalize to any number of pounds (MP2).
- Use the dashboard's teacher view to monitor for students who write expressions in different ways.

Sample Responses

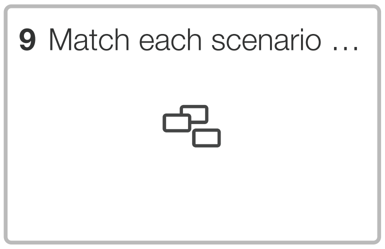
$p + 5$ or $1p + 5$ (or equivalent)

Student Supports

Students With Disabilities

Memory: Processing Time

To support students with working memory challenges, consider inviting them to record a strategy for finding an expression on a piece of blank paper so that they can refer back to it throughout the remainder of the lesson.



9 Match each scenario …

Teacher Moves

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- Circulate to monitor for students who use different ways to explain how to match each card, particularly how to decide if they want to use multiplication or addition. Invite these students to share their thinking during the discussion on Screen 10.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Sample Responses

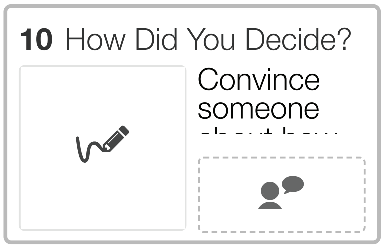
Image solution

Student Supports

Students With Disabilities

Executive Functioning: Eliminate Barriers

Chunk this activity into more manageable parts by inviting students to choose one scenario card at a time and matching it with a single expression card.


10 How Did You Decide?

Convince someone

Convince someone about how to group this card.

Teacher Moves

Facilitation

- Select and sequence several student responses.
- Facilitate a discussion to make connections between the situation and expression (MP2).
- Consider sharing that in the expression $15x$, 15 is a called a *coefficient* and in $x+15$, the x and the 15 are each called *terms*.
- If time allows, work together as a class to write a convincing argument about why $15x$ is the correct expression for this situation.

Discussion Questions

- *How can you check each equation?*
- *Why might someone think this matches with* $x+15$ *?*

Math Community

- Consider inviting students to think about what a student who responded differently might have been thinking.

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Sample Responses

Responses vary.

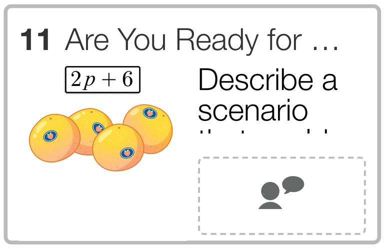
- This card has to go with $15x$ because if you order 2 pizzas that would be 30 dollars. $2+15$ is not 30 .
- If it is 15 dollars per pizza, then to figure out the cost of any number of pizzas, you would multiply 15 by how many pizzas you ordered. That is $15x$ because when a number and a variable are next to each other that means multiply.

Student Supports

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., I know that _______ matches this card because _________.).



11 Are You Ready for …

$\boxed{2p+6}$ Describe a scenario

Describe a scenario that could be represented by the expression $2p + 6$.

Create a table on paper if it helps you with your thinking.

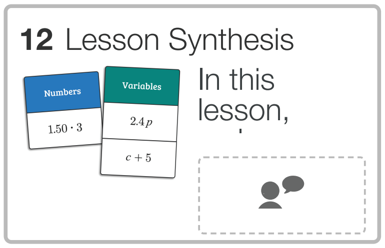
Teacher Moves

Facilitation

- Invite students who finish Screens 7–10 early to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion.
- If time allows, consider sharing a students' scenario and asking the class if they think it could be represented by $2p + 6$.

Sample Responses

Responses vary. I went to the store for oranges and chocolate. It costs $\$2$ per pound of oranges and $\$6$ for a giant chocolate bar.



12 Lesson Synthesis

In this lesson,

In this lesson, we've seen two types of expressions: expressions with **numbers** and expressions with **variables**.

When might each kind of expression be useful?

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to surface and summarize the advantages of expressions with variables and expressions with numbers.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their responses stronger and clearer based on the discussion.

Discussion Questions

- *When might someone want to write an expression with variables?*
- *When might someone want to write an expression with numbers?*

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Suggested Pacing: Screen 12

Sample Responses

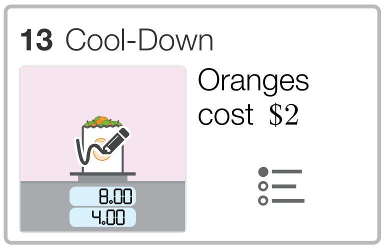
Responses vary.

Expressions with numbers are useful when you want to figure out a specific number of things, or you already know the information you need to solve a problem.

Expressions with variables are useful when you want to figure out what something would look like for any number, like the cost of any number of pounds of limes.

Student Supports

Students With Disabilities
Receptive Language: Processing Time
Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.



13 Cool-Down

Oranges cost $\$2$

Oranges cost $\$2$ per pound.

What is the cost of x pounds of oranges?

Teacher Moves

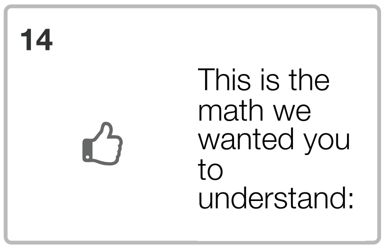
Support for Future Learning
If students struggle, plan to emphasize this when opportunities arise in Lesson 13, particularly on screens where students select an equation to represent a relationship.

Suggested Pacing: Screens 13–14

Sample Responses

$2x$

12 dollars



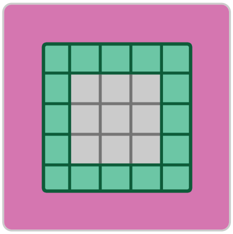
14

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can write an expression with a variable to represent a situation.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Border Tiles

Lesson 7: Equivalent Expressions

Overview

Students explore the concept of *equivalent expressions* (two different expressions that describe the same thing) in the context of the number of tiles that border a square.

Learning Goals

- Explain what it means for two expressions to be equivalent.
- Justify whether two expressions are equivalent.

Vocabulary

- equivalent expressions

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to introduce the concept of equivalent expressions (two different expressions that describe the same thing) in the context of the number of tiles that border a square. Students describe the number of tiles it would take to border any square and make sense of fictional students' expressions. This lesson supports the work students will do with equivalent expressions and the distributive property in Lessons 8 and 9.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to practice evaluating expressions at specific values of the variable, which introduces the idea that it is possible to represent a side length using a variable. Students will work directly with areas of rectangles in Activity 2 and again in Lessons 8 and 9.

Activity 1: Border Tiles (20 minutes)

The purpose of this activity is for students to explore the concept of equivalent expressions. Students explore the number of border tiles needed to surround an n-by-n square, and then make sense of fictional students' expressions for the number of border tiles. Students come to see equivalent expressions as different ways of describing the same thing.

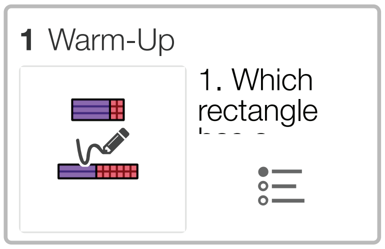
Activity 2: Rectangle Areas (10 minutes)

The purpose of this activity is for students to apply what they learned in Activity 1 to a different type of diagram: rectangles made up of x-tiles and one-tiles. These rectangles will set the foundation for the work students will do in Lessons 8 and 9 with the distributive property.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to summarize what it means for expressions to be equivalent.

Cool-Down (5 minutes)


1 Warm-Up

1. Which rectangle

1. Which rectangle has a greater area when $x = 4$?

Teacher Moves

Overview: In this lesson, students explore the concept of *equivalent expressions* (two different expressions that describe the same thing) in the context of the number of tiles that border a square. In this warm-up, students practice evaluating expressions at specific values of the variable. They will work directly with areas of rectangles in Activity 2 and again in Lessons 8 and 9.

Launch

- Invite students to work *individually.*
- Consider starting with the lesson paused and asking students how the rectangles are similar and how they are different.

Facilitation

- Give students 1–2 minutes to work independently and then share their responses with a partner.
- Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.
- Invite at least one student to share their strategy for calculating the area of each rectangle. Some students may calculate the area of each part of the rectangle separately: $3(4) + 3(2)$. Or they may multiply the entire length times the width: $3(6)$.

Suggested Pacing: Screen 1

Sample Responses

1. Rectangle B
2. Rectangle A

Explanations vary.

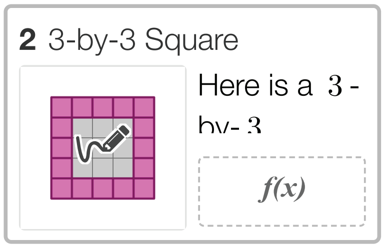
- The area of Rectangle A is always 6 plus 3 times x. The area of Rectangle B is always 12 plus 2 times x. When $x = 4$, Rectangle A's area is $6 + 12 = 18$ and Rectangle B's is $12 + 8 = 20$.
- Rectangle A is bigger by one x piece and Rectangle B is bigger by 6 of the small squares. So when $x = 4$, then 4 is less than 6, which means Rectangle B is bigger, but when $x = 8$, then 8 is more than 6, which means Rectangle A is bigger.

Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers
Use objects like tiles or graph paper to demonstrate the situation described on this screen.

Fine Motor Skills: Strategic Pairing
Allow students who struggle with fine motor skills to dictate use of the sketch tool as needed throughout the lesson and unit.



2 3-by-3 Square

Here is a 3-by-3

Here is a 3-by-3 square surrounded by pink tiles.

Without counting one by one, how many pink border tiles are there?

Teacher Moves

Overview: In Activity 1 (Screens 2–9), students explore the concept of *equivalent expressions* in the context of describing the number of tiles that border any sized square.

Launch

- Consider reading the prompt out loud and asking students what they think “without counting one by one” means.

Facilitation

- Encourage students to use the sketch tool to show how they grouped the tiles together to count.

Early Student Thinking

- Students might think that there are 12 border tiles because the perimeter of the gray square is 12.
- Consider asking these students to show where they see the 12 in the diagram.

Math Community

- Consider using the snapshot tool or dashboard’s teacher view to celebrate different ways that students counted the number of tiles.

Suggested Pacing: Screens 2–6

Sample Responses

16 border tiles

Explanations vary.

- There are 3 tiles on each side plus 4 at the corners.
- There are 5 tiles on the top and the bottom plus 3 on each side.

Student Supports

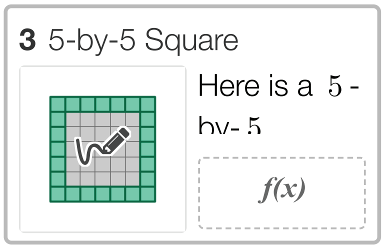
Students With Disabilities

Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Visual-Spatial Processing: Visual Aids

To support students in making sense of images, provide them graph paper to draw their own diagrams or highlight.



3 5-by-5 Square

Here is a 5 -by- 5

Here is a 5 -by- 5 square surrounded by green tiles.

1. Without counting one by one, how many green border tiles are there?

2. Use the sketch tool to show how you see it.

Teacher Moves

Facilitation

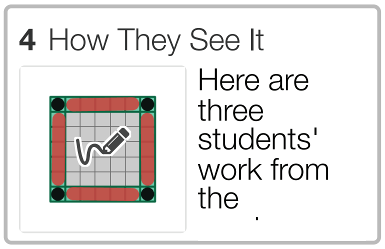
- Encourage students to use the sketch tool to show how they counted the squares.
- If time allows, invite students to look at a classmate’s sketch to see how it compares to the way they were thinking.

Math Community

- Consider using the snapshot tool or dashboard’s teacher view to celebrate different ways that students counted the number of tiles.

Sample Responses

24 border tiles



4 How They See It

Here are three students' work from the

Here are three students' work from the previous screen.

1. Select each student's name to see their work.

2. Discuss how all of these expressions are similar.

Teacher Moves

Facilitation

- When most students have responded, consider pausing the class to make connections between each student’s sketch and their expression.
- If it does not come up naturally, consider asking the first discussion question below.

Discussion Questions

- *Where do we see the side length of the square in Lucia’s expression? Kyrie’s? Manuel’s?*
- *Which student's thinking is most similar to yours?*

Math Community

- Consider renaming Lucia's, Kyrie’s, and Manuel’s strategies after the students in your class who used them.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary.

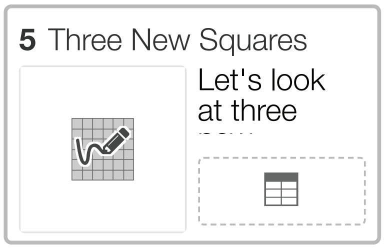
- They all show ways of counting the number of green border tiles.
- They all include at least one 5 (which is the size of the square).
- They all give the same answer: 24 border tiles.

Student Supports

Students With Disabilities

Conceptual Processing: Checks for Understanding
Invite students to talk aloud about their thinking to help them verbalize what concepts do and do not make sense.

Conceptual Processing: Eliminate Barriers
To assist students in recognizing the connections between this task and their work on the previous screen, consider inviting them to explain how they calculated one row of the table, such as the 9-by-9 square.



5 Three New Squares

Let's look at three

Let's look at three new squares one at a time.

Determine the number of border tiles for each square.

Use an expression if it helps you with your thinking.

Teacher Moves

Facilitation

- Consider using the dashboard's student view to show students that they can type expressions in the table, like $2 + 2$ or $4 \cdot 3$. Invite them to consider how the expressions on Screen 4 might be helpful here on Screen 5.
- Monitor for students who write expressions to calculate the number of border tiles. These may be helpful as students describe their thinking on the next screen.
- If you notice that students are using the feedback to guess-and-check the number of tiles, consider pausing and inviting students to think about how they could figure out the number of tiles without feedback.

Sample Responses

- 28
- 40
- 44

6 Describe a Strategy

Here is Lucia's

Here is Lucia's table from the previous screen.

Describe a strategy for determining the number of border tiles for an n-by-n square.

Teacher Moves

Facilitation

- Students might wonder what an n-by-n square is. Consider inviting these students to think back to Lesson 6 about what we might use a variable for.
- Monitor for students whose descriptions match Manuel's or Kyrie's expressions. Consider using the snapshot tool or dashboard's teacher view to share these descriptions as students are making sense of the expressions on Screens 7 and 8.

Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify with an incomplete description to support students in thinking about the level of detail needed in mathematical explanations like this one.

Sample Responses

Responses vary.

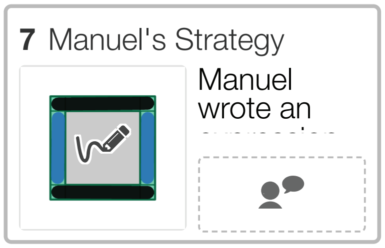
- Multiply the size of the square by 4 for the edges and then add 4 more for the corners.
- Add the size of the square two times for the sides, and then add two more than the size of the square for the top and the bottom.

Student Supports

Students With Disabilities

Memory: Processing Time

To support students with working memory challenges, consider inviting them to record their strategy for writing an expression on a piece of paper so that they can refer back to it throughout the remainder of the lesson.



7 Manuel's Strategy

Manuel wrote an

Manuel wrote an expression with variables to describe his strategy for an n-by-n square:

$$(n+2)+(n+2)+n+n$$

Show or describe how you see his expression in the diagram.

Teacher Moves

Facilitation

- Encourage students to read others’ responses and decide if others' descriptions were similar to or different from their own.
- Connecting expressions and diagrams may be new for students. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling (MP7).

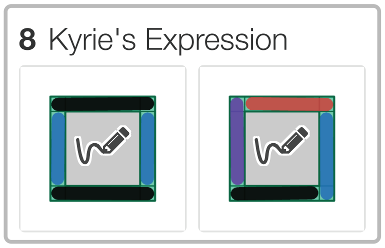
Suggested Pacing: Screens 7–9

Sample Responses

Responses vary. The $(n+2)$'s are the long black lines on the top and the bottom. The n's are the blue lines on the left and right side.

Student Supports

Students With Disabilities
Executive Functioning: Eliminate Barriers
To support organization, consider inviting students to substitute different numbers for n to help them make connections between expressions and Manuel's diagram.


8 Kyrie's Expression

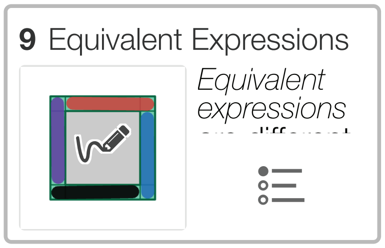
Teacher Moves

Facilitation

- To support students getting started, consider asking how Kyrie's sketch is similar to and different from Manuel's. Invite them to think about how they could write an expression for just the red line at the top.
- It's okay to lack consensus about Kyrie's expression at this stage. The activity will build toward consensus on Screen 9 when students consider why Kyrie's and Manuel's expressions are equivalent.

Sample Responses

$(n+1)+(n+1)+(n+1)+(n+1)$ (or equivalent)


9 Equivalent Expressions

Equivalent expressions

Equivalent expressions are different ways of describing the same thing. Here are two equivalent expressions:

Kyrie: $4(n+1)$

Manuel: $(n+2)+(n+2)+n+n$

Select another expression equivalent to $4(n+1)$.

Teacher Moves

Key Discussion Screen 🔑
There are two purposes of this discussion: 1) to introduce the term *equivalent expressions* and 2) to make sense of each student's expression for the number of border tiles (MP7).

Facilitation

- Encourage students to read the text on this screen aloud and to paraphrase it in their own words before responding.

- When most students have responded, facilitate a whole-class discussion with the goals described above.
- Consider sharing the correct response early in the discussion to focus on how you can tell if an expression is equivalent or not.

Discussion Questions

- *How can we tell that Kyrie and Manuel wrote equivalent expressions?*
- *What would* $4n + 4$ *look like on the diagram? Where would the* $4n$ *be? What about the* $+4$*?*
- *How can we tell that* $4n + 1$ *is not equivalent to* $4(n + 1)$*? Why might someone think that it is?*

Early Finishers

- Encourage students to write a different equivalent expression and to use the sketch tool to show their thinking.

Math Community

- Consider celebrating students who use different kinds of reasoning to make sense of equivalent expressions: drawing, plugging in numbers, thinking about the expressions themselves, etc.

Sample Responses

$4n + 4$

Image solution

Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers

To assist students in recognizing the connections between new problems and prior work, consider inviting students to connect Kyrie's and Manuel’s expressions to their sketches before responding.

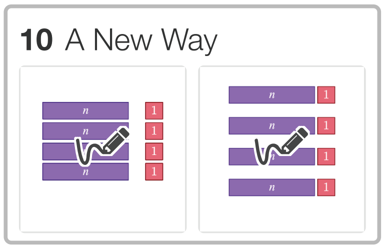
Receptive Language: Processing Time

Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners

Expressive Language: Eliminate Barriers

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.


10 A New Way

Teacher Moves

Overview: In Activity 2 (Screens 10–12), students apply what they learned in Activity 1 to a different type of diagram: rectangles made up of x-tiles and one-tiles.

Facilitation

- Give students one minute to share with a partner how the diagrams are similar and different before making their argument for why the expressions are equivalent (MP3).
- If time runs short, consider discussing this screen as a class.
- If students are curious, consider sharing that we can write $4(n+1)$ as $4(1n+1)$ because each group has a single n-tile.

Discussion Questions

- *How do we know these are equivalent expressions?*

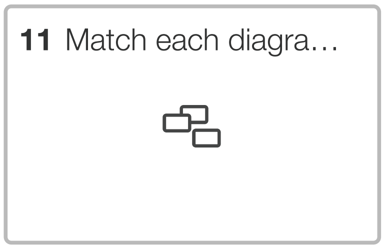
Math Community

- Celebrate students who make connections back to the border tiles in Activity 1 (MP3).

Suggested Pacing: Screens 10–12

Sample Responses

Responses vary. They are the same diagram, just split up in two different ways.


11 Match each diagra...

Teacher Moves

Facilitation

- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placements, and make revisions based on their conversation.

Sample Responses

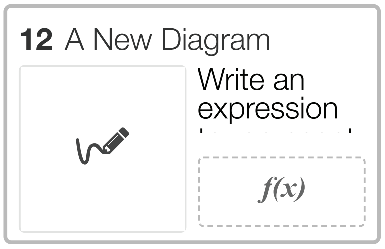
Image solution

Student Supports

Students With Disabilities

Executive Functioning: Eliminate Barriers

Chunk this activity into more manageable parts by inviting students to choose one expression card at a time and check if it matches with either diagram.



12 A New Diagram

Write an expression

Write an expression to represent this new diagram.

Try to write an expression none of your classmates will.

Teacher Moves

Facilitation

- Use the dashboard's teacher view to highlight each of the different expressions that students write.

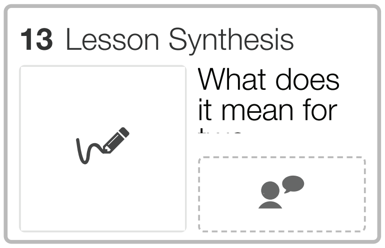
Early Finishers

- Encourage students to write as many equivalent expressions that represent this diagram as they can.

Sample Responses

Expressions vary.

- $6n + 3$
- $3(2n + 1)$
- $(2n + 1) + (2n + 1) + (2n + 1)$
- $3n + 3n + 3$



13 Lesson Synthesis

What does it mean for

What does it mean for two expressions to be *equivalent*?

Use the examples if they help you explain your thinking.

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to summarize what it means for expressions to be equivalent.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- As a class, create a definition of *equivalent expressions* based on the language students used in their responses.

Discussion Questions

- *How can we tell if expressions are equivalent?*
- *How could we make our definition stronger and clearer?*

Suggested Pacing: Screen 13

Routine (optional): Consider using the mathematical language routine Collect and Display to capture students' descriptions.

Sample Responses

Responses vary. Two expressions are equivalent if they are different ways of describing the same thing. For example, $3n+3$ and $3(n+1)$ are equivalent because they both show 3 n-tiles and 3 one-tiles.

Student Supports

Students With Disabilities

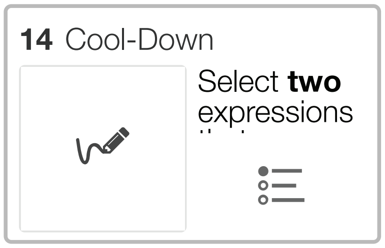
Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., For two expressions to be equivalent, they must ______.).



14 Cool-Down

Select **two** expressions

Select **two** expressions that are equivalent to $2n+4$.

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Support for Future Learning

If students struggle, plan to emphasize this when opportunities arise in Lesson 8, particularly in Activity 1 where students select which rectangles are equivalent.

Suggested Pacing: Screens 14–15

Sample Responses

- $n+n+1+1+1+1$

- $(n + 2) + (n + 2)$

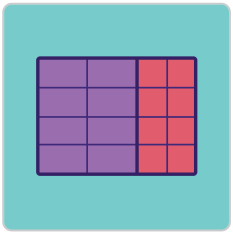
15

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain what it means for two expressions to be equivalent.
- I can justify whether two expressions are equivalent.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Products and Sums

Lesson 8: Distributive Property Part 1

Overview

Students use variable expressions to represent areas of rectangles in two different ways: as the sum of two areas and as the product of the two side lengths.

Learning Goals

- Use an area model to write different equivalent expressions.
- Use an area model to make sense of the distributive property.

Vocabulary

- product
- sum

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

This is the third lesson about expressions and the first of two lessons that focuses specifically on the distributive property of multiplication over addition. Students use variable expressions to represent areas of rectangles in two different ways: as the sum of two areas and as the product of the two side lengths. They then explore the areas of given rectangles and create their own rectangles in order to develop fluency.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to revisit how to calculate the area of a rectangle. They are also introduced to areas where one side is a variable length.

Activity 1: Rectangles and Equivalent Expressions (20 minutes)

The purpose of this activity is to introduce students to two ways of expressing the area of a rectangle. Students learn the terms *product* and *sum* as two ways of seeing the area: either as the product of the length and width, or as the sum of smaller areas. This activity connects back to the work students did with equivalent expressions in Lesson 7.

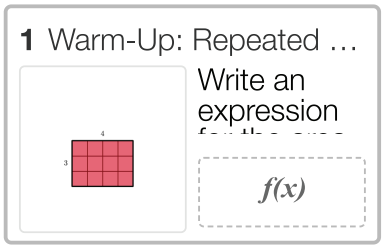
Activity 2: Create Your Own (10 minutes)

The purpose of this activity is for students to develop fluency with writing equivalent expressions that represent the area of a rectangle. In the Challenge Creator, students create their own rectangle, then challenge themselves and their classmates to write expressions that represent its area.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to summarize two or more ways of writing the area of a rectangle.

Cool-Down (5 minutes)



1 Warm-Up: Repeated …

Write an expression

Write an expression for the area of the rectangle.

Teacher Moves

Overview: In this lesson, students use variable expressions to represent areas of rectangles in two different ways: as the sum of two areas and as the product of the two side lengths. In this warm-up, students revisit how to calculate the area of a rectangle. They are also introduced to areas where one side is a variable length.

Facilitation

- Invite students to work *individually.*
- Give students 3–5 minutes to complete as many challenges as they can.
- Circulate to observe student strategies and offer help or encouragement where needed.

Discussion Questions

- *How did you calculate the area when one of the side lengths was a variable?*
- *What did you notice about the purple and red tiles? How are they similar? Different?*

Suggested Pacing: Screen 1

Sample Responses

This screen contains an unlimited number of challenges. The first few challenges are the same for each student; additional challenges are randomized.

The first three responses are:

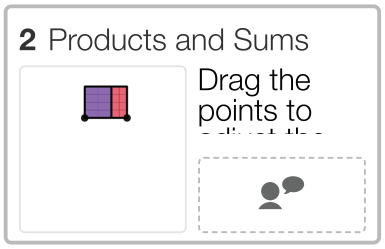
- 12
- $3x$
- $4x$

Student Supports

Students With Disabilities

Memory: Visual Aids

Consider inviting students with working memory challenges to record their work for each problem on a piece of blank paper to refer back to throughout the remainder of the lesson and unit.



2 Products and Sums

Drag the points to

Drag the points to adjust the rectangle.

What do you notice about the *product* expression?

What do you notice about the *sum* expression?

Teacher Moves

Overview: In Activity 1 (Screens 2–6), students are introduced to two ways of expressing the area of a rectangle.

Launch

- Start with the activity paused and demonstrate how to manipulate each movable point using the dashboard's student view.
- Review the terms *product* and *sum*, and ask: *Why do you think the expression on the left is called a product? Why do you think the expression on the right is called a sum?*

Facilitation

- Give students 2–3 minutes to change the structure of the rectangle and notice how it affects the product and sum (MP7). Consider asking: *Can you make the* $2x$ *into a* $3x$*?* $1x$*? Can you make the height* 5*?* 2*?*
- Facilitate a discussion to share students' noticings.
- If it does not come up naturally, consider asking if the two expressions are equivalent.

