Amplify Desmos Math NEW YORK

Teacher Edition Sampler

Grade 7

Inside you'll find:

- Program overview, scope and sequence, and correlations
- Complete sample lessons from Amplify Desmos Math
- Lesson plans from requested domains, partially designed

For Review Only.
Not Final Format.

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NEW YORK

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About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Cover illustration by Caroline Hadilaksono.

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Welcome reviewer

Welcome to your Amplify Desmos Math New York Teacher Edition sampler!

Amplify Desmos Math New York is the result of two groundbreaking research and development efforts in K–12 mathematics instruction led by the Amplify and Desmos Classroom teams. Merging the two teams in 2022 enabled us to build a new curriculum around the idea that all students deserve to engage in high-quality grade-level mathematics every day. Based on Illustrative Mathematics'® IM K–12 Math™, Amplify Desmos Math New York combines strong pedagogy, arresting design, and forward-looking collaborative technology to deliver a classroom experience that keeps students engaged and asking productive questions.

Every lesson in the Amplify Desmos Math digital platform has a corresponding lesson in the print teacher and student editions. While we are in the process of finalizing the print materials, we have provided exemplars highlighting the unique design and ease of use of the Amplify Desmos Math print resources. To provide content covering your specific domain requests, in this physical sampler we have included both robust Amplify Desmos Math lesson plans and partially designed lesson plans. However, all of the lessons can be reviewed in their complete forms online.

All Amplify Desmos Math lessons include:

- Easy-to-follow lesson plans, tested in classrooms across the country.
- Clear teaching suggestions and strategies, including math language routines.
- Recommended differentiation moves and practice sets.

Diagnostic, formative, and summative assessments are provided with each unit along with lesson-level checks for understanding.

Amplify and New York City have a long history of partnering to provide equitable, high-quality instruction to our next generation of leaders. We look forward to continuing this partnership with New York City Public Schools in middle school mathematics.

—Jason Zimba and the Amplify Desmos Math team

Amplify Desmos Math New York

Helping New York City teachers develop and celebrate student thinking

Deep and lasting learning occurs when students are able to make connections to prior thinking and experiences. This requires teachers to deliver math instruction that balances exploration and explanation, and that puts student thinking at the center of classroom instruction.

Amplify Desmos Math students are invited to explore the math that fills their everyday lives, while strengthening their knowledge of math facts, procedural skills, and conceptual knowledge. Using the Amplify Desmos Math print and digital lesson plans, teachers can confidently guide and instruct as they build on students' understandings to help them develop a better grasp of mathematics.

Amplify Desmos Math is **a truly student-centered program** built around three core tenets:

1 A strong foundation in problem-based learning is critical to developing deep conceptual understanding, procedural fluency, and application.

Students are introduced to interesting problems and leverage both their current understandings and problem-solving strategies to develop reasonable answers. The learning experience is an active one that leads students to explore, notice, question, solve, justify, explain, represent, and analyze. Teachers guide the process, supporting synthesis and sensemaking at the end of each lesson.

2 Technology can provide ongoing, enriched feedback that encourages students to persevere in problem solving.

Especially when new ideas are being introduced, Desmos Classroom technology shows students the meaning of their thinking in context, interpreting it mathematically rather than reducing it to a question of right or wrong. This creates a culture of going deep with mathematics and students as doers of mathematics, so that as learning progresses and correctness is the goal, incorrect answers become objects of curiosity rather than embarrassment. This information in response to student ideas is what we call “enriched feedback.” Amplify Desmos Math New York offers more enriched feedback than any other math program.

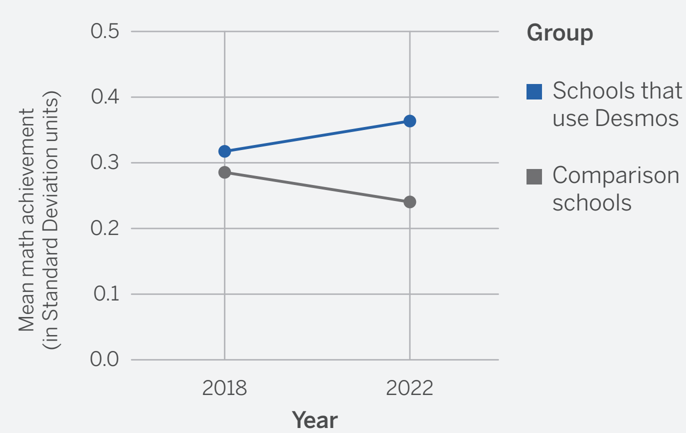
3 A commitment to access and equity should underpin every development decision.

All students can dive into problems on their own, and activities are designed to honor different approaches. Activities rely on collaboration and lots of hands-on, experiential learning.

And the program works.

Amplify Desmos Math New York expands on the Desmos Math 6–8 curriculum, which was recently proven to increase average math achievement in a study of more than 900 schools in nine states led by WestEd.

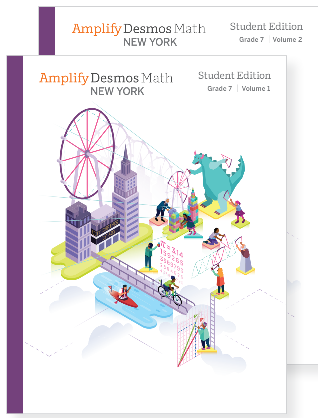
Mean Math Achievement for Desmos Schools and Matched Comparison Schools in 2018 and 2022



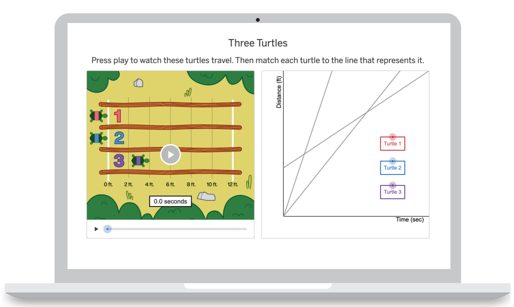
The Effect of Desmos Math Curriculum on Middle School Mathematics Achievement in Nine States. WestEd., (McKinney, D., Strother, S., Walters, K. & Schneider, S., 2023).

Amplify Desmos Math New York program resources

Student bundle includes:



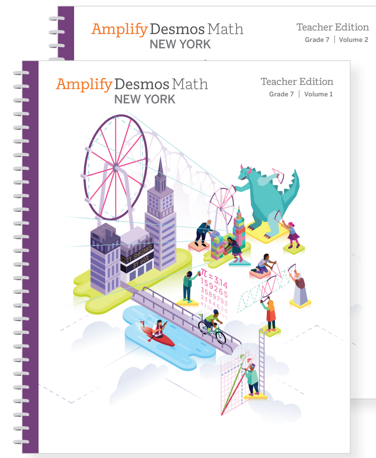
NY Student Edition,
multivolume, consumable



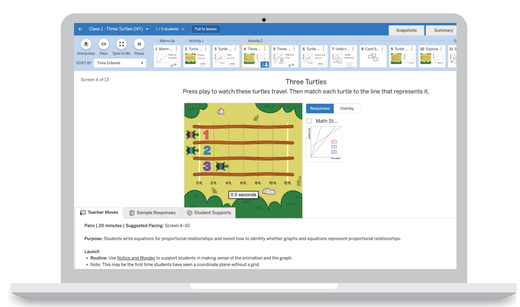
NY Digital Experience (English and Spanish), featuring:

- Interactive Student Activity Screens
- Enriched feedback
- Collaboration tools

Teacher bundle includes:



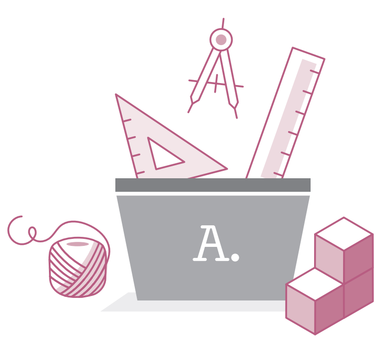
NY Teacher Edition,
multivolume, spiral-bound



NY Digital Experience (English and Spanish), featuring:

- Facilitation and progress monitoring tools
- Presentation Screens
- Instructional supports
- Assessment

Optional:



Middle School Manipulative Kit (Grades 6–8)

Extra Practice and Assessment Blackline Masters

Additional components and features may roll out over time.

Program architecture

Course

I mCLASS beginning-of-year diagnostic screener

I Interim Assesments

UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6	UNIT 7	UNIT 8
18 days	19 days	16 days	20 days	20 days	24 days	19 days	22 days

Unit

A Pre-Unit Check

A Sub-Unit Quiz

End-of-Unit Assessment A

- **Sub-Unit 1**: 1 2 3 4 5 6 7
- **Sub-Unit 2** (Practice Day): 8 9 10 11 12 PD
- **Sub-Unit 3** (Practice Day): 13 14 15 16 17 18 19 20 PD

Note: The number of sub-units and lessons vary from unit to unit; this depiction shows the general structure of a unit.

Lesson

Warm-up	Activity 1	Activity 2	Synthesis	Exit Ticket	Practice
5 min	15 min	15 min	5 min	5 min	timing varies
Independent, Pairs, Small Groups, Whole Class	Independent, Pairs, Small Groups, Whole Class	Independent, Pairs, Small Groups, Whole Class	Whole Class	Independent	Independent

Note: The number of activities and timing vary from lesson to lesson; this depiction shows the general structure of a lesson.

Key:
Independent
Small Groups
Pairs
Whole Class

Program Scope and Sequence

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 164 days total	Area and Surface Area 18 Instructional Days 3 Assessment Days 21 days total	Introducing Ratios 17 Instructional Days 3 Assessment Days 20 days total	Unit Rates and Percentages 16 Instructional Days 3 Assessment Days 19 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 158 days total	Scale Drawings 15 Instructional Days 3 Assessment Days 18 days total	Introducing Proportional Relationships 16 Instructional Days 3 Assessment Days 19 days total	Measuring Circles 13 Instructional Days 3 Assessment Days 16 days total	Proportional Relationships and Percentages 17 Instructional Days 3 Assessment Days 20 days total
Grade 8 150 days total	Rigid Transformation and Congruence 16 Instructional Days 4 Assessment Days 20 days total	Dilations, Similarity, and Introducing Slope 12 Instructional Days 4 Assessment Days 16 days total	Proportional and Linear Relationships 13 Instructional Days 3 Assessment Days 16 days total	Linear Equations and Linear Systems 17 Instructional Days 3 Assessment Days 20 days total

Unit 5	Unit 6	Unit 7	Unit 8
Decimal Arithmetic 18 Instructional Days 4 Assessment Days 22 days total	Expressions and Equations 19 Instructional Days 3 Assessment Days 22 days total	Positive and Negative Numbers 15 Instructional Days 3 Assessment Days 18 days total	Describing Data 19 Instructional Days 3 Assessment Days 22 days total
Operations With Positive and Negative Numbers 16 Instructional Days 4 Assessment Days 20 days total	Expressions, Equations, and Inequalities 21 Instructional Days 3 Assessment Days 24 days total	Angles, Triangles, and Prisms 16 Instructional Days 3 Assessment Days 19 days total	Probability and Sampling 19 Instructional Days 3 Assessment Days 22 days total
Functions and Volume 18 Instructional Days 4 Assessment Days 22 days total	Associations in Data 15 Instructional Days 2 Assessment Days 17 days total	Exponents and Scientific Notation 16 Instructional Days 3 Assessment Days 19 days total	The Pythagorean Theorem and Irrational Numbers 17 Instructional Days 3 Assessment Days 20 days total

Unit 1 Scale Drawings

In this unit, students examine and create scaled copies of figures, as well as explore scale drawings of real-life objects. In this unit, scaled copies have the same orientation as the original figure.

Pre-Unit

Sub-Unit 1 Scaled Copies

Sub-Unit 2 Scale Drawings

End-Unit

Unit 2 Introducing Proportional Relationships

In this unit, students learn what a proportional relationship is, how it is represented, and what types of contexts can be modeled by proportional relationships.

Pre-Unit

Sub-Unit 1 Proportional Relationships in Tables

Sub-Unit 2 Proportional Relationships in Equations

Sub-Unit 3 Proportional Relationships in Graphs

Sub-Unit 4 Using Proportional Relationships

End-Unit

Unit 3 Measuring Circles

In this unit, students explore the relationships between the radius, diameter, circumference, and area of a circle. Students also practice writing and using equations to calculate missing measurements, which they will revisit in Units 4 and 6.

Unit 4 Proportional Relationships and Percentages

In this unit, students apply what they have learned in Unit 2 about proportional relationships to solve problems involving fractional quantities and percent change.

Unit 5 Operations With Positive and Negative Numbers

In this unit, students extend what they learned in Grade 6 to add, subtract, multiply, and divide positive and negative numbers.

Unit 6 Expressions, Equations, and Inequalities

In this unit, students extend what they learned in Grade 6 about solving one-step equations to solve equations of the form px+q=r and p(x+q)=r, and equations that include expanding, factoring, or adding terms. Students also solve inequalities and graph their solutions on a number line.

Unit 7 Angles, Triangles, and Prisms

In this unit, students solve real-life and mathematical problems involving angle measures, volume, and surface area. Students also explore whether it is possible to draw no triangles, one triangle, or more than one triangle given three measures of sides or angles.

Pre-Unit

Sub-Unit 1 Angle Relationships

Sub-Unit 2 Drawing Triangles

Sub-Unit 3 Solid Geometry

End-Unit

Unit 8 Probability and Sampling

In the first section of the unit, students learn about probability as a way to describe the likelihood of unknown events and use simulations to estimate the probability of real-world situations. In the second section, students use samples to draw conclusions about and compare populations.

New York State Next Generation Mathematics Learning Standards, Grade 7

The following shows the alignment of Amplify Desmos Math to the New York State Next Generation Mathematics Learning Standards for Grade 7 Mathematics.

NY-7.RP	Ratios and Proportional Relationships	Lesson(s)
Analyze proportional relationships and use them to solve real-world and mathematical problems.		
NY-7.RP.1 *CCSS: 7.RP.A.1*	Compute unit rates associated with ratios of fractions.	7.1.04, 7.4.02, 7.4.03, 7.4 Practice Day 1, 7.4 Practice Day 2
NY-7.RP.2 *CCSS: 7.RP.A.2*	Recognize and represent proportional relationships between quantities.	7.1.02, 7.1 Practice Day 1, 7.2.02, 7.2.03, 7.2.04, 7.2.05, 7.2.06, 7.2.07, 7.2 Practice Day 1, 7.2.08, 7.2.09, 7.2.10, 7.2.11, 7.2.12, 7.2 Practice Day 2, 7.4.03, 7.4.04, 7.4 Practice Day 1, 7.4.09, 7.4 Practice Day 2
NY-7.RP.2a *CCSS: 7.RP.A.2.A*	Decide whether two quantities are in a proportional relationship.	7.1.01, 7.2.02, 7.2.03, 7.2.07, 7.2 Practice Day 1, 7.2.08, 7.2.10, 7.2 Practice Day 2, 7.3.01, 7.3.03, 7.3 Practice Day 1, 7.3.07, 7.3 Practice Day 2, 7.4 Practice Day 2
NY-7.RP.2b *CCSS: 7.RP.A.2.B*	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	7.1.04, 7.2.03, 7.2.05, 7.2.06, 7.2 Practice Day 1, 7.2.09, 7.2.10, 7.3.01, 7.3.03
NY-7.RP.2c *CCSS: 7.RP.A.2.C*	Represent a proportional relationship using an equation.	7.2.04, 7.2.05, 7.2.06, 7.2.07, 7.2 Practice Day 1, 7.2.10, 7.2 Practice Day 2, 7.4 Practice Day 2
NY-7.RP.2d *CCSS: 7.RP.A.2.D*	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.	7.2.09, 7.2 Practice Day 2
NY-7.RP.3 *CCSS: 7.RP.A.3*	Use proportional relationships to solve multistep ratio and percent problems.	7.1.07, 7.4.04, 7.4.05, 7.4.06, 7.4.07, 7.4 Practice Day 1, 7.4.08, 7.4.09, 7.4.10, 7.4.11, 7.4.12, 7.4 Practice Day 2, 7.8.12

NY-7.NS	The Number System	
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.		
NY-7.NS.1 *CCSS: 7.NS.A.1*	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers. Represent addition and subtraction on a horizontal or vertical number line.	7.5.02, 7.5.04, 7.5.05, 7.5 Practice Day 1, 7.5.09
NY-7.NS.1a *CCSS: 7.NS.A.1.A*	Describe situations in which opposite quantities combine to make 0.	7.5.01, 7.5 Practice Day 1
NY-7.NS.1b *CCSS: 7.NS.A.1.B*	Understand addition of rational numbers; $p + q$ is the number located a distance $\lvert q \rvert$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	7.5.02, 7.5.03, 7.5.04, 7.5 Practice Day 1
NY-7.NS.1c *CCSS: 7.NS.A.1.C*	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	7.5.02, 7.5.04, 7.5.05, 7.5 Practice Day 1
NY-7.NS.1d *CCSS: 7.NS.A.1.D*	Apply properties of operations as strategies to add and subtract rational numbers.	7.5.03, 7.5.04, 7.5 Practice Day 1, 7.5.10, 7.5 Practice Day 2
NY-7.NS.2 *CCSS: 7.NS.A.2*	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.	7.5.06, 7.5.09, 7.5 Practice Day 2
NY-7.NS.2a *CCSS: 7.NS.A.2.A*	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	7.5.07, 7.5 Practice Day 2
NY-7.NS.2b *CCSS: 7.NS.A.2.B*	Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(\frac{p}{q})=\frac{-p}{q}=\frac{p}{-q}$. Interpret quotients of rational numbers by describing real-world contexts.	7.5.08, 7.5 Practice Day 2
NY-7.NS.2c *CCSS: 7.NS.A.2.C*	Apply properties of operations as strategies to multiply and divide rational numbers.	7.5.08, 7.5.10, 7.5 Practice Day 2
NY-7.NS.2d *CCSS: 7.NS.A.2.D*	Convert a fraction to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	7.4.13
NY-7.NS.3 *CCSS: 7.NS.A.3*	Solve real-world and mathematical problems involving the four operations with rational numbers.	7.5 Practice Day 1, 7.5.10, 7.5 Practice Day 2, 7.5.11, 7.5.12, 7.5.13

New York State Next Generation Mathematics Learning Standards, Grade 7

NY-7.EE	Expressions and Equations	Lesson(s)
Use properties of operations to generate equivalent expressions.		
NY-7.EE.1 *CCSS: 7.EE.A.1*	Add, subtract, factor, and expand linear expressions with rational coefficients by applying the properties of operations	7.6.08, 7.6.09, 7.6.10, 7.6.11, 7.6 Practice Day 1, 7.6 Practice Day 2
NY-7.EE.2 *CCSS: 7.EE.A.2*	Understand that rewriting an expression in different forms in real-world and mathematical problems can reveal and explain how the quantities are related.	7.4.05, 7.4 Practice Day 1, 7.4.08, 7.4.10, 7.4 Practice Day 2, 7.6.08, 7.7.03, 7.7.04
Solve real-life and mathematical problems using numerical and algebraic expressions, equations, and inequalities.		
NY-7.EE.3 *CCSS: 7.EE.B.3*	Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate. Assess the reasonableness of answers using mental computation and estimation strategies.	7.5.05, 7.5 Practice Day 1, 7.5.10, 7.5 Practice Day 2, 7.5.11, 7.5.12, 7.5.13, 7.6.02, 7.6.03, 7.6.04, 7.6.11, 7.6.12, 7.6 Practice Day 1, 7.6 Practice Day 2, 7.8.02
NY-7.EE.4 *CCSS: 7.EE.B.4*	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	7.4.08, 7.4.10, 7.6.03, 7.6.04, 7.6.12, 7.6 Practice Day 1, 7.6.13, 7.6.14, 7.6.15, 7.6 Practice Day 2, 7.7.03, 7.7.04, 7.7 Practice Day 1
NY-7.EE.4a *CCSS: 7.EE.B.4.A*	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	7.6.05, 7.6.06, 7.6.07, 7.6.08, 7.6.11, 7.6.12, 7.6 Practice Day 1, 7.6 Practice Day 2
NY-7.EE.4b *CCSS: 7.EE.B.4.B*	Solve word problems leading to inequalities of the form $px + q > r$, $px + q \geq r$, $px + q \leq r$, or $px + q < r$, where p, q, and r are rational numbers. Graph the solution set of the inequality on the number line and interpret it in the context of the problem.	7.6.13, 7.6.14, 7.6.15, 7.6.16, 7.6.17, 7.6 Practice Day 2 **Alignment note:** NYS includes non-strict inequalities. Non-strict inequalities are addressed in the indicated lessons.

NY-7.G	Geometry	Lesson(s)
Draw, construct, and describe geometrical figures and describe the relationships between them.		
NY-7.G.1 *CCSS: 7.G.A.1*	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	7.1.01, 7.1.02, 7.1.03, 7.1.04, 7.1.05, 7.1 Practice Day 1, 7.1.06, 7.1.07, 7.1.08, 7.1.09, 7.1.10, 7.1 Practice Day 2
NY-7.G.2 *CCSS: 7.G.A.2*	Draw triangles when given measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	7.7.05, 7.7.06, 7.7.07, 7.7.08, 7.7 Practice Day 1
NY-7.G.3 *CCSS: 7.G.A.3*	Describe the two-dimensional shapes that result from slicing three-dimensional solids parallel or perpendicular to the base.	7.7.09
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.		
NY-7.G.4 *CCSS: 7.G.B.4*	Apply the formulas for the area and circumference of a circle to solve problems.	7.3.03, 7.3.04, 7.3 Practice Day 1, 7.3.06, 7.3.07, 7.3.08, 7.3.09, 7.3 Practice Day 2
NY-7.G.5 *CCSS: 7.G.B.5*	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	7.7.02, 7.7.03, 7.7.04, 7.7 Practice Day 1
NY-7.G.6 *CCSS: 7.G.B.6*	Solve real-world and mathematical problems involving area of two-dimensional objects composed of triangles and trapezoids. Solve surface area problems involving right prisms and right pyramids composed of triangles and trapezoids. Find the volume of right triangular prisms, and solve volume problems involving three-dimensional objects composed of right rectangular prisms.	6.1.01, 6.1.02, 6.1.08, 7.1 Practice Day 1, 7.7.10, 7.7.11, 7.7.12, 7.7.13, 7.7 Practice Day 2 **Alignment note:** NYS adds specific mention of trapezoids, right pyramids, right triangular prisms, and right rectangular prisms. These figures are addressed in the indicated lessons.

New York State Next Generation Mathematics Learning Standards, Grade 7

Statistics and Probability		Lesson(s)
Draw informal comparative inferences about two populations.		
NY-7.SP.1 *CCSS: aligned to parts of 6.SP.B.4, 6.SP.B.5.C*	Construct and interpret box-plots, find the interquartile range, and determine if a data point is an outlier.	6.8.12, 6.8.13, 6.8.14, 6.8.15, 6.8.16, 6.8 Practice Day 2, 7.8.13, 7.8.14 **Alignment note:** NYS adds specific mention of outliers. In Lesson 6.8.15, students interpret box plots that contain outliers, which is the expectation of this standard. **From the New York State Next Generation Mathematics Learning Standards:** "Note: Students in grade 7 are not expected to construct box-plots that include outliers in the data, but students are expected to interpret box-plots that may contain outliers."
NY-7.SP. *CCSS: 7.SP.B.3*	Informally assess the degree of visual overlap of two quantitative data distributions.	7.8.14, 7.8.15, 7.8 Practice Day 2
NY-7.SP. *CCSS: 7.SP.B.4*	Use measures of center and measures of variability for quantitative data from random samples or populations to draw informal comparative inferences about the populations.	7.8.14, 7.8.15, 7.8 Practice Day 2
Investigate chance processes and develop, use, and evaluate probability models.		
NY-7.SP.8 *CCSS: 7.SP.C.8*	Find probabilities of compound events using organized lists, sample space tables, tree diagrams, and simulation.	7.8.04, 7.8.05, 7.8 Practice Day 1
NY-7.SP.8a *CCSS: 7.SP.C.8.A*	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	7.8.06, 7.8 Practice Day 1, 7.8 Practice Day 2
NY-7.SP.8b *CCSS: 7.SP.C.8.B*	Represent sample spaces for compound events using methods such as organized lists, sample space tables, and tree diagrams. For an event described in everyday language, identify the outcomes in the sample space which compose the event.	7.8.06, 7.8 Practice Day 1, 7.8 Practice Day 2
NY-7.SP.8c *CCSS: 7.SP.C.8.C*	Design and use a simulation to generate frequencies for compound events.	7.8.07, 7.8.08, 7.8 Practice Day 1

The Standards for Mathematical Practice, New York State Next Generation Mathematics Learning Standards, Grade 7

The following shows the alignment of Amplify Desmos Math, Grade 7, to the Standards for Mathematical Practice for the New York State Next Generation Mathematics Learning Standards.

MP1 Make sense of problems and persevere in solving them. ***CCSS: MP1***	**Lesson(s)**
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	7.1.01, 7.1.10, 7.4.06, 7.4.08, 7.4.09, 7.5.05, 7.5.10, 7.5.11, 7.6.12, 7.7.05, 7.7.11, 7.8.10
MP2 Reason abstractly and quantitatively. ***CCSS: MP2***	
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	7.1.06, 7.1.07, 7.2.03, 7.2.05, 7.2.06, 7.2.08, 7.2.10, 7.4.04, 7.5.01, 7.5.02, 7.5.06, 7.5.07, 7.5.08, 7.5.11, 7.6.02, 7.6.03, 7.6.04, 7.6.12, 7.6.13, 7.7.03, 7.8.05, 7.8.07, 7.8.09
MP3 Construct viable arguments and critique the reasoning of others. ***CCSS: MP3***	
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	7.1.02, 7.1.03, 7.1.10, 7.2.12, 7.3.02, 7.3.05, 7.3.07, 7.4.02, 7.4.07, 7.4.09, 7.4.10, 7.4.13, 7.5.01, 7.5.10, 7.5.13, 7.6.01, 7.6.02, 7.6.08, 7.6.09, 7.6.14, 7.6.15, 7.6.16, 7.7.03, 7.7.09, 7.7.10, 7.8.01, 7.8.03, 7.8.04, 7.8.07, 7.8.09, 7.8.10, 7.8.12

The Standards for Mathematical Practice, New York State Next Generation Mathematics Learning Standards, Grade 7

MP4 Model with mathematics. ***CCSS: MP4***	**Lesson(s)**
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	7.2.11, 7.4.09, 7.4.12, 7.5.12, 7.5.13, 7.6.17, 7.8.07, 7.8.15
MP5 Use appropriate tools strategically. ***CCSS: MP5***	
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	7.4.03, 7.4.04, 7.4.12, 7.6.03, 7.7.12, 7.8.02, 7.8.06
MP6 Attend to precision. ***CCSS: MP6***	
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	7.1.02, 7.1.04, 7.1.05, 7.2.02, 7.2.04, 7.2.07, 7.2.08, 7.3.04, 7.3.05, 7.3.08, 7.4.02, 7.5.02, 7.5.04, 7.5.09, 7.6.04, 7.6.05, 7.6.07, 7.6.10, 7.7.02, 7.7.03, 7.7.11

MP7 Look for and make use of structure. *CCSS: MP7*	Lesson(s)
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	7.1.02, 7.1.04, 7.1.05, 7.2.02, 7.2.04, 7.2.07, 7.2.08, 7.3.04, 7.3.05, 7.3.08, 7.4.02, 7.5.02, 7.5.04, 7.5.09, 7.6.04, 7.6.05, 7.6.07, 7.6.10, 7.7.02, 7.7.03, 7.7.11
MP8 Look for and express regularity in repeated reasoning. ***CCSS: MP8***	
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	7.1.01, 7.1.05, 7.2.04, 7.2.06, 7.2.09, 7.3.01, 7.3.06, 7.3.09, 7.4.03, 7.4.05, 7.4.08, 7.5.02, 7.6.01, 7.6.08, 7.7.01, 7.7.05, 7.8.04, 7.8.05

GRADE 7

Amplify Desmos Math
NEW YORK

Teacher Edition Sample Lessons

In this section, two lesson samples showcase the unique design and ease of use of lesson plans found in the Amplify Desmos Math New York Teacher Edition. All Teacher Edition lessons will be created following this structure and design for the 2024-2025 school year.

Contents of this lesson:

- **Teacher Edition Overview**
- **Lesson 2.10: Three Turtles**
  Proportional Relationships and Graphs
- **Lesson 6.14: Unbalanced Hangers**
  Solutions to Inequalities

Powerful learning experiences with the flexibility you need in the classroom.

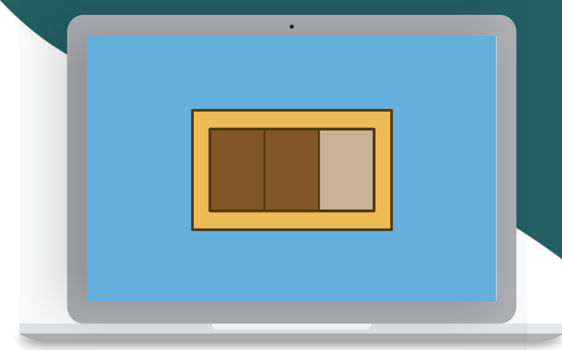
Every lesson in Amplify Desmos Math New York can be taught with students using print while the teacher projects digital Presentation Screens. For lessons that are best taught with students on devices, we make a clear recommendation and provide instructional guidance to support students using digital and on print. The robust collaboration tools, interactive visuals, and enriched feedback of Desmos technology are integral to daily learning as a whole class, pairs, or individuals.

Print and digital resources for every lesson!

Two types of lesson delivery

Digital Lesson RECOMMENDED

Assign Student Activity Screens for this lesson so students can explore visual potting soil models!



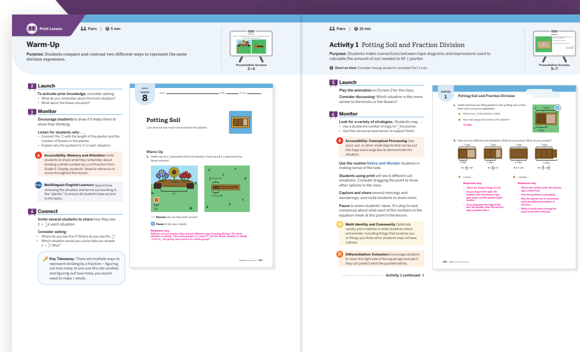
Digital recommended

Lesson goals best learned digitally

- Students use devices and interact with Student Activity Screens.
- Teachers present the Student Activity Screens to facilitate the lesson.
- Closely-aligned student print pages are available for off-line note taking or for students who may need to use print.
- About 75% of lessons

Print Lesson

Assign the Student Edition with Presentation Screens for this lesson.



Print

Lesson goals best learned with pencil-and-paper

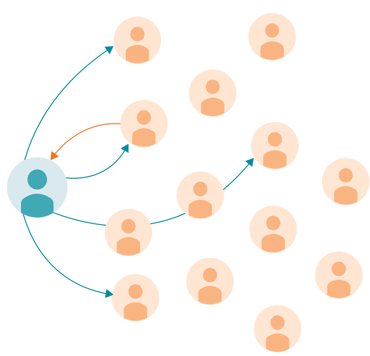
- Students interact with printed Student Edition pages and hands-on manipulatives (when applicable).
- Teachers present Presentation Screens to facilitate the lesson.
- Students must use consumable Student Edition.
- About 25% of lessons

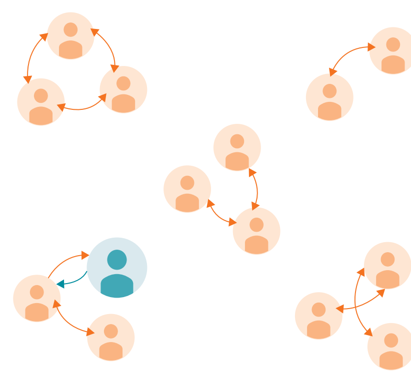
REVIEWER TIP

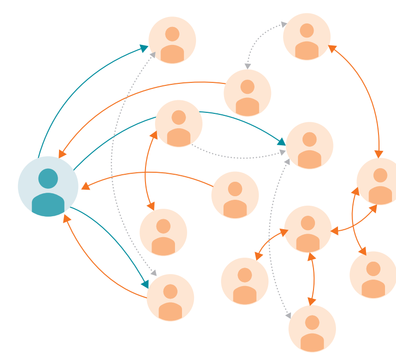
These print/digital flexibility enhancements can be found in Amplify Desmos Math New York lessons in this section and online, but are not yet available in the partially designed lesson plans.

Activities are designed to provide collaborative learning experiences.

Following a *Warm-Up*, a lesson includes two or more learning activities. All activities in Amplify Desmos Math New York utilize a *Launch, Monitor, Connect* framework to surface student thinking and spark interesting and productive discussions.







1 Launch

Teachers launch an activity and ensure students understand what's being asked. Launches are designed to ensure all students can access and engage with the problem.

2 Monitor

Students interact with each other to discuss and work out strategies for solving a problem. Teachers can better understand what their students are thinking so that they can choose their next move while students are working.

3 Connect

Students construct viable arguments and critique each other's reasoning.Then, at the end of the activity, they synthesize their learning with the teacher in a moment called the *Key Takeaway*.

Following all activities, each lesson wraps with *Synthesis and Summary* to consolidate thinking and refine strategies across activities. An *Exit Ticket* enables students to share how well they understood the math of the lesson and how they felt about learning that math.

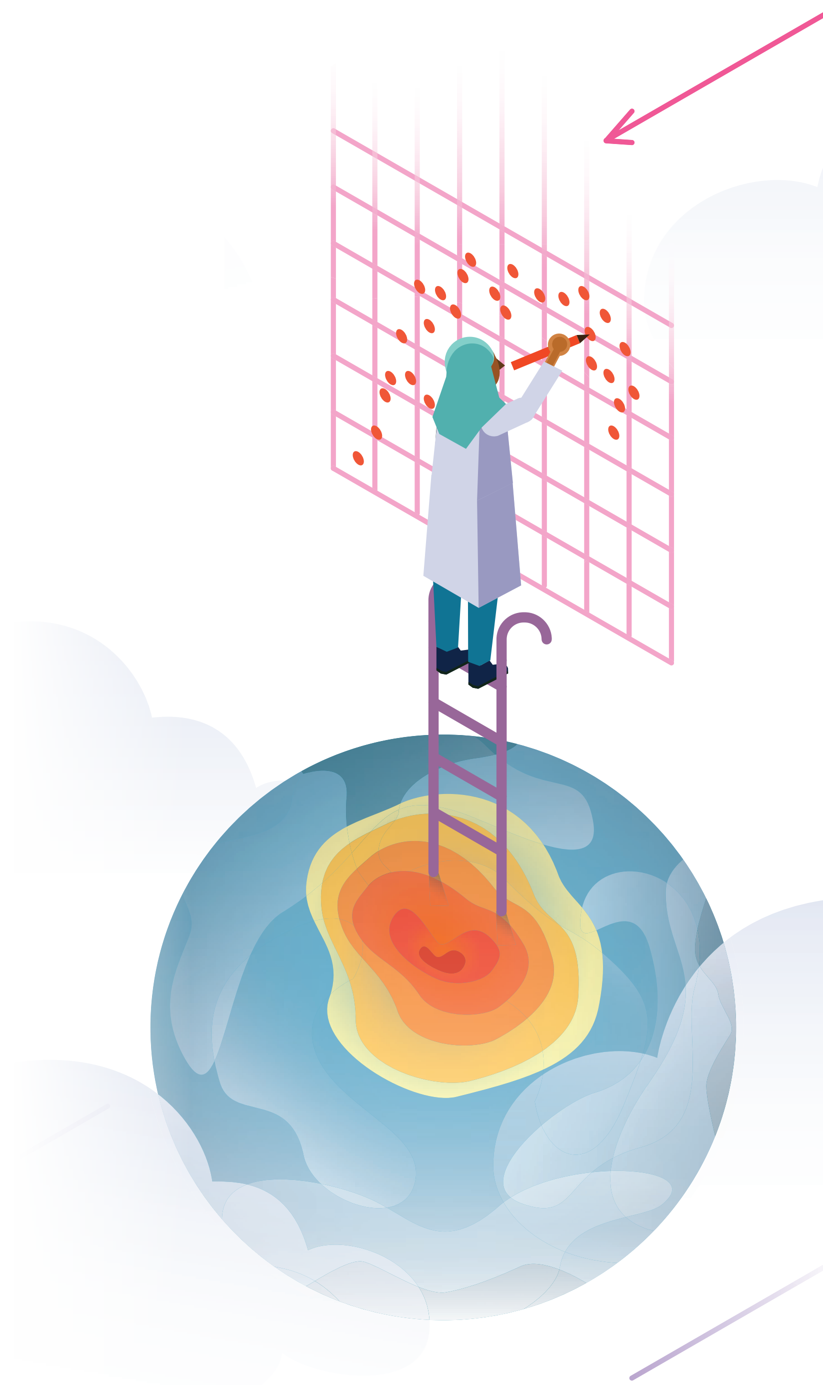
Every moment in the classroom is valuable.

Teachers play an active role as discussion facilitators, monitoring student work in real time, choosing moments to share and discuss, and synthesizing learning.

At Amplify, we want teachers to spend their time focused on their students, rather than preparing instruction and managing materials. Our comprehensive Teacher Edition and intuitive technology is designed with busy educators in mind.

Inside the Amplify Desmos Math New York Teacher Edition, you'll find:

- ***Unit at a Glance*** and ***Lesson at a Glance*** sections to quickly understand what to expect from a unit or lesson.
- ***Focus and Coherence*** information to connect today's goals to prior and future learning.
- A ***Prep Checklist*** to prepare materials for the day's lesson.
- ***Suggested pacing*** to allot the appropriate amount of time for each activity.
- ***Visuals of student pages and screens*** to streamline lesson planning.
- ***Practice problem item analysis*** to easily map learning to Depth of Knowledge (DOK) levels.

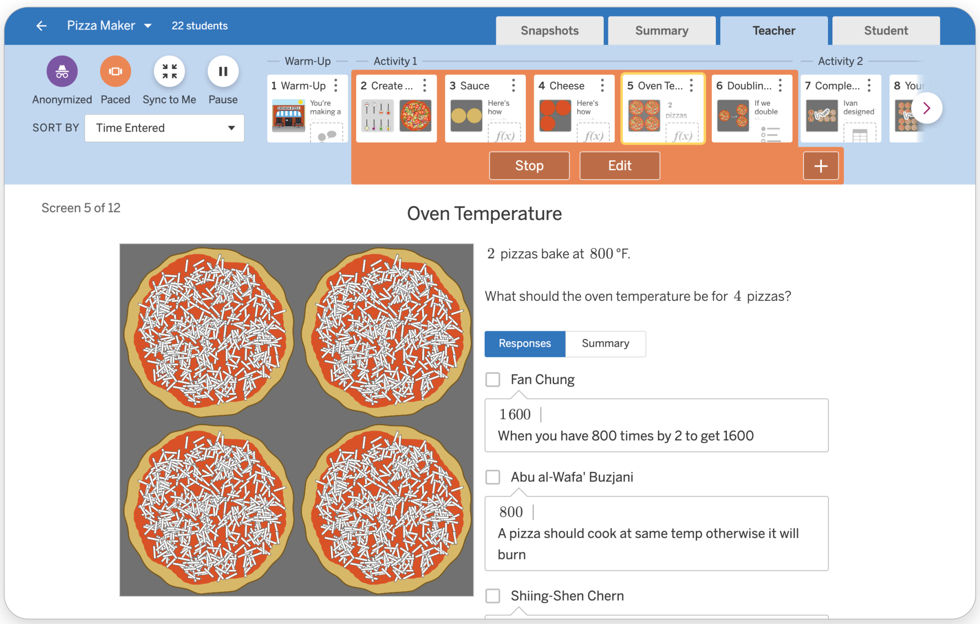


REVIEWER TIP

These time-saving enhancements can be found in Amplify Desmos Math New York lessons in this section and online, but are not yet available in the partially designed lesson plans.

Teacher facilitation tools enable dynamic interactions.

The teacher dashboard gives you insight into student thinking in real time, meaning you can select student work to display and discuss quickly and easily, and ask better questions to guide more productive discussions.

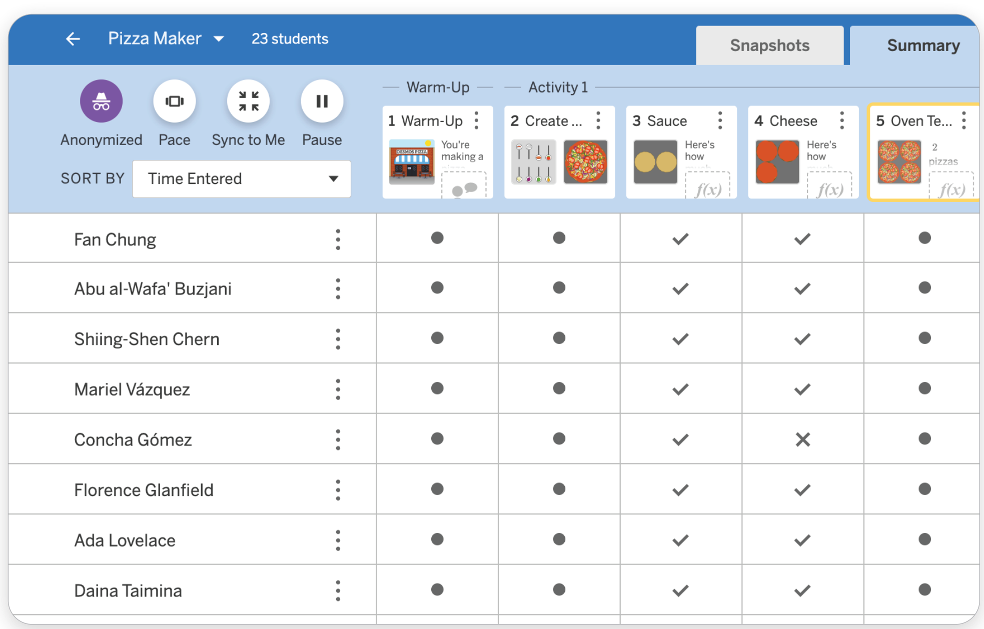


Teacher view and pacing

The Teacher view gives you access to student responses, student-facing content, teacher moves, and sample responses, as well as the ability to pace screens.

Summary view

The Summary view shows you where students are working. If a question is auto-scored it shows how they are doing, and the ability to look at individual student work.

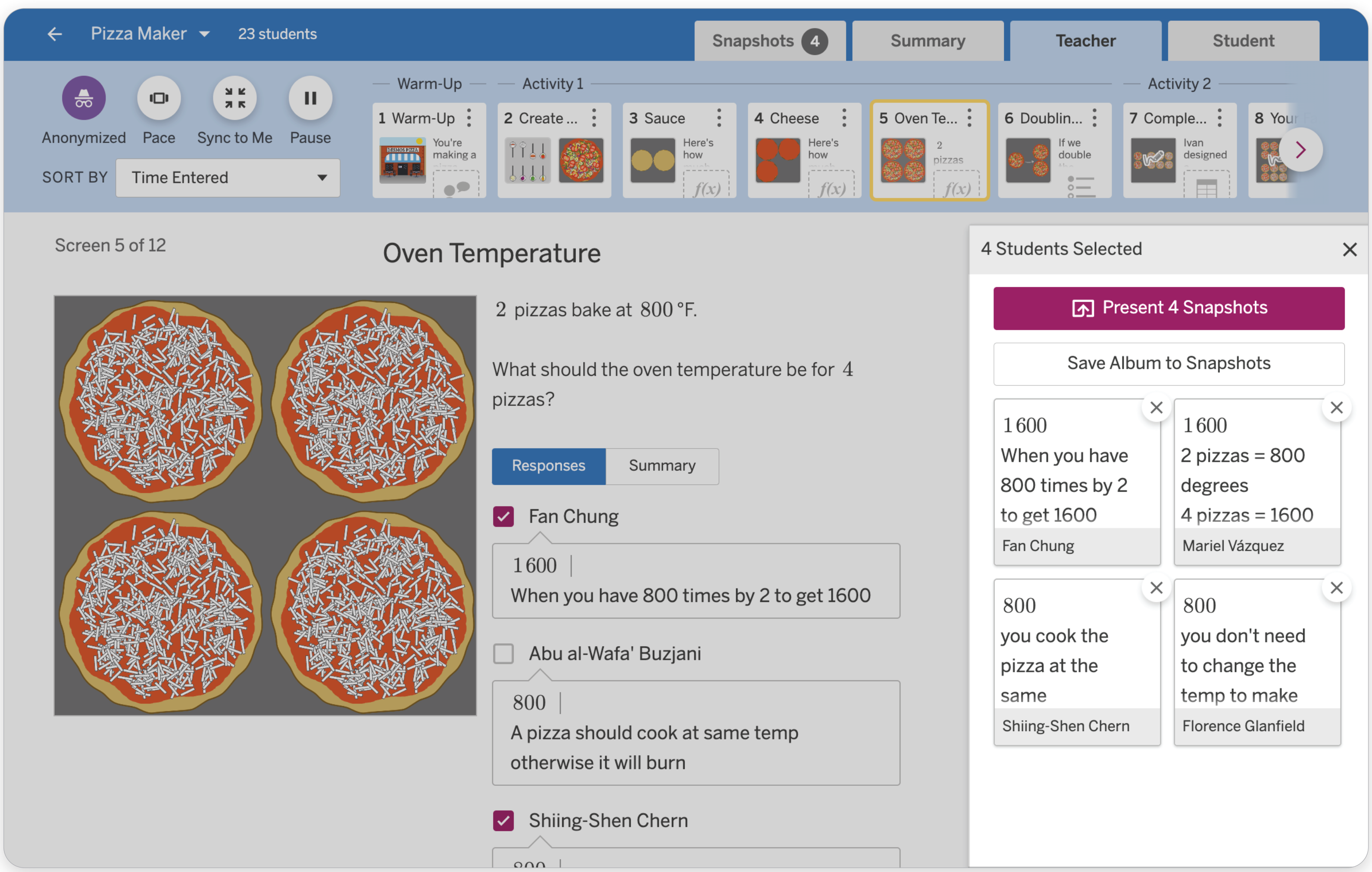


Snapshots

When you find student work you want to share, you can collect it in your snapshots and then show individual or even groups of students' responses to move the conversation in the direction you want. Names can be anonymized to protect students' identity.

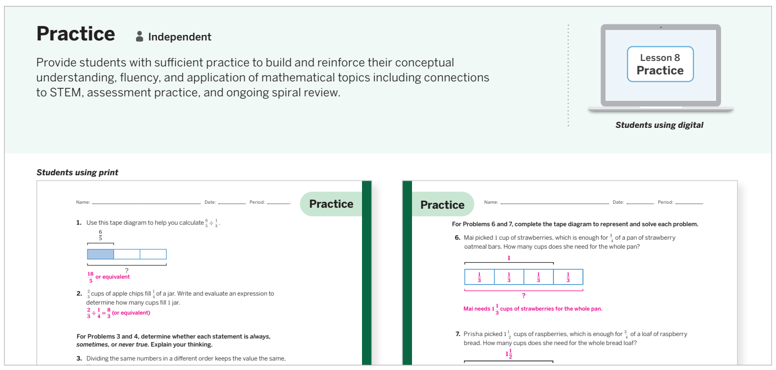
TRY IT OUT

Start your review at
amplify.com/math-review-nyc

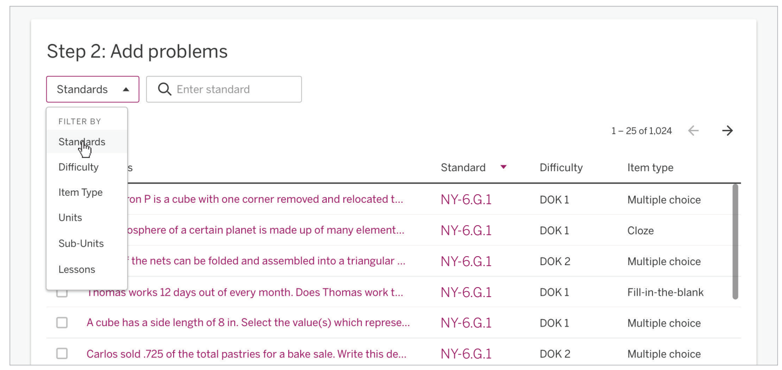


Practice makes progress.

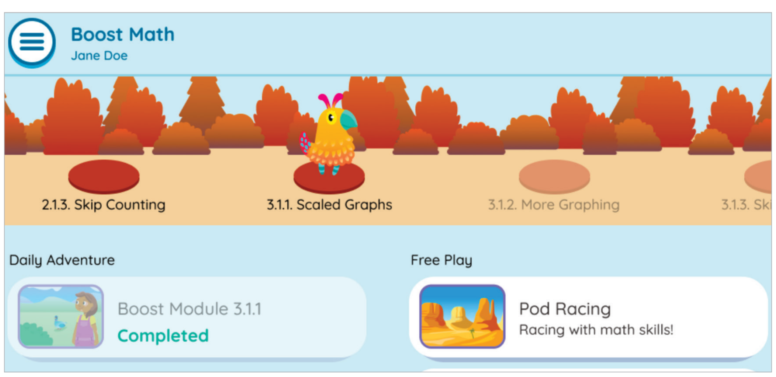
When it comes to cementing new learning into long-term understanding, ample practice opportunities are key. Amplify Desmos Math New York builds practice opportunities into both daily instruction and independent practice.



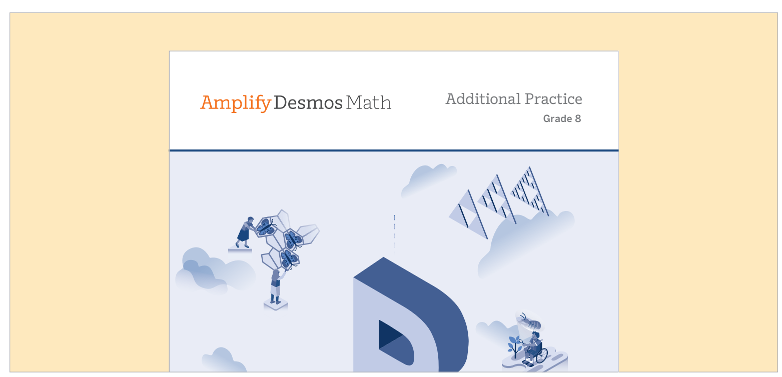
Daily practice problems for the day's lesson are included online and in the Student Edition, including fluency and test practice. This daily practice also includes *Spiral Review* to revisit formerly acquired math learning. A Depth of Knowledge (DOK) table is provided for practice problem item analysis and further insight into how students are doing conceptually.



An **online item bank** contains additional practice sets, or teachers can customize their own based on unit or sub-unit concepts and standards.



Boost Tutored Practice offers engaging, digital independent practice for students that provides access to grade-level math through personalized feedback that responds to student work to support their learning.



Additional Practice Blackline Masters contain additional practice problems to further address fluency, spiral review, and a variety of DOK questions in lesson learning, supporting differentiated practice based on the needs of students.

REVIEWER TIP

These practice enhancements can be found in Amplify Desmos Math New York lessons in this section, online, and in the Student Edition sampler, but are not yet available in the partially designed lesson plans.

In-the-moment instructional supports help teachers meet the needs of every learner.

Embedded instructional supports provide practical guidance for scaffolding or extending learning for all students using an asset-based approach.

D

Differentiation

Provides a lens with which to anticipate, view, and guide individual student work, including *Extensions* and *Differentiation Support* guidance. In addition, robust recommendations to *Support, Strengthen, and Stretch* are provided at the unit level.

A

Accessibility

Promotes main areas of cognitive functioning, including memory and attention, conceptual processing, visual-spatial processing, executive functioning, fine motor skills, and affective functioning.

ML/EL

Multilingual / English Learners

Provides math language development supports to help all students achieve the *Language Goal* of the lesson.

Math identity and Community

Highlights opportunities to recognize and celebrate the brilliance from all students.

Boost mini lessons

Offer teacher-led small group assistance to students who need more direct and explicit support to re-engage with grade-level math.

This just-in-time instruction is informed by assessment data such as pre-unit and sub-unit quizzes.

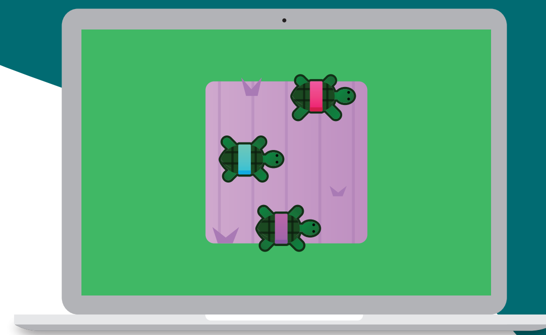
REVIEWER TIP

In-the-moment instructional supports are included in Amplify Desmos Math New York lessons in this section and online, but most are not yet available in the partially designed lesson plans.

Unit 2
Lesson
10

Digital Lesson RECOMMENDED

This is a digital lesson with additional print materials. A print option is also available.



Three Turtles

Proportional Relationships and Graphs

Let's use graphs and equations to compare proportional relationships.

Focus and Coherence

Today's Goals

1. **Goal:** Write an equation of a proportional relationship given a point on a graph.
2. **Language Goal:** Compare related proportional relationships on the same graph using mathematically precise language. **(Reading and Writing)**

The purpose of this lesson is for students to use graphs and equations to compare proportional relationships. Students write equations based on tables and graphs and differentiate between graphs and equations of relationships that are proportional and non-proportional. They compare multiple proportional relationships on the same coordinate plane to interpret the steepness of each graph in context. They reason abstractly to conclude that graphs can be used to compare constants of proportionality, even when the scale is not specified on each axis. **(MP2)**

Prior Learning

In Lesson 9, students analyzed graphs with multiple proportional relationships and interpreted the constants of proportionality in context.

Future Learning

In Lesson 11, students will solidify their understanding of the different representations of proportional relationships.

Standards

NY-7.RP.2b
Identify the constant of proportionality (unit rate) in tables, **graphs, equations,** diagrams, and verbal descriptions **of proportional relationships.**

NY-7.RP.2c
Represent proportional relationships with equations.

Also Addressing: **NY-7.RP.2, NY-7.RP.2a**

Mathematical Practices: MP2

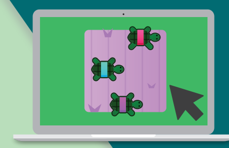
Rigor and Balance

- Students build **conceptual understanding** of the relationship between the constant of proportionality and the steepness of a proportional relationship when graphed.
- Students build **fluency** in recognizing the relationship between steepness and constant of proportionality by comparing multiple proportional relationships on the same coordinate plane.

Lesson at a Glance

~ 45 min

Standards: NY-7.RP.2, NY-7.RP.2a, NY-7.RP.2b, NY-7.RP.2c



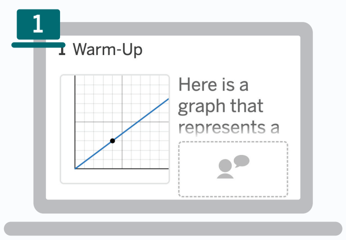
Why go digital?

Students connect turtles traveling to their distance-time graphs in real time.

Warm-Up

Pairs | 5 min

Students use their creativity to revisit the meaning of a point on a graph of a proportional relationship.

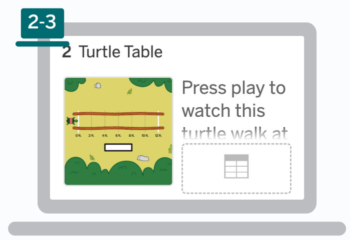


Pacing: Screen 1

Activity 1

Pairs | 10 min

Students connect information in a graph and a table to write an equation of a single proportional relationship and decide whether relationships represented by graphs are proportional.

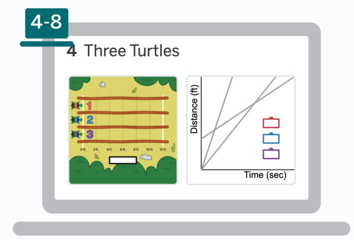


Pacing: Screens 2–3

Activity 2

Pairs | 20 min

Students write equations for proportional relationships and revisit how to identify whether graphs and equations represent proportional relationships.

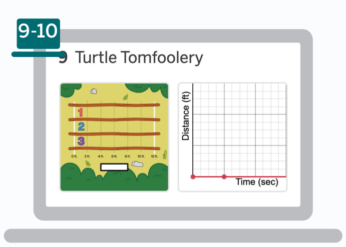


Pacing: Screens 4–8

Activity 3 (Optional)

Pairs | 10 min

Students create their own turtle race based on given conditions.

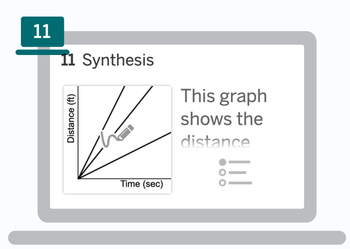


Pacing: Screens 9–10

Synthesis

Whole Class | 5 min

Students synthesize their understanding of comparing proportional relationships using graphs and equations.

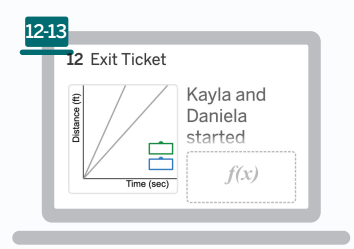


Pacing: Screen 11

Exit Ticket

Independent | 5 min

Students demonstrate their understanding of comparing proportional relationships by labeling two relationships on a graph and writing an equation.



Pacing: Screens 12–13

Prep Checklist

Assign the digital lesson. A print option is also available.

Student using digital:

- Digital Lesson

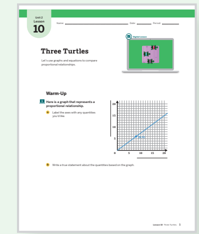
Student using print:

- Print Option in Student Edition
- Exit Ticket PDF
- Activity 2 Cards, one set per pair

Digital Lesson | Pairs | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students use their creativity to revisit the meaning of a point on a graph of a proportional relationship.



Students using print

1 Launch

Consider asking students what they remember about interpreting a point on a graph.

1 Connect

Invite students to share their axes labels and statements. Invite them to connect the statements to their graphs and critique each other's reasoning.

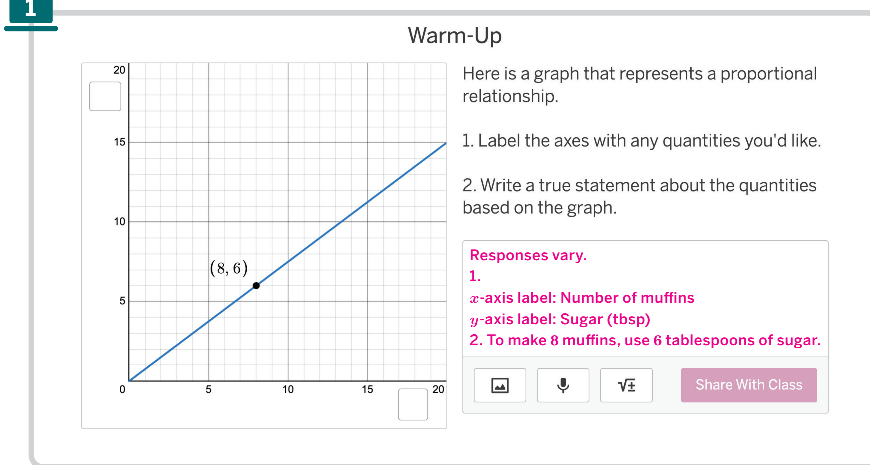
Emphasize early student thinking, including students who correctly mention the constant of proportionality $\frac{3}{4}$ and students whose statement swaps the meaning of the axes labels.

Consider asking students to make adjustments to their labels based on the feedback they received from their classmates and display these, as time permits.

Math Identity and Community Consider celebrating unique or creative choices of contexts as students share their axes labels and asking the authors to speak about their inspiration.

Students using digital

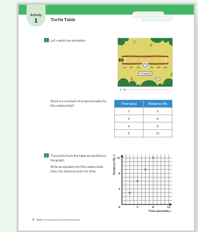
1



Pairs | 10 min | Pacing: Screens 2–3

Activity 1 Turtle Table

Purpose: Students connect information in a graph and a table to write an equation of a single proportional relationship and decide whether relationships represented by graphs are proportional.



Students using print

2 Launch

Play the digital animation of the turtle walking at a constant rate. **Note:** The distance is measured from the start at 0 ft to the turtle's head.

3 Monitor

Students using print will not receive automated, interpretive feedback on their equation. Consider checking in with these students before they move on.

D Differentiation

Look for students who:	Teacher Moves
Need support getting started. *(Screen 2)*	**Support:** Ask: "What do you need to multiply the time by to calculate the distance?"
Notice that the turtle travels 3 feet every 2 seconds. *(Screen 2)*	Consider asking, "What does this tell you about the constant of proportionality for this relationship?"
Compare the time to the distance to calculate the constant of proportionality, but do not switch variables in the equation. *(Screen 3)*	Remind students that when switching the relationship of the two variables, the variables in the equation must also switch.
Finish early. *(Screen 3)*	**Extension:** Encourage students to create both equations for this proportional relationship.

3 Connect

Emphasize that there are two constants of proportionality that represent this relationship, creating two different equations.

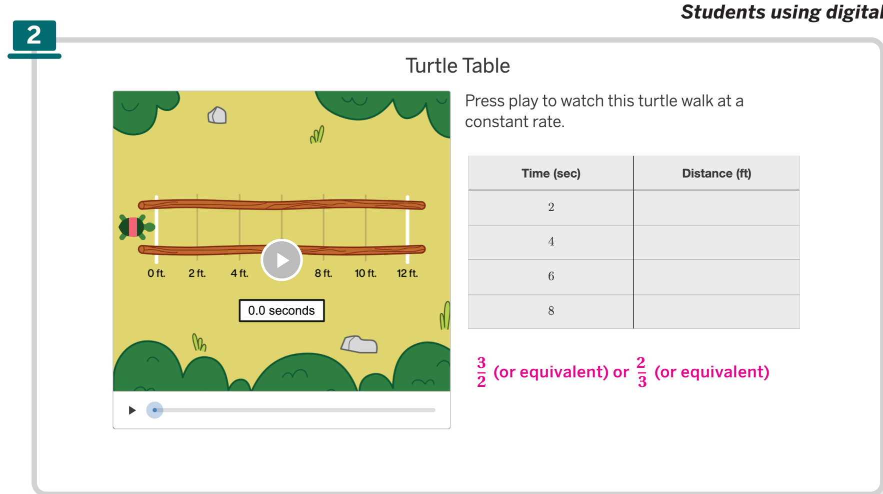
- Comparing the distance to the time, the constant of proportionality is $\frac{3}{2}$, giving an equation of $d = \frac{3}{2}t$.
- Comparing the time to the distance, the constant of proportionality is the reciprocal, $\frac{2}{3}$.
- Because the relationship of the two values switches, the variables in the equation must also switch, giving us $t = \frac{2}{3}d$.

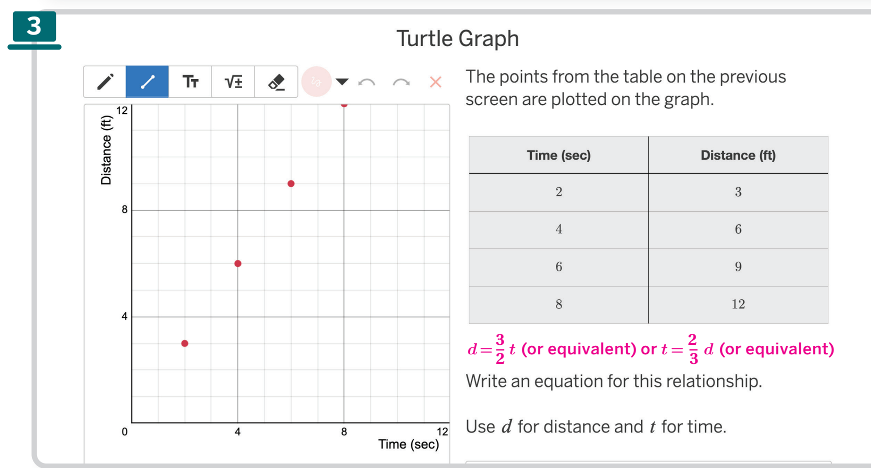
Discuss connections that students make between the constant of proportionality and each of the three representations on the screen.

To surface the Key Takeaway, consider asking,

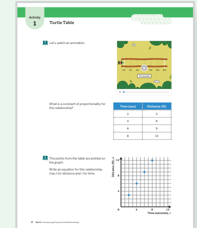
- "Where do you see $\frac{3}{2}$ in the table? In the equation? In the graph?
- What might the graph and equation look like for a turtle who is moving more quickly?
- What about a turtle who is moving more slowly?"

Students using digital





Key Takeaway: The constant of proportionality is visible in the table, the graph, and the equation.



Students using print

Activity 2 Three Turtles

Purpose: Students write equations for proportional relationships and revisit how to identify whether graphs and equations represent proportional relationships.

4 Launch

Play the the animation.

Use the Notice and Wonder routine to support students in making sense of the animation and the graph. Invite students to match each turtle to its graph.

Note: This may be the first time students are seeing a coordinate plane without a grid, requiring them to think abstractly. **(MP2)**

5 Monitor

Support getting started by asking, "What do you notice about where each turtle starts? How could this be represented on the graph?"

M/EL **Multilingual/English Learners:** Circulate and listen to students talk during pair work or group work. Jot down notes about common or important words and phrases (e.g., constant of proportionality), together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.

Use the dashboard to identify conflict or consensus among students' choices.

Pause to highlight one response. If there is conflict, consider selecting and sequencing 1–2 responses for each of the most popular choices, and invite those students to share their reasoning.

5 D Differentiation

Look for students who:	Teacher Moves
Identify that the graph for Turtle 3 represents a proportional relationship.	**Support:** Ask: "Where does Turtle 3 start compared to Turtles 1 and 2? How is this shown on the graph?"
Identify that the graph for Turtle 3 does not represent a proportional relationship, but does not connect the turtle's starting point in their explanation.	Consider asking, "What is similar and different about the graphs for each turtle? What makes the relationship for Turtle 3 non-proportional?"

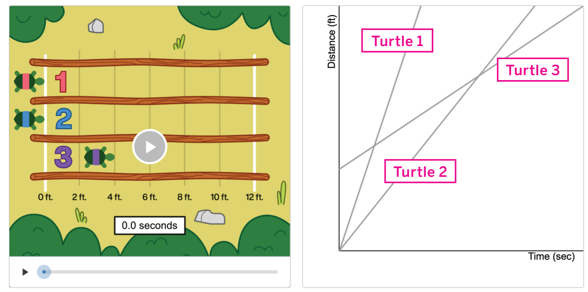
Activity 2 continued ›

Students using digital

4

Three Turtles

Press play to watch these turtles travel. Then match each turtle to the line that represents it.

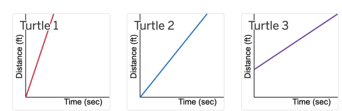


5

Proportional Relationships

Here are the graphs from the previous screen.