Discussion Questions

- *How is the product related to the rectangle?*
- *How is the sum related to the rectangle?*
- *Are the product and sum equivalent expressions? Why or why not?*

Math Community

- Consider celebrating variety and creativity in what students notice, including things that surprise you or that you think other students may not have noticed.

Routine: Consider using the routine Notice and Wonder to support students in making sense of the task.

Suggested Pacing: Screen 2

Sample Responses

Responses vary.

Product:

- I notice that the length is always the number in front.

- I notice that there is always a $+$ sign between the two parts inside the parentheses.
- I notice that the part inside the parentheses matches the width of the rectangle.

Sum:

- I notice that the number of purple tiles is the left part of the sum and the number of red tiles is the right part of the sum.
- I notice that it is like putting together two rectangles from the warm-up.
- I notice there is always a $+$ sign.

Student Supports

Students With Disabilities

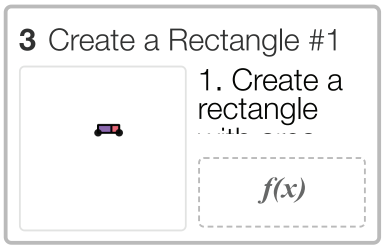
Fine Motor Skills: Strategic Pairing

Allow students who struggle with fine motor skills to dictate use of the sketch tool and draggable points as needed throughout the lesson and unit.

Multilingual Learners

Expressive Language: Eliminate Barriers

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.


3 Create a Rectangle #1

1. Create a rectangle

1. Create a rectangle with area $2(3x+4)$.

Teacher Moves

Launch

- Consider sharing with students that in the rest of the activity, we will be making and describing rectangles in two different ways, just like on the previous screen.

Facilitation

- Consider inviting students who struggle to start by just choosing one number in the product expression to focus on creating at a time, and then to press “Check My Rectangle” and use the feedback to revise.
- Monitor for different equivalent expressions that students write. When most students have responded, consider recording the product expression $2(3x+4)$ along with several equivalent expressions on the board for students to refer to throughout the lesson.

Math Community

- Consider celebrating variety and creativity in what students write as equivalent expressions.

Suggested Pacing: Screens 3–6

Sample Responses

Image solution

Expressions vary.

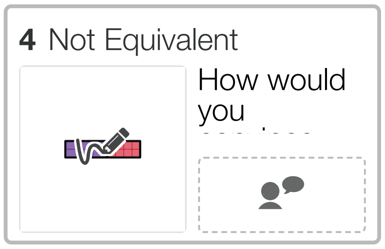
- $6x + 8$
- $2(4 + 3x)$
- $8 + 6x$
- $3x + 4 + 3x + 4$

Student Supports

Students With Disabilities

Executive Functioning: Eliminate Barriers

To support with organization in problem solving, consider chunking this activity by inviting students to focus on one number in the expression at a time.



4 Not Equivalent

How would you

How would you convince someone that $2(3x + 4)$ is **not** equivalent to $6x + 4$?

Teacher Moves

Facilitation

- Invite students to consider why someone might think these two expressions are equivalent before focusing on why they are not (MP3).

Discussion Questions

- *What could you change about* $6x + 4$ *to make it equivalent to* $2(3x + 4)$*?*

Math Community

- Consider inviting students to share what they think we can learn from looking at both correct and incorrect thinking.

Sample Responses

Responses vary.

- Count all the tiles. There are $6x$ and 8, not just 4.
- Multiply each of the numbers inside the parentheses by the number outside to check your expression.

Student Supports

Students With Disabilities

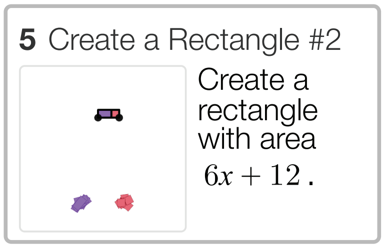
Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., $2(3x+4)$ is not equivalent to $6x+4$ because ______.).



5 Create a Rectangle #2

Create a rectangle with area $6x+12$.

Create a rectangle with area $6x+12$.

Teacher Moves

Facilitation

- Encourage students to create more than one rectangle with an area of $6x+12$. There are many correct rectangles.
- **Note:** The largest rectangle that can be created is $10(6x+8)$.

Sample Responses

Responses vary.

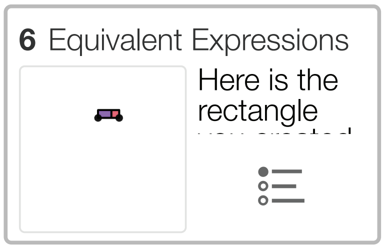
Image solution

Student Supports

Students With Disabilities

Executive Functioning: Eliminate Barriers

Invite students to plan how they will create their rectangle with a partner before trying it.



6 Equivalent Expressions

Here is the rectangle

Here is the rectangle you created on the previous screen.

Select **all** the expressions that are equivalent to $6x + 12$.

Adjust the rectangle if it helps you with your thinking.

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to surface strategies for determining if two expressions are equivalent.

Facilitation

- Consider displaying the distribution of responses, calling attention to any conflict or consensus you see.
- Discuss each possible response. Invite students to share their reasoning.
- If it does not come up naturally, consider asking: *How could you decide without using a rectangle?*
- Spend adequate time to ensure students can explain why $3(x + 4)$ and $6(x + 12)$ are incorrect.

Discussion Questions

- *How did you decide if this expression is equivalent? (MP7)*
- *Why might someone think this is equivalent?*

Early Finishers

- Encourage students to write an equivalent expression for every expression they did **not** select.

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Sample Responses

- $6(x + 2)$
- $3(2x + 4)$
- $2(3x + 6)$

Student Supports

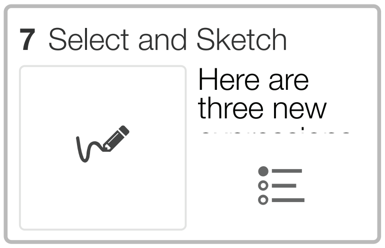
Students With Disabilities

Executive Functioning: Eliminate Barriers

To support organization in problem solving, consider chunking this activity by inviting students to consider each expression one at a time.

Receptive Language: Processing Time
Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners
Expressive Language: Eliminate Barriers
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



7 Select and Sketch

Here are three new

Here are three new expressions.

1. Select the **two** that are equivalent.

Teacher Moves

Overview: In Activity 2 (Screens 7–8), students develop fluency with writing equivalent expressions that represent the area of a rectangle.

Progress Check
- Use the dashboard's summary view to monitor students' thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Note: This is the first time students are searching for equivalent expressions without being given a rectangle.

Suggested Pacing: Screens 7–8

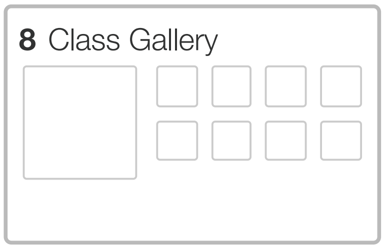
Sample Responses

$4(x+2)$ and $4x+8$

Sketches vary.

Student Supports

Students With Disabilities
Visual-Spatial Processing: Eliminate Barriers
Give students objects like pieces of paper or tiles to model each of the expressions.



8 Class Gallery

Teacher Moves

Overview: In this Challenge Creator, students create their own rectangle. Students then challenge themselves and their classmates to write equivalent expressions to represent the area of the rectangle.

Launch

- It may be helpful to demonstrate that students can rotate their rectangle using the icon in the top-right corner.

Facilitation

- Give students several minutes to create their own challenge and more time to solve their classmates' challenges.
- Encourage students to go back and review their classmates' responses to the challenge they created.
- While students are working, monitor for and highlight creative challenges and solutions.
- **Note:** We anticipate this Challenge Creator may take 10 minutes or more.

Math Community

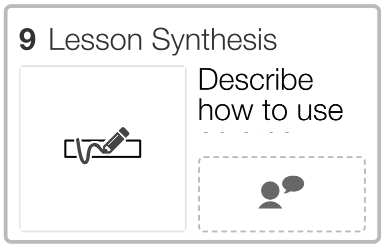
- Consider inviting students to share challenges they found particularly fun or creative.

Student Supports

Students With Disabilities

Executive Functioning: Eliminate Barriers

Consider helping students that are struggling to get started by identifying one or two challenges other students have created that might be a good starting point for them.



9 Lesson Synthesis

Describe how to use

Describe how to use an area model to write two or more equivalent expressions.

Use the example if it helps you show your thinking.

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to summarize two or more ways of writing the area of a rectangle (MP7).

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If it does not come up naturally, invite students to write at least two expressions to represent the rectangle on the left.
- If time allows, give students one minute to make their responses stronger and clearer based on the discussion.

Discussion Questions

- *How is ________'s expression related to the rectangle?*
- *How can we know that these expressions are equivalent?*

Math Community

- Invite students to share strategies for writing expressions that they've found most helpful and to attribute them to the students who shared them.

Suggested Pacing: Screen 9

Sample Responses

Responses vary. You can write one expression by adding up the areas of each smaller rectangle, and you can write another expression by multiplying the length by the width of the rectangle. In this rectangle, the product is $2(3x+4)$ and the sum is $6x+8$.

Student Supports

Students With Disabilities

Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., You can write an expression from an area model by ______.).

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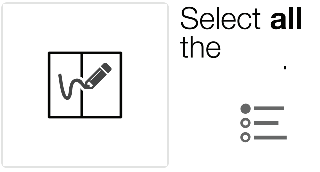
10 Cool-Down



Select **all** the

Select **all** the expressions that represent the area of the rectangle.

Teacher Moves

Support for Future Learning

If students struggle, plan to emphasize this when opportunities arise in Lesson 9, particularly during the warm-up and Activity 1.

Suggested Pacing: Screens 10–11

Sample Responses

- $5(x + 3)$
- $5x + 15$

11

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use an area model to write equivalent expressions.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Products, Sums, and Differences (NYC)

Lesson 9: Distributive Property Part 2

Purpose

This is the second of two lessons in which students study the distributive property. In this lesson, students strengthen the work they did in the previous lesson as well as extend their thinking to expressions that involve subtraction or expressions in which the common factor is a variable. Students may use area models or other strategies to support their thinking.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Expression Cards

- Print and cut one half sheet for each pair of students.

Materials

- Index cards or slips of colored paper
- Tape or glue (for attaching cards to the Student Worksheet)

Warm-Up (5 minutes)

Overview: Students revisit writing an expression for the area of a rectangle that includes variables.

Launch

- Invite students to work *individually.*
- Display Sheet 1 of the Teacher Projection Sheets.
- Consider asking: *What from the previous lesson might be helpful here?*

Facilitation

- Give students 1–2 minutes to think independently, then share their responses with a partner.
- To support students getting started, consider inviting them to cover up part of each rectangle and focus on one part of each rectangle at a time.
- Consider writing the area of each rectangle as a sum and a product on the board for students to refer to throughout the lesson.

Discussion Questions

- *Why might someone else have selected the other rectangle? What would you say to them?*
- *Rectangle A has an area of* $6x + 3$*. Someone said that* $6x + 3$ *is equal to* $9x$*. Do you agree?*

Math Community

- Ask students to imagine what someone who chose differently might have been thinking.

Activity 1: Card Sort (15 minutes)

Overview: Students connect expressions written as sums and products to area models, including expressions that have multiple variables and expressions in which the common factor is a variable.

Launch

- Invite students to work *in pairs.*
- Distribute one Student Worksheet and a set of cards to each pair of students. Optionally, distribute tape or glue to each pair of students.
- Review the instructions as a class, including revisiting what the terms *product* and *sum* mean.

Facilitation

- Give students several minutes to sort and then match the cards.
- Consider pausing to celebrate different ways that students sorted the cards in Problem 1 in order to call attention to different features of the expressions.
- Encourage students to justify each card placement before putting it on the worksheet.
- Consider inviting students to use color or arrows to make connections between the area model, product, and sum (MP2).
- Invite students to compare their matches with another pair and justify their reasoning before taping or gluing the expression cards on the worksheet.

Discussion Questions

- *How did you decide* whether $3x + 6$ matched row A or B?
- *How did you decide whether* $3a + 3b$ *matched row E or F?*
- *How was row C different from the other rows? How can we see that in your product and sum?*

Math Community

- Consider highlighting the value of changing one's mind by asking if any students revised their thinking as they completed the card sort.

Early Finishers

- Encourage students to draw an area model to represent each of the leftover cards.

Support for Students With Disabilities

Visual-Spatial Processing: Eliminate Barriers

To support students in distinguishing between expressions, consider inviting them to pick out all of the expressions with parentheses and explain to a partner how they are different from the other expressions before matching. It may also be helpful to read each expression aloud as a pair to ensure both partners are understanding the expression the same way.

Intermission (5 minutes)

Overview: Students use repeated reasoning to extend what they have learned about equivalent expressions involving addition to expressions involving subtraction (MP8).

Launch

- Display Sheet 2 of the Teacher Projection Sheets.

Facilitation

- Give students one minute to think independently, record their responses on the worksheet, share with a partner, and then share with the class.
- When the class has come to consensus on Problem 1, display Sheet 3 and repeat for the second problem.
- It may be helpful to substitute a value for each variable, such as $k = 10$ and $w = 3$ for students who are feeling unsure. While this does not *prove* equivalence, it can help students see how the two expressions are related.
- If students write expressions like $3(k - 2) + 0$ or $3(k - 2) \cdot 1$ for the first problem, celebrate the fact that these expressions *are* equivalent to $3(k - 2)$. Then highlight other correct responses, like $3k - 6$.

Support for Multilingual Learners
Receptive Language: Eliminate Barriers
Invite students to read each statement aloud and paraphrase what it means, clarifying any unknown words or concepts before answering.

Activity 2: Writing Equivalent Expressions (10 minutes)

Overview: Students develop fluency in writing equivalent expressions as products and as sums in a social way.

Launch

- Invite students to work *in pairs.*
- Depending on the choice of facilitation, it may be helpful to demonstrate one round for students as the routine may be new.

Facilitation

- There are several options for facilitating this activity:
    - Invite students to work on Problems 1 and 2 with a partner and justify their thinking before recording their responses. After 2–3 minutes, swap partners, compare solutions to the problems completed thus far, and collaborate on Problems 3 and 4. Repeat.

  - ○ Give students 3–4 minutes to work independently, then to select the problem they want to discuss most with a partner. When time is up, give each partner time to discuss the problem they chose.
  - ○ Invite one student to talk aloud as they think through one problem while the other student records their thinking and asks clarifying questions until both students agree on the correct expression. Switch for the next expression and repeat.
- If time allows, discuss the two problems students found most challenging as a class.

Support for Students With Disabilities
Conceptual Processing: Checks for Understanding
For expressions with subtraction, invite students to think of a similar expression with addition that they might be able to draw a rectangle to represent.

Lesson Synthesis (5 minutes)

Key Discussion 🔑
The purpose of this discussion is for students to consolidate strategies for convincing themselves that two expressions are equivalent.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their responses based on the discussion.

Discussion Questions
- *How would you say what ________ said in your own words?*
- *When would ______'s explanation be helpful? When might it be less helpful?*

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Support for Multilingual Learners
Expressive Language: Eliminate Barriers
Provide sentence frames to help students explain their thinking (e.g., I know $6 - 2x$ and $2(3 - x)$ are equivalent because ______.).

Cool-Down (5 minutes)

Support for Future Learning
Consider reviewing this question as a class before Practice Day 1 or offering individual support where needed during the practice day. Students will need to know how to write equivalent expressions on the End Assessment.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

6.6 Practice Day 1 (NYC)

Preparation

Scavenger Hunt Sheets

- Print one single-sided set.
- Shuffle the sheets and post them around the classroom, hallway, or outside. To reuse, consider laminating or using sheet protectors.

Student Workspace Sheet

- Print a double-sided workspace sheet for each student (or pair of students).

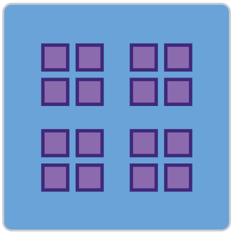
Facilitation

- Invite students to work individually or in pairs.
- In order to avoid too many students at one of the posted problems, either assign students different starting problems or invite students to start at the nearest problem and record their thinking on their workspace sheet.
- Encourage students at the same sheet to discuss their strategies and answers.
- When they are confident in their answer, invite students to search for their answer at the top of another scavenger hunt sheet, then answer the question on that sheet. If students cannot find their answer, invite them to revisit and revise their work.
- Students should continue until they have answered all 10 questions and are back to their starting point.

Early Finishers

- Invite students who solve all of the questions to work on the “Are You Ready for More?” question. The solution is not posted but is available on the answer key.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Powers

Lesson 10: What Are Exponents?

Overview

Students make sense of exponents, both when the base is a whole number and when it is a fraction.

Learning Goals

- Explain the meaning of an expression with an exponent, such as 3^5.
- Decide if two expressions involving exponents are equivalent and explain the reasoning.

Vocabulary

- exponent
- to the power of

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to introduce exponents, both when the base is a whole number and when it is a fraction. Students use visuals to explore how changing an exponent changes the value of an expression and engage in MP8 by connecting these visuals to expressions involving exponents. They use the language of equivalent expressions from earlier in the unit to write expressions with exponents as repeated multiplication and vice versa.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to use the Notice and Wonder routine to explore exponents in a visual way.

Activity 1: Exponents With Whole Number Bases (15 minutes)

The purpose of this activity is for students to use visuals and expressions to describe exponents with whole number bases. Students make connections between exponent notation and repeated multiplication. This activity also introduces students to the phrase “to the power of” as a way to describe exponent expressions.

Activity 2: Exponents With Fractional Bases (15 minutes)

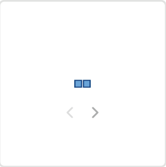
The purpose of this activity is for students to extend what they explored in Activity 1 to exponents with bases that are fractions. Students use reasoning similar to the reasoning they did with integers to calculate the value of exponent expressions like $\left(\frac{1}{2}\right)^3$ and $\left(\frac{1}{3}\right)^4$.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to summarize what they learned about exponents, particularly connecting exponents to repeated multiplication.

Cool-Down (5 minutes)

1 Warm-Up



Click the arrows to see the image change.

Click the arrows to see the image change.

Discuss what you notice and wonder.

Teacher Moves

Overview: In this lesson, students make sense of exponents, both when the base is a whole number and when it is a fraction. In this warm-up, students use the Notice and Wonder routine to explore exponents in a visual way.

Launch

- Invite students to work *in pairs.*
- Consider asking students if they have ever seen an expression that looks like this before.

Facilitation

- Give students 1–2 minutes to click the arrows and observe the image, discussing what they notice with a partner.
- Invite several students to share what they notice and wonder.

Discussion Questions

- *Have you ever seen an expression like this one before, with a little number up in the corner?*

Math Community

- Consider celebrating variety and creativity in what students notice and wonder, including things that surprise you or that you think other students may not have noticed.

Readiness Check (Problem 5)

- If most students struggled, consider reviewing this problem before beginning this lesson. Encourage students to think of why someone might have selected Duri or Nasir and to make arguments as a class.
- Consider creating an anchor chart or leaving Nasir's work visible for students to reference throughout the lesson.

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

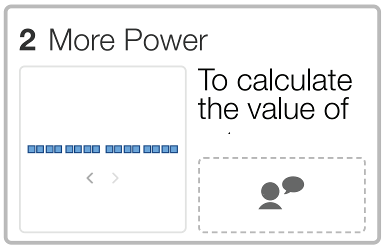
- I notice that there are always groups of 2 squares.
- I notice that the number goes in a pattern: 2, 4, 8, 16.
- I notice that the small number is not the same as the number of groups of 2.

- I wonder what the small number means.
- I wonder what 2^{10} would look like.
- I wonder if the number of tiles would ever fill up the screen.

Student Supports

Students With Disabilities
Fine Motor Skills: Strategic Pairing
Allow students who struggle with fine motor skills to dictate use of the sketch tool and clickable buttons as needed throughout the lesson and unit.

Memory: Visual Aids
Consider inviting students with working memory challenges to draw each image and expression on a piece of blank or graph paper to refer back to throughout the lesson and unit.



2 More Power

To calculate the value of

To calculate the value of 2^4 (2 *to the power of* 4), you can multiply $2 \cdot 2 \cdot 2 \cdot 2$.

What can you do to figure out the value of 2^5?

Teacher Moves

Overview: In Activity 1 (Screens 2–8), students use visuals and expressions to describe exponents with whole number bases.

Launch
- Consider reading the first sentence aloud and inviting students to practice saying the phrase "2 to the power of 4."

Facilitation
- Monitor for students who use different strategies for explaining, including using the visual or the repeated multiplication.

Suggested Pacing: Screens 2–5

Sample Responses

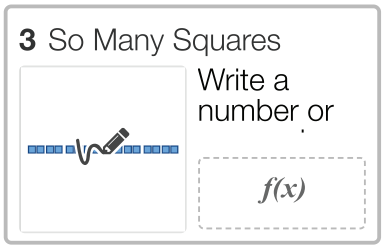
Responses vary.
- Double the number of blue tiles from 2^4.
- Multiply 2 five times, so $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Student Supports

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their strategy (e.g., First, ________. Then, _______.).



3 So Many Squares

Write a number or

Write a number or expression that is equivalent to 2^5.

Teacher Moves

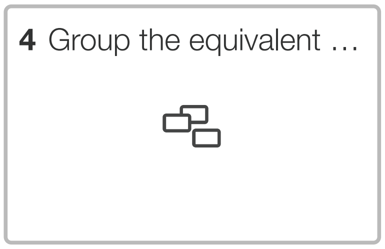
Facilitation

- Consider demonstrating how to type exponents in the math input using the ^ key (usually shift-6). Alternatively, consider showing students how to use the Desmos keypad and the a^b button to type exponents.
- Monitor for students who write expressions like $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ or $16 \cdot 2$ or $2^4 \cdot 2$.
- When most students have responded, use the snapshot tool or dashboard's teacher view to highlight student expressions and values for 2^5.

Sample Responses

Responses vary but are all equivalent to 32.

- 32
- $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
- $16 \cdot 2$
- $2^4 \cdot 2$



4 Group the equivalent …

Teacher Moves

This card sort is designed to highlight the difference between multiplication and exponents (MP7).

Facilitation

- Encourage students to share their reasoning with a partner and work to reach an agreement together about how to sort the cards.

Math Community
- Consider highlighting the value of changing one's mind by asking if any students revised their thinking as they were working on this card sort.

Sample Responses

Image solution

Student Supports

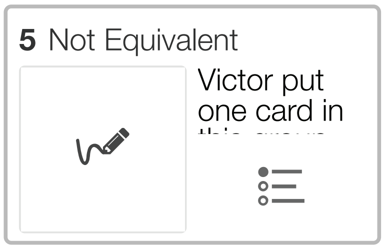
Students With Disabilities

Conceptual Processing: Eliminate Barriers

To assist students in recognizing connections between new problems and prior work, consider inviting them to go back to Screen 3 and asking: *How are these related to what we were trying to do on this screen?*

Executive Functioning: Eliminate Barriers

Chunk this activity into more manageable parts by inviting students to choose one card at a time and finding any matching equivalent expressions.



5 Not Equivalent

Victor put one card in

Victor put one card in this group that is not equivalent to the others.

Which card is **not equivalent** in this group?

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to make connections between exponents and repeated multiplication.

Facilitation
- Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.
- Facilitate a discussion to explain why each choice is or is not equivalent to the group (MP3).

Discussion Questions
- *How do you know __________ is/is not equivalent?*

Early Student Thinking
- Students may think that $2^4 \cdot 2$ is not equivalent because it does not have five 2 s.
- Consider asking: *How could I write* 2^4 *using multiplication?*

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Early Finishers

- Encourage students to write other expressions that are equivalent to this group.

Sample Responses

$2 + 2 + 2 + 2 + 2$

Explanations vary.

- The value of $2 + 2 + 2 + 2 + 2$ is 10, which is not equal to $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$.
- Exponents are like multiplication over and over again so $2 + 2 + 2 + 2 + 2$ doesn't belong because it has addition.

Student Supports

Students With Disabilities

Receptive Language: Processing Time

Read the prompt aloud for students who benefit from extra processing time.

Visual-Spatial Processing: Visual Aids

To support students in distinguishing between similar expressions, invite them to read each expression aloud and agree about what it says, or to choose expressions that look similar and explain how they are different before responding.

Multilingual Learners

Expressive Language: Eliminate Barriers

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

6 Challenge #1

Select an expression

Select an expression that is equivalent to 3^4.

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Progress Check

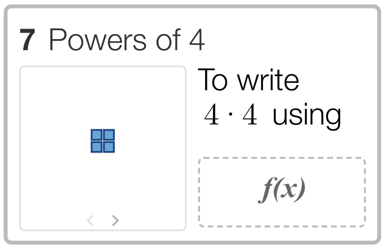
- Use the dashboard's summary view to monitor students' thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 6–8

Sample Responses

Responses vary.

- $3 \cdot 3 \cdot 3 \cdot 3$
- 81



7 Powers of 4

To write $4 \cdot 4$ using

To write $4 \cdot 4$ using exponents, you can write 4^2, where 2 is an *exponent*.

Enter $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ using exponents.

Teacher Moves

Progress Check

- Use the dashboard's summary view to monitor students' thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Note: This screen introduces the term *exponent.* If it makes sense in your class, consider also including the term *base* to describe the number that is being multiplied repeatedly.

Sample Responses

Responses vary.

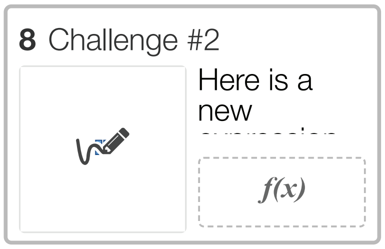
- 4^5
- $4^4 \cdot 4$

Student Supports

Students With Disabilities

Memory: Visual Aids

Consider inviting students with working memory challenges to draw the first two or three images and expressions on a piece of blank or graph paper to refer back to throughout the remainder of the lesson and unit.



8 Challenge #2

Here is a new

Here is a new expression: 4^3.

Write a number or expression that is equivalent to 4^3.

Teacher Moves

Facilitation

- Monitor for students who write expressions like $4 \cdot 4 \cdot 4$ or $16 \cdot 4$.
- When most students have responded, use the snapshot tool or dashboard's teacher view to highlight student expressions and values for 4^3.

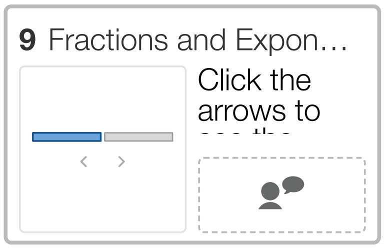
Early Finishers

- Encourage students to decide which is larger, 4^5 or 5^4, and then to convince a classmate.

Sample Responses

Responses vary, but are all equivalent to 64.

- 64
- $4 \cdot 4 \cdot 4$
- $16 \cdot 4$
- $4^2 \cdot 4$



9 Fractions and Expon...

Click the arrows to

Click the arrows to see the image change.

Write two things you know about $\left(\frac{1}{2}\right)^3$.

Teacher Moves

Overview: In Activity 2 (Screens 9–12), students extend what they explored in Activity 1 to exponents with bases that are fractions.

Launch

- Consider sharing the purpose of the activity with students and inviting them to predict how fractions might act differently from whole numbers.

Facilitation

- Give students 1–2 minutes to explore the interaction on the left, then to stop at $\left(\frac{1}{2}\right)^3$ and write two things they know.
- Monitor for students who write different things. Then use the snapshot tool or dashboard's teacher view to highlight students' responses.

Discussion Questions

- *Where do you see the* $\frac{1}{2}$ *in the visual of* $\left(\frac{1}{2}\right)^3$*?*
- *What do you think the* 3 *means in* $\left(\frac{1}{2}\right)^3$*?*

Suggested Pacing: Screen 9

Sample Responses

Responses vary.

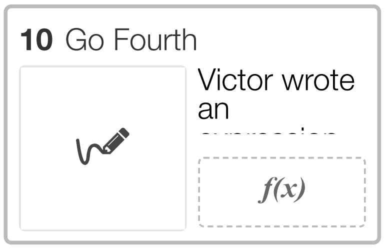
- I know it is smaller than 1 because 1 would be the whole rectangle.
- I know that the numbers keep splitting in half every time.
- I know that it has 1 blue block and 7 unshaded blocks.
- $\left(\frac{1}{2}\right)^3$ is 1 out of 8.

Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers

To assist students in recognizing the connections between new problems and prior work, consider asking them to reflect on how the image changes every time the exponent changes. Ask: *How is* $(\frac{1}{2})^2$ *similar to* $(\frac{1}{2})^1$*? How is it different?*


10 Go Fourth

Victor wrote an

Victor wrote an expression that is equivalent to $\left(\frac{1}{3}\right)^4$.

Write a different expression that is equivalent to $\left(\frac{1}{3}\right)^4$.

Teacher Moves

Launch

- Consider starting with the activity paused and giving students one minute to explain to a partner where they think the expression $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$ came from.

Facilitation

- Monitor for students who write different expressions or who struggle to multiply $\frac{1}{9}$ by $\frac{1}{9}$.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Suggested Pacing: Screens 10–12

Sample Responses

Responses vary but are all equivalent to $\frac{1}{81}$.

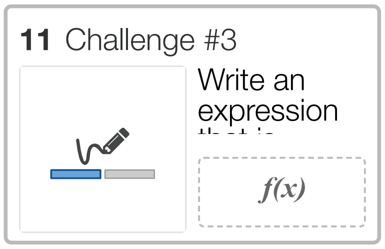
- $\frac{1}{81}$
- $\left(\frac{1}{3}\right)^3 \cdot \frac{1}{3}$
- $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^2$

Student Supports

Students With Disabilities

Conceptual Processing: Checks for Understanding
Invite students to talk aloud about each step of Victor's work to help them verbalize what concepts make and do not make sense.

Visual-Spatial Processing: Highlighting
To support students in making visual connections between each step of Victor's thinking, invite them to use colored pencils or highlighters to draw connections between each step.



11 Challenge #3

Write an expression

$f(x)$

Write an expression that is equivalent to $\left(\frac{1}{2}\right)^5$.

Teacher Moves

Progress Check

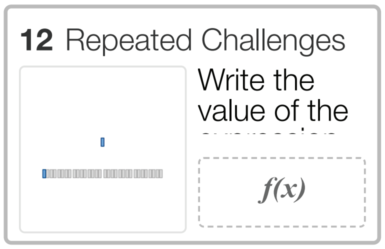
- Use the dashboard's summary view to monitor students' thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Note: This is the type of thinking that students will be asked to do on the next screen in the repeated challenges.

Sample Responses

Responses vary but are all equivalent to $\frac{1}{32}$.

- $\frac{1}{32}$
- $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
- $\left(\frac{1}{2}\right)^4 \cdot \frac{1}{2}$
- $\left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} \cdot \frac{1}{2}$



12 Repeated Challenges

Write the value of the

$f(x)$

Write the value of the expression 2^3.

(Challenges vary.)

Teacher Moves

How Repeated Challenges Work

- Students are presented with a variety of challenges one at a time (in this case, determining the value of a number to an exponent).
- The challenges typically increase in difficulty as they continue (in this case, by including bases that are fractions).

Facilitation

- Give students 5–7 minutes to complete as many challenges as they can.

- Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

Math Community
- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Sample Responses

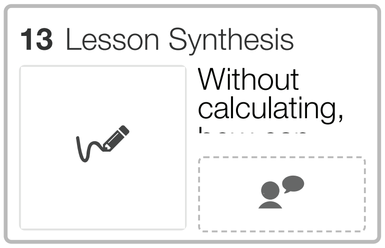
There are 10 total challenges.

The first few responses are:
- 8
- 16
- $\frac{1}{9}$

Student Supports

Students With Disabilities
Conceptual Processing: Checks for Understanding
Invite students to talk aloud about their thinking to help them verbalize which concepts make sense and which do not.

Students With Disabilities
Memory: Visual Aids
Consider inviting students with working memory challenges to record their work for each problem on a piece of blank paper to refer back to throughout the remainder of the lesson and unit.



13 Lesson Synthesis

Without calculating,

Without calculating, how can you tell whether expressions with exponents are equivalent?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to summarize what students have learned about exponents, particularly connecting exponents to repeated multiplication.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.

- If it does not come up naturally, invite students to share which expressions are equivalent and how they know (MP3).

Discussion Questions

- *Which of these expressions are equivalent? How do you know?*
- *What connections do you see between your classmates' strategies?*

Math Community

- Invite students to share information they have learned about exponents and to attribute them to the students who shared them.

Suggested Pacing: Screen 13

Sample Responses

Responses vary. You can tell if expressions with exponents are equivalent by checking the number of times a value is multiplied. 11^5 is equivalent to $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$ and $11^4 \cdot 11$ because they each multiply 11 five times.

Student Supports

Students With Disabilities

Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., You can tell whether expressions with exponents are equivalent by ______.).

---

14 Cool-Down

Teacher Moves

Support for Future Learning

If students struggle, plan to emphasize this when opportunities arise in Lessons 11 or 12, with a focus on connecting exponents to area and volume.

Suggested Pacing: Screens 14–15

Sample Responses

Image solution

15

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain what an expression with an exponent means (e.g., 3^5).
- I can decide whether two expressions that include exponents are equivalent.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Exponent Expressions (NYC)

Lesson 11: Exponents and Order of Operations

Purpose

Students use areas to make sense of more complex expressions with exponents, particularly evaluating expressions that include an exponent and one other operation. Students notice that expressions with the same numbers in different arrangements or expressions using different operations represent different visuals and different values. This is an extension of the work students did with order of operations in Grade 5.

Note: Students may know PEMDAS or another mnemonic for remembering the order of operations from previous grades. If students bring these up, consider discussing that PEMDAS can be misleading because it seems to show that multiplication should be evaluated before division, and addition before subtraction.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Cards

- Print and cut one half sheet for each pair of students.

Materials

- Tape or glue (for attaching cards to the Student Worksheet)

Warm-Up (5 minutes)

Overview: Students revisit what they learned about exponents in the previous lesson using the Which One Doesn’t Belong routine.

Launch

- Display Sheet 1 of the Teacher Projection Sheets.
- Invite students to select a representation that does not belong and explain why.

Facilitation

- Give students one minute of time to think quietly; ask them to discreetly indicate when they have selected one representation that does not belong and can explain why.
- Encourage students to look for more than one possibility.
- Then, give students two minutes to share their responses with a partner and work together to find at least one reason *each* representation doesn't belong.
- If it does not come up naturally, consider determining the value of each expression as a class.

Math Community

- Invite students to share how hearing others' perspectives can be helpful to everyone.

Readiness Check (Problem 6)

- If most students struggled, consider reviewing the order of operations and the purpose of parentheses. Create a class resource to refer to throughout the rest of the unit.

Activity 1: What's Missing? (15 minutes)

Overview: Students attend to the structure of different expressions as they match them with diagrams and calculate their values (MP7).

Launch

- Invite students to work *in pairs.*
- Distribute one Student Worksheet and a set of cards to each pair of students. Optionally, distribute tape or glue to each pair of students.
- Display Sheet 2 of the Teacher Presentation Sheets and review the instructions as a class.

Facilitation

- Encourage students to justify each card placement before putting it on the worksheet.
- Consider inviting students to use color or arrows to make connections between the diagram, expression, and value (MP2).
- Consider checking in with pairs of students or posting an answer key. This is also a good opportunity for students to practice justifying their thinking with another pair.

Discussion Questions

- *What is the same about each of these expressions?*
- *What is different about each of these expressions?*
- *How did you decide which diagram matched* $(3 + 5)^2$*? What was its value?*

Early Finishers

- Encourage students to create their own expression, diagram, and value.

Support for Students With Disabilities

Executive Functioning: Eliminate Barriers

To support with organization in problem solving, consider chunking this activity by inviting students to choose one card and to figure out which row it matches, then repeat.

Visual-Spatial Processing: Visual Aids

To support students with visualizing the values of each expression, provide them graph paper so they can draw models of each area to scale.

Intermission (5 minutes)

Key Discussion 🔑

Students use diagrams and reasoning to settle a dispute about the correct order of operations when using exponents.

Launch

- Display Sheet 3 of the Teacher Projection Sheets.

Facilitation

- Give students a minute to think independently, share with a partner, then discuss as a class.
- When the class has come to consensus on Problem 1, display Sheet 4 and repeat.
- It may be helpful to think about what expression Latifa evaluated [$(2 + 10)^2$] and what diagram from Problem 1 represents her thinking [Diagram C].
- Spend adequate time here for students to understand why Nicolas's value is correct and how this relates to the order of operations.

Support for Students With Disabilities

Visual-Spatial Processing: Visual Aids

To support students in making connections, provide printed copies of the Teacher Presentation Sheet for students to draw on or highlight.

Support for Multilingual Learners

Expressive Language: Eliminate Barriers

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

Activity 2: Partner Problems (10 minutes)

Overview: Students develop fluency in evaluating expressions with exponents in a social way.

Launch

- Invite students to work *in pairs.*
- Display Sheet 5 of the Teacher Projection Sheets.
- Assign a student in each pair to be Partner A and Partner B. Consider encouraging students to cut the worksheet down the middle and each work on their own half sheet.

Facilitation

- Give students several minutes to solve each problem, share their reasoning with their partner, and reach an agreement together about what the correct value is.
- Remind students that they need to show at least one intermediate step for each problem in addition to the value.

- Encourage students to check each row with their partner as they work. If they get different answers, invite them to work together to find and fix any errors until their answers match.
- If time allows, discuss the two problems students found most challenging as a class.
- It may be helpful to pause and review what an exponent to the power of 1 means.

Early Student Thinking

- If students struggle to decide in what order to complete the operations, consider inviting them to review their work in Activity 1 and sketch an area model first that could be represented by the expression.

Math Community

- Before beginning the activity, consider inviting students to think about what it should look like and sound like to support each other without giving answers. After the activity, allow one minute for students to reflect on what they and their partner did well and what they would improve.

Routine (optional): Consider pausing and using the routine Critique, Correct, Clarify to help students communicate about errors in their calculations.

Support for Students With Disabilities

Executive Functioning: Eliminate Barriers

To support with organization in problem solving, consider giving students lined paper or a graphic organizer to write their thinking one step at a time.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

The purpose of this discussion is to surface the importance of order of operations and what would be helpful to remember when evaluating expressions with exponents.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their responses based on the discussion.

Discussion Questions

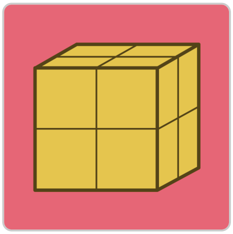
- *How can we decide what to do first when evaluating expressions with exponents?*
- *What mistakes did you make today that you or others might learn from?*

Routine (optional): Consider using the mathematical language routine Collect and Display to capture students' ideas about what is important to remember when evaluating expressions that include an exponent and other operations.

Cool-Down (5 minutes)

Support for Future Learning
If students struggle, consider spending extra time during Lesson 12 discussing order of operations when students are evaluating expressions with exponents and variables.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Squares and Cubes

Lesson 12: Exponent Expressions With Variables

Overview

Students extend what they have learned about numerical expressions with exponents to evaluate variable expressions that involve exponents and other operations.

Learning Goals

- Evaluate expressions that have a variable, an exponent, and one other operation for a given value of the variable.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to extend what they have learned about numerical expressions with exponents to evaluate variable expressions with exponents. Students use diagrams of squares and cubes to visualize what different expressions with exponents mean, then evaluate those expressions for different values of a variable. This lesson also revisits the concept of equivalent expressions.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to revisit what they have learned about using area to represent exponent expressions to the power of 2.

Activity 1: Variable Expressions With Area (10 minutes)

The purpose of this activity is for students to calculate the values of variable expressions that involve powers of 2. Students use area diagrams to calculate the values of expressions at specific values of a variable and consider how $(x+3)^2$ differs from x^2+3^2.

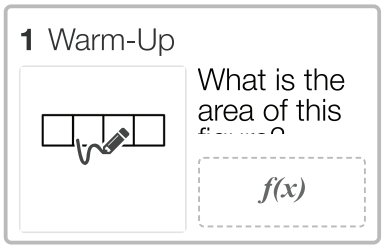
Activity 2: Cubes and Squares (20 minutes)

The purpose of this activity is for students to extend their thinking about powers of 2 to evaluate variable expressions that involve the power of 3. Students consider how $(2x)^3$ differs from $2x^3$, both in terms of their diagrams and their values for different values of a variable. This activity also supports students in developing fluency with a set of repeated challenges.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to summarize strategies for evaluating expressions that include exponents for a given value of a variable.

Cool-Down (5 minutes)



1 Warm-Up

What is the area of this

What is the area of this figure?

Teacher Moves

Overview: In this lesson, students extend what they have learned about numerical expressions with exponents to evaluate variable expressions that involve exponents and other operations. In this warm-up, students revisit what they have learned about using area to represent exponent expressions to the power of 2.

Launch

- Invite students to work *individually.*
- Consider asking: *What from the last lesson might help us here?*

Facilitation

- Give students 1–2 minutes to work independently, then share their responses with a partner.
- Monitor for students who calculate the area of each square and then multiply by 4, or who calculate the total width and calculate $5 \cdot 20$.

Discussion Questions

- *Is there another way you could have calculated the area?*
- *Where do you see each part of* $4(5)^2$ *in the diagram on the left?*

Early Finishers

- Encourage students to write equivalent expressions to $4(5)^2$ based on the diagram.

Suggested Pacing: Screen 1

Sample Responses

100 square units

Student Supports

Students With Disabilities

Fine Motor Skills: Strategic Pairing

Allow students who struggle with fine motor skills to dictate use of the sketch tool and clickable buttons as needed throughout the lesson and unit.

Visual-Spatial Processing: Visual Aids

To support students in visualizing the values of each expression, provide them graph paper so they can draw a model of the rectangle to scale.

2 Match each expressi…

Teacher Moves

Overview: In Activity 1 (Screens 2–5), students calculate the values of variable expressions that involve powers of 2.

Launch

- Consider inviting students to think about how these diagrams are similar to and different from the ones in the warm-up or from Lesson 11.
- Students may notice that these diagrams and expressions have variables. Consider asking: *What do you think it means if a diagram has a variable in it?*

Facilitation

- This card sort is designed for students to attend to the structure of the expressions and the diagrams. Consider monitoring for students who make comments about how parts of the expressions are seen in the diagrams (MP7).
- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- When most students have sorted, consider pausing and inviting students to justify to the class one or two matches in order to support students drawing their own diagrams if they are helpful later in the lesson.

Discussion Questions

- *How did you know _____ and ______ matched?*
- *How are* $x^2 + 4$ *and* $(x + 4)^2$ *different?*

Suggested Pacing: Screens 2–5

Sample Responses

Image solution

Student Supports

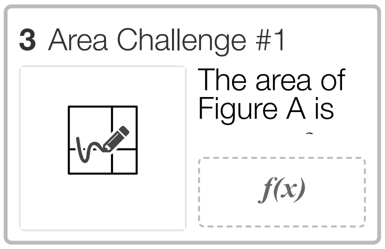
Students With Disabilities

Conceptual Processing: Eliminate Barriers

To assist students in recognizing the connections between new problems and prior work, consider asking: *How are these pictures similar and different from the ones we explored on the previous lesson?*

Executive Functioning: Eliminate Barriers

Chunk this activity into more manageable parts by inviting students to choose one expression card at a time and matching it to its diagram card.



3 Area Challenge #1

The area of Figure A is

The area of Figure A is $(x+3)^2$ square units.

What is the area when $x=4$?

Teacher Moves

Progress Check

- Use the dashboard’s summary view to monitor students’ thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

49 square units

Student Supports

Students With Disabilities

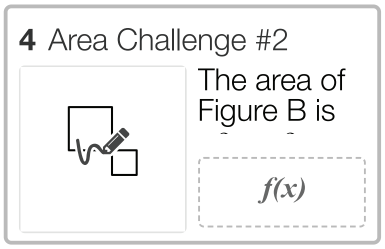
Memory: Visual Aids

Consider inviting students with working memory challenges to record their work for each problem on a piece of blank paper so that they can more easily compare their work on Screens 3 and 4.

Multilingual Learners

Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.



4 Area Challenge #2

The area of Figure B is

The area of Figure B is x^2+3^2 square units.

What is the area when $x=4$?

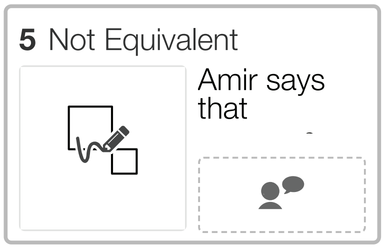
Teacher Moves

Facilitation

- To support students noticing that they are evaluating a different expression, consider asking them how this expression is different from the one on the previous screen.
- Use the dashboard’s summary view to monitor students’ thinking.

Sample Responses

25 square units


5 Not Equivalent

Amir says that

Amir says that $(x+3)^2$ and x^2+3^2 are equivalent.

Help him understand why they are **not** equivalent.

Use the sketch tool if it helps you show your thinking.

Teacher Moves

Facilitation

- Consider telling students that these diagrams and expressions are the same as the ones they saw on Screens 3 and 4.
- Encourage students to read their classmates' responses and compare their arguments to their own.
- If time allows, consider selecting and sequencing students' arguments for why these are not equivalent expressions (MP3).

Early Finishers

- Encourage students to write an expression that is equivalent to $(x+3)^2$.

Math Community

- Consider spending time discussing why Amir might think these expressions are equivalent to normalize that there is often a lot of correct thinking inside of incorrect ideas.

Sample Responses

Responses vary.

- When $x=4$, these expressions have different values so they can't be equivalent. $(4+3)^2=49$ and $x^2+3^2=25$.
- If you draw them, they make different pictures. $(x+3)^2$ is like Figure A and x^2+3^2 is like Figure B.

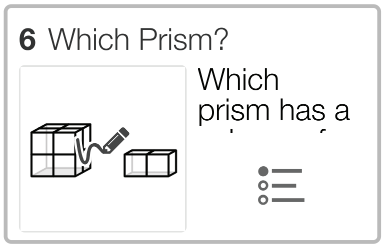
Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers

To assist students in recognizing the connections between this question and their work on Screens 3 and 4, consider asking: *How could your work on Screens 3 and 4 help you help Amir?*

Receptive Language: Processing Time
Read the prompt aloud for students who benefit from extra processing time.



6 Which Prism?

Which prism has a

Which prism has a volume of $(2x)^3$ cubic units?

Teacher Moves

Overview: In Activity 2 (Screens 6–12), students extend their thinking about powers of 2 to evaluate variable expressions that involve the power of 3.

Facilitation
- Consider giving students one minute to decide what they think with a partner. Then review as a whole class.
- If it does not come up naturally, consider asking how the parentheses affects which prism is represented and why (MP6).

Discussion Questions
- *What part of* $(2x)^3$ *do you think tells us we should be thinking about prisms and not areas like in Activity 1?*
- *Why are the parentheses important in the expression* $(2x)^3$*? What do they mean?*
- *What expression represents the volume of the other prism?*

Early Student Thinking
- Students may notice that Prism D has 2 cubes and Prism C has more than 2 cubes and so select Prism C.
- Consider asking these students what the volume of each prism would be if $x = 1$ and to compare that with the value of $(2x)^3$ if $x = 1$.

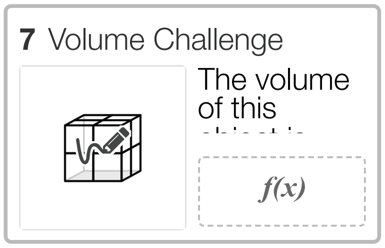
Suggested Pacing: Screen 6

Sample Responses

Prism C

Explanations vary.

- Prism C is like a cube that is $2x$ on each side, and that's what $(2x)^3$ means.
- If you plug in a number like $x=1$ then $(2\cdot 1)^3=2\cdot 2\cdot 2=8$. Prism D would only have a volume of 2 cubic units, not 8.



7 Volume Challenge

The volume of this

The volume of this object is $(2x)^3$ cubic units.

What is the volume when $x=3$?

Use paper if it helps you with your thinking.

Teacher Moves

Launch
- Consider sharing that on the next few screens students will explore strategies for calculating volumes of shapes like these.

Facilitation
- Encourage students to use the sketch tool and paper to help them with their thinking.
- Because students need to multiply $6\cdot 6\cdot 6$ in this problem, consider inviting them to write the multiplication out on paper and to confirm their calculations with a partner before checking.

Suggested Pacing: Screens 7–9

Sample Responses

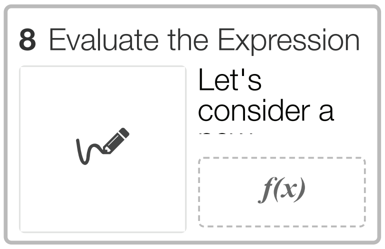
216 cubic units

Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers

Use objects like blocks to model Prism C. Ask: *How would we build this prism if* $x=3$*?*



8 Evaluate the Expression

Let's consider a

Let's consider a new expression: $2x^3$.

What is the value of $2x^3$ when $x = 5$?

Draw a diagram if it helps you with your thinking.

Teacher Moves

Facilitation

- **Note:** This is the first screen where students are asked to evaluate an expression without a diagram.
- To support students getting started, consider asking: *How is the expression* $2x^3$ *different from the expression on the previous screen,* $(2x)^3$*?*
- Consider monitoring for students who use strategies similar to Amir's or Chloe's on the next screen or other creative strategies. Invite these students to share their thinking during the discussion on Screen 9.

Sample Responses

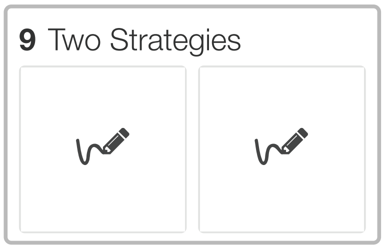
250

Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers

To assist students in recognizing the connections between new problems and prior work, consider asking: *How would a block prism of this look different from the prism on the previous screen?*



9 Two Strategies

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to make sense of two strategies for evaluating variable expressions at specific values of a variable.

Facilitation

- Encourage students to use the sketch tool to make connections between Amir's and Chloe's strategies.
- Spend adequate time discussing each strategy so that students have at least one strategy they feel comfortable with for evaluating these types of expressions.

Discussion Questions

- *How would you summarize Amir's strategy? Chloe's strategy?*
- *What are the advantages and disadvantages of Chloe's strategy? Amir's?*
- *How would each student approach evaluating* $2x^3$ *when* $x=10$*?*

Early Finishers

- Encourage students to use each strategy to evaluate $(2x)^3$ when $x=5$.

Math Community

- Consider renaming Amir's and Chloe's strategies after the students in your class who used them.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary. Both people calculated $5 \cdot 5 \cdot 5$ at some point, which shows the volume of one cube. Amir drew a picture and Chloe wrote out a sequence of numerical expressions. Chloe substituted 5 where x was and Amir made a 5-by-5-by-5 cube.

Student Supports

Students With Disabilities

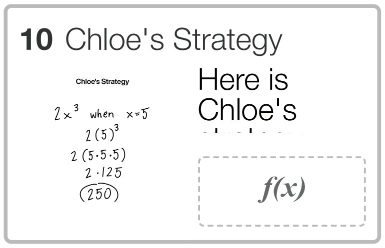
Receptive Language: Processing Time

Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners

Expressive Language: Eliminate Barriers

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



10 Chloe's Strategy

Chloe's Strategy

$2x^3$ when $x=5$

$2(5)^3$

$2(5 \cdot 5 \cdot 5)$

$2 \cdot 125$

$\boxed{250}$

Here is Chloe's

Here is Chloe's strategy from the previous screen.

On paper, use her strategy to determine the value of $2x^3$ when $x=\frac{1}{2}$.

Then enter your answer here.

Teacher Moves

Facilitation

- The purpose of this screen is for students to extend what they have learned to consider fractional side lengths.

Early Student Thinking

- Students may multiply 2 by x first and say that the value is $1^3 = 1$.
- Consider asking these students: *What operation did Chloe do first? How does this connect to Amir's picture?*

Suggested Pacing: Screens 10–12

Sample Responses

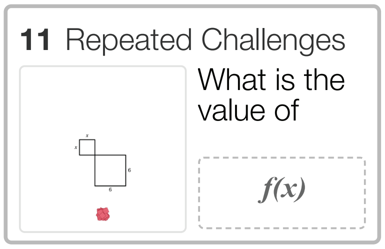
$\frac{1}{4}$ (or equivalent)

Student Supports

Students With Disabilities

Executive Functioning: Eliminate Barriers

To support with organization in problem solving, consider inviting students to number the steps of Chloe's strategy and then follow each step of her strategy with $x = \frac{1}{2}$ instead of $x = 5$.



11 Repeated Challenges

What is the value of

What is the value of $(x+6)^2$ when $x = 5$? (Challenges vary.)

Draw a diagram on paper if it helps you with your thinking.