1. Select *all* the distance-time relationships that are proportional.



2. Discuss your thinking with a classmate.

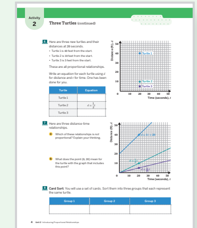
Turtle 1 and Turtle 2

Responses vary. The turtle with a head start (Turtle 3) has a distance-time relationship that is not proportional because the graph does not go through the origin. The other two turtles (Turtles 1 and 2) have distance-time relationships that are proportional because their graphs are straight lines that go through the origin

Pairs | 20 min | Pacing: Screens 6–8

Activity 2 Three Turtles (continued)

Purpose: Students write equations for proportional relationships and revisit how to identify whether graphs and equations represent proportional relationships.



Students using print

Students using digital

6-8 Monitor

Circulate to observe student strategies for writing equations from the graph and offer help or encouragement where needed.

MLR **Use the MLR2: Collect and Display routine** to gather students' ideas and create a class definition or anchor chart for equations that represent proportional relationships.

A **Accessibility: Executive Functioning** Chunk this activity into more manageable parts (e.g., sorting one card at a time), which will aid students who benefit from support with organizational skills in problem-solving.

Use the dashboard to identify conflict or consensus among students' choices.

Students using print will need copies of the Activity 2 cards.

Discuss student strategies to match the tables, graphs, equations, and word stories.

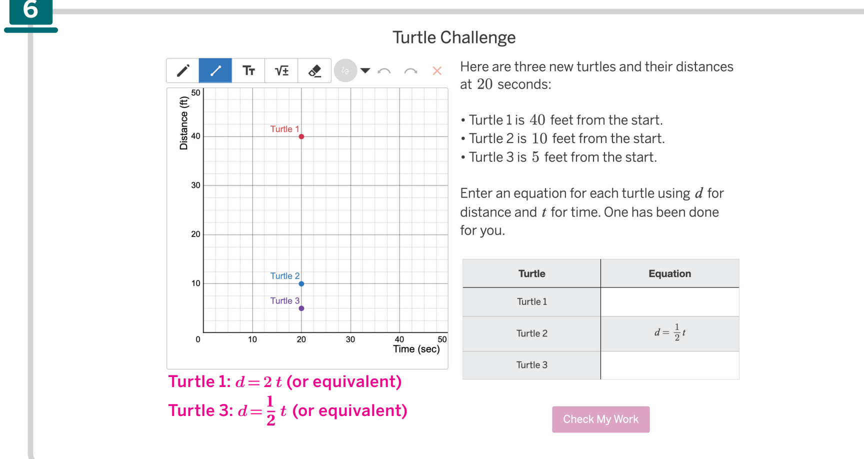
8 Connect

Display student responses and highlight one response if there is consensus. If there is conflict, consider selecting and sequencing 1–2 responses for each of the most popular choices and invite students to share their reasoning.

Invite students to share their strategies when matching different representations of the same turtle.

Consider asking, "Which was the most challenging card to sort? How did you decide where it should go?"

6



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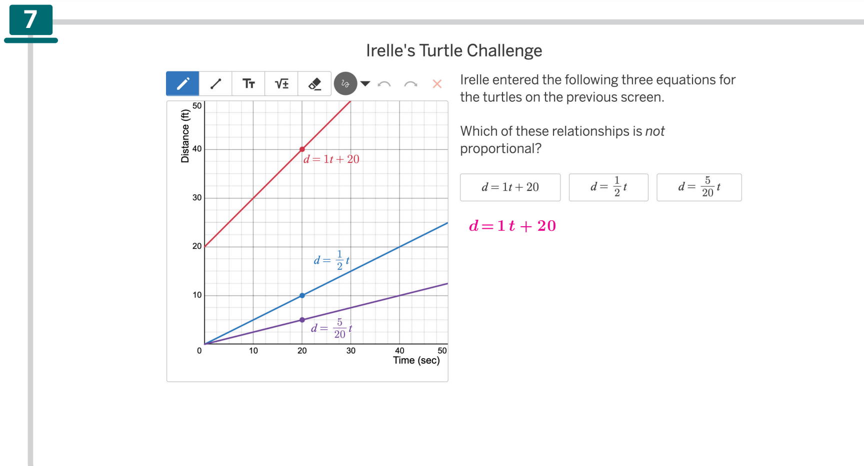

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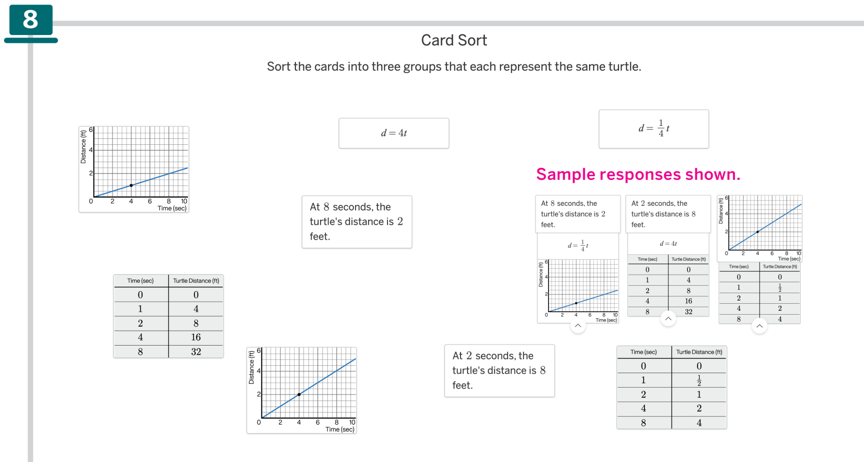
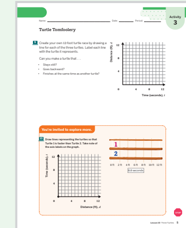


Digital Lesson | Pairs | 10 min | Pacing: Screens 9–10

Activity 3 Turtle Tomfoolery (Optional)

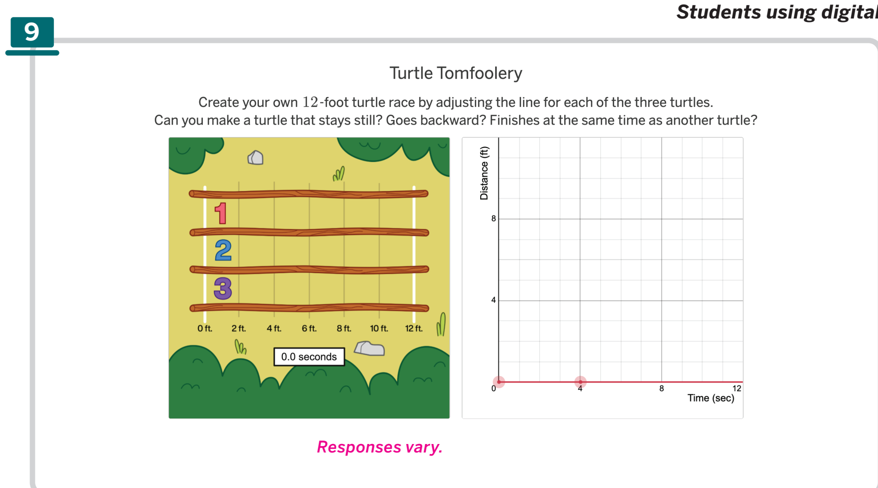
Purpose: Students create their own turtle race based on given conditions.

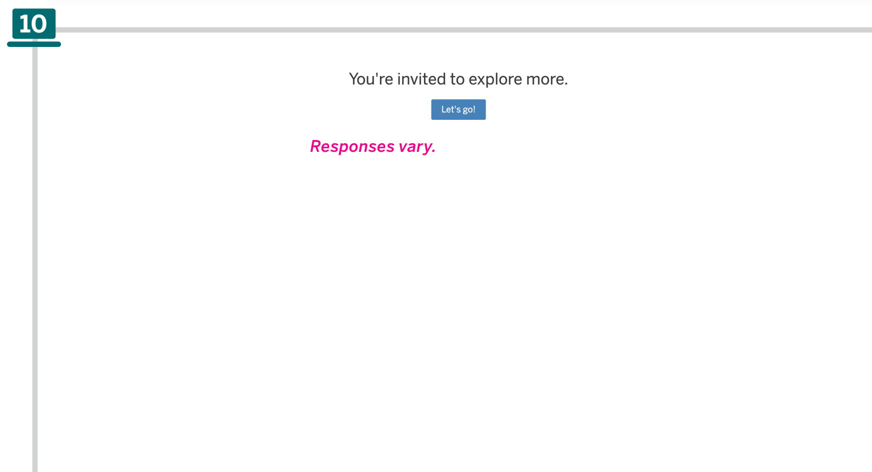
Short on time: Consider omitting Activity 3.



Students using print

Students using digital





9 Launch

Demonstrate how to use the tool and create three lines for three turtles.

Play the animation.

9 Monitor

Snapshot unique or creative graphs.

Math Identity and Community Consider highlighting creative strategies and naming those students explicitly.

Consider asking, "What do the titles of the axes on the graph represent?"

10 **Differentiation: Extension** Invite students who would like to explore further to complete this optional task. Encourage them to discuss their thinking with a partner.

9 Connect

Display snapshots of students' graphs. Ask students to predict what will happen to the turtles in each student's graph.

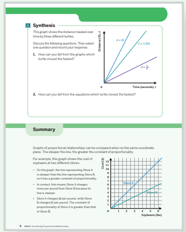
Consider asking:

- "How do you know which turtle moves the fastest when looking at a graph?" *Responses vary.* I can compare the relationships on the same coordinate plane. The fastest turtle is represented by the steepest line.
- "Can you determine which turtle moves the fastest even when the scale is not specified on each axis? Explain your thinking." *Responses vary.* Yes. Even without the scale, I can compare the constants of proportionality of each turtle's travels by comparing the steepness of each line.
- "What is alike and what is different between the proportional and non-proportional relationships we explored today?" *Responses vary.* Proportional and non-proportional relationships are similar in that they are represented by straight lines on a graph. These relationships are different in that proportional relationships pass through the point $(0, 0)$, while non-proportional relationships do not.
- "How can you represent a proportional relationship using different representations (e.g., descriptions, tables, graphs, and equations)?" *Responses vary.* If I have an equation in $y = rx$ form, then I know it is proportional and will pass through $(0, 0)$ on a graph. I can include a row in the table where both x and y are equal to 0.

Whole Class | 5 min | Pacing: Screen 11

Synthesis

Purpose: Students synthesize their understanding of comparing proportional relationships using graphs and equations.



Students using print

11 Synthesis

Invite students to respond and give them a minute to share their responses with a partner.

Display several students' responses.

Invite students to share the connections they see between responses or suggest revisions.

Lesson Takeaway: Graphs and equations help compare proportional relationships.

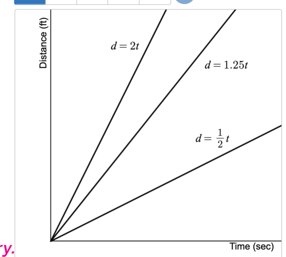
Summary

Share the Summary. Students can refer back to this throughout the unit and course.

Students using digital

11

Synthesis



This graph shows the distance traveled over time by three different turtles.

Discuss the following questions. Then select one question and record your response.

- How can you tell from the graphs which turtle moved the fastest?
- How can you tell from the equations which turtle moved the fastest?

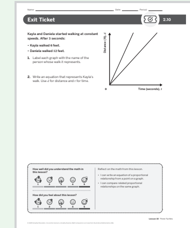
Responses vary.

- I can tell from the graphs which turtle moved fastest by comparing the steepness of the lines. When time is on the x-axis, a steeper line means the turtle moved at a faster rate.
- I can tell from the equations which turtle moved fastest by looking at the constant of proportionality. When the equations are all written for t in terms of d, a greater constant of proportionality means the turtle moved at a faster rate.

Digital Lesson | Independent | 5 min | Pacing: Screens 12–13

Exit Ticket

Purpose: Students demonstrate their understanding of comparing proportional relationships by labeling two relationships on a graph and writing an equation.



Students using print

12-13 Today's Goal

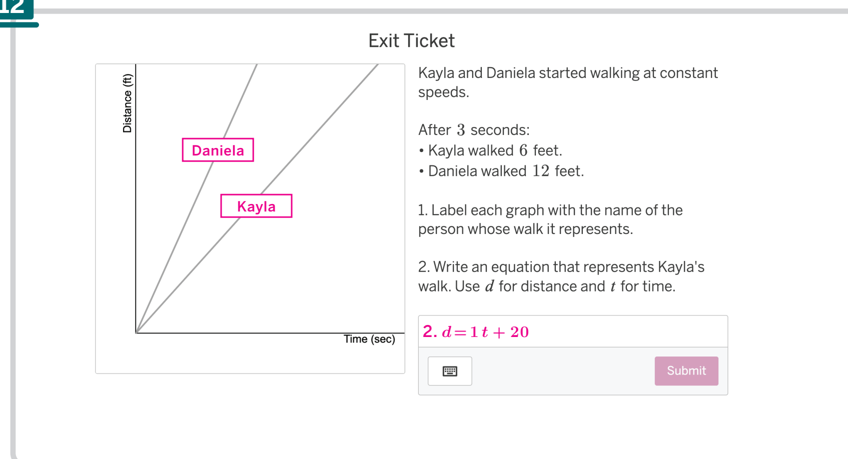
Goal: Write an equation of a proportional relationship given a point on a graph.

Language Goal: Compare related proportional relationships on the same graph using mathematically precise language. **(Reading and Writing)**

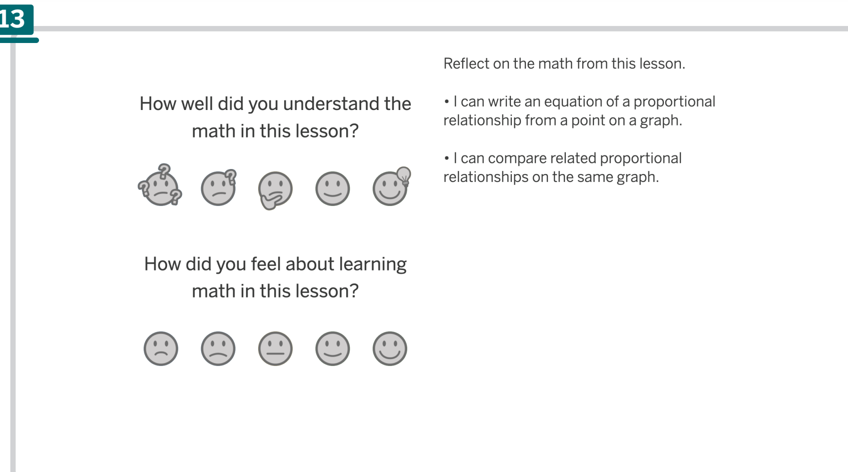
Support for Future Learning: If students struggle to write an equation, consider reviewing this Exit Ticket as a class before beginning Lesson 11, or offering individual support where needed during the lesson.

Students using digital

12



13



Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

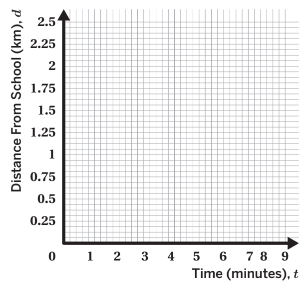
Students using digital

Students using print

Name: ______ Date: ______ Period: ______

Practice

For Problems 1–4, use this information. Mia and Jamal bike home from school at a steady pace. Mia bikes 1.25 kilometers and it takes her 4 minutes. Jamal bikes 1.75 kilometers and it takes him 7 minutes.



1. Create two lines that represent Mia and Jamal's journeys.
   Sample responses shown on graph.

2. Plot a point on each line when $t = 1$.
   Sample responses shown on graph.

3. Determine each value of r, the constant of proportionality.

Line	r
Mia	0.3125
Jamal	0.25

4. Who bikes faster? Explain your thinking.
   Mia; *Sample response:* Mia bikes faster because the constant of proportionality that represents the number of kilometers per minute that she travels is greater than Jamal's.

5. The point $\left(3, \frac{6}{5}\right)$ lies on a graph representing a proportional relationship. Select *all* of the points that also lie on the same graph.

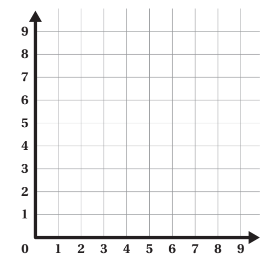
   A. $(1, 0.4)$
   B. $\left(1.5, \frac{6}{10}\right)$
   C. $\left(\frac{6}{5}, 3\right)$
   D. $\left(4, \frac{11}{5}\right)$
   E. $(15, 6)$

Additional Practice for this lesson is available online.

Practice

Name: ______ Date: ______ Period: ______

For Problems 6–9, use this information. At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for $5.40.



6. How much does the honey cost per ounce?
   0.45 dollars per ounce

7. How much honey can you buy for $1 dollar?
   About 2.2 ounces

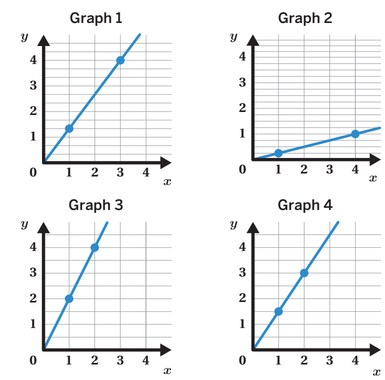
8. Write two different equations that represent this situation. Use h for ounces of honey and c for cost in dollars.
   $c = 0.45h$ (or equivalent) and $h = 2.2c$ (or equivalent)

9. Graph the line of one of your equations from the previous screen. Be sure to label the axes.
   Sample response shown on graph.

Spiral Review

For Problems 10–13, match each equation with its graph.

10. $y = \frac{1}{4}x$ Graph 2
11. $y = \frac{3}{2}x$ Graph 4
12. $y = 2x$ Graph 3
13. $y = \frac{4}{3}x$ Graph 1



Reflection

1. Put a smiley face next to a question you were stuck on and then figured out.
2. Use the space below to ask one question you have or to share something you are proud of.

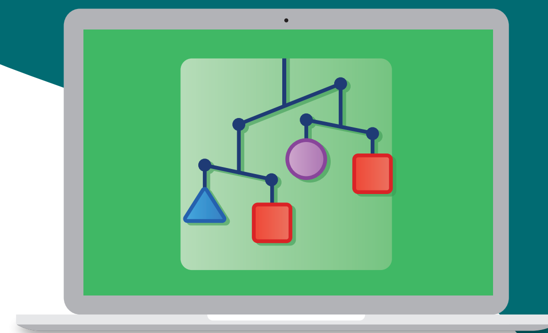
Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1, 2, 9	2	NY-7.RP.2
	3, 4, 6, 7	2	NY-7.RP.2b
Test Practice	5	2	NY-7.RP.2
	8	2	NY-7.RP.2c
Spiral Review			
Fluency	10–13	1	NY-7.RP.2

Unit 6
Lesson
14

Digital Lesson RECOMMENDED

This is a digital lesson. A print option is also available.



Unbalanced Hangers

Solutions to Inequalities

Let's solve inequalities using hanger diagrams.

Focus and Coherence

Today's Goals

1. **Goal:** Determine the solutions to an inequality with only positive numbers.

2. **Language Goal:** Compare and contrast solutions to equations and solutions to inequalities. **(Reading and Writing)**

Students determine solutions to inequalities using a familiar representation – hanger diagrams. Students connect solving an inequality with determining the values of x that will keep one side of a hanger heavier than the other and recognize that the solutions to inequalities are themselves inequalities.

Prior Learning

In Lesson 13, students revisited inequalities and were introduced to greater-than-or-equal-to ($\geq$) and less than-or-equal-to ($\leq$) symbols. They drew and labeled number lines to represent inequalities.

Future Learning

In Lesson 15, students will apply their understanding of inequalities to the context of budgeting.

Rigor and Balance

- Students strengthen their **procedural fluency** in determining solutions to an inequality through repeated challenges.
- Students **reason adaptively** by connecting the solutions to their inequality to the visual representation on the hanger.

Vocabulary

Review Vocabulary

solutions to an inequality

Standards

Addressing

NY-7.EE.4
Use variables to represent quantities in a real-world or **mathematical problem, and construct simple** equations and **inequalities to solve problems by reasoning about the quantities.**

Also Addressing: **NY-7.EE.4b**

Mathematical Practices: MP3

Building On	Building Toward
NY-6.EE.5	**NY-7.EE.4b**

Amplify Desmos Math **NEW YORK**
Lesson Sample

Lesson at a Glance

~ 45 min

Standards: NY-7.EE.4, NY-7.EE.4b

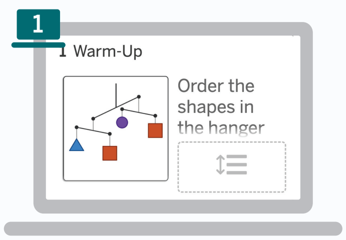
Why go digital?

Students receive interpretive feedback as they solve inequalities in real time.

Warm-Up

Pairs | 5 min

Students revisit hanger diagrams and reason about relative weights of objects using an unbalanced hanger. **Think, Pair, Share**

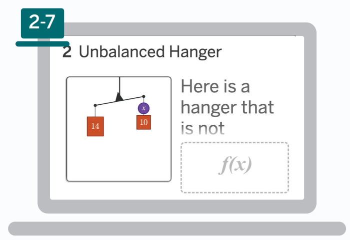


Pacing: Screen 1

Activity 1

Pairs | 10 min

Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities.

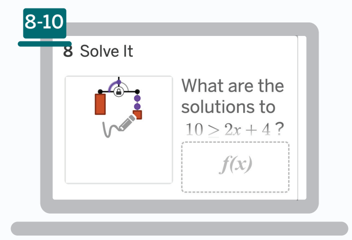


Pacing: Screens 2–7

Activity 2

Pairs | 15 min

Students develop strategies for determining solutions to inequalities, making connections to equations and using unbalanced hangers to help them revise their thinking. **Decide and Defend**

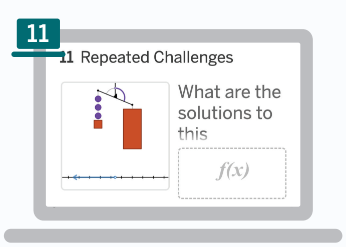


Pacing: Screens 8–10

Activity 3

Independent or Pairs | 5 min

Students practice solving inequalities with positive values using a set of repeated challenges (digital) or partner problems (print).

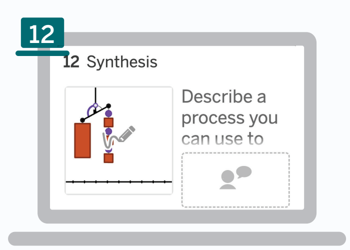


Pacing: Screen 11

Synthesis

Whole Class | 5 min

Students synthesize their understanding about how to solve an inequality.

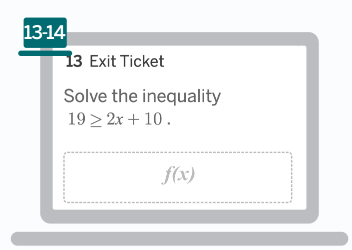


Pacing: Screen 12

Exit Ticket

Independent | 5 min

Students demonstrate their understanding by solving an inequality and explaining their thinking.



Pacing: Screens 13-14

Prep Checklist

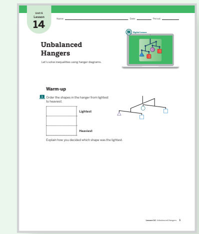
Assign the digital lesson. A print option is also available.

Student using digital:	Student using print:
Digital Lesson	Print Option in Student Edition
	Exit Ticket PDF

Digital Lesson | Pairs | 5 min | Pacing: Screen 1

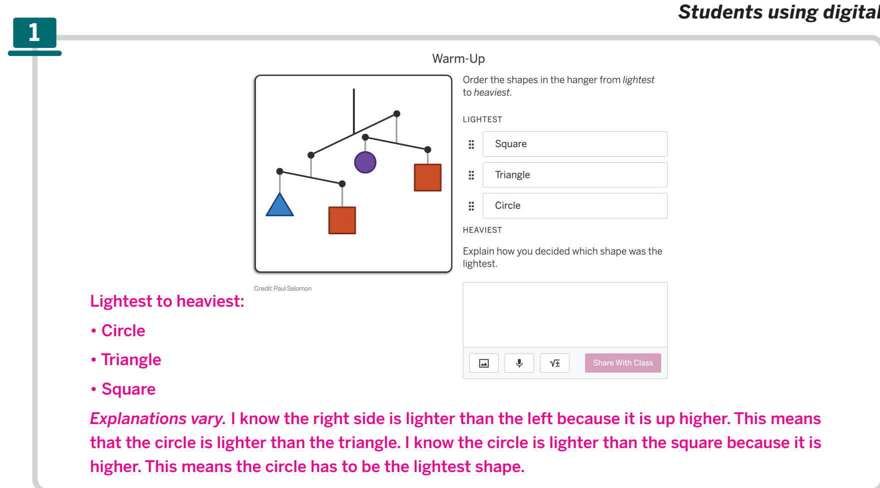
Warm-Up

Purpose: Students revisit hanger diagrams and reason about relative weights of objects using an unbalanced hanger.



Students using print

Students using digital



1 Launch

Invite students to share what they notice about the hanger.

Use the Think-Pair-Share routine to support collaborative learning. Invite students to think independently, then to take turns sharing with a partner.

M/EL **Multilingual/English Learners** Provide sentence frames to help students explain their strategy (e.g., "The____ is lighter/heavier than the ____ because ____.").**(Speaking and Listening)**

A **Accessibility: Visual-Spatial Processing** For students using digital, consider providing access to the Student Edition throughout this lesson, which contains printed versions of the hanger diagrams for students to draw on or highlight.

1 Connect

Display a few student explanations. Invite several students to share their reasoning.

Consider asking, "Which side of an unbalanced hanger is heavier? How do you know?" if this idea does not come up naturally.

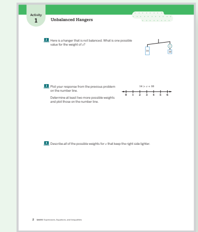
Math Identity and Community Invite students to celebrate a classmate's thinking that was strong, clear, or creative.

Key Takeaway: Unbalanced hanger diagrams can be a visual representation for inequalities.

Pairs | 10 min | Pacing: Screen 2–7

Activity 1 Unbalanced Hangers

Purpose: Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities.



Students using print

2 Launch

Invite students to share what they notice and wonder about the hanger. Listen for and amplify words such as *heavier*, *lighter*, *tilted*, and *unbalanced*.

Note: In prior lessons, students solved problems to make a hanger balance. In this lesson, students will make sense of unbalanced hangers.

2-4 Monitor

Use the dashboard to identify students you would like to check in with during the activity.

Encourage students to record several values that do and do not make the left side heavier.

Invite students using print to check their values with a partner in place of the interpretive feedback provided in digital.

D Differentiation

Look for students who:	Teacher Moves
Need support getting started. *(Screen 3)*	**Support:** Encourage them to make connections between the dots on the number line and the hanger.
List specific numbers ($1, 2, 5$, etc). *(Screen 4)*	Consider asking, "Are there other solutions that are not listed here yet? How could we incorporate those?"
Write any number less than 4. *(Screen 4)*	Consider asking, "How might we represent that using symbols?"
Write $x < 4$. *(Screen 4)*	**Extension:** Challenge students to describe the largest and the smallest solutions.

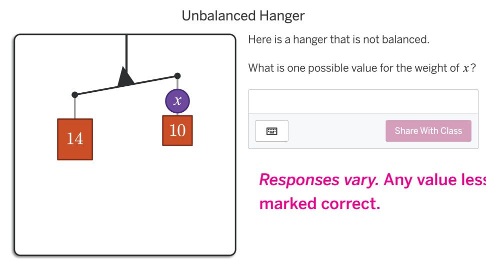
M/EL Multilingual/English Learners Provide sentence frames to help students explain their reasoning (e.g., "All of the possible weights are _______ because ________.").

Pause to have students share their responses before having them continue with the activity.

Activity 1 continued ›

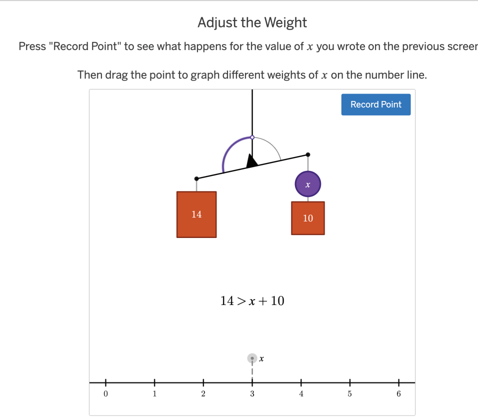
Students using digital

2

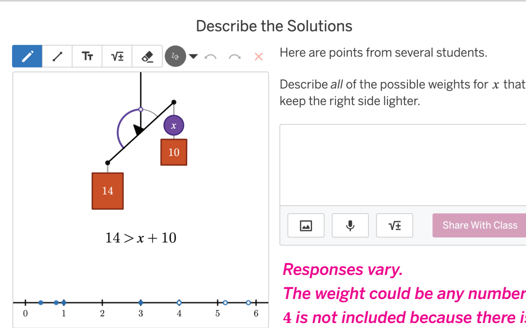


Responses vary. Any value less than 4 will be marked correct.

3



4

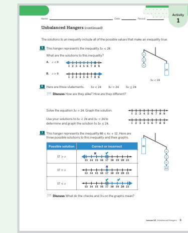


Responses vary.
The weight could be any number less than 4.
4 is not included because there is a $<$ sign, not a $\leq$ sign.

Digital Lesson | Pairs | 15 min | Pacing: Screens 5–7

Activity 1 Unbalanced Hangers (continued)

Purpose: Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities.



Students using print

5-6 Monitor

Use the Decide and Defend routine to support students in making arguments about the symbolic and graphical representations of the solutions to the inequality on Screen 5. **(MP3)**

Display the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus.

Encourage students to make arguments and justify their reasoning for each response.

Consider asking (if these ideas do not come up naturally):
- "What would the hanger diagram look like for each of the responses?"
- "What do solutions to inequalities look like graphically? Symbolically?"
- "What strategies can we use to solve inequalities like $3x \le 24$?" *(Screen 6)*

Math Identity and Community Celebrate students who shared even though they were not sure yet.

7

Students using print analyze three sample results while students using digital press "Try It" to see results.

Encourage connections by asking:
- "How could you solve this if it was an equation?"
- "Why do you think there are two checks or Xs?"

Accessibility: Executive Functioning
To support students with carrying out multiple steps, invite students to record their thinking in the Student Edition for the duration of this lesson.

7 Connect

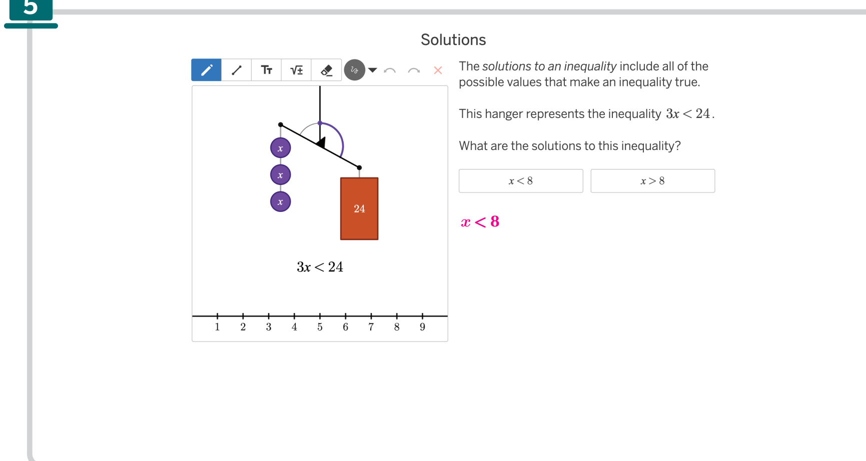
Invite students to share solutions they tried and what they observed as a result.

Consider asking, "How did you use the checks and x's to help edit your response?"

Key Takeaway: Solving an inequality is similar to solving an equation. The solutions to an inequality are also an inequality because there is more than one value that makes the inequality true.

Students using digital

5



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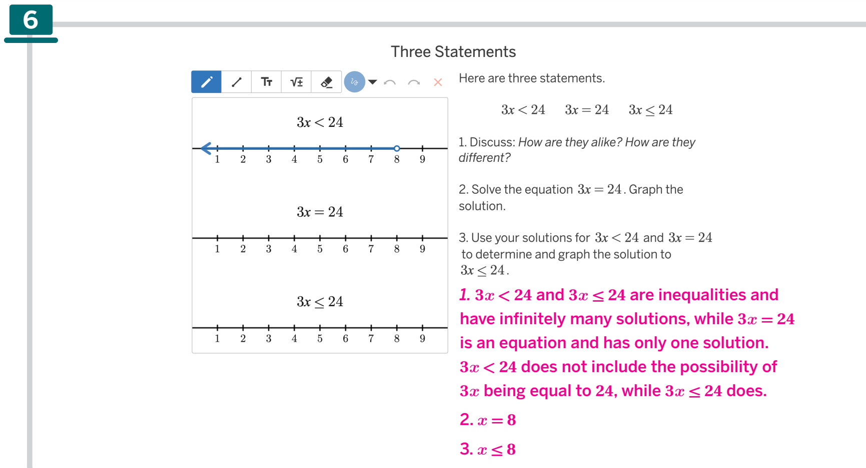

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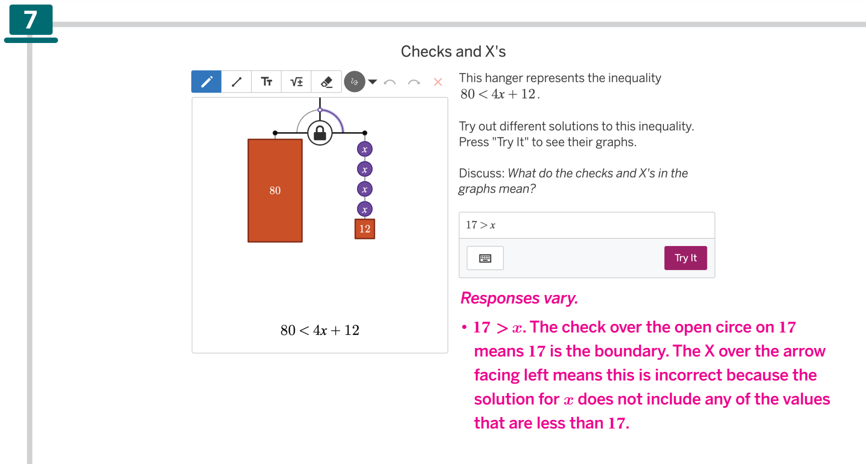
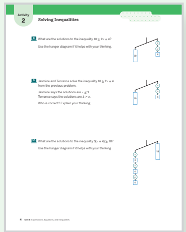


Pairs | 15 min | Pacing: Screens 8–10

Activity 2 Solving Inequalities

Purpose: Students develop strategies for determining solutions to inequalities, making connections to equations and using unbalanced hangers to help them revise their thinking.



Students using print

Students using digital

8 Launch

Display the hanger and the inequality $10 \geq 2x + 4$.

Consider asking, "What might be your first step in solving this inequality?"

9 Monitor

M/EL **Multilingual/English Learners** Encourage students to use gestures or hand motions to help them predict what the hanger should look like. **(Reading and Listening)**

Look and listen for students who justify the response "Both" in creative ways on Screen 9. Consider highlighting these during discussion in the Connect.

Capture several responses to share during the Connect.

D **Differentiation**

Look for students who:	Teacher Moves
Substitute values for x and test whether they make the inequality true.	Consider asking, "What strategies do we have from solving equations that could support us in identifying all of the solutions to this inequality?"
Connect the inequality to the hanger by annotating the image.	Celebrate this visual thinking. Consider asking, "How does this connect to ___'s strategy of distributing, subtracting, and then dividing on both sides?"
Use a variety of algebraic strategies or reasoning to determine the solutions, such as the Distributive Property or making moves on both sides of the inequality.	**Extension:** Consider asking, "What are other ways we could write these solutions?"

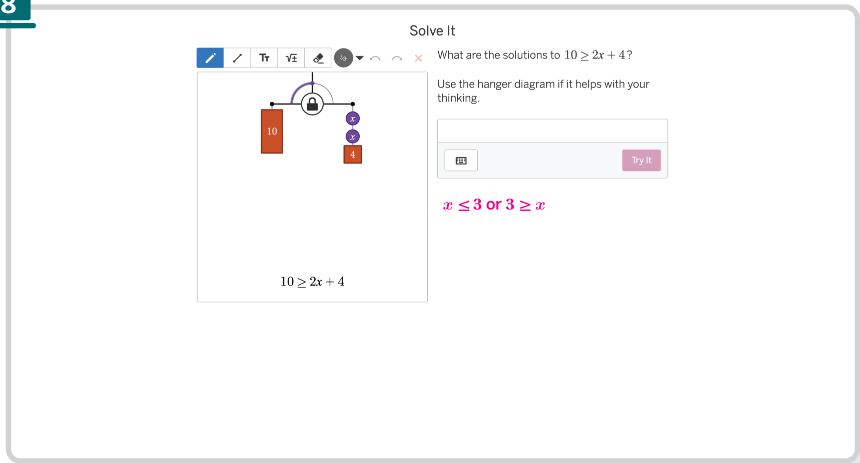
10 Connect

Display captured strategies. Consider starting with a solution where a student substituted a value of x to test the result.

Consider asking, "How is solving an inequality like solving an equation? How is it different?"

Math Identity and Community Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

8



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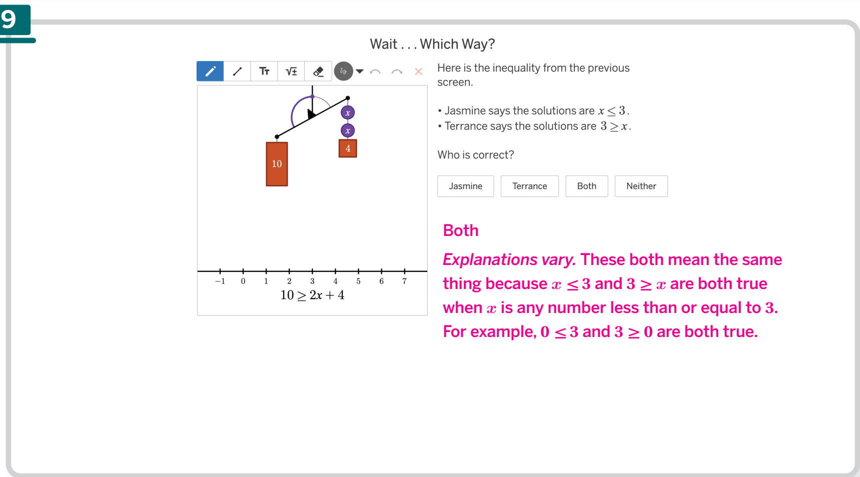

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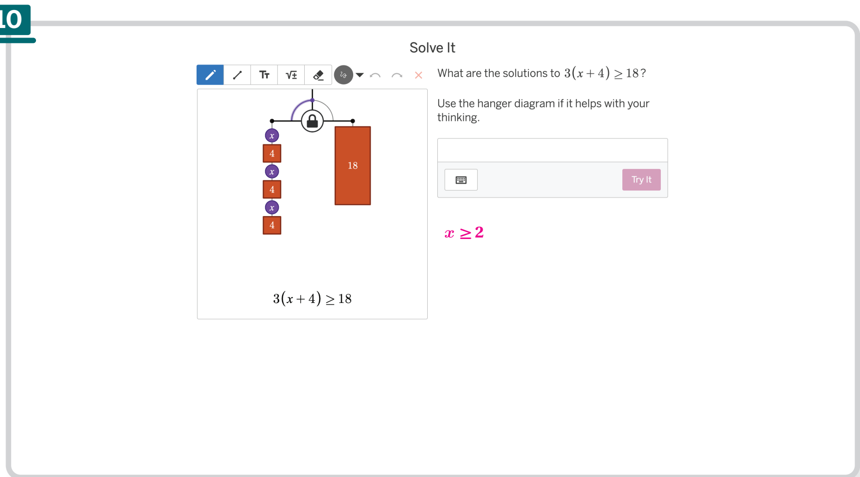
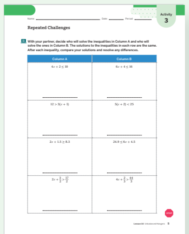


Activity 3 Repeated Challenges

Purpose: Students practice solving inequalities with positive values using a set of repeated challenges (digital) or partner problems (print).

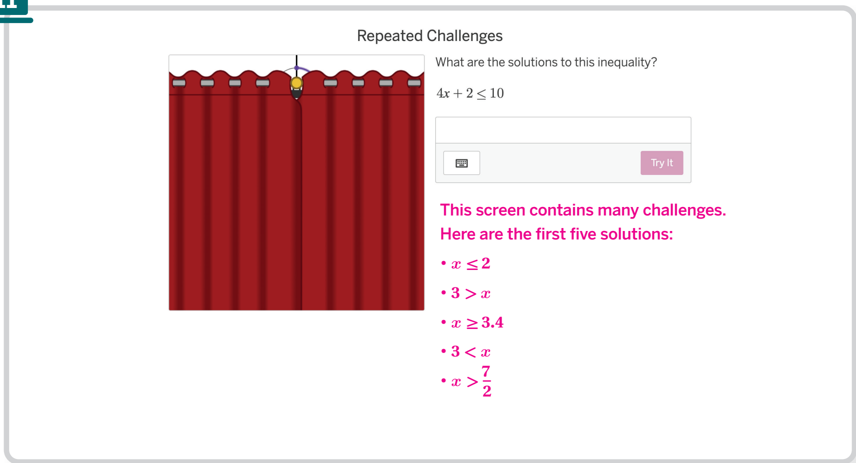
Short on time: Consider omitting this activity.



Students using print

Students using digital

11



11 Launch

Invite students to set their own goal for how many challenges they would like to complete, sharing that sense-making is more important than speed.

11 Monitor

Students using print will need a partner for this activity. They will solve and compare solutions to pairs of problems with the same solutions in place of receiving interpretive feedback.

Circulate to observe student strategies, listen to partner discussions, and offer help or encouragement where needed.

Note: Use the dashboard's teacher view to see how many challenges each student has completed correctly.

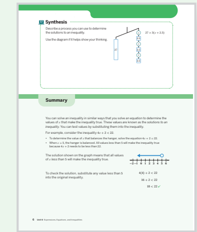
11 Connect

Invite students to share strategies that they found helpful or mistakes they made while solving these challenges.

Whole Class | 5 min | Pacing: Screen 12

Synthesis

Purpose: Students synthesize their understanding about how to solve an inequality.



Students using print

12 Synthesis

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, including:

- Connecting the inequality to the hanger.
- Distributing the 3 first and solving as if this were an equation in order to determine the boundary value.
- Substituting values for x and testing if they make the inequality true.

Math Identity and Community Invite students to name strategies they found most helpful and attribute them to the students who shared them.

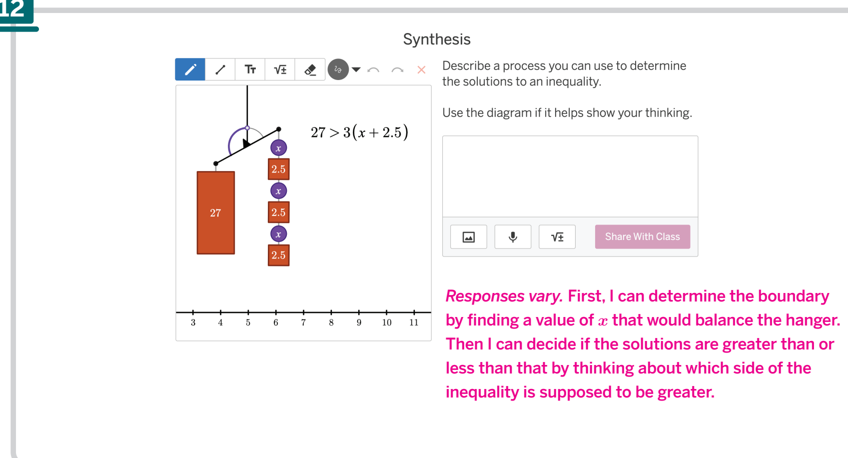
Lesson Takeaway: An inequality can be solved similarly to an equation, allowing the boundary value to be determined. Values can then be tested to determine the solutions that make the inequality true.

Summary

Share the Summary. Students can refer back to this throughout the unit and course.

Students using digital

12

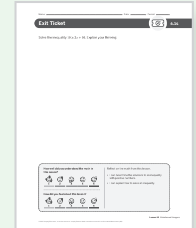


Digital Lesson

Independent | 5 min | Pacing: Screens 13–14

Exit Ticket

Purpose: Students demonstrate their understanding by solving an inequality and explaining their thinking.



Students using print

Students using digital

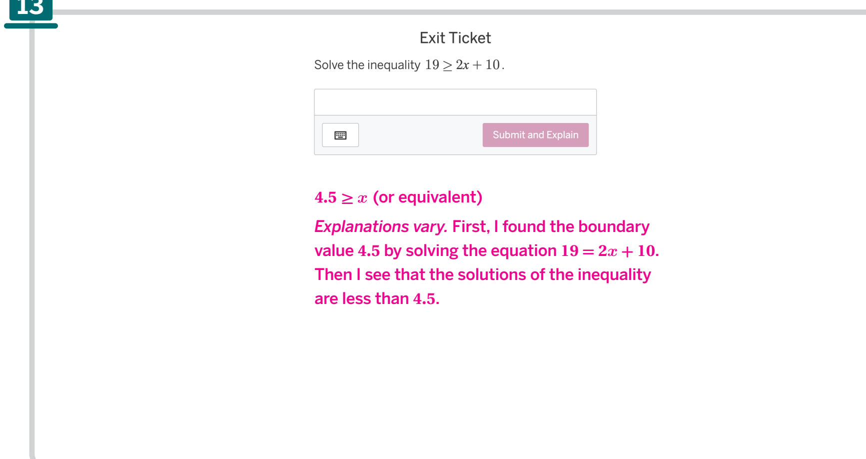
13-14 Today's Goal

Goal: Determine the solutions to an inequality with only positive numbers.

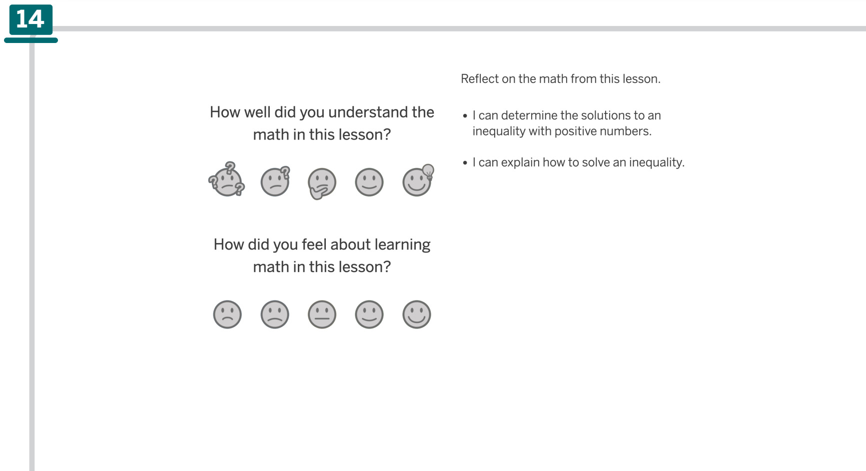
Language Goal: Compare and contrast solutions to equations and solutions to inequalities. **(Reading and Writing)**

Support for Future Learning: If students struggle with solving the inequality, plan to emphasize this when opportunities arise over the next several lessons. For example, spend extra time in Lesson 15 discussing strategies for solving the inequality that represents each situation.

13



14



Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

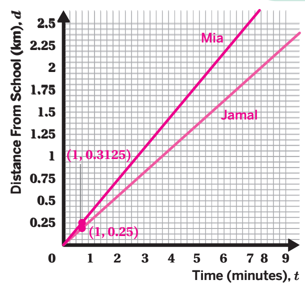
Students using digital

Students using print

Name: Date: Period:

Practice

For Problems 1–4, use this information. Mia and Jamal bike home from school at a steady pace. Mia bikes 1.25 kilometers and it takes her 4 minutes. Jamal bikes 1.75 kilometers and it takes him 7 minutes.



1. Create two lines that represent Mia and Jamal's journeys.
   Sample responses shown on graph.

2. Plot a point on each line when $t = 1$.
   Sample responses shown on graph.

3. Determine each value of r, the constant of proportionality.

Line	r
Mia	0.3125
Jamal	0.25

4. Who bikes faster? Explain your thinking.
   ***Mia; Sample response:* Mia bikes faster because the constant of proportionality that represents the number of kilometers per minute that she travels is greater than Jamal's.**

5. The point $\left(3, \frac{6}{5}\right)$ lies on a graph representing a proportional relationship. Select *all* of the points that also lie on the same graph.

   A. $(1, 0.4)$
   B. $\left(1.5, \frac{6}{10}\right)$
   C. $\left(\frac{6}{5}, 3\right)$
   D. $\left(4, \frac{11}{5}\right)$
   E. $(15, 6)$

Additional Practice for this lesson is avilable online.

Practice

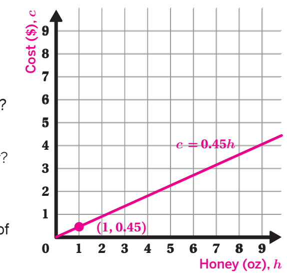
Name: Date: Period:

For Problems 6–9, use this information. At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for \$5.40.

6. How much does the honey cost per ounce?
   0.45 dollars per ounce

7. How much honey can you buy for a dollar?
   About 2.2 ounces

8. Write two different equations that represent this situation. Use h for ounces of honey and c for cost in dollars.
   $c = 0.45h$ (or equivalent) and $h = 2.2c$ (or equivalent)



9. Graph the line of one of your equations from the previous screen. Be sure to label the axes.
   Sample response shown on graph.

Spiral Review

For Problems 10–13, match each equation with its graph.

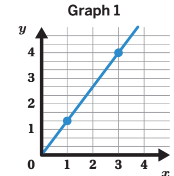
10. $y = \frac{1}{4}x$ **Graph 2**

11. $y = \frac{3}{2}x$ **Graph 4**

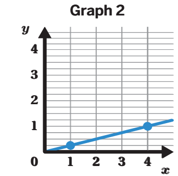
12. $y = 2x$ **Graph 3**

13. $y = \frac{4}{3}x$ **Graph 1**

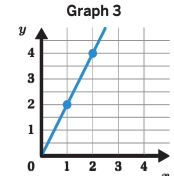
Graph 1



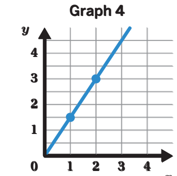
Graph 2



Graph 3



Graph 4



Reflection

1. Put a smiley face next to a question you were stuck on and then figured out.
2. Use the space below to ask one question you have or to share something you are proud of.

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1, 2, 9	2	NY-7.RP.2
	3, 4, 6, 7	2	NY-7.RP.2b
Test Practice	5	2	NY-7.RP.2
	8	2	NY-7.RP.2c
Spiral Review			
Fluency	10–13	1	NY-7.RP.2

GRADE 7

Unit 2
Lesson Plans

Teacher lesson plans from Unit 2 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Ratios and Proportional Relationships** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 7 Unit 2

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1

1 Paint
Explain what equivalent ratios mean in the context of paint mixtures.

2 Balloon Float
Identify patterns in tables that represent proportional relationships.

3 Sugary Drinks
Determine the constant of proportionality for a proportional relationship represented in a table and explain what it means.

Practice Day

Practice Day 1
Practice the concepts and skills developed during Lessons 1–7. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

Quiz: Sub-Unit 1
Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2

8 DinoPops
Explain that a proportional relationship can be represented by points on a line that includes the origin..

9 Gallon Challenge
Interpret points on the graph of a proportional relationship, including (1, k) where k is the constant of proportionality.

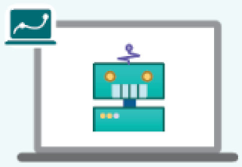
Summative Assessment

End-of-Unit Assessment
Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing: 16 days | Short on time? See pacing considerations below.

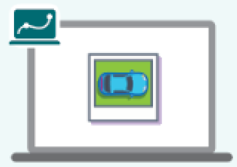
Pre-Unit Check: (Optional)
12 Lessons: 45 min each
2 Practice Days: 45 min each

1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



4 Robot Factory

Explain that when the value of one quantity in a proportional relationship is 1, the other quantity is the constant of proportionality.

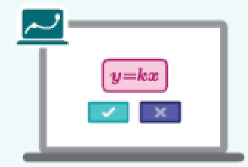


5 Snapshots

Connect each part of an equation of the form y=kx to the situation it represents.

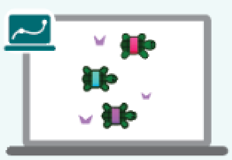
6 Two and Two

Use the word reciprocal to explain the relationship between the two constants of proportionality in a proportional relationship.



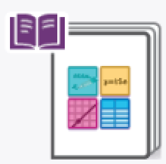
7 All Kinds of Equations

Justify whether a relationship is proportional or not by looking at the structure of an equation.



10 Three Turtles

Write an equation of a proportional relationship given a point on a graph.



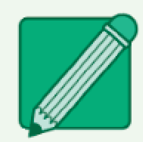
11 Four Representations

Create four different representations of a proportional relationship (description, table, graph, equation).

12 Water Efficiency

Use proportional relationships to analyze a problem about water usage.

Practice Day



Practice Day 2

Practice the concepts and skills developed during Lessons 1–12. Consider using this time to prepare for the upcoming Quiz.

Pacing Considerations

Lesson 1: This lesson supports students in developing their understanding of key ideas about proportional relationships, which will be addressed in more depth in upcoming lessons. If most students demonstrate a strong understanding of proportional relationships in Problems 1 and 7 of the Pre-Unit Check, this lesson may be omitted.

Lesson 5: This lesson supports students in developing fluency writing and using equations to make sense of proportional relationships. If students show a strong understanding working with equations of proportional relationships in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how each part of an equation of a proportional relationship relates to a situation in Lessons 6, 7, and 10.

Lesson 9: This lesson supports students in developing fluency working with graphs of a proportional relationship. If students show a strong understanding working with graphs of proportional relationships in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how points on graphs of proportional relationships can empower us to better understand our society throughout the unit.

Lesson 11: This lesson gives students an opportunity to connect descriptions, tables, equations, and graphs of proportional relationships. There is no new content introduced in this lesson.

Lesson 12: This lesson gives students an opportunity to use their creativity and personal experiences to apply the concepts they learned in this unit to explore whether baths or showers use more water. There is no new content introduced in this lesson.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Paint

Lesson 1: Using Equivalent Ratios

Overview

Students surface initial informal conversations about the key ideas in proportional relationships.

Learning Goals

- Explain what equivalent ratios mean in the context of paint mixtures.
- Use equivalent ratios to generate the same color paints.

Vocabulary

- equivalent ratios

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to surface initial, informal conversations about the key ideas in proportional relationships before the next lesson introduces the terms for those ideas. In this lesson, students will determine which paint mixtures involve equivalent ratios, and justify that they are equivalent. They will build on their Grade 6 understanding of representing ratios with tables and diagrams by connecting and comparing equivalent ratios using a variety of representations.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to familiarize students with the context of the lesson. In this warm-up, students will learn how to make various shades of green paint by mixing green and white cups of paint together.

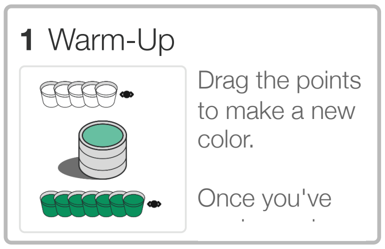
Activity 1: Color Match (30 minutes)

The purpose of this activity is to develop an informal understanding of equivalent ratios by building on students' Grade 6 understanding of representing ratios using tables. In this activity, students will use ratio tables to mix paint to be the same color as a given paint color. They will also decide which mixtures from a list will make the same paint color.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to help students connect what they know about scale factors from Unit 1 to equivalent ratios.

Cool-Down (5 minutes)



Drag the points to make a new color.

Once you've made a color you like, continue to the next screen.

Teacher Moves

Lesson Overview: The purpose of this lesson is to revisit the concept of equivalent ratios from Grade 6. This lesson also informally introduces some key ideas in *proportional relationships* before the next lesson, where students explore the term more formally.

Warm-Up Purpose: Students are introduced to the context of making a shade of paint by mixing two or more colors together.

Facilitation: Invite students to create their favorite shade of green by dragging the number of cups of green paint and white paint. We will use this color on the following screens to informally explore equivalent ratios.

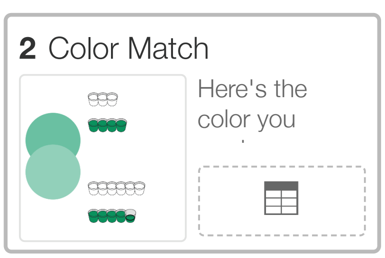
Pacing: Consider using pacing to restrict students to Screens 1–5.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors and allow students who struggle with fine motor skills to dictate physical manipulation of the sliders as needed.



Here's the color you made.

Brielle wants to match your color.

How many cups of green should she mix with 4 cups of white paint to make the same color?

Teacher Moves

Activity 1 Purpose: Students compare ratios of paint to determine if they are equivalent and determine how much paint is needed to create an equivalent ratio.

Facilitation: Invite students to find other combinations of paints that will produce the same shade as the one they chose on Screen 1.

Consider using informal language as students develop their own strategies for how to make different mixtures that produce the same color. The phrase *equivalent ratios* will be more formally introduced on the next screen.

Connection to Readiness Check: If Problem 7 of the readiness check suggests that some students have unfinished learning with determining equivalent ratios, consider pausing for a longer discussion here. Consider asking students to share different representations they used to make sense of these ratios.

Sample Responses

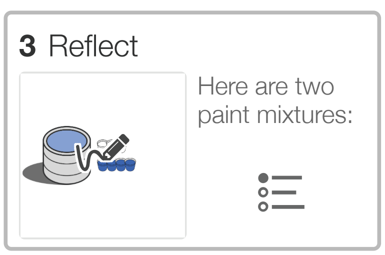
Responses vary.

(The number of white cups is doubled, so the number of green cups should also be doubled.)

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Direct students to the scientific calculator button to ensure inclusive participation.



Here are two paint mixtures:

- 3 cups of white and 4 cups of blue.
- 6 cups of white and 8 cups of blue.

Both mixtures make the same color because they are in EQUIVALENT RATIOS.

Which of these mixtures is also in an equivalent ratio?

Teacher Moves

This is a possible discussion screen.

Facilitation: While students are working, consider selecting and sequencing responses that highlight several different strategies for determining whether two ratios are equivalent.

After most students have had a chance to respond, facilitate a whole-class discussion to surface strategies for figuring out equivalent ratios. Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. To surface different strategies, consider asking a question like: *How can you tell if two different paint mixtures will make the same color?*

Consider snapshotting imprecise or unfinished explanations. During the discussion, highlight the strengths of these explanations by asking students to identify what parts of each explanation they found to be valuable.

Connection to Readiness Check: If Problem 4 of the readiness check suggests that some students have unfinished learning with determining equivalent ratios, consider pausing for a longer discussion here. Consider highlighting students' strategies for finding equivalent ratios and recording them for students to refer to throughout the lesson and unit.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

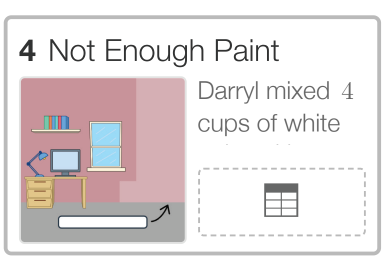
Sample Responses

1.5 white, 2 blue

Responses vary.

Using a unit rate:
- I figured out that for each mixture, there is 0.75 cup of white paint for each cup of blue paint, so that means for 2 cups of blue paint, there must be $0.75 \cdot 2 = 1.5$ cups of white paint.

Using a scale factor:
- I started with the ratio in the picture and took half of each color, which is 1.5 cups of white and 2 cups of blue.



Darryl mixed 4 cups of white paint with 6 cups of red paint, but didn't have enough to finish painting his wall.

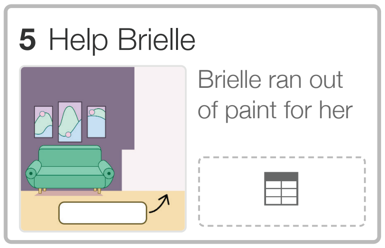
How much red paint would he need to add to 1 cup of white paint to match the color?

Teacher Moves

Facilitation: Consider waiting to discuss this screen until most students have completed Screen 5.

Sample Responses

$\frac{6}{4}$ or 1.5 (or equivalent)



Brielle ran out of paint for her room.

Fill in the table so that the new mixture will match the original paint color.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface strategies for how to use equivalent ratios to create paint mixtures of the same color.

Facilitation: Once students have had a chance to complete the table, project the task using the dashboard's student view.

Consider facilitating a whole-class discussion around questions like:
- *How can you figure out how many cans of red paint you need? White paint?*
- *What are some ways to check that the mixtures will make the same color?*

Spend adequate time here to ensure that students understand how to use equivalent ratios to make paint mixtures before they create and solve each other's challenges on the next screen.

Early Finishers: Encourage students who finish Screens 1–5 early to determine how many cups of red and white paint you would need for 1 cup of blue paint.

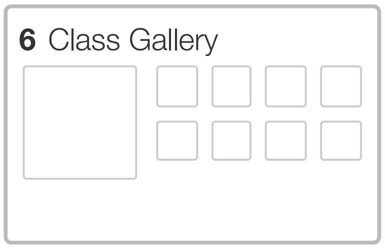
Sample Responses

3 cups of red paint, $\frac{14}{3}$ cups of white paint

Student Supports

Students With Disabilities
- *Executive Functioning: Visual Aids*

Create an anchor chart for public display that describes strategies for calculating equivalent ratios for future reference.



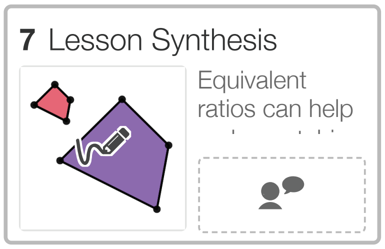
Teacher Moves

Facilitation: Invite students to create their own challenges and solve challenges created by their classmates. Encourage students to also take some time to review responses to their own.

While students are working, monitor for creative challenges that may expand your students' understanding of equivalent ratios. Consider highlighting unique or creative challenges using the dashboard's teacher view.

Note: We anticipate this Challenge Creator may take 15 minutes or more.

Pacing: Consider using pacing to restrict students to this screen.



Equivalent ratios can help make matching paint colors.

Where do you see equivalent ratios in scale drawings?

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

Lesson Synthesis Purpose: Students connect what they know about scale factors from Unit 1 to equivalent ratios.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses or suggest revisions.

Routine (optional): Consider using the mathematical language routine Collect and Display.

Sample Responses

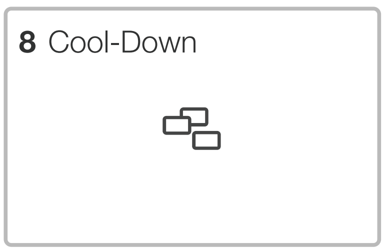
Responses vary.

All pairs of corresponding sides between the two scaled figures are in an equivalent ratio.

Student Supports

Multilingual Learners
• *MLR 8 (Discussion Supports)*
Revoice student explanations using appropriate mathematical language.

Students with Disabilities
• *Conceptual Processing: Eliminate Barriers*
Assist students in recognizing the connections between new problems and prior work. Students may benefit from a review of different representations to activate prior knowledge.



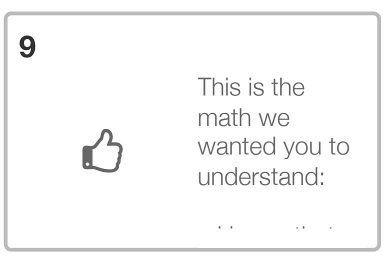
Teacher Moves

Support for Future Learning: Students will have more chances to develop their understanding of equivalent ratios in the upcoming lessons, particularly in Lesson 2 and 3.

Pacing: Consider using pacing to restrict students to Screens 8–9.

Sample Responses

Image solution

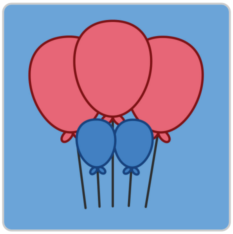


This is the math we wanted you to understand:

• I know that two paint mixtures will look the same if the ingredients are in equivalent ratios.

• I can use equivalent ratios to generate the same color paints.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Balloon Float

Lesson 2: Introducing Proportional Relationships With Tables

Overview

Students are introduced to the concept of a proportional relationship by looking at tables of equivalent ratios.

Learning Goals

- Identify patterns in tables that represent proportional relationships.
- Use a table to calculate unknown quantities in a proportional relationship.

Vocabulary

- proportional relationship

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to introduce the concept of a proportional relationship by looking at tables of equivalent ratios. Students learn that they can use multiplicative strategies to obtain unknown values for quantities that are in a proportional relationship.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to elicit the idea that you can use a table to see patterns between related quantities, which will be useful later in the lesson when students discuss how to use tables to learn about proportional relationships (MP7).

Activity 1: Balloon Float (15 minutes)

The purpose of this activity is to introduce students to proportional relationships. Students also learn strategies to complete tables when given a proportional relationship, including using a scale factor and using a unit rate.

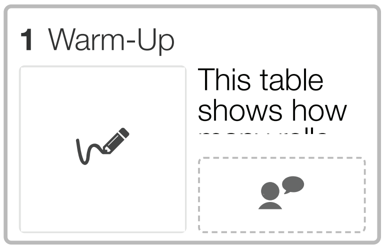
Activity 2: Proportional Relationships (15 minutes)

The purpose of this activity is to introduce students to the term *proportional relationship* and apply their understanding to analyze several tables (MP6).

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate the distinctions between proportional relationships and non-proportional relationships.

Cool-Down (5 minutes)



This table shows how many rolls of paper towels a store receives when they order different numbers of cases.

What are three things you notice about the table?

Teacher Moves

Lesson Overview: The purpose of the warm-up is to elicit the idea that you can use a table to see patterns between related quantities, which will be useful later in the lesson when students discuss how to use tables to learn about proportional relationships (MP7).

Warm-Up Purpose: Students use a table to identify patterns between related quantities, which will be useful when they discuss how to use tables to analyze proportional relationships later in the lesson.

Facilitation: Arrange students into pairs. Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses (Notice and Wonder). If possible, annotate the table as students share their thinking.

After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. See the sample responses for ideas and questions that may surface during the discussion. If patterns across or down the table do not come up naturally, consider introducing one or more as noticings from a student in a previous class or year.

Connection to Readiness Check: If Problem 1 suggests that some students have unfinished learning with scaling in tables, consider pausing for a longer discussion here. Consider inviting students to use the sketch tool to add a new pair of values to the table.

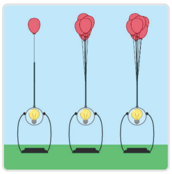
Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- Each row is in an equivalent ratio to the other rows.
- There are 12 rolls of paper towels in every case.
- If I multiply every number in the first column by 12, I get the number in the second column.
- The numbers in the bottom three rows seem to be "scaled copies" of the numbers in the top row. For example, I could get the numbers 5 and 60 by multiplying the numbers in the top row by 5.

2 Balloon Float



Helium balloons can make objects float, but

Helium balloons can make objects float, but too many balloons will make objects fly away!

In this activity, we'll figure out how many balloons it takes to float various objects.

Press the play button. Watch the **middle** lightbulb float.

Continue to the next screen when you're ready.

Teacher Moves

Activity 1 Purpose: Students are introduced to the idea of a proportional relationship. Students also learn strategies to complete tables when given a proportional relationship.

Facilitation: Consider starting with this activity paused and asking students what experiences they have had with helium balloons and what they think it means to float an object. [Floating an object means getting it to stay right in the center of the screen, not falling to the ground or flying away.]

Invite students to watch the animation on their screen, and then to continue to the next screen.

Pacing: Consider using pacing to restrict students to Screens 2–3.

Student Supports

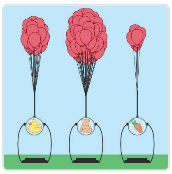
Students With Disabilities

• *Conceptual Processing: Processing Time*
Check in with individual students, as needed, to assess for comprehension during each step of the activity.

• *Memory: Processing Time*
Provide sticky notes or mini whiteboards to aid students with working memory challenges.

• *Visual-Spatial Processing: Visual Aids*
Provide printed copies of the representations for students to draw on or highlight.

3 Make Them Float



On the previous

On the previous screen, the lightbulb weighed 2 ounces and required 6 balloons to float.

This table shows the weight of three new objects. Enter the number of balloons needed to float each object. (All balloons carry the same weight.)

Teacher Moves

Facilitation: Allow students one minute of quiet work time, followed by three minutes of work time with their partner.

If students are having difficulty getting started, consider referring back to the warm-up and asking a question like: *Which of the patterns we noticed in the warm-up might be useful here?*

While students are working, consider looking ahead to the two student strategies on Screen 4. If you are able to, find actual students in your class who took those two approaches. Invite each student to show their thinking on the board. Consider using these students' work as the basis for discussion rather than the fake students on Screen 4.

Sample Responses

Rubber duck: 30 balloons
Toy bear: 18 balloons
Carrot: 9 balloons

---

4 Two Strategies

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to understand two methods that can be used to complete tables where two quantities are in the same ratio.

Facilitation: Consider sharing with students that this screen contains two pieces of student work from the previous screen, and each one shows a different approach to solve the problem. Give students 2–3 minutes to analyze the strategies, invite them to summarize each students' strategy with their partner, and then hold a discussion to explain both strategies.

Consider questions like:

- *Why does Ariel's strategy work? Why does Emma's strategy work?*
- *When might Ariel's strategy be easier to use? What about Emma's strategy?*

• *Why might it be helpful to add a row that represents floating a 1-ounce object?*

Students might use terms like “scale factor” or “unit rate,” but it is not necessary at this point. Vocabulary for describing these relationships will be formalized later in the lesson and the unit.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

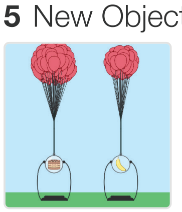
Responses vary.

Ariel might finish the table by scaling the numbers in the first row (2 ounces and 6 balloons). Determining how many balloons would float the toy bear would require a scale factor of 3, resulting in 18 balloons. Determining how many balloons would float the carrot would require a scale factor of 1.5, resulting in 9 balloons.

Emma might finish the table by multiplying each weight by 3 to get the number of balloons. This relationship can be explained in terms of the *unit rate* (balloons per ounce), which tells us how many balloons are required to carry one ounce of weight.

---

5 New Objects



Here are some new objects.

Here are some new objects.

Complete the table so that the new objects float.

Teacher Moves

Progress Check: This is a great place to check student progress to see if they can apply the strategies from the previous screen. The banana offers a useful check on whether students can multiply the weight by 3 to get the number of balloons. The cake offers a useful check on whether students can apply a scale factor to a known row to complete a new row. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Pacing: Consider using pacing to restrict students to Screens 5–6.

Sample Responses

Cake: 20 ounces
Banana: 10 balloons

---

6 Proportional Relation…
When two quantities are always in an equivalent

When two quantities are always in an equivalent ratio, it is called a *proportional relationship*.

Which of these two tables represents the proportional relationship between weight and balloons?

Teacher Moves

Activity 2 Purpose: The purpose of this activity is to introduce students to the term *proportional relationship* and apply their understanding to analyze several tables (MP6).

Facilitation: Consider starting with the activity paused and introducing the term introducing the term *proportional relationship*. Consider reading the first statement aloud as a class and asking several students to explain what they think a proportional relationship is in their own words. Consider recording these explanations for students to refer back to throughout the lesson and unit. Then, invite students to read the question and select the appropriate table. Facilitate a discussion as needed.

Early Student Thinking: Students may notice that all of the number of balloons in Table 1 are 3 more than the weight and select Table 1 as the proportional relationship. Consider encouraging these students to consider the number of balloons per ounce for each row of the table and asking something like: *If 3 ounces require 6 balloons, how many balloons do you need to hold each ounce?*

Pacing: Consider using pacing to restrict students to Screens 6–9.

Sample Responses

Table 2

Responses vary.

The number of balloons is always three times the weight in ounces. That means they are always in a 3-to-1 ratio, which means the relationship is proportional.

7 Card Sort

Teacher Moves

Facilitation: Encourage students to share their reasoning with a partner and work to reach an agreement together about how to sort the cards.

If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Sample Responses

Image solution

Student Supports

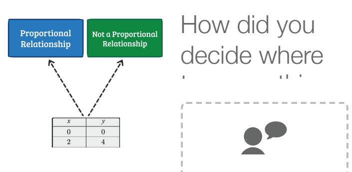
Students With Disabilities

• *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the representations for students to draw on or highlight.

---

8 How Did You Decide?



How did you decide where to group this card?

What feature(s) of this table were important?

Teacher Moves

This is a possible discussion screen.

Facilitation: While students are working, select and sequence several explanations using the snapshots tool.

After students have responded to this screen, consider pausing the lesson and facilitating a whole-class discussion to surface ideas about features of a proportional relationship.

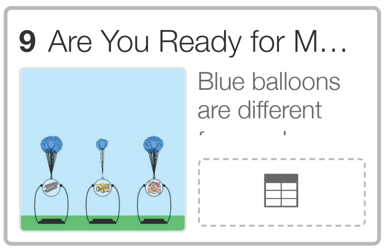
The following ideas may surface during the discussion:

- A proportional relationship has equivalent ratios.
- Many proportional relationships include the point $(0, 0)$, but this does not guarantee that the relationship is proportional.
- In each proportional relationship, there is one number that can be used to multiply the numbers in one column to get the numbers in the other column.

Sample Responses

Responses vary.

This is not a proportional relationship. If it were a proportional relationship, I could multiply all the x-values by the same number and get all the y-values. That is not possible for this relationship.



Blue balloons are different from red balloons. 8 blue balloons can float a 10-ounce stapler.

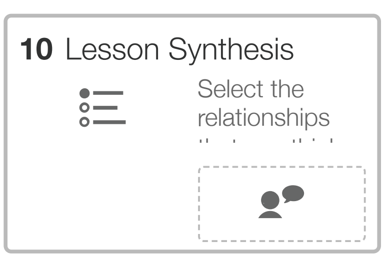
Complete the table so that each object floats.

Teacher Moves

This screen is designed as an extra challenge for students who finish Screens 6–8 before the class discussion on Screen 10. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

Toy truck: 12 balloons
Jelly beans: 35 ounces



Select the relationships that you think are proportional.

Then answer: What determines whether a relationship is proportional or not?

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their ideas about what makes a relationship proportional.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Select and sequence several students' text responses to display using the dashboard's teacher view or snapshot tool.

Follow with a whole-class discussion to 1) settle on criteria for what determines a proportional relationship as a class and 2) use the criteria to discuss the multi-select options one at a time.

Note: Answers to the multi-select may be different depending on the assumptions made about the situation. For instance, a person's time and total distance during a race is a proportional relationship if the person runs at a perfectly consistent rate, but is *not* a proportional relationship if the person's rate changes during the race.

Routine (optional): Consider using the mathematical language routine Collect and Display.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. (Different choices could be correct depending on the assumptions that students make.)

- **A person's height in feet and that person's height in inches:** Proportional
- **The number of cookies baked and the minutes in the oven:** Not proportional
- **The grams of flour needed to make different amounts of bread:** Proportional
- **A person's time and total distance as they run a marathon:** Proportional if the person runs at a consistent pace for the entire race. Otherwise, not proportional.
- **The gallons of gasoline purchased and the total cost:** Proportional if the total cost is based upon a price per gallon and doesn't include any fees. Otherwise, not proportional.

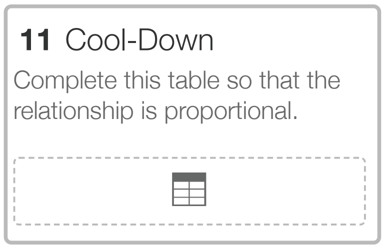
A relationship is proportional if the two quantities are always in the same ratio. For example, if the total cost of gasoline is based on a consistent price (like $\$3$ per gallon), then the gallons of gasoline purchased and the total cost would always be in the same ratio. That relationship would be proportional.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time. This may include reading the information in the table and in the graph.



Complete this table so that the relationship is proportional.

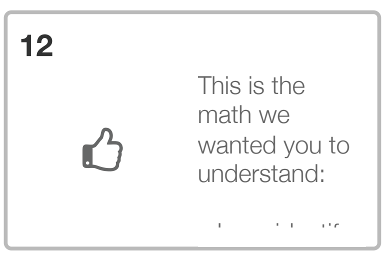
Teacher Moves

Support for Future Learning: Students will have more chances to develop their understanding of using proportional relationships to determine unknown values in the upcoming lessons, particularly in Lesson 3 and 5.

Pacing: Consider using pacing to restrict students to Screens 11–12.

Sample Responses

- $x = 3$, $y = 10$
- $x = 60$, $y = 200$



This is the math we wanted you to understand:

- I can identify patterns in tables that represent proportional relationships.
- I can use a table to calculate unknown quantities in a proportional relationship.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Sugary Drinks (NYC)

Lesson 3: Constant of Proportionality

Purpose

The purpose of this lesson is for students to determine, interpret, and use *constants of proportionality* to make sense of proportional relationships.

Preparation

Student Worksheet:

- *Activity 1 and 2*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Warm-Up (5 minutes)

Purpose: Introduce the term *constant of proportionality*.

Facilitation: Consider starting the lesson by asking students what they know about the relationship between sugar and health. Display the Teacher Projection Sheet and read the first sentence as a class. As time allows, invite students to notice and wonder about the values in the table before revealing the sentences.

Then, give students 1–2 minutes of quiet think-time to decide how to complete each sentence.

Follow with a whole-class discussion to introduce the term *constant of proportionality*.

Consider asking: *In what ways is* 30 *an important number in this relationship?*

Ideas that may surface during the discussion:

- We always multiply or divide by 30 between the number of days and the grams of sugar
- 30 is the maximum number of grams of sugar *per day*.

Then, share with students that the number 30 in this relationship is called a *constant of proportionality*. Consider asking students to think of other examples of constants of proportionality that they've encountered in earlier lessons.

Note: Every proportional relationship has two constants of proportionality. In this example, grams of sugar is proportional to the number of days, so one constant of proportionality is 30 grams of sugar per day. The other constant of proportionality for this relationship represents the number of days per gram of sugar: at maximum, children should consume a gram of sugar every $\frac{1}{30}$ of a day. Students will explore these two constants of proportionality and their relationship in Lesson 6 of this unit.

Activity 1: Orange Juice (10 minutes)

Purpose: Students calculate and interpret a constant of proportionality in a proportional relationship.

Facilitation: Consider beginning the activity by asking students: *Do you think the relationship between volume of orange juice and amount of sugar will be proportional?*

Once the class has agreed that the relationship is proportional, distribute one double-sided sheet (Activity 1 and 2) to each pair of students and invite students to complete Activity 1, then continue to Activity 2. Ensure that calculators are available throughout the lesson.

Spend adequate time here to ensure students understand how to calculate a constant of proportionality for a proportional relationship. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Activity 2: Other Sugary Drinks (20 minutes)

Purpose: Students use constants of proportionality to compare relationships and determine what a constant of proportionality means in context (MP2).

Facilitation: Students may begin this activity at different times as they complete Activity 1. As with orange juice, students can assume that each beverage's volume is proportional to the amount of sugar it contains. While students are working, monitor for different approaches to Question 2, including calculating and comparing a constant of proportionality for each drink.

After 5–7 minutes of work time, consider facilitating a whole-class discussion to surface strategies for comparing proportional relationships, especially strategies that use a constant of proportionality.

Invite several students to share their thinking on Question 2, asking questions like:

- *How might we compare the sugariness of these drinks without completing the table?*
- *How might constants of proportionality help us determine which drink is the most sugary?*

The following ideas may surface during the discussion:

- The personal bottle of carbonated soda is a similar size to the glass bottle of apple cider, but it has more than double the amount of sugar.
- There is a 12-oz. energy drink and a 12-oz. carbonated soda, so the one that has more sugar at that size is the more sugary drink.
- The highest constant of proportionality corresponds to the drink that's most sugary.
- The constant of proportionality represents the amount of sugar per (1) ounce.

Then, discuss responses to Question 4 to surface strategies for how to determine whether relationships are proportional.

Consider asking questions like:

- *How is the table in Question 4 different from the other tables we've seen in this lesson?*
- *How can you use a table to decide if a relationship is proportional?*

The following ideas may surface during the discussion:

- Previous tables involved different sizes of the same recipe. This table represents four different recipes. Some candies might be more sugary than others, so it is unlikely that the relationship between weight and sugar across different candies will be proportional.
- We can calculate how much sugar per (1) gram is in each candy. When we do, we can see that there is no single constant of proportionality, so the relationship is not proportional.

Lesson Synthesis (5 minutes)

Purpose: Students justify whether a table represents a proportional relationship or not.

Distribute one double-sided half sheet of the lesson synthesis and cool-down to each student. Give students 2–3 minutes to respond and then one minute to share their responses with a classmate. Invite several students to share their reasoning, monitoring for opportunities to use the phrase *constant of proportionality*.

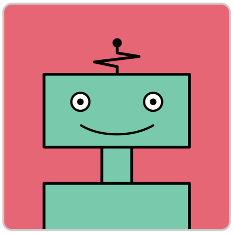
Routine (optional): Consider using the mathematical language routine Collect and Display.

After the discussion, invite students to complete the cool-down individually on their worksheet.

Cool-Down (5 minutes)

If students struggle to determine the constant of proportionality, plan to emphasize this when opportunities arise over the next several lessons. For example, consider pausing on Screen 5 of Lesson 4 to make connections between the constant of proportionality and the 2 in the table.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Robot Factory

Lesson 4: Proportional Relationships and Equations

Overview

Students are supported in writing equations for proportional relationships. This may be one of the first times that students encounter equations with two variables.

Learning Goals

- Explain that when the value of one quantity in a proportional relationship is 1, the other quantity is the constant of proportionality.
- Write equations to represent proportional relationships.

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to support students in writing equations for proportional relationships. This may be one of the first times that students encounter equations with two variables. This lesson is intended to expose students to the power of using equations as representations of proportional relationships. In the next lesson, students will practice writing and interpreting equations for different scenarios.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce the context of creating a robot and discuss the importance of the point $(1, k)$ in tables of proportional relationships.

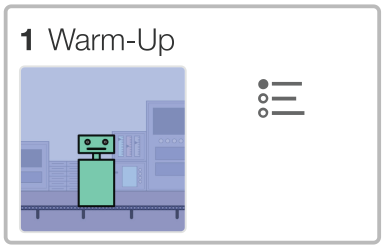
Activity 1: Robots (30 minutes)

The purpose of this activity is for students to develop and use equations to produce robots en masse to fit their design. As the lesson progresses, students choose different accessories for their robot and move from selecting the correct equation to writing their own equations. Students use structure (MP7) and repeated reasoning (MP8) to help them make sense of and write these equations.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their ideas about how to write an equation from a table of a proportional relationship.

Cool-Down (5 minutes)



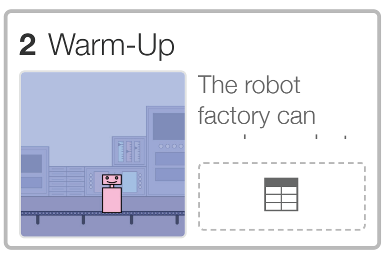
Teacher Moves

Lesson Overview: The purpose of this lesson is to make sense of writing equations for proportional relationships. Note: This may be one of the first times that students encounter equations with two variables.

Warm-Up Purpose: Screens 1–2 orient students to the scenario and discuss the importance of the point $(1, k)$ in tables of proportional relationships.

Facilitation: Arrange students into pairs. Consider starting with the screen paused and sharing with students that this lesson is about designing a robot and assembling scaled copies of the robot. Then unpause and invite students to select a color. Continue to the next screen when the class is ready.

Pacing: Consider using pacing to restrict students to Screens 1–2.



The robot factory can produce robots of all different sizes.

The shade you chose is made by mixing two paint colors (as shown in the table below).

Complete the table to color several different robots.

Teacher Moves

This is a possible discussion screen.

The purpose of this discussion is to connect the constant of proportionality to the value opposite 1 in the table and to surface strategies for determining that value.

Facilitation: Circulate to observe student strategies and offer help or encouragement where needed.

Follow with a whole-class discussion around a question like: *What is the constant of proportionality for this relationship and where can you see it in the table?* Invite several students to share their strategies for determining this value and to critique each other's reasoning.

Spend adequate time here to ensure that all students have at least one strategy for determining the constant of proportionality from a table.

If it does not come up naturally, consider discussing whether or not students believe it is a coincidence that the constant of proportionality is $\frac{3}{4}$ and the number opposite 1 is $\frac{3}{4}$.

Early Finishers: Consider encouraging students who finish this screen early to determine the constant of proportionality using as many different strategies as they can find and explain their reasoning to a classmate.

Sample Responses

Cups of white paint: 6, $\frac{3}{4}$, $\frac{30}{4}$

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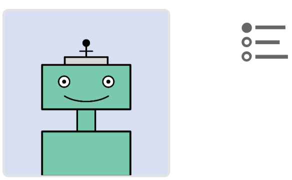
3 Robot Hat



Teacher Moves

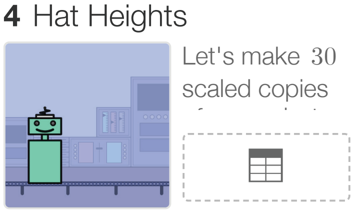
Activity 1 Purpose: On Screens 3–11, students use structure (MP7) and repeated reasoning (MP8) to develop and use equations to produce robots en masse to fit their design. This activity is intended to expose students to the power of using equations.

Facilitation: Consider sharing with students that they will be in charge of creating 30 scaled copies of their robot, and that their robots will be accessorized with hats, legs, and arms. Invite students to select a hat, then continue to the next screen.

Pacing: Consider using pacing to restrict students to Screens 3–11.

---

4 Hat Heights



Let's make 30 scaled copies of your robot.

The width of your robot is 3 inches. Its hat is 6 inches off the ground.

Complete the table to place hats on the first six robots.

Teacher Moves

Facilitation: Consider demonstrating that students can type expressions like $2+2$ into the table using the dashboard's student view.

Monitor for students who write expressions such as $2.18 \cdot 2$ in the table. Consider highlighting these tables using the snapshots tool or the dashboard's teacher view while students work on Screens 5–6.

Sample Responses

Heights for placing hat: 8, 2, 7, 4.36, 3.52, 6.85

Student Supports

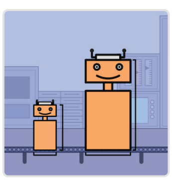
Students With Disabilities

• *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time. This may include reading the information in the table and in the graph.

---

5 Write Instructions



Some of your work on the

Some of your work on the first few robots is shown below.

Teacher Moves

On this screen, students gather their thoughts before they encounter equations for the relationship between the robot's width and the height for placing its hat on the next screen.

Sample Responses

Responses vary.

To know the height for placing the robot's hat, take its width and double it.

Student Supports

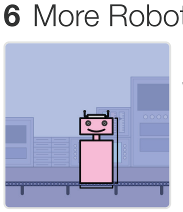
Multilingual Learners

• *MLR 3 (Critique, Correct, and Clarify)*

Instead of using these discussion questions, use a "Critique a Flawed or Partial Response" strategy by having students improve upon a statement, like "Multiply by 2."

---

6 More Robots, More H…



Instructions with words are

Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Choose an equation that tells the factory the relationship between a robot's width, w, and the height for placing its hat, h.

Teacher Moves

This is a possible discussion screen.

Facilitation: Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. When most students have completed this screen, facilitate a whole-class discussion to interpret the meaning of the correct equation.

Consider asking questions like:

- *How would you explain what the equation* $h = 2w$ *means in a sentence?*
- *Why might someone think the equation* $w = 2h$ *is correct? What would you say in response?*

If time allows, consider trying the equation $h = w + 3$ using the dashboard's student view in order to highlight that an equation should successfully predict the height of **all** robots. This equation successfully calculates the height for the first robot but not for later ones.

Sample Responses

$h = 2w$

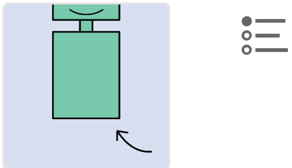
Student Supports

Students With Disabilities

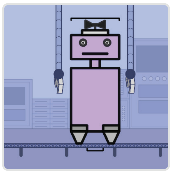
- *Conceptual Processing: Eliminate Barriers*

Assist students in recognizing the connections between new problems and prior work. Students may benefit from a review of different representations to activate prior knowledge.

7 Robot Legs



8 Leg Day



Your robot is 3 inches wide. Its

Your robot is 3 inches wide. Its legs are 1 inch apart.

For each robot, enter the distance between the legs.

Teacher Moves

Facilitation: Monitor for students who use the following strategies to determine the distance between the legs: 1) divide the width by 3 or 2) multiply the width by $\frac{1}{3}$.

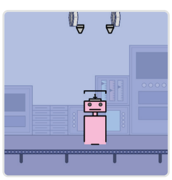
Consider capturing examples of each strategy using the snapshots tool or highlighting different ways students represented the distance using the dashboard's teacher view.

Sample Responses

Distance between legs: 2, $\frac{5}{3}$, $\frac{1}{3}$

---

9 More Robots, More L...



Write an equation so

$f(x)$

Write an equation so that the factory can finish adding legs to your robots.

Use d for the distance between the legs and w for the robot's width.

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

The purpose of this discussion is to surface strategies for writing an equation for a proportional relationship.

Facilitation: While students are working, select and sequence several different incorrect and correct equations using the snapshots tool. If possible, capture different forms of the correct equation (such as $d = \frac{1}{3}w$ and $d = \frac{w}{3}$). When most students have completed this screen, pause the class and facilitate a whole-class discussion.

Display several equations, and ask students to justify if the equation will build the robots correctly and to critique others' reasoning. Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit. If it does not come up naturally, consider asking students: *Where do we see the* $\frac{1}{3}$ *in the table?*

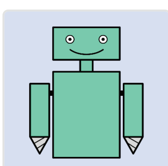
Note: Students who do not use the variables w and d in their equation or type $d=\frac{1}{3w}$ will see a ⚠ symbol. Consider demonstrating to students how to type a fraction multiplied by a variable (e.g., $\frac{2}{7}x$).

Sample Responses

$d=\frac{w}{3}$ or $d=\frac{1}{3}w$ (or equivalent)

---

10 Robot Arms



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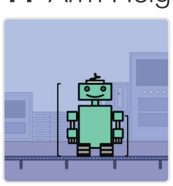
11 Arm Height



This robot has a height of 9

f(x)

This robot has a height of 9 inches. Its arms are 5 inches off the ground.

Write an equation so that the factory can proportionally attach arms to your robots.

Use a for the height to place the arms and r for the robot height.

Teacher Moves

This is a possible discussion screen.

Facilitation: Encourage students who finish this screen early to compare their equation to a classmate's and discuss any similarities and differences. They can also show off the robot they made. Once most students have attempted this screen, consider asking a couple of students to share strategies for writing this more complex equation given only one pair of values in a table.

This relationship is challenging and students may need support to write an equation to represent it. Students will have several more opportunities to practice over the subsequent lessons. Encourage students to submit their equation even if they are unsure; they can edit their equation if the robots are not assembling correctly.

Note: Students who do not use the variables a and r in their equation will see a ⚠ symbol.

Sample Responses

$a = \frac{5}{9}r$ (or equivalent)

Student Supports

Students With Disabilities

• *Conceptual Processing: Eliminate Barriers*
Assist students in recognizing the connections between new problems and prior work. Students may benefit from a review of different representations to activate prior knowledge.

---

12 Lesson Synthesis

Examine the information

The constant of proportionality is 2.

An equation for this relationship is $h = 2w$.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their ideas about how to write an equation from a table of a proportional relationship.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses or suggest revisions.

Routine (optional): Consider using the mathematical language routine Compare and Connect, displaying several students' responses and asking students: *What is the same and what is different?* Consider recording strategies and connections for students to revisit over the next several lessons.

Pacing: Consider using pacing to restrict students to this screen.

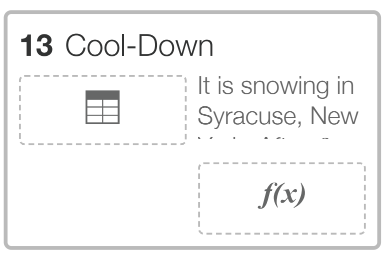
Sample Responses

Responses vary.

• First, find the constant of proportionality for the relationship. Then write an equation that looks like: (2nd variable) = (constant of proportionality) · (1st variable).
• Use the given values to determine the second value when the first value is 1. For instance, if you know the pair $(9, 5)$, then divide both values by 9 to get $\left(1, \frac{5}{9}\right)$. This number, $\frac{5}{9}$, is the constant of proportionality, which you can use to write the equation.

Student Supports

Multilingual Learners
• *MLR 8 (Discussion Supports)*
Revoice student explanations using appropriate mathematical language.



It is snowing in Syracuse, New York. After 2 hours, 1 inch of snow has fallen. The snow falls at the same rate.

Complete the table.

Then write an equation for the amount of snow, s, that has fallen after h hours.

Teacher Moves

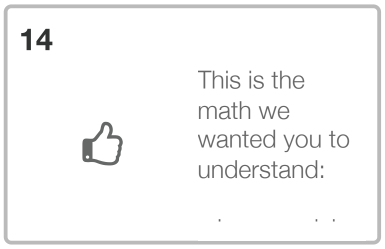
Support for Future Learning: Students will have more chances to develop their understanding of writing equations for proportional relationships in the upcoming lessons, particularly in Lesson 5.

Pacing: Consider using pacing to restrict students to Screens 13–14.

Sample Responses

Snow: 0.5, 3.5

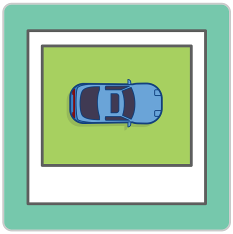
Equation: $s = 0.5h$ (or equivalent)



This is the math we wanted you to understand:

- I can explain where to find the constant of proportionality as a value in a table.
- I can write equations to represent proportional relationships.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Snapshots

Lesson 5: More Equations of Proportional Relationships

Overview

Students develop fluency writing and using equations to make sense of proportional relationships in a variety of contexts.

Learning Goals

- Connect each part of an equation of the form $y = kx$ to the situation it represents.
- Use an equation to solve problems involving a proportional relationship.

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop fluency writing and using equations to make sense of proportional relationships in a variety of contexts.

Lesson Summary

Warm-Up (5 minutes)

The purpose of this warm-up is to introduce students to the context of Activity 1 and surface that snapshots may not be enough to know a whole story. Students are given four images, and they will write a story based on those images.

Activity 1: Travel Times (10 minutes)

The purpose of this activity is for students to explore the power of an equation to predict any pair of values in a proportional relationship. Students will also develop a deeper understanding of the connection between an equation and a table of values that represent the same relationship.

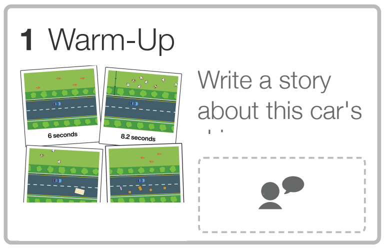
Activity 2: Cakes and Bears (20 minutes)

The purpose of this activity is for students to practice interpreting and using equations of proportional relationships in two new contexts. The activities introduce new contexts, and for the first time, do not provide tables; students who still need tables can create tables for themselves. As students use equations of the form $y = kx$ to reason about quantitative situations, they engage in MP2.

Lesson Synthesis (5 minutes)

The purpose of this synthesis is for students to explain what they know about interpreting equations by creating their own situation. Students are given an equation and are asked to write a situation and explain what each value or variable in the equation represents in their situation.

Cool-Down (5 minutes)



Write a story about this car's drive.

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to develop fluency writing and using equations to make sense of proportional relationships in a variety of contexts.

Warm-Up Purpose: Students are introduced to the context of Activity 1 and come to realize that a set of snapshots may not be enough to know a whole story.

Facilitation: Arrange students into pairs. Invite students to use their creativity to tell a story about this car's drive and to include information they see in the four snapshots. Consider highlighting unique or creative stories using the snapshots tool or the dashboard's teacher view.

Consider asking students: *Can we know the whole story from these snapshots? Why or why not?*

Routine: Consider using the routine Tell a Story to support students in using their knowledge and creativity to make sense of a situation.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

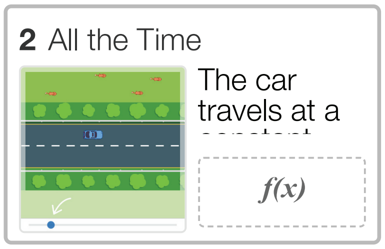
Responses vary. The car was driving along and minding its own business when it witnessed some foxes near a bunch of sheep only about 2 seconds away. The foxes were heading for the sheep. Oh no! Later in the drive, the car encountered a mattress that had fallen off of a purple car as it was driving. Pretty soon afterwards, it passed a bunch of boxes and a yoga mat that a person was throwing out of their roof as they drove.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., First, the car ______________. Then, the car ______).



The car travels at a constant speed. After 6 seconds, it travels 180 meters.

Write a proportional equation to find the car's distance, d, at any time, t.

Teacher Moves

Activity 1 Purpose: On Screens 2–3, students explore the power of an equation to predict any pair of values in a proportional relationship.

Facilitation: Consider starting with the activity paused and asking students what they remember from the previous lesson about writing equations for proportional relationships. Then unpause and invite students to write an equation to predict the distance the car traveled at any time. If students are having difficulty getting started, consider asking: *What is the constant of proportionality in the relationship?*

Once students have entered a correct equation and have access to the entire story, encourage them to find several interesting points and test the points in their equation.

If time allows, consider briefly highlighting common times that students selected, discussing why students selected them, and testing those points in the equation.

Pacing: Consider using pacing to restrict students to Screens 2–3.

Sample Responses

$d = 30t$

Responses vary.

All rows of the table will include times from 0 to 40 seconds and distances from 0 to $1\,200$ meters.

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

Check in with individual students, as needed, to assess for comprehension during each step of the activity.

3 Tables and Equations

Ethan says that a row in a table

An equation for this relationship is $d = 30t$.

Teacher Moves

This is a possible discussion screen.

Facilitation: If students are having difficulty getting started, consider asking: *How are a picture and a video similar and different?*

Once students have had an opportunity to respond to this screen, encourage students to discuss their response with a partner to find similarities and differences among their responses.

Facilitate a whole-class discussion to compare and contrast equations and tables using Ethan's analogy. If time allows, consider recording students' thinking on a chart to refer to throughout the lesson.

See the sample responses for ideas that may surface during the discussion.

Sample Responses

Responses vary.

- A row in a table is just one pair of values in a relationship and an equation can tell you about all possible pairs of values.
- A row in a table is a single moment. An equation can provide all of the moments.

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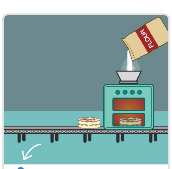
4 Cake Calculations



A cake recipe says to use 3

$f(x)$

A cake recipe says to use 3 cups of flour for every 2 cakes.

Write a proportional equation to calculate the amount of flour needed, f, for any number of cakes, c.

Explain what the constant of proportionality means in this situation.

Teacher Moves

Activity 2 Purpose: On Screens 4-8, students practice interpreting and using equations of proportional relationships in two new contexts.

Facilitation: Consider starting with the activity paused and asking students to share their experience baking cakes. Then invite students to determine an equation for the relationship between flour and number of cakes (MP2). If students are struggling, consider encouraging them to make a table of values to help find the constant of proportionality.

Once students have entered a correct equation and explained what the constant of proportionality means, consider inviting students to drag the point to any spot and test those values in the equation.

Pacing: Consider using pacing to restrict students to Screens 4–8.

Sample Responses

$f = \frac{3}{2}c$ (or equivalent)

Responses vary.

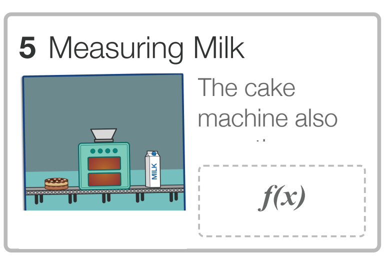
- The $\frac{3}{2}$ means that you need 3 cups of flour for every 2 cakes.
- The $\frac{3}{2}$ means that each cake uses 1.5 cups of flour for each cake.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.



The cake machine also uses the equation $m = 6c$, where c is the number of cakes and m is ounces of milk.

How many ounces of milk are needed to bake 12 cakes?

Teacher Moves

This is a possible discussion screen.

Facilitation: Once most students have completed this screen, consider facilitating a brief whole-class discussion to interpret the equation and surface strategies for solving the problem.

Consider asking questions like:

- *What does the number 6 mean in this equation?*
- *How could you use the equation to figure out how many ounces of milk are needed?*

Routine (optional): Consider using the routine Clarify, Critique, Correct to help students communicate about errors and ambiguities in math ideas and language. If possible, use a student response in anonymize mode. If not, consider using the response "You need 2 ounces of milk because $6 \cdot 2 = 12$."

Sample Responses

72 ounces

Explanations vary. I substituted 12 for c in the equation $m = 6c$. From there, I calculated $6 \cdot 12 = 72$, so $m = 72$.

---

6 Equations and Centi…

Teacher Moves

Encourage students to share their reasoning with a partner and work to reach an agreement together about how to sort the cards.

If time allows, consider asking pairs to compare their card sorts, justify their card placements, and make revisions based on their conversation.

While students are working, monitor for students who determine and interpret the constant of proportionality as the length of one gummy bear.

Sample Responses

Image solution

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity.

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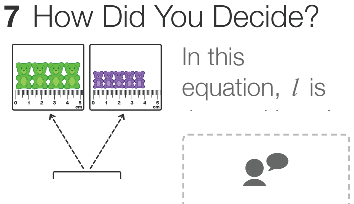
7 How Did You Decide?



In this equation, l is

In this equation, l is the total length and g is the number of gummy bears.

How did you decide which image to group with this card?

Describe your strategy.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to connect each part of an equation of a proportional relationship to the context.

Facilitation: While students are working, consider selecting and sequencing responses that highlight different ways of thinking about the equation using the snapshots tool.

When most students have had a chance to respond to this screen, consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Consider asking students why someone might have chosen each of the two equations. Facilitate a whole-class conversation using these questions to guide the discussion:

- *What does the* $\frac{5}{4}$ *mean in this situation?*
- *How did you decide which image matches this equation?*

See the sample responses for ideas that may surface during the discussion.

Consider celebrating the act of revising one's thinking by asking if any students have changed their minds about the placement of cards on the card sort. Give students a moment to return to the previous screen and adjust their responses as time allows. Students will have more opportunities to practice equations over the subsequent lessons.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Responses vary.

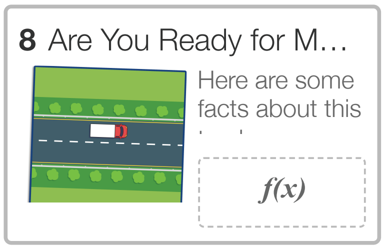
- I grouped it with the large gummy bears because each of those is longer than 1 cm wide, and $\frac{5}{4}$ is the width of one gummy bear.
- My strategy was to try the numbers in the equation. 4 does not equal $\frac{5}{4} \cdot 5$, so the right-hand image was not correct.
- The constant of proportionality is $\frac{5}{4}$, which means the length is 5 centimeters for every 4 gummy bears.

Student Supports

Multilingual Learners

- *MLR 8 (Discussion Supports)*

Revoice student explanations using appropriate mathematical language.



Here are some facts about this truck:

- It travels at an average rate of 50 miles per hour.
- It can travel 6 miles for each gallon of gas.

How many hours can the truck travel without stopping if it has a full tank of 150 gallons?

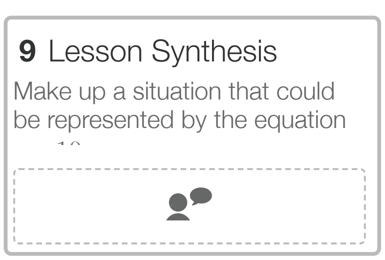
Teacher Moves

This screen is designed as an extra challenge for students who finish Screens 4–7 before the class discussion on Screen 7. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

18 hours

Explanations vary. Since the truck can travel 6 miles for each gallon and can hold 150 gallons of gas, it can travel $150 \cdot 6 = 900$ miles on each tank of gas. Traveling 50 miles per hour, a truck could travel for $\frac{900}{50} = 18$ hours.



Make up a situation that could be represented by the equation $r = 10p$.

Be sure to explain what r, 10, and p represent in your situation.

Teacher Moves

This is a key discussion screen.

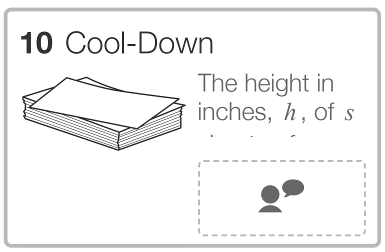
Lesson Synthesis Purpose: Students explain what they know about interpreting equations by creating their own situation given an equation.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between students' equations or suggest revisions.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. The amount of rice I need, r, is 10 times the amount of peas I need, p, for a giant pot of peas and rice.



The height in inches, h, of s sheets of paper can be described by the equation $h = 0.004s$.

What does the 0.004 mean in this situation?

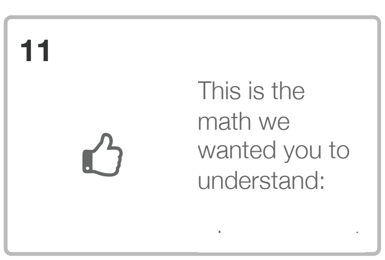
Teacher Moves

Support for Future Learning: If students struggle to interpret the meaning of the constant of proportionality, consider spending extra time during the next lesson's warm-up discussing the meaning of the constant of proportionality, or reviewing this cool-down as a class.

Pacing: Consider using pacing to restrict students to Screens 10–11.

Sample Responses

Responses vary. The 0.004 means that each sheet of paper is 0.004 inches tall.



This is the math we wanted you to understand:

- I can connect each part of an equation of the form $y = kx$ to the situation it represents.
- I can use an equation to solve problems involving a proportional relationship.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Two and Two (NYC)

Lesson 6: Two Equations for Each Relationship

Purpose

The purpose of this lesson is for students to determine two constants of proportionality for the same proportional relationship and explain how those constants are related. In particular, students use the word *reciprocal* to describe the relationship between the two constants.

Preparation

Worksheet

- *Activity 1*: Print one double-sided sheet for each pair of students.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Materials

- Scissors for each pair of students (optional)

Warm-Up (10 minutes)

Purpose: Students connect a proportional relationship in a table and an equation, and see that there is more than one way to write the equation for this proportional relationship.

Facilitation: This warm-up has two parts. For part one, display Sheet 1 of the Teacher Projection Sheets. Give students 2 minutes of quiet think-time to determine their choices. Then, display Sheet 2 of the Teacher Projection Sheets. Give students two minutes of quiet think-time to gather their ideas about the dispute and a few minutes to discuss with a partner.

While students are working, monitor for strategies students may have used to match the equations to the tables. Students may have substituted values from the table into an equation or multiplied the values in the first column by a constant to get corresponding values in the second column.

Facilitate a whole-class conversation to elicit strategies for matching an equation to a table, and for students to see that it's possible to have more than one equation for a table of values. Consider asking open-ended questions about Carlos's and Alexis's reasoning like: *Can anyone share why they think Carlos is correct?* If it does not come up naturally, consider asking: *Is it possible to have more than one equation for a table of values?*

Connection to Readiness Check: If Problem 2 suggests that some students have unfinished learning with calculating unit rates, consider revisiting techniques for finding unit rates here.

Early Student Thinking: Some students may not realize initially that both Alexis and Carlos are correct. Consider holding off on revealing the final answer until various ways of thinking about the problem have been shared with the class.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Activity 1: Jayden's Cooler (25 minutes)

Purpose: Students practice finding constants of proportionality and equations for proportional relationships.

Facilitation: Arrange students into pairs. Distribute one double-sided Student Worksheet to each pair of students and ask them to cut the sheet so that each partner gets a half-sheet (or cut the half sheets in advance). Share with students that each partner has a different problem to work on for the front side of the sheet and invite them to complete the front side of the sheet individually.

Once partners have finished Problems 1 and 2, ask them to swap papers and check each other's responses. When partners feel confident in their responses, invite them to discuss the similarities and differences that they notice between their problem sets. Once most students have discussed similarities and differences, invite everyone to complete Problems 3–5 on their own and then to discuss with a partner.

Facilitate a whole-class discussion to introduce the word *reciprocal* and to help students see why there are two constants of proportionality and two equations for each proportional relationship.

Consider asking any of these questions as they make sense:
- *How did you find an equation for each table?*
- *Where do you see the constant of proportionality in the table and in the equation?*

Invite several students to share their response to the question: *What is the relationship between the two constants of proportionality?* If it doesn't come up naturally, share with students that we call these numbers reciprocals. Consider recording a definition and examples from this lesson for students to refer to during the remainder of the activity and unit.

Connections to Grade 6

These equations can help students see why the constants of proportionality are reciprocals:

$$w = \frac{8}{5}t$$
$$\left(\frac{5}{8}\right)w = \left(\frac{5}{8}\right)\cdot\frac{8}{5}t$$
$$\left(\frac{5}{8}\right)w = t$$
$$t = \left(\frac{5}{8}\right)w$$

This argument builds on work in Grade 6, where students worked with expressions and equations, so this may be familiar to students.

Lesson Synthesis (5 minutes)

Facilitation: Distribute one double-sided half sheet of the lesson synthesis and cool-down to each student.

Give students 2–3 minutes to respond to this question, followed by a few minutes to share their responses with their partner. Follow with a whole-class discussion about the relationship between the two constants of proportionality. Consider supporting students in using the word *reciprocal* in their responses.

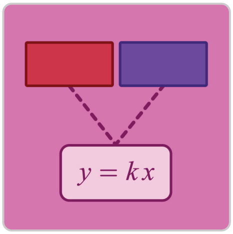
Connection to Readiness Check: If Problem 3 suggests that some students have unfinished learning with interpreting unit rates, consider pausing to discuss the meaning of the two constants of proportionality in each relationship.

After the discussion, invite students to complete the cool-down individually on their worksheet.

Cool-Down (5 minutes)

If students struggle to identify both constants of proportionality, consider making time to explicitly revisit these ideas, such as asking students to write two equations for any relationship they think is proportional during Activity 1 of Lesson 7.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



All Kinds of Equations

Lesson 7: Equations of Proportional Relationships

Overview

Students develop strategies for deciding whether a relationship is proportional or not based on the structure of its equation (MP7).

Note: This lesson includes a paper supplement.

Learning Goals

- Justify whether a relationship is proportional or not by looking at the structure of an equation.

Materials

- Blank paper

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop strategies for deciding whether a relationship is proportional or not based on the structure of its equation (MP7).

Lesson Plan

Warm-Up (5 minutes)

The purpose of the warm-up is for students to surface strategies for making a table from an equation in order to decide whether a relationship is proportional or not.

Activity 1: Stories, Equations, Tables (15 minutes)

The purpose of this activity is for students to develop an intuition for types of equations that do and do not represent proportional relationships by diving deep into a couple of examples.

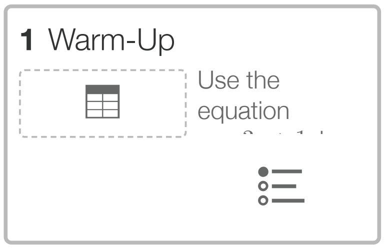
Activity 2: Equations and Proportionality (15 minutes)

The purpose of this activity is for students to generalize the forms of equations that do and do not represent proportional relationships. By the end of this activity, students should be able to describe that only equations that can be written in the form $y =$ something $\cdot x$ represent proportional relationships.

Lesson Synthesis (5 minutes)

The purpose of this synthesis is for students to consolidate and refine their ideas about types of equations that represent proportional relationships and types that do not.

Cool-Down (5 minutes)



Use the equation $y = 3x + 1$ to complete the table.

Then decide if the equation represents a proportional relationship.

Teacher Moves

This is a possible discussion screen.

Lesson Overview: The purpose of this lesson is for students to develop strategies for deciding whether a relationship is proportional or not based on the structure of its equation.

Warm-Up Purpose: Students surface strategies for making a table from an equation in order to decide if a relationship is proportional or not.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and asking students what they remember about how to decide from a table whether a relationship is proportional. Then unpause and give students two minutes to work quietly. Consider following with a discussion by inviting students to share their strategies for making a table from the equation.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

x: 0, 1, 3, 5, 2.5
y: 1, 4, 10, 16, 8.5

Not proportional

Responses vary.

- The constant of proportionality is not the same for each row.
- In order for y to equal 4, you need to multiply 1 by 4. In order for y to equal 10, you need to multiply 3 by $\frac{10}{3}$.

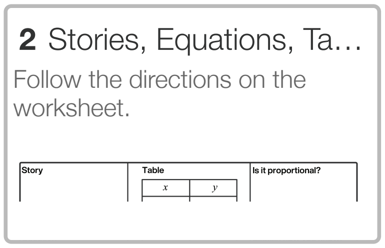
Student Supports

Support for Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Direct student attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.

Multilingual Learners
- *MLR 3 (Critique, Correct, and Clarify)*

Instead of using these discussion questions, use a "Critique a Flawed or Partial Response" strategy by having students improve upon a statement, like "It's proportional because each x-value is multiplied by three".



Follow the directions on the worksheet.

Then continue to the next screen.

Teacher Moves

Activity 1 Purpose: On Screens 2–3, students develop an intuition for types of equations that do and do not represent proportional relationships by diving deep into a couple of examples.

Facilitation: Distribute one double-sided paper supplement to each pair of students; ask them to cut the sheet in half so that each partner has two scenarios. Invite students to complete each set of representations and share their thinking with their partner, then move on to Screen 3. Consider waiting until most students have completed Screen 3 to discuss strategies as a class.

Monitor for students who use the following strategies to decide if the relationship is proportional:
- Look for equivalent ratios using the table.
- Consider the structure of the equation.
- Make sense of the situation using the description of the context.

Note: Students must press a button that says, “I’m ready!” to see the details of this screen.

Materials: Pairs will need a double-sided copy of the paper supplement (either cut in half in advance or provide scissors for students to cut in half).

Pacing: Consider using pacing to restrict students to Screens 2–3.

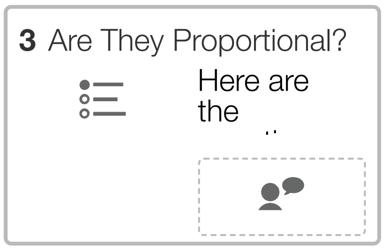
Sample Responses

See the paper supplement answer key.

Student Supports

Support for Multilingual Learners
- *Reading: MLR 2 Collect and Display*

Listen for and collect vocabulary and phrases students use to describe what makes each table unique, such as the *constant of proportionality* and *structure of the equations*. Collect and display the language for the whole class to use as a reference, and remind students to borrow language from the display as needed. This will help students notice the patterns and generalize the form that an equation should have in order to represent a proportional relationship.



Here are the equations from the worksheet.

Select **all** the equations that represent a proportional relationship.

Then explain one way to decide if an equation represents a proportional relationship.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface some types of equations that do and do not represent proportional relationships.

Facilitation: Encourage students to work together to reach an agreement before responding. Use the snapshot tool to capture several different explanations including some imprecise or unfinished explanations.

Then facilitate a whole-class discussion to develop and display an initial list of types of equations that represent a proportional relationship and types that do not. Ask students to justify their hypotheses and critique each other's reasoning.

Consider questions like:
- *Do you think all equations that look like this represent proportional relationships? Why or why not?*
- *Where can you see the constant of proportionality in the equations you chose?*

If it does not come up naturally, consider highlighting that both equations for proportional relationships are of the form $y = kx$.

Early Finishers: Consider inviting students who finish Screens 2–3 early to create their own equations and ask their partner whether or not it represents a proportional relationship.

Sample Responses

$y = 12x$

$$y = \frac{1}{2}x$$

Explanations vary.

One way to decide if a relationship is proportional is to look at the equation and see if it has the form $y =$ (something) x.

Student Supports

Support for Multilingual Learners
Speaking: MLR 8 Discussion Supports
• Amplify mathematical uses of language to communicate about the similarities and differences between equations that represent proportional and non-proportional relationships. Use sentence frames to aid student responses to the final question, such as “I noticed that all the equations of the proportional relationships have _____ in common” or “Equations of proportional relationships always _____.”

---

4 Equations and Propo…

Teacher Moves

Activity 2 Purpose: On Screens 4–6, students generalize the forms of equations that do and do not represent proportional relationships. By the end of this activity, students should be able to describe that only equations that can be written in the form $y =$ something $\cdot x$ represent proportional relationships.

Facilitation: Give students 5–7 minutes of time to work. Encourage students to use scratch paper to make tables of x- and y-values for any equation they are not sure how to sort. While students are working, consider anonymizing the class and displaying the dashboard's teacher view so that groups can see when they have correctly completed the sort. Invite students to continue to the next screen when they feel confident in their responses.

Early Student Thinking: Students may think that $0.04x = y$ and $y = \frac{x}{4}$ are not equations of proportional relationships because they aren’t in the form $y = kx$. Consider addressing this idea during the whole-class discussion on Screen 5.

Materials: Students may need scratch paper to create tables or test values as they sort each equation.

Pacing: Consider using pacing to restrict students to Screens 4–6.

Sample Responses

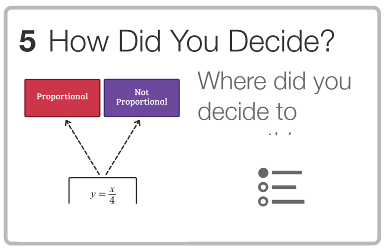
Image solution

Student Supports

Support for Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this task into more manageable parts (e.g., sorting one card at a time) to aid students who benefit from support with organizational skills in problem-solving.



Where did you decide to group this card?

Teacher Moves

This is a possible discussion screen.

Facilitation: While students are working, select and sequence several student responses using the snapshots tool. Consider monitoring for students who express the equation as $y = 0.25x$ or $y = \frac{1}{4}x$, or students who discuss the connection between multiplying by $\frac{1}{4}$ and dividing by 4, its reciprocal.

When most students have had a chance to respond to this screen, consider facilitating a whole-class discussion to add to the initial list from earlier in the lesson. Connect the equation $y = \frac{x}{4}$ to the other equations that represent proportional relationships by asking: *Is it possible to write this equation in the same form as the other proportional equations? If so, how? If not, why not?*

It may be useful to reference student responses from the card sort, particularly for the equation $y = \frac{4}{x}$.

Consider celebrating the act of revising one's thinking by asking if any students have changed their minds about the placement of cards on the card sort. Give students a moment to return to the previous screen and adjust their responses as time allows.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

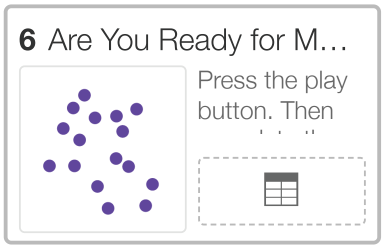
Sample Responses

Proportional

Explanations vary. The constant of proportionality is $\frac{1}{4}$ because dividing by 4 is another way of saying multiplying by $\frac{1}{4}$.

Student Supports

Support for Multilingual Learners
Speaking: MLR 8 Discussion Supports
- Amplify mathematical uses of language to communicate about the similarities and differences between equations that represent proportional and non-proportional relationships. Use sentence frames to aid student responses to the final question, such as “I noticed that all the equations of the proportional relationships have _____ in common” or “Equations of proportional relationships always _____.”



Press the play button. Then complete the table.

Teacher Moves

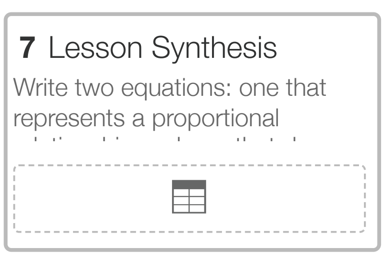
This screen is designed as an extra challenge for students who finish Screens 4–5 before the class discussion on Screen 5. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

Stage 1: 2 circles
Stage 2: 4 circles
Stage 3: 8 circles
Stage 4: 16 circles

Not proportional. *Explanations vary.*

- The constant of proportionality is not the same in every row.
- There is no number that I can multiply each stage number by to always get the correct number of circles.



Write two equations: one that represents a proportional relationship and one that does not.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their ideas about types of equations that represent proportional relationships and types that do not.

Facilitation: Give students 2–3 minutes to write equations and respond to the prompt and another minute to share their equations and explanation with a partner.

Select and sequence several student equations and explanations to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion to share students' thinking.

Routine (optional): Consider using the mathematical language routine Collect and Display, either creating a new display or updating a display.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

Proportional relationship:

- $d = 58t$
- $a = 0.12B$
- $W = \frac{n}{2}$

Not a proportional relationship:

- $d = t + 58$
- $a = 0.12B - 5$
- $W = \frac{2}{n}$

Explanations vary. Equations of proportional relationships can always be made to have the form $y =$ something $\cdot x$, and the "something" is the constant of proportionality.

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8 Cool-Down

Select **all** of the proportional relationships.

Select **all** of the proportional relationships.

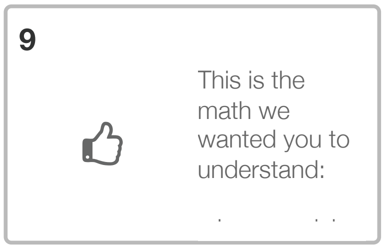
Teacher Moves

Support for Future Learning: If students struggle to select all the proportional relationships, consider reviewing this cool-down as a class before the practice day, or offering individual support where needed during the practice day.

Pacing: Consider using pacing to restrict students to Screens 8–9.

Sample Responses

- $1.08x = y$
- $y = 8x$
- $y = \frac{x}{8}$



This is the math we wanted you to understand:

- I can explain why a relationship is proportional or not by looking at the equation.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

7.2 Practice Day 1 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided copy for each student.

Cards

- Print one set of cards for the whole class (option 1), one set of cards for each group of students (option 2), or two sets of cards for the entire class (option 3).

Instructions

Option 1: Stations

This structure allows students to collaborate and helps them manage their time by breaking the task into smaller, more manageable chunks. It also can increase engagement by incorporating opportunities for students to move around the classroom.

Print and cut out one set of cards. Place several cards at each station and arrange students into groups of 3–4. Give each student the Student Workspace Sheet to complete as they solve the cards at their station. Instruct students to move from station to station after a set amount of time.

Option 2: Group Questions

This structure supports student collaboration and focuses students' attention on one problem at a time. Arrange students into groups of 2–3. Print and cut out one set of cards for each group. Give each student the Student Workspace Sheet to complete as they work together to solve each of the task cards. Consider posting the answer key, or walking around with it and providing feedback to students as they work.

Option 3: Solve and Swap

This structure supports student collaboration with many different partners and allows for movement around the classroom throughout the activity. Students are accountable to discuss their problem and to ask for support from their partner in non-threatening one-on-one interactions.

Print and cut out two sets of cards for the entire class. Be sure you have one card per student. Students circulate the class with their card and then pair up with a classmate. Each student in the pair solves the problem on their partner's card, collaborating as needed, and records their thinking on their workspace sheets. If a pair of students winds up with a problem they've already solved, they should compare strategies and solutions with their partner.

When both students have completed their problems, they swap cards, then stand up with a hand up, and find another classmate to pair up with. Repeat the process.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

DinoPops

Lesson 8: Introducing Graphs of Proportional Relationships

Overview

Students explore what a proportional relationship looks like graphically.

Learning Goals

- Explain that a proportional relationship can be represented by points on a line that includes the origin.
- Justify whether a graph represents a proportional relationship or not.

Vocabulary

- coordinate plane
- origin

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to explore what a proportional relationship looks like graphically. This is the first time in Grade 7 that students encounter a graph. Students will reason abstractly and quantitatively (MP2), connecting lollipop boxes to points in the coordinate plane.

Lesson Summary

Warm-Up (5 minutes)

The purpose of this warm-up is to introduce the context of a lollipop factory and establish that the relationship between box width and box height for these lollipops is proportional.

Activity 1: DinoPops (15 minutes)

The purpose of this activity is for students to recognize that a proportional relationship on a graph is represented by points on a line that includes the origin. Students investigate the relationship between box height and width for two different types of lollipops and use a graph to make predictions about other boxes that would fit each lollipop type. They use these properties to spot points that are not in the relationship. This may be the first time that students have used the phrase *the origin* to refer to the point $(0, 0)$.

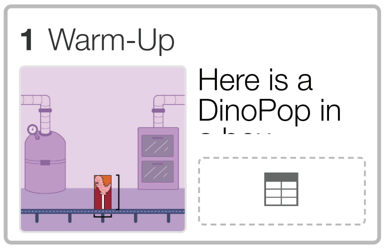
Activity 2: Proportional or Not? (15 minutes)

The purpose of this activity is for students to justify whether a graph represents a proportional relationship or not. Students analyze a graph of a relationship where the points form a line, but that line does not pass through the origin, and discuss whether or not it represents a proportional relationship. They then apply this knowledge in a card sort.

Lesson Synthesis (5 minutes)

The purpose of this synthesis is for students to demonstrate their understanding of what the graph of a proportional relationship looks like.

Cool-Down (5 minutes)



Here is a DinoPop in a box.

DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.

Complete the table to make boxes that fit a DinoPop.

Teacher Moves

Lesson Overview: The purpose of this lesson is to explore what a proportional relationship looks like graphically. Note: This is the first time in Grade 7 that students encounter a graph.

Warm-Up Purpose: Students familiarize themselves with the context of a lollipop factory and establish that the relationship between box width and box height for these lollipops is proportional.

Facilitation: Arrange students into pairs. Consider starting with the activity paused, sharing the context of the lesson with students, and asking them to guess the height of the record-holding world's tallest lollipop. [See's Candies made a lollipop that was approximately 199 inches tall in 2012.] Then, unpause and invite students to determine measurements for other boxes that would fit a DinoPop.

Once most students have completed this screen, consider asking: *Is the relationship between box width and height proportional? Why or why not?*

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

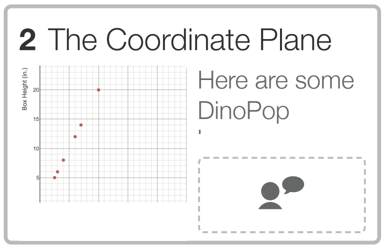
Responses vary.

Some possible response are: $(1, 2)$, $(99, 198)$, $(55.5, 111)$.

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Begin with a demonstration of the first problem to provide access to students who benefit from clear and explicit instructions. Check in with individual students, as needed, to assess for comprehension during each step of the activity.



Here are some DinoPop boxes.

Press the "Play" button and watch the animation.

What do you notice? What do you wonder?

Teacher Moves

This is a possible discussion screen.

Activity 1 Purpose: On Screens 2–6, students come to recognize that a proportional relationship on a graph is represented by points on a line that includes the origin. They use these properties to predict new points in the relationship and spot points not in the relationship.

Facilitation: Give students two or three minutes to play the animation, select different choices, and record what they notice and wonder. Consider encouraging students to slide the animation backwards and forwards at their own pace. Then invite them to discuss their noticings and wonderings with a partner.

Consider asking students if they remember the phrase *coordinate plane*, and if so, what it means.

See the sample responses for ideas and questions that may surface during the discussion. Consider naming noticings you hear after the students who share them and using those names throughout the rest of the lesson and unit.

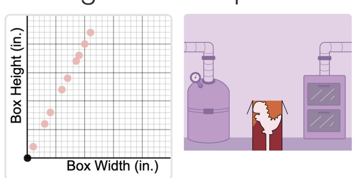
Routine: Consider using the routine Notice and Wonder to support students in making sense of the task.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

- I notice that each point on the graph corresponds to one lollipop box.
- I notice that the axes on the graph are the same as the headings in the table from the warm-up.
- I notice that you can draw a straight line through all of the points.
- I notice that the imaginary line passes through the lower-left corner of the graph, $(0, 0)$.
- I wonder if all of the DinoPop boxes will be on this imaginary line.
- I wonder if other types of boxes make points that line up nicely.
- I wonder if other types of boxes form a line through $(0, 0)$.

3 Drag the black point t…



Teacher Moves

Facilitation: Consider starting with the screen paused, using the dashboard's student view to drag the movable point to a random location, and asking students to predict what they think will happen when you press "Try It."

Then, unpause and give students several minutes to find points that fit and do not fit a DinoPop. This is an opportunity for students to look for and make use of structure (MP7).

Connection to Readiness Check: If Problem 6 suggests that some students have unfinished learning with plotting or interpreting coordinates, consider pausing to discuss the meaning of the coordinates of the movable point.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

Image solution

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

Begin with a demonstration of the first problem to provide access to students who benefit from clear and explicit instructions. Check in with individual students, as needed, to assess for comprehension during each step of the activity.

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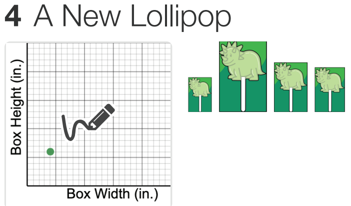
4 A New Lollipop



Here are several TriceraPops in boxes.

The graphed point represents ONE of these boxes. Tell a classmate what you know about this box.

Then add points to the graph to represent some other TriceraPop boxes.

Teacher Moves

This is a possible discussion screen.

Facilitation: Consider reading the prompt aloud as a class. Then, give pairs a minute to discuss what they know about the box of the

TriceraPop. If students are having difficulty getting started, consider asking them what they think each of the numbers in the point $(4, 6)$ means. Then, invite students to add points to the graph that they believe represent more TriceraPop boxes.

If time allows, consider displaying the overlay on the dashboard's teacher view and asking several students to share strategies they used to predict other points that would represent TriceraPop boxes. Students will have an opportunity to verify their predictions on the next screen.

Consider asking students to predict whether or not they think the point $(0, 0)$, also called the origin, would be marked correct or incorrect and to justify their reasoning. This may be the first time that students have heard the phrase *the origin* to refer to this point.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

I know that this box has a width of 4 inches and a height of 6 inches.

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the representations for students to draw on or highlight.

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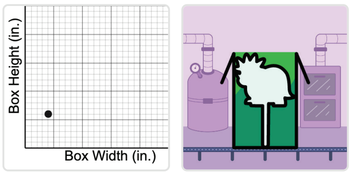
5 TriceraPops



Teacher Moves

Facilitation: Give students a few minutes to find boxes that fit a TriceraPop using strategies they heard or developed. Then, invite students to continue to the next screen when they have determined a few other correct box sizes. Consider displaying the overlay on the dashboard's teacher view to see all students' points at once.

Pacing: Consider adjusting the pacing to restrict students to Screens 5–6.

Sample Responses

Responses vary.

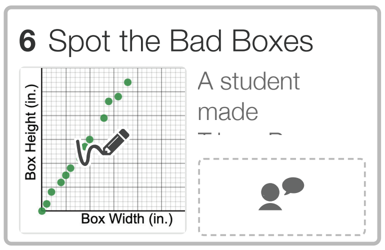
Image solution

Student Supports

Students With Disabilities

• *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors and allow students who struggle with fine motor skills to dictate physical manipulation of the coordinate point as needed.



A student made TriceraPop boxes on the previous screen.

Some of the boxes were not a good fit.

Describe how to use the graph to find the bad boxes. Use the sketch tool if it helps you to show your strategy.

Teacher Moves

Facilitation: While students are working, monitor for those who use appropriate tools strategically by making a line through the origin to help determine which boxes are bad.

If time allows, consider displaying some sketches or explanations to emphasize the power of the graph as a representation in this scenario. Consider revisiting the idea from Screen 4 that a proportional relationship is represented by a line through the origin.

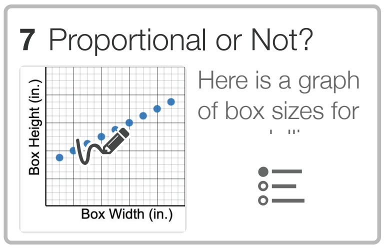
If it has not come up already, consider intentionally using the term *origin* to describe the point $(0, 0)$.

Early Finishers: Encourage students who finish early to return to Screen 5 and create other bad boxes that might be tough to spot.

Sample Responses

Responses vary.

• Sketch a line that goes through the origin and most of the other points. The few points that do not fall on this line are the bad boxes.
• $(0, 0)$ might be a bad box (in addition to boxes that are not on the line). Even though it fits the pattern by being on the line, a box that is 0 inches wide and 0 inches high is not a real box.



Here is a graph of box sizes for a new lollipop.

Is there a proportional relationship between the height and the width of these boxes?

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to build an understanding that forming a line is not enough to determine that a relationship is proportional. It must also pass through the origin.

Activity 2 Purpose: On Screens 7–9, students justify whether a graph represents a proportional relationship or not.

Facilitation: Give students one minute to decide whether the relationship is proportional or not, and explain their thinking. Then give them another minute to share their reasoning with a partner. Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. While students are working, select and sequence sketches and explanations that highlight both correct and incorrect solutions.

Then facilitate a whole-class discussion. Consider asking students to make an argument for a choice they did not select. At the end of the discussion, consider asking students to name particular students whose thinking influenced their own. Spend adequate time here making sure students understand that this relationship is not proportional, and have several strategies to explain why.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language. If possible, use a student response in anonymize mode. If not, consider using the response "It is proportional because the dots are all in a line."

Pacing: Consider using pacing to restrict students to Screens 7–9.

Sample Responses

No

Responses vary.