Teacher Moves

How Repeated Challenges Work

Students are presented with a variety of challenges one at a time (in this case, evaluating a variable expression that includes exponents at a specific value of the variable).

Facilitation

- Give students 5–7 minutes to complete as many challenges as they can.
- Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

- Encourage students who have at least 8 correct to explore the “Are You Ready for More?”.

Math Community
- Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Sample Responses

This screen contains an unlimited number of challenges. The first few challenges are the same for each student; additional challenges are randomized.

The first few responses are:
- 49
- 45
- 32

Student Supports

Students With Disabilities
Conceptual Processing: Checks for Understanding
Invite students to talk aloud about their thinking to help them verbalize which concepts make sense and which do not.

Memory: Visual Aids
Consider inviting students with working memory challenges to record their work for each problem on a piece of blank paper to refer back to throughout the remainder of the lesson and unit.

12 Are You Ready for …

Consider the

Consider the expressions $(x+5)^2$ and x^2+5^2.

Amir says they **always** have the same value.
Chloe says they **never** have the same value.

Who is correct?

Teacher Moves

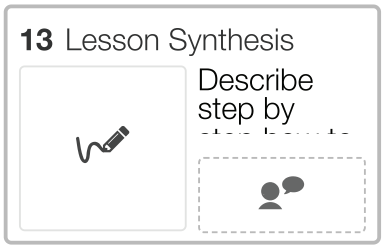
Facilitation
- Students may wonder if this screen is exactly like Screen 5 where they argued that two expressions are not equivalent. Consider inviting them to see how Amir’s and Chloe’s arguments are different than before.

- Encourage students to share responses with each other in place of a whole-class discussion.

Sample Responses

Neither

Explanations vary. They have the same value if $x = 0$, so they sometimes have the same value.



13 Lesson Synthesis

Describe step by

Describe step by step how to calculate the value of $(x+1)^3$ when $x = 3$
.

Use the sketch tool if it helps you show your thinking.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to summarize strategies for evaluating expressions that include exponents for a given value of a variable.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- Consider creating an anchor chart or other public display with students' strategies for the class to refer to throughout the unit.

Discussion Questions
- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*
- *What are the advantages of _________'s strategy?*

Routine (optional): Consider using the mathematical language routine Collect and Display to gather and share students' ideas.

Suggested Pacing: Screen 13

Sample Responses

Responses vary.

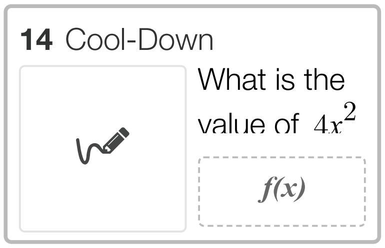
- Substitute the number 3 where the x is in the expression. First, we need to add $3 + 1$, which is 4. Then calculate 4^3. 4 to the power of 3 means $4 \cdot 4 \cdot 4$, which is $16 \cdot 4 = 64$.

- Draw a picture of a cube that is $3 + 1$ on each side. That means the cube is 4 by 4 by 4. Volume is the base area times the height; $4 \cdot 4 = 16$ and $16 \cdot 4 = 64$.

Student Supports

Students With Disabilities
Receptive Language: Processing Time
Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners
Expressive Language: Eliminate Barriers
Provide sentence frames to help students explain their thinking (e.g., To calculate the value of $(x+1)^3$ when $x = 3$, first ______. Then, ______.).



14 Cool-Down

What is the value of $4x^2$

What is the value of $4x^2$ when $x = 3$?

Draw a diagram if it helps you with your thinking.

Teacher Moves

Support for Future Learning
If students struggle, consider reviewing this screen as a class before Practice Day 2 or offering individual support where needed during the Practice Day. Students will need to be able to evaluate expressions at specific values of their variables on the End Assessment.

Suggested Pacing: Screens 14–15

Sample Responses

36

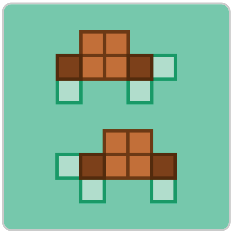
15

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can determine the value of an expression that has a variable, an exponent, and addition, subtraction, multiplication, or division for a specific value of the variable.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Turtles All the Way

Lesson 13: Stories and Tables and Variables, Oh My!

Overview

Students extend their work with variable expressions in earlier lessons to explore relationships between two variables. Students use both tables and equations to represent relationships and learn the terms *independent variable* and *dependent variable* to describe each part of a relationship.

Learning Goals

- Understand what independent and dependent variables are in a relationship.
- Use a table or an equation to represent a relationship between two variables.

Vocabulary

- dependent variable
- independent variable

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to extend the work students did with variable expressions in earlier lessons to explore relationships between two variables. Students use both tables and equations to represent relationships and learn the terms *independent variable* and *dependent variable* to describe each part of a relationship. This lesson also builds on the work students did in Units 2 and 3 with ratios as students use ratio reasoning to describe relationships and create tables.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to attend to details in the pattern they will explore in Activity 1. It also informally introduces the idea of measuring and counting something as it changes, which will be formalized with variables in Activity 1.

Activity 1: Turtles, Turtles, Turtles (15 minutes)

The purpose of this activity is to introduce ways to explore a relationship between two variables. Students examine the relationship between the number of turtles, t, and the number of green tiles, g, by first using words and then using tables and equations. This is an extension of the work from Lesson 6, where students wrote expressions in one variable to describe situations. This activity also introduces the terms *independent variable* and *dependent variable* as ways of describing each of the quantities in a relationship. Students will continue to explore tables and equations of relationships throughout this lesson and unit.

Activity 2: Border Tiles Revisited (15 minutes)

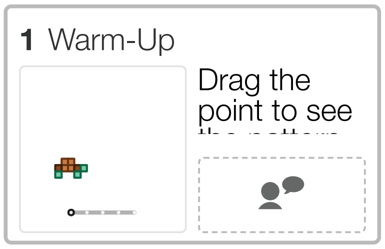
The purpose of this activity is for students to apply what they learned in Activity 1 to the pattern from Lesson 7: Border Tiles. Students can choose the dependent variable they want to investigate, then use words, tables, and equations to describe the relationship for the variable they chose.

Note: After Screen 6, students will see the same prompts but have different responses based on the dependent variable they chose on Screen 6.

Lesson Synthesis (5 minutes)

The purpose of this discussion is to make connections between tables, equations, and images that show the same relationship.

Cool-Down (5 minutes)



1 Warm-Up

Drag the point to see

Drag the point to see the pattern.

What different things can you count in this pattern?

Teacher Moves

Overview: In this lesson, students extend their work with variable expressions in earlier lessons to explore relationships between two variables. In this warm-up, students attend to details in the pattern they will explore in Activity 1.

Launch

- Invite students to work *in pairs.*
- Consider starting with the activity paused, dragging the point, and discussing what students think “different things you can count” means.
- It may be helpful to brainstorm one example as a class, such as number of turtles or number of dark brown tiles, before giving students time to think on their own.

Facilitation

- Give students 1–2 minutes to brainstorm a list together and then share out as a class.
- Consider keeping a list of what students wrote here to revisit during Activity 1 where students are introduced to assigning variables to these quantities and again when students learn the terms *independent variable* and *dependent variable*.

Discussion Questions

- *What is changing as you drag the point? What else is changing?*

Math Community

- Consider celebrating variety and creativity in what students write, including things that surprise you or that you think other students may not have noticed.

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

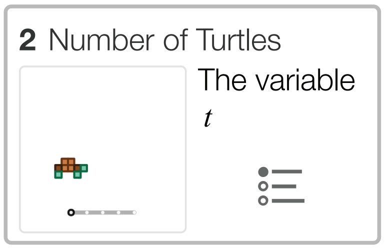
- The number of turtles
- The number of green squares
- The number of tiles in total
- The size of one turtle
- The number of lines in the picture
- The number of rows of turtles

Student Supports

Students With Disabilities

Fine Motor Skills: Strategic Pairing

Allow students who struggle with fine motor skills to dictate use of the sketch tool and movable points as needed throughout the lesson and unit.



2 Number of Turtles

The variable t

The variable t represents the number of turtles.

1. Choose another variable.

Teacher Moves

Overview: In Activity 1 (Screens 2–5), students are introduced to ways of exploring a relationship between two variables. This activity also introduces the terms *independent variable* and *dependent variable* as ways of describing each of the quantities in a relationship.

Launch

- Spend time making connections between what students wrote in the warm-up and the variables written here.
- Consider asking: *Why do you think someone might have chosen the variable* t *to represent the number of turtles? Could they have chosen a different letter?*

Facilitation

- Before they choose a variable, give students one minute to discuss with a partner what they think each variable means about the image.
- It is okay if students are still early in their thinking about describing relationships like these. They will have more practice throughout this lesson and in the next few lessons.

Suggested Pacing: Screens 2–5

Sample Responses

Responses depend on the variable chosen.

$a =$ **total area of the turtles**

The total area always goes up by 9 every time you add a turtle.

$h =$ **height of a turtle**

The height of a turtle is always 3 no matter how many turtles there are.

$g =$ **number of green squares**

There are 3 green squares on every turtle so it's always 3 times the number of turtles.

Student Supports

Students With Disabilities

Conceptual Processing: Checks for Understanding

Invite students to talk aloud about their thinking to help them verbalize which concepts make sense and which do not.

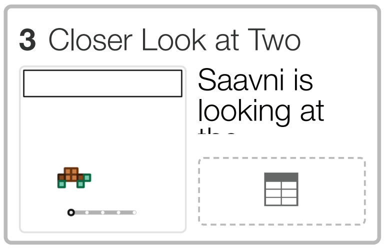
Multilingual Learners

Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase what they think each variable represents on the turtle to support students with sensemaking.

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., When I change the value of t by 1, the [other variable] ______________.).



3 Closer Look at Two

Saavni is looking at

Saavni is looking at the relationship between the number of turtles, t, and the number of green squares, g.

She made a table to help find a pattern.

1. Fill in the missing values in the table.

2. Discuss any patterns you see with a classmate.

Teacher Moves

Facilitation

- Circulate to listen to students' strategies for filling in the missing values. Monitor for students who make connections between this and the work with equivalent ratios in Units 2 and 3.

Sample Responses

6

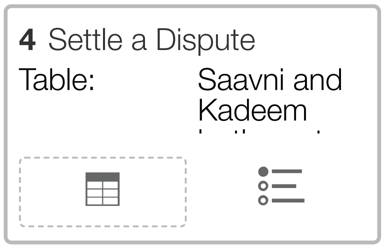
9

12

15

Student Supports

Students With Disabilities
Conceptual Processing: Eliminate Barriers
Demonstrate the steps for the screen by inviting a student to explain how the row $t=1$ and $g=3$ connects to the turtle image before students respond.



4 Settle a Dispute

Table: Saavni and Kadeem

Table:

Teacher Moves

Facilitation

- When most students have responded, display the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.
- If there is not a consensus, invite students to make arguments and justify their reasoning for each response (MP3).
- **Note:** This may be the first time students encounter equations with two variables.

Discussion Questions

- *How did you decide which equation was correct?*
- *When might writing an equation be useful?*

Early Student Thinking

- Students might notice that both equations involve the same numbers and variables and select "Both."
- Consider asking these students to plug in the numbers from the first row of the table into each equation and share what they notice.

Math Community

- Consider inviting students to think about what a student who responded differently might have been thinking.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Saavni's $\left(g=3t\right)$

Explanations vary.

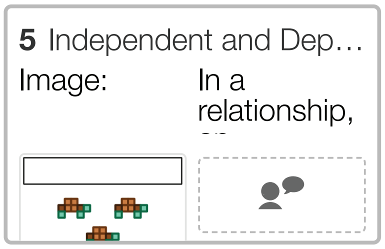
- I used the numbers from the table. Saavni's equation kept being true and Kadeem's equation didn't. For example, 6 equals $3(2)$, but 3 doesn't equal $6(2)$.
- If you think about it, Saavni's equation says that the green tiles is 3 times the number of turtles. Every turtle has 3 green tiles, so her equation makes sense.

Student Supports

Students With Disabilities

Visual-Spatial Processing: Eliminate Barriers

To support students with distinguishing between similar equations, invite pairs to read each equation aloud and discuss how they are similar and different before responding.



5 Independent and Dep…

Image:

In a relationship,

Image:

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to make connections between the new terms *independent variable* and *dependent variable,* and the work students have done thus far.

Facilitation

- Select and sequence several student responses using the snapshot tool.
- Consider starting the discussion by reading the text aloud and inviting students to paraphrase what each variable means in their own words. Then, create a class definition for each term that students can refer to throughout the lesson and unit.
- If possible, make connections between the list students generated in the warm-up or the variables from Screen 2 and the concept of dependent variables.

Discussion Questions

- *Of the ideas we have seen so far in this lesson, which ideas could we call dependent variables?*
- *Where do you see the independent variable in the table? In the equation?*

Early Finishers

- Encourage students to choose a different dependent variable and write an equation for the relationship between t and the variable they chose.

Math Community

- Consider starting with students' language or revisiting students' ideas from earlier screens as you create a class definition of each term.

Routine (optional): Consider using the mathematical language routine Collect and Display in order to gather students' ideas before writing a class definition.

Sample Responses

Responses vary.

- The number of tiles in total
- The size of one turtle
- The number of lines in the picture
- The number of rows of turtles

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time.

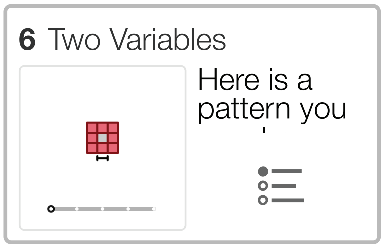
Multilingual Learners

Expressive Language: Eliminate Barriers

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

Expressive Language: Visual Aids

Create a visual display with definitions and examples of *independent variable* and *dependent variable* to aid in explanations and reasoning. If possible, use students' informal language as part of the display.



6 Two Variables

Here is a pattern you

Here is a pattern you may have seen before.

The **independent variable** is n, the side length of the gray square.

Pick a dependent variable you would like to explore.

Then describe in words how changing n affects the variable you chose.

Teacher Moves

Overview: In Activity 2 (Screens 6–10), students apply what they learned in Activity 1 to the pattern from Lesson 7: Border Tiles.

Note: After Screen 6, students will see the same prompts but have different responses based on the dependent variable they chose on Screen 6.

Launch

- Consider asking: *Why might it make sense that the length of the gray square is the independent variable?*

Facilitation

- Before they choose a variable, give students one minute to discuss with a partner what they think each dependent variable means about the image.

Suggested Pacing: Screens 6–10

Sample Responses

Responses depend on the variable chosen.

$t =$ **total area of the tiles**

The total area gets bigger and bigger every time. It is like a big square.

$g =$ **area of the gray tiles**

The area of the gray tiles kind of grows in a square. It doesn't go up by the same number every time.

$p =$ **perimeter of the gray square**

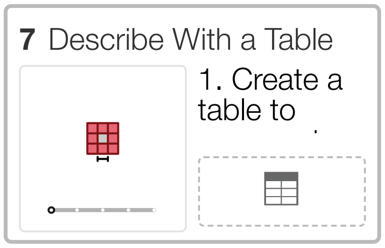
The perimeter of the gray square goes up by 4 every time you make n bigger by 1.

Student Supports

Students With Disabilities

Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase what they think each variable represents on the pattern to support students with sensemaking.



7 Describe With a Table

1. Create a table to .

1. Create a table to represent the relationship between n and your variable.

2. Discuss any patterns you see with a partner.

n = side length of gray square

Teacher Moves

Facilitation

- It may be helpful to rearrange students into groups that chose the same variable so that they can compare their thinking throughout this activity.

Early Student Thinking

- Students may measure a variable that is different than the one they chose on the previous screen.
- Consider asking students to show you how they counted the number for $n = 2$.

Sample Responses

Responses depend on the variable chosen.

$t =$ **total area of the tiles**

$9, 16, 25, 36, 49$

$g =$ **area of the gray tiles**

$1, 4, 9, 16, 25$

$p =$ **perimeter of the gray square**

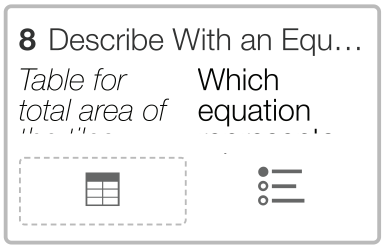
$4, 8, 12, 16, 20$

Student Supports

Students With Disabilities

Memory: Processing Time

To support students with working memory challenges, consider inviting them to record their work as they are working on a piece of blank paper to refer back to throughout the remainder of the lesson.



8 Describe With an Equ…

Table for total area of

Which equation

Table for total area of the tiles:

Teacher Moves

Facilitation

- To support students getting started, consider asking: *How could you use a row from the table to help you decide?*
- When most students have responded, consider facilitating a short conversation to discuss strategies students used to decide which equation represented their pattern.

Discussion Questions

- *How did you decide which equation was correct?*
- *What could you do to show that the other equations do not represent the pattern?*

Math Community

- Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Sample Responses

Responses depend on the variable chosen.

$t =$ **total area of the tiles**

$t = (n+2)^2$

$g =$ **area of the gray tiles**

$g = n^2$

$p =$ **perimeter of the gray square**

- $p = 4n$

Explanations vary.

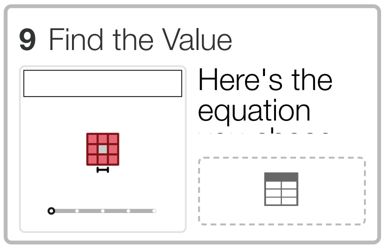
- I used numbers from the table and tested them in each equation. Only one of the equations was true for all the numbers I tested.
- I thought about what each variable meant and decided which one matched the pattern I saw.

Student Supports

Students With Disabilities

Visual-Spatial Processing: Eliminate Barriers

To support students with distinguishing between similar equations, invite pairs to read each equation aloud and discuss how they are similar and different before responding.



9 Find the Value

Here's the equation

Here's the equation you chose on the previous screen.

What is the value of your variable when $n = 10$?

Teacher Moves

Progress Check

- Use the dashboard's summary view to monitor students' thinking.
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

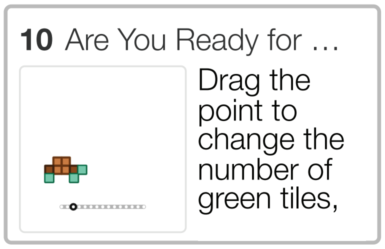
Sample Responses

Responses depend on the variable chosen.

$t =$ **total area of the tiles**
144

$g =$ **area of the gray tiles**
100

$p =$ **perimeter of the gray square**
40


10 Are You Ready for …

Drag the point to change the number of green tiles,

Drag the point to change the number of green tiles, g.

On paper:

1. Explain what the independent variable is.
2. Choose a dependent variable to study.
3. Describe the relationship between the independent and dependent variable.

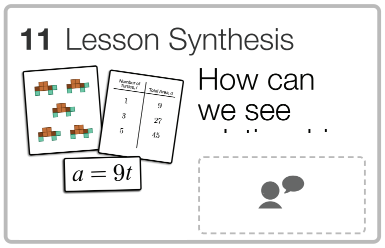
Teacher Moves

Facilitation

- The purpose of this screen is for students to consider what might happen if the independent variable and dependent variable were swapped.
- If time allows, consider inviting all students to drag the point and discuss at least the first two bullet points.

Sample Responses

1. The number of green tiles, g.
2. *Responses vary.* The number of turtles, t.
3. *Responses vary depending on responses to 2.* Every time the number of green tiles gets to 3 more, the number of turtles jumps up by 1. For example, if $g = 4$, the number of turtles is still 1, but once it gets to $g = 6$, the number of turtles jumps to 2.


11 Lesson Synthesis

How can we see

$a = 9t$

How can we see relationships between independent and dependent variables in tables and equations?

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to make connections between tables, equations, and images that show the same relationship.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their responses stronger and clearer based on the discussion.

Discussion Questions

- *What makes sense to you about each strategy? What does not make sense?*
- *What connections do you see between your classmates' strategies?*

Math Community

- Invite students to share strategies they've found most helpful and attribute them to the students who shared them.

Suggested Pacing: Screen 11

Sample Responses

Responses vary. Tables show a relationship between the variables in each row. The left column is the independent variable and the right column is the dependent variable. For example, there are 3 turtles in the image and they have 27 total squares, so the table has the row $t = 3$ and $a = 27$. An equation shows the relationship in general. In this case, the independent variable is 9 times the dependent variable, so we can write the equation $a = 9t$.

Student Supports

Students With Disabilities

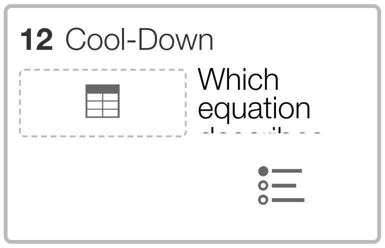
Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., Tables show these relationships by ______. Equations show these relationships by ______.).



12 Cool-Down

Which equation

Which equation describes the same relationship in this table?

Teacher Moves

Support for Future Learning

If students struggle, plan to emphasize this when opportunities arise in Lesson 15. Consider spending extra time on the card sort making connections between tables, equations, and graphs of the same relationship.

Sample Responses

$c = 2.5h$

Explanations vary. I plugged in the row $h = 1$ and $c = 2.5$ into every equation. $2.5 = 2.5(1)$, but 2.5 does not equal $5(1)$ or $(1) + 5$, and 1 does not equal $2.5 \cdot (2.5)$

13

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I understand what the independent and dependent variables are in a relationship.
- I can use a table or an equation to represent a relationship.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Representing Relationships

Lesson 14: Interpreting Graphs of Relationships

Overview

Students are introduced to graphs as a way of representing relationships between two variables.

Note: This lesson includes a paper supplement.

Learning Goals

- Represent relationships using tables and graphs.

Materials

- Supplement (half sheet for each student)

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to introduce graphs as a way of representing relationships between two variables. This extends the work students did in Grade 5 graphing points in the first quadrant. Students explore relationships within a growing pattern and examine what those relationships look like when represented with an image, a table, and a graph. Students also plot points in the first quadrant and interpret what points mean about a situation.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to attend to the details of the pattern they will be exploring throughout the lesson using the Notice and Wonder routine.

Activity 1: Introducing Graphs (15 minutes)

The purpose of this activity is for students to revisit graphs from Grade 5 and use it in a new way: representing relationships between variables. Students create tables for the relationship between the side length and the area of the purple rectangles as the pattern grows, then use those tables to make connections between tables and graphs of the same relationship.

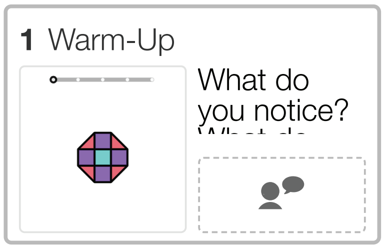
Activity 2: Plotting Points (15 minutes)

The purpose of this activity is for students to put into practice what they learned in Activity 1 as they explore different relationships within the same growing pattern. Students plot points and consider what making a graph of a relationship can highlight. Students also consider a relationship where one of the variables does not change (in this case, the relationship between the side length and the area of the red triangles).

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to summarize the connections between tables, graphs, and images of patterns.

Cool-Down (5 minutes)



1 Warm-Up

What do you notice?

What do you notice? What do you wonder?

Teacher Moves

Overview: In this lesson, students are introduced to graphs as a way of representing relationships between two variables. In this warm-up, students use the Notice and Wonder routine to attend to the details of the pattern they will be exploring throughout the lesson.

Launch

- Invite students to work *in pairs.*

Facilitation

- Give students 1–2 minutes to drag the point and record what they notice and wonder.
- Invite several students to share their noticings and wonderings.

Math Community

- Consider celebrating variety and creativity in what students notice and wonder, including things that surprise you or that you think other students may not have noticed.

Readiness Check (Problems 7 and 8)

- If students struggled, consider reviewing Problem 8 as a class before Activity 1. If it does not come up naturally, consider asking students how to write each student's point on the graph as a coordinate pair and what that means about the number of pencils and erasers they each have.

Suggested Pacing: Screen 1

Sample Responses

Responses vary.

- I notice that there are always three colors.
- I notice that the middle is a square that keeps growing.
- I notice that there are always 4 red triangles.

- I wonder when the shape would get so big it takes over the screen.
- I wonder how many squares I would need to build each shape.
- I wonder if there is more green, purple, or red on each shape.

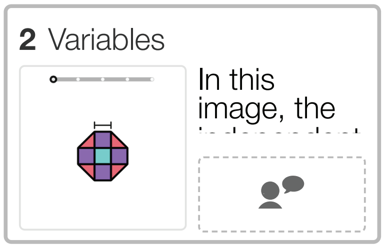
Student Supports

Students with Disabilities

Fine Motor Skills: Strategic Pairing

Allow students who struggle with fine motor skills to dictate use of the sketch tool and movable points as needed throughout the lesson and unit.

English Language Learners
Expressive Language: Eliminate Barriers
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



2 Variables

In this image, the

In this image, the independent variable is the side length of the purple rectangles, n .

What dependent variables could you explore?

Teacher Moves

Overview: In Activity 1 (Screens 2–6), students revisit graphs from Grade 5 and use it in a new way: representing relationships between variables.

Launch
- Invite students to work *in pairs.*
- Revisit the definition of *independent variable* and *dependent variable* as a class.
- Consider making connections to the previous lesson by asking: *How are these relationships similar to and different from the ones we studied last class?*

Facilitation
- Give students one minute to discuss with a partner and then respond.
- Use the snapshot tool or dashboard's teacher view to highlight several different examples of dependent variables.

Early Student Thinking
- Students may say that the area is a dependent variable. Ask these students to consider the different areas that may be measured in this image. Then discuss with theses students the benefit of defining the different areas precisely (MP6).

Discussion Questions
- *Why does it make sense that the side length is the independent variable?*
- *Could this be a dependent variable? Why or why not?*

Suggested Pacing: Screen 2

Sample Responses

Responses vary.

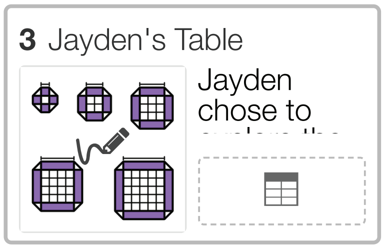
- Total area of the shape
- Number of green squares
- Height of the shape

Student Supports

Multilingual Learners

Expressive Language: Visual Aids

Review a visual display with definitions and examples of *independent variable* and *dependent variable* to aid in explanations and reasoning. If it makes sense, add students' language from this screen to the display.



3 Jayden's Table

Jayden chose to

Jayden chose to explore the relationship between the side length, n, and the total area of purple rectangles, p.

They want to represent this relationship with a table.

Fill in the missing values.

Teacher Moves

Launch

- Distribute one half-sheet supplement to each student to use throughout the lesson.

Facilitation

- To support students' getting started, consider asking: *Which part of the image shows when* $n = 2$*? What is the area of the purple rectangles?*

Early Student Thinking

- Some students may continue to write 4 for each row because there are always 4 purple rectangles.
- Consider sharing that we are calculating the total number of square units instead of the number of rectangles.

Materials: Supplement

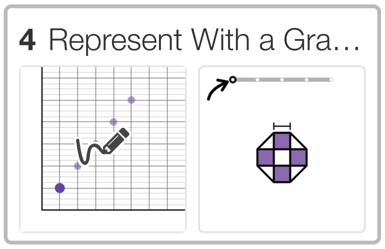
Suggested Pacing: Screens 3–6

Sample Responses

8
12
16
20

Student Supports

Students With Disabilities
Conceptual Processing: Eliminate Barriers
Demonstrate the steps for this screen by inviting a student to explain how the row $n = 1$ and $p = 4$ connects to the image before students respond.


4 Represent With a Gra…

Teacher Moves

Facilitation

- Circulate to listen to pair conversations. Monitor for connections students share that might be useful during the discussion on Screen 5.

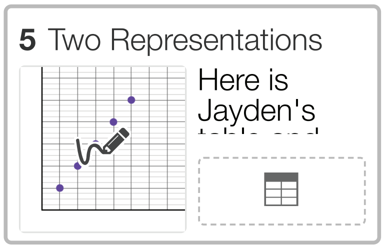
Sample Responses

Responses vary. Each point in the graph matches one of the images. The number going horizontally is the side length and the number going vertically is the area of the purple rectangles.

Student Supports

Students With Disabilities
Receptive Language: Processing Time
Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners
Expressive Language: Eliminate Barriers
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.
Encourage students to use the sketch tool to show their thinking if they are not sure of the words.



5 Two Representations

Here is Jayden's

Here is Jayden's table and Rebecca's graph.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface connections between tables and graphs of the same relationship.

Facilitation

- Select and sequence student responses and sketches.
- Facilitate a whole-class discussion, inviting several students to justify how these represent the same relationship (MP3).
- Consider connections that are as detailed as noticing that they both use the same variables, n and p (MP6).

Discussion Questions

- *Where do you see the row with* $n = 2$ *and* $p = 8$ *in the graph?*
- *Here is a point in the graph. Where do we see that information in the table? What does it mean about the pattern?*

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

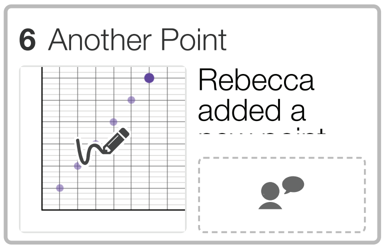
Sample Responses

Responses vary. I know these represent the same relationship because they show the same information in a different way. For example, you can see the row with $n = 2$ and $p = 8$ in the graph 2 units over and 8 units up. The 2 shows the side length and the 8 shows the area of the purple rectangles.

Student Supports

Students With Disabilities
Executive Functioning: Eliminate Barriers
To support focus, consider chunking this activity by inviting students to choose one row of the table and explain how the graph shows the same relationship.

English Language Learners
Expressive Language: Eliminate Barriers
Provide sentence frames to help students explain their thinking (e.g., The table and graph show the same relationship because ______.).



6 Another Point

Rebecca added a

Rebecca added a new point at $(6, 24)$ to the graph.

Explain what those values mean in this context.

Teacher Moves

Progress Check

- Use the dashboard's summary view to monitor students' thinking, particularly the level of precision in their responses (MP6).
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- If time allows, consider creating an ideal response as a class using the routine below.

Early Finishers

- Encourage students to predict what n would have to be for the area of the purple squares to be 100 square units.

Routine (optional): Consider using the mathematical language routine Clarify, Critique, Correct to help students understand the level of precision needed to interpret a point on a graph in context.

Sample Responses

Responses vary. It means that when the side length is 6 units, the area of the purple rectangles is 24 square units.

Student Supports

Students With Disabilities

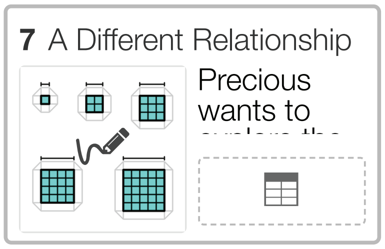
Receptive Language: Processing Time

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., The point $(6, 24)$ means ______.).



7 A Different Relationship

Precious wants to

Precious wants to explore the relationship between the side length, n, and the area of the teal square, s.

Fill in this table.

Teacher Moves

Overview: In Activity 2 (Screens 7–11), students put into practice what they learned in Activity 1 as they explore different relationships within the same growing pattern.

Launch

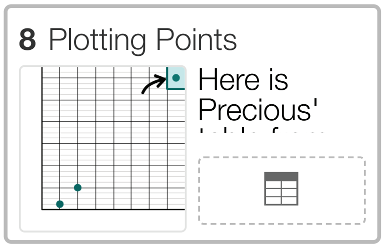
- Consider asking students: *Do you think this pattern will grow in the same way as the purple rectangles? Why or why not?*

Facilitation

- Encourage students to use the feedback on the screen to help them revise their thinking.

Sample Responses

4
9
16
25



8 Plotting Points

Here is Precious'

Here is Precious' table from the previous screen.

Plot points to represent the last two rows.

Teacher Moves

Facilitation

- To support students getting started, consider asking: *What does the table tell us about where the first point should go?*
- Encourage students to use the feedback to revise their thinking.
- **Note:** This may be the first time students encounter coordinates in Grade 6. If it does not come up naturally, consider using this screen as an opportunity to review coordinate notation of points on a graph.

Discussion Questions

- *What values in a table would this point represent?*

- *What do you think the coordinates will be if I place the point [here]?*

Sample Responses

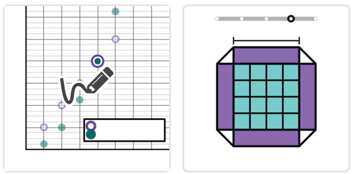
Image solution

Student Supports

Students With Disabilities
Visual-Spatial Processing: Visual Aids
To support students in making connections between the table and the graph, give them time to discuss how the two points already graphed are connected to the table. Ask: *Which row in the table is the second point? Where do you see the* 2*? Where do you see the* 4*?*

---

9 Two Relationships



Teacher Moves

Facilitation

- When most students have responded, consider pausing the class and discussing the advantages of creating a graph of a relationship.

Discussion Questions

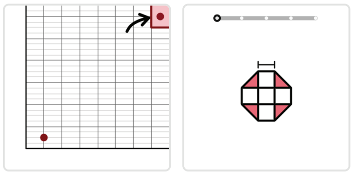
- *Do you think there are any other points that belong to both relationships? Why or why not?*
- *Why do you think someone might want to make a graph of a relationship?*
- *What is more helpful about an image? A table? A graph?*

Sample Responses

Responses vary. It means that when the side length is 4 units, both the purple and teal parts have the same area. The area is 16 square units.

---

10 A Third Relationship



Teacher Moves

Facilitation

- Consider encouraging students to use the supplement to count the area of the red triangles.
- Consider using the dashboard's teacher view or snapshot tool to highlight several different student graphs.

Discussion Questions

- *How is this relationship different from other ones we have explored in this lesson? How is it similar?*

- *How does a graph show when one variable is changing and the other one is not changing?*

Early Student Thinking

- Students may notice that there are 4 red triangles and plot points at $(n, 4)$.
- Consider asking these students: *What is the area of each red triangle? Is it less than, greater than, or equal to 1 square unit?*

Sample Responses

Image solution

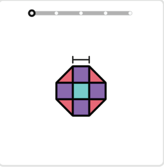
Student Supports

Students With Disabilities

Conceptual Processing: Eliminate Barriers

To assist students in recognizing the connections between new problems and prior work, consider asking: *How do the three points at $n = 1$ connect to the pattern when $n = 1$?*

11 Are You Ready for …



On paper, draw an image to represent when:

On paper, draw an image to represent when:

- $n = 0$.
- $n = 10$.

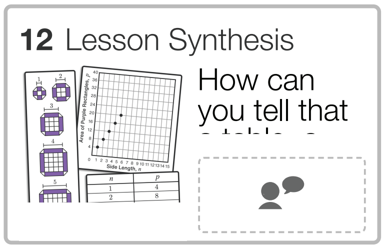
Teacher Moves

Facilitation

- Invite students to use the supplement to draw their image.
- For $n = 10$, encourage students to label each side length instead of drawing an image to scale.
- Encourage students to share responses with each other in place of a whole-class discussion or to convince a classmate who has not tried this problem of their thinking.

Sample Responses

Image solution



12 Lesson Synthesis

How can you tell that

How can you tell that a table, a graph, and an image show the same relationship?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to summarize connections between tables, graphs, and images of patterns.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their responses stronger and clearer based on the discussion.

Discussion Questions

- *Where do you see information from the image in the table? In the graph?*

Math Community

- Invite students to share their favorite representation and why.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Suggested Pacing: Screen 12

Sample Responses

Responses vary. They each should have the same information. Each row of the table represents part of the image and a point on the graph. For example, the image shows that when the side length is 4 units, the purple area is 16 square units. This shows up in the table in the row $n = 4$ and $p = 16$ and on the graph in the point $(4, 16)$.

Student Supports

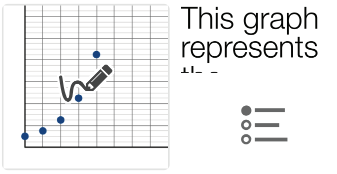
Students With Disabilities
Receptive Language: Processing Time
Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners
Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., You can tell a table, a graph, and an image show the same relationship by ______.).

---

13 Cool-Down



This graph represents

This graph represents the relationship between time, t, and the number of mosquitoes, m.

Select the table that represents the same relationship as the graph.

Teacher Moves

Support for Future Learning
If students struggle, plan to emphasize this when opportunities arise in Lesson 15, particularly in Activity 1 when students are making connections between tables and graphs.

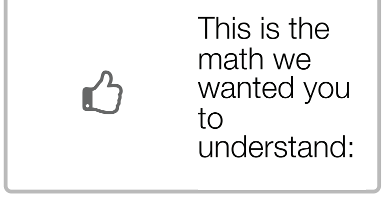
Suggested Pacing: Screens 13–14

Sample Responses

$t = 0, 1, 2, 3$

$m = 2, 3, 5, 9$

14



This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can represent relationships using tables and graphs.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Connecting Representations (NYC)

Lesson 15: Tables, Equations, and Graphs of Relationships

Purpose

This is the third of four lessons about representing relationships in tables, graphs, equations, and stories. In this lesson, students focus on making connections between different representations of the same relationship and on the details in each representation that indicate what the story may represent. This lesson is also an opportunity to discuss the advantages of each representation for making sense of a relationship.

Preparation

Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Cards

- Print and cut one half sheet for each pair of students.

Materials

- Tape or glue (for attaching cards to the Student Worksheet)

Warm-Up (5 minutes)

Overview: Students revisit the connection between tables and graphs from the previous lesson.

Launch

- Invite students to work *individually.*
- Display Sheet 1 of the Teacher Projection Sheets.
- To support students in sense-making, consider asking: *How are the tables similar? Different?*

Facilitation

- Give students 1–2 minutes to think independently and then share their responses with a partner.
- Invite students to share their choice and justify their thinking.

Discussion Questions

- *How can you tell which number in a table goes where in a graph?*
- *How would the graph look different for Table A? How do you know?*

Math Community

- Consider inviting students to think about what a student who responded differently might have been thinking: *Why might someone have chosen Table A?*

Activity 1: What's Missing? (20 minutes)

Overview: Students attend to details to match tables, graphs, and equations to situations (MP2).

Launch

- Invite students to work *in pairs.*
- Distribute one Student Worksheet and a set of cards to each pair of students. Optionally, distribute tape or glue to each pair of students.
- Display Sheet 2 of the Teacher Projection Sheets and review the instructions as a class.
- Introduce the words paletas (pa·le·tas, a Mexican frozen treat made from fresh natural fruits) and piraguas (pi·ra·gua, a Puerto Rican shaved ice dessert) if they are new for students.

Facilitation

- This lesson structure may be familiar to students from earlier in the unit. Consider some of these facilitation moves to support student-to-student dialogue:
  - Students take turns selecting one card and justifying to their partner which story it matches with. When both partners agree, add the card to their paper.
  - Students put all the cards in a stack. Together they select one at a time and decide where it goes before adding the card to their paper.
- Consider checking in with pairs or posting the answer key. This is also a good opportunity for students to practice justifying their thinking with another pair (MP3).

Early Student Thinking

- Students may use what they learned earlier in the unit with expressions and wonder if it makes sense to call these representations *equivalent.*
- Consider sharing that even though these representations show the same relationship in a different way, we do not call them equivalent.

Discussion Questions

- *What details in the table, graph, and equation were most helpful in deciding which situation they matched with?*
- *In the second row, where do we see the* 2.50 *in the table? In the graph? In the equation?*

Math Community

- Consider inviting students to discuss with their partner what good pair work will look like before beginning the activity. After the activity, they can reflect on what they did well and what they would improve on.

Early Finishers

- Encourage students to begin Activity 2.

Support for Students With Disabilities

Executive Functioning: Eliminate Barriers

To support with organization in problem solving, consider chunking this activity by inviting students to choose one card and to figure out which row it matches, then repeat.

Visual-Spatial Processing: Visual Aids

To support students in making connections, consider providing highlighters for them to show similar information shown in a table, graph, equation, and situation.

Activity 2: Rough Draft Graph (10 minutes)

Overview: Students make sense of a graph of a relationship by analyzing a graph with errors and then making their own.

Launch

- Invite students to work *individually* with the support of a partner.
- Consider spending time at the beginning of this activity normalizing mistakes by asking questions like: *Why is it important for us to learn from our own and others' mistakes? How can we react to our own and others' mistakes in a way that helps them feel safe?*

Facilitation

- Give students several minutes to make sense of Sora's rough draft, then to make their own.
- Encourage students to share their second drafts with several different classmates in order to help make their second drafts stronger and clearer.
- When most students have created second drafts, facilitate a discussion and record students' ideas about what to remember when creating graphs, including mistakes people might make.

Early Finishers

- Encourage students to think of a product they might want to sell, then to create a graph that represents the money they would earn selling that product.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in their math ideas and language.

Support for Students With Disabilities

Executive Functioning: Eliminate Barriers

To support organization, consider chunking the activity by inviting students to first scale the graph and then to plot each point. Provide graph paper for students who want to create a graph that is larger than the one on the Student Worksheet.

Lesson Synthesis (5 minutes)

Key Discussion 🔑

The purpose of this discussion is to summarize the connections between tables, equations, and graphs of the same relationship.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students time to revise their response based on the discussion.

Discussion Questions

- *Which representation do you find most challenging? Least challenging? Why?*
- *What makes sense to you? What does not make sense?*
- *How would you say what ________ said in your own words?*

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Support for Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames to help students explain their thinking (e.g., You can see the same relationship in the _____ and the _____ by looking at ______.).

Cool-Down (5 minutes)

Support for Future Learning

Students will have more chances to make sense of representations for relationships in Lesson 16.

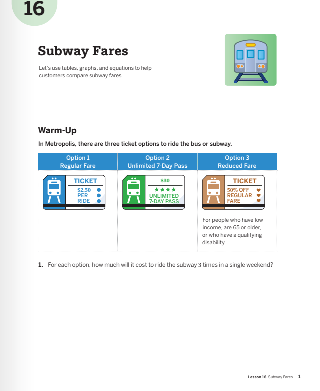
Unit 6

Lesson

16

Print Lesson

Assign the Student Edition with Presentation Screens for this lesson.



Unit 6
Lesson
16

Name: Date: Period:

Subway Fares

Let's use tables, graphs, and equations to help customers compare subway fares.

Warm-Up

In Metropolis, there are three ticket options to ride the bus or subway.

Option 1 Regular Fare	Option 2 Unlimited 7-Day Pass	Option 3 Reduced Fare
TICKET $2.50 PER RIDE	$30 ★★★★ UNLIMITED 7-DAY PASS	TICKET 50% OFF REGULAR FARE For people who have low income, are 65 or older, or who have a qualifying disability.

1. For each option, how much will it cost to ride the subway 3 times in a single weekend?

Lesson 16 Subway Fares 1

Subway Fares

Applying Relationships

Let's use tables, graphs, and equations to help customers compare subway fares.

Focus and Coherence

Today's Goals

1. **Goal:** Create graphs, tables, and equations to represent situations.
2. **Language Goal:** Interpret representations to analyze an issue in society. **(Reading, Writing, Speaking, and Listening)**

Students use tables, graphs, and equations to interpret relationships and help customers make decisions about what type of transportation ticket to buy. Students are also invited to stand in the shoes of others, considering the potential impact of changing fares on customers with different needs. **(MP2)**

Prior Learning

In Lessons 13–15, students used tables, graphs, and equations to represent and compare proportional and nonproportional relationships.

Future Learning

In Grade 7, students will use multiple representations (tables and graphs) to decide whether two quantities are in a proportional relationship and use proportional relationships to solve multistep problems.

Rigor and Balance

- Students **apply** their understanding about ratios and rate to compare transportation tickets using tables, graphs, and equations.

Standards

Addressing

NY-6.EE.9
Use variables to represent two quantities in a real-world problem that change in relationship to one another.
Given a verbal context and an equation, identify the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Also Addressing: **NY-6.RP.1**, **NY-6.RP.2**, **NY-6.RP.3**, **NY-6.RP.3a**, **NY-6.EE.6**

Mathematical Practices: MP2, MP4

Building Toward

NY-7.RP.2

Print Lesson

Lesson at a Glance

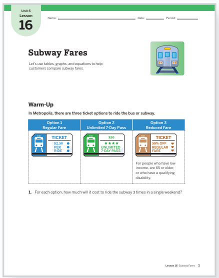
~ 45 min

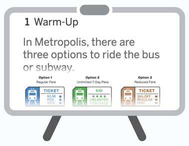
Standard(s): NY-6.EE.9, NY-6.RP.1, NY-6.RP.2, NY-6.RP.3, NY-6.RP.3a, NY-6.EE.6

Warm-Up

Pairs | 5 min

Students use the **Think-Pair-Share** routine to make sense of the context they will explore in this lesson: different fare options for riding the subway or bus.





1 Warm-Up

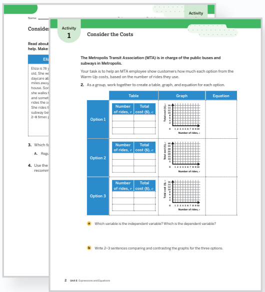
In Metropolis, there are three options to ride the bus or subway.

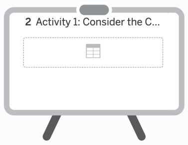
Presentation Screen 1

Activity 1

Small Groups | 15 min

Students create representations for the relationship between the number of rides and the total cost of each fare option and make decisions about which fare option to choose. **(MP2)**





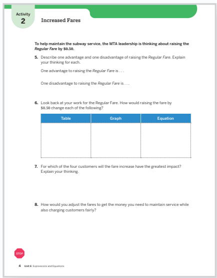
2 Activity 1: Consider the C...

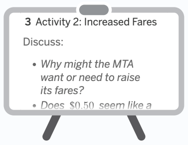
Presentation Screen 2

Activity 2

Small Groups | 15 min

Students use their representations from Activity 1 to analyze an issue in society: the impacts of raising transit fares. **(MP4)**





3 Activity 2: Increased Fares

Discuss:

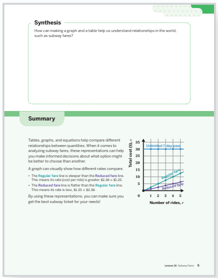
- *Why might the MTA want or need to raise its fares?*
- *Does $0.50 seem like a*

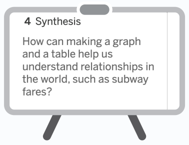
Presentation Screen 3

Synthesis

Whole Class | 5 min

Tables, graphs, and equations are all useful tools to analyze and compare relationships and make decisions. **MLR 1: Stronger and Clearer Each Time**





4 Synthesis

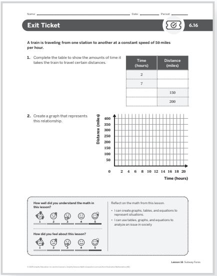
How can making a graph and a table help us understand relationships in the world, such as subway fares?

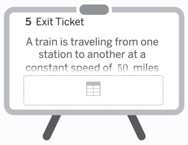
Presentation Screen 4

Exit Ticket

Independent | 5 min

Students create a table and a graph to represent the relationship between distance and time of a train traveling at a constant rate.





5 Exit Ticket

A train is traveling from one station to another at a constant speed of 50 miles

Presentation Screen 5

Prep Checklist

Students will use their Student Editions. Display the Teacher Presentation Screens. Go online to access the Teacher Presentation lesson screens.

This lesson includes:

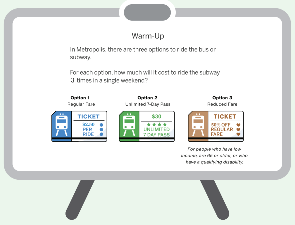
Student Edition

Exit Ticket PDF

Large sheets of paper or graph paper (optional)

Warm-Up

Purpose: Students use the **Think-Pair-Share** routine to make sense of the context they will explore in this lesson: different fare options for riding the subway or bus.



Presentation Screen
1

Launch

To activate background knowledge, consider asking:

- "Does anyone have experience riding the subway or bus?"
- "What do you think an unlimited 7-day pass means?"

Use the Think-Pair-Share routine giving students one minute to think individually before sharing with their partner.

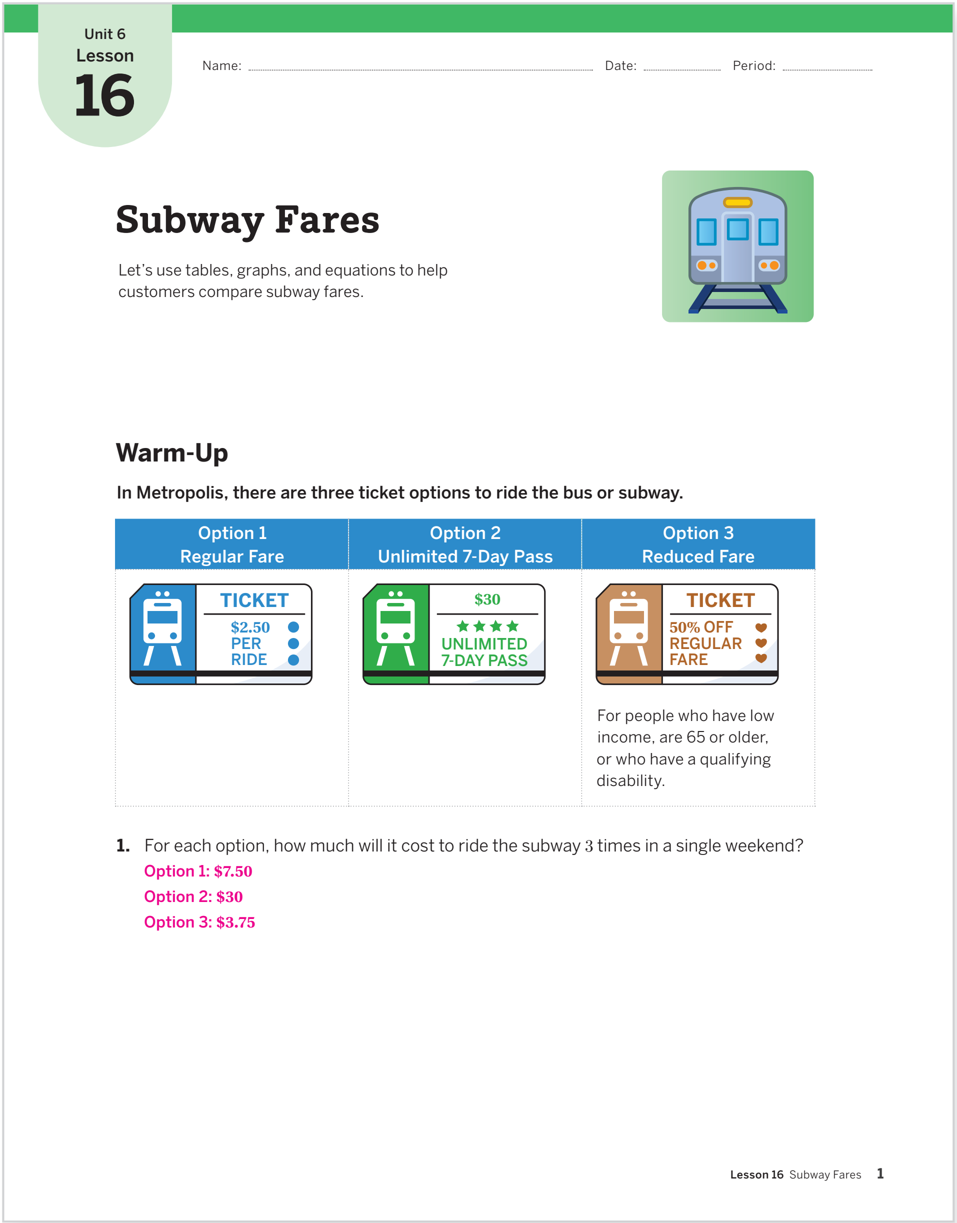
Accessibility: Conceptual Processing
To support students getting started, encourage them to consider how much one trip would cost first.

Connect

Invite students to share strategies for calculating the cost of each option, particularly what "50% off the regular fare" means and why it costs $30 for only three rides with Option 2.

Consider asking:

- "When might Option 1 be a good choice? When might Option 2 be a good choice?"
- "Who might qualify for Option 3?"



Unit 6
Lesson
16

Name: Date: Period:

Subway Fares

Let's use tables, graphs, and equations to help customers compare subway fares.

Warm-Up

In Metropolis, there are three ticket options to ride the bus or subway.

Option 1 Regular Fare	Option 2 Unlimited 7-Day Pass	Option 3 Reduced Fare
TICKET $2.50 PER RIDE	$30 ★★★★ UNLIMITED 7-DAY PASS	TICKET 50% OFF REGULAR FARE For people who have low income, are 65 or older, or who have a qualifying disability.