- I see that this relationship includes a box that is 6 inches by 6 inches and also a box that is 2 inches by 4 inches. Those are not equivalent ratios, so the relationship cannot be proportional.

- Some boxes are short and wide, and others are tall and narrow. Therefore, the width and height are not proportional to one another.
- The points do lie on a single line, but that line does not include the origin, so the relationship is not proportional.

Student Supports

Multilingual Learners

- *MLR 3 (Critique, Correct, and Clarify)*

Instead of using these discussion questions, use a "Critique a Flawed or Partial Response" strategy by having students improve upon a statement, like "It is not proportional because it is not low enough in the graph."

---

8 Card Sort

Teacher Moves

Facilitation: While students are working, consider anonymizing the class and displaying the dashboard's teacher view so that groups can see when they have correctly completed the sort.

Monitor for students who use the strategies named during earlier discussions.

Sample Responses

Image solution

Student Supports

Support for Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this task into more manageable parts (e.g., sorting one card at a time) to aid students who benefit from support with organizational skills in problem-solving.

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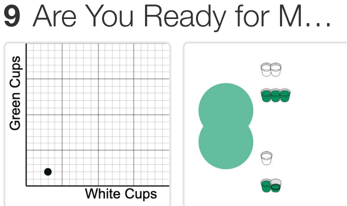
9 Are You Ready for M…



Teacher Moves

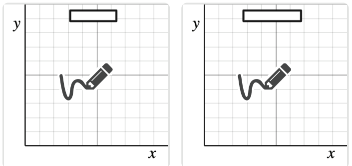
This screen is designed as an extra challenge for students who finish Screens 7–8 before the class discussion on Screen 10. Consider inviting these students to share their graphs with each other in place of a whole-class discussion.

Sample Responses

Responses vary.

Image solution

10 Lesson Synthesis



Teacher Moves

🔑 *This is a key discussion screen.* 🔑

Lesson Synthesis Purpose: Students demonstrate their understanding of what the graph of a proportional relationship looks like.

Facilitation: Give students 2–3 minutes to draw and a minute to discuss the prompt with a partner. Select and sequence several student responses to display using the dashboard's teacher view or the snapshot tool. Follow with a brief whole-class discussion to celebrate creative or interesting examples of graphs, or proportional relationships and not proportional relationships. Consider asking students whether they think each selected sketch represents a proportional relationship or not.

Consider closing by revisiting how to identify a proportional relationship from its graph (namely that all points lie on a line that goes through the origin).

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses and explanations vary.

- A relationship is proportional if all its points lie on a single line that goes through $(0, 0)$.
- A relationship is not proportional if all its points lie on a single line that does not go through the origin.
- A relationship is not proportional if it has points that do not lie on a single line.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Gallon Challenge

Lesson 9: Interpreting Graphs of Proportional Relationships

Overview

Students develop fluency working with graphs of a proportional relationship and identify the constant of proportionality using a graph.

Learning Goals

- Interpret points on the graph of a proportional relationship, including $(1, k)$ where k is the constant of proportionality.
- Identify the constant of proportionality from the graph of a proportional relationship.

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop fluency working with graphs of a proportional relationship and identify the constant of proportionality using a graph. Students analyze graphs of different vehicles' gas mileages and reason about how the concept of gas mileage is connected to the constant of proportionality.

Lesson Summary

Warm-Up (10 minutes)

The purpose of this warm-up is to introduce the context and establish that a graph can be thought of as a collection of all the possible points that make a relationship true. Students take several imaginary drives using different amounts of gas and imagine what the graph of all possible drives would look like.

Activity 1: Gas Mileage (10 minutes)

The purpose of this activity is for students to develop strategies for determining the gas mileage of a vehicle using its graph. Students may use two strategies: 1) dividing the maximum distance by the number of gallons or 2) looking on the graph for the maximum distance when the number of gallons is 1.

Activity 2: Constant of Proportionality (15 minutes)

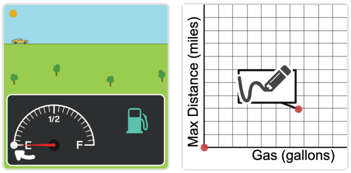
The purpose of this activity is for students to use the strategies from Activity 1 to determine the constant of proportionality from a graph. Students compare different vehicles to determine which has the best gas mileage and then calculate the constant of proportionality for several vehicles.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their strategies for using a graph to determine the constant of proportionality.

Cool-Down (5 minutes)

1 Warm-Up



Teacher Moves

Lesson Overview: The purpose of this lesson is for students to develop fluency working with graphs of proportional relationships and identify the constant of proportionality using a graph.

Warm-Up Purpose: Screens 1–2 introduce students to the context and establish that a graph can be thought of as a collection of all the possible points that make a relationship true.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and asking students what they know about gas in cars (how it is measured, gas mileage, buying gas, running out of gas, etc.). Use the dashboard's student view to demonstrate how to drag the movable point and observe the result. Then unpause and invite students to take several different drives to collect at least three different points.

Once most students have collected several points, give students two minutes to discuss what they notice and wonder with a partner. Consider facilitating a brief whole-class discussion for students to share their thoughts and make connections with others' ideas (Notice and Wonder).

See the sample responses for ideas and questions that may surface during the discussion.

Pacing: Consider using pacing to restrict students to Screens 1–2.

Sample Responses

Responses vary.

- I notice that the points all line up.
- I notice that the number of miles is always 20 times the gallons.
- I notice that you can drive further when there is more gas in the tank to start.
- I wonder why you can't add more than half a tank of gas.
- I wonder what is the furthest you could drive on one tank of gas.

Student Supports

Students With Disabilities

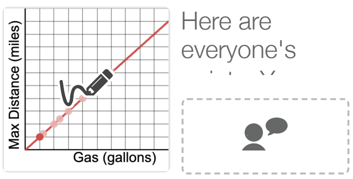
- *Executive Functioning: Graphic Organizers*

Provide students a T-chart to record what they notice and wonder before they are expected to share their ideas with others.

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time. This may include reading the information in the table and in the graph.

2 Warm-Up



Here are everyone's points. Yours are dark red.

What would the graph look like if it included every possible point for this car?

Teacher Moves

This is a possible discussion screen.

Facilitation: Give students a minute to respond and another minute to read others' responses.

While students are working, select and sequence several student responses using the snapshots tool. Monitor for students who connect the idea of points to lines or students who explain their reasoning in interesting ways. Then facilitate a brief discussion to help students see and describe why a line can be thought of as a collection of all the possible points.

This may also be an opportunity to highlight the power of graphs to represent all values in a relationship at once.

Sample Responses

Responses vary. It would look like a line through the origin.

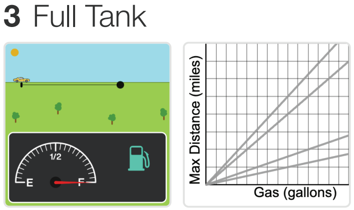
Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., If every point was included, the graph would look like ________.).

3 Full Tank



Teacher Moves

Activity 1 Purpose: On Screens 3–4, students develop strategies for determining the gas mileage of a vehicle using its graph.

Facilitation: Consider starting with this activity paused and calling attention to the axis labels by asking students what they think the phrase "maximum distance" means (MP6). Then, invite students to select the line that represents this car and continue to the next screen when they are ready.

While students are working, monitor for students who discuss the importance of the point $(12, 240)$ on the graph.

Question to push students' thinking: *How could you change the situation to make a different line correct?*

Pacing: Consider using pacing to restrict students to Screens 3–4.

Sample Responses

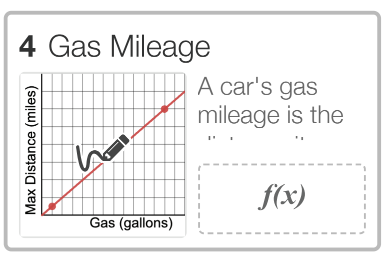
Image solution

Student Supports

Multilingual Learners

- *MLR 7 (Compare and Connect)*

After students share their approaches for selecting a line, ask groups to discuss, "What is similar, and what is different?" between the approaches. Ask students to describe what worked well with their approach and what might make an approach more complete or easier to understand.



A car's gas mileage is the distance it can go using 1 gallon of gas (measured in miles per gallon).

Based on the graph, what is this car's gas mileage?

Teacher Moves

This is a possible discussion screen.

Facilitation: While students are working, select and sequence several different explanations using the snapshots tool.

Monitor for students who use the following strategies:

- Divide the maximum distance by the number of gallons (e.g., $\frac{240}{12}$).
- Look on the graph for the maximum distance when the number of gallons is 1 .

Then, facilitate a whole-class discussion to surface strategies for determining gas mileage. Ask students to justify their reasoning and critique each other's reasoning. If both strategies do not come up naturally, consider introducing one as a strategy from a student in a previous class or year.

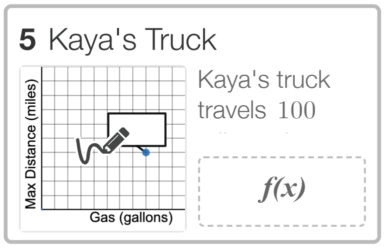
Consider naming these strategies after the students who use them and using those names throughout the rest of the lesson and unit.

Sample Responses

20 miles per gallon

Explanations vary.

- The point $(1, 20)$ on the graph means that the car can drive 20 miles using 1 gallon of gas.
- You can use the point $(12, 240)$ on the graph and divide 240 by 12 to know the number of miles for each gallon of gas.



Kaya's truck travels 100 miles using 8 gallons of gas.

What is the gas mileage for her truck?

Teacher Moves

Activity 2 Purpose: On Screens 5–8, students determine the constant of proportionality from a graph using the strategies that surfaced in the previous activity: 1) using the point $(1, k)$ or 2) dividing the y-value by the x-value for any point on the line.

Facilitation: Consider starting with the activity paused and asking students what types of vehicles they think get better gas mileage: cars or trucks. [It depends.]

Then unpause and invite students to use any of the strategies they discussed to determine the gas mileage for Kaya's truck and to continue to the next screen when they are ready.

Pacing: Consider using pacing to restrict students to Screens 5–8.

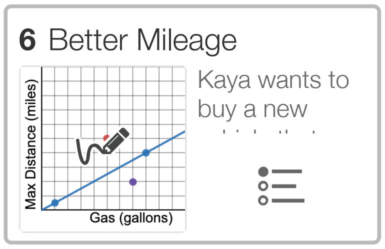
Sample Responses

12.5 miles per gallon

Student Supports

Multilingual Learners

• *MLR 2 (Collect and Display)*
Circulate and listen to students talk during pair work or group work. Jot down notes about common or important words and phrases (e.g. constant of proportionality), together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.



Kaya wants to buy a new vehicle that gets better gas mileage than her truck.

Which vehicle should she pick?

Teacher Moves

Facilitation: Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

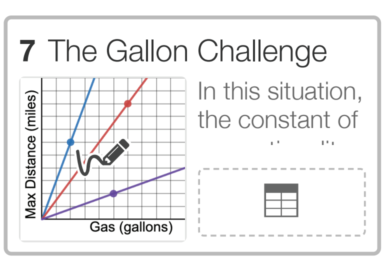
Monitor for students who draw a line to represent each vehicle and compare the graphs when the gallons of gas are 1, or students who calculate the gas mileage for each vehicle from the given point.

Sample Responses

Vehicle A

Explanations vary.

• Vehicle B's gas mileage is less than 25 miles per gallon because the line for Vehicle B goes through a point lower on the graph than $(1, 25)$. Vehicle A's gas mileage is 25 miles per gallon.
• Using Vehicle A, you can go 125 miles using 5 gallons of gas. Using Vehicle B, you can't even go 75 miles.



In this situation, the constant of proportionality is the vehicle's gas mileage.

What is the constant of proportionality for each relationship?

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to revisit the two strategies from earlier in the lesson and connect them to the idea of constant of proportionality.

Facilitation: Encourage students to try different strategies and revise their thinking until they have a strategy that is consistently successful. Asking students to find the constant of proportionality for multiple vehicles encourages looking for and expressing regularity in repeated reasoning (MP8). Once most students have completed this screen, follow with a whole-class discussion.

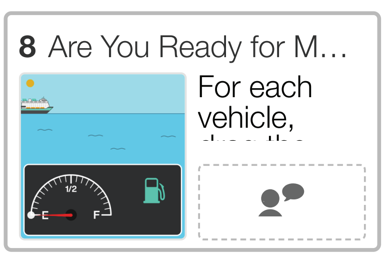
Consider questions like:

- *Why is the constant of proportionality like the gas mileage in this situation?*
- *Who used a different strategy and arrived at the same constant of proportionality?*
- *Which strategy was easier to use for Vehicle C? Vehicle D? Vehicle E? Why?*

If the strategies were named after specific students earlier, consider revisiting the names here.

Sample Responses

Vehicle C: 60 miles per gallon
Vehicle D: 30 miles per gallon
Vehicle E: 8 miles per gallon



For each vehicle, drag the movable point to add gas to the tank and see how far it can travel.

Teacher Moves

This screen is designed as an extra challenge for students who finish Screens 5–7 before the class discussion on Screen 7. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

Responses vary.

- The plane gets 0.5 miles per gallon and carries 100 passengers.
- The cruise ship gets 0.02 miles per gallon and carries 3000 passengers.
- The train gets 0.2 miles per gallon and carries 300 passengers.

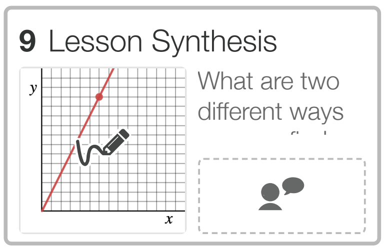
- A full cruise ship and train can carry passengers the same distance per gallon of gas. The plane is less efficient.
- If a car is carrying 4 people and gets 20 miles per gallon, then it is more efficient than these options, but a car with one person is less efficient than a plane, cruise, ship, or train.

Student Supports

Students With Disabilities

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



What are two different ways you can find a constant of proportionality using a graph?

Use the graph on the left if it helps you with your thinking.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: On this screen, students consolidate and refine their strategies for using a graph to determine the constant of proportionality.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses or suggest revisions.

Routine (optional): Consider using the mathematical language routine Collect and Display.

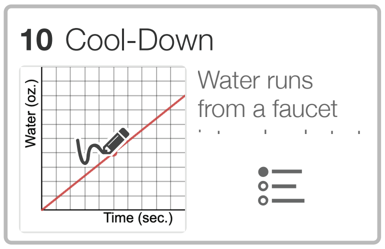
Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

1. Figure out what the y-value is when the x-value is 1. In this case, the graph passes through the point $(1, 2)$, so the constant of proportionality is 2.

2. Divide a y-value by its corresponding x-value to find out how much y there is per x. For example, $\frac{12}{6} = 2$.



Water runs from a faucet into a bucket at a steady rate.

The relationship between the amount of water in the bucket and time is proportional.

Select ALL of the true statements.

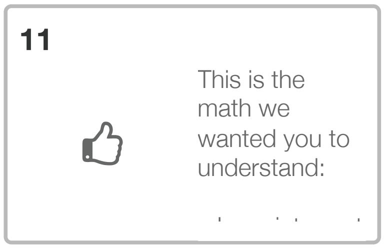
Teacher Moves

Support for Future Learning: If students struggle to determine and interpret the constant of proportionality from a graph, plan to emphasize this when opportunities arise over the next several lessons. Consider spending extra time during Lesson 10's warm-up discussing the constant of proportionality of the graph.

Pacing: Consider using pacing to restrict students to Screens 10–11.

Sample Responses

- After 1 second, there are 4 ounces of water in the bucket.
- The point $(1, 4)$ is on the graph of the line.
- A constant of proportionality for this relationship is 4.



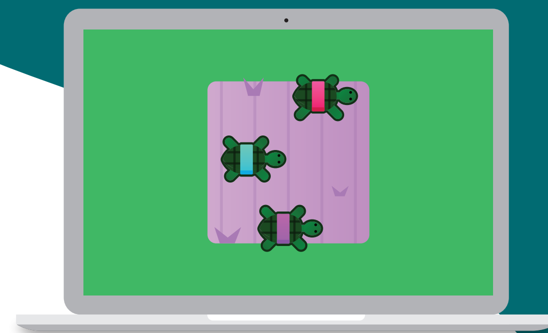
This is the math we wanted you to understand:

- I can interpret points on the graph of a proportional relationship.
- I can identify the constant of proportionality from a graph of a proportional relationship.

Unit 2
Lesson
10

Digital Lesson RECOMMENDED

This is a digital lesson with additional print materials. A print option is also available.



Three Turtles

Proportional Relationships and Graphs

Let's use graphs and equations to compare proportional relationships.

Focus and Coherence

Today's Goals

1. **Goal:** Write an equation of a proportional relationship given a point on a graph.
2. **Language Goal:** Compare related proportional relationships on the same graph using mathematically precise language. **(Reading and Writing)**

The purpose of this lesson is for students to use graphs and equations to compare proportional relationships. Students write equations based on tables and graphs and differentiate between graphs and equations of relationships that are proportional and non-proportional. They compare multiple proportional relationships on the same coordinate plane to interpret the steepness of each graph in context. They reason abstractly to conclude that graphs can be used to compare constants of proportionality, even when the scale is not specified on each axis. **(MP2)**

Prior Learning

In Lesson 9, students analyzed graphs with multiple proportional relationships and interpreted the constants of proportionality in context.

Future Learning

In Lesson 11, students will solidify their understanding of the different representations of proportional relationships.

Standards

NY-7.RP.2b

Identify the constant of proportionality (unit rate) in tables, **graphs, equations,** diagrams, and verbal descriptions **of proportional relationships.**

NY-7.RP.2c

Represent proportional relationships with equations.

Also Addressing: **NY-7.RP.2, NY-7.RP.2a**

Mathematical Practices: MP2

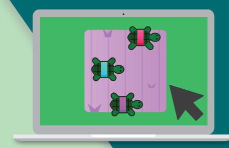
Rigor and Balance

- Students build **conceptual understanding** of the relationship between the constant of proportionality and the steepness of a proportional relationship when graphed.
- Students build **fluency** in recognizing the relationship between steepness and constant of proportionality by comparing multiple proportional relationships on the same coordinate plane.

Lesson at a Glance

~ 45 min

Standards: NY-7.RP.2, NY-7.RP.2a, NY-7.RP.2b, NY-7.RP.2c



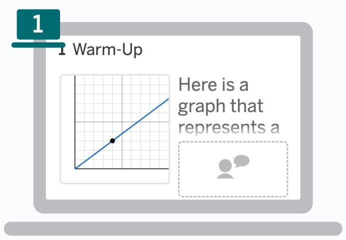
Why go digital?

Students connect turtles traveling to their distance-time graphs in real time.

Warm-Up

Pairs | 5 min

Students use their creativity to revisit the meaning of a point on a graph of a proportional relationship.

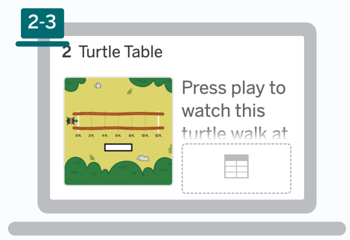


Pacing: Screen 1

Activity 1

Pairs | 10 min

Students connect information in a graph and a table to write an equation of a single proportional relationship and decide whether relationships represented by graphs are proportional.

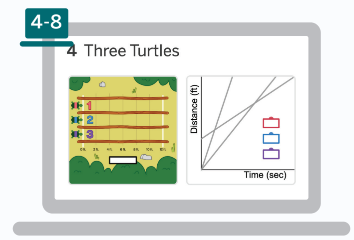


Pacing: Screens 2–3

Activity 2

Pairs | 20 min

Students write equations for proportional relationships and revisit how to identify whether graphs and equations represent proportional relationships.

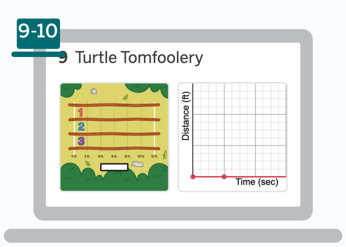


Pacing: Screens 4–8

Activity 3 (Optional)

Pairs | 10 min

Students create their own turtle race based on given conditions.

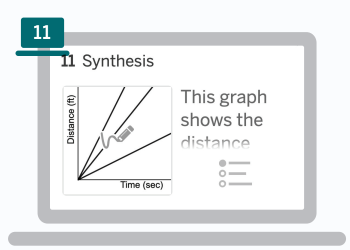


Pacing: Screens 9–10

Synthesis

Whole Class | 5 min

Students synthesize their understanding of comparing proportional relationships using graphs and equations.

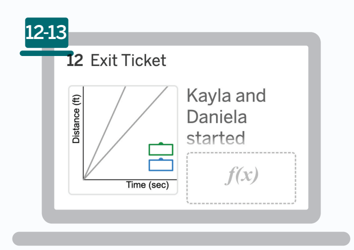


Pacing: Screen 11

Exit Ticket

Independent | 5 min

Students demonstrate their understanding of comparing proportional relationships by labeling two relationships on a graph and writing an equation.



Pacing: Screens 12–13

Prep Checklist

Assign the digital lesson. A print option is also available.

Student using digital:

- Digital Lesson

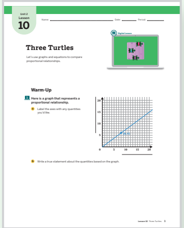
Student using print:

- Print Option in Student Edition
- Exit Ticket PDF
- Activity 2 Cards, one set per pair

Digital Lesson | Pairs | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students use their creativity to revisit the meaning of a point on a graph of a proportional relationship.



Students using print

1 Launch

Consider asking students what they remember about interpreting a point on a graph.

1 Connect

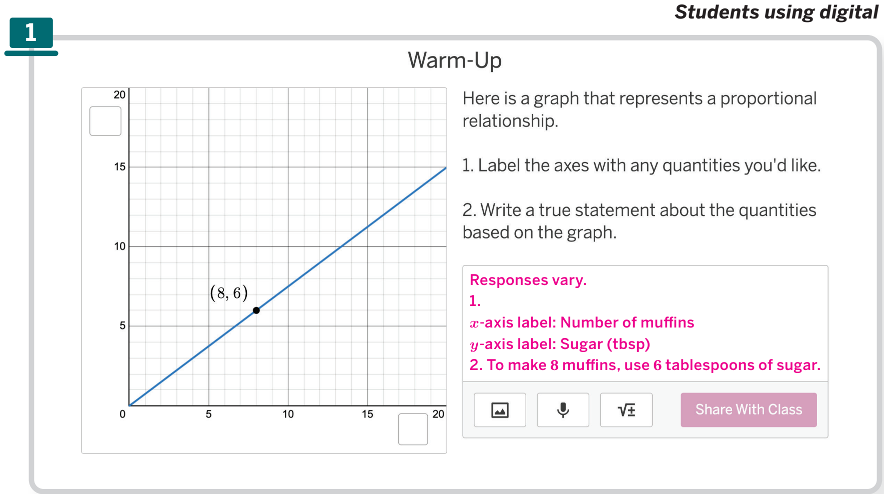
Invite students to share their axes labels and statements. Invite them to connect the statements to their graphs and critique each other's reasoning.

Emphasize early student thinking, including students who correctly mention the constant of proportionality $\frac{3}{4}$ and students whose statement swaps the meaning of the axes labels.

Consider asking students to make adjustments to their labels based on the feedback they received from their classmates and display these, as time permits.

Math Identity and Community Consider celebrating unique or creative choices of contexts as students share their axes labels and asking the authors to speak about their inspiration.

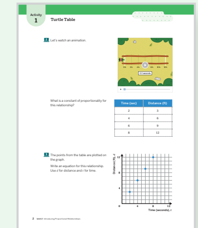
Students using digital



Pairs | 10 min | Pacing: Screens 2–3

Activity 1 Turtle Table

Purpose: Students connect information in a graph and a table to write an equation of a single proportional relationship and decide whether relationships represented by graphs are proportional.



Students using print

2 Launch

Play the digital animation of the turtle walking at a constant rate. **Note:** The distance is measured from the start at 0 ft to the turtle's head.

3 Monitor

Students using print will not receive automated, interpretive feedback on their equation. Consider checking in with these students before they move on.

D Differentiation

Look for students who:	Teacher Moves
Need support getting started. *(Screen 2)*	**Support:** Ask: "What do you need to multiply the time by to calculate the distance?"
Notice that the turtle travels 3 feet every 2 seconds. *(Screen 2)*	Consider asking, "What does this tell you about the constant of proportionality for this relationship?"
Compare the time to the distance to calculate the constant of proportionality, but do not switch variables in the equation. *(Screen 3)*	Remind students that when switching the relationship of the two variables, the variables in the equation must also switch.
Finish early. *(Screen 3)*	**Extension:** Encourage students to create both equations for this proportional relationship.

3 Connect

Emphasize that there are two constants of proportionality that represent this relationship, creating two different equations.

- Comparing the distance to the time, the constant of proportionality is $\frac{3}{2}$, giving an equation of $d = \frac{3}{2}t$.
- Comparing the time to the distance, the constant of proportionality is the reciprocal, $\frac{2}{3}$.
- Because the relationship of the two values switches, the variables in the equation must also switch, giving us $t = \frac{2}{3}d$.

Discuss connections that students make between the constant of proportionality and each of the three representations on the screen.

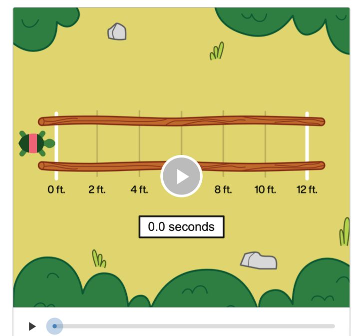
To surface the Key Takeaway, consider asking,

- "Where do you see $\frac{3}{2}$ in the table? In the equation? In the graph?
- What might the graph and equation look like for a turtle who is moving more quickly?
- What about a turtle who is moving more slowly?"

Students using digital

2

Turtle Table



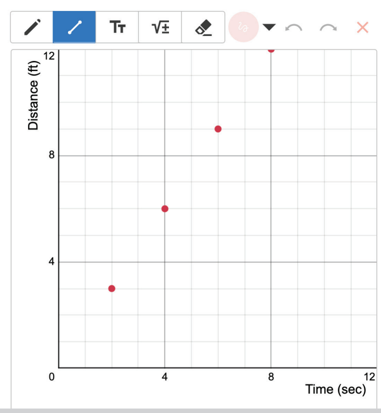
Press play to watch this turtle walk at a constant rate.

Time (sec)	Distance (ft)
2	
4	
6	
8	

$\frac{3}{2}$ (or equivalent) or $\frac{2}{3}$ (or equivalent)

3

Turtle Graph



The points from the table on the previous screen are plotted on the graph.

Time (sec)	Distance (ft)
2	3
4	6
6	9
8	12

$d = \frac{3}{2}t$ (or equivalent) or $t = \frac{2}{3}d$ (or equivalent)

Write an equation for this relationship.

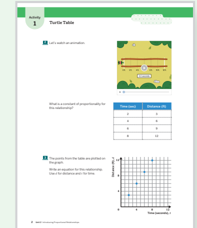
Use d for distance and t for time.

Key Takeaway: The constant of proportionality is visible in the table, the graph, and the equation.

Digital Lesson | Pairs | 20 min | Pacing: Screens 4–5

Activity 2 Three Turtles

Purpose: Students write equations for proportional relationships and revisit how to identify whether graphs and equations represent proportional relationships.



Students using print

4 Launch

Play the the animation.

Use the Notice and Wonder routine to support students in making sense of the animation and the graph. Invite students to match each turtle to its graph.

Note: This may be the first time students are seeing a coordinate plane without a grid, requiring them to think abstractly. **(MP2)**

5 Monitor

Support getting started by asking, "What do you notice about where each turtle starts? How could this be represented on the graph?"

> M/EL **Multilingual/English Learners:** Circulate and listen to students talk during pair work or group work. Jot down notes about common or important words and phrases (e.g., constant of proportionality), together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.

Use the dashboard to identify conflict or consensus among students' choices.

Pause to highlight one response. If there is conflict, consider selecting and sequencing 1–2 responses for each of the most popular choices, and invite those students to share their reasoning.

5 D Differentiation

Look for students who:	Teacher Moves
Identify that the graph for Turtle 3 represents a proportional relationship.	**Support:** Ask: "Where does Turtle 3 start compared to Turtles 1 and 2? How is this shown on the graph?"
Identify that the graph for Turtle 3 does not represent a proportional relationship, but does not connect the turtle's starting point in their explanation.	Consider asking, "What is similar and different about the graphs for each turtle? What makes the relationship for Turtle 3 non-proportional?"

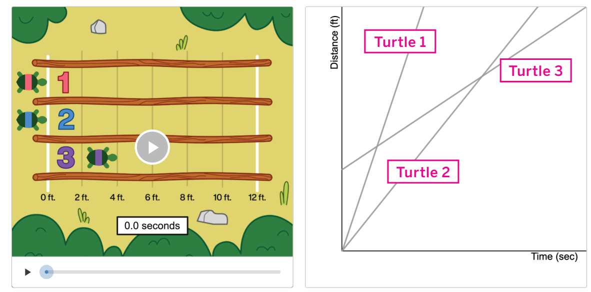
Activity 2 continued ›

Students using digital

4

Three Turtles

Press play to watch these turtles travel. Then match each turtle to the line that represents it.

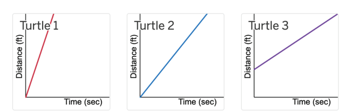


5

Proportional Relationships

Here are the graphs from the previous screen.

1. Select *all* the distance-time relationships that are proportional.



2. Discuss your thinking with a classmate.

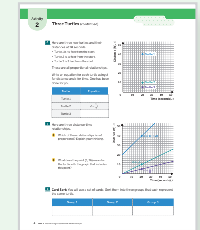
Turtle 1 and Turtle 2

Responses vary. **The turtle with a head start (Turtle 3) has a distance-time relationship that is not proportional because the graph does not go through the origin. The other two turtles (Turtles 1 and 2) have distance-time relationships that are proportional because their graphs are straight lines that go through the origin**

Pairs | 20 min | Pacing: Screens 6–8

Activity 2 Three Turtles (continued)

Purpose: Students write equations for proportional relationships and revisit how to identify whether graphs and equations represent proportional relationships.



Students using print

6-8 Monitor

Circulate to observe student strategies for writing equations from the graph and offer help or encouragement where needed.

MLR **Use the MLR2: Collect and Display routine** to gather students' ideas and create a class definition or anchor chart for equations that represent proportional relationships.

A **Accessibility: Executive Functioning** Chunk this activity into more manageable parts (e.g., sorting one card at a time), which will aid students who benefit from support with organizational skills in problem-solving.

Use the dashboard to identify conflict or consensus among students' choices.

Students using print will need copies of the Activity 2 cards.

Discuss student strategies to match the tables, graphs, equations, and word stories.

8 Connect

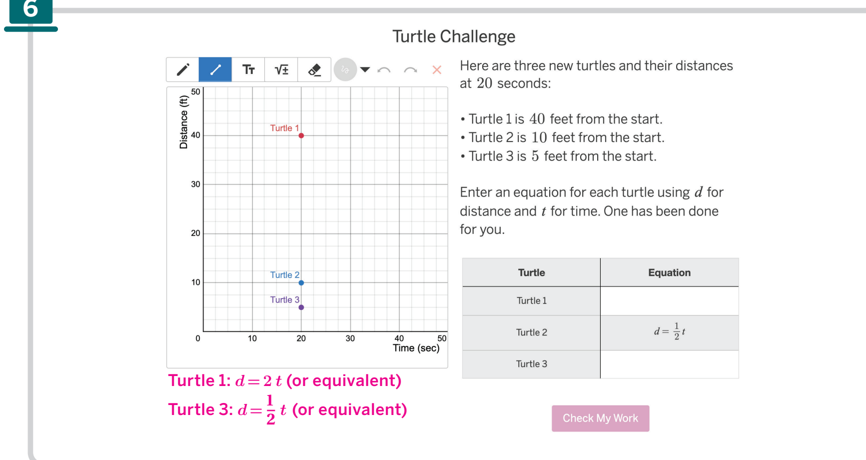
Display student responses and highlight one response if there is consensus. If there is conflict, consider selecting and sequencing 1–2 responses for each of the most popular choices and invite students to share their reasoning.

Invite students to share their strategies when matching different representations of the same turtle.

Consider asking, "Which was the most challenging card to sort? How did you decide where it should go?"

Students using digital

6



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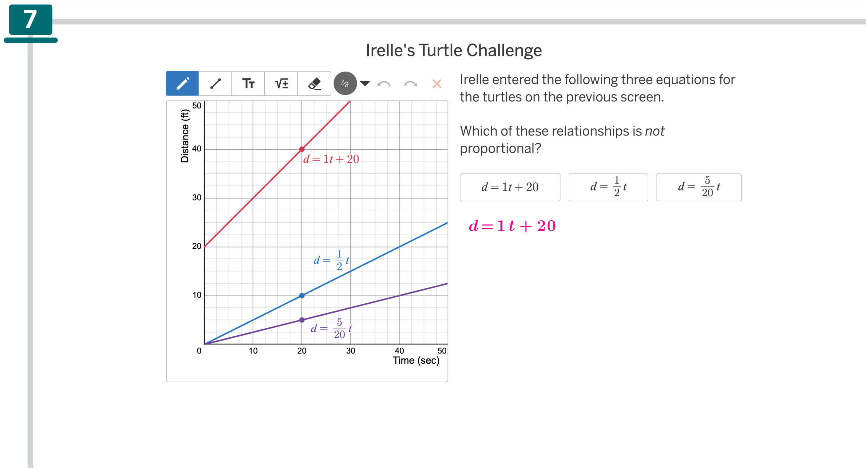

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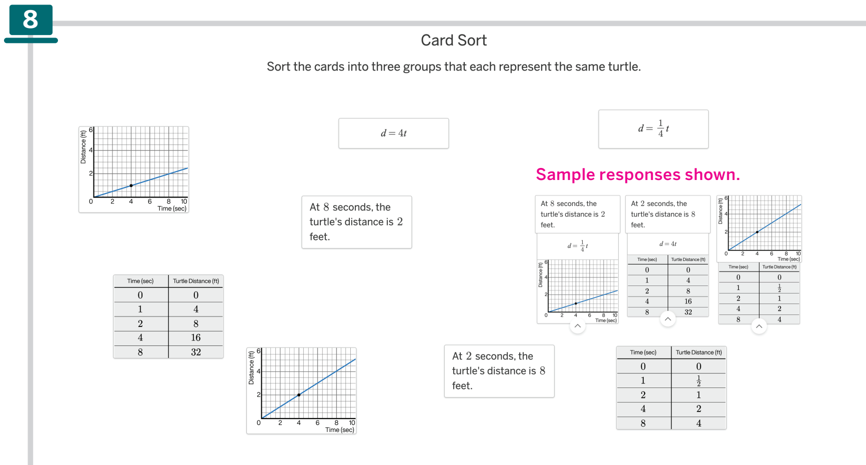
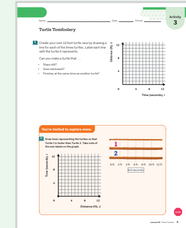


Digital Lesson | Pairs | 10 min | Pacing: Screens 9–10

Activity 3 Turtle Tomfoolery (Optional)

Purpose: Students create their own turtle race based on given conditions.

Short on time: Consider omitting Activity 3.



Students using print

9 Launch

Demonstrate how to use the tool and create three lines for three turtles.

Play the animation.

9 Monitor

Snapshot unique or creative graphs.

Math Identity and Community Consider highlighting creative strategies and naming those students explicitly.

Consider asking, "What do the titles of the axes on the graph represent?"

10 **D Differentiation: Extension** Invite students who would like to explore further to complete this optional task. Encourage them to discuss their thinking with a partner.

9 Connect

Display snapshots of students' graphs. Ask students to predict what will happen to the turtles in each student's graph.

Consider asking:

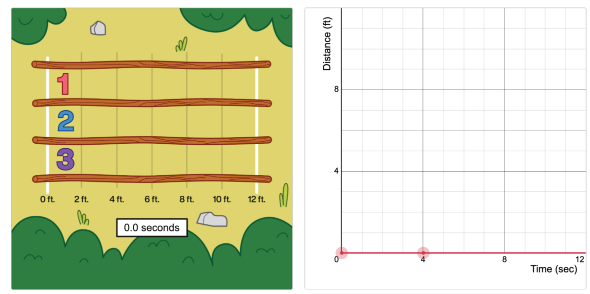
- "How do you know which turtle moves the fastest when looking at a graph?" *Responses vary.* I can compare the relationships on the same coordinate plane. The fastest turtle is represented by the steepest line.
- "Can you determine which turtle moves the fastest even when the scale is not specified on each axis? Explain your thinking." *Responses vary.* Yes. Even without the scale, I can compare the constants of proportionality of each turtle's travels by comparing the steepness of each line.
- "What is alike and what is different between the proportional and non-proportional relationships we explored today?" *Responses vary.* Proportional and non-proportional relationships are similar in that they are represented by straight lines on a graph. These relationships are different in that proportional relationships pass through the point $(0, 0)$, while non-proportional relationships do not.
- "How can you represent a proportional relationship using different representations (e.g., descriptions, tables, graphs, and equations)?" *Responses vary.* If I have an equation in $y = rx$ form, then I know it is proportional and will pass through $(0, 0)$ on a graph. I can include a row in the table where both x and y are equal to 0.

Students using digital

9

Turtle Tomfoolery

Create your own 12-foot turtle race by adjusting the line for each of the three turtles.
Can you make a turtle that stays still? Goes backward? Finishes at the same time as another turtle?



Responses vary.

10

You're invited to explore more.

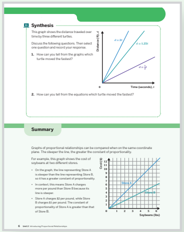
Let's go!

Responses vary.

Whole Class | 5 min | Pacing: Screen 11

Synthesis

Purpose: Students synthesize their understanding of comparing proportional relationships using graphs and equations.



Students using print

11 Synthesis

Invite students to respond and give them a minute to share their responses with a partner.

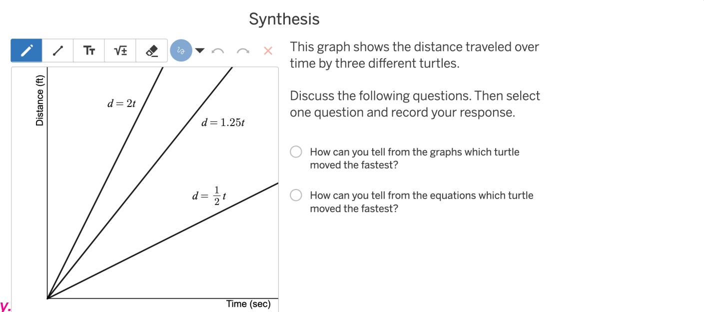
Display several students' responses.

Invite students to share the connections they see between responses or suggest revisions.

Lesson Takeaway: Graphs and equations help compare proportional relationships.

Students using digital

11



Responses vary.

- **I can tell from the graphs which turtle moved fastest by comparing the steepness of the lines. When time is on the x-axis, a steeper line means the turtle moved at a faster rate.**
- **I can tell from the equations which turtle moved fastest by looking at the constant of proportionality. When the equations are all written for t in terms of d, a greater constant of proportionality means the turtle moved at a faster rate.**

Summary

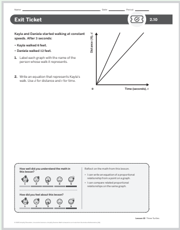
Share the Summary. Students can refer back to this throughout the unit and course.

Digital Lesson

Independent | 5 min | Pacing: Screens 12–13

Exit Ticket

Purpose: Students demonstrate their understanding of comparing proportional relationships by labeling two relationships on a graph and writing an equation.



Students using print

12-13 Today's Goal

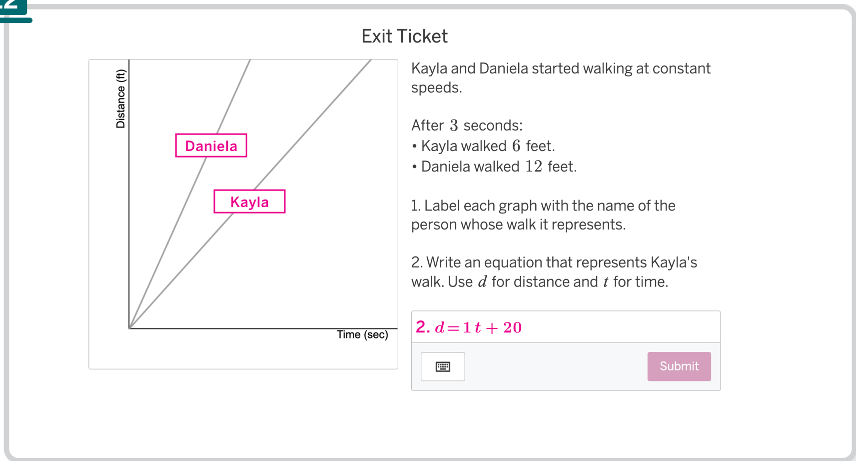
Goal: Write an equation of a proportional relationship given a point on a graph.

Language Goal: Compare related proportional relationships on the same graph using mathematically precise language. **(Reading and Writing)**

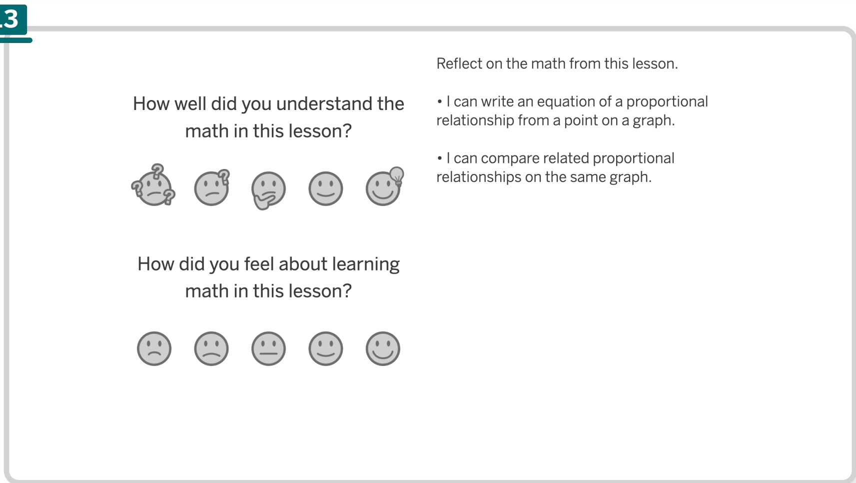
Support for Future Learning: If students struggle to write an equation, consider reviewing this Exit Ticket as a class before beginning Lesson 11, or offering individual support where needed during the lesson.

Students using digital

12



13



Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

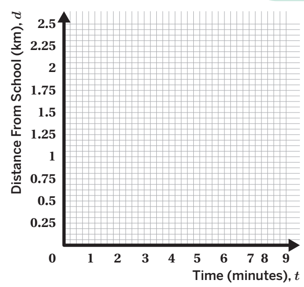
Students using digital

Students using print

Name: ______ Date: ______ Period: ______

Practice

For Problems 1–4, use this information. Mia and Jamal bike home from school at a steady pace. Mia bikes 1.25 kilometers and it takes her 4 minutes. Jamal bikes 1.75 kilometers and it takes him 7 minutes.



1. Create two lines that represent Mia and Jamal's journeys.
   Sample responses shown on graph.
2. Plot a point on each line when $t = 1$.
   Sample responses shown on graph.
3. Determine each value of r, the constant of proportionality.

Line	r
Mia	0.3125
Jamal	0.25

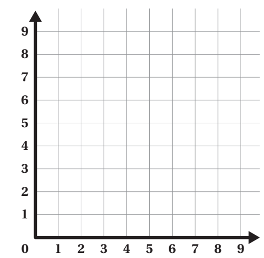
4. Who bikes faster? Explain your thinking.
   Mia; *Sample response:* Mia bikes faster because the constant of proportionality that represents the number of kilometers per minute that she travels is greater than Jamal's.
5. The point $\left(3, \frac{6}{5}\right)$ lies on a graph representing a proportional relationship. Select *all* of the points that also lie on the same graph.
   - A. $(1, 0.4)$
   - B. $\left(1.5, \frac{6}{10}\right)$
   - C. $\left(\frac{6}{5}, 3\right)$
   - D. $\left(4, \frac{11}{5}\right)$
   - E. $(15, 6)$

Additional Practice for this lesson is avilable online.

Practice

Name: ______ Date: ______ Period: ______

For Problems 6–9, use this information. At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for $5.40.



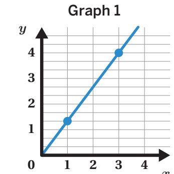
6. How much does the honey cost per ounce?
   0.45 dollars per ounce
7. How much honey can you buy for $\$1$ dollar?
   About 2.2 ounces
8. Write two different equations that represent this situation. Use h for ounces of honey and c for cost in dollars.
   $c = 0.45h$ (or equivalent) and $h = 2.2c$ (or equivalent)
9. Graph the line of one of your equations from the previous screen. Be sure to label the axes.
   Sample response shown on graph.

Spiral Review

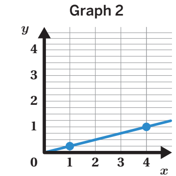
For Problems 10–13, match each equation with its graph.

10. $y = \frac{1}{4}x$ ____ **Graph 2**
11. $y = \frac{3}{2}x$ ____ **Graph 4**
12. $y = 2x$ ____ **Graph 3**
13. $y = \frac{4}{3}x$ ____ **Graph 1**

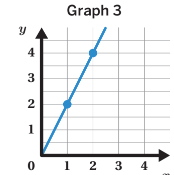
Graph 1



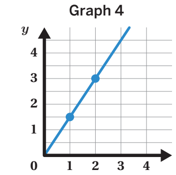
Graph 2



Graph 3



Graph 4



Reflection

1. Put a smiley face next to a question you were stuck on and then figured out.
2. Use the space below to ask one question you have or to share something you are proud of.

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1, 2, 9	2	NY-7.RP.2
	3, 4, 6, 7	2	NY-7.RP.2b
Test Practice	5	2	NY-7.RP.2
	8	2	NY-7.RP.2c
Spiral Review			
Fluency	10–13	1	NY-7.RP.2

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Four Representations (NYC)

Lesson 11: Connecting Descriptions, Tables, Graphs, and Equations

Purpose

The purpose of this lesson is for students to create and connect descriptions, tables, equations, and graphs of proportional relationships.

Preparation

Worksheet

- *Activity 1–2*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Warm-Up: Which One Doesn’t Belong? (5 minutes)

Purpose: Identify similarities and differences between representations of a proportional relationship.

Facilitation: Arrange students into pairs. Display Sheet 1 of the Teacher Projection Sheets. Give students one minute of time to think quietly. Ask them to indicate when they have noticed one representation that does not belong and can explain why; encourage students to look for more than one possibility. When the minute is up, give students two minutes to share their responses with their partner, and then to work together to find at least one reason each representation doesn't belong.

Facilitate a whole-class discussion to describe the ways each representation does not belong. Consider asking students: *Could these all represent the same relationship?* (Which One Doesn’t Belong).

If it does not come up naturally, discuss the constant of proportionality in each representation. [The graph has a constant of proportionality of 6, while the other representations all have a constant of proportionality of 12.]

Activity 1: Stronger and Clearer Each Time (10 minutes)

Purpose: Students create and write a detailed description of a proportional relationship between quantities that they choose. They will use an interactive structure to strengthen and clarify their ideas and their descriptions.

This is a significant change from previous activities where students were given two quantities and had to decide if the relationship between the two was proportional or not. In this activity, students create a situation in which two quantities are in a proportional relationship, which is an important step of modeling with mathematics (MP4).

Facilitation: Distribute one double-sided worksheet to each student and share the purpose of the activity.

Display Sheet 2 of the Teacher Projection Sheets, and invite students to select two items from different lists. Some of the items on the lists will be familiar and others may be unfamiliar. Consider taking time to clarify the meaning of any unknown words or asking students to do some research on their meanings.

Then facilitate the instructional routine Stronger and Clearer Each Time. This routine will ask students to draft an initial response, use successive pair conversations to strengthen and clarify their ideas and their descriptions, and write a final version. In each pair conversation, students should first check to see if they agree that the relationships described are proportional.

It may make sense to approve of students' choices of variables since they will use this relationship in the next activity. For example, "legs" and "earthworms" may not make for a very interesting relationship.

Activity 2: Four Representations (20 minutes)

Purpose: Students create four representations of the proportional relationship they defined in Activity 1.

Facilitation: Invite students to use the relationship they developed to create four representations of their proportional relationship on the second side of their worksheet.

If students need more support, consider projecting the warm-up for students to use as an example as they work.

Early Finishers: As time allows, invite students who finish early to trade their work with a classmate to give each other feedback about their analysis.

Early Student Thinking: Students may choose numbers in their table that end up being challenging to use for calculating values in their table or for scaling their graph. Consider asking these students why their numbers are unfriendly, what numbers would be friendlier, or how they can adjust their representation to fit these values.

Students may choose scales for their axes that are not appropriate for their context (e.g., scaling the graph of the number of legs on a centipede by ones). Consider asking these students: *What scale would be most helpful for your data?*

Lesson Synthesis (5 minutes)

Purpose: Decide what makes a relationship proportional and calculate the constant of proportionality for a proportional relationship in an unfamiliar context.

Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion to help students understand how they can identify the constant of proportionality in every representation.

Cool-Down (5 minutes)

If students struggle to show the constant of proportionality in each representation, consider making time to explicitly revisit these ideas. A strong understanding of how the constant appears in a table, graph, and equation will support students in the End Assessment.

Water Efficiency (NYC)

Lesson 12: Let's Put It to Work

Purpose

The purpose of this lesson is for students to use proportional relationships to explore whether baths or showers use more water.

Note: This lesson is designed for 90 minutes and may take multiple class periods to complete.

Preparation

Worksheet

- *Activity 1:* Print one double-sided worksheet for each student.
- *Activities 2 & 3:* Print one single-sided worksheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Warm-Up (10 minutes)

Purpose: Introduce students to the main task of the lesson and brainstorm the quantities they might need as they develop a solution.

Facilitation: Display the Teacher Projection Sheet. Consider asking students why it is important to save water. Then ask them what their first instinct is—*does a shower or a bath use more water?* Record students' votes for all to see.

Then, distribute one copy of Activity 1 to each student and ask students to copy the large questions from the projection sheet to the "Warm-Up" section of the student worksheet and record their guess.

Next, give students 1–2 minutes of quiet think-time to list the information they need to answer the question more precisely. Invite students to share their list with a partner, and then with the whole class. Encourage students to add or delete items on their list as they hear other students' thinking.

During the discussion, invite students to help each other attend to precision (MP6) with questions like: *Why is it important to know the size of the bathtub?* or *What unit(s) would you use to measure how much water a shower uses?*

Next, invite students to make estimates with units for the information they need. Some quantities, like the length of a shower, may be simpler than others for students to estimate. Consider discussing more challenging measurements as a class. For discussions about flow rate, bringing or displaying a 1-gallon jug may help. Ask questions like: *How long do you think it would take for a shower head to fill the jug? Are there any experiences you can draw on for getting a reasonable estimate?*

Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify to help students attend to precision in their descriptions of quantities.

Activity 1: Bath vs. Shower (30 minutes)

Purpose: Students make assumptions and estimates to model the bath-shower situation and use their knowledge of proportional reasoning to decide whether baths or showers use more water.

Facilitation: Share with students that they will come up with a solution to the problem introduced in the warm-up and eventually make a poster to share their answer.

Arrange students in groups. Their task is to answer the question: *Which uses more water and by how much?* Invite groups to begin by deciding which quantities they will need and what values to use for those quantities. Assure students that they can change these assumptions later.

Consider providing some of the values below for students who are stuck or who request this information:

- **Typical shower length:** 11 minutes (5 minutes is recommended during a drought.)
- **Flow rate for showers:** 1.9–2.5 gallons per minute (up to 5.5 gallons per minute for older shower heads)
- **Time to fill a bath:** 5–8 minutes
- **Flow rate for bathtub faucets:** 4–7 gallons per minute
- **Volume of typical bathtubs filled to the top:** 120–180 gallons of water
- **Interior of a typical bathtub:** 30–32 inches wide, 55–60 inches long, 18–24 inches deep
- There are approximately 230 cubic inches in 1 gallon of water.

Circulate to support groups as they work. When applicable, encourage students to use their understanding of proportional relationships to aid them in their task. For instance, if a group is adjusting their assumptions about the length of a shower, it may be useful to write an equation to represent the amount of water used.

After students have had some time to think about all three questions, invite them to transition to making a poster with their group. Guidelines for the poster are on the worksheet.

Activity 1: Synthesis (5 minutes)

Purpose: Surface assumptions that students made as they created their models of the bath/shower situation and which representations they found most helpful (MP4).

Facilitation: Give students 1–2 minutes of quiet time to think. Invite students to share their responses with their group and decide which assumptions they thought were most important for their work. Consider asking each group to share with the class.

If you are completing this lesson over two class periods, this is a great stopping point for the first class period.

Activity 2: Warm-Up (5 minutes)

Purpose: Students revisit their posters before getting feedback on their work from others.

Facilitation: Give students 1–2 minutes of quiet time to think. Invite students to share their responses with their group and decide how they plan to revise their posters in Activity 2. Consider asking each group to share with the class.

Activity 2: Gallery Tour (15 minutes)

Purpose: Students see their peers' different conclusions, and the assumptions and strategies that led to those conclusions.

Facilitation: Distribute a copy of the Activity 2–3 worksheet to each student. Invite groups to display their posters and then do a gallery tour where all students circulate the classroom looking at the work of their peers. Once students have returned to their seats, give 2–3 minutes of silent think-time to answer the two questions on their worksheet.

Follow with a brief class discussion to help students understand the different assumptions and methods that different groups used on this problem. Invite students to share aspects from their classmates' work that they found interesting.

Routine: Use the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Activity 3: Revisions and Reflection (15 minutes)

Purpose: Students consider how the process of making assumptions, doing calculations, and applying proportional reasoning offers them insight into a real-life context.

Facilitation: Invite students to work with their groups from Activity 1 and use insights from Activity 2 to revise their poster and add at least one additional representation (graph, equation, table).

When students have completed their revisions, invite individuals to respond to the reflection questions and then to share their responses with their group. Follow with a whole-class discussion.

Push students to go beyond listing actions that would reduce water usage, such as taking shorter showers, and instead consider which changes have the biggest impact. Ask a question like: *For a person who prefers showering, how does reducing their shower time compare to reducing the flow rate of their shower head?*

Lesson Synthesis (5 minutes)

Purpose: Highlight connections between these contexts and the major ideas of this unit.

Facilitation: Give students 2–3 minutes to respond to the synthesis questions and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. If it does not come up naturally, ask students how proportional relationships were—or could have been—helpful in solving this problem.

Cool-Down (5 minutes)

If students struggle to answer a question using constants of proportionality, consider reviewing this cool-down as a class before students engage in the End Assessment.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

7.2 Practice Day 2 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided sheet for each student.

Task Cards

- *Option 1 (Stations)*: Print two sets of task cards for the whole class.
- *Option 2 (Task Cards)*: Print one set of task cards for each group of 2–3 students.

Instructions

Option 1: Stations

This structure allows students to collaborate and helps them manage their time by breaking each task into smaller, more manageable chunks. It also increases engagement by giving students opportunities to move around the classroom.

Arrange students into groups of 3–4. Give each student the student workspace sheet to complete as they work together to solve the task at each station.

Options for student movement:

- As students finish a station, instruct students to move from station to station.
- After a set amount of time, instruct students to move as a group from station to station.
- After a set amount of time, instruct students to move to a new station such that no one from their previous group is in their new group.

Option 2: Task Cards

This structure supports student collaboration and focuses students' attention on one task at a time.

Arrange students into groups of 2–3. Print one set of task cards for every group of students. Give each student the student workspace sheet to complete as they work together to solve each of the tasks.

Consider posting the answer key, or walk around with it and provide feedback to students as they work.

GRADE 7

Unit 4
Lesson Plans

Teacher lesson plans from Unit 4 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Ratios and Proportional Relationships** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 7 Unit 4

Teacher Edition Sampler

Unit at a Glance

Key
- Print Lessons
- Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1

1 Mosaics
Visualize benchmark percentages on a grid (e.g., 20% = 1/5).

2 Peach Cobbler
Determine and use a constant of proportionality to solve problems involving fractional quantities.

3 Sticker Sizes
Determine an unknown value in a proportional relationship using a table.

Practice Day

Practice Day 1
Practice the concepts and skills developed during Lessons 1–7. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

Quiz: Sub-Unit 1
Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2

8 Tax and Tip
Calculate the original amount, the new amount, or the percent change in the context of sales tax and tip.

9 Minimum Wage
Use proportional relationships and percent change to analyze an issue in society.

Practice Day

Practice Day 2
Practice the concepts and skills developed during Lessons 1–13. Consider using this time to prepare for the upcoming Quiz.

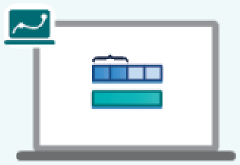
Summative Assessment

End-of-Unit Assessment
Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing: 17 days | **Short on time?** See pacing considerations below.

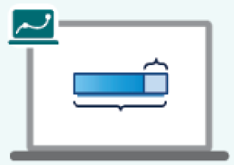
Pre-Unit Check: (Optional)
13 Lessons: 45 min each
2 Practice Days: 45 min each

1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



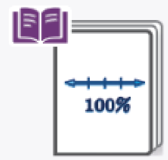
4 More and Less

Use tape diagrams and tables to represent situations that involve adding or subtracting a percent from the initial value (or from 100%).



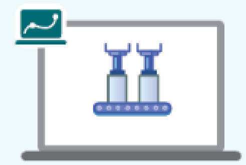
5 All the Equations

Use equations to represent situations involving a percent increase or decrease.



6 100%

Use double number lines to represent situations involving a percent increase or decrease.



7 Percent Machines

Calculate the original amount given the new amount and a percent change for one-step and multistep percent problems.

10 Cost of College

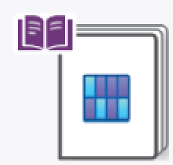
Solve problems about minimum wage and the cost of college over time, including calculating both costs and percentages.

11 Bookcase Builder

Explain that percent error is an error as a percentage of the correct measurement.

12 Posing Percent Problems

Generate and answer questions about a real-world situation involving percent increase or decrease.



13 Decimal Deep Dive

Convert a fraction to a decimal using long division.

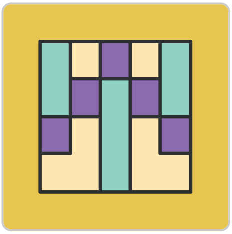
Pacing Considerations

Lesson 1: This lesson supports students in surfacing what they know about relationships between percents and fractions. This sets them up for success as they explore percent increase and decrease in the unit. If students show a strong understanding of working with percentages in Problems 3–5 of the Pre-Unit Check, this lesson may be omitted.

Lessons 9-10: These lessons give students an opportunity to use what they've learned about proportional relationships and percent change to analyze issues in our society. If students show a strong understanding of working with percent change in earlier lessons in the unit, one of these lessons may be omitted. If omitted, be sure to discuss how percent change can empower us to better understand our society throughout the unit.

Lesson 12: This lesson gives students an opportunity to apply what they've learned about increasing and decreasing by a percentage to generate and answer questions about the society we live in. There is no new content introduced in this lesson.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Mosaics

Lesson 1: Working With Fractions and Percentages

Overview

Students discover what they already know about the relationships between percents and fractions. This sets them up for success as they explore percent increase and decrease in the unit.

Note: This lesson is optional.

Learning Goals

- Visualize benchmark percentages on a grid (e.g., $20\% = \frac{1}{5}$).
- Calculate the percentage of a quantity as a rate per 100 (e.g., 8% of 20).

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to surface what students already know about the relationships between percents and fractions, and to set them up for success as they explore percent increase and decrease in the unit. Students analyze and then create their own mosaics and determine what percent of each mosaic is shaded different colors.

Lesson Summary

Warm-Up (5 minutes)

The purpose of this warm-up is for students to reason about percentages as parts of a whole represented by 100%.

Activity 1: Percent Shaded (15 minutes)

The purpose of this activity is for students to develop strategies for figuring out what percent of a whole is shaded. This activity surfaces two specific strategies: 1) determining the percentage represented by equally sized parts and 2) multiplying the decimal representation by the total number of parts, though students may also use other strategies. Students analyze existing mosaics and create mosaics to fit certain criteria.

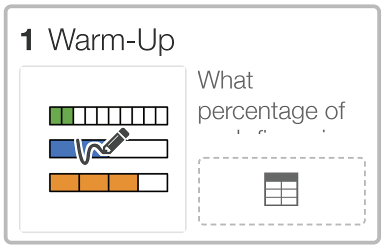
Activity 2: Create Your Own Mosaic (15 minutes)

The purpose of this activity is for students to practice calculating the percentage of a number and express their creativity by creating their own mosaics. This activity includes a Challenge Creator in which students create their own mosaic, then calculate the percentage that is shaded each color. Students are presented with one of their classmates' mosaics and are asked to determine the percentages.

Lesson Synthesis (5 minutes)

The purpose of this synthesis is for students to consolidate and refine their ideas about how to visualize and calculate the percentage of a number.

Cool-Down (5 minutes)



What percentage of each figure is shaded?

Be prepared to explain your reasoning.

Teacher Moves

Lesson Overview: The purpose of this lesson is to surface what students already know about the relationships between percents and fractions, and to set students up for success as they explore percent increase and decrease in the unit.

Warm-Up Purpose: Students reason about percentages as parts of a whole represented by 100%.

Facilitation: Consider starting with the lesson paused and asking students what they remember about the concept of percentages. Consider reviewing the meaning of a few benchmark percentages, such as 25%. Then, unpause and invite students to think about the percentage of each figure that is shaded. If students are having difficulty getting started, consider asking: *What percent would a completely shaded bar represent? How do you know?* When students have recorded their thinking, consider reviewing each figure one at a time, eliciting several different responses for the percentage, then asking students to share their reasoning.

Routine (optional): Consider using the routine *Number Talk*. Before students enter the activity, conduct the number talk using a whiteboard or another large display so you can show each tape diagram one at a time. If it does not come up naturally, consider asking students: *What percent would a completely shaded bar represent? How do you know?*

Readiness Check: If Problems 3 and 4 of the Readiness Check suggest that some students have unfinished learning about the relationship between fractions, decimals, and percentages, consider reviewing these questions as a class and creating an anchor chart of common benchmark percents (e.g., 5%, 10%, 20%, 25%) and their equivalent decimals and fractions.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

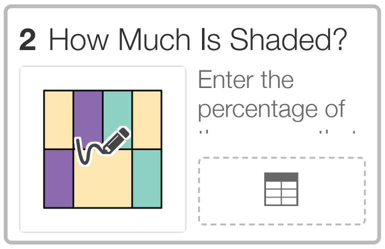
Figure A: 20%
Figure B: 50%
Figure C: 75%

Student Supports

Students With Disabilities

• *Visual-Spatial Processing: Visual Aids*
Provide printed copies of the representations for students to draw on or highlight.

• *Conceptual Processing: Eliminate Barriers*
Direct student attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.



Enter the percentage of the square that is shaded each color.

Teacher Moves

Activity 1 Purpose: On Screens 2–4, students develop strategies for figuring out what percent of a whole is shaded. This activity surfaces two specific strategies: 1) determining the percentage represented by equally sized parts and 2) multiplying the decimal representation by the total number of parts, though students may also use other strategies.

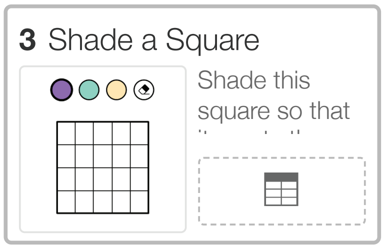
Facilitation: Arrange students into pairs. Consider starting with the activity paused and directing their attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson and unit. Note: If students type 15%, the calculator will write $15\% \text{ of}$.

Unpause and invite students to pay attention to the strategies they use to determine the percentage of the mosaic that is shaded each color. Then continue to the next screen.

Pacing: Consider using pacing to restrict students to Screens 2–4.

Sample Responses

Purple: 25%
Teal: 25%
Yellow: 50%



Shade this square so that it meets the requirements in the table below.

Teacher Moves

Facilitation: If needed, use the dashboard's student view to demonstrate how to select different colors and how to remove a color (either by double clicking on a part or selecting the eraser).

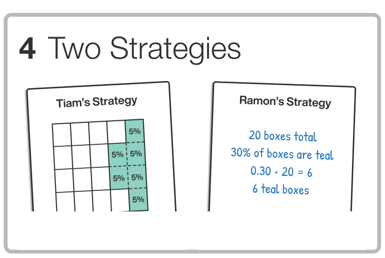
If students are having difficulty getting started, consider asking: *How would you know if a picture was* 25% *purple?*

While students are working, select and display several student squares using the snapshots tool in order to highlight students' creativity. If you are able to, look for students in your class who use each of the strategies that will be presented on Screen 4, and ask them to show their thinking on the board or use the snapshots tool to display their thinking.

Sample Responses

Responses vary.

- 5 boxes will be shaded purple (potentially a full row).
- 6 boxes will be shaded teal.
- 9 boxes will be shaded yellow.



Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to make sense of two strategies for determining the percentage of a number.

Facilitation: Give students 1–2 minutes to think quietly about the two strategies and one minute to discuss the prompt on the screen with a partner. Then, facilitate a whole-class discussion.

Consider beginning the discussion by asking if students used either of the strategies and renaming them after students in your class. Then, ask students to explain each strategy and how it could be applied to determine how to shade 45% yellow. If time permits, consider highlighting unique or creative mosaics from Screen 3 using the snapshots tool or the dashboard's teacher view. Ask the author to speak about their inspiration.

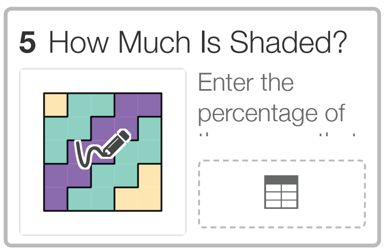
Question to push students' thinking: *In what situations might Tiam's strategy be more useful? What about Ramon's strategy?*

Early Finishers: Encourage students who have extra time before the whole-class discussion to discuss the advantages and disadvantages of each strategy.

Student Supports

Students With Disabilities
- *Executive Functioning: Graphic Organizers*

Provide a Venn diagram so students can compare the similarities and differences between the students' strategies.



Enter the percentage of the square that is shaded each color.

Teacher Moves

Activity 2 Purpose: On Screens 5–6, students practice calculating the percentage of a quantity and express their creativity by creating their own mosaics.

Facilitation: Invite students to use the strategies discussed to figure out the percentage of the square that is shaded each color. If students are having difficulty getting started, consider encouraging them to think of the square as a grid made up of smaller boxes. Ask: *How many boxes is the square broken into, and how many boxes are each color?*

Students who successfully calculate the percentage of the square that is shaded each color should continue to the Challenge Creator on the next screen.

Pacing: Consider using pacing to restrict students to Screens 5–6.

Sample Responses

Purple: 36%
Teal: 48%
Yellow: 16%

Student Supports

Students With Disabilities
- *Executive Functioning: Graphic Organizers*

Provide a graphic organizer throughout this activity so students can organize and record their calculations to determine the percentage of the square that is shaded each color.

6 Class Gallery

Teacher Moves

Facilitation: In this Challenge Creator, students create their own mosaic, then calculate the percentage that it is shaded each color. Students are presented with one of their classmates' mosaics and are asked to determine the percentages.

Give students several minutes to create their own challenge and more time to solve the challenges of others. After a set amount of time, encourage students to also take some time to review responses to their own. While students are working, monitor for creative challenges and solutions.

Note: We anticipate this Challenge Creator may take 15 minutes or more.

7 Lesson Synthesis

Describe a strategy to visualize and calculate 20% of 40.

Use the sketch tool if it helps you to show your thinking.

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

Lesson Synthesis Purpose: On this screen, students consolidate and refine their ideas about how to visualize and calculate the percentage of a number.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. While students are working, monitor for students who:

- Create an 8-by-5 or 4-by-10 rectangle and mark 8 of the 40 squares.
- Create a rectangle divided into 5 parts, and mark or write that each part represents 8.

Ask selected students who used each strategy to share, sequenced in this order. Follow with a brief whole-class discussion in which students share connections they see between responses.

If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Readiness Check: If Problem 5 of the Readiness Check suggests that some students have unfinished learning about calculating the percentage of a number, consider spending extra time here and asking a question like: *How could we use this strategy to calculate* 20% *of* 80 *or* 20% *of* 150*?*

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- I can make a rectangle that is 8-by-5 since that has 40 squares. Because 20% is the same as $\frac{1}{5}$, each one of the five rows of my rectangle represents 20% of the whole rectangle. Each of the five rows has 8 squares, so 20% of 40 is 8.

- Because 20% is equivalent to $\frac{1}{5}$, I can make a box around 5 squares. Each one of those squares represents 20% of the box. Since that box itself represents 40, each of the squares represents $40 \div 5$ or 8.

Student Supports

Students With Disabilities
- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors and allow students who struggle with fine motor skills to dictate physical manipulation of the sketch tool as needed.

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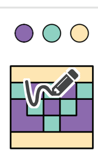
8 Cool-Down



What percentage of

$f(x)$

What percentage of the square is yellow?

Teacher Moves

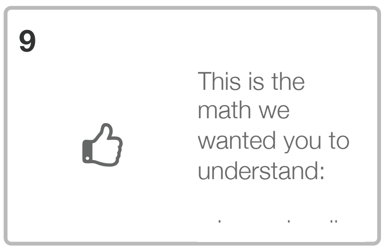
Support for Future Learning: If students are struggling to calculate the percentage that is shaded yellow, consider reviewing questions similar to the warm-up and the cool down as students work on Lessons 2 and 3 to prepare them to revisit percentages in Lesson 4.

Pacing: Consider using pacing to restrict students to Screens 8–9.

Sample Responses

40%

Explanations vary. There are five rows in the design. Therefore, each row makes up 20%. Since two full rows are yellow, the design is 40% yellow.



This is the math we wanted you to understand:

- I can visualize a percentage on a grid.
- I can calculate the percentage of a number.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Peach Cobbler (NYC)

Lesson 2: Rates and Ratios With Fractions

Purpose

In this lesson, students use strategies from Unit 2 to compare relationships that involve fractional quantities. Students use constants of proportionality to decide which of three recipes is the least sweet and to scale up a recipe to serve different numbers of people.

Preparation

Worksheet

- *Activity 1 and 2*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Warm-Up (5 minutes)

Purpose: Surface students' strategies for dividing whole numbers and fractions into fractional parts. Students will use these strategies to calculate the amount of sugar per serving in Activity 1.

Facilitation: Display each of the four Teacher Project Sheets one at a time. Use the instructional routine Number Talk to help students look for and make use of the structure of the expressions to develop and name strategies (MP7).

Readiness Check: If most students struggled with Problems 1 and 2 of the Readiness Check, consider reviewing these questions as a class and highlighting several different strategies students used, including strategies that involve calculating a unit rate. Consider creating a display of student strategies for students to reference as they engage in this unit.

Activity 1: Which Recipe? (15 minutes)

Purpose: Students apply what they learned in Unit 2 about constants of proportionality to make an argument about recipes involving fractional quantities (MP3).

Facilitation: Arrange students into pairs. Consider starting the activity by asking students about their experiences using recipes. Then, invite students to spend a minute comparing the three recipes. Give them 5–10 minutes to answer Problems 1–3.

If students are having difficulty getting started, consider sharing that one serving is the amount for one person. Ask a question like: *How much sugar is in one serving of Recipe A?* Some students may benefit from manipulatives, or examples of tape diagrams or double number lines as they reason with fractional quantities.

Early Student Thinking: Some students may believe that Recipe C is the least sweet because they are considering the total amount of sugar. Encourage these students to think about whether or not one serving of Recipe C would be less sweet, sweeter, or just as sweet as one serving of Recipe A.

While students are working, monitor for students who use different strategies to compare the recipes, especially students who determine the amount of sugar per serving in each recipe.

Then, facilitate a whole class discussion to 1) create a class argument for which recipe is least sweet (MP3) and 2) surface strategies for determining the amount of sugar per serving in each recipe. Consider asking several students to record their strategies publicly for other students to refer to throughout the lesson and naming the strategies after these students. If it does not come up naturally, consider asking: *Why is the amount of sugar per serving an example of a constant of proportionality?*

Early Finishers: Encourage students who finish Problems 1–3 early to choose one of the other recipes to adjust so that it is just as sweet as the one they chose for Amara.

Activity 2: Adjusting a Recipe (15 minutes)

Purpose: Students use a constant of proportionality to adjust a recipe involving fractional quantities.

Facilitation: Invite students to choose one of the strategies discussed during Activity 1 to help a different student, Jamar, make Recipe B for his family. Give students 5–10 minutes to answer Problems 1–3 of Activity 2. Consider encouraging students to use their knowledge about their own families to decide how many servings they think Jamar should make.

Then, facilitate a whole-class discussion around Problems 2 and 3. Consider highlighting several students' recipes using different numbers of servings and asking students what they notice and wonder. If it does not come up naturally, consider asking students what examples of proportional relationships they see in each recipe.

Spend adequate time surfacing strategies that students used to determine how many spoonfuls of lemon juice Jamar needs. Students may divide the amount of lemon juice by $\frac{1}{8}$ or multiply both quantities by 8.

Lesson Synthesis (5 minutes)

Purpose: Students describe strategies for comparing relationships that involve fractional quantities.

Facilitation: Give students 2–3 minutes to respond and a minute to share their strategies with a partner. Follow with a brief whole-class discussion in which students describe their strategies and make connections between strategies. If it does not come up naturally, consider highlighting a strategy that involves calculating the number of cups of carrots needed for each liter of soup.

If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Cool-Down (5 minutes)

Support for Future Learning: If students struggle to compare two relationships with fractions, consider reviewing the cool-down as a class before students begin Lesson 3.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Sticker Sizes

Lesson 3: Revisiting Proportional Relationships

Overview

Students use tools to determine unknown values in proportional relationships, some of which involve fractional quantities. This lesson ties back to what students learned about scaled copies in Unit 1 and proportional relationships in Unit 2.

Learning Goals

- Determine an unknown value in a proportional relationship using a table.

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to use tools to determine unknown values in proportional relationships, some of which involve fractional quantities. This lesson ties back to what students learned about scaled copies in Unit 1 and proportional relationships in Unit 2.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to the situation of creating different-size stickers from a logo and to recall what they know about scaled copies.

Activity 1: Scaling Stickers (15 minutes)

The purpose of this activity is for students to make connections between scale factor and constant of proportionality, and discuss when each is useful for figuring out missing measurements. Students use tables to organize their thinking and discuss different tools and strategies for figuring out the appropriate height and width of stickers of different sizes.

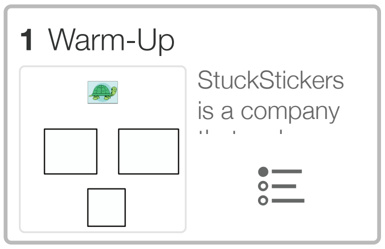
Activity 2: Sticker Sheets (15 minutes)

The purpose of this activity is for students to practice the strategies surfaced in Activity 1 for determining an unknown value in a proportional relationship that involves fractional quantities. Students use the cost for a couple of sheets of stickers to predict the cost of more stickers and determine how many sheets can be purchased given a target cost.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their ideas about using a table to determine missing values.

Cool-Down (5 minutes)



StuckStickers is a company that makes stickers of all different sizes.

Which size will make a scaled copy of this logo?

Teacher Moves

Lesson Overview: The purpose of this lesson is to use constants of proportionality as tools to determine unknown values in proportional relationships, some of which involve fractional quantities. This lesson ties back to what students learned about scaled copies in Unit 1 and proportional relationships in Unit 2.

Warm-Up Purpose: Students are introduced to the scenario of creating different-size stickers from a logo and recall what they know about scaled copies.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and asking students if they have ever created their own logo or sticker. Then, unpause and invite students to select which size would be a scaled copy and defend their choice.

Display the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Consider selecting and sequencing 1–2 responses for each of the most popular choices to display, and invite those students to share their reasoning in order to resurface the properties of scaled copies: 1) you need to multiply by the same scale factor or 2) the sides need to be in the same ratio.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

6 in. by 8 in.

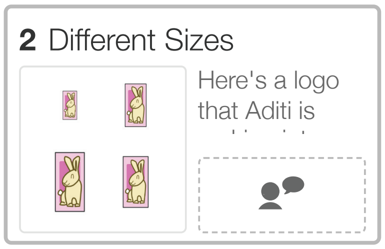
Explanations vary.

- This is the only sticker size where the image stays the same. In the others, the image is squished a little.
- This is the only sticker size where you can multiply the length and width of the original logo by 2 and get the larger version.

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Assist students in recognizing the connections between new problems and prior work. Students may benefit from a review of different representations to activate prior knowledge.



Here's a logo that Aditi is making into stickers.

Enter the missing values so that the logo looks the same on each sticker.

Then describe your strategy.

Teacher Moves

Activity 1 Purpose: On Screens 2–5, students make connections between scale factor and constant of proportionality, and discuss when each is useful for figuring out missing measurements. Students use tables to organize their thinking.

Facilitation: Consider starting with the activity paused and using the dashboard's student view to demonstrate that you can type mixed fractions; type the integer part, then a space, and then the fractional part. Next, unpause and invite students to figure out the missing measurements and describe their strategy (MP8).

Encourage students to read others' responses or discuss their response with a partner, and decide if others' strategies were similar to or different from their own.

If possible, consider monitoring for students who use strategies similar to the ones described on Screen 4 in order to make connections between these strategies later in the lesson.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Left image: 11

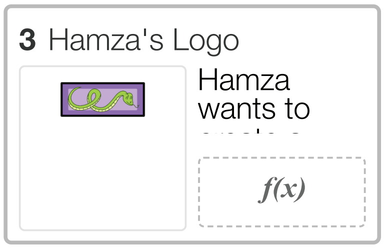
Right image: $3\frac{1}{5}$

Explanations vary. I noticed that the height is 2 times as long as the width. So, to find the height, I multiplied the width by 2.

Student Supports

Multilingual Learners
- *MLR 7 (Compare and Connect)*

After students share their approaches for missing measurements, ask groups to discuss *what is similar* and *what is different* between the approaches. Ask students to describe what worked well with their approach and what might make an approach more complete or easier to understand.



Hamza wants to create a bumper sticker with his design.

Bumper stickers usually have a height of $3\frac{1}{2}$ inches.

What will the width of his bumper sticker be?

Teacher Moves

Facilitation: If students are having difficulty getting started, consider asking: *How is the goal of this screen similar to or different from the previous screen?*

The purpose of this screen is for students to try a problem before sharing tools and strategies. Monitor for students who create tables, double number lines, drawings, or use a calculator to support their thinking. (MP5)

If you notice students guessing and checking, encourage them to continue to Screen 4, read the strategies there, and then revisit their work on this screen.

Pacing: Consider using pacing to restrict students to Screens 3–5.

Sample Responses

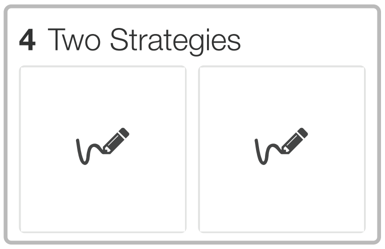
$8\frac{3}{4}$ inches

Student Supports

Students With Disabilities

• *Conceptual Processing: Processing Time*

Check in with individual students, as needed, to assess for comprehension during each step of the activity.



Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to make sense of two strategies for using tables to calculate unknown values with fractional quantities.

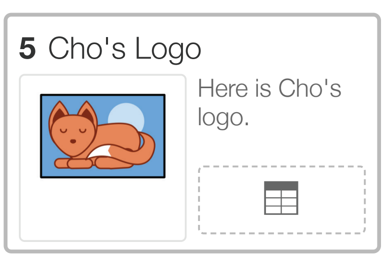
Facilitation: Give students 1–2 minutes to think quietly about the two strategies and one minute to discuss the prompt on the screen with a partner. Then, facilitate a whole-class discussion.

Consider beginning the discussion by asking if students used either of the strategies and renaming them after students in your class. Then ask students to explain each strategy. This is also an opportunity for students to share other tools that they used to determine the width of Hamza's bumper sticker (tape diagram, double number line, etc.)

If it does not come up naturally, connect each strategy to words from previous units ("scale factor" for Polina's Strategy and "constant of proportionality" for Jamir's).

Question to push students' thinking: *In what situations might Polina's strategy be more useful? What about Jamir's strategy?*

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.



Here is Cho's logo.

They want to create stamps of their logo.

If the stamp is $\frac{3}{5}$ inches wide, what does the height need to be?

Teacher Moves

Progress Check: This is a great place to check students' progress calculating unknown values in proportional relationships. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

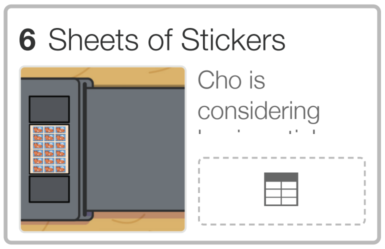
If it has not come up already, consider displaying and discussing visual tools students may have learned previously, like tape diagrams or double number lines.

Early Finishers: Encourage students who finish Screens 2–5 early to use different tool and to determine which tool is more effective for this problem and why. (MP5)

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Sample Responses

$\frac{2}{5}$



Cho is considering buying stickers by the sheet.

Four sheets cost $\$14$.

How much would $11\frac{1}{2}$ sheets cost?

Teacher Moves

Activity 2 Purpose: On Screens 6–9, students practice the strategies surfaced in Activity 1 for determining an unknown value in a proportional relationship that involves fractional quantities.

Facilitation: Consider starting with the activity paused and using the dashboard's student view to show how the visual feedback works, asking students to interpret how the printer is figuring out how many sheets to print. Then, unpause and invite students to figure out how much $11\frac{1}{2}$ sheets would cost.

Pacing: Consider using pacing to restrict students to Screens 6–9.

Sample Responses

$\$40.25$

Student Supports

Students With Disabilities

• *Memory: Processing Time*

Provide sticky notes or mini whiteboards to aid students with working memory challenges.

Cho has $\$70$ budgeted for stickers.

How many sheets can they purchase?

Teacher Moves

Progress Check: This is a great place to check students' progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

20 sheets of stickers

---

8 Sheets of Stickers

Which equations

Which equations could represent the relationship between the total cost, t, and number of sheets of stickers, s?

Teacher Moves

This is a possible discussion screen.

Facilitation: When students have responded to this screen, give them one minute to think of one equation they said could represent the relationship and one relationship they said could not and to share their reasoning with a classmate.

Then, facilitate a brief whole-class discussion to surface what features are important when writing the equation for a proportional relationship.

Consider ending the discussion by asking: *Why might an equation be useful in this situation?*

Sample Responses

$$t = \frac{7}{2}s$$

$$s = \frac{2}{7}t$$

---

9 Are You Ready for M…

It takes about $1\frac{1}{4}$ seconds for light to travel from Earth to the Moon.

It takes about $1\frac{1}{4}$ seconds for light to travel from Earth to the Moon.

The Moon is $238\,000$ miles from Earth.

The Sun is $94\,500\,000$ miles from Earth.

On paper, determine about how long it would take for light to get from the Sun to Earth. You may use a calculator.

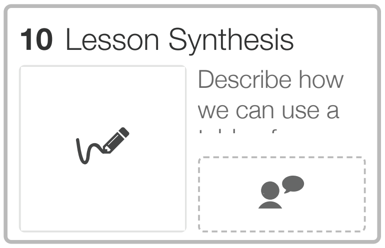
Teacher Moves

Facilitation: This screen is designed as an extra challenge for students who finish Screens 6–8 before the class discussion on Screen 8.

Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

About 500 seconds (or $8\frac{1}{3}$ minutes)



Describe how we can use a table of a proportional relationship to determine missing values.

Use the table on the left if it helps you with your thinking.

Teacher Moves

This is a key discussion screen.

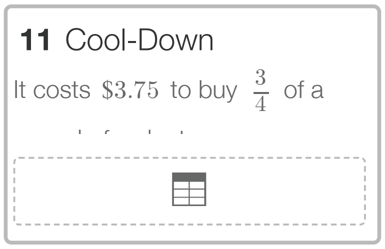
Lesson Synthesis Purpose: Students consolidate and refine their ideas about using a table to determine missing values.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. You can figure out a constant of proportionality using the top row. In this table, $\frac{1}{2}$ is a constant of proportionality. Then you can use that to either multiply or divide to figure out the missing number in the second row.



It costs $\$3.75$ to buy $\frac{3}{4}$ of a pound of walnuts.

How many pounds of walnuts can you purchase with $\$11.25$?

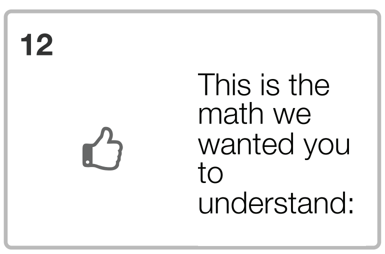
Teacher Moves

Support for Future Learning: Students will continue to use tables to calculate unknown values in the remainder of the unit.

Pacing: Consider using pacing to restrict students to Screens 11–12.

Sample Responses

$2\frac{1}{4}$ (or equivalent)



This is the math we wanted you to understand:

- I can use a table to determine an unknown value in a proportional relationship.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

More and Less

Lesson 4: Percent Increase and Decrease With Tape Diagrams

Overview

Students visualize what it means to increase or decrease by a percentage, and use that visual to help them calculate unknown values.

Learning Goals

- Use tape diagrams and tables to represent situations that involve adding or subtracting a percent from the initial value (or from 100%).
- Determine the new amount given the original amount and a percent increase or decrease.

Vocabulary

- percent increase
- percent decrease

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to visualize what it means to increase or decrease by a percentage, and use that visual to help them calculate unknown values. They see both tape diagrams and tables used to represent percent increase and percent decrease, and they use these representations to solve problems.

Lesson Summary

Warm-Up (5 minutes)
The purpose of the warm-up is for students to make sense of the structure of tape diagrams, which they may have seen in Grade 6. Students will consider how to use these diagrams to represent a relationship between two variables in the rest of the lesson.

Activity 1: Granola Bars (20 minutes)
The purpose of this activity is for students to visualize percent increase and decrease, and answer questions using the context of granola bars. Students can solve the problems in this activity using any representation, including but not limited to tape diagrams, double number lines, and tables.

Activity 2: Calculating Percentages (10 minutes)
The purpose of this activity is for students to apply the concepts from Activity 1 to new contexts. Students also distinguish between a percentage of a number and percent change.

Lesson Synthesis (5 minutes)
The purpose of the synthesis is for students to consolidate and refine their ideas about strategies for calculating percent increase or decrease.

Cool-Down (5 minutes)

1 Warm-Up

Which one doesn't belong?

Which one doesn't belong?

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to visualize what it means to increase or decrease by a percentage, and use that visual to help them calculate unknown values.

Warm-Up Purpose: Students make sense of the structure of tape diagrams, which they may have seen in Grade 6.

Facilitation: Arrange students into pairs. Use the routine Which One Doesn't Belong to support students in noticing the structure of tape diagrams, and validate different noticings. If it does not come up naturally, invite students to verbally express each diagram as an equation describing the relationship between x and y. Consider asking students: *What is an equation that represents each diagram?*

See the sample responses for ideas and questions that may surface during the discussion.

Early Finishers: Encourage students who finish early to write two equations for each relationship between x and y.

Readiness Check: If Problem 6 of the Readiness Check suggests that some students struggle with reasoning about percent increase or decrease of an original amount, consider reviewing this problem as a class before engaging in this lesson and asking a question like: *How much total sugar does it have now?*

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- **Upper left:** The only one where y is a square. $y = \frac{4}{3}x$ or $x = \frac{3}{4}y$
- **Upper right:** The only one where x is exactly half of y. $y = 2x$ or $x = \frac{1}{2}y$
- **Lower left:** The only one where x is longer than y. $y = \frac{4}{5}x$ or $x = \frac{5}{4}y$

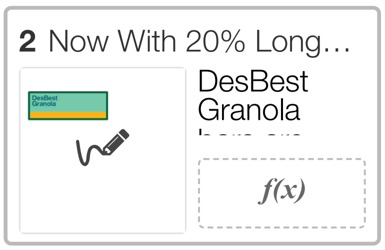
- **Lower right:** The only one where x and y are the same length. $y = x$

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., ____________ doesn't belong because _____________.).



DesBest Granola bars are now 20% longer.

If the original bar was 15 centimeters long, how long is the new granola bar?

Use paper if it helps you with your thinking.

Teacher Moves

Activity 1 Purpose: On Screens 2–6, students visualize percent increase and decrease, and answer questions using the context of granola bars.

Facilitation: Consider starting with the activity paused and asking students where they have seen phrases like "20% more" or "20% longer" in their lives. Then, unpause and invite students to figure out how long the new granola bar is before continuing on to the next screen.

If students are having difficulty getting started, consider asking: *How long is a 20% piece of this granola bar? How do you know?*

This may be the first time students encounter a percent increase. The feedback on this screen is designed to support students in visualizing what it means to grow by a percentage. Encourage students to use blank paper and the feedback on the screen to help them to revise their thinking.

Pacing: Consider using pacing to restrict students to Screens 2–6.

Sample Responses

18 centimeters

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*
Check in with individual students, as needed, to assess for comprehension during each step of the activity.

- *Receptive Language: Processing Time*
Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.

Here is how DeAndre found the length of the 15-centimeter granola bar after a 20% increase.

Explain what he may have been thinking.

Teacher Moves

This is a possible discussion screen.

Facilitation: Identify any students who used a strategy similar to DeAndre's. Consider renaming the strategy after these students.

Encourage students to read others' responses and/or discuss their response with a partner, and decide if others' explanations were similar to or different from their own.

Then facilitate a brief discussion around questions like: *Why do you think DeAndre decided to divide the rectangle into fifths? Where did the number 3 come from?*

Sample Responses

Responses vary. Since 5 copies of 20% make 100%, DeAndre split 15 cm into 5 pieces that were 3 cm each. Since this bar has 20% more, it is 3 more than 15, or 18.

Student Supports

Multilingual Learners

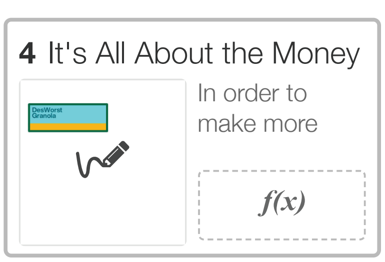
• *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., DeAndre may have thought that ________.).

Students With Disabilities

• *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the image to review prior to implementation of this activity.



In order to make more money, DesWorst Granola bars are now 10% shorter.

If the original bar was 15 centimeters long, how long is the new granola bar?

Use paper if it helps you with your thinking.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface different tools and strategies students used to determine 10% less than 15.

Facilitation: While students are working, monitor for different expressions students use and select them using the snapshots tool. (MP5)

When most students have completed this screen, facilitate a whole-class discussion. Ask students to justify their strategy and critique each other's reasoning.

Consider asking students: *How did you know whether to add or subtract? How do you know how to use the* 1.5 *cm to find the length of the new granola bar? How did your expression help to answer the question?*

Question to push students' thinking: *If the new granola bar were* 1.5 *cm long, what percent decrease would DesWorst have used?*

Sample Responses

13.5 centimeters

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*
Check in with individual students, as needed, to assess for comprehension during each step of the activity.

- *Receptive Language: Processing Time*
Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.

Here is how Afia found the length of the 15-centimeter granola bar after a 10% decrease.

Explain what she may have been thinking.

Teacher Moves

Facilitation: Identify any students who used a strategy similar to Afia's. Consider renaming the strategy after these students.

Encourage students to read others' responses and/or discuss their response with a partner, and decide if others' explanations were similar to or different from their own.

If time allows, consider asking students: *How is this table similar to or different from tables we have used so far this year?*

Sample Responses

Responses vary. She figured out that 10% less is another way of saying 90% there, so she multiplied by the constant of proportionality 0.9 to figure out how long the new bar was.

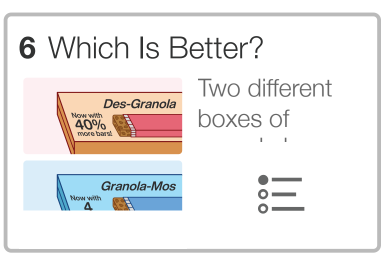
Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., Afia may have thought that ________.).

Students With Disabilities
- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity.



Two different boxes of granola bars are the same price.

One box has 40% more bars; the other has 4 more bars.

Which is a better deal?

Teacher Moves

Facilitation: Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. If time allows, consider highlighting one response from a student who selected each option. If it does not come up naturally, consider asking: *What information do we need in order to figure out how many bars are* 40% *more?*

Sample Responses

Not enough information

Explanations vary.

- I don't know how many bars there are, so I don't know how much 40% more is.
- If the number of bars is small, then 4 more bars is better. If the number of bars is large, like 20, then 40% would be better, so it depends.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time. This may include reading the information in the table and in the image.

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7 Card Sort

Teacher Moves

This is a possible discussion screen.

Activity 2 Purpose: On Screens 7–10, students apply what they learned in Activity 1 to new contexts. Students also distinguish between a percentage of a number and percent change.

Facilitation: Consider starting with the activity paused and asking students what they notice about the two diagrams. Then, unpause and invite students to match each situation to one of the diagrams (MP2) and then continue to the next screen. Encourage students to discuss what words or phrases clue them about whether the scenario represents percent increase or decrease.

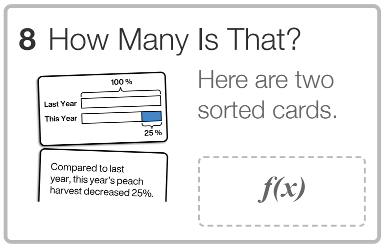
When most students have completed the card sort, pace all students to this screen and facilitate a brief discussion about what phrases helped students figure out where to match each scenario.

Routine (optional): Consider using the mathematical language routine Collect and Display to capture phrases that indicate percent increase or decrease.

Pacing: Consider using pacing to restrict students to Screens 7–10.

Sample Responses

Image solution



Here are two sorted cards.

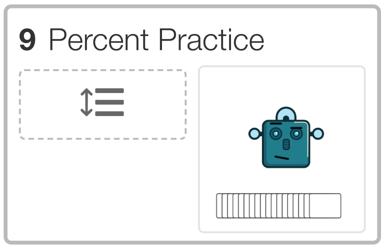
If 80 pounds of peaches were harvested last year, how many pounds were harvested this year?

Teacher Moves

Progress Check: This is a great place to check students' progress with using strategies for calculating the amount after a percent decrease. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

60 pounds



Teacher Moves

Facilitation: Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

Monitor for students who use different tools to support their thinking, such as a table, a tape diagram, or a calculator. Consider pausing the class to share and discuss new strategies or tools that emerge. (MP5)

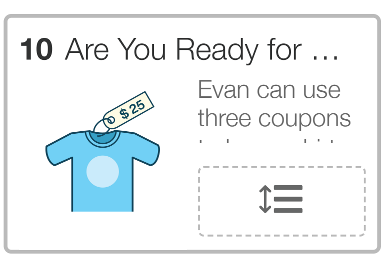
Sample Responses

From least to greatest:
- 25% of 20
- 75% less than 60
- 100% of 20
- 25% more than 20
- 25 more than 20

Student Supports

Students With Disabilities
- *Memory: Processing Time*

Provide sticky notes or mini whiteboards to aid students with working memory challenges.



Evan can use three coupons to buy a shirt.

The store will apply his coupons one at a time.

Order his coupons so that he gets the lowest price.

Teacher Moves

Facilitation: This screen is designed as an extra challenge for students who finish Screens 7–9 before the class discussion on Screen 11. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

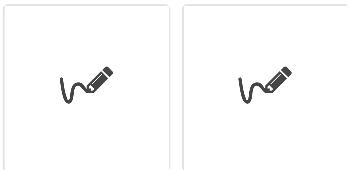
There are two correct responses.

First coupon to last coupon: 20% off, 5% off, $\$10$ off or

First coupon to last coupon: 5% off, 20% off, $\$10$ off

Explanations vary. We want the biggest discount, so the $\%$ offs should use the largest number, which is at the beginning.

11 Lesson Synthesis



Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their ideas about strategies for calculating percent increase or decrease.

Facilitation: Give students 2–3 minutes to think quietly and one minute to share their responses with a partner. Then, ask several students to share how the strategies are similar and different. Consider asking: *Where do we see the* 40% *in each strategy?*

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. These strategies are the same because they both calculate 60% of $\$30$. In the tape diagram, you think of 60% as $\frac{3}{5}$ and figure out that fraction of $\$30$. In the table, you multiply $\$30$ by 0.6, which is another way of writing 60%.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students discuss similarities and differences in the strategies (e.g., These strategies are the same because ________. These strategies are different because ________.).

Students With Disabilities

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this synthesis.

12 Cool-Down

The number of fish in a pond decreased by 10% this year

$f(x)$

The number of fish in a pond decreased by 10% this year compared to last year.

Last year, there were 60 fish in the pond.

How many fish are in the pond this year?

Teacher Moves

Support for Future Learning: This idea will be reinforced over the next several lessons.

Pacing: Consider using pacing to restrict students to Screens 12–13.

Sample Responses

54 fish

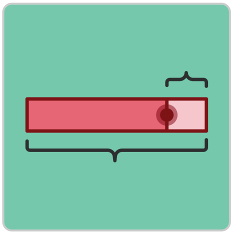
13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use tape diagrams and tables to represent adding or subtracting a percentage from 100%.
- I can determine the new amount if I know the original amount and the percent change.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



All the Equations

Lesson 5: Percent Increase and Decrease With Equations

Overview

Students represent situations involving percent increase or decrease using equations and make connections between different ways to write that equation.

Learning Goals

- Use equations to represent situations involving a percent increase or decrease.

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to represent situations involving percent increase or decrease using equations and to make connections between different ways to write that equation. The focus in this unit is for students to write equations and understand their connection to a context. In a later unit, students will use equations to solve problems.

Lesson Summary

Warm-Up (5 minutes)
The purpose of this warm-up is for students to visualize increasing and decreasing by the percentages they will represent with equations in the next activity.

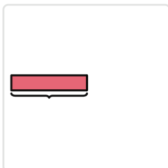
Activity 1: Writing Equations (15 minutes)
The purpose of this activity is for students to use what they know about representing situations involving percent increase and decrease to make sense of equations for these situations. This activity ends with students writing their own equation to represent a situation.

Activity 2: Using Equations (15 minutes)
The purpose of this activity is for students to connect equations to percent increase and decrease situations. Students will then use these equations to deepen their understanding of the connection between the equation and the context.

Lesson Synthesis (5 minutes)
The purpose of the synthesis is for students to connect equations to situations involving increasing or decreasing by a percentage.

Cool-Down (5 minutes)

1 Warm-Up



Drag the point to make this rectangle 72% shorter.

Drag the point to make this rectangle 72% shorter.

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to represent situations involving percent increase or decrease using equations and to make connections between different ways to write that equation.

Warm-Up Purpose: Students visualize increasing and decreasing by the percentages they will represent with equations in Activity 1.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and asking students to estimate how long the new rectangle will be: *Will the amount shaded be larger or smaller than half of the larger rectangle?* It's okay—even desirable—to lack consensus at this stage. Then, unpause and invite students to drag the movable point.

Note: The purpose of this screen is for students to get a sense of what 72% shorter looks like. Encourage students who get close to 72% to continue to the next screen.

Readiness Check: If Problem 7 of the Readiness Check suggests that some students have unfinished learning around equivalent expressions and the distributive property, consider reviewing this question as a class and creating an anchor chart of this question as an example of which expressions are and are not equivalent.

Pacing: Consider using pacing to restrict students to Screens 1–2.

Sample Responses

Image solution

Student Supports

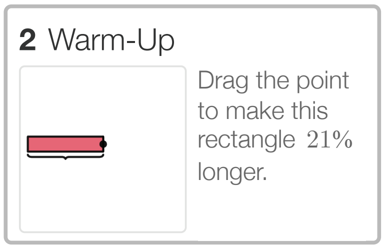
Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Direct students' attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



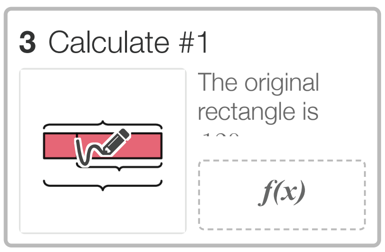
Drag the point to make this rectangle 21% longer.

Teacher Moves

Early Finishers: Encourage students who finish Screens 1–2 early to drag the point to a new spot and guess what percentage longer or shorter the new rectangle is than the original.

Sample Responses

Image solution



The original rectangle is 120 centimeters.

Calculate the length of a rectangle that is 72% shorter.

Teacher Moves

Activity 1 Purpose: On Screens 3–6, students use what they know about using tape diagrams to represent situations involving percent increase and decrease to make sense of equations for these situations. This activity ends with students writing their own equation to represent a situation.

Facilitation: This is the first time in this unit where students are calculating a length after a percent decrease that does not use a benchmark percentage (10%, 20%, etc.). Consider starting with the activity paused and reviewing how to calculate the percentage of a value.

If students are having difficulty getting started, consider asking them how they would approach the problem if the percentage were 70% or 75% in order to draw out strategies. Encourage students to use blank paper and the feedback to help them revise their thinking.

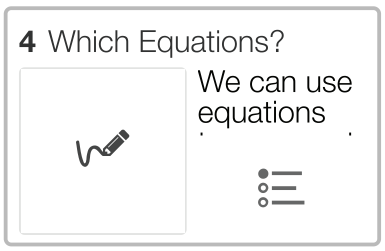
Monitor for students who use the following strategies to calculate the length of the shorter rectangle:
- Calculate the remaining percentage of the rectangle's length (28%).
- Calculate 72% of 120 and subtract it from 120.
- Represent 72% as the decimal 0.72 or represent 28% as the decimal 0.28.

Consider asking these students to record their thinking for other students to see and refer to during the discussion on Screen 4.

Pacing: Consider using pacing to restrict students to Screens 3–4.

Sample Responses

33.6 centimeters



We can use equations to represent relationships involving percent increase and decrease.

Select the **three** equations that represent the relationship between the length of the original rectangle, b, and the length of the new rectangle, c.

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

The purpose of this discussion is to make connections between different equations that represent the same scenario.

Facilitation: Encourage students to share their reasoning for each equation with a partner and to work together to reach an agreement about which three equations represent the relationship. If students are having difficulty getting started, consider inviting them to test the values 120 and 33.6 from the previous screen in different equations and see what they notice.

When most students have selected three equations, display the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Discuss each equation one by one. Consider asking a student who selected that equation to explain their reasoning. Then discuss whether or not the equation would represent the situation and why. Consider recording correct equations that emerge for all students to reference throughout the unit.

Consider ending the discussion with a question like: *How are each of the three correct equations related?*

Early Finishers: Encourage students who finish Screens 3–4 early to test the equations they chose by creating their own rectangle, b, and determining the length of the shorter rectangle, c.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Sample Responses

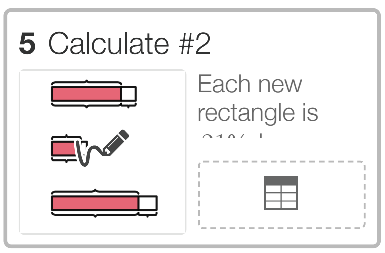
- $c = 0.28b$
- $(1 - 0.72)b = c$
- $c = 1b - 0.72b$

Student Supports

Multilingual Learners

- *MLR 7 (Compare and Connect)*

After students share their selections for which equations represent this relationship, ask groups to discuss *what is similar* and *what is different* between the equations. Ask students to describe where they see each part of the equations in the image.



Each new rectangle is 21% longer than the original.

Complete the table with the length of each new rectangle.

Teacher Moves

Facilitation: Invite students to use any strategies that have surfaced so far to calculate the length of these new rectangles. Students will use repeated reasoning (MP8) to calculate each new length before they write an equation on the next screen.

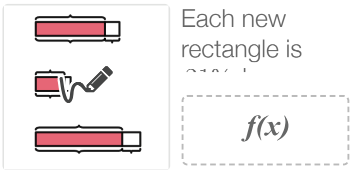
Progress Check: This is a great place to check students' progress on calculating percent increases. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Pacing: Consider using pacing to restrict students to Screens 5–6.

Sample Responses

- 145.2 cm
- 60.5 cm
- 181.5 cm

6 Write an Equation



Each new rectangle is 21% longer than the original.

Write an equation to represent the relationship between the length of an original rectangle, b, and the length of the new rectangle, c.

Teacher Moves

This is a possible discussion screen.

Facilitation: This relationship is challenging, and students may need support writing an equation to represent it. Students will have several more opportunities to practice over the subsequent lessons. Encourage them to submit their equation even if they are unsure; they can edit their equation if the rectangles are not the correct lengths.

Select and sequence several different correct and incorrect equations. Then, facilitate a brief discussion about whether or not each equation represents the relationship and why. If it does not come up during the activity, ask students: *Are the relationships in this lesson proportional? Why or why not?*

Early Finishers: Encourage students who finish Screens 5–6 early to write a different correct equation for this relationship.

Note: Students who do not use the variables b and c in their equation will see a ⚠ symbol.

Sample Responses

Responses vary.

- $c = 1.21b$
- $c = 1b + 0.21b$
- $c = (1 + 0.21)b$

---

7 Card Sort

Teacher Moves

Activity 2 Purpose: On Screens 7–11, students use equations similar to the ones in Activity 1 to determine unknown values.

Facilitation: Encourage students to share their reasoning with a partner and to work together to reach an agreement about how to sort the cards. If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Note: Students will use the equations from the card sort screen to answer questions later. The following screens contain information to support students who may not have sorted the cards correctly.

Pacing: Consider using pacing to restrict students to Screens 7–11.

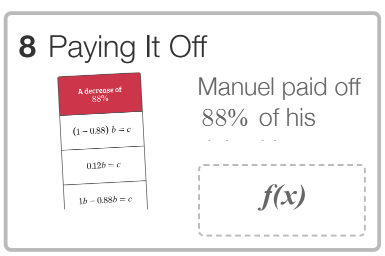
Sample Responses

Image solution

Student Supports

Multilingual Learners

• *Speaking: MLR 8 Discussion Supports.*
Use this routine to support whole-class discussion. Provide sentence frames for students to use when they share the equation they matched to each situation (e.g., Situation ____ matches with equation ____ because ____). Call students' attention to how the percent increase or decrease is represented in the equation and the situation.



Manuel paid off 88% of his debt. He originally owed $\$1950$. How much does Manuel owe now?

Use one of the equations from the card sort if that helps you with your thinking.

Teacher Moves

This is a possible discussion screen.

Facilitation: Consider encouraging students to decide whether the $\$1950$ represents b or c in each equation before determining how much Manuel owes.

While students are working, monitor for students who create tape diagrams, make tables, use the equation, or create their own strategy to determine how much Manuel owes. When students have completed this screen, facilitate a whole-class discussion to make sense of the three equations in the image and surface strategies for determining the new value.

Consider asking questions like:

• *Is the amount he owes now more or less than half of what he owed originally? How do you know?*

- *What does the 0.12 represent in the equation? Where does it come from?*

Sample Responses

234 dollars

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

Review an image or video, or lead a discussion about paying off debt in order to activate prior knowledge of the context.

- *Receptive Language: Processing Time*

Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.

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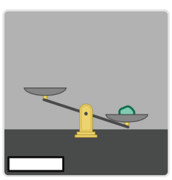
9 Are You Ready for M…



An astronaut brought a 16-

An astronaut brought a 16-gram sample of goo from another planet to her laboratory.

When the goo is exposed to light, the amount of goo increases by the same percentage every hour.

Complete the table.

Teacher Moves

Facilitation: This screen is designed as an extra challenge for students who finish Screens 7–8 before the class discussion on Screen 8. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

121.5

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10 Lesson Synthesis

Here are two equations that could be used to

Here are two equations that could be used to solve a problem about **percent increase** or **percent decrease**.

Select an equation and write a story about a situation it could represent.

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

Lesson Synthesis Purpose: Students connect equations to situations involving increasing or decreasing by a percentage.

Facilitation: Give students 2–3 minutes to respond and one minute to share their story with a partner.
Consider highlighting unique or creative contexts and asking the authors to speak about their inspiration.

Routine (optional): Consider using the routine Tell a Story to support students in using their knowledge and creativity to make sense of a situation.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- Did you know that if you do not cut hair, it can grow 25% longer in a month?
- My friend always eats 20% of whatever I bring for lunch.

---

11 Cool-Down

Jayla's bank account increased by 7% this year.

$f(x)$

Jayla's bank account increased by 7% this year.

Write an equation to represent the relationship between the amount that Jayla started with, b, and the amount she has now, c.

Teacher Moves

Support for Future Learning: If a few students struggle with this cool-down, offer them individual support during Activity 2 of Lesson 6. If a large portion of the class struggles, consider facilitating a whole-class discussion about the different equations representing a percent increase or decrease during the discussion for Problem 6 of Lesson 6, Activity 1.

Pacing: Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Responses vary.

- $1b + .07b = c$
- $1.07b = c$

- $c = (1 + 0.07)b$

12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can write an equation to represent adding or subtracting a percentage from 100%.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

100% (NYC)

Lesson 6: Percent Increase and Decrease With Double Number Lines

Purpose

The purpose of this lesson is for students to practice calculating the original amount, the new amount, or the percent change given the other two quantities. This lesson also helps students connect the double number line, which they may have encountered in Grade 6, to other representations.

Preparation

Student Worksheet

- *Activity 1–2:* Print one double-sided worksheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one double-sided half sheet for each student.

Warm-Up (10 minutes)

Purpose: Students interpret the language of situations involving percent increase and decrease, and connect these situations to double number lines.

Facilitation: Display Sheet 1 of the Teacher Projection Sheets. Use the first two reads described in the math language routine Three Reads to support students in making sense of the situation (MP1). If it does not come up naturally, consider asking students: *Is the 600 grams the weight before or after the increase? How do you know?*

Then, display Sheet 2 of the Teacher Projection Sheets. Invite students to share what they remember about these types of diagrams (double number lines). Give students 2–3 minutes to think quietly, then one minute to share their reasoning with a partner.

Follow with a whole-class discussion to explain what the parts of the double number line represent in this situation. Consider asking questions like:

- *Why does the 600 grams align with the 120%?*
- *What does the value above 100% represent?*

Readiness Check: If most students struggled with Problem 8 of the Readiness Check, consider reviewing this question as a class and creating an anchor chart that includes an example of a double number line.

Activity 1: Double Number Lines (15 minutes)

Purpose: Students use a double number line to calculate the original amount, the new amount, or the percent change given the other two quantities. This activity also reinforces the idea that the original amount is represented by 100%.

Facilitation: Here are several options for structuring student work time on Problems 1–4. They are intended to support students in making sense of situations involving percent change (MP1):

- **Option 1:** Arrange students into pairs: Partner A and Partner B. Partner A reads the question aloud. Partner B describes the important information in the situation and identifies what value represents 100% (sometimes this will be unknown). Both partners work together to complete the diagram and answer the question. Roles switch after each problem.
- **Option 2:** Arrange students into pairs. Give partners 3–5 minutes to answer Problem 1 together, reading the situation aloud and identifying the important information. When time is up, ask students to find a different partner and answer Problem 2. Repeat after each problem.
- **Option 3:** Arrange the class into six groups. Assign each group one of the six problems (alternatively, arrange the class into three groups, assigning two problems per group). Give groups 3–5 minutes to make sense of and answer their problem. Then, form new groups of six students, with one person who is an expert on each problem. Give them 5–10 minutes to answer the problems one by one, with the expert acting as a coach for their problem.

Once students have completed the front side of the worksheet, give them 3–5 minutes to discuss what the original and new amounts represent in Problems 5 and 6 and to create their own double number lines to answer the questions.

Once most students have completed Problem 6, display Sheet 3 of the Teacher Projection Sheets and facilitate a whole-class discussion to help students connect the double number line to other representations they have seen in this unit. Ask questions like:

- *Where do you see the same information in the double number line, table, and equation?*
- *Where do you see the 100% in each representation? What number does it align with?*
- *When might each representation be most useful? Why?*
- *Which representation made the most sense to you? Why?*

Activity 2: Green Sea Turtles (10 minutes)

Purpose: Students represent a situation three different ways and use the representations to calculate the original amount given the other two quantities.

Facilitation: Consider starting the activity by asking students why it might be helpful to make more than one representation for a situation. Then, invite students to create each representation and then answer the question.

Facilitate a brief whole-class discussion to highlight student thinking and to support students in connecting the representations. Consider selecting work that is incorrect and asking students: *How would the question need to be different for this work to be correct?*

Lesson Synthesis (5 minutes)

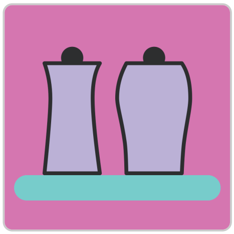
Purpose: Students solidify and refine their understanding of how to represent a percent increase.

Facilitation: Give students several minutes to think about and record their responses. Consider asking students to justify their reasoning about whose diagram is correct. Ask students: *How would the question need to be different for Abdullah's double number line to be correct?*

Cool-Down (5 minutes)

Connection to Future Learning: Double number lines will not be specifically addressed in future lessons. Students will have more opportunities to practice calculating original amounts, new amounts, and percent change using other representations in Lesson 7 and beyond.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Percent Machines

Lesson 7: Calculating Unknowns With Percentages

Overview

Students practice calculating the original value, the new value, or the percent change given the other two quantities. This is the first lesson in which students experience multistep percent problems, which will set them up for success when they make sense of tax and tip in Lesson 8.

Learning Goals

- Calculate the original amount given the new amount and a percent change for one-step and multistep percent problems.

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to practice calculating the original value, the new value, or the percent change given the other two quantities. Students practice this in the context of one-step and two-step percent machines. This is the first lesson in which students experience multistep percent problems, which will set them up for success when they make sense of tax and tip in Lesson 8.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to percent machines, which take an input value and increase or decrease that value by a percentage to produce an output. Students also reason about whether increasing and then decreasing by the same percentage balance each other out.

Activity 1: Percent Challenges (15 minutes)

The purpose of this activity is for students to practice calculating the new value and percentage, and then develop and analyze strategies for working backwards to calculate the original value given the new value and percent change.

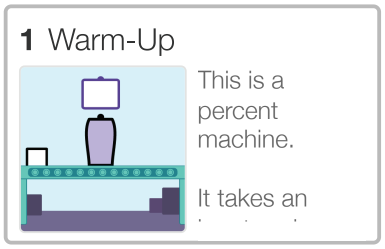
Activity 2: Challenge Creator (15 minutes)

The purpose of this activity is for students to express their creativity by creating their own challenge and to practice calculating the original value given the new value and a percent change.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their ideas about determining the original value given a percent change and the final value.

Cool-Down (5 minutes)



This is a percent machine.

It takes an input and increases it by 50% to make an output.

Press "Try It" to see what we mean.

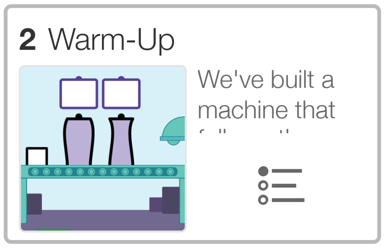
Teacher Moves

Lesson Overview: The purpose of this lesson is for students to practice calculating the original value, new value, or percent change given the other two quantities. This is also the first lesson in which students experience multi-step percent problems, which will set them up for success when they make sense of tax and tip in Lesson 8.

Warm-Up Purpose: On Screens 1–2, students are introduced to percent machines, which take an input value and increase or decrease that value by a percentage to produce an output. They also reason about whether increasing and then decreasing by the same percentage balance each other out.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and playing the animation for the class using the dashboard’s student view. Ask students how they think this percent machine works. Then, unpause and invite students to replay the video on their own or immediately continue to the next screen.

Pacing: Consider using pacing to restrict students to Screens 1–2.



We've built a machine that follows these instructions:

- Increase by 50%
- Decrease by 50%

Do you think the machine's output will be less than, greater than, or equal to the input?

Teacher Moves

This is a possible discussion screen.

Facilitation: Once students have had a chance to respond and explain their thinking, consider displaying the distribution of responses using the dashboard’s teacher view. Then facilitate a brief whole-class discussion to hear students' reasoning. Ask students to justify whether they believe the output will be greater than, less than, or equal to the input and to critique each other's reasoning (MP3).

It's okay—even desirable—to lack consensus at this stage. The activity will build toward consensus on Screen 3 where students will calculate one specific output of this two-step machine.

Early Finishers: Encourage students who finish early to test specific values to make their argument stronger and clearer.

Sample Responses

Responses and explanations vary.

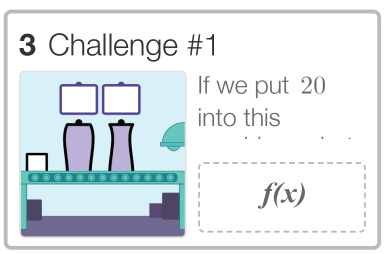
The output will be less than the input because we are decreasing a larger value, so the final value will be smaller.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time. This may include reading the information in the percent machine.



If we put 20 into this machine, what will come out?

Teacher Moves

Activity 1 Purpose: On Screens 3–7, students first practice calculating the new value and percentage, and then develop and analyze strategies for working backwards to calculate the original value given the new value and percent change.

Facilitation: Consider starting with the activity paused and sharing with students that they will encounter a variety of challenges with percent machines. Invite students to keep track of strategies they use that might be helpful to others in the class. Then, unpause and invite students to test their reasoning from the warm-up on Challenge #1 and then to continue through each challenge at their own pace.

Pacing: Consider using pacing to restrict students to Screens 3–7.

Sample Responses

15

Student Supports

Support for Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Direct student attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.

- *Executive Functioning: Graphic Organizers*

Provide students a graphic organizer to record the original value, the percent increase or decrease, and the new value so that they can organize their thinking on each challenge.

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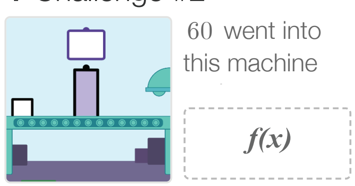
4 Challenge #2



60 went into this machine and 72 came out.

What percent increase did this machine use?

Teacher Moves

Progress Check: This is a great place to check students' progress on reasoning with percentages. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking: Some students may calculate $\frac{72}{60} = 1.2$ and enter that the percent increase as 2%. Invite these students to examine how increasing 60 by 2% changes its value and how that compares to the change they were looking for.

Sample Responses

20%

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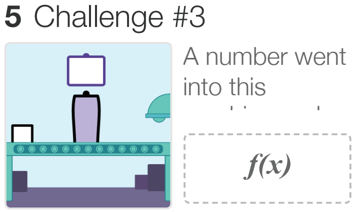
5 Challenge #3



A number went into this machine and 36 came out.

What number went in?

Teacher Moves

Facilitation: If students have difficulty getting started, consider asking whether they think the input is greater than, less than, or equal to 36. Invite students who are struggling after several attempts to continue to the next screen and then return to this screen to revise their response.

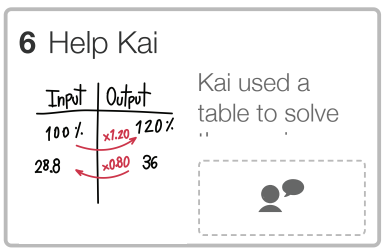
Monitor for students who use the following strategies to determine the original value:

- Double number line
- Tape diagram
- Table
- Equation (e.g., $36 = 1.2x$)

Consider capturing and displaying student thinking recorded on paper using the picture snapshots tool and waiting until the conversation on the following screen to discuss these strategies.

Sample Responses

30



Kai used a table to solve the previous challenge.

He made an error.

1. Describe something Kai did well.

2. Describe what you would change.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface multiple strategies for how to calculate an original value given a new value and a percent change. This is also an opportunity to clarify any confusion on Challenges #1–3 that may have arisen.

Facilitation: Once most students have completed this screen, consider pacing all students to this screen and pausing. Begin by surfacing all of the different things that Kai did well before asking students what they would change about Kai's solution and why (MP3).

If time allows, either display the picture snapshots of students' strategies from Screen 5 or ask students to describe how they determined the original value. Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Ideas that may surface during the discussion:

- Increasing by 20% and decreasing by 20% does not balance each other out.

- The constant of proportionality should be the same within each relationship (in this case, 1.20).
- It can be helpful to think of the input as 100%.
- You can divide by the constant of proportionality in order to work backwards to figure out the input.

Consider ending the discussion by asking students what they learned from analyzing incorrect thinking and inviting them to revise their response on Screen 5, if needed, before beginning Activity 2.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language.

Sample Responses

Responses vary.

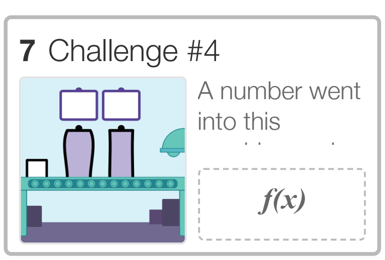
Something Kai did well:
- Figured out that the output was 120% of the original.
- Put the 36 in the output column of the table.
- Figured out that the constant of proportionality is 1.20.
- Got a value less than 36.

Something I would change:
- Keep the 1.2 (the constant of proportionality) the same in both rows.
- Divide by 1.2 to work backwards instead of taking away 20% to get 0.80.

Student Supports

Students With Disabilities
- *Executive Functioning: Visual Aids*

Create an anchor chart for public display that describes how to use a table to determine an unknown original value for future reference.



A number went into this machine and 57 came out.

What number went in?

Teacher Moves

Facilitation: The purpose of this screen is to introduce students to multi-step machines in which they need to determine the original value. These types of machines may arise in the Challenge Creator in Activity

2. If you are running short on time, consider skipping this screen or using it as a challenge for students who complete Screens 3–6 early in order to spend adequate time on Activity 2.

Sample Responses

40

---

8 Class Gallery

Teacher Moves

Activity 2 Purpose: Students express their creativity by creating their own challenge and also practice calculating the original value given the new value and a percent change.

Facilitation: Consider starting with the activity paused and sharing with students that they will create their own challenge and solve challenges created by their classmates. In this Challenge Creator, students select a one-step or two-step machine, decide on a percent change and output, and then calculate the input. Students are presented with one of their classmates' outputs and percentages, and must determine the input.

Give students several minutes to create their own challenge and more time to solve the challenges of others. After a set amount of time, encourage students to also take some time to review responses to their own. While students are working, monitor for creative challenges and solutions.

Note: We anticipate this Challenge Creator may take 15 minutes or more.

Pacing: Consider using pacing to restrict students to this screen.

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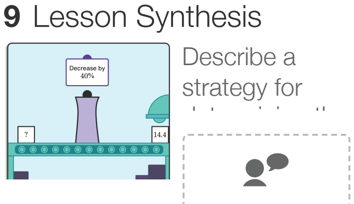
9 Lesson Synthesis



Describe a strategy for determining the input when you know the percent change and the output.

Use this example if it helps you to explain your thinking.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: On this screen, students consolidate and refine their ideas about determining the original value given a percent change and the final value.

Facilitation: Before students respond to the prompt, consider asking them to quickly estimate what they believe the value of the input might be here in order to surface whether or not the value should be less than, greater than, or equal to the final value. Then, give students 2–3 minutes to respond and one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses or suggest revisions.

Pacing: Consider using pacing to restrict students to this screen.

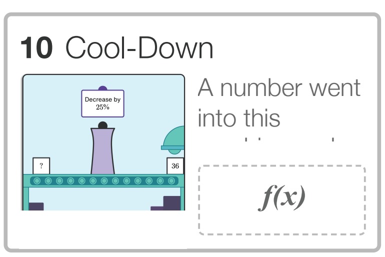
Sample Responses

Responses vary. To figure out the original value, you need to know what the going-forward relationship is. You can write this in a table, or as an equation, or in a double number line. In this example, the equation is $x \cdot 0.60 = 14.4$. Then, divide by 0.6 to determine the input.

Student Supports

Multilingual Learners
- *MLR 2 (Collect and Display)*

Circulate and listen to students talk during pair work or group work. Jot down notes about common or important words and phrases (e.g., decimal equal to the percentage, constant of proportionality, multiply or divide) together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.



A number went into this machine and 36 came out.

What number went in?

Teacher Moves

Support for Future Learning: Consider offering individual support to students who struggle with the cool-down before students take the quiz, or conduct a whole class discussion before the next class.

Early Student Thinking: Students who enter 45 may have calculated 25% of 36 and added that to 36.

Pacing: Consider using pacing to restrict students to Screens 10–11.

Sample Responses

48

11

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can determine the original amount if I know the new amount and the percent change for one-step and multistep problems.

7.4 Practice Day 1 (NYC)

Preparation

Student Workspace

- Print one double-sided copy for each student.

Cards

- *Solve and Swap:* Print and cut enough sets of cards so that there is one card for each student (e.g., two sets for a class of 24; three sets for a class of 36).
- *Group Questions:* Print and cut one set of cards for each group of students.

Instructions

Option 1: Solve and Swap

This structure supports student collaboration with many different partners and allows for movement around the classroom. Students are positioned as experts as they discuss each problem and support one another.

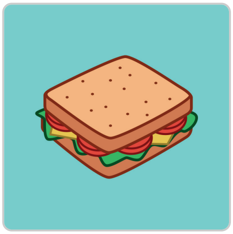
- Print and cut out enough sets of cards so that there is one card for each student.
- Give each student a card and invite them to answer the question on their worksheet.
- Invite students to circulate the class with their card and pair up with a classmate. In the pair, each student should solve the problem on their partner's card, collaborating as needed.
- If a pair of students end up with a problem they've already solved, invite them to compare strategies and solutions with their partner.
- When both students have completed their problems, invite them to swap cards, stand up with a hand up, and find another classmate to pair up with.
- Repeat the process.
- If students are waiting for a partner, invite them to try the "Are You Ready for More?" task at the bottom of their worksheet.

Option 2: Group Questions

This structure supports student collaboration and focuses students on one problem at a time.

- Arrange students into *groups of 2–3.* Print and cut out one set of 12 cards for each group.
- Invite students to select one card to work on at a time as a group.
- Give each student the Student Workspace Sheet to complete as they work together. Encourage students to justify their reasoning as they discuss their strategies.
- If time allows, invite students to order the cards from what they think will be more challenging to what they think will be less challenging. This helps them prioritize if they are not able to answer all 12 questions.
- Consider posting the answer key, or walking around with it and providing feedback to students as they work.
- If groups complete all 12 cards, invite them to try the "Are You Ready for More?" task.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Tax and Tip

Lesson 8: Multistep Percent Problems

Overview

Students apply what they've learned in the first section of the unit to solve multistep percent problems in a common context: sales tax and tip.

Learning Goals

- Calculate the original amount, the new amount, or the percent change in the context of sales tax and tip, including situations that involve multiple percent increases or decreases (e.g., sales tax and a discount).

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to apply what they've learned in the first section of the unit to solve multistep percent problems in a common context: sales tax and tip. This lesson ends with an opportunity for students to practice calculating the total after two percentages are applied (e.g., a discount followed by sales tax).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to the concept of sales tax.

Activity 1: Tax and Tip (30 minutes)

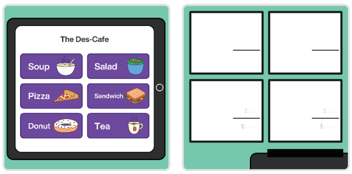
The purpose of this activity is for students to apply what they've learned to calculate the total amount in situations that involve multiple percent increases or decreases (e.g., sales tax and a discount). In addition, students write expressions to represent both sales tax and tip.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their ideas about calculating the total amount in multistep percent problems.

Cool-Down (5 minutes)

1 Warm-Up



Teacher Moves

Lesson Overview: The purpose of this lesson is for students to apply what they've learned in the first section of the unit to solve multistep percentage problems in a common context: sales tax and tip.

Warm-Up Purpose: On Screens 1–2, students are introduced to the concept of sales tax and determine the tax rate from a set of receipts.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and asking students if they have ever had the experience of something costing more than they expected because of tax. Students may bring knowledge of tax and tip from their own experience or the experience of their communities.

Then, unpause and invite students to select several items from the menu to figure out what is wrong with the receipts and then to continue to the next screen.

Pacing: Consider using pacing to restrict students to Screens 1–2.

Student Supports

Students With Disabilities

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.

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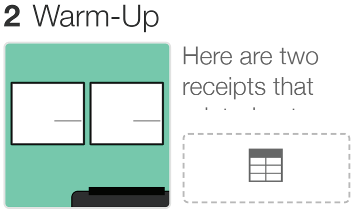
2 Warm-Up



Here are two receipts that printed out correctly.

Teacher Moves

This is a possible discussion screen.

Facilitation: While students are working, select several correct and incorrect student responses to display using the dashboard's teacher view or snapshot tool. Encourage students to read others' responses and decide if others' strategies were similar to or different from their own.

Follow with a whole-class discussion to introduce tax and decide which expressions represent the sales tax and what other values, such as 0.07 and 1.07, represent in this context. Spend adequate time here to ensure that students understand what the tax rate is in this situation and why. Students will use this information during Activity 1.

Ideas that may surface during the discussion:
- Sales tax is a percentage of the cost of the item, so it depends on the original cost of the item.
- Different cities and states have different tax rates.
- The number 1.07 is the constant of proportionality between the price of the item and the total cost.
- The number 0.07 is the tax rate written as a decimal rather than a percentage.

Note: Students can use the scientific calculator button near the top of the screen to help them calculate the tax rate.

Early Finishers: Encourage students who finish Screens 1–2 early to use others' responses to make their explanation stronger and clearer or to research the tax rate in their own community and calculate the total cost using their tax rate.

Sample Responses

7%

Explanations vary.

- I divided the total cost by the price to figure out the constant of proportionality, which was 1.07. The 1 represents 100%, and the $.07$ represents a 7% increase.
- I divided the change ($\$0.56$) by the original price ($\$8.00$), which was 0.07. This as a percent is 7%.

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Direct students' attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.

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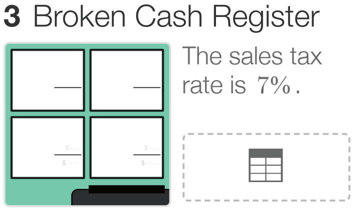
3 Broken Cash Register



The sales tax rate is 7%.

Determine the remaining values for these receipts.

Teacher Moves

Activity 1 Purpose: On Screens 3–8, students apply what they've learned about percentages to calculate the total amount in situations that involve multiple percent increases or decreases (e.g., sales tax and a discount).

Facilitation: Invite students to use the sales tax rate they discussed during the warm-up to figure out the values on the missing receipts and then to continue to the next screen.

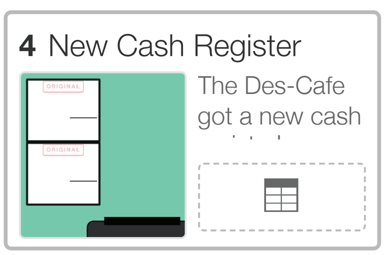
Note: Students will have different tables depending on the items they selected on Screen 1.

Pacing: Consider using pacing to restrict students to Screens 3–8.

Sample Responses

Responses vary.

Soup: Tax $\$0.35$; Total $\$5.35$
Pizza: Tax $\$1.26$; Total $\$19.26$
Donut: Tax $\$0.14$; Total $\$2.14$
Salad: Tax $\$0.56$; Total $\$8.56$
Sandwich: Tax $\$0.91$; Total $\$13.91$
Tea: Tax $\$0.07$; Total $\$1.07$



The Des-Cafe got a new cash register!

Write expressions that the cash register can use to determine the tax and the total for any item.

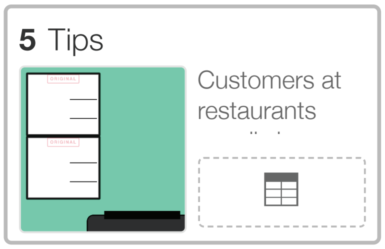
Teacher Moves

Progress Check: This is a great place to check students' progress on using repeated reasoning to write expressions for percent problems (MP8). Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking: Some students may notice that there is a 7% sales tax and write the expressions $0.7c$ and $1.7c$. Consider asking students what the difference between $0.7c$ and $0.07c$ means about the percent increase.

Sample Responses

Tax: $0.07c$
Total: $1.07c$



Customers at restaurants usually leave a tip for the server.

Sometimes, restaurants add an 18% tip to the bill automatically.

Complete the table to program the cash register.

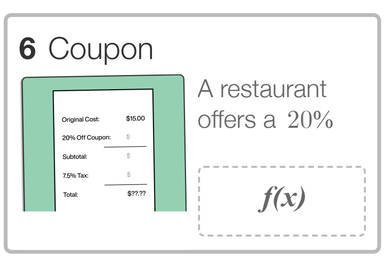
Teacher Moves

Facilitation: Consider pausing for a moment when most students have begun this screen to discuss the concept of tip.

Ideas that may surface during the discussion:
- Tip is a percentage of the cost of the item, so the amount changes depending on the original cost of the item.
- There are different customs for tipping in different countries and industries.
- People do not often tip an exact amount, but a value close to a certain percentage.
- Some people use rules like double the tax to figure out how much to tip.

Sample Responses

18% **tip:** $0.18T$
Total with tip: $1.18T$



A restaurant offers a 20% off coupon.

The tax rate is 7.5%.

If an item is listed at $\$15$, how much does it cost after the coupon and tax have been applied?

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

The purpose of this discussion is to surface strategies for applying multiple percentages and for reasoning about non-integer percentages.

Facilitation: While students are working, select and sequence several student responses and explanations using the snapshots tool.

When most students have responded, facilitate a whole-class discussion around students' strategies. Consider naming powerful strategies that arise after the students who use them and using those names throughout the rest of the lesson and unit. Record several strategies for students to refer back to as they complete the repeated practice on the next screen.