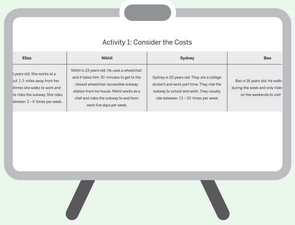
1. For each option, how much will it cost to ride the subway 3 times in a single weekend?
   Option 1: $7.50
   Option 2: $30
   Option 3: $3.75

Lesson 16 Subway Fares 1

Small Groups | 15 min

Activity 1 Considering the Cost

Purpose: Students create representations for the relationship between the number of rides and the total cost of each fare option and make decisions about which fare option to choose. **(MP2)**


Presentation Screen 2

Launch

To activate prior knowledge, consider asking "Why might it be helpful for an MTA employee to have a table, a graph, and an equation of the fare options? How might they be helpful to customers?"

Invite students to work as a group to create representations for each fare option. Consider providing large sheets of paper or graph paper to help group members see each others' thinking

Math Identity and Community Consider giving students time to discuss what they think they can contribute to their group, such as organization, asking good questions, creating graphs, making connections between representations, personal experience, etc.

Monitor

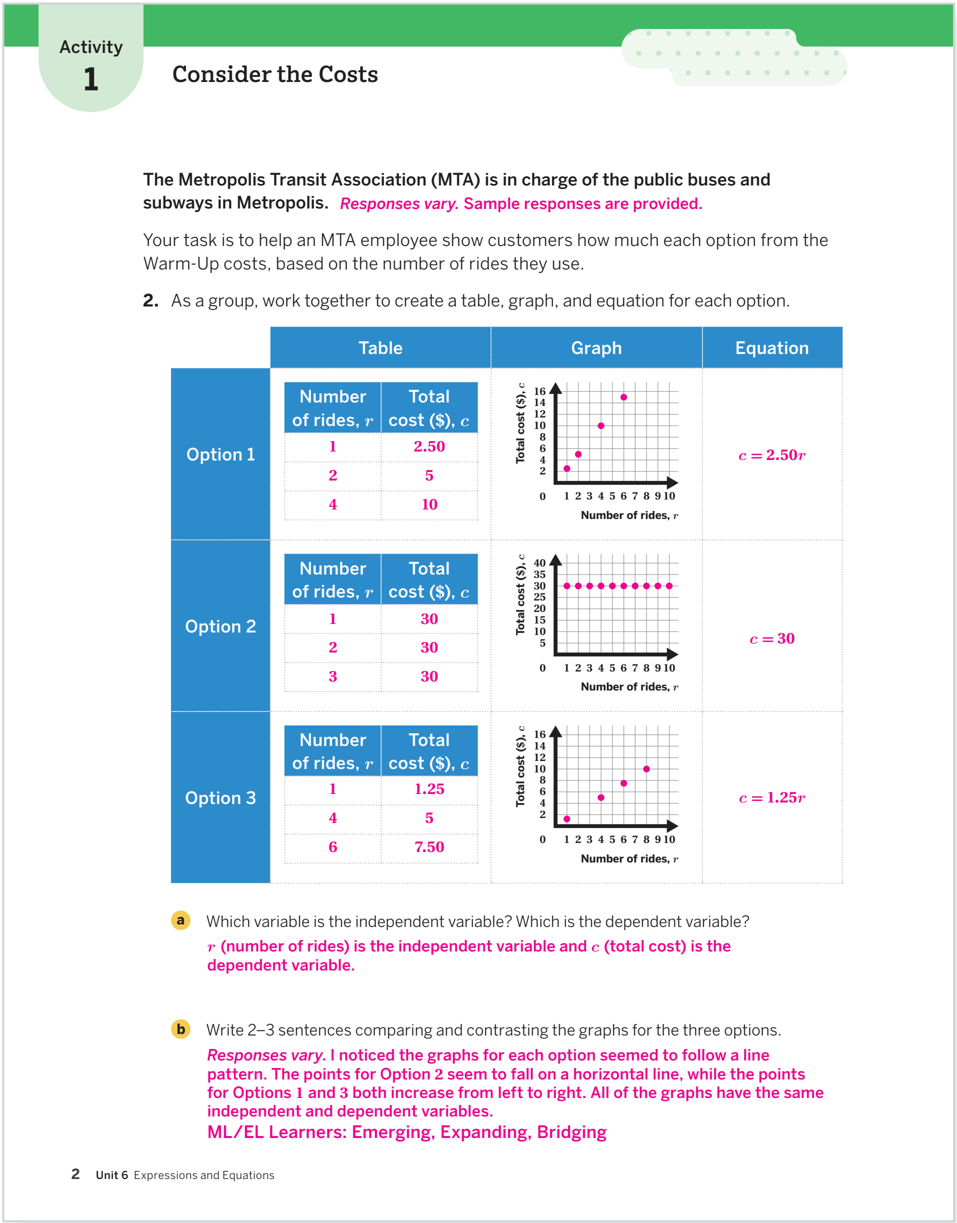
Encourage students to spend 7–10 minutes to create graphs, tables, and equations for each fare option. **(MP2)**

Differentiation

Look for students who . . .	Teacher Moves
Wonder how to create a table and a graph for the unlimited fare.	**Support:** Consider asking, "How much would it cost to ride the subway once? Twice? 10 times?"
Swap the variables in their equations.	**Support:** Invite them to substitute 1 for the number of rides into their equation. Ask, "Does the cost match the values in your table and graph?"
Use unit rates to determine the cost of one ride in Options 1 and 3 and apply the unit rate to complete the table and write the equation.	Invite them to share why this strategy works for Options 1 and 3, but not Option 2

Pause to share students' ideas around creating their representations before students begin on Problem 3.

Activity 1 continued ›


Activity 1

Consider the Costs

The Metropolis Transit Association (MTA) is in charge of the public buses and subways in Metropolis. *Responses vary.* Sample responses are provided.

Your task is to help an MTA employee show customers how much each option from the Warm-Up costs, based on the number of rides they use.

2. As a group, work together to create a table, graph, and equation for each option.

	Table		Graph	Equation
	Number of rides, r	Total cost ($), c		
Option 1	1	2.50		$c = 2.50r$
	2	5		
	4	10		
Option 2	1	30		$c = 30$
	2	30		
	3	30		
Option 3	1	1.25		$c = 1.25r$
	4	5		
	6	7.50		

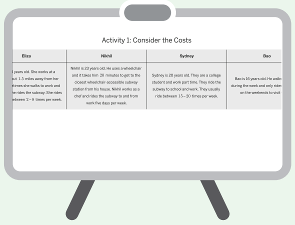
a. Which variable is the independent variable? Which is the dependent variable?
r (number of rides) is the independent variable and c (total cost) is the dependent variable.

b. Write 2–3 sentences comparing and contrasting the graphs for the three options.
Responses vary. I noticed the graphs for each option seemed to follow a line pattern. The points for Option 2 seem to fall on a horizontal line, while the points for Options 1 and 3 both increase from left to right. All of the graphs have the same independent and dependent variables.
ML/EL Learners: Emerging, Expanding, Bridging

2 **Unit 6** Expressions and Equations

Activity 1 Considering the Cost (continued)

Purpose: Students create representations for the relationship between the number of rides and the total cost of each fare option and make decisions about which fare option to choose. **(MP2)**



Presentation Screen
2

Monitor

Encourage students to use any of the three representations their group created to answer Problem 3.

Consider asking, "Does your customer qualify for the reduced fare? How do you know?"

M/EL **Multilingual/English Learners** Invite students to describe or act out each customer's information in their own words to make sense of it.

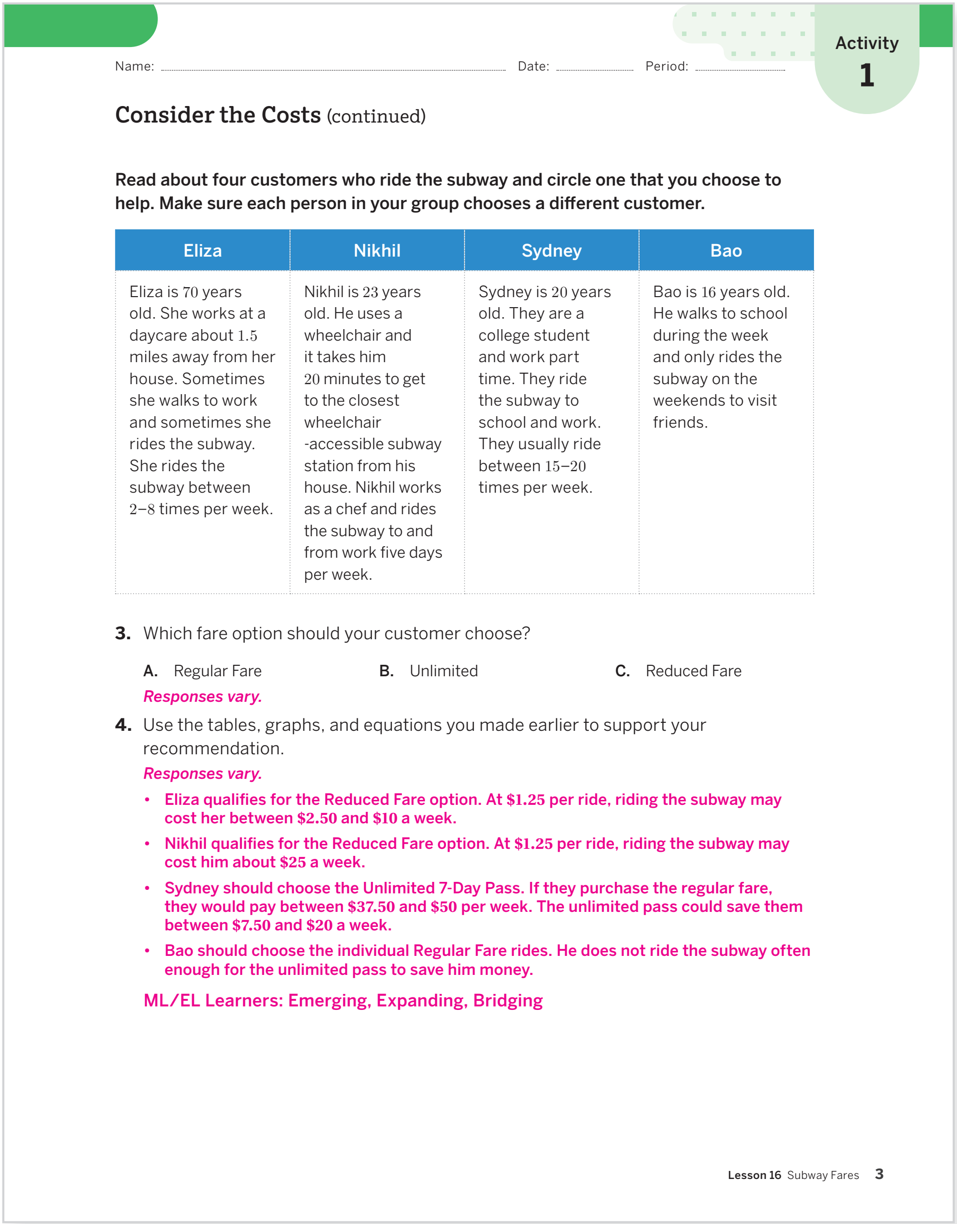
Connect

Invite students to share which option they selected for each customer and what evidence they used to support their claim.

Consider asking:
- "How are your tables for each option similar? How are they different?"
- "Which representation(s) helped you decide which option to choose for your customer? How did you use them?"

Math Identity and Community Consider asking, "Why do you think it's important to learn about different customers with different needs?"

Key Takeaway: Each representation (table, graph, and equation) can be used to compare fare options and to determine which fare option is the best choice depending on a person's individual needs.



Activity
1

Name: ______________ Date: ________ Period: ________

Consider the Costs (continued)

Read about four customers who ride the subway and circle one that you choose to help. Make sure each person in your group chooses a different customer.

Eliza	Nikhil	Sydney	Bao
Eliza is 70 years old. She works at a daycare about 1.5 miles away from her house. Sometimes she walks to work and sometimes she rides the subway. She rides the subway between 2–8 times per week.	Nikhil is 23 years old. He uses a wheelchair and it takes him 20 minutes to get to the closest wheelchair -accessible subway station from his house. Nikhil works as a chef and rides the subway to and from work five days per week.	Sydney is 20 years old. They are a college student and work part time. They ride the subway to school and work. They usually ride between 15–20 times per week.	Bao is 16 years old. He walks to school during the week and only rides the subway on the weekends to visit friends.

3. Which fare option should your customer choose?

A. Regular Fare B. Unlimited C. Reduced Fare

Responses vary.

4. Use the tables, graphs, and equations you made earlier to support your recommendation.

Responses vary.
- **Eliza qualifies for the Reduced Fare option. At $1.25 per ride, riding the subway may cost her between $2.50 and $10 a week.**
- **Nikhil qualifies for the Reduced Fare option. At $1.25 per ride, riding the subway may cost him about $25 a week.**
- **Sydney should choose the Unlimited 7-Day Pass. If they purchase the regular fare, they would pay between $37.50 and $50 per week. The unlimited pass could save them between $7.50 and $20 a week.**
- **Bao should choose the individual Regular Fare rides. He does not ride the subway often enough for the unlimited pass to save him money.**

ML/EL Learners: Emerging, Expanding, Bridging

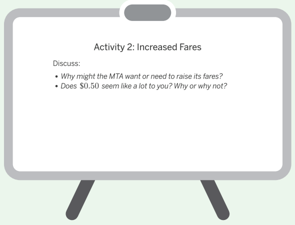
Lesson 16 Subway Fares 3

Small Groups | **15 min**

Activity 2 Increased Fares

Purpose: Students use their representations from Activity 1 to analyze an issue in society: the impacts of raising transit fares. **(MP4)**

Short on time: Consider having students complete Problem 6, then discuss Problems 7 and 8 as a group.


Activity 2: Increased Fares

Discuss:

- *Why might the MTA want or need to raise its fares?*
- *Does $\$0.50$ seem like a lot to you? Why or why not?*

Presentation Screen
3

3 Launch

Consider discussing: "Why might the MTA want or need to raise its fares? Does $\$0.50$ seem like a lot to you? Why or why not?"

M/EL **Multilingual/English Learners** Group students with different strengths, including social, mathematical, and language strengths, to support each other in making sense of each of the questions and craft a response.

3 Monitor

Encourage students to use their work from Activity 1 to support their recommendations about who would be most impacted.

A **Accessibility: Conceptual Processing** To support students getting started, consider asking, "What do you think the new cost of each option would be?"

D **Differentiation**

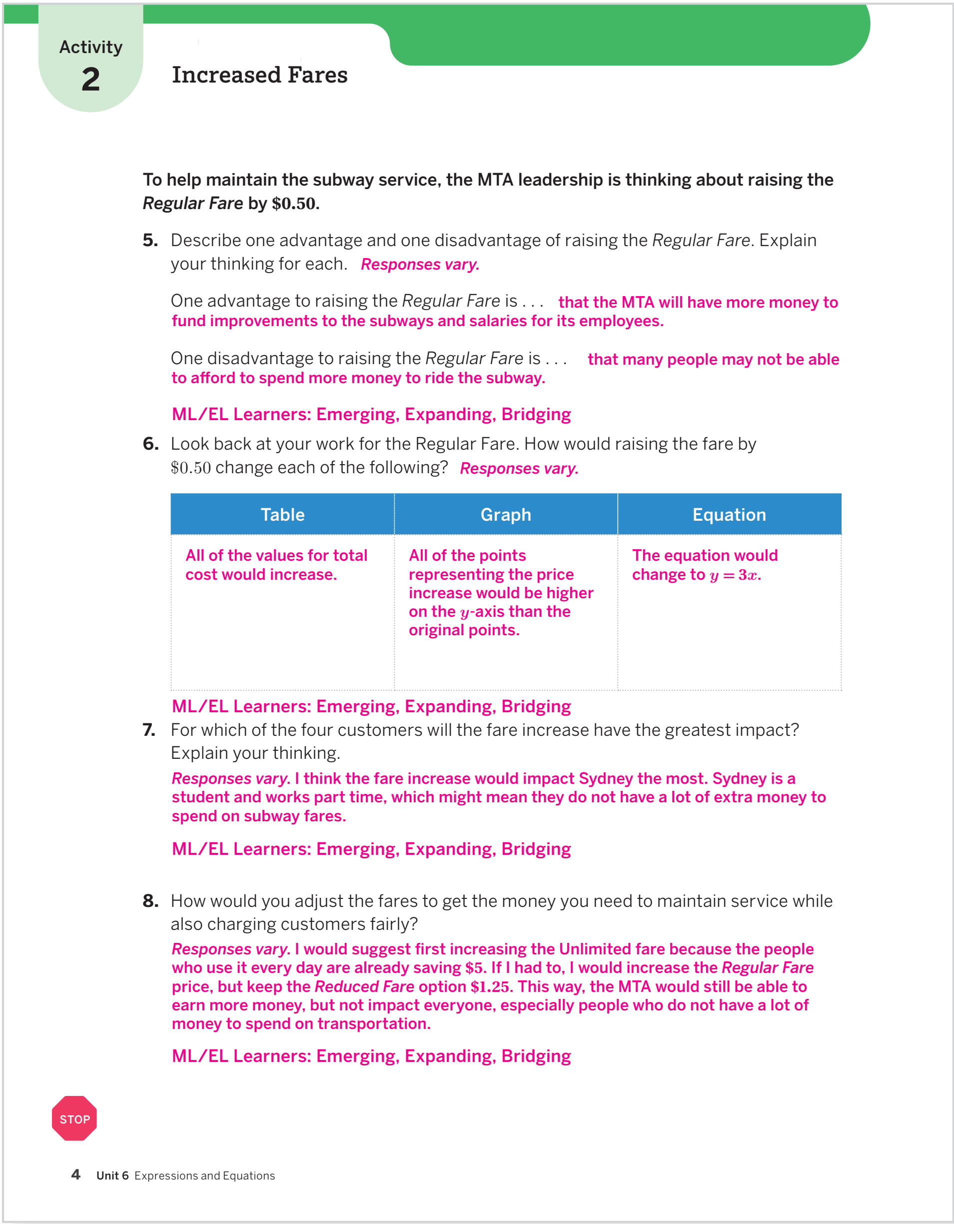
Look for students who . . .	Teacher Moves
Thinking the total cost would increase by $\$0.50$ total regardless of the number of rides.	**Support:** Consider asking, "How much would each ride cost after the increase? Two rides?"
Modeling the increase using a graph, table, or equation to reason about its impact.	**Extension:** Consider asking, "By what percent did the cost of one ride increase? What would be the new cost of the unlimited pass if it increased by the same amount?"

Pause to share students' ideas when most groups have responded to Problem 7.

3 Connect

Invite groups of students to share who they think would be most impacted by the increased fare and share the ideas students have for what the MTA should do.

Math Identity and Community Celebrate students who use information about each of the customers or their own sense of fairness to support them in their reasoning.


Activity 2 Increased Fares

To help maintain the subway service, the MTA leadership is thinking about raising the *Regular Fare* by $0.50.

5. Describe one advantage and one disadvantage of raising the *Regular Fare*. Explain your thinking for each. ***Responses vary.***

One advantage to raising the *Regular Fare* is . . . **that the MTA will have more money to fund improvements to the subways and salaries for its employees.**

One disadvantage to raising the *Regular Fare* is . . . **that many people may not be able to afford to spend more money to ride the subway.**

ML/EL Learners: Emerging, Expanding, Bridging

6. Look back at your work for the Regular Fare. How would raising the fare by $\$0.50$ change each of the following? ***Responses vary.***

Table	Graph	Equation
All of the values for total cost would increase.	**All of the points representing the price increase would be higher on the y-axis than the original points.**	**The equation would change to $y = 3x$.**

ML/EL Learners: Emerging, Expanding, Bridging

7. For which of the four customers will the fare increase have the greatest impact? Explain your thinking.

***Responses vary.* I think the fare increase would impact Sydney the most. Sydney is a student and works part time, which might mean they do not have a lot of extra money to spend on subway fares.**

ML/EL Learners: Emerging, Expanding, Bridging

8. How would you adjust the fares to get the money you need to maintain service while also charging customers fairly?

***Responses vary.* I would suggest first increasing the Unlimited fare because the people who use it every day are already saving $5. If I had to, I would increase the *Regular Fare* price, but keep the *Reduced Fare* option $1.25. This way, the MTA would still be able to earn more money, but not impact everyone, especially people who do not have a lot of money to spend on transportation.**

ML/EL Learners: Emerging, Expanding, Bridging

STOP

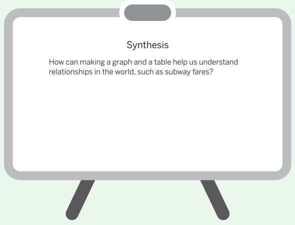
4 **Unit 6** Expressions and Equations

Key Takeaway: An equation, table, or graph can model increasing the price of the Regular Fare ticket. In the equation, the coefficient will increase by $\$0.50$. In the table and graph, the y-values will increase proportionally to the number of tickets purchased.

Whole Class | 5 min

Synthesis

Key Takeaway: Tables, graphs, and equations are all useful representations to analyze and compare relationships and make decisions.



Synthesis

How can making a graph and a table help us understand relationships in the world, such as subway fares?

Presentation Screen
4

4 Synthesis

Encourage students to look back at Activity 1, to compare the tables and graphs for each option.

Use the routine **MLR1: Stronger and Clearer Each Time** to help students develop their ideas and language.

Look for a variety of ideas, including:
- Values in a table can be seen in a graph.
- A table helps organize thinking and graphs help visualize relationships.

Consider asking:
- "What other situations might graphs or tables be helpful for?"
- "When is a table more helpful? When is a graph more helpful?'
- "How can graphs and tables be used to help to compare relationships?"

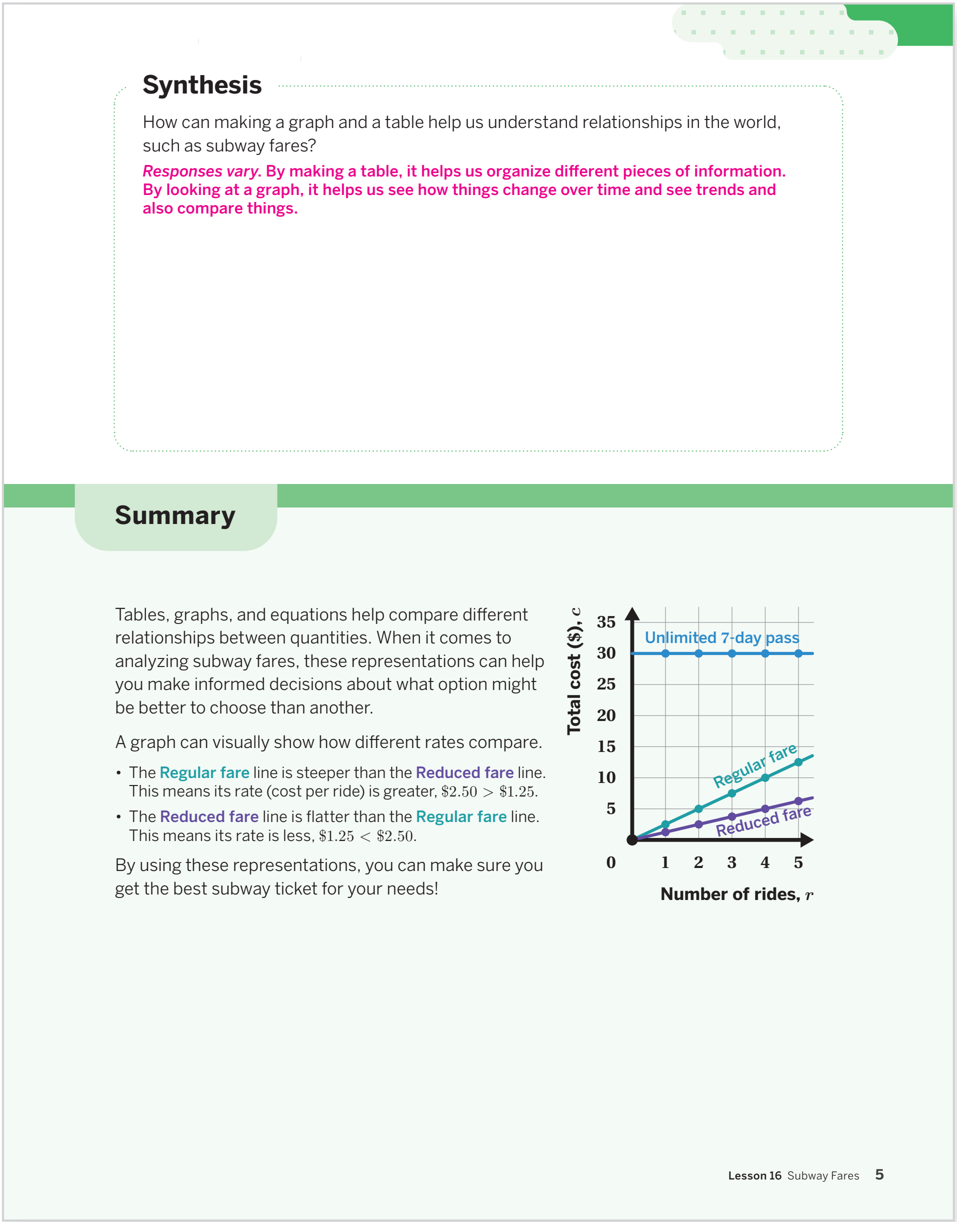
If time allows, invite students to make their response stronger and clearer.

M/EL **Multilingual/English Learners** Provide sentence frames to help them explain their thinking (e.g., Making a table and a graph can help us by ______.).

Lesson Takeaway: Tables, graphs, and equations are all useful representations of two-variable relationships. They can be used to analyze and compare relationships to make decisions.

Summary

Invite students to read the Summary. Share that students can refer back to the Summary throughout the unit and grade.



Synthesis

How can making a graph and a table help us understand relationships in the world, such as subway fares?

Responses vary. **By making a table, it helps us organize different pieces of information. By looking at a graph, it helps us see how things change over time and see trends and also compare things.**

Summary

Tables, graphs, and equations help compare different relationships between quantities. When it comes to analyzing subway fares, these representations can help you make informed decisions about what option might be better to choose than another.

A graph can visually show how different rates compare.

- The **Regular fare** line is steeper than the **Reduced fare** line. This means its rate (cost per ride) is greater, $\$2.50 > \$1.25$.
- The **Reduced fare** line is flatter than the **Regular fare** line. This means its rate is less, $\$1.25 < \$2.50$.

By using these representations, you can make sure you get the best subway ticket for your needs!

Lesson 16 Subway Fares 5

Independent | 5 min

Exit Ticket

Purpose: Students create a table and a graph to represent the relationship between distance and time of a train traveling at a constant rate.

Presentation Screen
5

5 Learning Goals

Goal: Create graphs, tables, and equations to represent situations.