Consider ending the discussion by asking: *If you are using a coupon, do you think you should calculate the tip based on the original cost or the subtotal after using the coupon? Why or why not?*

Note: This is the first time in the unit that students experience a non-integer percent, 7.5%. Consider specifically identifying this and asking students to describe how this affects their calculations.

Early Finishers: Encourage students who finish early to write an expression to calculate the total using only one step.

Sample Responses

$\$12.90$

Explanations vary. To determine the price after the coupon, I multiplied 15 by 0.8 and got 12. Then, to determine the price after tax, I multiplied $\$12$ by 1.075 to get 12.9.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.

- *Executive Functioning: Eliminate Barriers*

Chunk this problem into more manageable parts (e.g., presenting one step at a time), which will aid students who benefit from support with organizational skills in problem-solving.

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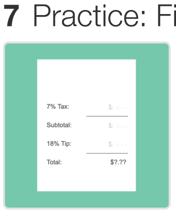
7 Practice: Find the Total



This meal costs $\$25.00$.

$f(x)$

This meal costs $\$25.00$.

A 7% sales tax is applied, followed by an 18% tip.

What is the total with tax and tip?

Teacher Moves

Facilitation: Give students 5–10 minutes to complete as many challenges as they can. Students are not expected to complete all challenges. Consider setting a goal for the total number of correct problems students should get or invite students to set their own goal.

Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

Consider pausing the class to celebrate students who persisted through struggle (e.g., "I saw a student struggling on the first few problems, and because they kept at it, they're crushing it now!").

Note: Students will need to make sense of each new problem as the problems vary between tax and tip and coupon and tax (MP1).

Sample Responses

This screen contains many challenges.

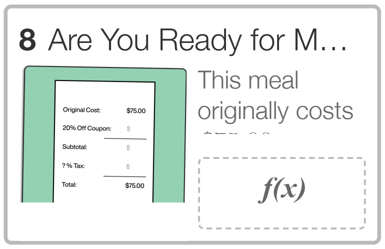
Responses vary.

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

Check in with individual students, as needed, to assess for comprehension during each step of this screen.



This meal originally costs $\$75.00$.

Fatima used a 20% off coupon and then tax was added.

The total after coupon and tax was $\$75.00$.

What was the tax rate?

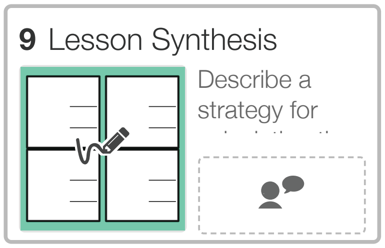
Teacher Moves

Facilitation: This screen is designed as an extra challenge for students who meet their goal on Screen 7 before the class discussion on Screen 9. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

25%

Explanations vary. I wrote the equation $75 \cdot 0.8 \cdot x = 75$ and then solved it by dividing. I found that x must be 0.25, which means the tax rate is 25%.



Describe a strategy for calculating the total for any item after a discount and tax.

Use the receipts if they help you with your thinking.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their ideas about calculating the total amount in multistep percent problems.

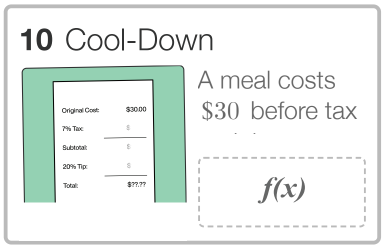
Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. First, figure out the subtotal with the discount, which usually means multiplying by a number less than 1. Then, use the answer that you got to figure out the total with tax. This is usually a number greater than 1. Be careful about tax percentages like 6.25%, which can be confusing.



A meal costs $\$30$ before tax and tip.

There is a 7% sales tax. After the tax, a 20% tip is added.

What is the total after tax and tip?

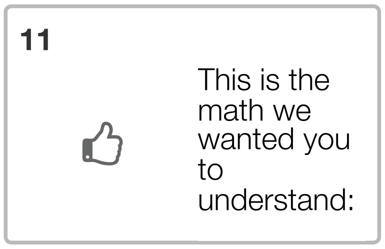
Teacher Moves

Support for Future Learning: Consider checking in with students who struggle on this cool-down when they work on stations during the Practice Day, or review this question as a class before the end of the unit.

Pacing: Consider using pacing to restrict students to Screens 10–11.

Sample Responses

$\$38.52$



This is the math we wanted you to understand:

- I can solve multistep problems about sales tax and tip.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Minimum Wag (NYC)

Lesson 10: Real-World Situations Involving Percent Increase, Part 1

Purpose

The purpose of this lesson is for students to use what they have learned about proportional relationships and percent change to analyze an issue in society. Students use their sense of fairness to decide whether or not existing systems for paying restaurant servers are fair. They then design their own pay system.

Preparation

Worksheet

- *Activity 1 and 2*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Warm-Up (5 minutes)

Purpose: Introduce students to the concept of minimum wage, including how minimum wage is sometimes calculated differently for workers who receive tips and those who do not.

Facilitation: Before displaying Sheet 1 of the Teacher Projection Sheets, consider asking students what they know about minimum wage. Share with students that minimum wage is a dollar amount per hour that represents the smallest hourly wage a company can pay its workers. The minimum wage can vary depending on the city and state.

Then, display Sheet 1 and give students one minute to take a guess for the two missing values. Invite several students to share their estimates briefly and to discuss whether the minimum wage for workers who receive tips is greater than, less than, or equal to the minimum wage for other workers.

Next, display Sheet 2. Give students a minute to think quietly about whether or not they think this system is fair and another minute to share their reasoning with a partner. Then, facilitate a brief whole-class discussion.

Ideas that may surface in this discussion:

- This is fair because the reduced hourly wage balances out the pay from tips.
- This is unfair because a worker who receives tips could make more money from tips than a worker who does not receive tips.
- This is unfair because workers cannot control how much money they make in tips.

Activity 1: Waiting Tables (15 minutes)

Purpose: Students solve a multistep problem that involves percentages in order to understand an issue.

Facilitation: Arrange students into groups of four and distribute one double-sided worksheet to each student. Consider sharing with students that most restaurant servers' pay is determined by two factors: the hourly minimum wage where they live and the amount they make in tips, which depends on the cost of the food their customers order.

Give groups one minute for each member to select a different person and another minute to read the description of their person silently. Then, invite students to share key details about their person with their group before starting Problem 1 on the worksheet. The purpose of this share-out is to support students in making sense of the problem before they calculate (MP1).

If students are having difficulty getting started, consider the following:

- Give groups a few minutes to make a plan about how to calculate the pay for any person.
- Work through a new fictional example as a class.
- Ask students who have started to share their strategies with the class.
- Ask individual students questions like: *How much money would your person make from minimum wage alone? How much money would your person make per table?*

Consider posting the answer key, or walking around with it and providing feedback to students as they work.

When all group members have calculated how much their person makes in a typical week along with the effect of increasing the tip percentage by 5%, give them a few minutes to discuss the prompts at the bottom of the first page. To provide students with real-life references for discussion, consider researching minimum wage or how pay works for tipped workers in your community.

Then, facilitate a whole-class discussion for students to justify whether or not they believe the system is fair and to critique each other's reasoning (MP3). This is an opportunity for students to bring in what they and their communities value. For example, students may reference personal details as they justify whether or not the situation is fair.

Early Student Thinking: Some students may use what they know about percent increase to calculate the amount that servers make in tips (e.g., the bill is multiplied by 118% instead of 18%). Consider asking these students whether they think servers make all of the money that customers pay or only the tip.

Early Finishers: Encourage students who finish early to support their teammates or to research the approach to tipped and non-tipped workers in your city and state.

Materials: Allow students to use calculators to ensure inclusive participation in this activity.

Activity 2: What's Fair? (15 minutes)

Purpose: Students use their creativity and knowledge of proportional relationships to develop a system for paying workers (MP4).

Facilitation: Consider sharing with students that some restaurants are considering removing tips entirely, and that in this activity, they are going to think about which systems for paying restaurant workers are the most fair. Then, invite students to decide which of the people from Activity 1 would be happy working at a restaurant without tips.

Consider arranging students into pairs as they design their own payment systems. Encourage them to incorporate factors that seem important to them that might not have been present in any of the other systems, such as years of work experience.

Here are two options for debriefing this activity:

1. Select several students who created different systems to share their calculations, and ask the class to decide whether or not they think the system is fair. This is an opportunity to celebrate that there are different ways to solve a problem.
2. Ask students to trade and discuss their systems with a different pair of students.

Early Finishers: Encourage students who finish Activity 2 early to revise their system, incorporating feedback and ideas from their group, or to make a clear argument about why their system is more fair than any of the alternatives.

Lesson Synthesis (5 minutes)

Purpose: Students describe the process for combining minimum wage and money earned from restaurant tips to determine a server's weekly pay.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Follow with a brief whole-class discussion in which students share their descriptions.

If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Cool-Down (5 minutes)

Support for Future Learning: Students will revisit these concepts in Lesson 10: Cost of College.

Cost of College (NYC)

Lesson 10: Real-World Situations Involving Percent Increase, Part 2

Purpose

In this lesson, students use what they have learned about calculating percent increase to solve problems about the cost of college over time. Students analyze how costs have increased over time compared to wages and think critically about what it means for college and minimum wage to increase by dramatically different percentages over time. Students use this information to make predictions and recommendations for the future.

Preparation

Worksheet

- *Activity 1 and 2*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Warm-Up (5 minutes)

Purpose: Surface students' knowledge about how prices change over time and strategies for calculating percent change.

Facilitation: Display Sheet 1 of the Teacher Projection Sheets and give students one minute to estimate the average price of a movie ticket in 2010. Invite several students to briefly share their estimates, and record their estimates to emphasize the range of responses.

Then, display Sheet 2 of the Teacher Projection Sheets, and ask students whether or not the actual value is surprising. Give students one minute to calculate the percent increase and one minute to compare their response with a partner. Then, facilitate a brief whole-class discussion to share students' strategies.

Note: Students may be surprised about the average movie costs. Consider using this as an opportunity to discuss as a class how costs can differ dramatically depending on what part of the United States a person is living in.

Activity 1: Minimum Wage vs. Cost of College (15 minutes)

Purpose: Students compare the changes in minimum wage and cost of college since 1990.

Facilitation: Arrange students into pairs. Consider starting the activity by asking students what they remember about minimum wage. If it has not come up naturally, consider discussing how minimum wage differs depending on the state and city. If time allows, consider looking up minimum wages in your city or state and using those numbers in place of the federal minimum wage.

Then, give students 10 minutes to complete Problems 1–3 on the front side of the worksheet.

Students will select different pairs of years, so they will have different responses to Problem 1. Consider displaying the percent increase between minimum wage and cost of college for several different pairs of years on the board.

When students have completed all three problems, facilitate a whole-class discussion to surface what students learned about society through the activity.

Consider asking questions like:

- *What did you learn about minimum wage and the cost of college over time?*
- *What in this activity was surprising to you?*
- *What are some reasons the rates of increase might be different?*

Then, consider using students' thinking on Problem 3 to answer the question: *Was college affordable for a person working minimum wage in 1990? What about in 2017?* (MP3).

Early Finishers: Encourage students who finish early to choose a neighboring state that has a different minimum wage and to compare those percentages with the ones they already calculated.

Materials: Provide students with calculators to ensure inclusive participation in the activity.

Activity 2: Future Cost (15 minutes)

Purpose: Students use an equation to predict the cost of college several years in the future.

Facilitation: Give students 10 minutes to work on Problems 1–4. Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed. Consider listening for examples of early student thinking and discuss them as a class.

Then, facilitate a whole-class discussion around the question: *What would a fair minimum wage be?* Consider beginning by asking students to share the strategies they used to determine how much college will cost when they graduate from high school. Then, invite several students to share what they believe a fair minimum wage would be, and record their responses. Ask these students to justify their reasoning (MP3).

Early Student Thinking: Students may notice that the percent increase is 2.94% and write the equation $y = 1.294x$. Consider asking these students what an equation for a 25% percent increase would be and to compare it to the equation they wrote. Other students may assume that you can predict the cost of college five years in the future by multiplying the percent increase by 5 and using that percent increase. Consider asking these students to think of a similar situation with simpler numbers and to decide if this strategy will still be true.

Are You Ready for More? (optional)

This activity is designed as an extra challenge for students who finish Activity 2 before the class discussion. Consider inviting these students to research Madam C.J. Walker and to share responses with each other in place of a whole-class discussion.

Lesson Synthesis (5 minutes)

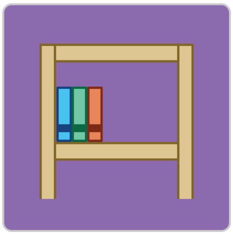
Purpose: Students practice using repeated reasoning to predict how a number will change over multiple years.

Give students 2–3 minutes to respond and a minute to share their responses with a partner. Follow with a brief whole-class discussion in which students share strategies for calculating 10 years in the future. If time allows, give students one minute to revise their response based on the conversation.

Cool-Down (5 minutes)

Support for Future Learning: Students will have an opportunity to practice calculating multiple percent increases during the Practice Day.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Bookcase Builder

Lesson 11: Percent Error

Overview

Students are introduced to real-world contexts in which error occurs. They consider why percent error is useful, and they practice calculating percent error and deciding whether or not a measurement is within an acceptable range.

Learning Goals

- Explain that percent error is an error as a percentage of the correct measurement.
- Decide whether a value is within an acceptable percent error.

Vocabulary

- percent error

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to introduce students to real-world contexts in which error occurs. This lesson asks students to consider why percent error is useful and supports students with practice calculating percent error and deciding whether or not a measurement is within an acceptable range.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to encounter a context involving two errors and to decide which error is larger. This lays the groundwork for students to consider why percent error is a useful tool.

Activity 1: Percent Error (15 minutes)

The purpose of this activity is for students to come to understand that percent error is one way to describe how far above or below a desired measurement a value is. Students begin this activity by figuring out ranges of screws that work for different bookcases and then analyze their results.

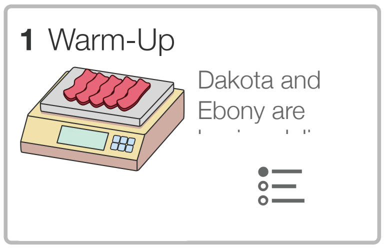
Activity 2: Will It Fit? (15 minutes)

The purpose of this activity is for students to practice deciding whether or not a value is within a range of acceptable percent errors. Students will be presented with a series of different challenges in which they need to determine whether or not a particular shelf length will fit given a desired shelf length and acceptable percent error.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their ideas about how to determine a range of acceptable values within a percent error.

Cool-Down (5 minutes)



Dakota and Ebony are buying deli meat.

Dakota orders 2 ounces of ham, but ends up with 2.3 ounces.

Ebony orders 10 ounces of turkey, but ends up with 9.5 ounces.

Whose order had a bigger error?

Teacher Moves

This is a possible discussion screen.

Lesson Overview: The purpose of this lesson is to introduce students to real-world contexts in which error occurs. This lesson asks students to consider why percent error is useful and supports students with practice determining percent error and whether a measurement is within an acceptable range.

Warm-Up Purpose: Students encounter a context involving two errors and are asked to decide which error is larger. This lays the groundwork for students to consider why percent error is a useful tool.

Facilitation: Consider starting with the activity paused and asking students whether they have ever had an experience at a store similar to Dakota and Ebony's experience. Then, unpause and invite students to decide whose order had a bigger error and to explain their reasoning.

Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Display 1–2 responses for each and invite those students to share their reasoning. It's okay—even desirable—to lack consensus at this stage. The purpose of the conversation is to surface the challenge of comparing the errors based on different original amounts. If it does not come up naturally, ask students: *When comparing the error, does the fact that Dakota and Ebony ordered different amounts of meat matter?*

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses and explanations vary.

- Dakota's order has a bigger error. The error on her order is 15% compared to 5% on Ebony's order.
- Ebony's order has a bigger error. She got 0.5 ounces less than she expected.

- Dakota's order has a bigger error. I multiplied Dakota's order by 5 so that I could better compare them.
- Ebony's order has a bigger error. Getting less than you ordered is always worse than getting more than you ordered.

Student Supports

Support for Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Direct students' attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.

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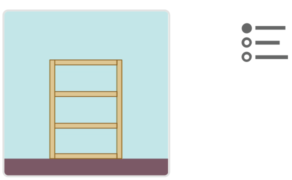
2 Bookcase Builder



Teacher Moves

Activity 1 Purpose: On Screens 2–6, students come to understand that percent error is one way to describe how far above or below a desired measurement a value is.

Facilitation: Arrange students into pairs. Consider starting with the activity paused, reading the prompt aloud as a class, and asking students if this description matches their experience putting together furniture. Share with students that in this lesson, they will be discussing the common real-world situation of measurements that are not perfectly accurate. Then, unpause and invite students to try each screw and to continue to the next screen.

Pacing: Consider using pacing to restrict students to Screens 2–4.

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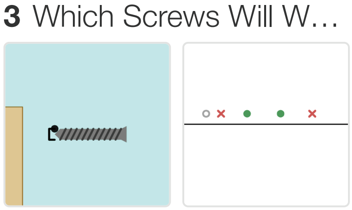
3 Which Screws Will W…



Teacher Moves

Facilitation: Consider using the dashboard's student view to demonstrate how to change the size of the screw by dragging the movable point. Encourage students to determine the largest and smallest screw sizes that will work.

After 1–2 minutes, invite students to continue to the next screen.

Sample Responses

All screws between 9 and 11 mm work and result in a green circle on the graph.

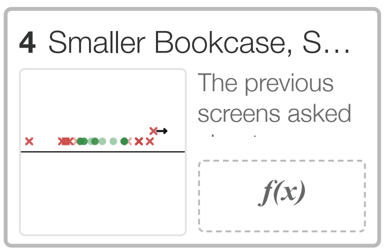
All others do not work and result in a red X on the graph.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors and allow students who struggle with fine motor skills to dictate physical manipulation of the draggable point as needed.



The previous screens asked about a bookcase designed for 10 mm screws.

Now consider a different bookcase designed for 6 mm screws.

Enter different screw sizes to figure out the smallest and largest sizes that will work for this bookcase.

Teacher Moves

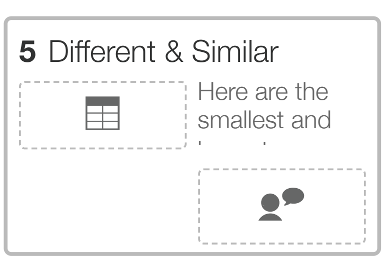
Facilitation: After 1–2 minutes, consider asking students to share what they believe are the smallest and largest sizes that will work.

Early Finishers: Encourage students who finish Screens 2–4 early to think of a rule that might be used to figure out what screws work for any desired size screw.

Sample Responses

All screws between 5.4 and 6.6 mm work and result in a green circle on the graph.

All others do not work and result in a red X on the graph.



Here are the smallest and largest acceptable screw sizes for the two previous screens.

How are the acceptable screw sizes different in each row? How are they similar?

Teacher Moves

This is a possible discussion screen.

Facilitation: Invite students to compare their prediction from Screen 4 with the information in the table, and then write about ways the acceptable screw sizes are the same and different.

Then, facilitate a whole-class discussion around the question: *What is similar about the range of acceptable values for both the* 6 *mm screw and the* 10 *mm screw?*

Some ideas that may surface during the discussion:
• It makes sense that a smaller object should have a smaller acceptable range.
• The desired width is exactly in the middle between the smallest and largest acceptable widths.
• The acceptable range is between $\frac{1}{10}$ (or 10%) smaller and the same amount larger than the desired value for both screws.

Consider ending the discussion by asking students to discuss what they think the smallest and largest acceptable width would be for a 15 mm screw.

Pacing: Consider using pacing to restrict students to Screens 5–6, one at a time.

Sample Responses

Responses vary.

Ways they are different:
• The range of acceptable screws is different for each situation. When 10 mm is desired, it can be 1 mm bigger or smaller. When 6 mm is desired, it can only be 0.6 mm bigger or smaller.

Ways they are similar:
• In both situations, the screws are allowed to be bigger or smaller by a specific amount.
• In both situations, that amount is 10% of the desired screw size.

---

6 Percent Error

A factory tries to make a 6 mm screw. It ends up being 5.7 mm

A factory tries to make a 6 mm screw. It ends up being 5.7 mm instead.

One way to describe the error is 0.3 mm.

Another way is to describe it as a PERCENT ERROR.

This screw has a percent error of $\frac{0.3}{6} = 5\%$.

Complete the table to decide if each screw will work in its bookcase (its percent error must be 10% or less).

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

The purpose of this discussion is to introduce the concept of percent error and surface strategies for calculating percent error.

Facilitation: Consider starting with this screen paused and using the dashboard's student view to read the note aloud as a class, asking students to say what percent error means in their own words. Ask students how percent error is similar to and different from other ideas they have learned in this unit. Then, invite students to calculate the percent error of each screw in order to decide whether or not it will work.

When students have responded, facilitate a whole-class discussion to hear several students' strategies for determining whether or not each screw will work.

Consider ending the discussion by asking: *When do you think calculating percent error might be useful in your life or in the lives of people in your community?*

Early Finishers: Encourage students who finish this screen early to create their own examples and quiz their partner to decide whether or not the screw will work.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Sample Responses

Row 2: **Percent error**: 9%. **Will it work?** Yes.
Row 3: **Percent error**: 20%. **Will it work?** No.
Row 4: **Percent error**: 8%. **Will it work?** Yes.

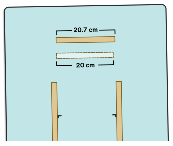
Student Supports

Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., calculating one problem at a time), which will aid students who benefit from support with organizational skills in problem-solving.

7 Shelves



Nikolai is making a shelf

Nikolai is making a shelf for his new bookcase.

He wants it to be 20 centimeters long. It ends up being 20.7 centimeters long.

The acceptable percent error is 5% for a shelf to fit.

Will the shelf fit?

Teacher Moves

Activity 2 Purpose: On Screens 7–9, students practice deciding whether or not a value is within a range of acceptable percent errors.

Facilitation: Invite students to use the strategies they tried on the previous screen to figure out if the shelf will fit or not, and to explain their reasoning. Then, have students continue to the next screen.

Pacing: Consider using pacing to restrict students to Screens 7–9.

Sample Responses

Yes

Explanations vary. The error is 0.7 centimeters and the original is 20 centimeters. 0.7 is 3.5% of 20, which is less than 5%.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., The shelf will/will not fit because _______.).

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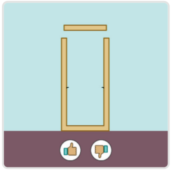
8 Challenge Your Shelf



Determine whether this shelf will or will not fit given

Determine whether this shelf will or will not fit given the acceptable error.

- Shelf length: 20.2 cm
- Desired shelf length: 20 cm
- Acceptable error: 5%

Teacher Moves

Facilitation: Give students 5–10 minutes to complete as many challenges as they can. Consider setting a goal for the best streak students should get or asking students to set their own goal.

Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed. Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Sample Responses

This screen contains many challenges. Here are the first five answers:

- The shelf fits.
- The shelf doesn't fit.
- The shelf doesn't fit.
- The shelf fits.
- The shelf doesn't fit.

---

9 Are You Ready for M…

Nikolai is building shelves for bookcases of different sizes,

Nikolai is building shelves for bookcases of different sizes, allowing for a 25% error.

Select the graph in which the green area represents good shelves.

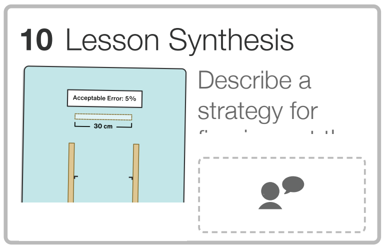
Teacher Moves

Facilitation: This screen is designed as an extra challenge before the class discussion on Screen 10. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

The left graph

Explanations vary. The acceptable range will get bigger as the desired length gets bigger. For example, when the desired length is 10 cm, the acceptable error will be 2.5 cm above or below 10 cm. However, when the desired length is 20 cm, the acceptable error will be 5 cm.



Describe a strategy for figuring out the range of values that are acceptable when you know the desired measurement and an acceptable percent error.

Use the example if it helps you with your thinking.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: On this screen, students consolidate and refine their ideas about how to determine a range of acceptable values within a percent error.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- Calculate 5% of 30 cm, which is 1.5 cm. Then calculate $30+1.5$ and $30-1.5$.
- Calculate 95% and 105% of 30 cm, which is 28.5 and 31.5.
- Write expressions for 95% and 105% of the shelf length: $0.95s$ and $1.05s$.

Student Supports

Multilingual Learners

- *MLR 2 (Collect and Display)*

Circulate and listen to students talk during pair work or group work. Jot down notes about common or important words and phrases (e.g., percent error, distance away from the value you wanted, etc.) together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.

11 Cool-Down
To be labeled a jumbo egg, an egg is supposed
$f(x)$

To be labeled a jumbo egg, an egg is supposed to weigh 2.5 ounces.

Rafael buys a carton of jumbo eggs and finds that one egg weighs 2.4 ounces.

What is the percent error?

Teacher Moves

Supports for Future Learning: If students struggle, consider inviting these students to spend extra time on the section of the Practice Day that involves percent error, or reviewing this question before students begin the Practice Day.

Pacing: Consider using pacing to restrict students to Screens 11–12.

Sample Responses

4%

12
This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain what percent error is and how to calculate it.
- I can decide whether a value is within an acceptable percent error.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Posing Percent Problems

Lesson 12: Analyze Data Using Percentages

Overview

Students apply what they've learned about increasing and decreasing by a percentage to generate and answer questions about the society in which we live.

Note: This lesson is designed for 90 minutes and may take multiple class periods to complete.

Learning Goals

- Generate and answer questions about a real-world situation involving percent increase or decrease.
- Select and analyze statistics about the larger society in which we live.

Materials

- Tools for creating a visual display
- Graph paper

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to apply what they've learned about increasing and decreasing by a percentage to generate and answer questions about the society in which we live. First, students generate interesting and answerable questions related to either prison populations or the wage gap in the U.S. based on given statistics. Then, students work in pairs to create a visual display to answer their questions using the information given.

Lesson Summary

Warm-Up (5 minutes)
The purpose of the warm-up is for students to practice writing questions from a set of statistics and develop characteristics of a good question. Completing this warm-up is intended to help students better understand the types of questions that are interesting and answerable with the information provided.

Activity 1: Stronger and Clearer Each Time (15 minutes)
The purpose of this activity is for students to generate questions about a real-world situation involving percent increase or decrease. Students choose a topic that interests them and use the paper supplement to write two questions about the topic that they could figure out using the information they have and whose answer is not already given. They then work with several classmates to refine their questions (MP4).

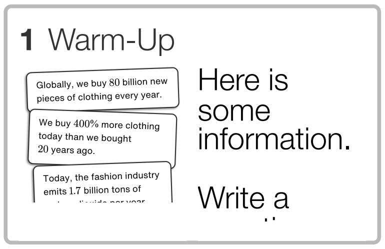
Activity 2: Make a Poster (20 minutes) + Synthesis (5 minutes)
The purpose of this activity is for students to use what they know about percent increase and decrease to select and analyze statistics in order to answer the questions they generated in Activity 1. Students work together to create at least one representation of the information that they select to help them answer each question (MP5).

Activity 3: Warm Up (5 minutes) + Gallery Tour (15 minutes)
The purpose of this activity is for students to see their peers' different conclusions, and the assumptions and strategies that led to those conclusions.

Activity 4: Revisions and Reflection (15 minutes)
The purpose of this activity is for students to revises their posters, and add at least one additional representation. Students then reflect on the process of making assumptions, doing calculations, and applying proportional reasoning to gain insight into a real-life context.

Lesson Synthesis (5 minutes)
The purpose of the synthesis is for students to generate new questions about the society in which we live.

Cool-Down (5 minutes)



Here is some information.

Write a question that you could figure out using this information and whose answer is not already given.

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to apply what they've learned about increasing and decreasing by a percentage to generate and answer questions about the society in which we live.

Warm-Up Purpose: Students practice writing questions from a set of statistics and develop characteristics of a good question.

Facilitation: Consider starting with the activity paused and reading the three statistics aloud. Consider asking students what these statistics make them think of in their own lives. Then, invite students to write a question that 1) they can figure out using the information and 2) is not already answered in the information cards.

Routine: Use the mathematical language routine Critique, Correct, Clarify to support students to write questions that meet the criteria. Consider recording words or phrases students use to describe what makes a good question for students to refer back to in the rest of the lesson.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

Questions that meet both criteria:
- How many pieces of clothing did we buy 20 years ago?
- How many tons of carbon dioxide did we emit 20 years ago?
- How many more tons of carbon dioxide do we emit now?

Questions we could not figure out from this information:
- How much clothing does the average person buy?
- How much space does a ton of carbon dioxide take up?

Questions whose answers are given:
- How much carbon dioxide does the fashion industry emit per year?
- How many pieces of clothing do we buy every year?

Student Supports

Students With Disabilities

• *Conceptual Processing: Eliminate Barriers*
Direct students' attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.

Multilingual Learners
• *MLR 5 (Co-Craft Questions and Problems)*
Before answering the questions in this activity, have students write possible mathematical questions about the activity. In pairs, students can compare their questions. Then invite pairs to share their questions with the class.

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2 Stronger and Clearer …

• The average White man in America earned $\$60\,388$ in 2017 and $\$58\,879$ in 2000.

• In America between 2000 and 2017, Black women's earnings increased by 2.57% on average to $\$36\,735$.

• In 2017, the average White woman in America earned about 23% less than the average White man.

• In 2000, the average White woman in America earned $\$42\,591$.

• Compared to the average White man, the average Black man in America earned 27% less in 2000 and 30.3% less in 2017.

Teacher Moves

Activity 1 Purpose: Students generate questions about a real-world situation involving percent increase or decrease and then work with several classmates to refine their questions.

Note: Conversations about identity can be very different for different students, in different classrooms, and in different communities. As students share their perspectives on fair wages, consider a) the culture of your classroom b) how you will facilitate a discussion where students may bring perspectives different from their peers and from your own, and c) how you will check-in with students throughout and after the discussion.

Facilitation: Consider starting with the activity paused, displaying the situation titles without the statistics, and asking students what they think the phrases “wage gap” and “prison population” mean. Then distribute one copy of the paper supplement to each student. Give students 2–3 minutes to read the statistics either on the screen or the worksheet and to identify words or phrases they are unsure of. Spend adequate time here to clarify these phrases before students generate their own questions.

Then, invite students to choose a topic that interests them and write two questions about the topic that they could figure out using the information they have and whose answer is not already given (MP4). Use the mathematical language routine Stronger and Clearer Each Time to help students refine their questions.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

Wage Gap
- Is the wage gap for Black women and White women closing at the same rate?
- How much did the average White woman make per year in 2017?
- How much did the average Black man make per year in 2017?

Prison Populations
- Did the prison populations of the U.S. and China change at the same rate between 1986 and 2018?
- How many people were in U.S. prisons in 2018?
- What was the prison population in India in 1986?

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.

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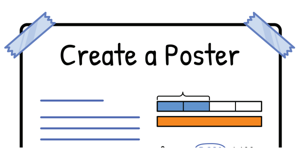
3 Percent Problem Poster



Teacher Moves

Activity 2 Purpose: Students use what they know about percent increase and decrease to select and analyze statistics in order to answer the questions they generated in Activity 1.

Facilitation: Arrange students into pairs of students who choose the same topic. Invite pairs to select two questions from the four they've written and to work together to use what they've learned so far in this unit to answer those questions. Then, they create a poster to visualize and organize their thinking.

Students can keep track of their progress using the checklist on the Supplement worksheet.

Monitor for students use different representations (tables, graphs, equations) to help them answer their question (MP5).

Pacing: Consider using pacing to restrict students to this screen.

Student Supports

Students With Disabilities

• *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., presenting one part at a time), which will aid students who benefit from support with organizational skills in problem-solving.

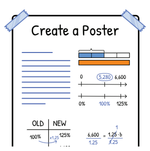
Multilingual Learners

• *Receptive/Expressive Language: Peer Tutors*

Pair students with their previously identified peer tutors to aid them in comprehension and expression of understanding.

---

4 Activity 3: Warm-Up



Look back at your poster.

1. What are

Look back at your poster.

1. What are you most proud of from your work so far?

2. What might make your poster stronger or clearer?

Teacher Moves

Activity 2 Purpose: Students use what they know about percent increase and decrease to select and analyze statistics in order to answer the questions they generated in Activity 1.

Facilitation: Arrange students into pairs of students who choose the same topic. Invite pairs to select two questions from the four they've written and to work together to use what they've learned so far in this unit to answer those questions. Then, they create a poster to visualize and organize their thinking.

Students can keep track of their progress using the checklist on the Supplement worksheet.

Pacing: Consider using pacing to restrict students to this screen.

Student Supports

Students With Disabilities

• *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., presenting one part at a time), which will aid students who benefit from support with organizational skills in problem-solving.

Multilingual Learners

- *Receptive/Expressive Language: Peer Tutors*

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5 Lesson Synthesis

What new questions do you have about these topics after seeing the work of other groups?

What new questions do you have about these topics after seeing the work of other groups?

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

Lesson Synthesis Purpose: Students generate new questions about the society we live in.

Facilitation: Give students 2–3 minutes to review other students' posters, then one minute to write new questions they have about these topics. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- Why is the wage gap closing for White women but not for Black men?
- How is the U.S.'s high rate of incarceration impacting the wage gap?
- Why does the U.S. put such a high percent of its population in prisons?

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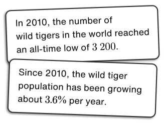
6 Cool-Down



Here are two facts.

1. Write a question

Here are two facts.

1. Write a question that you could figure out using this information and whose answer is not already given.

2. Answer your question.

Teacher Moves

Support for Future Learning: Students will continue to work with real-world situations involving percent increase and decrease in the Practice

Day.

Pacing: Consider using pacing to restrict students to Screens 5–6.

Sample Responses

Responses vary.

How many wild tigers are there in the world today?

Decimal Deep Dive (NYC)

Lesson 13: Writing Fractions as Decimals

Purpose

The purpose of this lesson is to provide students with a strategy for converting fractions to decimals. This builds on their understanding of using decimals in equations to represent percent increase and decrease. The skills that students build in this lesson lay the foundation for a series of lessons on rational and irrational numbers in Math 8.

Preparation

Student Worksheet

- *Activity 1–3:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Warm-up (5 minutes)

Overview: Students connect back to mosaics to estimate a non-integer percentage.

Launch

- Display Screen 1 of the Teacher Presentation Screens.
- Invite students to share what they notice and wonder about the image.

Facilitation

- Give students one minute to think independently, then share their responses with a partner.
- Invite several students to share their estimates and justify their reasoning.
- It's okay—even desirable—to lack consensus at this stage. The lesson will build toward consensus in Activity 1 when students use long division to write $\frac{5}{8}$ as a decimal.

Math Community

- Consider celebrating noticings at different levels of specificity, like “I know it has to be more than 50%.”

Activity 1: Carlos’s Calculations (10 minutes)

Overview: Students analyze one strategy for writing fractions as decimals and compare calculations that result in terminating and repeating decimals.

Launch

- Invite students to work *in pairs.*
- Display Screen 2 of the Teacher Presentation Screens.
- Distribute one Student Worksheet to each student.

Facilitation

- Give students 1–2 minutes to discuss Carlos's thinking with a partner.
- Invite students to discuss and then finish Carlos's calculations on their worksheet.

- Display Screen 3 and play the reveal. Invite students to share what was surprising.
- Consider returning to Screen 1 and answering the question: *What percentage is purple?*

- Display Screen 4. Give students 2–3 minutes to work and compare with a classmate.

- Display Screen 5. Give students one minute to revise their thinking and answer Problem 1.3.
- If it does not come up naturally, consider sharing that decimals that stop, like 0.625, are called *terminating decimals* and decimals that go on forever in a pattern, like $0.181818...$, are called *repeating decimals* and can be written as $0.\overline{18}$.

Possible Discussion Questions

- *How would you summarize Carlos's strategy?*
- *What was challenging about Carlos's strategy? What was surprising about it?*

Activity 2: Terminating or Repeating? (10 minutes)

Overview: Students practice writing fractions as decimals.

Launch

- Invite pairs to split up the problems so that each partner answers 2 of the 4 problems.

Facilitation

- Give students several minutes to work independently, then one minute to trade calculations with a partner and give each other feedback.
- Consider facilitating a brief whole-class discussion to share strategies and challenges.

Discussion Questions

- *Was there anything surprising that you noticed as you wrote these fractions as decimals?*

Activity 3: Decimal Deep Dive (15 minutes)

Overview: Students use their creativity and practice converting fractions to decimals.

Launch

- Arrange students in groups of 2–3. Invite students to work individually and to discuss ideas and strategies in their group.
- Give students one minute to make sense of the table.
- Ask: *Why do you think* $\frac{5}{8}$ *is on the left under "three digits"? Why is* $\frac{2}{11}$ *where it is?*
- Invite students to write the fractions from Activity 2 in the table.

Facilitation

- Give students several minutes to test out different fractions.
- The goal of this task is to complete as many of the cells in the table as they can. It is unlikely that students will find an example for each part of the table in the time allotted.
- As students find new fractions that fit in the table, invite them to share with the class by recording on a public display or using another method.

Math Community

- This activity is designed so that students can feel successful no matter how many fractions they find and to celebrate many different student responses.
- Consider pausing the class to celebrate students who persisted through struggle or who had creative approaches (e.g., "I saw a student struggling, and because they kept at it, they're crushing it now!").

Lesson Synthesis (5 minutes)

Overview: Students refine their ideas about how to convert a fraction into a decimal.

Facilitation

- Give students 1–2 minutes to respond and one minute to share their responses with a partner.
- Invite several students to share their thinking.
- If time allows, give students one minute to revise their response based on the discussion.

Math Community

- Invite students to share the advice they've found most helpful and to attribute them to the students who shared them.

Routine (optional): Consider using the mathematical language routine Clarify, Critique, Correct with a starting description like "use long division" in order to support students in writing robust descriptions of strategies.

Cool-Down (5 minutes)

Support for Future Learning

- Students will revisit these ideas in the following grade. Writing fractions as decimals is not included in the End Assessment.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

7.4 Practice Day 2 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided copy for each student.

Task Cards

- Print one set of cards for the whole class (option 1) or one set of cards for each group of students (option 2).

Instructions

Option 1: Stations

Print and cut out one single–sided set of task cards. Place each task card at different locations around the room and arrange students into groups of 3–4.

Give each student the Student Workspace Sheet to record their work as they solve the task at each station. Invite students to rotate to a new station after a set amount of time.

Option 2: Task Cards

Arrange students into groups of 2–3. Print one set of task cards for every group of students.

Give each student the Student Workspace Sheet to complete as they work together to solve each of the tasks.

Consider posting the answer key, or walking around with it and providing feedback to students as they work.

GRADE 7

Unit 6 Lesson Plans

Teacher lesson plans from Unit 6 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 7 Unit 6

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

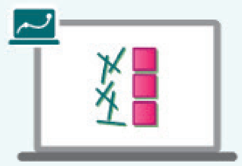
Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

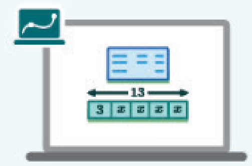
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



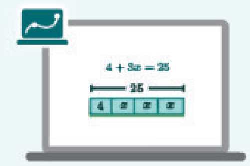
1 Toothpicks and Tiles

Use patterns to determine unknown values in relationships that are predictable but not proportional.



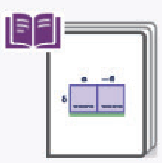
2 Smudged Receipts

Interpret a tape diagram that represents a relationship in context.



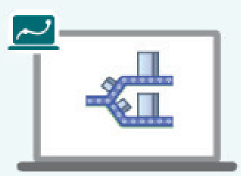
3 Equations

Connect tape diagrams, equations, and verbal descriptions in context.



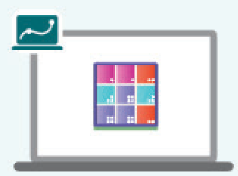
8 Factoring and Expanding

Expand and factor expressions that include positive and negative coefficients.



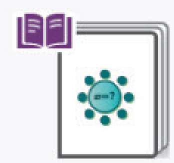
9 Always-Equal Machines

Write equivalent expressions that involve factoring, expanding, and reordering terms.



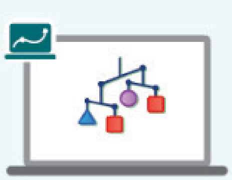
10 Collect the Squares

Write equivalent expressions with fewer terms by expanding and adding terms.



11 Equation Roundtable

Solve equations that involve adding and expanding expressions.

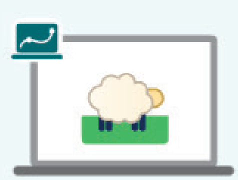


14 Unbalanced Hangers

Determine the solutions to an inequality with only positive numbers.

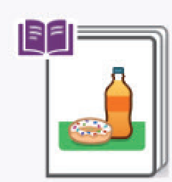
15 Budgeting

Write an inequality to represent a context.



16 Shira the Sheep

Solve inequalities with positive and negative coefficients, and graph their solutions.

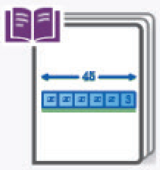


17 Write Them and Solve Them

Critique solutions to an inequality, including whether fractional or negative values are reasonable.

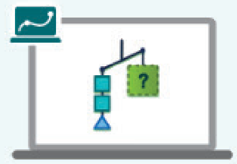
Pacing: 21 days | **Short on time?** See pacing considerations below.

Pre-Unit Check: (Optional)
17 Lessons: 45 min each
2 Practice Days: 45 min each
1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



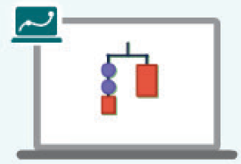
4 Seeing Structure

Write equations of the form px+q=r and p(x+q)=r to represent situations and determine their solutions using a tape diagram.



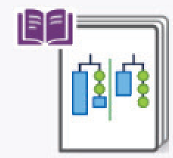
5 Balancing Moves

Determine an unknown value in a hanger diagram, and explain their strategy.



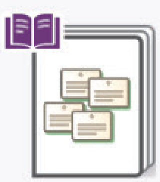
6 Balancing Equations

Connect moves on hanger diagrams to moves that determine the value of a variable in an equation.



7 Keeping It True

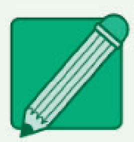
Solve equations of the form px+q=r and p(x+q)=r that involve positive and negative numbers, and explain the strategy.



12 Community Day

Write and solve equations to represent situations involving positive and negative numbers.

Practice Day



Practice Day 1

Practice the concepts and skills developed during Lessons 1–12. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

Quiz: Sub-Unit 1

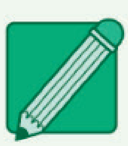
Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2

13 I Saw the Signs

Understand and use the symbols and to describe inequalities.

Practice Day



Practice Day 2

Practice the concepts and skills developed during Lessons 1–17. Consider using this time to prepare for the upcoming Quiz.

Summative Assessment

End-of-Unit Assessment

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing Considerations

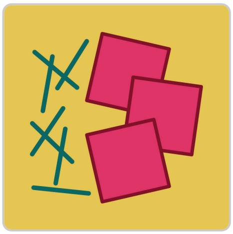
Lesson 1: This lesson surfaces what students know about finding unknowns in proportional and nonproportional relationshipsand sets up ideas for the rest of the unit. These ideas will be addressed in more depth in upcoming lessons

Lessons 2-3: The purpose of these lessons are for students to revisit tape diagrams and surface strategies for using tapediagrams to determine unknown values. If students show a strong understanding of matching tape diagrams with expressions in Problem 1 of the Pre-Unit Check, these two lessons may be condensed into one class period by doing Activity 1 of Lesson 2 followed by Screens 1–6 of Lesson 3.

Lesson 12: This lesson gives students an opportunity to apply what they've learned about writing, representing, and solving equations. There is no new content introduced in this lesson.

Lesson 14: This lesson supports students in making sense of inequalities by revisiting balanced hangers, a familiar representation. If students show a strong understanding of inequalities in earlier lessons, this lesson may be omitted.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Toothpicks and Tiles

Lesson 1: Nonproportional Relationships

Overview

Students extend what they know about proportional relationships to explore patterns in other types of relationships.

Learning Goals

- Use patterns to determine unknown values in relationships that are predictable but not proportional.

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to extend what they know about proportional relationships to explore patterns in other types of relationships. Students use visual patterns and number patterns to make predictions and determine unknown values in the relationship between the stage number of a figure, the number of toothpicks that border it, and the number of tiles that border it.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to make sense of a visual pattern by counting the number of border toothpicks and the number of border tiles.

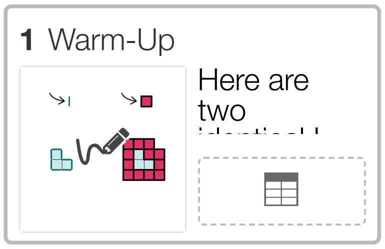
Activity 1: Patterns and Predictions (30 minutes)

The purpose of this activity is for students to use patterns to make predictions about the number of toothpicks and tiles in different designs. Students determine the number of border tiles at a specific stage and the stage that will have a specific number of tiles. Students also get to make their own designs and make predictions about them.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their ideas about how to make predictions about a pattern that is predictable but not proportional.

Cool-Down (5 minutes)



Here are two identical L-shape figures.

The left figure is bordered with toothpicks. The right figure is bordered with tiles.

How many toothpicks are used? How many tiles?

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to extend what they know about proportional relationships to explore patterns in other types of relationships.

Warm-Up Purpose: Students make sense of a visual pattern by counting the number of border toothpicks and the number of border tiles.

Facilitation: Consider starting with the lesson paused and asking students what they notice and wonder about the image on the left half of the screen. Discuss as a class what a border toothpick is and what a border tile is. Then unpause and invite students to determine the number of border toothpicks and tiles, and to use the sketch tool to show how they counted.

Give students one minute to respond. Then facilitate a whole-class discussion to surface all of the different ways students counted toothpicks and tiles. Highlight unique or creative sketches using the snapshot tool or dashboard's teacher view.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Border toothpicks: 8

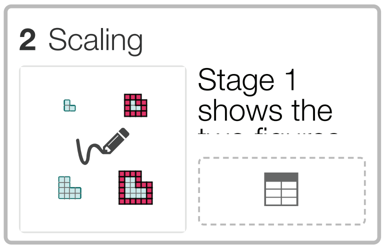
Border tiles: 12

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate the use of the sketch tool throughout this lesson.



Stage 1 shows the two figures from the previous screen.

Stage 2 is the same, but the figure has grown by a scale factor of 2.

How many border toothpicks and tiles are in Stage 2?

Teacher Moves

Activity 1 Purpose: On Screens 2–9, students use patterns to make predictions about the number of toothpicks and tiles in different designs.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and sharing with students that they will use what they notice to make predictions. Then unpause and invite students to figure out the number of toothpicks and tiles in Stage 2. Encourage students to develop a strategy or use a strategy from the warm-up so that they don't need to count one by one.

Pacing: Consider using pacing to restrict students to Screens 2–4.

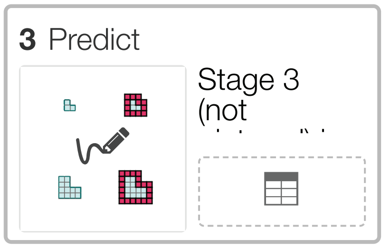
Sample Responses

Border toothpicks: 16
Border tiles: 20

Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the representations for students to draw on or highlight.



Stage 3 (not pictured) is made by applying a scale factor of 3 to the original figure.

Predict how many border toothpicks and tiles are used in Stage 3.

Teacher Moves

Facilitation: Circulate to observe student strategies and listen to discussions. Encourage students to use paper on this screen and throughout the activity as they make their predictions. Monitor for students who use the number of toothpicks to help them with their prediction for the number border tiles or students who use other creative strategies, as these strategies will be useful on subsequent screens. Invite students to share these strategies with the class during the discussion on Screen 4.

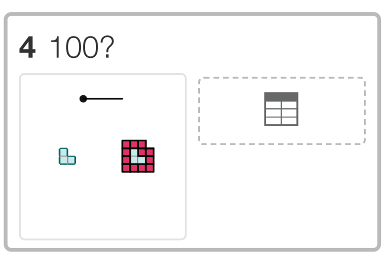
Note: In this design, each stage will have 4 more tiles than toothpicks. One way to explain why is to invite students to consider the relationship between toothpicks and tiles. There is one tile for every toothpick, with a few exceptions. There are 5 extra tiles in the corners that don't correspond to any toothpicks, marked yellow in this image. The tile marked green has the opposite effect: 2 toothpicks that match the same tile. The result is a net of 4 more tiles than toothpicks. This is a pattern that holds for many different designs, but not all.

Sample Responses

Border toothpicks: 24
Border tiles: 28

Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*
Provide graph paper for students to draw on as they predict Stage 3.



Drag the slider to look at Stages 1–3.

Will there ever be a stage with exactly 100 **toothpicks**?

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface the reasoning students use to determine whether or not there will be a stage with exactly 100 toothpicks or tiles.

Facilitation: Encourage students to share the thinking behind their choices with a classmate. When most students have responded, consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. For each of the most popular choices, consider inviting students to share their reasoning and to build on others' reasoning (MP3).

Early Finishers: Encourage students who finish Screens 2–4 early to predict which stage will have 100 toothpicks, 100 tiles, or both.

Routine (optional): Consider using one or more rounds of the routine [Stronger and Clearer Each Time] to help students develop their ideas and language.

Sample Responses

100 **toothpicks:** No. (The number of toothpicks is always a multiple of 8, and 100 is not a multiple of 8.)

100 **tiles:** Yes. (The number of tiles is always 4 more than the number of toothpicks. There will be a stage that has 96 toothpicks because 96 is a multiple of 8, so there will be a stage that has $96 + 4 = 100$ tiles.)

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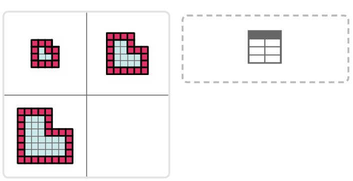
5 Use 100 Tiles



You said there is a stage that uses 100 border tiles.

Which stage number?

Teacher Moves

Facilitation: Consider starting with the activity paused and sharing that our task is to use any strategy to determine *what stage* will use exactly 100 border tiles. Then unpause and invite students to devise a strategy and try it.

Pacing: Consider using pacing to restrict students to Screens 5–9.

Sample Responses

Stage 12

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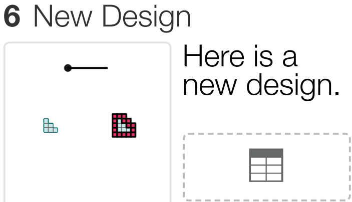
6 New Design



Here is a new design.

Teacher Moves

This is a possible discussion screen.

Facilitation: While students are working, select and sequence several student responses using the snapshots tool. Monitor for students whose descriptions use different levels of precision. Once most students have responded, consider pausing the class and facilitating a whole-class discussion to analyze different students' strategies for calculating the number of border tiles.

Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson.

Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify to support students to write stronger and clearer explanations.

Sample Responses

Responses vary. There are 12 more toothpicks at each stage, so there would be 60 toothpicks at Stage 5. Then I added 4 to get 64 because there are always 4 more tiles than toothpicks.

Student Supports

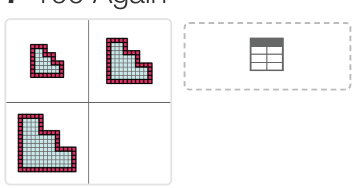
Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., First, ________. Then, _______.).

---

7 100 Again



Here is the design from the previous screen.

Which stage uses 100 border tiles?

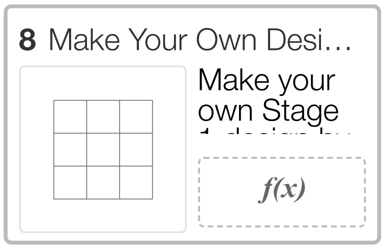
Teacher Moves

Progress Check: This is a great place to check students' progress determining an unknown stage number. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

If students are having difficulty getting started, consider asking them how many toothpicks and tiles they think Stage 5 would have, and then to use the patterns they notice.

Sample Responses

Stage 8



Make your own Stage 1 design by pressing the grid squares.

Predict how many border tiles are used in Stage 4.

The border toothpicks and tiles for Stages 1–3 are shown.

Teacher Moves

Facilitation: The purpose of this screen is for students to notice patterns by trying out lots of different designs. Encourage students to create several different designs. Challenge students to create designs that have many different numbers of tiles in Stage 4.

While students are working, the teacher dashboard will show their designs in real time. Use the snapshots tool to capture and highlight unique and creative designs.

If time allows, follow with a brief discussion of what students noticed as they made several different designs (MP8).

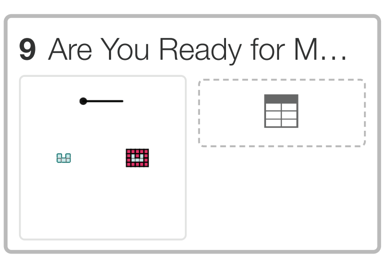
Sample Responses

Responses vary.

Student Supports

Students With Disabilities
- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate creation of their design.



There's something unusual about the number of border tiles in Stage 1 of this design.

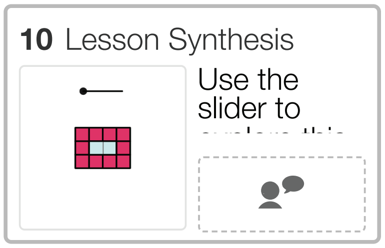
Why is Stage 1 different from Stages 2 and beyond?

Teacher Moves

Facilitation: This screen is designed as an extra challenge for students who finish Screens 5–9 before the class discussion on Screen 10. Consider inviting these students to share responses with each other in place of a whole class discussion.

Sample Responses

Responses vary. The difference between Stage 1 and all the other stages occurs in the top row of tiles. In Stages 2 and beyond, there is an even number of tiles in the top row (8, 10, 12, etc.). But when I move the slider to Stage 1, the pattern breaks: instead of 6 tiles, there are 5 because some of the tiles "collide."



Use the slider to explore this new pattern.

Describe how to determine the number of border tiles at any stage.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their ideas about how to make predictions about a pattern that is predictable but not proportional.

Facilitation: Give students 2–3 minutes to drag the movable point and respond, plus one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or the snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

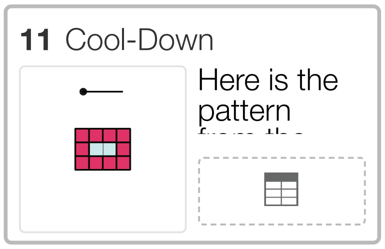
Responses vary. You can figure out the number of toothpicks because it's like the perimeter of the shape, or 6 times the stage number. The number of tiles is 4 more than the number of toothpicks.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., First, ________. Then, _______.).



Here is the pattern from the lesson synthesis.

Teacher Moves

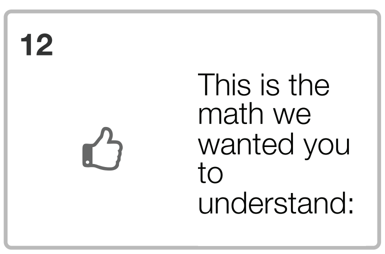
Support for Future Learning: Students will have more chances to develop their understanding of non-proportional relationships throughout the unit.

Pacing: Consider using pacing to restrict students to Screens 11–12.

Sample Responses

No

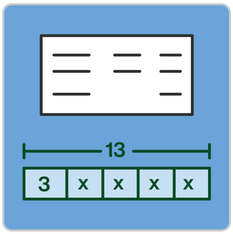
Explanations vary. Each stage adds 6 new tiles. If I keep adding 6 tiles, later stages will have 46 and 52 tiles, but not 50.



This is the math we wanted you to understand:

- I can use patterns to determine unknown values.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Smudged Receipts

Lesson 2: Connecting Contexts and Tape Diagrams

Overview

Students revisit tape diagrams and surface strategies for using tape diagrams to determine unknown values in situations. Students used tape diagrams in Unit 4 to reason about percent increases and decreases. In this lesson, students use tape diagrams to informally represent situations of the form $px + q = r$ and $p(x + q) = r$. Students will be introduced to these types of equations formally in Lessons 3 and 4.

Learning Goals

- Interpret a tape diagram that represents a relationship in context.
- Use a tape diagram to determine an unknown value in context.

Vocabulary

- tape diagram

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.

- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to revisit tape diagrams and to surface strategies for using tape diagrams to determine unknown values in situations. Students used tape diagrams in Unit 4 to reason about percent increases and decreases. In this lesson, students use tape diagrams to informally represent situations of the form $px+q=r$ and $p(x+q)=r$. Students will be introduced to these types of equations formally in Lessons 3 and 4.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to revisit tape diagrams and surface strategies for reasoning about unknowns in tape diagrams.

Activity 1: Smudged Receipts (15 minutes)

The purpose of this activity is for students to connect tape diagrams to scenarios involving smudged receipts, and use the tape diagrams to determine unknown costs.

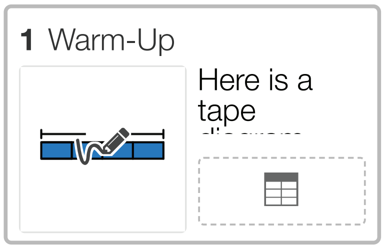
Activity 2: Ordering Food (15 minutes)

The purpose of this activity is for students to connect situations to tape diagrams (MP2), including comparing and contrasting situations that lead to expressions like $4x+6$ and $4(x+6)$, though this lesson does not explicitly include expressions or equations. Students will be introduced to expressions like these in Lesson 3.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to use their creativity to connect tape diagrams and stories.

Cool-Down (5 minutes)



Here is a tape diagram. What are possible values for a and b?

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to revisit tape diagrams and to surface strategies for using tape diagrams to determine unknown values in situations.

Warm-Up Purpose: Students revisit tape diagrams and surface strategies for reasoning about unknowns in tape diagrams.

Facilitation: Consider starting with the lesson paused and asking students what they notice and wonder about the diagram on the left, and to share if they have seen similar diagrams before. Students may connect these tape diagrams to the diagrams that they used in Unit 4. Then unpause and invite students to figure out possible values for a and b in the tape diagram.

If students are having difficulty getting started, consider asking students how long they think each piece of the tape diagram would be and why.

While students are working, select and sequence several correct and incorrect student responses using the snapshots tool. Then facilitate a brief whole-class or pair discussion around a question like: *Which of these values will make the tape diagram true? Why or why not?*

Readiness Check: If Problem 1 of the Readiness Check suggests that some students have unfinished learning about tape diagrams, consider reviewing what the 28 in the tape diagram means before unpausing students.

Early Finishers: Encourage students who finish early to adjust their values for a and b to a pair that they think no one else in the class will write.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- $a=4$, $b=3$. I drew a vertical line to chop the diagram after one $a+b$ piece. I knew this distance had to be 7 because 7 is $\frac{1}{4}$ of 28.

- $a = 5$, $b = 2$. I made a represent 5. Since there are four as, together they have a distance of 20. To make 28, b needs to be 2.
- $a = 1$, $b = 6$. I realized that if four copies of $(a + b)$ made 28, one copy would make 7 because $28 \div 4 = 7$. Then I picked numbers that made $a + b = 7$.

Student Supports

Students With Disabilities

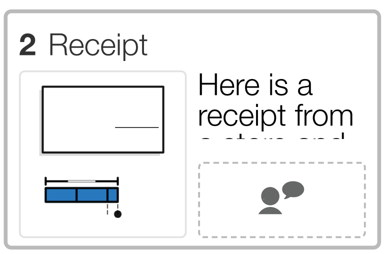
- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate use of the sketch tool as needed throughout the lesson.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., These values make sense because_____.).



Here is a receipt from a store and a tape diagram that represents it.

Move the slider around to see what happens.

What does each part of the tape diagram represent?

Teacher Moves

Activity 1 Purpose: On Screens 2–6, students connect tape diagrams to scenarios involving smudged receipts and use the tape diagrams to determine unknown costs.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and sharing the purpose of the activity. Invite students to drag the slider to a position that feels right to them based on the values in the receipt, then to respond to the prompts.

Early Student Thinking: Some students may correctly connect $\$3.00$ with pasta but not describe that it is the cost of pasta. Consider asking these students what it is about the pasta that the 3 represents. Then invite them to revise their response to make it stronger and clearer.

Pacing: Consider using pacing to restrict students to Screens 2–4.

Sample Responses

Responses vary.

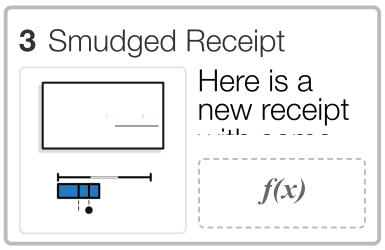
The 3s represent the price of each box of pasta.
The x represents the price of the cheese.
The length of the diagram represents the total cost of all the items.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate physical manipulation of movable points throughout this lesson.



Here is a new receipt with some smudged numbers.

Use the slider to determine the price of a cantaloupe.

Enter the price below.

How would you figure out the price of a cantaloupe **without** using the slider?

Teacher Moves

This is a possible discussion screen.

Facilitation: When most students have completed this screen, facilitate a brief whole-class discussion to surface strategies students might use to figure out the value of x without the slider.

Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Sample Responses

$\$3.50$

Explanations vary.

- I know that the total is $\$9$ and the honeydew is $\$2$, so there is $\$7$ left for two cantaloupes. Each cantaloupe has to be $\$3.50$.
- The total is 9. 2 comes from the small piece of tape, so the length of the two x pieces of tape must be 7. I could divide by 2 and figure out that each x piece is 3.5.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., You could figure out the price of a cantaloupe by _____.).

---

4 Diagrams and Receipts

Teacher Moves

Facilitation: Encourage students to share their reasoning with a partner and work together to reach an agreement about how to pair the cards.

If time allows, consider asking students to compare their matches, justify their card placement, and make revisions based on their conversation.

Early Finishers: Encourage students who finish Screens 2–4 early to create their own receipt for the unpaired diagram.

Sample Responses

Image solution

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate how to sort the cards.

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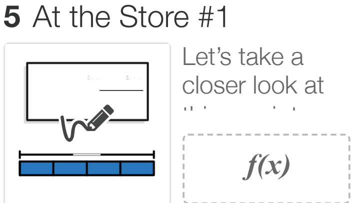
5 At the Store #1



Let's take a closer look at this receipt.

What is the price of juice at this store?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Progress Check: This is a great place to check students' progress determining unknown values using a tape diagram. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Pacing: Consider using pacing to restrict students to Screens 5–6.

Sample Responses

$\$3.25$

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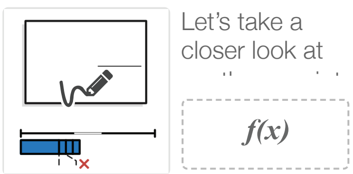
6 At the Store #2



Let's take a closer look at another receipt.

What is the price of beans?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface strategies for using a tape diagram to determine an unknown value in a context.

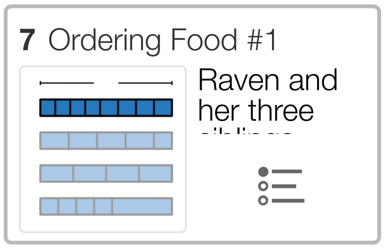
Facilitation: If students are having difficulty getting started, consider asking how long the 3 x-values together are. While students are working, circulate to observe student strategies, listen to discussions, and offer help or encouragement where needed. Monitor for students who used strategies discussed earlier in the lesson or new strategies.

When most students have completed this screen, facilitate a whole-class discussion. Consider asking students to share strategies that were successful and strategies that were not successful, and record students' successful strategies visually so that they can refer to them throughout the remainder of the lesson.

Early Finishers: Invite students who finish Screens 5–6 early to practice describing their strategy verbally so that it is clear and concise, or to come up with a different strategy that would also get the same solution.

Sample Responses

$\$3.10$



Raven and her three siblings were given $\$44$ to order takeout food.

One restaurant they are considering, Burrito Express, charges a $\$6$ service fee for their order.

Select a diagram that could help determine how much each sibling can spend on their meal.

Teacher Moves

Activity 2 Purpose: On Screens 7–9, students connect situations to tape diagrams (MP2), including comparing and contrasting situations that lead to expressions like $4x+6$ and $4(x+6)$, though this lesson does not explicitly include expressions or equations. Students will be introduced to expressions like these in Lesson 3.

Facilitation: Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.

Routine (optional): Consider using the mathematical language routine Which One Doesn't Belong to help students notice features of each representation.

Pacing: Consider using pacing to restrict students to Screens 7–8.

Sample Responses

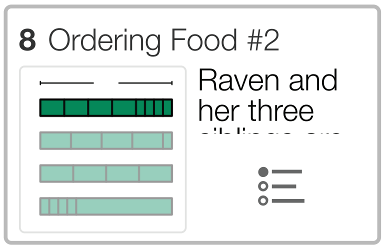
Diagram B

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the tape diagrams on Screens 7–8 for students to draw on or highlight.



Raven and her three siblings are also considering using their $\$44$ to order from Salads-R-Us.

This restaurant charges a $\$3$ service fee for each meal.

Select a diagram that could help determine how much each sibling can spend on their meal.

Explain how this diagram represents the situation.

Teacher Moves

This is a possible discussion screen.

Facilitation: Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Hold a brief discussion, if time permits. Ask students to share their reasoning for how they connected diagrams to situations on Screens 7–8, and invite students to critique each other's reasoning (MP3).

If it does not come up naturally, consider asking students what key words or phrases they paid attention to in each situation. Consider also asking students if there is more than one diagram that could represent either situation (MP2).

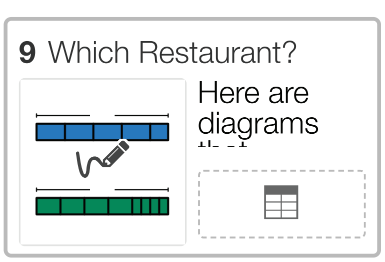
Early Finishers: Encourage students who finish Screens 7–8 to consider what system of charging fees they think is most fair and why. If students are curious why a restaurant might charge a service fee per meal, consider asking them what costs different restaurants might have that would cause them to charge fees differently.

Sample Responses

Diagram A or Diagram C

Explanations vary.

- Diagram A has 4 pieces of tape with a 3, which is like the four $\$3$ service fees. There are also 4 y pieces, which is like the amount that each sibling can spend on food.
- Diagram C has 4 pieces of tape, one for each sibling. Each piece is $y+3$ long because there is a $\$3$ service fee for each meal, but they don't know how much they are going to spend on food (y).



Here are diagrams that represent the situations on the previous screens.

Figure out the value of x and y in the diagrams. Enter your values in the table below.

Explain to a neighbor what your values say about which restaurant the siblings should choose.

Teacher Moves

This is a possible discussion screen.

Facilitation: When most students have completed this screen, consider reviewing it as a class. Ask students to share strategies they used to figure out the unknown in each diagram and what the value of the unknown means in the situation.