Language Goal: Interpret representations to analyze an issue in society. **(Reading, Writing, Speaking, and Listening)**

Support for Future Learning: If students struggle, consider reviewing this problem as a class before Practice Day 2 or offering individual support where needed during the practice day.

Name: ______ Date: ______ Period: ______

Exit Ticket 6.16

A train is traveling from one station to another at a constant speed of 50 miles per hour.

1. Complete the table to show the amounts of time it takes the train to travel certain distances.

Time (hours)	Distance (miles)
2	**100**
7	**350**
3	150
4	200

2. Create a graph that represents this relationship.

How well did you understand the math in this lesson?

1 2 3 4 5

How did you feel about this lesson?

1 2 3 4 5

Reflect on the math from this lesson.

- I can create graphs, tables, and equations to represent situations.
- I can use tables, graphs, and equations to analyze an issue in society

Lesson 16 Subway Fares

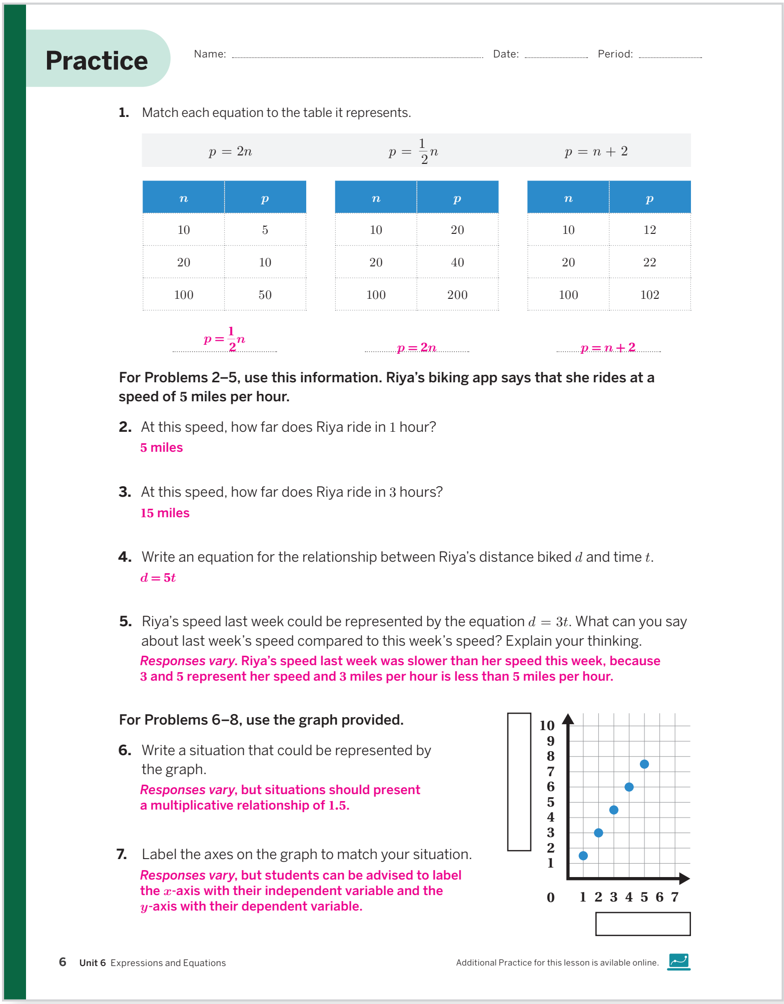
Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Students using digital

Students using print



Practice

Name: .... Date: .... Period: ....

1. Match each equation to the table it represents.

$p = 2n$	$p = \frac{1}{2}n$	$p = n + 2$

n	p
10	5
20	10
100	50

$p = \frac{1}{2}n$

n	p
10	20
20	40
100	200

$p = 2n$

n	p
10	12
20	22
100	102

$p = n + 2$

For Problems 2–5, use this information. Riya's biking app says that she rides at a speed of 5 miles per hour.

2. At this speed, how far does Riya ride in 1 hour?
5 miles

3. At this speed, how far does Riya ride in 3 hours?
15 miles

4. Write an equation for the relationship between Riya's distance biked d and time t.
$d = 5t$

5. Riya's speed last week could be represented by the equation $d = 3t$. What can you say about last week's speed compared to this week's speed? Explain your thinking.
***Responses vary.* Riya's speed last week was slower than her speed this week, because 3 and 5 represent her speed and 3 miles per hour is less than 5 miles per hour.**

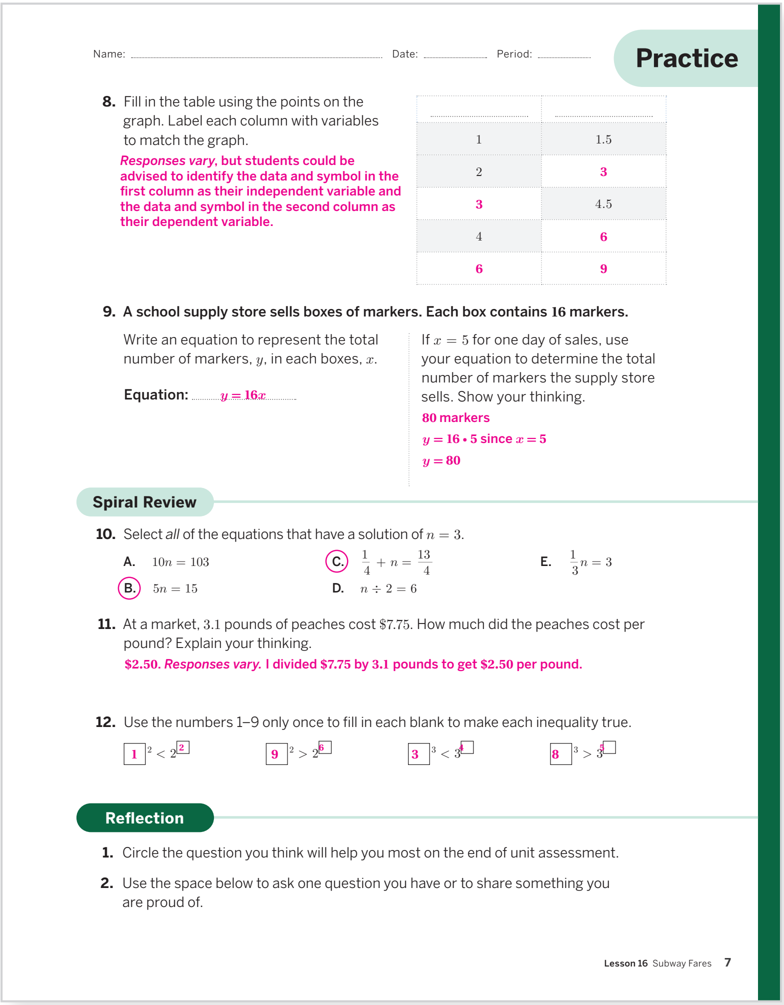
For Problems 6–8, use the graph provided.

6. Write a situation that could be represented by the graph.
***Responses vary*, but situations should present a multiplicative relationship of 1.5.**

7. Label the axes on the graph to match your situation.
***Responses vary*, but students can be advised to label the x-axis with their independent variable and the y-axis with their dependent variable.**

6 **Unit 6** Expressions and Equations

Additional Practice for this lesson is avilable online.



Name: .... Date: .... Period: ....

Practice

8. Fill in the table using the points on the graph. Label each column with variables to match the graph.
***Responses vary*, but students could be advised to identify the data and symbol in the first column as their independent variable and the data and symbol in the second column as their dependent variable.**

....	....
1	1.5
2	**3**
3	4.5
4	**6**
6	**9**

9. A school supply store sells boxes of markers. Each box contains 16 markers.

Write an equation to represent the total number of markers, y, in each boxes, x.

Equation: $y = 16x$

If $x = 5$ for one day of sales, use your equation to determine the total number of markers the supply store sells. Show your thinking.
80 markers
$y = 16 \cdot 5$ **since** $x = 5$
$y = 80$

Spiral Review

10. Select *all* of the equations that have a solution of $n = 3$.

A. $10n = 103$
[✓ B.] $5n = 15$
[✓ C.] $\frac{1}{4} + n = \frac{13}{4}$
D. $n \div 2 = 6$
E. $\frac{1}{3}n = 3$

11. At a market, 3.1 pounds of peaches cost $7.75. How much did the peaches cost per pound? Explain your thinking.
$2.50. *Responses vary.* I divided $7.75 by 3.1 pounds to get $2.50 per pound.

12. Use the numbers 1–9 only once to fill in each blank to make each inequality true.

$\boxed{1}^2 < 2^{\boxed{2}}$ $\boxed{9}^2 > 2^{\boxed{6}}$ $\boxed{3}^3 < 3^{\boxed{4}}$ $\boxed{8}^3 > 3^{\boxed{5}}$

Reflection

1. Circle the question you think will help you most on the end of unit assessment.

2. Use the space below to ask one question you have or to share something you are proud of.

Lesson 16 Subway Fares 7

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1	1	NY-6.EE.9
	2, 3	1	NY-6.RP.3
	4, 5	2	NY-6.EE.9
	6–8	3	NY-6.RP.3
Test Practice	9	2	NY-6.EE.9, NY-6.EE.2c
Spiral Review			
Fluency	10	1	NY-6.EE.7
	11	1	NY-6.RP.3
	12	3	NY-6.EE.1

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

6.6 Practice Day 2 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided copy for each student.

Task Cards

- *Option 1 (Stations)*: Print two single-sided sets of task cards for the entire class .
- *Option 2 (Level Up)*: Print one single-sided set of task cards for each group of 3–4 students.

Instructions

Option 1: Stations

- Arrange students into *groups of 2–4.*
- Distribute one Student Workspace Sheet to each student.

Options for student movement:

- Instruct students to move from station to station after they finish a station.
- After a set amount of time, instruct students to move as a group from station to station.
- After a set amount of time, instruct students to move to a new station such that no one from their previous group is in their new group.

Students can move to the “Are You Ready for More?” station when they have finished all the others. Students should record their responses on a separate sheet of paper.

Option 2: Level Up

- Arrange students into *groups of 2–3.*
- Distribute one Student Workspace Sheet to each student.
- Share with students that there are four tasks and one “Are You Ready for More?” task.
- Distribute one copy of “Dance-a-Thon” to each group.
- Once the group completes “Dance-a-Thon,” review their thinking. Consider choosing one student at random in the group to share the group’s ideas.
- Once the group has successfully completed “Dance-a-Thon”, invite them to pick up a copy of “Equivalent Expressions” to complete together.
- Continue this process until students have completed all four tasks. If students finish early, invite them to complete the “Are You Ready for More?” task on a separate sheet of paper.

GRADE 6

Unit 7
Lesson Plans

Teacher lesson plans from Unit 7 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

NOTE: *We have included only those lessons from Unit 7 that cover the standards in the Expressions, Equations, and Inequalities domain.*

Grade 6 Unit 7

Teacher Edition Sampler

Unit at a Glance

Key

📖 **Print Lessons**

💻 **Digital Lessons**

Assess and Respond

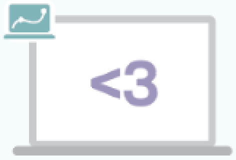
Pre-Unit Check
(Optional)

Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1

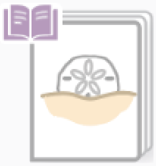
1 Can You Dig It?

Explain what positive and negative numbers are.



2 Digging Deeper

Identify and plot positive and negative numbers on the number line.



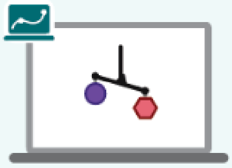
3 Order in the Class

Compare positive and negative numbers using the terms greater than, less than, and opposite, and the symbols > and <.

Sub-Unit 2

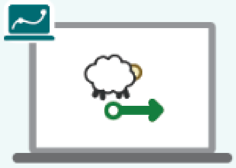
6 Tunnel Travels

Connect verbal, symbolic, and number line representations of inequalities in context.



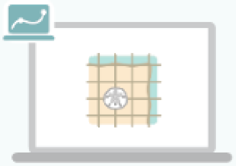
7 Comparing Weights

Write and interpret inequalities that represent relationships between weights on an unbalanced hanger.



8 Shira's Solutions

Draw and label a number line diagram to represent the solutions to an inequality.



9 Sand Dollar Search

Explain how the coordinate plane extends to include positive and negative numbers.

Summative Assessment

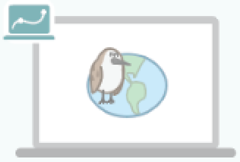
End-of-Unit Assessment

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing: 16 days | **Short on time?** See pacing considerations below.

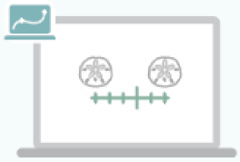
Pre-Unit Check: (Optional)
12 Lessons: 45 min each
2 Practice Days: 45 min each

1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



4 Sub-Zero

Explain what positive numbers, negative numbers, and 0 mean in context.



5 Distance on the Number Line

Understand the absolute value of a number as its distance from 0 on the number line.

Practice Day

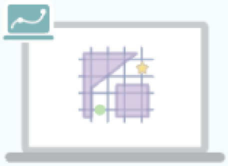
Practice Day 1

Practice the concepts and skills developed during Lessons 1–5. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

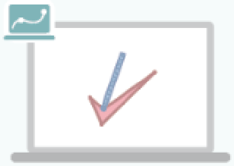
Quiz: Sub-Unit 1

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.



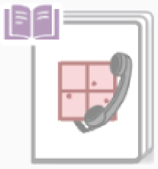
10 The A-maze-ing Coordinate Plane

Plot points with positive and negative coordinates on grids with different scales.



11 Polygon Maker

Draw a polygon in the coordinate plane given the coordinates for its vertices.



12 Graph Telephone

Use points with positive and negative coordinates on a graph to make sense of situations in context.

Practice Day

Practice Day 2

Practice the concepts and skills developed during Lessons 1–12. Consider using this time to prepare for the upcoming Quiz.

Pacing Considerations

Lesson 1: This lesson supports students in describing values less than 0 on a number line, which will be addressed in more depth in upcoming lessons.

Lesson 3: This lesson supports students in comparing positive and negative rational numbers. If students show a strong understanding comparing numbers on Problem 4 of the Pre-Unit Check, this lesson may be omitted. If omitted, be sure to discuss the word sign and how to use a number line to compare and order positive and negative numbers.

Lesson 7: The purpose of this lesson is for students to practice interpreting and writing inequalities in preparation for understanding solutions of inequalities in the next lesson. If most students show a strong understanding of inequalities in Problem 4 of the Pre-Unit Check and in Lesson 6, this lesson may be omitted.

Lesson 12: This lesson gives students an opportunity to apply the concepts they learned in this unit to plot and interpret coordinates in order to make sense of situations in context. There is no new content introduced in this lesson.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Tunnel Travels

Lesson 6: Graphing Inequalities

Overview

This is the first of three lessons about inequalities with variables. The purpose of this lesson is to introduce inequalities with variables and connect verbal descriptions, symbols, and number line representations of inequalities. This lesson builds on the work students did in Lessons 1–5 comparing numbers using inequality symbols and plotting numbers on the number line.

Learning Goals

- Connect verbal, symbolic, and number line representations of inequalities in context.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

This is the first of three lessons about inequalities with variables. The purpose of this lesson is to introduce inequalities with variables and connect verbal descriptions, symbols, and number line representations of inequalities. This lesson builds on the work students did in Lessons 1–5 comparing numbers using inequality symbols and plotting numbers on the number line.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to inequalities with variables and to give students an opportunity to be creative.

Activity 1: Inequalities in Context (20 minutes)

The purpose of this activity is for students to connect verbal descriptions, symbols, and number line representations of inequalities. Students use the contexts of vehicles fitting in a tunnel and children riding an amusement park ride to make connections between each representation.

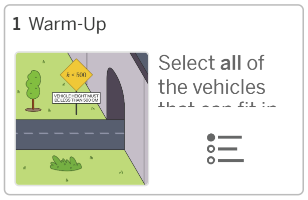
Activity 2: Inequalities Out of Context (10 minutes)

The purpose of this activity is for students to apply what they learned in Activity 1 to make sense of inequality symbols and graphs out of context. This activity also introduces students to inequalities with the variable on the right side, like $19 < x$.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to surface the advantages and disadvantages of different representations of inequalities as they apply what they learned to a new context.

Cool-Down (5 minutes)



1 Warm-Up

Select **all** of the vehicles

Select **all** of the vehicles that can fit in this tunnel.

Teacher Moves

Overview: This is the first of three lessons about inequalities with variables. The purpose of this lesson is to introduce inequalities with variables and connect verbal descriptions, symbols, and number line representations of inequalities. In this warm-up, students are introduced to inequalities with variables and have an opportunity to be creative.

Launch

- Consider starting with the lesson paused, displaying the dashboard's student view, and asking students what they notice and wonder about the words and symbols on the sign.
- Invite students to share any experiences they have with height limits or similar signs.
- Consider using a meter stick or measuring tape and asking students questions like: *Would you fit in the tunnel? How do you know? Would a vehicle as tall as the classroom fit in the tunnel?*

Facilitation

- If there is no consensus, consider displaying the distribution of responses during the discussion on Screen 2.

Materials (optional): Meter stick or measuring tape to help students visualize 500 cm.

Suggested Pacing: Screens 1–2

Sample Responses

The vehicles that are 300 cm tall and 495 cm tall.

Student Supports

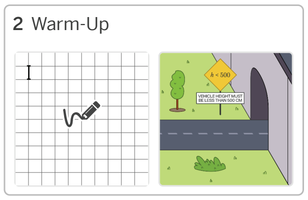
Students With Disabilities

• *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts by inviting students to choose one vehicle at a time and decide if it fits in the tunnel.

• *Conceptual Processing: Eliminate Barriers*

Consider using toy cars or other manipulatives to simulate cars fitting into and not fitting into a tunnel.



2 Warm-Up

Teacher Moves

Facilitation

- While students are working, select and sequence several student vehicles using the snapshot tool.

Discussion Questions

- *Will this vehicle fit? How do you know?*
- *How can you use the scale on the grid to help you decide if your vehicle fits?*
- *How could we change the sign so that this vehicle fits (or doesn't fit)? How would the words change? How would the symbols change?* (MP2)

Early Finishers

- Encourage students to research the height of a car they are familiar with (in centimeters) to see if it would fit in the tunnel.

Sample Responses

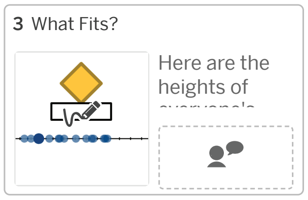
Drawings vary.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Strategic Pairing*

Allow students who struggle with fine motor skills to dictate use of the sketch tool as needed throughout the lesson.



3 What Fits?

Here are the heights of

Here are the heights of everyone's vehicles that fit in the tunnel.

What do you think a graph of all the vehicle heights that fit would look like?

Teacher Moves

Key Discussion Screen

- The purpose of this discussion is to make connections between individual points that make an inequality true and the graph of all points, and to surface the advantages of each representation.

Overview: In Activity 1 (Screens 3–7), students connect verbal descriptions, symbols, and number line representations of inequalities.

Facilitation

- While students are working, monitor for creative or concise text responses and sketches.
- When most students have responded, facilitate a whole-group discussion.

Discussion Questions

- *Why do you think there are no dots to the right of* 500*?*
- *Is there a faster way to show all the heights than making dots?*
- *Where do you see "less than" in the symbols, words, and number line?*
- *What are the advantages of the symbols? Words? Number line?*

Suggested Pacing: Screens 3–7

Sample Responses

Responses vary.

- It would look like a lot of dots squished together all to the left of 500.
- All the vehicle's heights would make a thick line left of 500 that stopped at 0 since you can't have a vehicle with a negative height.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

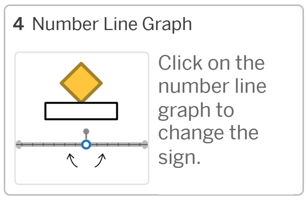
Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., A graph of all the vehicles that fit would look like _____.).

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



4 Number Line Graph

Click on the number line graph to change the sign.

Click on the number line graph to change the sign.

What do you notice? What do you wonder?

Teacher Moves

Facilitation

- Invite students to explore creating different signs.
- Consider encouraging students to try recreating the sign from Screens 1–3.

- Use the routine Notice and Wonder to surface what students noticed and wondered about the diagram.

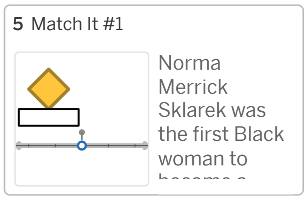
Student Supports

Students With Disabilities
- *Executive Functioning: Graphic Organizers*

Provide students a T-chart to record what they notice along with the inequality they used before they are expected to share their ideas with others.

Multilingual Learners
- *Routine: Collect and Display*

Circulate and listen to students talk as they describe what they notice about each representation. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.


5 Match It #1

Norma Merrick Sklarek was the first Black woman to

Sources:

- *Pioneering Women of American Architecture*
- *Parkopedia*

Teacher Moves

Facilitation
- Encourage students to use the feedback on the screen to help them revise their thinking.
- Consider using the language routine Three Reads to help students make sense of the text on this screen.

Sample Responses

Image solution

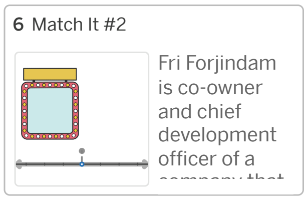
Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Demonstrate an incorrect solution like $h > 300$ to provide access to students who benefit from clear and explicit instructions.

- *Executive Functioning: Visual Aids*

Create an anchor chart for public display that includes the symbols, the verbal description, and the graph of an inequality for future reference.


6 Match It #2

Fri Forjindam is co-owner and chief development officer of a

Sources:

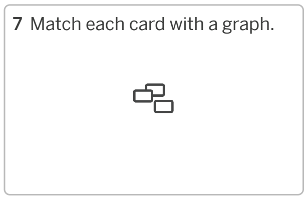
- *Fast Company*
- *Bollywood Parks Dubai*

Teacher Moves

Progress Check
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Image solution


7 Match each card with a graph.

Teacher Moves

Facilitation
- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to use the structure of each representation to sort the cards (MP7).
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Early Finishers
- Encourage students to create their own set of 3 or 6 cards, then share the cards with the class and discuss whether or not the representations tell the same story.

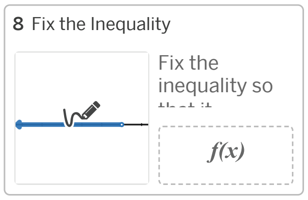
Sample Responses

Image solution

Student Supports

Students With Disabilities
- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts by inviting students to choose one graph at a time and find all of the cards that match with it.


8 Fix the Inequality

Fix the inequality so

Fix the inequality so that it matches the graph.

Teacher Moves

Overview: In Activity 2 (Screens 8–10), students apply what they learned in Activity 1 to make sense of inequality symbols and graphs out of context.

Launch
- Consider starting with the activity paused and sharing the purpose of the activity.
- Give students one minute to look at the graph and inequality before responding.

Suggested Pacing: Screens 8–10

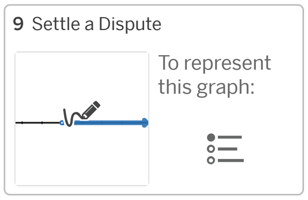
Sample Responses

$x < 23$ or $23 > x$

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Invite students to press "Check My Work" without fixing the inequality, and then share with a partner what is incorrect about the inequality before fixing it.


9 Settle a Dispute

To represent this graph:

To represent this graph:

- Martina wrote the inequality $20 < x$.
- Nasir wrote the inequality $x < 20$.

Whose inequality is correct?

Teacher Moves

Facilitation

- While students are working, select and sequence 1–2 responses for each of the most popular choices.
- Once most students have responded, consider displaying the distribution of responses, calling attention to any conflict or consensus you see.

Discussion Questions

- *How did you decide whose inequality was correct?*
- *Is there another correct way to describe this inequality?*

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Martina

Explanations vary. Since the graph is to the right, x has to be all the numbers that are greater than 20, which means that 20 is less than x.

Student Supports

Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Before students respond, ask them what they could do to test if an inequality is correct. If it does not come up naturally, consider inviting students to choose a number that is shaded on the graph and test it in each inequality.

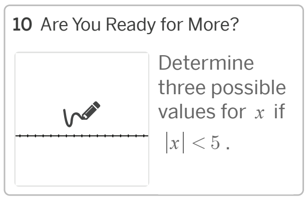
Multilingual Learners

- *Receptive Language: Processing Time*

Read the prompt aloud or invite students to read the prompt in pairs so they can practice speaking and listening to phrases like "20 is less than x" and "x is less than 20."

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., _____ is correct because ________.).


10 Are You Ready for More?

Determine three possible values for x if $|x| < 5$.

Determine three possible values for x if $|x| < 5$.

1. Plot these values on the number line.
2. Plot as many other possible values for x as you can.

Teacher Moves

Facilitation

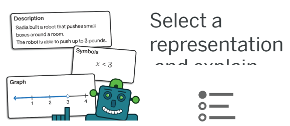
- Invite students who finish Screens 8–10 early to explore this screen.
- Encourage students to share responses with each other in place of a whole-class discussion.

Sample Responses

Responses vary. The number line should be shaded between -5 and 5.

Image solution

11 Lesson Synthesis



Select a representation and explain how it shows that Sadia's robot can push a 2-pound box.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of the discussion is to compare the advantages and disadvantages of different representations of inequalities as they apply what they learned to a new context.

Facilitation
- Give students 2–3 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses for each representation to display using the dashboard's teacher view or snapshot tool.

Discussion Questions
- *What are the advantages of the words? The symbols? The graph?*
- *What are the disadvantages of the the words? The symbols? The graph?*

Suggested Pacing: Screen 11

Sample Responses

Responses vary.

- From the description, I know that Sadia's robot can push a 2-pound box because it says it can push up to 3 pounds and 2 is less than 3.
- From the symbols, I know that Sadia's robot can push a 2-pound box because $2 < 3$.
- From the graph, I know that Sadia's robot can push a 2-pound box because 2 is in the shaded part of the number line.

Student Supports

Students With Disabilities
- *Receptive Language: Processing Time*

Read the prompt and the information in each representation aloud for students who benefit from extra processing time.

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., From the _____, I know that ____.).
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

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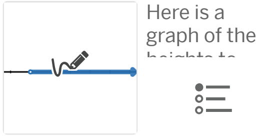
12 Cool-Down



Here is a graph of the heights to ride a roller coaster.