Consider concluding the discussion with a question like: *If you were Raven and her siblings, where would you choose to eat? Why?*

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

$x = 9.50$, $y = 8$

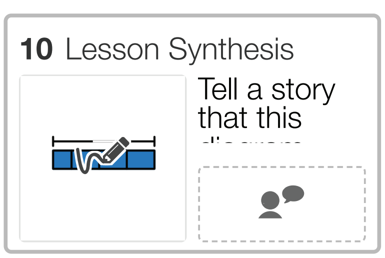
Explanations vary. If Raven and her siblings want the most money to spend on food, they should order from Burrito Express because each person would have $\$1.50$ more to spend on food ($\$9.50$ versus $\$8$).

Student Supports

Multilingual Learners

- *Receptive/Expressive Language: Peer Tutors*

Pair students to aid them in making sense of the explain prompt and expressing their understanding before the whole-class discussion.



Tell a story that this diagram could represent.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students use their creativity to connect tape diagrams and stories.

Facilitation: Give students 2–3 minutes to respond and one minute to share their stories with a partner. Consider highlighting unique or creative stories using the snapshots tool or the dashboard's teacher view. Ask the author to speak about their inspiration and how they know the tape diagram represents the story.

Follow with a brief whole-class discussion in which students share connections they see between stories. If time allows, give students one minute to make their story more clearly connected to the tape diagram based on the conversation.

Routine: Consider using the routine Tell a Story to support students in using their knowledge and creativity to make sense of a situation.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. The Jones family has four children. One of them is 4 years old. The other three are triplets. Their ages add up to 22.

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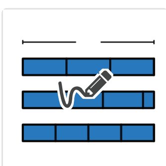
11 Cool-Down



Neel and his sister are

Neel and his sister are making gift bags for a party.

Neel puts 3 pencil erasers in each bag. His sister puts x stickers in each bag. After filling 4 bags, they have used a total of 36 items.

Which diagram best represents the story?

Teacher Moves

Support for Future Learning: If students struggle to connect situations and tape diagrams, plan to revisit this when opportunities arise during Lesson 3. Consider spending extra time discussing how diagrams connect to situations in Lesson 3, Activity 1.

Pacing: Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Diagram C

$x = 6$

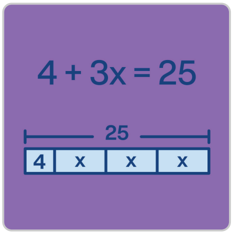
12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can connect a tape diagram to a story.
- I can use a tape diagram to figure out an unknown value.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Equations

Lesson 3: Representing Contexts With Tape Diagrams and Equations

Overview

Students use what they know about tape diagrams and situations to make sense of equations of nonproportional relationships. Students both connect different representations of the same situation and create their own representations.

Learning Goals

- Connect tape diagrams, equations, and verbal descriptions in context.
- Write an equation to represent a tape diagram or a context.

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to use what they know about tape diagrams and situations to make sense of equations of nonproportional relationships. Students both connect different representations of the same situation and create their own representations.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to connect what they have learned about tape diagrams in Lesson 2 to equations.

Activity 1: Connecting Representations (15 minutes)

The purpose of this activity is for students to use equations and tape diagrams to make sense of a situation. Students answer a question about a situation involving baking cookies using any representation (MP5). Students also create an equation and tape diagram to represent a similar situation.

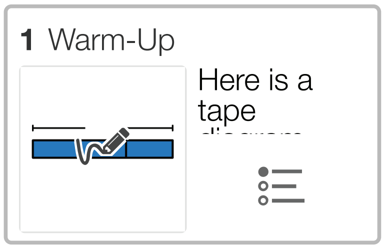
Activity 2: Missing Representations (15 minutes)

The purpose of this activity is for students to practice connecting situations, tape diagrams, and equations. Students also create their own representations for missing cards in the card sort, including writing a story to represent a tape diagram and equation.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to connect stories, equations, and tape diagrams with their solution.

Cool-Down (5 minutes)



Here is a tape diagram and three equations.

Select the equation that matches the tape diagram.

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to use what they know about tape diagrams and situations to make sense of equations of non-proportional relationships.

Warm-Up Purpose: Students connect what they have learned about tape diagrams in Lesson 2 to equations.

Facilitation: Consider starting with the lesson paused, displaying the dashboard's student view, and asking students what they notice about the tape diagram on the left. Next, unpause and give students one minute to decide which equation they think matches the tape diagram and another minute to share their reasoning with a partner. Then, give pairs another minute to work together to draw a tape diagram for a different equation.

If students are having difficulty getting started, consider asking them to figure out what the value of x would be in the tape diagram and to test that value in each equation.

Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Then, facilitate a brief whole-class conversation to surface how students decided which equation to choose. If it does not come up naturally, consider asking questions like: *What would the tape diagram for* $3x+6=30$ *look like? What about* $3+6x=30$*?*

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

$3(x+6)=30$

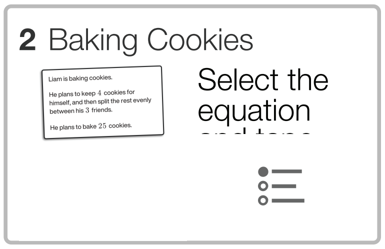
Image solution for the follow-up question.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate use of the sketch tool throughout this lesson.



Select the equation and tape diagram that match Liam's situation.

Teacher Moves

Activity 1 Purpose: On Screens 2–5, students to use equations and tape diagrams to make sense of a situation. Students answer a question about a situation involving baking cookies using any representation (MP5).

Facilitation: Arrange students into pairs. Consider starting with the activity paused and sharing the purpose with students and that these equations are unrelated to the ones in the warm-up. Read the situation aloud as a class and ask students to paraphrase what the situation is about with a partner. Then invite students to decide which representations match Liam's situation and to continue to the next screen.

Progress Check: This is a great place to check students' progress matching situations, equations, and tape diagrams. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Pacing: Consider using pacing to restrict students to Screens 2–5.

Sample Responses

$4 + 3x = 25$

Student Supports

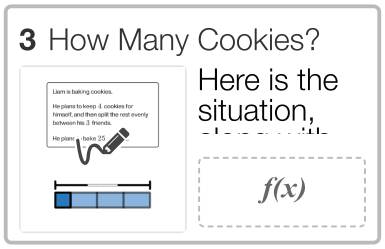
Students With Disabilities

- *Receptive Language: Processing Time*

Read the situation aloud. Students who both listen to and read the information will benefit from extra processing time. If time allows, give students one minute to explain what the situation is about and what information is important before responding.

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the choices for students to draw on or highlight.



Here is the situation, along with an equation and a tape diagram that matches it.

How many cookies should each of Liam's friends receive?

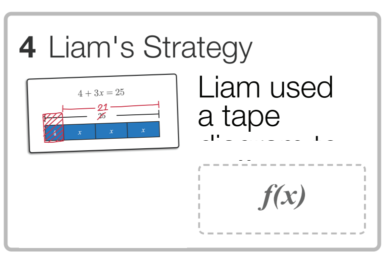
Teacher Moves

Facilitation: Circulate to observe different tools students use to answer Liam's question. Some students may use the equation, others may use the tape diagram, and some may reason directly from the situation (MP5). Encourage students to check their solution in both the tape diagram and equation.

Readiness Check: If Problem 2 of the Readiness Check suggests that some students have unfinished learning about writing and solving equations to represent situations, consider paying attention to how students decide what the solution is on this screen. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

7 cookies



Liam used a tape diagram to solve his equation.

His first step is shown.

Then he wrote a new equation.

What equation do you think he wrote?

Why might this equation be helpful?

Teacher Moves

Facilitation: If students are having difficulty getting started, consider asking them to draw what they think Liam's second tape diagram would look like. It may also be helpful to explain how each of the parts of the equation $4 + 3x = 25$ is related to the original tape diagram.

Sample Responses

Responses vary.

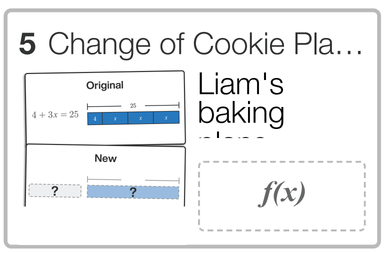
- $3x = 21$. This equation is helpful because it's getting closer to an equation like "$x =$ ". It also helps to see that x must be 7 because $3 \cdot 7 = 21$.
- $x + x + x = 21$. This equation is helpful because it shows that an unknown number added three times is 21 , so the number must be 7 .
- $4 + 3x - 4 = 25 - 4$. This equation helps keep track of the steps that Liam used to solve.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., This equation might be helpful because ________.).



Liam's baking plans changed. He ended up baking 33 cookies. He kept 5 for himself and split the rest evenly between 7 friends.

What will the new equation be?

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface connections between the situation, equation, and tape diagram.

Facilitation: While students are working, select and sequence several student responses using the snapshots tool. Monitor for different ways students describe both the equation and the tape diagram, and for students who make connections between the situation and the other representations (MP2).

When most students have responded, consider displaying several different examples of student thinking using the snapshots tool or the dashboard’s teacher view. Facilitate a whole-class discussion around questions like: *How are the new equation and tape diagram connected? How can we see the parts of the story in each representation?*

Sample Responses

Responses vary.

The new equation will be $5 + 7x = 33$.

The new tape diagram will look like one section with 5 and then 7 sections labeled x . The total (33) will be on top.

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Provide graph paper so students can draw their own tape diagram before they respond to the text prompt.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

---

6 Multiple Representati…

Teacher Moves

Activity 2 Purpose: On Screens 6–10, students practice connecting situations, tape diagrams, and equations. Students also create their own representations for missing cards in the card sort.

Facilitation: Consider starting with the activity paused and asking students how many stories, tape diagrams, and equations they notice in the card sort. Share that they will create one group that has all three representations and two groups that are missing either an equation or a story.

Then unpause and invite students to sort the cards. Encourage students to share their reasoning with a partner and work to reach an agreement together about how to sort the cards.

If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Note: Students will see several correct and incorrect pairings on Screens 7–9.

Pacing: Consider using pacing to restrict students to Screens 6–10.

Sample Responses

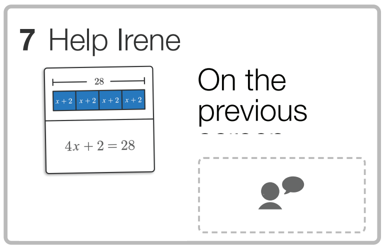
Image solution

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the tape diagrams and situations for students to draw on or highlight. These will be useful throughout this activity.



On the previous screen, Irene matched these two cards.

What would you tell Irene to convince her that these cards don't match?

Teacher Moves

This is a possible discussion screen.

Facilitation: When all students have responded, consider pacing students to this screen and facilitating a brief whole-class discussion about how students know that the tape diagram and equation do not match.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language.

Sample Responses

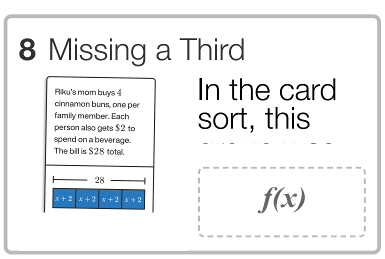
Responses vary. In $4x + 2 = 28$, the 2 is by itself, so that means that the 4 groups are only x. The 2 is its own section.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., These do not match because ________.).



In the card sort, this group was missing an equation.

Enter an equation that matches the two cards.

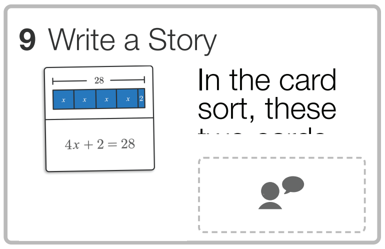
Teacher Moves

Progress Check: This is a great place to check students' progress writing equations. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

If students are having difficulty getting started, consider encouraging students to revisit the warm-up and look at what equation they matched with the tape diagram. (Note: If you advise students to return to the warm-up, make sure the activity pacing allows them to do so.)

Sample Responses

$4(x+2)=28$ (or equivalent)



In the card sort, these two cards did not have a story to go with them.

Write a story that matches these two cards.

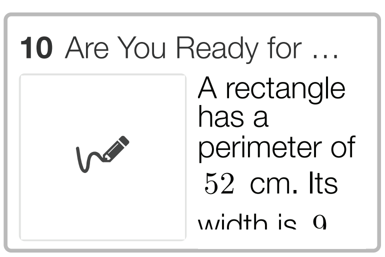
Teacher Moves

Facilitation: Consider highlighting unique or creative stories using the snapshots tool or the dashboard's teacher view. Ask the author to speak about their inspiration.

Routine: Consider using the routine Tell a Story to support students in using their knowledge and creativity to make sense of a situation.

Sample Responses

Responses vary. I was playing around at home with a scale. I put 4 bricks on the scale, and then my 2-pound cat jumped on the scale too. The scale read: 28 pounds.



A rectangle has a perimeter of 52 cm. Its width is 9 cm.

On paper, write an equation and/or make a tape diagram to represent the situation.

Then determine the length of the rectangle.

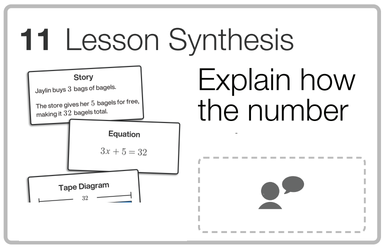
Teacher Moves

Facilitation: This screen is designed as an extra challenge for students who finish Screens 6–9 before the class discussion on Screen 11. Consider inviting these students to share their equations, tape diagrams, and solutions with each other in place of a whole-class discussion.

Sample Responses

$2(x + 9) = 52$ (or equivalent)

The length of the rectangle is 17 cm.



Explain how the number 9 is important in each representation.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students connect stories, equations, and tape diagrams with their solution.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Routine (optional): Consider using the mathematical language routine Collect and Display.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- In the story, there must be 9 bagels in each bag. If you have 3 bags of 9 and add 5 bagels, you get 32 bagels.
- In the equation, 9 makes sense as the value for x because $3(9)+5 = 32$.
- In the tape diagram, 9 is the length of the x segments because $9 + 9 + 9 + 5 = 32$.

Student Supports

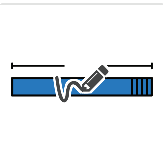
Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., In the equation, _______. In the tape diagram, _______. In the story, ______).

- *Expressive Language: Visual Aids*
Create or review an anchor chart that publicly displays each of the representations to aid in students' explanations.

12 Cool-Down



Ella ran 6 times

Ella ran 6 times around her school building. Then she ran 4 miles home.

Her phone told her that she ran 7 miles total.

Select an equation that represents this situation.

Teacher Moves

Support for Future Learning: Students will have more chances to develop their understanding of equations, situations, and solutions in the upcoming lessons, particularly Lesson 4 and Lesson 12.

Pacing: Consider using pacing to restrict students to Screens 12–13.

Sample Responses

$6x + 4 = 7$
$x = 0.5$ (or equivalent)

13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can connect tape diagrams, equations, and stories.
- I can write an equation to represent a tape diagram or a story.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Seeing Structure (NYC)

Lesson 4: Practice With Tape Diagrams and Equations

Purpose

The purpose of this lesson is for students to use structure to analyze and solve a question in context and interpret the meaning of its solution. Students compare and contrast equations of the form $px+q=r$ and $p(x+q)=r$, and situations that lead to these equations (MP7). Students then use tape diagrams to answer questions about these situations.

Preparation

Worksheet

- *Warm-Up:* Print one single-sided sheet for each student.
- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Warm-Up (10 minutes)

Purpose: Students notice structure in equations of the form $px+q=r$ and $p(x+q)=r$ (MP7).

Facilitation: Arrange students into pairs. Display Sheet 1 of the Teacher Projection Sheets and distribute one Warm-Up worksheet to each student. Give students 3–4 minutes to look at all six equations and respond to the prompts on paper.

Give students another two minutes to share how they grouped the equations with a partner, along with their reasoning. Consider asking students to share with their partner how the tape diagrams for each group would look similar or different.

Then, facilitate a whole-class discussion to highlight all of the different patterns students noticed. Consider visualizing students' groupings on the board, including how they would describe each group. If it does not come up naturally, consider asking if any students grouped the equations based on whether there were parentheses or not, and how the parentheses would change the tape diagram.

Support for Students With Disabilities
Conceptual Processing: Processing Time
Check in with individual students to check for understanding as needed at each step of the activity.

Activity 1: Which Diagram? (10 minutes)

Purpose: Students connect the features of a situation to an equation, a tape diagram, and a solution, and interpret the meaning of the solution in the situation (MP2).

Facilitation: Distribute one Student Worksheet to each student. Give students a minute to examine the two tape diagrams and consider how they are similar and different. Then give students 5–10 minutes to analyze each of the three situations.

There are two options for facilitating this activity:

- **Option 1:** Ask students to determine individually which diagram matches the situation, and then compare and work together to write the equation, solution, and meaning of the solution.
- **Option 2:** Arrange students into pairs. They take turns describing their thinking on a situation out loud as the other student records their thinking on paper, asking questions if they disagree.

Circulate to observe student strategies, and offer help or encouragement where needed. Consider posting the answer key, or walking around with it and providing feedback to students as they work.

Then, facilitate a whole-class discussion to surface strategies students used to decide which diagram matched each situation and the meaning of the solution. Consider giving away the correct solutions in order to focus the conversation on students' reasoning. Consider highlighting students who use connections to their own lives in order to help them decide.

Early Finishers: Invite students who finish early to write their own situation that matches one of the diagrams, then trade situations with another student to figure out which diagram they selected.

Support for Students With Disabilities
Conceptual Processing: Eliminate Barriers
Demonstrate the steps for the activity by reading Problem 1 aloud and determining which diagram matches it as a class before students begin.

Activity 2: Write Your Own (15 minutes)

Purpose: Students use the features of a situation to write their own question, then determine a solution and write an equation. Students also write their own situations.

Facilitation: Consider starting this activity by asking students what they remember about how to write good questions or what they think makes a good question at the end of a situation. This builds on the work students did writing questions in Unit 4, Lesson 12.

Then give students 5–10 minutes to write and answer questions for Problems 1–2. Consider consulting with pairs about their solutions to Problems 1 and 2 before they begin the "Are You Ready for More?" problem.

Progress Check: This is a great place to check students' progress writing equations from situations. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Support for Multilingual Learners
Receptive/Expressive Language: Processing Time
Students who benefit from extra processing time would also be aided by reading each situation aloud, either in pairs or as a class.

Lesson Synthesis (5 minutes)

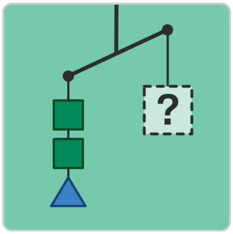
Purpose: Students compare tape diagrams for similar equations with different structures.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Ask students to show their thinking on the board. Follow with a brief whole-class discussion to compare and contrast the two diagrams. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Cool-Down (5 minutes)

Support for Future Learning: If students struggle with this cool-down, they will have more opportunities to write equations from contexts, particularly in Lesson 12 and Practice Day 1.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Balancing Moves

Lesson 5: Introduction to Balanced Hangers

Overview

Students develop strategies for figuring out an unknown weight by creating balanced hangers with fewer objects. This is the first of a two-lesson sequence. In Lesson 6, students connect determining weights on balanced hangers to solving equations.

Learning Goals

- Determine an unknown value in a hanger diagram, and explain their strategy.
- Describe and use moves that will keep a hanger balanced (e.g., adding or removing equal items from each side).

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop strategies for figuring out an unknown weight by creating balanced hangers with fewer objects. This is the first of a two-lesson sequence. In Lesson 6, students connect determining weights on balanced hangers to solving equations.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to understand the concept of balanced hangers and recognize that hangers are balanced when the total weight on each side is equal.

Activity 1: Unknown Weights (15 minutes)

The purpose of this activity is for students to use different strategies to determine unknown weights in balanced hangers. Students develop their own strategies for creating simpler hangers and analyze two different strategies: removing the same shape from both sides and dividing hangers into equal groups.

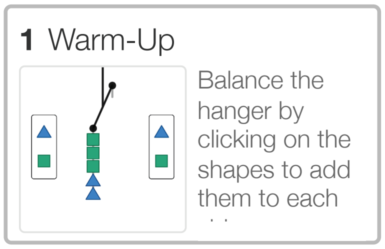
Activity 2: Create Your Own Hanger (15 minutes)

The purpose of this activity is for students to express their creativity by creating their own hangers and practice determining the weight of an unknown shape in a hanger. Creating a balanced hanger may take several minutes, including several rounds of adjusting objects and weights (MP6).

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to refine their strategies for creating balanced hangers with fewer objects (MP7).

Cool-Down (5 minutes)



Balance the hanger by clicking on the shapes to add them to each side.

Be sure to make the sides different.

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to develop strategies for figuring out an unknown weight by creating balanced hangers with fewer objects.

Warm-Up Purpose: Students are introduced to the concept of balanced hangers and recognize that hangers are balanced when the total weight on each side is equal.

Facilitation: Consider starting with the lesson paused and displaying the dashboard's student view. Ask students what they think would happen if we added 5 triangles to the right side. Demonstrate how to add shapes to each side and how to reset each side.

Then unpause and invite students to create hangers that balance. Encourage students who figure out one way to create a balanced hanger to find a different way to balance the hanger.

Monitor for several different ways of creating balanced hangers, including hangers that use the same weights in different orders and hangers that use different sets of weights on each side.

After 2–3 minutes, facilitate a brief whole-class discussion around a question like: *How can you predict if a hanger will be balanced or not?* Students may bring in their knowledge of hangers from their closets or from Grade 6 to support their arguments.

Pacing: Consider using pacing to restrict students to this screen.

Materials (optional): Consider bringing belts, socks, or ties placed on each end of a hanger as a concrete model showing how a hanger will tilt if the sides are not balanced.

Sample Responses

Responses vary. The hanger is balanced when the total weight on each side of the hanger is the same.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate creation of the hanger as needed.

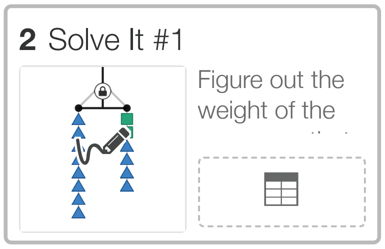


Figure out the weight of the square so that the hanger stays balanced.

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Activity Purpose: On Screens 2–6, students use different strategies to determine unknown weights in balanced hangers.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and asking students what they think would happen if each square weighed 100 pounds. Then unpause and invite students to figure out how much the square would weigh if the hanger were balanced. Encourage students to use the sketch tool to show their thinking.

If students are having difficulty getting started, consider encouraging them to take a guess and see what happens, then revise their thinking.

Monitor for students who use different correct and incorrect strategies for determining the weight of the square, including strategies similar to Adnan's strategy on Screen 3.

Pacing: Consider using pacing to restrict students to Screens 2–6.

Sample Responses

4 pounds

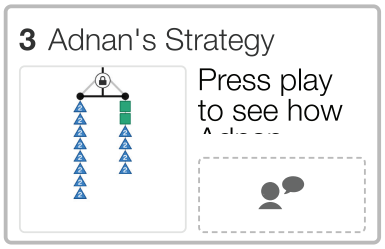
Responses vary. I crossed off 4 triangles from each side, leaving 8 pounds balanced with 2 squares, so each square must weigh 4 pounds.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate use of the sketch tool as needed throughout this lesson.



Press play to see how Adnan determined the weight of a square.

Describe this strategy.

Teacher Moves

Facilitation: Consider inviting students to think about what Adnan's new hanger would look like. Encourage students to read others' responses and/or discuss their response with a partner, and decide if others' descriptions were similar to or different from their own.

Consider publicly celebrating students who tried a strategy similar to Adnan's, and rename the strategy after these students.

Sample Responses

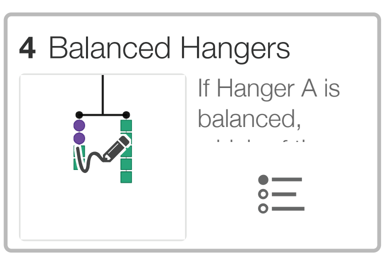
Responses vary. Adnan first crossed off 4 triangles from each side, so only squares were left on the right. Then he divided each side into 2 groups because there were 2 squares.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., First, Adnan ________. Then, he _______.).



If Hanger A is balanced, which of the following hangers will also be balanced?

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

The purpose of this discussion is to surface different ways to create (or not create) hangers with fewer objects that are still balanced.

Facilitation: Circulate to listen to discussions and offer help or encouragement where needed. Encourage students to be prepared to share why they think a hanger will or will not be balanced.

When most students have responded, consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Then, facilitate a whole-class discussion to share reasons why each hanger is or is not balanced.

Question to push students' thinking: *Which of these hangers would you use to help you determine the weight of one circle?*

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

- C
- D
- F
- G

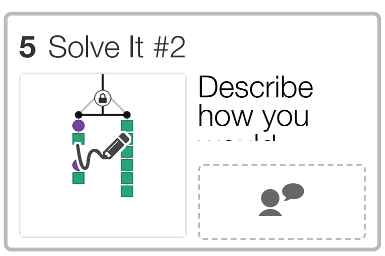
Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the choices for students to draw on or highlight.

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning during the discussion (e.g., Hanger ___ is/is not balanced because ________.).



Describe how you would figure out the weight of a circle.

Teacher Moves

Facilitation: Consider sharing with students that there are many different ways to figure out the weight of a circle, and encourage them to use a strategy that makes sense to them.

Monitor for students who use different correct and incorrect strategies for determining the weight of the square, including strategies similar to Zoe's strategy on Screen 6, in anticipation of the class discussion on Screen 6.

Sample Responses

Responses vary. First, I crossed off 40 pounds from each side, leaving 2 circles balanced with 80 pounds. Then, I divided both sides by 2

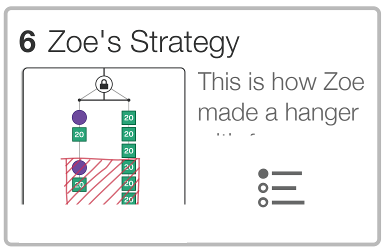
and found that one circle was balanced with 40 pounds.

Student Supports

Multilingual Learners

• *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., First, ________. Then, _______.).



This is how Zoe made a hanger with fewer objects.

Will the new hanger be balanced?

Teacher Moves

This is a possible discussion screen.

Facilitation: Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Consider selecting and sequencing 1–2 responses for each of the most popular choices to display, and invite those students to share their reasoning.

Spend adequate time here to ensure students understand Zoe's strategy in order to support them in making sense of strategies for solving tricky hangers they might see in the upcoming Challenge Creator.

Early Finishers: Encourage students who finish Screens 2–6 early to create their own hanger with an unknown weight, then trade with a classmate. This will be good practice for the Challenge Creator.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language. If possible, use a student response in anonymize mode. If not, consider using the response "No. You took away different things from each side."

Sample Responses

Yes

Responses vary. Zoe split each side of the hanger into halves, so it will be still be balanced. What remains on each side is equal to what was taken away.

Student Supports

Multilingual Learners
• *MLR 3 (Clarify, Critique, Correct)*
Instead of using these discussion questions, use a "Critique a Flawed or Partial Response" strategy by having students improve upon a statement, like "She got rid of part of the picture."

7 Class Gallery

Teacher Moves

Activity Purpose: Students express their creativity by creating their own hangers, and practice determining the weight of an unknown shape in a hanger.

Facilitation: Consider starting with the activity paused and sharing with students that they will create their own challenge and then solve challenges created by their classmates.

In this activity, students create a balanced hanger. Students are presented with their classmates' hangers and are asked to determine the weight of one object.
Creating a balanced hanger may take several minutes, including several rounds of adjusting objects and weights (MP6).

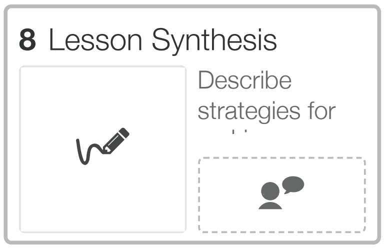
Give students several minutes to create their own challenge and more time to solve the challenges of others. After a set amount of time, encourage students to also take some time to review responses to their own. While students are working, monitor for creative challenges and solutions. Celebrate both simple and more complicated challenges.

Note: We anticipate this Challenge Creator may take 15 minutes or more.

Pacing: Consider using pacing to restrict students to this screen.

Student Supports

Students With Disabilities
• *Fine Motor Skills: Peer Tutors*
Allow students who struggle with fine motor skills to dictate creation of the hanger as needed.



Describe strategies for making a balanced hanger with fewer objects.

Use the diagrams if they help you with your explanation.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their strategies for creating balanced hangers with fewer objects (MP7).

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool.

Follow with a brief whole-class discussion in which students share connections they see between responses. If it does not come up naturally, consider revisiting the strategies identified throughout the class, particularly those named after individual students.

Routine (optional): Consider using the mathematical language routine *Collect and Display*.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

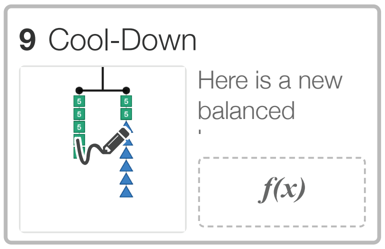
- Take away the same objects from both sides.
- Divide both sides by two.

Student Supports

Multilingual Learners

- *MLR 2 (Collect and Display)*

Circulate and listen to students talk during pair work or group work. Jot down notes about common or important words and phrases (e.g.,"groups" and "remove from both sides") together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the unit.



Here is a new balanced hanger.

What is the weight of a triangle?

Use the sketch tool if it helps you with your thinking.

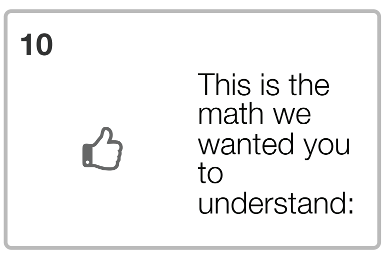
Teacher Moves

Support for Future Learning: If students struggle with this cool-down, they will have more opportunities to determine unknown weights in balanced hangers, particularly in Lessons 6 and 7.

Pacing: Consider using pacing to restrict students to Screens 9–10.

Sample Responses

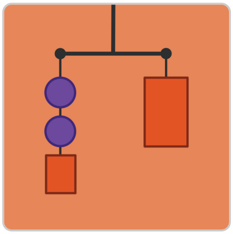
2.5 pounds



This is the math we wanted you to understand:

- I can figure out an unknown value in a hanger diagram and explain my strategy.
- I can make moves to keep a hanger balanced.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Balancing Equations

Lesson 6: Solving Equations With Balanced Hangers

Overview

Students understand strategies for solving an equation and connect those to balanced moves on a hanger.

Learning Goals

- Connect moves on hanger diagrams to moves that determine the value of a variable in an equation.
- Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive rational numbers.

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to understand strategies for solving an equation and connect those to balanced moves on a hanger. Students also create their own hangers to match equations, and shift from solving for unknowns in hangers toward solving for unknowns in equations.

Lesson Summary

Warm-Up (5 minutes)
The purpose of the warm-up is for students to revisit balanced hangers and connect balanced hangers to equations.

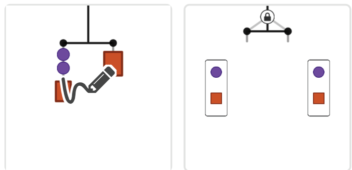
Activity 1: Connecting Hangers to Equations (15 minutes)
The purpose of this activity is for students to connect moves on hanger diagrams with moves used to determine the value of a variable in an equation. Students develop their own strategies and examine strategies of fictional students, including removing the same objects from both sides of a hanger and splitting the hanger into groups.

Activity 2: Solving Equations (15 minutes)
The purpose of this activity is for students to shift from solving for unknown weights of objects on hangers toward solving for unknowns in equations. Students can draw a hanger diagram if it supports their thinking. This activity ends with a repeated practice screen where students are asked to determine the value of x in several different equations.

Lesson Synthesis (5 minutes)
The purpose of the synthesis is for students to connect solving equations to solving for the weight of an object on a balanced hanger.

Cool-Down (5 minutes)

1 Warm-Up



Teacher Moves

Lesson Overview: The purpose of this lesson is for students to understand strategies for solving an equation and connect those to balanced moves on a hanger.

Warm-Up Purpose: Students revisit balanced hangers and connect balanced hangers to equations.

Facilitation: Consider starting with the lesson paused, and ask students what they notice and wonder about Hanger A and its equation with a question like: *Where do you see each part of the equation in the hanger?* Consider using the dashboard's student view to demonstrate what happens when you select multiple purple circles and multiple red squares.

Then unpause and invite students to create different balanced hangers. Use the dashboard's teacher view or snapshots tool to highlight several different balanced and unbalanced hangers.

Consider asking students to predict whether or not they think each hanger will balance and to explain their reasoning. Encourage students to share any connections they make between the hangers and their equations, or what they notice about each of the hangers that balance.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- x and 2 on the left and 5 on the right.
- $2x$ on the left and 6 on the right.

Student Supports

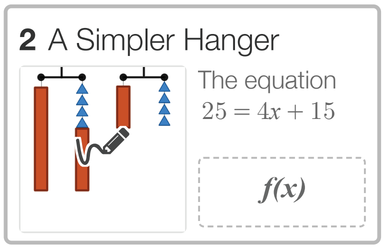
Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate the creation of a hanger as needed.

- *Conceptual Processing: Eliminate Barriers*

Assist students in recognizing the connections between this problem and the work they did in Lesson 5. Students may benefit from a review of hanger diagrams to activate prior knowledge.



The equation $25 = 4x + 15$ represents Hanger A.

1. Write an equation to represent Hanger B.

Teacher Moves

Activity 1 Purpose: On Screens 2–6, students connect moves on hanger diagrams with moves used to determine the value of a variable in an equation.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and sharing the purpose with students. Then, unpause and invite students to write an equation and explain what someone might have done if they created Hanger B from Hanger A.

Encourage students to read others' responses and/or discuss their response with a partner and decide if other descriptions were similar to or different from their own.

Readiness Check: If Problem 4 of the Readiness Check suggests that some students have unfinished learning about solving one-step equations, consider spending time reviewing Problems 4.1–4.3 before beginning this activity.

Pacing: Consider using pacing to restrict students to Screens 2–6.

Sample Responses

1. $10 = 4x$

2. *Responses vary.* To get from Hanger A to Hanger B, you could take away 15 from each side.

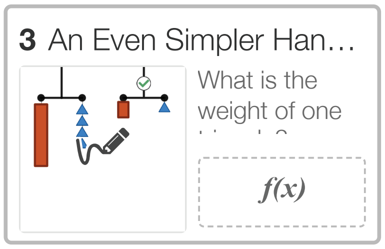
Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the representations for students to draw on or highlight throughout this lesson.

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., To get from Hanger A to Hanger B, _______.).



What is the weight of one triangle?

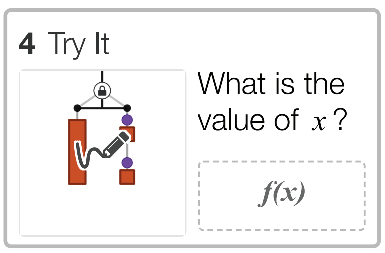
Teacher Moves

Progress Check: This is a great place to check students' progress with determining unknown weights. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Note: Students can check the equation they wrote on Screen 2 against the equation for Hanger B on this screen.

Sample Responses

2.5 pounds



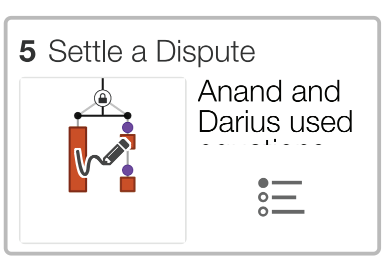
What is the value of x?

Teacher Moves

Facilitation: Encourage students to use the sketch tool to show their thinking as they determine the weight of a circle. Select and sequence successful and unsuccessful strategies, including removing 6 from each side and splitting the hanger into two groups, similar to Darius's and Anand's strategies on Screen 6. Consider asking questions like: *What did this student do correctly? Will this hanger stay balanced? How do you know?*

Sample Responses

4



Anand and Darius used equations to figure out the value of x.

Darius wrote the equation $14 = 2x + 6$.

Anand wrote the equation $14 = 2(x + 3)$.

Who is correct?

Teacher Moves

This is a possible discussion screen.

Facilitation: Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Consider selecting and sequencing 1–2 responses for each of the most popular choices to display, and invite those students to share their reasoning in order to surface that there are multiple ways to be correct.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Both

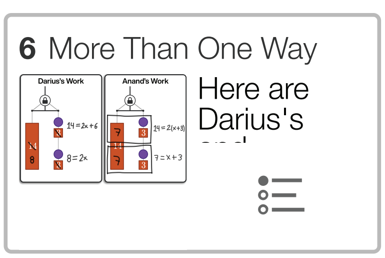
Responses vary. Anand noticed that there are two groups of $x + 3$, and Darius noticed that there are two xs and a total of 6 on the right side of the hanger. These both represent the same total objects in different ways.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., _____ is/are correct because ________.).



Here are Darius's and Anand's work.

Select a question to answer.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to celebrate multiple ways of solving an equation and to make connections between steps used for solving and balanced moves on a hanger.

Invite students to find a classmate who answered a different question from theirs, and then share their response and reasoning with each other. Then facilitate a whole-class discussion examining each student's strategy. Consider sharing students' work from Screen 4 in place of or in addition to Darius's and Anand's work during the discussion, and refer to each work by the student's name.

To set students up for success in Activity 2 where they will solve equations without visuals, spend adequate time here ensuring that students make the connection between each equation and the balanced move it represents. Consider asking questions like: *What would each student's next step be to figure out the weight of a circle? Why?*

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Early Finishers: Encourage students who finish Screens 2–6 early to create their own equation, then trade equations with a classmate and draw a hanger to match.

Sample Responses

Responses vary.

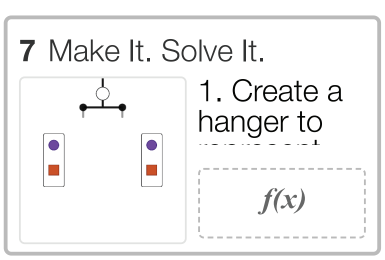
- Darius wrote $8 = 2x$ because he subtracted 6 from each side of the hanger.
- Anand wrote $7 = x + 3$ because he divided both sides of the hanger by 2.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Use the mathematical language routine Stronger and Clearer Each Time to help students discuss and refine their ideas before discussing as a class.



1. Create a hanger to represent $7 = 4x + 2$.

2. What is the value of x that makes the equation true?

Teacher Moves

Activity 2 Purpose: On Screens 7–9, students shift from solving for unknown weights of objects on hangers toward solving for unknowns in equations. Students can draw a hanger diagram if it supports their thinking.

Progress Check: This is a great place to check students' progress on connecting hangers and equations. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Question to push students' thinking: *Does it matter which side the* 7 *is on?*

Pacing: Consider using pacing to restrict students to Screens 7–9.

Sample Responses

1. Image solution

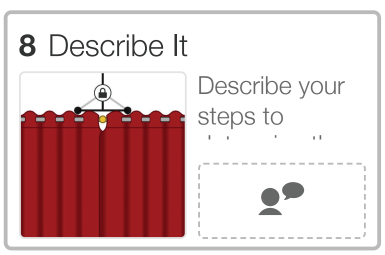
2. $x = \frac{5}{4}$ (or equivalent)

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate creation of the hanger as needed.



Describe your steps to determine the value of x that makes the equation $3(x + 200) = 3000$ true.

Use paper if it helps you with your thinking.

Teacher Moves

Facilitation: This screen is one example of what students will experience on Screen 9, which includes many challenges. Consider using the dashboard's teacher view or snapshots tool to highlight several different approaches, including students who drew their own hanger to represent the situation. Connect these strategies back to the strategies that were named during the Screen 6 discussion about Anand and Darius.

Sample Responses

Responses vary.

- Divide both sides by 3 to get $x + 200 = 1000$
- Subtract 200 from each side to get $x = 800$.

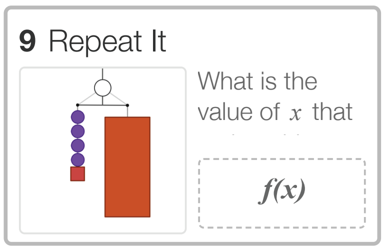
$x = 800$

Student Supports

Students With Disabilities

• *Executive Functioning: Graphic Organizers*

Provide students a graphic organizer to record their work on Screens 8–9 to support their organizational skills.



What is the value of x that makes this equation true?

[Equations vary]

Teacher Moves

Facilitation: Give students 5–10 minutes to complete as many challenges as they can. Consider asking students to set their own goal for how many they would like to complete.

Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed.

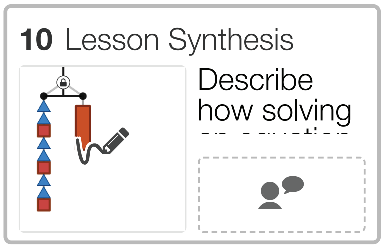
Consider pausing the class to celebrate students who persisted through struggle (e.g., "I saw a student struggling on the first few screens, and because they kept at it, they're crushing it now!").

Note: Check the dashboard's teacher view to see how many challenges each student has completed correctly.

Sample Responses

This screen contains many challenges. Here are the first five answers:

- 2
- 3
- 2.3
- 4.1
- $\frac{7}{4}$



Describe how solving an equation is like solving for the weight of an object on a balanced hanger.

Use the diagram if it helps you with your thinking.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students connect solving equations to solving for the weight of an object on a balanced hanger.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool.

Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Pacing: Consider using pacing to restrict students to this screen.

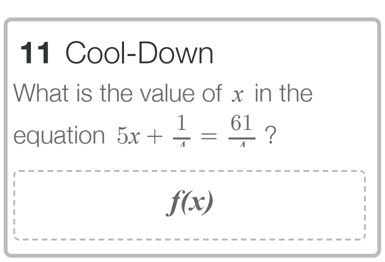
Sample Responses

Responses vary. For both an equation and a hanger puzzle, you need to keep both sides balanced. You can subtract weight from each side or divide each side by the same number to create a balanced hanger or simpler equation.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their thinking (e.g., Solving an equation is like solving for an unknown weight because ________.).



What is the value of x in the equation $5x + \frac{1}{4} = \frac{61}{4}$?

Teacher Moves

Support for Future Learning: If students struggle with figuring out the value of x, plan to revisit this when opportunities arise in Lesson 7. Consider spending more time during Activity 1 of Lesson 7 connecting the moves on the hanger in Problem 1 with the equation steps in Problems 2 and 3.

Pacing: Consider using pacing to restrict students to Screens 11–12.

Sample Responses

$x = 3$

12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can connect balancing moves on hangers to solving equations.
- I can solve equations with positive numbers.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Keeping It True (NYC)

Lesson 7: Solving Equations

Purpose

The purpose of this lesson is for students to describe and use strategies for solving equations of the forms $px + q = r$ or $p(x + q) = r$ that involve positive and negative numbers. This is the first lesson where students are not given a visual model (either a tape diagram or a hanger diagram) to support their reasoning.

Preparation

Student Worksheet

- *Activity 1–3:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Equation Cards

There are two options for using these equation cards:

- *Option 1:* Print and cut one single-sided copy of the Equation Cards for each pair of students. These cards can be reused if you have multiple classes.
- *Option 2:* Print (and do not cut out) one single-sided copy of the Equation Cards for each student or pair of students. Laminate or place the sheets in sheet protectors to reuse with multiple classes. Allow students to write on them with dry erase markers.

Warm-Up: Number Talk (10 minutes)

Purpose: Students surface strategies for solving an equation for the value of x. This is the first time students solve equations with negative numbers in this unit.

Facilitation: Display the Teacher Projection Sheets one at a time. Use the instructional routine Number Talk to help students surface and name strategies for determining the value of x in each equation (MP7).

Some students may reason about the value of x using logic. For example, in $-2x = 6$, the x must be -3 since $-2 \cdot -3 = 6$. Other students may reason about the value of x by changing the value of each side of the equation equally (e.g., dividing each side of $-2x = 6$ by -2 to get the result $x = -3$). Highlight both of these strategies during the discussion where possible.

Activity 1: Keep It True (10 minutes)

Purpose: Students connect their work with balanced hangers from Lessons 5 and 6 to steps for solving an equation. This is the first time students write out strategies for solving equations step by step.

Facilitation: Distribute one Student Worksheet to each student. Consider asking students what they know about hangers and equations, and share that in this activity, they are going to record each step of their strategy as they solve. Then invite students to complete Problems 1–4.

After several minutes, consider asking students to compare their work with another student, justify their responses, and make revisions based on their conversation. Consider creating a visual display that lists students' words to describe their steps (e.g., divide each side by #), or using the mathematical language routine Collect and Display.

Consider naming the strategies for checking solutions after the students who use them and using those names throughout the lesson and unit.

Progress Check: This is a great place to check students' progress solving equations. Encourage students to use paper to draw a hanger diagram, like they did in Lesson 6, to help them with their thinking. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Readiness Check: If Problem 3 of the Readiness Check suggests that some students have unfinished learning about checking solutions to equations, consider spending extra time discussing responses to Problem 4.

Early Finishers: Encourage students who finish early to try solving each problem using a different set of steps than the ones listed on the worksheet.

Support for Multilingual Learners
Expressive Language: Visual Aids
Create or review an anchor chart that publicly displays possible steps for solving an equation.

Activity 2: Less and More Difficult (5 minutes)

Purpose: Students use the structure of equations to decide what they see as "less difficult to solve" and "more difficult to solve" when looking at equations (MP7).

Facilitation: Arrange students into pairs and distribute one set of Equation Cards to each pair. Give students one minute of time to look quietly at the cards and then 2–3 minutes to share their reasoning with a partner.

Then, facilitate a brief whole-class discussion to surface strategies for solving different types of equations in order to prepare students for Activity 3.

Consider asking questions like these to further the discussion: *What did you think would be difficult about equation ___? What is a first step that you can take to solve this type of problem?*

Remind students that their opinion of the difficulty level may change as they grow more confident and that this confidence takes practice and time.

Support for Students With Disabilities
Receptive Language: Processing Time
Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.

Activity 3: Solve 'em (10 minutes)

Purpose: Students practice solving equations. If time is short, invite students to solve one or two of the equations instead of four.

Give students 3–5 minutes of quiet think-time to get started and then 3–5 minutes to discuss and work with their partner.

Facilitate a brief whole-class discussion to close the activity. Consider asking some of the following questions to further the discussion:

- *Which equations were more difficult to solve than you expected?*
- *Which were less difficult to solve than you expected?*
- *What strategies or steps were helpful in solving the equations?*

Support for Students With Disabilities
Executive Functioning: Eliminate Barriers
Chunk this activity into more manageable parts (e.g., presenting one problem at a time), which will aid students who benefit from support with organizational skills in problem-solving.

Lesson Synthesis (5 minutes)

Purpose: Students use the structure of equations to create their own difficult equation and justify why they think it is difficult to solve.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Follow with a brief whole-class discussion in which students share why they believe these equations may be difficult and their advice for solving. If time allows, record advice along with students' names for students to refer to in future lessons.

Cool-Down (5 minutes)

Support for Future Learning: If students struggle to solve each equation, consider reviewing these problems as a class before beginning Lesson 8, or spending extra time during Lesson 8, Activity 2 discussing students' strategies and highlighting common errors.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Factoring and Expanding (NYC)

Lesson 8: Options for Solving One Equation

Purpose

The purpose of this lesson is for students to extend what they know about distributing and solving to factor and expand expressions, and then solve equations in multiple ways and compare the results.

Preparation

Student Worksheet

- *Activity 1–3:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Warm-Up (5 minutes)

Purpose: Students compare and contrast visual representations and equations in factored and expanded form. This is intended to prepare students to compare and contrast equations in factored and expanded form in the lesson.

Facilitation: Display Sheet 1 of the Teacher Projection Sheets. Give students one minute to think quietly. Ask them to indicate when they have noticed a choice that does not belong and can explain why. Encourage students to look for more than one possibility. When the minute is up, give students two minutes to share their responses with their partner. If time permits, have students work together to find at least one reason why each choice doesn't belong (this routine is described in detail here: Which One Doesn't Belong).

Follow with a whole-class discussion. If it does not come up naturally, ask students to explain which of the representations does not show the same relationship as the others and how they know.

Readiness Check: If Problem 5 of the Readiness Check shows that some students have unfinished learning about equivalent expressions, consider spending more time on the warm-up explaining why $3(x + 5)$ is not equivalent to $3x + 5$ but is equivalent to $3x + 15$.

Support for Students With Disabilities
Visual-Spatial Processing: Visual Aids
Provide printed copies of the representations for students to draw on or highlight.

Activity 1: Factoring Puzzles (10 minutes)

Purpose: Students practice factoring and expanding expressions.

Facilitation: Arrange students into pairs. Display Sheet 2 of the Teacher Projection Sheets. Share that in this activity we are going to be looking at two ways of writing expression: factored form and expanded form. Give students 1–2 minutes to notice and wonder quietly about the two diagrams and another minute to share with a partner. Then, ask students what they think *factored* and *expanded* mean and how these words connect to the diagram. Consider asking students what they think belongs in each blank and to share their reasoning.

Consider displaying Sheet 3 of the Teacher Projection Sheets or filling in the blanks together as a class. Students may make the connection between these diagrams and the area of a rectangle. Leave either image displayed so that students can reference them as they complete Activity 1.

Then invite students to complete each of the puzzles on the Student Worksheet. Here are several options for completing this activity:

- Students complete each puzzle on their own, then compare and justify their responses and make revisions based on their conversation.
- Students in each pair take turns proposing how to fill in one blank and explaining their reasoning. Once their partner agrees, they fill in the blank and switch.
- Students in each pair take turns filling in one blank at a time. One student explains their reasoning out loud, while the other writes down their thinking and asks questions if they are unsure or disagree.

Early Student Thinking: Students might combine the numbers using addition or subtraction rather than multiplication. Consider asking these students what the operation $5(3)$ or $5(a)$ represents.

Students may determine the correct values with incorrect signs. Consider asking these students to review their work, looking specifically at the sign of each number to make sure that they make sense.

Note: Students who struggle with visual processing may be supported by labeling all four side lengths of the rectangle in each diagram.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the two forms.

Activity 2: Step by Step by Step by Step (10 minutes)

Purpose: Students compare and contrast strategies to solve equations of the form $p(x + q) = r$.

Facilitation: Display Sheet 4 of the Teacher Projection Sheets and give students 1–2 minutes to discuss with a partner how they might solve the equation. Then invite them to look at Sadia's and Amir's thinking on Sheet 5 and answer Problems 1 and 2. Follow with a whole-class discussion.

Consider asking students what they like about each strategy. It is important to draw out that neither solution is better than the other; these are two ways to accomplish the same task of solving for x. If it does not come up naturally, consider asking if the solution to each equation will be the same and why (MP3).

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Support for Multilingual Learners
Expressive Language: Eliminate Barriers
Provide sentence frames to help students explain their reasoning (e.g., These are/are not both correct because ________.).

Activity 3: Different First Steps (10 minutes)

Purpose: Students practice solving equations both by expanding and by dividing a common factor.

Facilitation: Display Sheet 6 of the Teacher Projection Sheets, and ask students what they think it means to "expand first" or "divide first." Consider connecting to Sadia and Amir or to students in the class who used similar strategies. Then complete the first step of each strategy as a class.

Next, invite students to complete each strategy on their own worksheet and to put a check in the empty square if their solutions are the same. Consider asking students a question like: *What might I do if my solutions are different?*

If students have difficulty getting started on Problems 1 and 2, consider asking them how this problem is similar to Activity 2 (MP8). Even though the numbers get more challenging, the reasoning for each problem is the same.

Consider polling the class to see which strategy students prefer and asking those students to explain why they liked the method they chose.

Early Finishers: Encourage students who finish early to create an equation they believe is much less difficult to complete by expanding first and an equation they believe is much less difficult to complete by dividing first, and to convince a classmate that this is true.

Lesson Synthesis (5 minutes)

Purpose: Students refine and consolidate their ideas about the two strategies surfaced in this lesson, as well as their advantages and disadvantages.

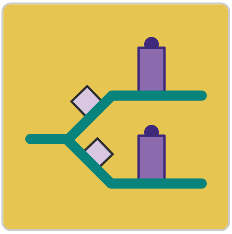
Give students 2–3 minutes to respond and a minute to share their responses with a partner. Then facilitate a brief whole-class discussion to surface the advantages and disadvantages of each strategy.

If time allows, create a visual display with the advantages and disadvantages of each strategy.

Cool-Down (5 minutes)

Support for Future Learning: If students struggle to solve the equation, plan to revisit this when opportunities arise in Lesson 11 and Practice Day 1. Consider spending extra time on Problems 2 and 3 of Set 1 in Activity 1 of Lesson 11.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Always-Equal Machines

Lesson 9: Equivalent Expressions

Overview

Students understand that equivalent expressions have the same value for any value of the variable. Students will reason about equivalent expressions that involve factoring, expanding, and reordering terms. This will support students in making sense of more complex expressions later in the unit. This lesson does not contain any like terms, which students will see in Lesson 10.

Learning Goals

- Write equivalent expressions that involve factoring, expanding, and reordering terms.
- Justify whether or not factored, expanded, or reordered expressions are equivalent.

Vocabulary

- equivalent expression

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to understand that equivalent expressions have the same value for any value of the variable. Students will reason about equivalent expressions that involve factoring, expanding, and reordering terms. This will support students in making sense of more complex expressions later in the unit. This lesson does not contain any like terms, which students will see in Lesson 10.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to revisit the context of number machines and reason about what it means for two expressions to be equivalent: they have equal outputs for all values.

Activity 1: Equivalent Expressions (15 minutes)

The purpose of this activity is for students to identify and write equivalent expressions by factoring, expanding, and reordering terms. Students also analyze common mistakes made when expanding expressions that involve several negative values.

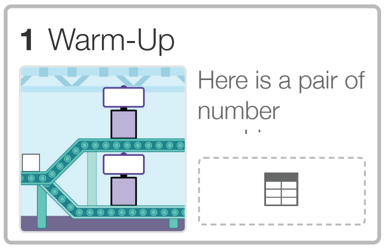
Activity 2: More Than One Way (15 minutes)

The purpose of this activity is for students to factor, expand, and reorder terms to write more than one equivalent expression for each situation. This activity includes a card sort where students match equivalent expressions.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their ideas about how to decide whether two expressions are equivalent or not.

Cool-Down (5 minutes)



Here is a pair of number machines.

Press "Try It" to see how they work.

Teacher Moves

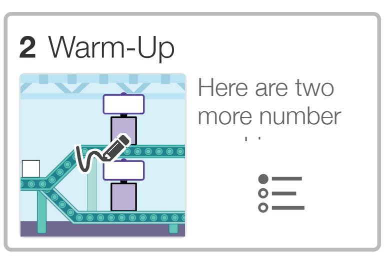
Lesson Overview: The purpose of this lesson is for students to understand that equivalent expressions have the same value for any value of the variable. Students will reason about equivalent expressions that involve factoring, expanding, and reordering terms.

Warm-Up Purpose: Students revisit the context of number machines and reason about what it means for two expressions to be equivalent: that they have equal outputs for all values.

Facilitation: Consider starting with the lesson paused and using the dashboard's student view to press "Try It." Ask students what they notice and wonder about the two number machines and their inputs and outputs. If it does not come up naturally, discuss how the machines calculated each output.

Then, ask students if they think the outputs of the two machines will always be equal. Unpause and invite students to explore the question on their own. When students have tried several different inputs, invite them to continue to Screen 2.

Pacing: Consider using pacing to restrict students to Screens 1–2.



Here are two more number machines.

When will these machines have equal outputs?

Teacher Moves

Facilitation: Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Consider selecting and sequencing 1–2 responses for each of the most popular choices to display, and invite those students to share their reasoning in order to surface students' strategies for deciding when machines have equal outputs.

If time allows, give students one minute to revise their response and explanation (MP6).

Early Finishers: Encourage students who finish Screens 1–2 early to create their own pair of expressions that would always have equal outputs, then convince a classmate that this is true.

Sample Responses

Sometimes

Responses vary. These machines will only have equal outputs when the input is 2.

Student Supports

Students With Disabilities

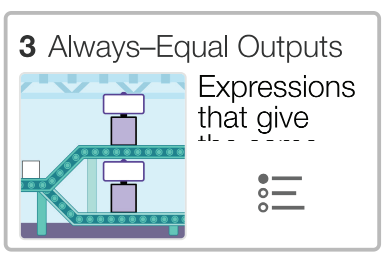
- *Executive Functioning: Visual Aids*

Create an anchor chart for public display that describes what inputs, outputs, and equivalent expressions are for future reference. Consider including examples of expanding and factoring, and adding to the chart throughout the lesson.

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



Expressions that give the same output for every input are called equivalent expressions.

Are these equivalent expressions?

Teacher Moves

This is a possible discussion screen.

Activity 1 Purpose: On Screens 3–7, students identify and write equivalent expressions using factoring, expanding, and reordering terms.

Facilitation: Consider starting with the activity paused, reading the definition of equivalent expressions aloud, and asking students to paraphrase it in their own words. Then unpause and invite students to decide if the two expressions in the machines are equivalent and to explain how they know.

While students are working, select and sequence several student responses using the snapshots tool. Monitor for responses that mention specific inputs and outputs, and responses that use factoring or expanding.

Then, facilitate a whole-class discussion around questions like: *How can we tell if two expressions will be equivalent? Is testing a few examples enough? Why or why not?*

Pacing: Consider using pacing to restrict students to Screens 3–7.

Sample Responses

Yes

Responses vary. If I expand $3(4x-5)$, I get $12x-15$. These expressions are equivalent, so they will give equal outputs for any input.

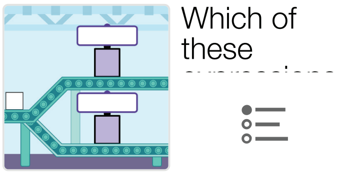
Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., These are/are not equivalent expressions because ________.).

4 Challenge #1



Which of these expressions is equivalent to $2(10 - x)$?

Teacher Moves

Progress Check: This is a great place to check students' progress with writing equivalent expressions by expanding. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking: Each incorrect choice is intended to highlight a common thought students may have at this stage of learning. Consider encouraging students who select an incorrect choice to reflect on why the outputs are not always the same and what advice they would give their future selves or other students in this situation.

Sample Responses

$20 - 2x$

Student Supports

Students With Disabilities

- *Executive Functioning: Graphic Organizers*

Provide students a graphic organizer to record their work on Screens 4–7 to support with organizational skills.

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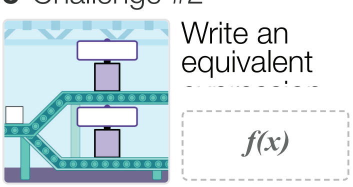
5 Challenge #2



Write an equivalent expression for $-5(8 - 2x)$.

Teacher Moves

Facilitation: Encourage students to use paper or draw a diagram, like they did in Lesson 8, to help them with their thinking on each of these challenges and to revise their expression based on the feedback from the number machines.

Sample Responses

Responses vary. $-40 + 10x$

6 Your Favorite Mistake

Three students

Three students made mistakes writing an equivalent expression for $-5(8-2x)$.

Select your favorite mistake.

Teacher Moves

This is a possible discussion screen.

Facilitation: Consider displaying the distribution of students' favorite mistakes using the dashboard's teacher view. This is an opportunity to emphasize that incorrect, imprecise, and unfinished thinking is valuable. Ask students to explain what each student already knows or did correctly, their mistake, and why it was their favorite.

If time allows, create a visual display with common errors for writing equivalent expressions that students can refer to throughout the lesson and unit.

Sample Responses

Responses and explanations vary.

- Zion's expression is incorrect because you can't just switch the order of the terms with subtraction. $2x-8$ is not equivalent to $8-2x$.
- Juliana's expression is incorrect because she multiplied $-2x$ by 5, not -5.
- Nickolas's expression isn't equivalent because he didn't multiply -5 by $2x$. He only multiplied -5 by 8.

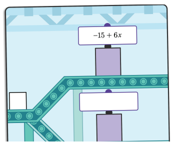
Student Supports

Multilingual Learners

- *MLR 3 (Clarify, Critique, Correct)*

Consider using the mathematical language routine Clarify, Critique, Correct. Use a "Critique a Flawed or Partial Response" strategy by having students improve upon a statement like "Juliana has the wrong numbers."

7 Challenge #3



Select **all** of the .

Select **all** of the expressions equivalent to $-15 + 6x$.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface ways to create (and not create) equivalent expressions.

Facilitation: While students are working, select several student explanations using the snapshots tool, sequenced in the order the expressions appear in the list.

When most students have responded, consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Then discuss expressions one at a time, asking students to justify why the expression is or is not equivalent to $-15 + 6x$ and to critique each other's reasoning (MP3). Consider creating a T-chart and placing each expression under "equivalent" or "not equivalent" as you discuss.

If time allows, add to the visual display ways to create equivalent expressions and common errors.

Early Finishers: Encourage students who finish Screens 3–7 early to practice how they would convince a classmate that each of the expressions they selected on Screen 7 is equivalent to $-15 + 6x$ and that none of the others are.

Routine (optional): Consider using one or more rounds of the mathematical language routine Stronger and Clearer Each Time to help students refine their ideas.

Sample Responses

- $-3(5 - 2x)$
- $3(2x - 5)$
- $6x + (-15)$

Explanations vary. I know $3(2x - 5)$ is equivalent because $3 \cdot 2x = 6x$ and $3 \cdot -5 = -15$, so the $6x$ is positive and the 15 is negative.

8 Card Sort

Teacher Moves

Activity 2 Purpose: Students factor, expand, and reorder terms to write more than one equivalent expression for each situation.

Facilitation: Encourage students to use paper to record their thinking and to attend to precision as they sort the cards.

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.

If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Pacing: Consider using pacing to restrict students to Screens 8–11.

Sample Responses

Image solution

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

Check in with individual students, as needed, at each step of the activity to check for understanding.

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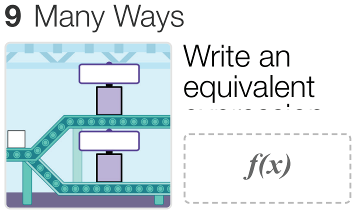
9 Many Ways



Write an equivalent expression for $50-35x$.

Try to create more than one.

Teacher Moves

Progress Check: This is a great place to check students' progress with writing equivalent expressions through factoring or reordering terms. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Responses vary.

- $-35x+50$
- $5(10-7x)$

- $10(5 - 3.5x)$

10 Equivalent Expressi…

Teacher Moves

This is a possible discussion screen.

Facilitation: Encourage students to write as many different equivalent expressions as they can. The screen checks up to 10 expressions.

While students are working, select both equivalent and not equivalent expressions using the snapshots tool.

Then facilitate a whole-class discussion around questions like:
- *Is this expression equivalent? How do you know?*
- *What strategy might someone have used to create this expression?*

Sample Responses

Responses vary.

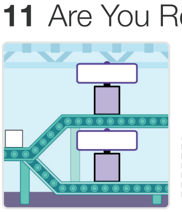
- $-16 + 64x$
- $-1(-64x + 16)$
- $16(4x - 1)$

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Begin with a demonstration of the first problem to provide access to students who benefit from clear and explicit instructions.

11 Are You Ready for …



Make a **never-**

Make a **never-equal** machine that will never give the same output for any input.

Teacher Moves

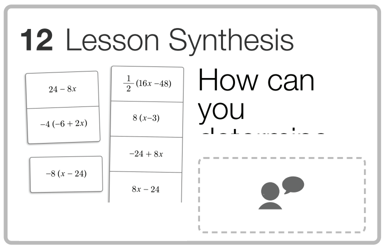
Facilitation: This screen is designed as an extra challenge for students who finish Screens 8–10 before the class discussion on Screen 10. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

Responses vary.

- x
- $x+1$

These are never equal because one machine will always output a value that is one greater than the other machine.



How can you determine whether two expressions are equivalent to each other?

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their ideas about how to decide whether two expressions are equivalent or not.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Routine (optional): Consider using the mathematical language routine Collect and Display.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. Two expressions are equivalent to each other if one can be rearranged, factored, or expanded to be the same as the other.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., First, ________. Then, _______.).

13 Cool-Down

Write an equivalent expression for

f(x)

Write an equivalent expression for $-5(3-2x)$.

Teacher Moves

Support for Future Learning: If students struggle with writing an equivalent expression, plan to revisit this when opportunities arise in Lesson 10. Consider spending extra time during Activity 1 of Lesson 10 (where expressions in factored form are introduced) discussing strategies for rewriting these in expanded form and what errors might come up.

Pacing: Consider using pacing to restrict students to Screens 13–14.

Sample Responses

Responses vary. $-15+10x$

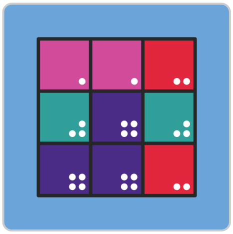
14

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can write equivalent expressions.
- I can explain whether or not two expressions are equivalent.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Collect the Squares

Lesson 10: Adding Expressions

Overview

Students practice writing expressions with fewer terms by adding and expanding terms. This lesson builds on work students did with equivalent expressions in Grade 6 and is intended to support students in solving more complex equations in Lesson 11.

Learning Goals

- Write equivalent expressions with fewer terms by expanding and adding terms.

Vocabulary

- term

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.
- ☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to practice writing expressions with fewer terms by adding and expanding terms. This lesson builds on work students did with equivalent expressions in Grade 6 and is intended to support students in solving more complex equations in Lesson 11.

Lesson Summary

Warm-Up: Number Talk (10 minutes)

The purpose of the warm-up is for students to solve equations by writing equivalent expressions with fewer terms and to explore why writing an expression with fewer terms is helpful.

Activity 1: Collect the Squares (25 minutes)

The purpose of this activity is for students to practice writing equivalent expressions with fewer terms by expanding and adding terms. Students select expressions from a 2-by-2 or a 3-by-3 grid of squares, and write their sum using the fewest number of terms.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to explain how to write an expression with fewer terms.

Cool-Down (5 minutes)

1 Warm-Up: Number Talk

Solve this equation mentally.

$$\boxed{6x = 12}$$

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to practice writing expressions with fewer terms by adding and expanding terms. This lesson builds on work students did with equivalent expressions in Grade 6 and is intended to support students in solving more complex equations in Lesson 11.

Warm-Up Purpose: Students notice that they can solve equations by writing equivalent expressions with fewer terms and explore why this may be helpful.

Facilitation: Consider starting with the lesson paused and sharing with students that we are going to engage in a Number Talk. Then use the dashboard's student view to project Screens 1–5, one screen at a time. Use the instructional routine Number Talk to help students surface and name strategies for determining the value of x in each equation (MP7).

Notes:
- Share with students that the expression on Screen 2 has two terms, and ask students what they think *terms* mean in order to support students in discussing their reasoning throughout the lesson.
- The first three equations and the second two equations have the same solution to support students in looking for and making use of the structure of the equations.
- If it does not come up naturally, ask a question to elicit why adding terms when possible is useful for solving equations, like: *Why might it be helpful to write an expression with fewer terms before solving?*
- Consider giving students one extra minute to discuss with a partner how many terms there are on the left side of the equation on Screen 5 and which ones we can add together, and to share their reasoning.

Pacing: Consider using pacing to restrict students to Screens 1–5, one screen at a time.

Sample Responses

$x = 2$

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators in the warm-up to ensure inclusive participation.