1. Select the sign that matches this graph.

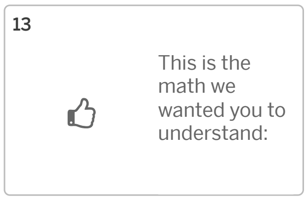
Teacher Moves

Support for Future Learning
- Students will have more chances to develop their understanding of inequalities in the upcoming lessons, particularly Lessons 7 and 8.

Suggested Pacing: Screens 12–13

Sample Responses

- You must be more than 48 inches to ride.
- $h > 48$



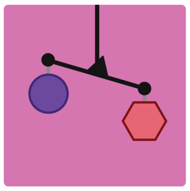
13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can show the same information about an inequality using words, symbols, and a number line.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Comparing Weights

Lesson 7: Writing Inequalities

Overview

Students develop a deeper understanding of using inequality symbols to describe unknown values. Students use the hanger representation from earlier in Grade 6 to interpret and write inequalities that involve one variable and more than one variable.

Learning Goals

- Write and interpret inequalities that represent relationships between weights on an unbalanced hanger.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop a deeper understanding of using inequality symbols to describe unknown values. Students use the hanger representation from earlier in Grade 6 to interpret and write inequalities that involve one variable and more than one variable. This lesson also digs deeper into the different ways of writing an inequality to describe the same relationship (e.g., $s < 8$ and $8 > s$).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to unbalanced hangers and for students to use a hanger to compare the relative weights of objects.

Activity 1: One-Variable Inequalities (15 minutes)

The purpose of this activity is for students to interpret inequalities that represent relationships between weights on unbalanced hangers. This builds on the work students did in Lesson 6 connecting contexts and inequalities represented in words, symbols, and graphs. This activity also introduces students to the idea that there is more than one correct way to write an inequality that includes a variable.

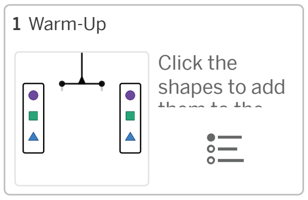
Activity 2: Many-Variable Inequalities (15 minutes)

The purpose of this activity is for students to extend their understanding to write inequalities using only variables to compare weights. Students use their creativity to create unbalanced hangers and write inequalities comparing unknown weights.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to make connections between an unbalanced hanger with unknown weights and an inequality written in symbols with variables.

Cool-Down (5 minutes)



1 Warm-Up

Click the shapes to add

Click the shapes to add them to the hanger.

Which object is heaviest?

Teacher Moves

Overview: In this lesson, students develop a deeper understanding of using inequality symbols to describe unknown values. In this warm-up, students are introduced to unbalanced hangers and use a hanger to compare the relative weights of objects.

Launch
- Consider starting with the activity paused.
- Use the dashboard's student view to demonstrate how to add weights and reset each side.

Facilitation
- After 2–3 minutes, encourage several students to share their reasoning.
- If it doesn't come up naturally, consider asking students how they know which side of the hanger is heavier.
- **Note:** As students explain their reasoning, it may be helpful to write inequalities that students describe on the board for future reference. For example, $c < s$ would represent the circle on the left and the square on the right, where the square is heavier than the circle.

Early Finishers
- Encourage students to use a different strategy to defend their argument, or to try and create a balanced hanger.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Suggested Pacing: Screen 1

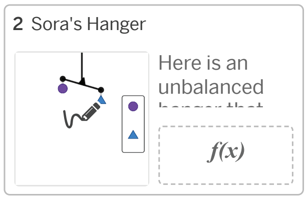
Sample Responses

Square

Explanations vary. The square is heavier than the circle and the square is heavier than the triangle, so it might be the heaviest object.

Student Supports

Students With Disabilities
• *Conceptual Processing: Eliminate Barriers*
Demonstrate the steps for the warm-up by inviting a student to suggest two different shapes and then discuss as a class which is heavier and how they know.



2 Sora's Hanger

Here is an unbalanced

Here is an unbalanced hanger that Sora made.

The circle weighs 5 pounds. What is a possible weight for the triangle?

Teacher Moves

Overview: In Activity 1 (Screens 2–5), students interpret inequalities that represent relationships between weights on unbalanced hangers.

Launch
- Arrange students *into pairs.*
- Consider starting with the activity paused, displaying the dashboard's student view and inviting students to share what they notice and wonder about the hanger.
- If it does not come up naturally, consider asking: *Which object is heavier? How do you know?*

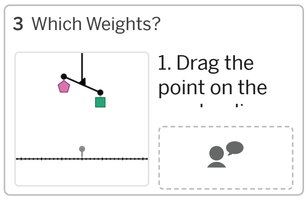
Facilitation
- Encourage students to read others' responses and decide if others' weights make sense for this hanger.

Suggested Pacing: Screens 2–5

Sample Responses

Responses vary.

- 7 pounds
- 100 pounds
- 5.1 pounds



3 Which Weights?

1. Drag the point on the

1. Drag the point on the number line to adjust the weight of the square.

2. Describe all of the possible weights for the square that will make it **heavier** than the pentagon.

Teacher Moves

Facilitation
- While students are working, select and sequence several student responses using the snapshot tool.
- Consider building on students' thinking on this screen during the discussion on the next screen.

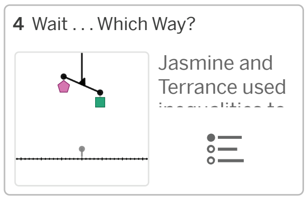
Sample Responses

Responses vary. Any weight more than 8 grams will make the square heavier than the pentagon.

Student Supports

Students With Disabilities
- *Executive Functioning: Graphic Organizers*

Provide students a chart to record what they notice and wonder as they drag the point on the number line before they respond to the prompt.



4 Wait . . . Which Way?

Jasmine and Terrance used

Jasmine and Terrance used inequalities to describe all the weights for the square, s, that are heavier than the pentagon.

- Jasmine says it can be written as $8 < s$.
- Terrance says it can be written as $s > 8$.

Who is correct?

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to make connections between symbols that represent inequalities and unknown weights on hangers, and to surface why it is possible to write more than one inequality to represent a hanger.

Facilitation
- Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.
- Select and sequence several responses using the snapshots tool.
- Invite those students to share their reasoning about which inequality represents the hanger (MP2).

Discussion Questions
- *How would you say each students' inequality in words?*
- *How can both students be correct?*
- *What would a hanger for the inequality $s < 8$ look like?*

Sample Responses

Both

Explanations vary. Terrance's inequality says that s has to be more than 8, which matches the hanger. Jasmine's says that 8 has to be less than s, which is another way of saying that s has to be more than 8.

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Before students respond, ask them what they could do to test if an inequality is correct. If it does not come up naturally, consider inviting students to choose a number that is shaded on the graph and test it in each inequality.

Multilingual Learners
- *Receptive Language: Processing Time*

Read the prompt aloud or invite students to read the prompt in pairs so they can practice speaking and listening to phrases like "8 is less than s" and "s is greater than 8."

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., _____ is correct because ________.).

---

5 Match each card with a hanger.

Teacher Moves

Facilitation
- Encourage students to share their reasoning with a partner and work to reach an agreement about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Early Student Thinking
- Students may recognize that $10 > h$ and $h > 10$ both have the same symbol and assume they say the same thing.
- Consider inviting these students to choose a specific value for h or to say out loud what each inequality means.

Early Finishers
- Encourage students who finish early to create their own set of cards that they think might be tricky.

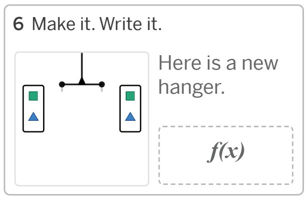
Sample Responses

Image solution

Student Supports

Students With Disabilities
- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts by inviting students to choose one hanger at a time and find all of the cards that match it.



6 Make it. Write it.

Here is a new hanger.

Here is a new hanger.

1. Add objects to each side of the hanger.

2. Write an inequality to represent the hanger.

Teacher Moves

Overview: In Activity 2(Screens 6–8), students extend their understanding to write inequalities using only variables to compare weights.

Launch
- Consider sharing with students that they will use their creativity to write inequalities with no numbers in them at all.

Facilitation
- Give students 2–3 minutes to make different hangers and write inequalities to represent them.
- If students are having difficulty getting started, consider inviting them to create a simple unbalanced hanger and writing an inequality together first (MP1).

Early Student Thinking
- Some students may create hangers with more than one variable on one side and write the weight as st instead of $s + t$. Consider asking these students to make up weights for the square and the triangle and then check to see if $s \cdot t$ is equal to the total they expect.

Suggested Pacing: Screen 6

Sample Responses

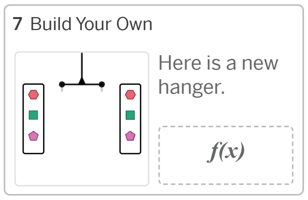
Responses vary.

- $s > 0$
- $s < t$
- $t > 2s$
- $5s > s + t$

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Demonstrate this task by inviting a student to suggest a hanger to create and then discuss as a class an inequality that describes that hanger.


7 Build Your Own

Here is a new hanger.

Here is a new hanger.

1. Add objects to each side of the hanger.

2. Write an inequality to represent the hanger.

Teacher Moves

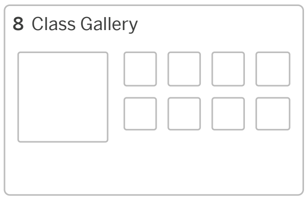
Progress Check
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- It may be helpful to remind students that they do not need to include all three shapes in their hanger.

Suggested Pacing: Screens 7–8

Sample Responses

Responses vary.

- $h > s$
- $h + p < s + h + p$
- $2h > 3p$


8 Class Gallery

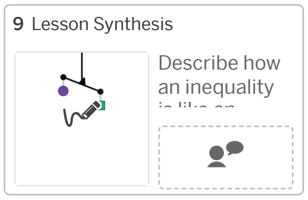
Teacher Moves

Overview: In this Challenge Creator, students build their own unbalanced hanger. Students then challenge themselves to write an inequality to represent their and their classmates' hangers.

Facilitation
- Give students several minutes to create their own challenge and more time to solve their classmates' challenges.
- Encourage students to go back and review their classmates' responses to the challenge they created.
- While students are working, monitor for and highlight creative challenges and solutions.
- **Note:** We anticipate this Challenge Creator may take 10 minutes or more.

Math Community
- Consider inviting students to share challenges they found particularly fun or creative.

Recommended Pacing: Screen 8



9 Lesson Synthesis

Describe how an inequality

Describe how an inequality is like an unbalanced hanger.

Use the diagram if it helps you with your thinking.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is for students to make connections between an unbalanced hanger with unknown weights and an inequality written in symbols with variables.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions
- *What is similar about the inequality and the hanger?*
- *Why might we want to describe an inequality with symbols?*

Suggested Pacing: Screen 9

Sample Responses

Responses vary. An unbalanced hanger shows which weight is heavier. An inequality shows which number is greater. For example, since the circle is lighter than the square, $c < s$ makes sense because these symbols mean that whatever weight the circle is, it is less than the weight of the square.

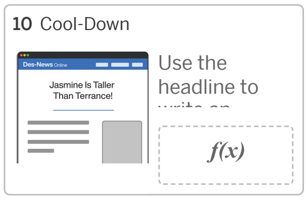
Student Supports

Students With Disabilities
- *Receptive Language: Processing Time*

Read the prompt aloud for students who benefit from extra processing time.

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



10 Cool-Down

Jasmine Is Taller Than Terrance!

Use the headline to

f(x)

Use the headline to write an inequality describing Jasmine's height, j, and Terrance's height, t.

Teacher Moves

Support for Future Learning
- Students will have more chances to develop their understanding of writing inequalities in the next lesson.

Suggested Pacing: Screens 10–11

Sample Responses

$j > t$

Responses vary. 61 inches.

11

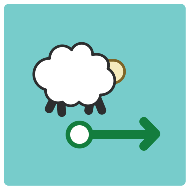
👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can write and interpret inequalities to describe unbalanced hangers.

Digital Lesson

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Shira's Solutions

Lesson 8: Solutions to Inequalities

Overview

Students consolidate and apply what they have learned about inequalities on the number line and learn the term *solution to an inequality*.

Learning Goals

- Draw and label a number line diagram to represent the solutions to an inequality.
- Explain why an inequality may have infinitely many solutions.
- Justify whether a value is a solution to a given inequality.

Vocabulary

- solution to an inequality

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to consolidate and apply what they have learned about inequalities on the number line and learn the term *solution to an inequality*. Students continue to connect graphs, verbal descriptions, and symbolic representations of inequalities. This lesson also formally introduces the idea that there are an infinite number of solutions to inequalities of the form $x < c$ or $x > c$.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce the context of Shira the Sheep and surface that there are multiple inequalities that include some of the same solutions.

Activity 1: Connecting Graphs and Inequalities (15 minutes)

The purpose of this activity is for students to write inequalities that include specific solutions, which are represented by blades of grass. Students solve a series of challenges in which they write inequalities to control how Shira the Sheep moves. This activity also informally addresses the boundary point (e.g., why 5 is not a solution to the inequality $x<5$).

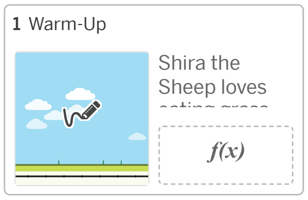
Activity 2: Solutions to an Inequality (15 minutes)

The purpose of this activity is for students to make connections between graphs, numeric solutions, and symbolic representations of inequalities. Students also investigate the number of solutions to an inequality.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to reflect on what it means for a number to be a solution to an inequality.

Cool-Down (5 minutes)



1 Warm-Up

Shira the Sheep loves

Shira the Sheep loves eating grass.

Try out different inequalities to see what happens.

Teacher Moves

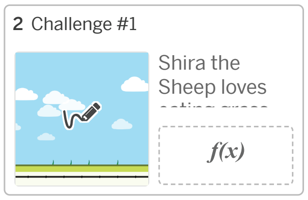
Overview: In this lesson, students consolidate and apply what they have learned about inequalities on a number line and learn the term *solution to an inequality*. The warm-up introduces the context of Shira the Sheep eating grass, and surfaces that there are multiple inequalities that include some of the same solutions.

Launch
- Invite students to press "Try It" and observe what happens.

Facilitation
- Give students 1–2 minutes to try different inequalities and then share with a partner what they notice and wonder.
- Encourage students to try different inequality symbols and different values.
- After 2–3 minutes, invite students to share inequalities that they thought had an interesting or surprising result, and any inequalities that made Shira eat all the grass.

Math Community
- Consider highlighting unique or creative inequalities or equations using the snapshot tool or the dashboard's teacher view.
- Ask the author to speak about what happened when they tried that inequality or equation.

Suggested Pacing: Screen 1



2 Challenge #1

Shira the Sheep loves

Shira the Sheep loves eating grass.

Fix this inequality to help her eat all the grass.

Teacher Moves

Overview: In Activity 1(Screens 2–6), students write inequalities that include specific solutions, which are represented by blades of grass.

Launch
- Arrange students *into pairs.*
- Consider starting with the activity paused and sharing with students that there is often more than one inequality that will help Shira eat all the grass.
- Invite students to try several different ways to fix the inequality.

Facilitation
- If a student is having difficulty getting started, encourage them to press "Try It," then to use the feedback to revise the inequality.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Suggested Pacing: Screens 2–6

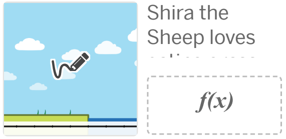
Sample Responses

Responses vary.

- $11 > x$

- $-11 < x$

3 Challenge #2



Shira the Sheep loves

Shira the Sheep loves eating grass.

She does not like water.

Fix this inequality to help her eat all the grass without falling in the water.

Teacher Moves

Math Community

- Remind students that there are several interesting ways to be right and wrong on this screen.
- Consider inviting students to share with a partner what they learned from inequalities that did not work.

Sample Responses

Responses vary.

- $x < 10$
- $6 > x$

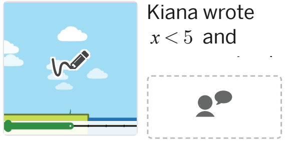
Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Invite students to share what is different about Challenge #2 before responding.

4 Reflect



Kiana wrote $x < 5$ and

Kiana wrote $x < 5$ and was surprised that there was one blade of grass remaining.

Explain why 5 is not a solution to this inequality.

Teacher Moves

Facilitation

- Invite students to go back to Screen 3 and enter $x < 5$ to see what Kiana saw.
- When most students have responded, consider pausing for a discussion in order to surface what happens at the boundary point of an inequality (MP6).

Discussion Questions

- *What do you think "solution" means here?*
- *What are solutions to $x < 5$?*
- *How could we change Kiana's inequality so 5 is a solution?*

Sample Responses

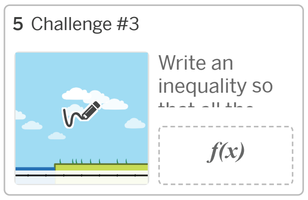
Responses vary. The inequality $x < 5$ is true for all numbers that are less than 5. Since 5 is not less than 5, it is not a solution to this inequality.

Student Supports

Students With Disabilities

• *Receptive Language: Processing Time*
Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners
• *Expressive Language: Eliminate Barriers*
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



Write an inequality so that all the blades of grass are solutions and the water has no solutions.

Teacher Moves

Facilitation

- While students are working, select several successful and unsuccessful inequalities using the snapshot tool.
- Consider pausing the class to discuss these inequalities. Ask students to justify which inequalities will help Shira and which will not, and to critique each other's reasoning (MP3).
- If it does not come up naturally, consider highlighting different ways of writing a successful inequality (e.g., $x > -4.5$ and $-4.5 < x$).

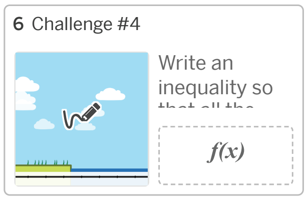
Sample Responses

Responses vary.

- $x > -4.5$
- $-4.3 < x$

Student Supports

Students With Disabilities
• *Visual-Spatial Processing: Visual Aids*
Invite students to share what is different about Challenge #3 before responding.



Write an inequality so that all the blades of grass are solutions and the water has no solutions.

Teacher Moves

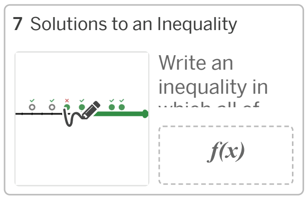
Early Finishers

- Encourage students to revisit Challenges 1–4 and find other ways to solve each challenge.

Sample Responses

Responses vary.

- $x < -5.3$
- $-5.35 > x$


7 Solutions to an Inequality

Write an inequality in

Write an inequality in which all of the solid points and none of the open points are solutions.

Teacher Moves

Overview: In Activity 2 (Screens 7–12), students make connections between graphs, numeric solutions, and symbolic representations of inequalities. They also investigate the number of solutions to an inequality.

Launch
- Consider starting with the activity paused and creating a class definition of the term *solution to an inequality*.

Math Community
- Celebrate several different inequalities that meet the conditions, including any unique or creative solutions.

Suggested Pacing: Screens 7–12

Sample Responses

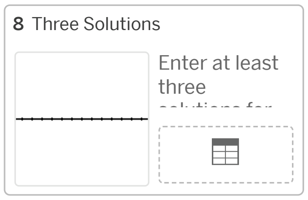
Responses vary.

- $x > -2$
- $-2.5 < x$

Student Supports

Students With Disabilities
• *Conceptual Processing: Processing Time*
Demonstrate an incorrect solution like $x > 2$ and then invite students to share with a partner what is incorrect about the inequality before fixing it.

• *Receptive Language: Processing Time*
Read the prompt aloud for students who benefit from extra processing time.


8 Three Solutions

Enter at least three

Enter at least three solutions for the inequality $2.7 > x$.

Teacher Moves

Progress Check
- Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.
- Encourage students to use the feedback to make revisions as necessary.

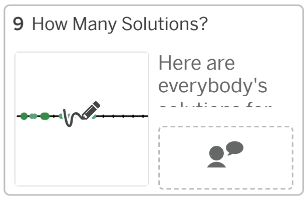
Early Student Thinking
- Students may recognize the symbol $>$ and assume that x has to be greater than 2.7.
- Consider inviting these students to replace x with a number they think is a solution and to decide if that statement is true or false.

Sample Responses

Responses vary.

- 2

- 0
- -5.5



9 How Many Solutions?

Here are everybody's

Here are everybody's solutions for $2.7 > x$. Yours are bold.

Describe how many solutions this inequality has and how you know.

Teacher Moves

Key Discussion Screen 🔑
- The purpose of this discussion is to surface that inequalities have infinite solutions and to reason about solutions to inequalities written as $\# > x$ (MP2).

Facilitation
- When most students have responded, consider pausing the class and facilitating a whole-class discussion.
- Consider inviting students to sketch the graph of all of the solutions using the sketch tool.

Discussion Questions
- *How do you know* 2 *is a solution to this inequality, but* 3 *is not?*
- *What does it mean to have infinite solutions? Is this special to this inequality?*
- *How do we show the infinite solutions on the graph?*

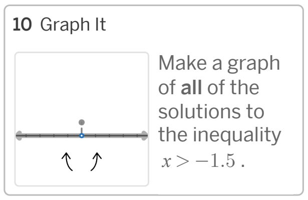
Routine (optional): Consider using the mathematical language routine Collect and Display.

Sample Responses

Responses vary. There are infinite solutions because a solution can be any number that is less than 2.7 and there are infinitely many of those.

Student Supports

Multilingual Learners
• *Expressive Language: Eliminate Barriers*
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



10 Graph It

Make a graph of **all** of the solutions to the inequality $x > -1.5$.

Make a graph of **all** of the solutions to the inequality $x > -1.5$.

Teacher Moves

Facilitation
- Consider inviting students to share several solutions to the inequality $x > -1.5$ and where they see them in the graph with a partner.

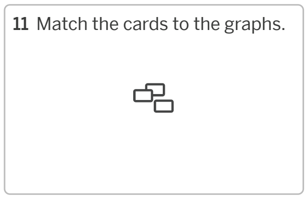
Sample Responses

Image solution

Student Supports

Students With Disabilities

• *Fine Motor Skills: Strategic Pairing*
Allow students who struggle with fine motor skills to dictate physical manipulation of the numberline as needed.


11 Match the cards to the graphs.

Teacher Moves

Facilitation
- Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

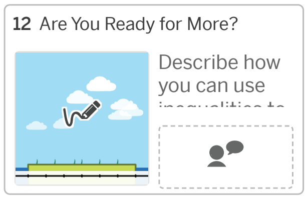
Early Student Thinking
- Students may match each inequality with one graph.
- Consider inviting students to test some of the solutions they see in the graph by inserting them into the inequality and checking if they make the inequality true.

Sample Responses

Image solution

Student Supports

Students With Disabilities
• *Executive Functioning: Eliminate Barriers*
Chunk this activity into more manageable parts by inviting students to choose one graph at a time and find all of the cards that match it.


12 Are You Ready for More?

Describe how you can use

Describe how you can use inequalities to help Shira the Sheep eat all of the grass without falling in the water.

Teacher Moves

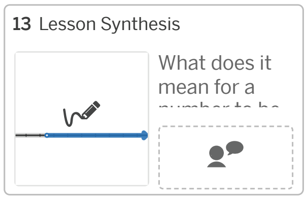
Facilitation
- This screen is designed as an extra challenge for students who finish Screens 7–11 before the class discussion on Screen 13.
- Consider inviting these students to share responses with each other in place of a whole-class discussion.

Note: Some students may make a connection to absolute value from Lesson 5 as they work on this screen.

Sample Responses

Responses vary.

- $x > -3$ and $x < 3$
- The absolute value of x is less than 3.
- $|x| < 3$


13 Lesson Synthesis

What does it mean for a

What does it mean for a number to be a solution to an inequality?

Teacher Moves

Key Discussion Screen
- The purpose of this discussion is for students to reflect on what it means for a number to be a solution to an inequality.

Facilitation
- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Select and sequence several student responses to display.
- If time allows, give students one minute to make their response stronger and clearer based on the discussion.

Discussion Questions
- *How can you test if a number is a solution to an inequality?*

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Suggested Pacing: Screen 13

Sample Responses

Responses vary. It means that that number makes the inequality true. It's also in the shaded part of the graph. For example, -1 is a solution to $x > -2.5$ because $-1 > -2.5$ and because -1 is in the shaded part of the graph.

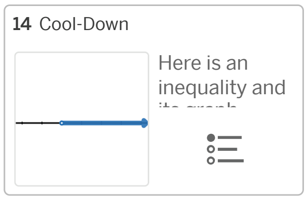
Student Supports

Students With Disabilities
- *Receptive Language: Processing Time*

Consider reading the prompt aloud and inviting one or more students to paraphrase it in their own words to support students who benefit from both reading and listening.

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., For a number to be a solution to an inequality, __________.).


14 Cool-Down

Here is an inequality and

Here is an inequality and its graph.

Select **all** of the numbers that are solutions.

Teacher Moves

Support for Future Learning
- If students struggle, consider reviewing this screen as a class before Practice Day 2 or offering individual support where needed during the practice day.

Suggested Pacing: Screens 14–15

Sample Responses

- 0

- 5
- 20

15

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can draw and label a number line diagram that represents the solutions to an inequality.
- I can explain how many solutions an inequality can have.
- I can justify whether or not a value is a solution to a given inequality.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

6.7 Practice Day 2 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided copy for each student.

Task Cards

- *Option 1 (Stations)*: Print two single-sided sets of task cards for the entire class (10 cards total). Cut out the cards on the “Sort It” task, if desired.
- *Option 2 (Level Up)*: Print one single-sided set of task cards for each group of 2–3 students. Cut out the cards on the “Sort It” task, if desired. Make a pile for each task card in a central location for students to drop off and pick up.

Instructions

Option 1: Stations

Arrange students into groups of 2–4. Distribute one Student Workspace Sheet to each student to complete as they solve each of the tasks.

Options for student movement:

- Instruct students to move from station to station after they finish a station.
- After a set amount of time, instruct students to move as a group from station to station.
- After a set amount of time, instruct students to move to a new station such that no one from their previous group is in their new group.

Students should only move to the “Are You Ready for More?” station when they have finished all the others.

Option 2: Level Up

Arrange students into groups of 2–3. Distribute one Student Workspace Sheet to each student to complete as they solve each of the tasks.

Share with students that there are four tasks in this activity and one “Are You Ready for More?” task. Distribute one copy of “Plot It” to each group. Once the group completes “Plot It,” review their thinking. Consider choosing one student at random in the group to share the group’s ideas. Once the group has successfully completed “Plot It”, invite them to pick up a copy of “Finish It” to complete together.

Continue this process until students have completed all four tasks. If students finish early, invite them to complete the “Are You Ready for More?” task.

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