- *Memory: Processing Time*

Provide sticky notes or mini whiteboards for students with working memory challenges to record each solution during the warm-up.

2 Warm-Up: Number Talk

Here is the previous equation.

$\boxed{6x = 12}$

Now solve this equation mentally.

Sample Responses

$x = 2$

3 Warm-Up: Number Talk

Here are the previous equations.

$\boxed{6x = 12}$

$\boxed{3x + 3x = 12}$

Sample Responses

$x = 2$

4 Warm-Up: Number Talk

Here are the previous equations.

$\boxed{6x = 12}$

$\boxed{3x + 3x = 12}$

Teacher Moves

Early Student Thinking: Some students may notice that we have added the terms in the first three equations and believe that $6x + 3 = 27$ is equivalent to $9x = 27$ in the fourth equation. Consider asking these students to see if their value of x makes the original equation true.

Sample Responses

$x = 4$

5 Warm-Up: Number Talk

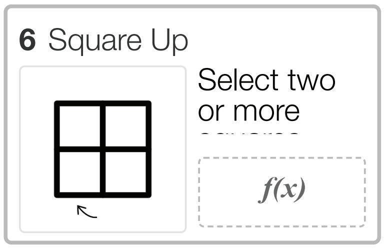
Here are the previous equations.

$\boxed{6x = 12}$

$\boxed{3x + 3x = 12}$

Sample Responses

$x = 4$



Select two or more squares.

Write their sum using the fewest number of terms.

Teacher Moves

Activity 1 Purpose: On Screens 6–11, students practice writing equivalent expressions with fewer terms by expanding and adding terms.

Facilitation: Consider starting with the activity paused and using the dashboard's student view to demonstrate how to select squares. Consider selecting two squares and asking students how the expression at the bottom of the graph changes. Then unpause and invite students to select two or more squares and write their sum. Encourage students to use paper to record their expressions and their thinking.

Circulate to observe student strategies and offer help or encouragement where needed. If students are having difficulty getting started, consider drawing a new 2-by-2 grid of squares (two that contain constants and two that contain multiples of x) and asking students which terms we can add together and why.

Pacing: Consider using pacing to restrict students to Screens 6–8 and opening pacing to Screens 9–11 as students become successful on earlier screens.

Sample Responses

Responses vary depending on the student's choice of expressions.

- $-2x + -10x = -12x$
- $4x + 1 + -2x = 2x + 1$

Student Supports

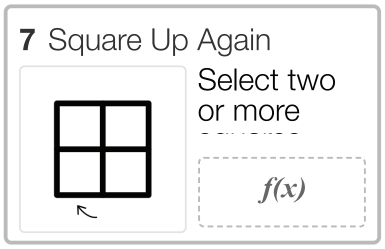
Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate which squares to select throughout this lesson.

- *Receptive Language: Processing Time*

Read all statements aloud and give examples of what it means to write an expression "using the fewest number of terms."



Select two or more squares.

Write their sum using the fewest number of terms.

Teacher Moves

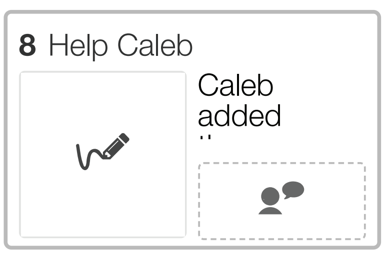
Progress Check: This is a great place to check students' progress on writing equivalent expressions with fewer terms. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Monitor for students who select different groups of squares to share their thinking during the discussion on Screen 8.

Sample Responses

Responses vary depending on students' choice of expressions.

- $-2x - 2 + -3 = -2x - 5$
- $8x + 8 + 3(x + 1) = 11x + 11$



Caleb added these expressions, but he made an error.

Find the error and explain why it is incorrect.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to make sense of a common error when writing an expression with fewer terms.

Facilitation: While students are working, select several student responses using the snapshot tool. When most students have responded, facilitate a whole-class discussion about Caleb's thinking.

Consider starting by asking students to share what is correct about Caleb's thinking (e.g., he reordered the terms, he added $-2x$ and $8x$, etc.). Then ask what advice students would give to Caleb to see his error, and to share all the reasons someone might make that error. This is an opportunity to highlight the value of incorrect thinking as a way to make sense of a problem and clarify common sources of error.

If time allows, consider discussing what the sum of the other two squares ($3(x + 1)$ and -3) would be and what errors someone might make.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language.

Sample Responses

Responses vary. In the last step, Caleb added $6x$ and 6 together, but you can't because we don't know what x is.

Student Supports

Multilingual Learners

• *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., The error is ________ because ______.).

---

9 Collect All the Squares

Select at least two squares at a time.

Write their sum using the fewest number of terms.

Collect all of the squares.

Teacher Moves

Facilitation: The purpose of this screen is for students to gain fluency in writing expressions with fewer terms. There are many different ways of completing the challenges on Screens 9–10. Consider using the dashboard's teacher view or the snapshot tool to highlight different groups of squares that students select.

If students are struggling, consider one of these options:

• Arrange students into pairs or groups of 3–4. Encourage students to work together to write the sum using the fewest number of terms and to justify their reasoning to their group before pressing "Check My Work." Repeat as a group until all of the squares have been collected.
• Pause the activity. Display the screen using the dashboard's student view. Ask students to select at least two squares. Give students time to add the expressions quietly, then invite them to share their strategies. Repeat as a class until all of the squares have been collected.

Note: The colors and white dots in the bottom-right corner of each square indicate the set of expressions for which a student wrote the sum.

Sample Responses

Responses vary depending on the student's choice of expressions.

Student Supports

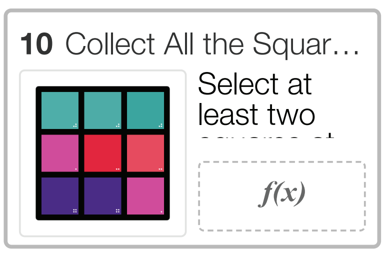
Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., asking students to collect one row or 2 squares), which will aid students who benefit from support with organizational skills in problem-solving.

- *Executive Functioning: Graphic Organizers*

Provide a graphic organizer for students to record their thinking for each set of expressions they choose.



Select at least two squares at a time.

Write their sum using the fewest number of terms.

Collect all of the squares.

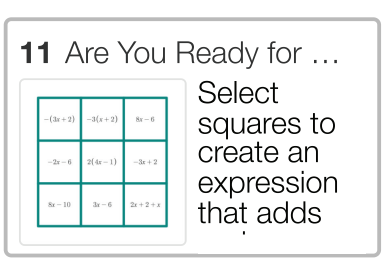
Teacher Moves

Facilitation: Consider pausing the class to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

There are expressions on both Screen 9 and Screen 10 whose sum is 0. Consider challenging students to figure out which expressions those are as they work.

Sample Responses

Responses vary depending on the student's choice of expressions.



Select squares to create an expression that adds up to $5x - 8$.

Create as many different expressions as you can.

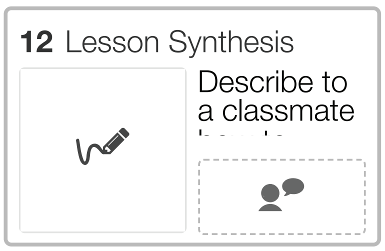
Teacher Moves

Facilitation: This screen is designed as an extra challenge for students who finish Screens 6–10 before the class discussion on Screen 12. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

Responses vary.

- $-3(x+2)+2(4x-1)$
- $-(3x+2)+8x-6$
- $8x-10+-3x+2$



Describe to a classmate how to write an equivalent expression with fewer terms.

Use the expression in the sketch if it helps with your thinking.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students explain how to write an expression with fewer terms.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses or suggest revisions.

Note: This may be the first time students see an expression with a term in factored form. This is an opportunity to discuss potential errors when writing equivalent expressions.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. First, look for places where you can expand the expression. In the example, -2 times $6x-4$ is $-12x+8$. Then, add together any terms that are alike. In this case, $5x$ and $-12x$ are alike, so $5x+-12x=-7x$. The final expression is $-7x+8$.

Student Supports

Multilingual Learners

• *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., First, ________. Then, _______.).

---

13 Cool-Down

Write each expression with fewer terms.

$f(x)$

Write each expression with fewer terms.

$10x - 2x$

Teacher Moves

Support for Future Learning: Students will have more chances to develop their understanding of writing expressions with fewer terms in Lesson 11 and Practice Day 1.

Pacing: Consider using pacing to restrict students to Screens 13–14.

Sample Responses

- $8x$
- $14x - 7$

14

This is the math we wanted you to understand:

This is the math we wanted you to understand:

• I can write equivalent expressions with fewer terms.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Equation Roundtable (NYC)

Lesson 11: Solving Equations by Adding Terms and Expanding

Purpose

The purpose of this lesson is for students to practice solving equations that involve adding terms and expanding expressions.

Preparation

Student Worksheet

- *Activity 1:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Warm-Up (5 minutes)

Purpose: Students recognize that there is often more than one useful first step in solving an equation and surface several of these first steps.

Facilitation: Display Sheet 1 of the Teacher Projection Sheets. Consider starting by asking students what they first notice about the equation. Then invite students to think quietly about what first step they would take to solve it. After one minute, give students a minute to share their response and reasoning with a partner.

Then, facilitate a brief whole-class discussion to surface several possible first steps. Consider listing each option publicly for students to refer to throughout the lesson. If time allows, consider solving the equation using several different first steps and comparing their solutions, or ask students to choose which first step they prefer and share their reasoning.

Activity 1: Equation Roundtable (30 minutes)

Purpose: Students practice solving equations that involve adding and expanding expressions in a structure that allows them to get feedback from their peers and revise their thinking.

Facilitation: Arrange students in groups of 3–4. Distribute one Student Worksheet to each student. Display the Teacher Projection Sheets to help facilitate the lesson.

There are several options for facilitating this activity:

1. Display Sheet 3 of the Teacher Projection Sheets. Each student copies the first equation from Set 1 onto their sheet and writes only the first step toward solving the equation. After everyone in their group has written down a first step, students pass their sheet to the student on their left and receive a new sheet from the student on their right. They check the work on this new sheet and ask the step's author to justify their thinking. Once both students agree that this step is correct, each student completes the next step on their classmate's sheet and

passes the paper again. Students continue this process until every student in the group has solved for the value of x. Students then get their original sheet back and check the work on their sheet. Repeat the process with the next problem.

2. Display page 3 of the Teacher Projection Sheets. Each student copies the first equation from Set 1 onto their sheet and takes all of the steps to solve for the value of x. When all students in the group are finished, students pass their sheet to the student on their left and receive a new sheet from the student on their right. They check the work on this new sheet and ask the solution's author to justify their thinking. Once both students agree that these steps are correct, each student completes the next problem on their classmate's sheet and passes the paper again. Repeat the process with the next problem.

3. Display pages 3–6 of the Teacher Projection Sheets one at a time so that each student from the group can copy a different equation from Set 1 onto their sheet. Each student writes only the first step toward solving their equation. After everyone in their group has written down a first step, students pass their sheet to the student on their left and receive a new sheet from the student on their right. They check the work on this new sheet and ask the step's author to justify their thinking. Once both students agree that this step is correct, each student completes the next step on their classmate's sheet and passes the paper again. Students continue this process until every worksheet in the group has been solved for the value of x. Students then get their original sheet back and check the work on their sheet. Repeat the process with the next set of problems.

Remind students to push each other to explain how they know the equation is still balanced after each step. For example, a student might say they are combining two terms on one side of the equation, which means the equation is still balanced because the value of that side did not change, only the way it is written did.

When most groups have finished Set 1, facilitate a whole-class discussion to surface interesting strategies that students used and the advice they would give themselves or others when solving.

Consider asking questions like:

- *Did your partner ever make a move different than the one you expected them to? Describe it.*
- *What advice would you give yourself or others about what to look out for when solving equations like these?*

Then give students another 5–10 minutes to complete Set 2. Consider changing facilitation structures if it makes sense for your class. If time is short, consider asking students to only complete Set 1, or to complete fewer problems from each set.

Early Finishers: Encourage groups that finish early to work together to figure out a solution path for each equation that uses the fewest number of steps.

Support for Students With Disabilities
Conceptual Processing: Processing Time
For students who benefit from extra processing time, provide them the problems to review prior to implementation of this activity.

Support for Multilingual Learners
Expressive Language: Visual Aids
Create or review an anchor chart that publicly displays common solving steps, along with vocabulary like *expand*, *factor*, *term*, and *equivalent expression* to aid students in explaining their reasoning.

Lesson Synthesis (5 minutes)

Purpose: Students consolidate their ideas about all of the possible first steps for solving complex equations, including adding terms and expanding expressions.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Follow with a brief whole-class discussion in which students share several possible first steps. If time allows, consider asking students to choose which first step they prefer and share their reasoning.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition or anchor chart.

Support for Multilingual Learners
Expressive Language: Eliminate Barriers
Provide sentence frames to help students explain their reasoning (e.g., I prefer _____ because ________.).

Cool-Down (5 minutes)

Support for Future Learning: If students struggle with identifying the error, consider checking in with individual students as they solve equations during Practice Day 1, or reviewing this problem as a class before beginning Practice Day 1.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Community Day (NYC)

Lesson 12: Using Equations to Solve Problems

Purpose

This lesson is an opportunity for students to use everything they have learned so far in the unit to represent and answer questions about situations in context (MP2). Students make sense of verbal descriptions of situations, and compare and contrast situations in similar contexts (MP1).

Note: This lesson is designed for 90 minutes and may take multiple class periods to complete.

Preparation

Student Worksheet

- *Activities 1–3:* Print one sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Materials

- Tools for creating a visual display

Warm-Up (5 minutes)

Purpose: Students connect a visual representation and an equation to a description of a situation.

Facilitation: Display Sheet 1 of the Teacher Projection Sheets and read the situation out loud. Give students one minute to think quietly about Problem 1 and another minute to share their reasoning with a partner and work together on Problem 2.

While students are working, monitor for students who write equivalent equations like $3(a + 2) = 24$ and $3a + 6 = 24$.

Then facilitate a brief whole-class discussion in order to surface how students decided which diagram to choose. Consider asking questions like: *What words in the situation helped you decide which diagram to choose? How do you know the other one does* ***not*** *represent this situation?*

Early Finishers: Encourage students who finish early to write a story that matches the tape diagram they did not select.

Activity 1: Three Reads (10 minutes)

Purpose: Students use the routine Three Reads to make sense of a situation described in words, to represent the situation with a visual, and to answer a question about it. This reasoning is an example of what students will do on their own in Activity 2.

Facilitation: Arrange students into pairs. Distribute one Student Worksheet to each student. Consider starting by asking students what they think a Community Day might look like, and gather students' knowledge and experiences about similar events.

Then facilitate the Three Reads routine. Here is a brief summary of the routine. A full description of the routine can be found here.

1. Display Sheet 2 of the Teacher Projection Sheets and read the situation aloud. Give students one minute to discuss with their partner what the situation is about and share with the class.
2. Display Sheet 3. Consider asking a student to read the situation aloud or to read the situation in pairs. Then give students 1–2 minutes to draw a visual to represent the situation. This will not contain any numbers yet.
3. Display Sheet 4, which contains the revealed quantities in the problem. Read the situation once more and give students 2–3 minutes to adjust their diagram and answer the question. Consider asking a question like: *What strategy would you try first to answer this question?*

Early Student Thinking: Students may be unsure how to interpret the phrase "within a week." Consider asking these students to choose a day of the week for the story and figure out when Kyrie would be finished by. It may also be helpful for students to draw a diagram to show the number of days Kyrie would be working on invitations.

Support for Students With Disabilities
Conceptual Processing: Eliminate Barriers
Assist students in recognizing the connections between the thinking in this problem and prior work. Students may benefit from a review of different representations to activate prior knowledge.

Support for Multilingual Learners
Receptive Language: Models
Use physical invitations or images to aid students in understanding the situation.

Activity 2: Similar Problems (25 minutes)

Purpose: Students practice creating visuals in order to write and solve equations for situations in context.

Facilitation: Arrange students in groups of 2–3. Share with students that this activity continues the Community Day context from Activity 1. Share that each set in this activity has two situations (A and B) that are very similar to each other but not exactly the same. Then share that each group will select one set of situations to explore and that the sets go in order of difficulty. Invite students to choose one set that they believe would present a meaningful challenge to them and to work together to create a poster for the set that they chose.
Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed. Monitor for students who discuss specific words to help them figure out the differences between the two situations and who make connections between the situations and their own lives.

Consider pausing the class and highlighting different visual representations of the same situation to highlight multiple valid ways of thinking or to share strategies or words that students found helpful in distinguishing the two scenarios.

When groups are finished, invite students to pair with another group and discuss their posters, or use the mathematical language routine Compare and Connect.

Support for Multilingual Learners
Receptive/Expressive Language: Peer Tutors
Pair students to aid them in comprehension and expression of understanding of each situation.

Activity 2 Synthesis (5 minutes)

Purpose: Surface strategies that students used as they created equations for each situation and which representations they found most helpful (MP1).

Facilitation: Give students 1–2 minutes of quiet think-time. Invite students to share their responses with their group and what helped them decide what equation to write for each situation. Consider asking each group to share with the class.

If you are completing this lesson over two class periods, this is a great stopping point for the first class period.

Activity 3 Warm-Up (5 minutes)

Purpose: Students revisit their posters before getting feedback on their work from others.

Facilitation: Give students 1–2 minutes of quiet think-time. Invite students to share their responses with their group and decide how they plan to revise their posters in Activity 4. Consider inviting each group to share with the class.

Activity 3: Gallery Tour (15 minutes)

Purpose: Students see their peers' different conclusions and the strategies that led to those conclusions.

Facilitation: Invite groups to display their posters and then do a gallery tour where all students circulate the classroom looking at the work of their peers. Once students have returned to their seats, give them 2–3 minutes of silent think-time to answer the two questions on their worksheet.

Follow with a brief class discussion to help students understand the representations and strategies different groups used to make sense of their situations. Invite students to share aspects from their classmates' work that they found interesting.

Routine: Use the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Activity 4: Revisions and Reflection (15 minutes)

Purpose: Students consider how the process of making sense of a situation, doing calculations, and applying proportional reasoning offers them insight into a real-life context.

Facilitation: Invite students to work with their groups from Activity 2 and use insights from Activity 3 to revise their poster and add a way of checking that their solutions are correct for each situation.

When students have completed their revisions, invite individuals to respond to the reflection questions and then to share their responses with their group. Follow with a whole-class discussion.

Lesson Synthesis (5 minutes)

Purpose: Students reflect on what they think is important to remember about using equations and visual representations to solve problems in context.

Facilitation: Give students two minutes to respond and one minute to share their responses with their group. Then facilitate a brief whole-class discussion to create a list of what students think is important to remember.

Cool-Down (5 minutes)

Support for Future Learning: If students struggle to answer the question in context, consider making time to explicitly revisit these ideas before the Quiz. Students will have the opportunity to answer questions like these during Practice Day 1.

7.6 Practice Day 1 (NYC)

Preparation

Student Workspace

- Print one double-sided copy for each student.

Cards

- *Solve and Swap:* Print and cut enough sets of cards so that there is one card for each student (e.g., two sets for a class of 24; three sets for a class of 36).
- *Group Questions:* Print and cut one set of cards for each group of students.

Instructions

Option 1: Solve and Swap
This structure supports student collaboration with many different partners and allows for movement around the classroom. Students are positioned as experts as they discuss each problem and support one another.

- Print and cut out enough sets of cards so that there is one card for each student.
- Give each student a card and invite them to answer the question on their worksheet.
- Invite students to circulate the class with their card and pair up with a classmate. In the pair, each student should solve the problem on their partner's card, collaborating as needed.
- If a pair of students end up with a problem they've already solved, invite them to compare strategies and solutions with their partner.
- When both students have completed their problems, invite them to swap cards, stand up with a hand up, and find another classmate to pair up with.
- Repeat the process.
- If students are waiting for a partner, invite them to try the "Are You Ready for More?" task at the bottom of their worksheet.

Option 2: Group Questions
This structure supports student collaboration and focuses students on one problem at a time.

- Arrange students into *groups of 2–3.* Print and cut out one set of 12 cards for each group.
- Invite students to select one card to work on at a time as a group.
- Give each student the Student Workspace Sheet to complete as they work together. Encourage students to justify their reasoning as they discuss their strategies.
- If time allows, invite students to order the cards from what they think will be more challenging to what they think will be less challenging. This helps them prioritize if they are not able to answer all 12 questions.
- Consider posting the answer key, or walking around with it and providing feedback to students as they work.
- If groups complete all 12 cards, invite them to try the "Are You Ready for More?" task.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

I Saw the Signs

Lesson 13: Inequalities on the Number Line

Overview

Students build on their knowledge from Grade 6 about graphing inequalities and are introduced to two new symbols, greater-than-or-equal-to ($\geq$) and less-than-or-equal-to ($\leq$).

Learning Goals

- Understand and use the symbols $\leq$ and $\geq$ to describe inequalities.
- Draw and label a graph on the number line that represents an inequality.

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to build on their knowledge from Grade 6 about graphing inequalities and to introduce them to two new symbols, greater-than-or-equal-to ($\geq$) and less-than-or-equal-to ($\leq$). Students use words, symbols, and graphs to describe restrictions for riding amusement park rides.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to attend to precision in language as they examine common ways of describing greater-than and greater-than-or-equal-to in the context of height requirements at an amusement park (MP6). The formal terms and symbols will be introduced during Activity 1.

Activity 1: Riding the Rides (20 minutes)

The purpose of this activity is for students to connect words, symbols, and graphs as ways of describing who can ride a specific amusement park ride, making sense of quantities and their relationships in context (MP2). Students use an open circle or closed circle in the graphs of inequalities to indicate whether the boundary point is included. They also connect the inclusion of the boundary point to the symbols $\leq$ and $\geq$.

Activity 2: Inequalities Out of Context (10 minutes)

The purpose of this activity is for students to apply what they have learned to make sense of inequalities out of context. Students write inequalities to represent graphs and consider the difference between $19 < x$ and $x < 19$.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to connect words, symbols, and a graph representing the same inequality.

Cool-Down (5 minutes)

Here are two signs for two different rides at an amusement park.

Habib is exactly 60 inches tall.

Which ride can he go on?

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to build on their knowledge from Grade 6 about graphing inequalities, and to introduce them two new symbols, greater-than-or-equal-to ($\geq$) and less-than-or-equal-to ($\leq$).

Warm-Up Purpose: Students attend to precision in language as they examine common ways of describing greater-than and greater-than-or-equal-to in the context of height requirements at an amusement park (MP6).

Facilitation: Consider starting with the lesson paused and asking students if they have ever been too tall or too short to be allowed to do something. Consider sharing that some parks have restrictions for safety that only let people of a certain height ride specific rides. Then unpause and give students one minute to select which rides Habib can go on and another minute to explain their reasoning to a partner.

Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Invite students to share their reasoning for each of the most popular choices. This conversation will continue on Screen 2 as students see symbols that represent each sign.

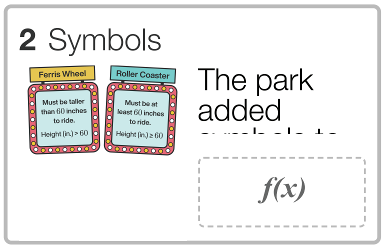
If it does not come up naturally, consider asking questions like: *How are these two signs different? In real life, do you think this situation makes sense? Why or why not?*

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Roller coaster

Explanations vary. Habib *is* 60 inches tall, so he can ride the roller coaster, but he is not taller than 60 inches, so he can't ride the Ferris wheel.



The park added symbols to make the signs clearer.

The symbol $>$ means greater than.

The symbol $\geq$ means greater than or equal to.

What is the shortest height Makayla can be and still ride both rides?

Teacher Moves

Activity 1 Purpose: On Screens 2–7, students to connect words, symbols, and graphs as ways of describing who can ride a specific amusement park ride, making sense of quantities and their relationships in context (MP2).

Facilitation: Consider starting this activity by continuing the conversation from the warm-up. Read the text aloud and ask students how the two symbols in the sign are similar and different.

Give students one minute to either record what they think the shortest height is on the screen or to share it with a partner. Surface several different student responses along with their reasoning.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- 61 inches
- 60.00001 inches

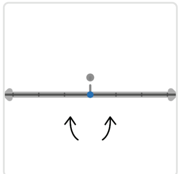
Student Supports

Multilingual Learners

- *Expressive Language: Visual Aids*

Create an anchor chart that publicly displays language and symbols that represent less-than, less-than-or-equal-to, greater-than, and greater-than-or-equal-to to aid in explanations and reasoning.

3 Number Line Graph



Teacher Moves

Facilitation: Consider sharing with students that they can drag the movable point, click on either side of the line, and click on the dot. Invite students to play with the graph and share with a partner all of the things that they notice about the graph. After 2–3 minutes, invite students to continue to the next screen.

Readiness Check: If Problem 6 of the Readiness Check suggests that some students have unfinished learning about graphing inequalities, consider reviewing Problem 6.1 as a class before continuing.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the inequalities.

Pacing: Consider using pacing to restrict students to Screens 3–4.

Sample Responses

Responses vary. I notice that when the blue circle is filled in, the symbol is $\leq$ or $\geq$, and when the circle is empty, the symbol is $<$ or $>$.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Allow students who struggle with fine motor skills to dictate manipulation of the movable point and arrows throughout this lesson.

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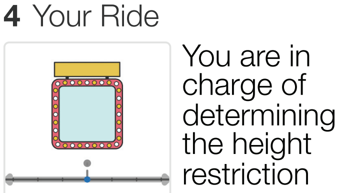
4 Your Ride



You are in charge of determining the height restriction for your ride.

Use the number line to create a sign for your ride.

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to surface connections between the words, symbols, and graphs and what they mean about who can ride the ride.

Facilitation: Give students several minutes to explore the number line and to decide what sign they want for their ride. Encourage students to play, making lots of changes to the graph and seeing how the sign changes in response. While students are working, select several student graphs using the snapshot tool.

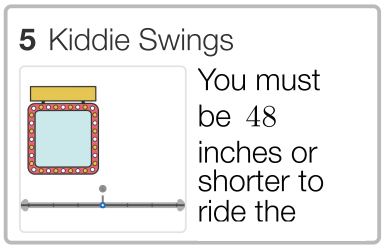
When most students have responded, give students one minute to share the graph they selected for their ride with a partner and the connections they notice between the words, symbols, and graph.

Then facilitate a whole-class discussion using student responses in the snapshot tool or the dashboard's student view. Consider asking questions like: *Where do you see the same information in the words, symbol, and graph? What do you think would change if . . . ?*

If it does not come up naturally, consider asking students how the words and symbol change when you change the graph from having a filled-in circle to an empty circle.

Sample Responses

Responses vary.



You must be 48 inches or shorter to ride the kiddie swings.

Make a graph on the number line to represent the possible heights for this ride.

Teacher Moves

Progress Check: This is a great place to check students' progress on connecting inequalities described in words with their graphs. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Pacing: Consider using pacing to restrict students to Screens 5–7.

Sample Responses

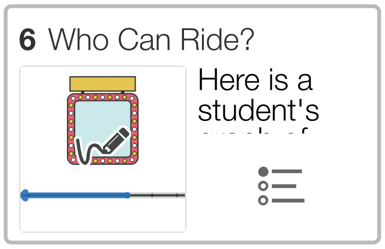
Image solution

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

Begin with a demonstration of an incorrect solution to this problem to provide access to students who benefit from clear and explicit instructions. Check in with individual students, as needed, to assess for comprehension as they finish the activity.



Here is a student's graph of the possible heights of riders on the kiddie swings.

Luis is allowed to ride.

Omar is 6 inches shorter than Luis.

Can Omar ride this ride?

Teacher Moves

This is a possible discussion screen.

Consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Select and sequence 1–2 responses for each of the most popular choices to display, and invite those students to share their reasoning.

The thinking students do on this screen will support them as they solve more complex inequalities later in the unit.

Sample Responses

Yes

Explanations vary. People who are 48 inches tall or shorter are allowed to ride the Kiddie Swings. Luis is allowed to ride, so he must be 48 inches tall or less. Anyone shorter than Luis would also be allowed to ride.

Student Supports

Multilingual Learners

• *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., Omar can/cannot ride the ride because ________.).

---

7 Sort the cards into fo...

Teacher Moves

Facilitation: Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards. If time runs short, consider continuing to Activity 2.

Sample Responses

Image solution

Student Supports

Students With Disabilities

• *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., asking students to focus on one number line at a time), which will aid students who benefit from support with organizational skills in problem-solving.

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8 Make It Work

Fix the inequality

$f(x)$

Fix the inequality so that it represents the graph shown.

Teacher Moves

Activity 2 Purpose: On Screens 8–11, students apply what they have learned to make sense of inequalities out of context.

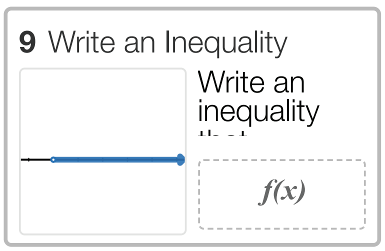
Facilitation: Consider starting with the activity paused and sharing the purpose of the activity. Give students one minute to look at the graph and inequality before responding. Then unpause and invite students to fix the inequality and to continue to the next screen.

Note: Students can type the $\leq$ symbol by typing $<$ $=$ or by using the keyboard icon on the right of the math input.

Pacing: Consider using pacing to restrict students to Screens 8–11.

Sample Responses

$x \leq 23$ or $23 \geq x$



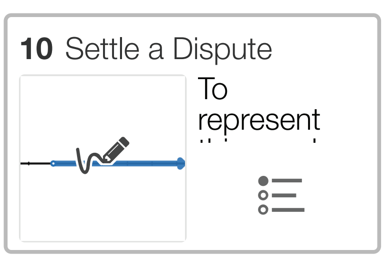
Write an inequality that represents this graph.

Teacher Moves

Progress Check: This is a great place to check students' progress writing inequalities to represent graphs on a number line. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

$x > 19$ or $19 < x$



To represent this graph:

- Tiara wrote the inequality $19 < x$.
- Devon wrote the inequality $x < 19$.

Whose inequality is correct?

Teacher Moves

This is a possible discussion screen.

Facilitation: While students are working, select and sequence 1–2 responses for each of the most popular choices. Once most students have responded, consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.

Then use the snapshot tool to display the selected responses and invite students to share their reasoning.

If it does not come up naturally, ask students if there is another correct way to write this inequality.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

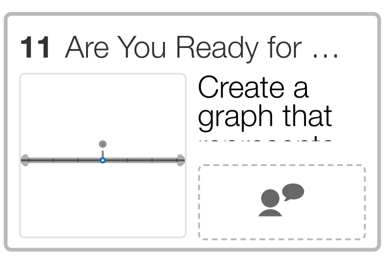
Sample Responses

Tiara

Explanations vary. Since the graph is to the right, x has to be all the numbers that are greater than 19, which means that 19 is less than x.

Student Supports

Multilingual Learners
• *Expressive Language: Eliminate Barriers*
Provide sentence frames to help students explain their reasoning (e.g., _____ is correct because ________.).



Create a graph that represents all the possible solutions to the inequality $x + 10 \leq 25$.

Explain how you determined where to place the movable point.

Teacher Moves

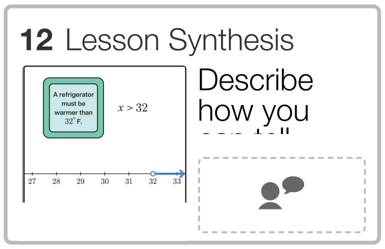
Facilitation: This screen is designed as an extra challenge for students who finish Screens 8–10 before the class discussion on Screen 12. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Note: The thinking on this screen is similar to the thinking all students will do in Lesson 14.

Sample Responses

Image solution

Responses vary. I put the movable point on 15 because that is the largest number that works. If you put it on 16, then $16 + 10$ is not less than or equal to 25.



Describe how you can tell from each representation that 32 **isn't** included in the inequality.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students connect words, symbols, and a graph representing the same inequality.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Follow with a brief whole-class discussion in which students discuss each representation. If it does not come up naturally, consider asking a question like: *Why are each of these three representations useful or powerful?*

Routine (optional): Consider using the mathematical language routine Collect and Display.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. In the equation, the symbol is $>$, not $\geq$. In the graph, there is an open circle, so 32 is not included. In the sign, it says "warmer than", which is like "more than." 32 is not more than 32.

Student Supports

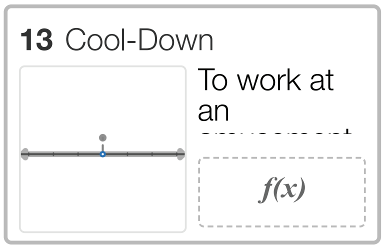
Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their strategy (e.g., In the equation, _______. In the number line, _______. In the sign, ______).

- *Expressive Language: Visual Aids*

Create or review an anchor chart that publicly displays each of the representations to aid in students' explanations.



To work at an amusement park, employees must be at least 14 years old.

1. Make a graph on the number line to represent the possible ages of employees at this park.

2. Write an inequality to represent this situation.

Teacher Moves

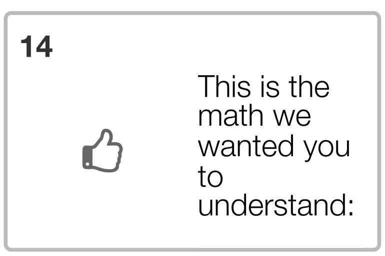
Support for Future Learning: If students struggle with writing or graphing an inequality based on the situation, plan to emphasize this when opportunities arise over the next several lessons. For example, plan to spend extra time making connections between the inequality and the situations in Activity 1 of Lesson 15.

Pacing: Consider using pacing to restrict students to Screens 13–14.

Sample Responses

Image solution

$x \geq 14$ or $14 \leq x$

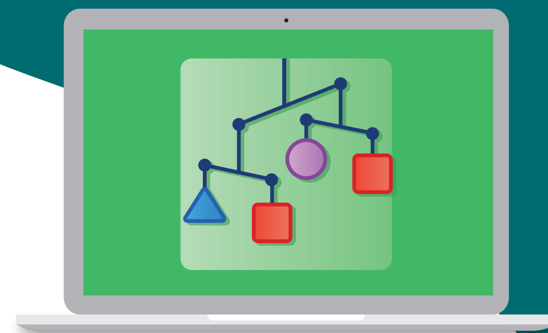


This is the math we wanted you to understand:

- I understand and can use the symbols $\leq$ and $\geq$.
- I can graph inequalities on a number line.

Unit 6
Lesson
14

Digital Lesson RECOMMENDED
This is a digital lesson. A print option is also available.



Unbalanced Hangers

Solutions to Inequalities

Let's solve inequalities using hanger diagrams.

Focus and Coherence

Today's Goals

1. **Goal:** Determine the solutions to an inequality with only positive numbers.
2. **Language Goal:** Compare and contrast solutions to equations and solutions to inequalities. **(Reading and Writing)**

Students determine solutions to inequalities using a familiar representation – hanger diagrams. Students connect solving an inequality with determining the values of x that will keep one side of a hanger heavier than the other and recognize that the solutions to inequalities are themselves inequalities.

Prior Learning

In Lesson 13, students revisited inequalities and were introduced to greater-than-or-equal-to $(\geq)$ and less than-or-equal-to $(\leq)$ symbols. They drew and labeled number lines to represent inequalities.

Future Learning

In Lesson 15, students will apply their understanding of inequalities to the context of budgeting.

Vocabulary

Review Vocabulary

solutions to an inequality

Standards

Addressing

NY-7.EE.4
Use variables to represent quantities in a real-world or **mathematical problem, and construct simple** equations and **inequalities to solve problems by reasoning about the quantities.**

Also Addressing: **NY-7.EE.4b**

Mathematical Practices: MP3

Building On	Building Toward
NY-6.EE.5	**NY-7.EE.4b**

Rigor and Balance

- Students strengthen their **procedural fluency** in determining solutions to an inequality through repeated challenges.
- Students **reason adaptively** by connecting the solutions to their inequality to the visual representation on the hanger.

Lesson at a Glance

~ 45 min

Standards: NY-7.EE.4, NY-7.EE.4b

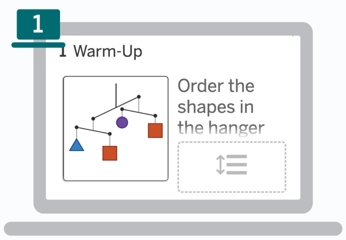
Why go digital?

Students receive interpretive feedback as they solve inequalities in real time.

Warm-Up

Pairs | 5 min

Students revisit hanger diagrams and reason about relative weights of objects using an unbalanced hanger. **Think, Pair, Share**

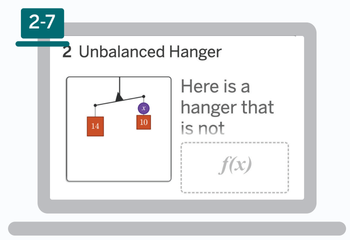


Pacing: Screen 1

Activity 1

Pairs | 10 min

Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities.

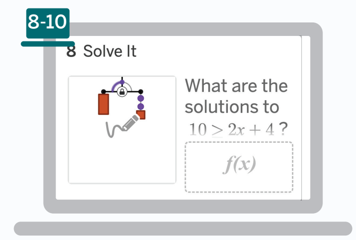


Pacing: Screens 2–7

Activity 2

Pairs | 15 min

Students develop strategies for determining solutions to inequalities, making connections to equations and using unbalanced hangers to help them revise their thinking. **Decide and Defend**

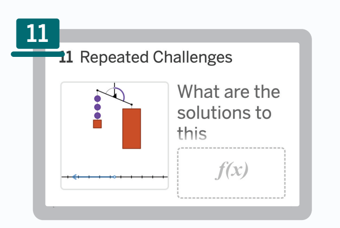


Pacing: Screens 8–10

Activity 3

Independent or Pairs | 5 min

Students practice solving inequalities with positive values using a set of repeated challenges (digital) or partner problems (print).

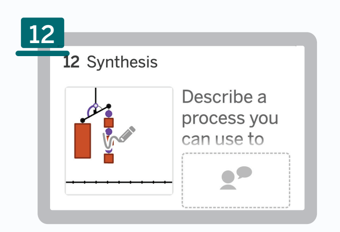


Pacing: Screen 11

Synthesis

Whole Class | 5 min

Students synthesize their understanding about how to solve an inequality.

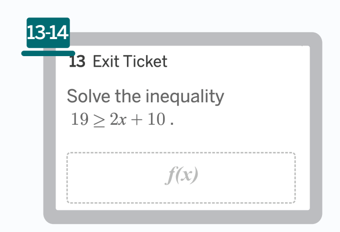


Pacing: Screen 12

Exit Ticket

Independent | 5 min

Students demonstrate their understanding by solving an inequality and explaining their thinking.



Pacing: Screens 13-14

Prep Checklist

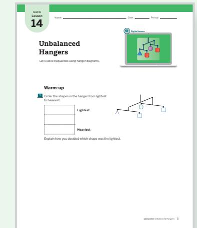
Assign the digital lesson. A print option is also available.

Student using digital:	**Student using print:**
Digital Lesson	Print Option in Student Edition
	Exit Ticket PDF

Digital Lesson | Pairs | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students revisit hanger diagrams and reason about relative weights of objects using an unbalanced hanger.



Students using print

Students using digital

1 Launch

Invite students to share what they notice about the hanger.

Use the Think-Pair-Share routine to support collaborative learning. Invite students to think independently, then to take turns sharing with a partner.

M/EL **Multilingual/English Learners** Provide sentence frames to help students explain their strategy (e.g., "The____ is lighter/heavier than the ____ because ____.").**(Speaking and Listening)**

A **Accessibility: Visual-Spatial Processing** For students using digital, consider providing access to the Student Edition throughout this lesson, which contains printed versions of the hanger diagrams for students to draw on or highlight.

1 Connect

Display a few student explanations. Invite several students to share their reasoning.

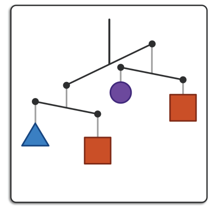
Consider asking, "Which side of an unbalanced hanger is heavier? How do you know?" if this idea does not come up naturally.

Math Identity and Community Invite students to celebrate a classmate's thinking that was strong, clear, or creative.

Key Takeaway: Unbalanced hanger diagrams can be a visual representation for inequalities.

1

Warm-Up



Credit: Paul Salomon

Order the shapes in the hanger from *lightest* to *heaviest*.

LIGHTEST

- Square
- Triangle
- Circle

HEAVIEST

Explain how you decided which shape was the lightest.

Share With Class

Lightest to heaviest:

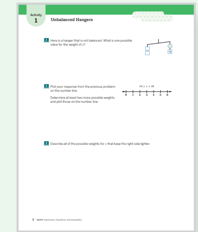
- Circle
- Triangle
- Square

Explanations vary. I know the right side is lighter than the left because it is up higher. This means that the circle is lighter than the triangle. I know the circle is lighter than the square because it is higher. This means the circle has to be the lightest shape.

Pairs | 10 min | Pacing: Screen 2–7

Activity 1 Unbalanced Hangers

Purpose: Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities.



Students using print

Students using digital

2 Launch

Invite students to share what they notice and wonder about the hanger. Listen for and amplify words such as *heavier*, *lighter*, *tilted*, and *unbalanced*.

Note: In prior lessons, students solved problems to make a hanger balance. In this lesson, students will make sense of unbalanced hangers.

2-4 Monitor

Use the dashboard to identify students you would like to check in with during the activity.

Encourage students to record several values that do and do not make the left side heavier.

Invite students using print to check their values with a partner in place of the interpretive feedback provided in digital.

D Differentiation

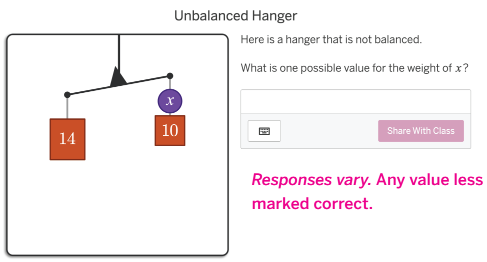
Look for students who:	Teacher Moves
Need support getting started. *(Screen 3)*	**Support:** Encourage them to make connections between the dots on the number line and the hanger.
List specific numbers ($1, 2, 5$, etc). *(Screen 4)*	Consider asking, “Are there other solutions that are not listed here yet? How could we incorporate those?”
Write any number less than 4. *(Screen 4)*	Consider asking, “How might we represent that using symbols?”
Write $x < 4$. *(Screen 4)*	**Extension:** Challenge students to describe the largest and the smallest solutions.

M/EL Multilingual/English Learners Provide sentence frames to help students explain their reasoning (e.g., “All of the possible weights are ______ because _______.”).

Pause to have students share their responses before having them continue with the activity.

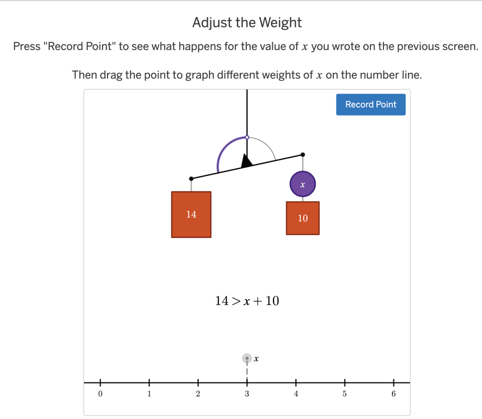
Activity 1 continued >

2

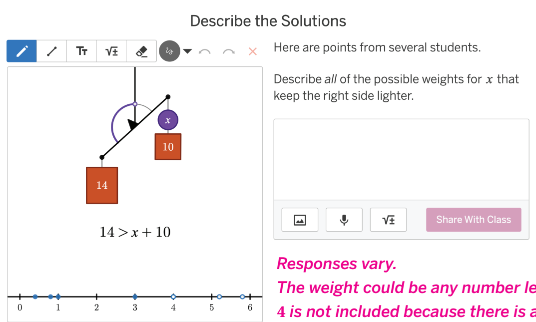


Responses vary. **Any value less than 4 will be marked correct.**

3



4

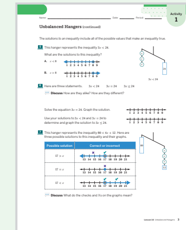


Responses vary.
The weight could be any number less than 4. 4 is not included because there is a $<$ sign, not a $\leq$ sign.

Digital Lesson | Pairs | 15 min | Pacing: Screens 5–7

Activity 1 Unbalanced Hangers (continued)

Purpose: Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities.



Students using print

5-6 Monitor

Use the Decide and Defend routine to support students in making arguments about the symbolic and graphical representations of the solutions to the inequality on Screen 5. **(MP3)**

Display the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus.

Encourage students to make arguments and justify their reasoning for each response.

Consider asking (if these ideas do not come up naturally):

- "What would the hanger diagram look like for each of the responses?"
- "What do solutions to inequalities look like graphically? Symbolically?"
- "What strategies can we use to solve inequalities like $3x \leq 24$?" *(Screen 6)*

Math Identity and Community Celebrate students who shared even though they were not sure yet.

7

Students using print analyze three sample results while students using digital press "Try It" to see results.

Encourage connections by asking:

- "How could you solve this if it was an equation?"
- "Why do you think there are two checks or Xs?"

A Accessibility: Executive Functioning
To support students with carrying out multiple steps, invite students to record their thinking in the Student Edition for the duration of this lesson.

7 Connect

Invite students to share solutions they tried and what they observed as a result.

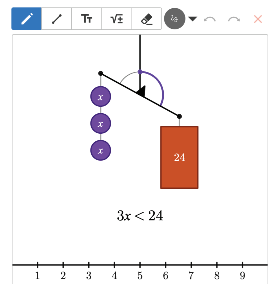
Consider asking, "How did you use the checks and x's to help edit your response?"

Key Takeaway: Solving an inequality is similar to solving an equation. The solutions to an inequality are also an inequality because there is more than one value that makes the inequality true.

Students using digital

5

Solutions



The *solutions to an inequality* include all of the possible values that make an inequality true.

This hanger represents the inequality $3x < 24$.

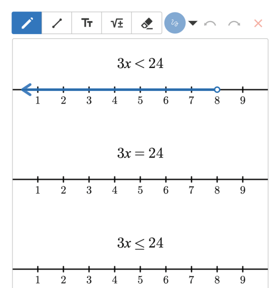
What are the solutions to this inequality?

$x < 8$	$x > 8$

$x < 8$

6

Three Statements



Here are three statements.

$3x < 24$ $3x = 24$ $3x \leq 24$

1. Discuss: *How are they alike? How are they different?*

2. Solve the equation $3x = 24$. Graph the solution.

3. Use your solutions for $3x < 24$ and $3x = 24$ to determine and graph the solution to $3x \leq 24$.

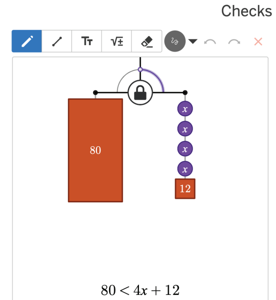
1. $3x < 24$ and $3x \leq 24$ are inequalities and have infinitely many solutions, while $3x = 24$ is an equation and has only one solution. $3x < 24$ does not include the possibility of $3x$ being equal to 24, while $3x \leq 24$ does.

2. $x = 8$

3. $x \leq 8$

7

Checks and X's



This hanger represents the inequality $80 < 4x + 12$.

Try out different solutions to this inequality. Press "Try It" to see their graphs.

Discuss: *What do the checks and X's in the graphs mean?*

$17 > x$

Try It

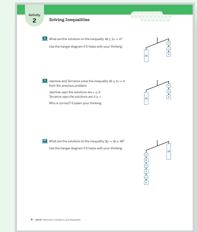
Responses vary.

- **$17 > x$. The check over the open circe on 17 means 17 is the boundary. The X over the arrow facing left means this is incorrect because the solution for x does not include any of the values that are less than 17.**

Pairs | 15 min | Pacing: Screens 8–10

Activity 2 Solving Inequalities

Purpose: Students develop strategies for determining solutions to inequalities, making connections to equations and using unbalanced hangers to help them revise their thinking.



Students using print

8 Launch

Display the hanger and the inequality $10 \geq 2x + 4$.

Consider asking, "What might be your first step in solving this inequality?"

9 Monitor

M/EL **Multilingual/English Learners** Encourage students to use gestures or hand motions to help them predict what the hanger should look like. **(Reading and Listening)**

Look and listen for students who justify the response "Both" in creative ways on Screen 9. Consider highlighting these during discussion in the Connect.

Capture several responses to share during the Connect.

D Differentiation

Look for students who:	Teacher Moves
Substitute values for x and test whether they make the inequality true.	Consider asking, "What strategies do we have from solving equations that could support us in identifying all of the solutions to this inequality?"
Connect the inequality to the hanger by annotating the image.	Celebrate this visual thinking. Consider asking, "How does this connect to ___'s strategy of distributing, subtracting, and then dividing on both sides?"
Use a variety of algebraic strategies or reasoning to determine the solutions, such as the Distributive Property or making moves on both sides of the inequality.	**Extension:** Consider asking, "What are other ways we could write these solutions?"

10 Connect

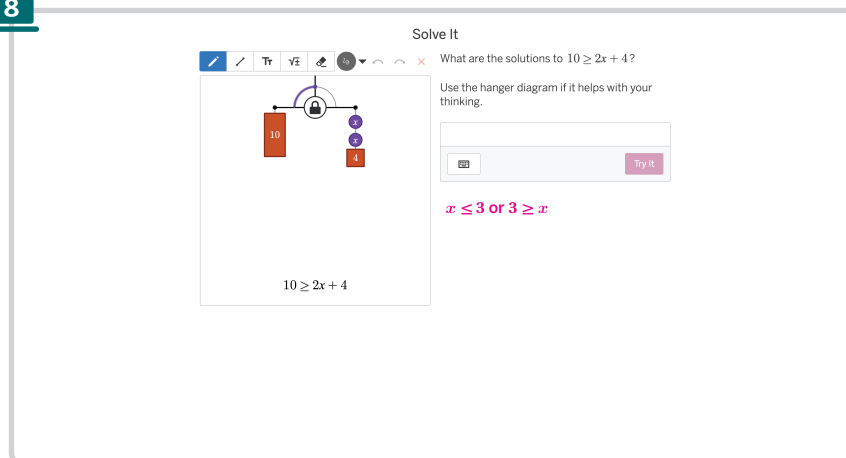
Display captured strategies. Consider starting with a solution where a student substituted a value of x to test the result.

Consider asking, "How is solving an inequality like solving an equation? How is it different?"

Math Identity and Community Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

Students using digital

8



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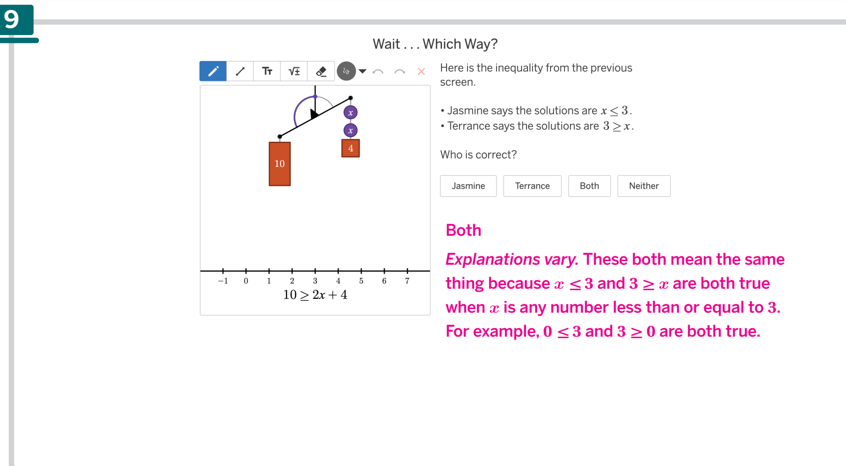

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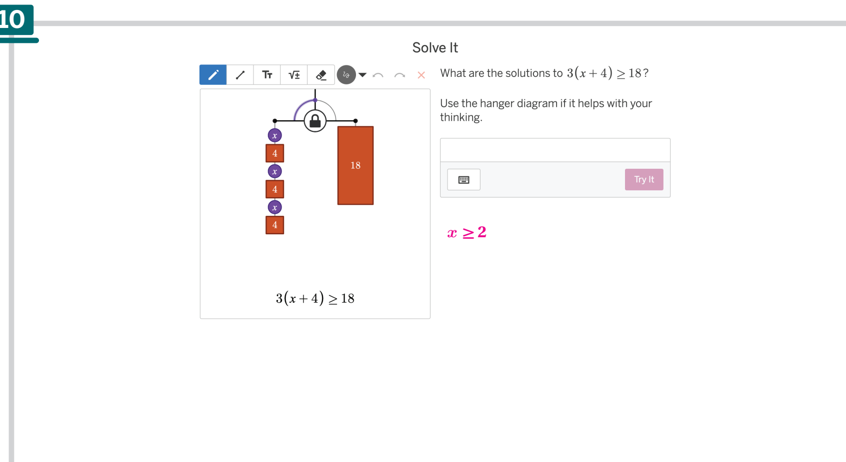


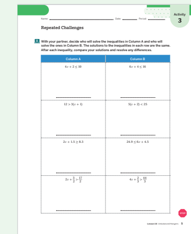
Digital Lesson

Pairs/ Independent | 5 min | Pacing: Screen 11

Activity 3 Repeated Challenges

Purpose: Students practice solving inequalities with positive values using a set of repeated challenges (digital) or partner problems (print).

Short on time: Consider omitting this activity.



Students using print

11 Launch

Invite students to set their own goal for how many challenges they would like to complete, sharing that sense-making is more important than speed.

11 Monitor

Students using print will need a partner for this activity. They will solve and compare solutions to pairs of problems with the same solutions in place of receiving interpretive feedback.

Circulate to observe student strategies, listen to partner discussions, and offer help or encouragement where needed.

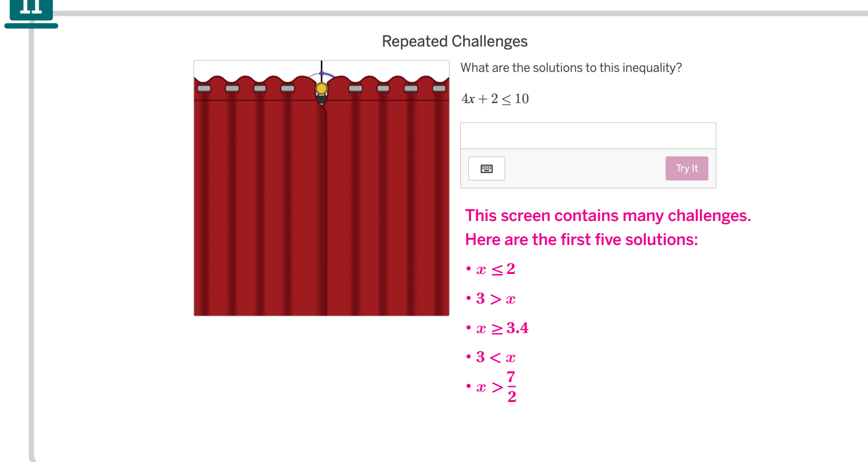
Note: Use the dashboard's teacher view to see how many challenges each student has completed correctly.

11 Connect

Invite students to share strategies that they found helpful or mistakes they made while solving these challenges.

Students using digital

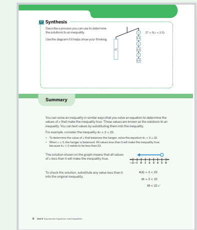
11



Whole Class | 5 min | Pacing: Screen 12

Synthesis

Purpose: Students synthesize their understanding about how to solve an inequality.



Students using print

12 Synthesis

Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, including:

- Connecting the inequality to the hanger.
- Distributing the 3 first and solving as if this were an equation in order to determine the boundary value.
- Substituting values for x and testing if they make the inequality true.

Math Identity and Community Invite students to name strategies they found most helpful and attribute them to the students who shared them.

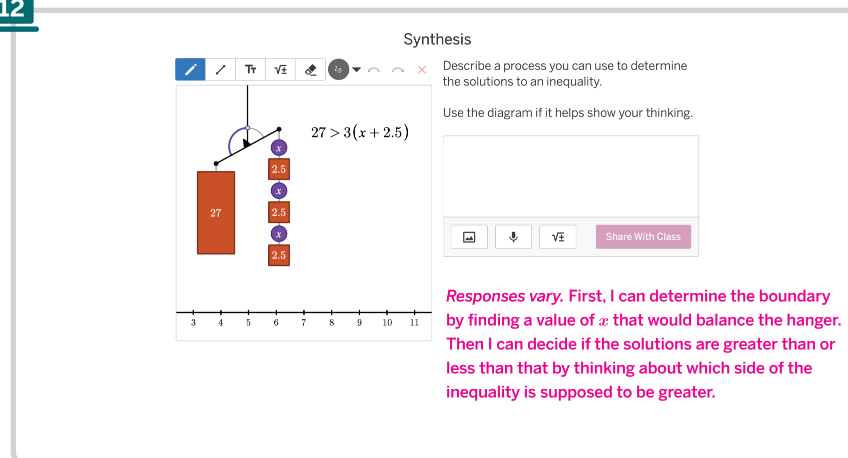
Lesson Takeaway: An inequality can be solved similarly to an equation, allowing the boundary value to be determined. Values can then be tested to determine the solutions that make the inequality true.

Summary

Share the Summary. Students can refer back to this throughout the unit and course.

Students using digital

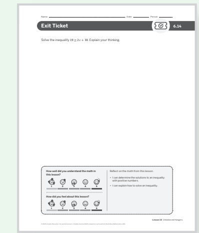
12



Digital Lesson | Independent | 5 min | Pacing: Screens 13–14

Exit Ticket

Purpose: Students demonstrate their understanding by solving an inequality and explaining their thinking.



Students using print

13-14 Today's Goal

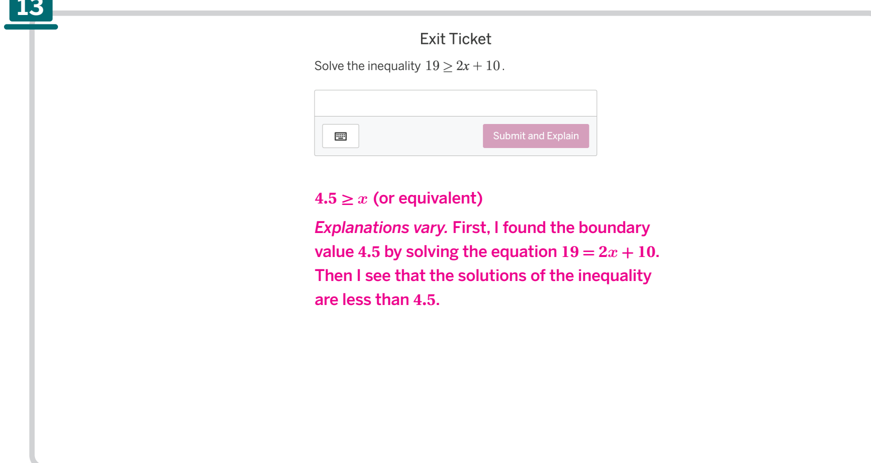
Goal: Determine the solutions to an inequality with only positive numbers.

Language Goal: Compare and contrast solutions to equations and solutions to inequalities. **(Reading and Writing)**

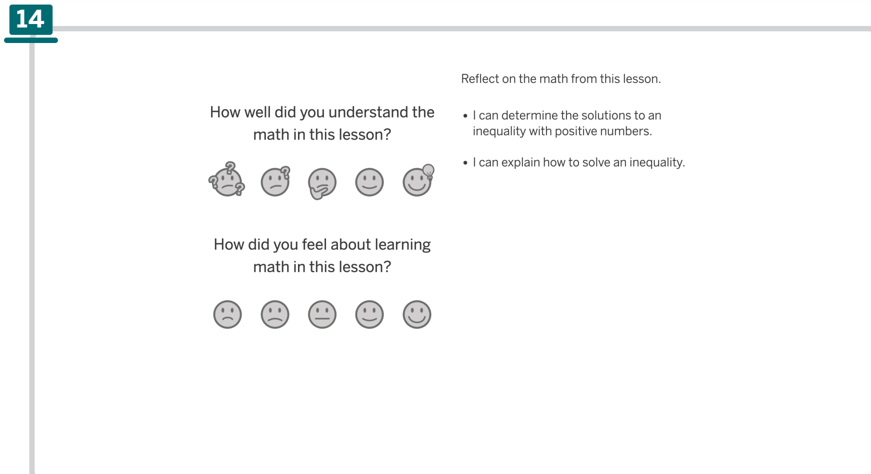
Support for Future Learning: If students struggle with solving the inequality, plan to emphasize this when opportunities arise over the next several lessons. For example, spend extra time in Lesson 15 discussing strategies for solving the inequality that represents each situation.

Students using digital

13



14



Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

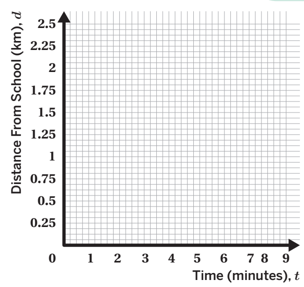
Students using digital

Students using print

Name: ______ Date: ______ Period: ______

Practice

For Problems 1–4, use this information. Mia and Jamal bike home from school at a steady pace. Mia bikes 1.25 kilometers and it takes her 4 minutes. Jamal bikes 1.75 kilometers and it takes him 7 minutes.



1. Create two lines that represent Mia and Jamal's journeys.
   Sample responses shown on graph.

2. Plot a point on each line when $t = 1$.
   Sample responses shown on graph.

3. Determine each value of r, the constant of proportionality.

Line	r
Mia	0.3125
Jamal	0.25

4. Who bikes faster? Explain your thinking.
   Mia; *Sample response:* Mia bikes faster because the constant of proportionality that represents the number of kilometers per minute that she travels is greater than Jamal's.

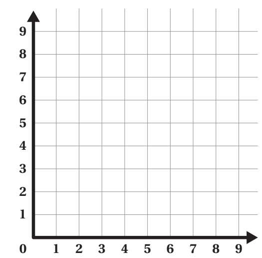
5. The point $\left(3, \frac{6}{5}\right)$ lies on a graph representing a proportional relationship. Select *all* of the points that also lie on the same graph.
   A. $(1, 0.4)$
   B. $\left(1.5, \frac{6}{10}\right)$
   C. $\left(\frac{6}{5}, 3\right)$
   D. $\left(4, \frac{11}{5}\right)$
   E. $(15, 6)$

Additional Practice for this lesson is available online.

Practice

Name: ______ Date: ______ Period: ______

For Problems 6–9, use this information. At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for $5.40.



6. How much does the honey cost per ounce?
   0.45 dollars per ounce

7. How much honey can you buy for $1 dollar?
   About 2.2 ounces

8. Write two different equations that represent this situation. Use h for ounces of honey and c for cost in dollars.
   $c = 0.45h$ (or equivalent) and
   $h = 2.2c$ (or equivalent)

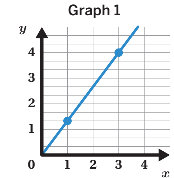
9. Graph the line of one of your equations from the previous screen. Be sure to label the axes.
   Sample response shown on graph.

Spiral Review

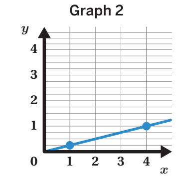
For Problems 10–13, match each equation with its graph.

10. $y = \frac{1}{4}x$ Graph 2
11. $y = \frac{3}{2}x$ Graph 4
12. $y = 2x$ Graph 3
13. $y = \frac{4}{3}x$ Graph 1

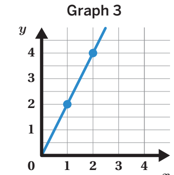
Graph 1



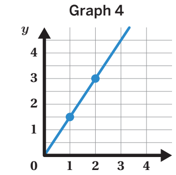
Graph 2



Graph 3



Graph 4



Reflection

1. Put a smiley face next to a question you were stuck on and then figured out.
2. Use the space below to ask one question you have or to share something you are proud of.

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1, 2, 9	2	NY-7.RP.2
	3, 4, 6, 7	2	NY-7.RP.2b
Test Practice	5	2	NY-7.RP.2
	8	2	NY-7.RP.2c
Spiral Review			
Fluency	10–13	1	NY-7.RP.2

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Budgeting (NYC)

Lesson 15: Solving Inequalities in Context

Purpose

The purpose of this lesson is for students to use what they have learned to write and solve inequalities related to situations about budgeting and spending money. For each inequality, students are encouraged to solve a related equation, then use the solution and the context to determine the solutions to the inequality. Each situation removes one layer of scaffolding until students write and solve an inequality without other prompting questions.

Preparation

Student Worksheet

- *Activity 1–2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Warm-Up (5 minutes)

Purpose: Students use the routine Three Reads to make sense of a real-world situation and surface that there is more than one reasonable solution. Students will continue to explore this situation in Activity 1.

Facilitation: Arrange students into pairs. Then facilitate the Three Reads routine. Here is a brief summary of the routine:

1. Display Sheet 1 of the Teacher Projection Sheets and read the situation aloud. Give students one minute to discuss with their partner what the situation is about and share with the class.
2. Display Sheet 2. Consider asking a student to read the situation aloud or to read the situation in pairs. Then give students 1–2 minutes to choose values they believe make sense. Consider sharing with students that many answers are reasonable, and encourage them to use any knowledge they have from their lives to decide what feels reasonable.
3. Display Sheet 3, which contains a question. Read the situation once more and give students 2–3 minutes to answer the question for the values they chose.

If it does not come up naturally, consider asking a question like: *If Maia sold one more magazine than your answer, would she still reach her goal? What about 5 more magazines? 10?*

Support for Multilingual Learners
Receptive Language: Eliminate Barriers
Provide printed copies of the situation for students to annotate or highlight.

Activity 1: Maia's Magazines (15 minutes)

Purpose: Students answer a question about a situation modeled by an inequality, first by solving an equation and then by reasoning about the set of all the possible solutions.

Facilitation: Distribute one Student Worksheet to each student. Consider sharing that this activity is still about Maia and her magazines. Then give students 5–10 minutes to work with a partner on Problems 1–5. If students are having difficulty getting started, consider asking questions like: *If she sold* 5 *magazines, would she be able to buy the ball? What about* 10*?*

Then facilitate a whole-class discussion around Problems 3 and 5 in order to surface how students decided which inequality sign and value made sense in their solutions.

Consider asking questions like these to further the discussion:

- *What number did you use in your inequality? What does that number mean in the situation?*
- *How did you decide which inequality sign to use?*

Early Finishers: Encourage students who finish Activity 1 early to create their own story about Maia and to write an inequality that represents the solutions to their story.

Support for Multilingual Learners
Receptive/Expressive Language: Peer Tutors
Pair students to aid them in comprehension of each situation.

Activity 2: Bao's Budgeting (15 minutes)

Purpose: In the previous activity, students were encouraged to make sense of a situation by writing and solving an equation, and then extending that to an inequality. In this activity, students write inequalities to represent situations about budgeting and determine their solutions.

Facilitation: Consider sharing with students that all of the situations in this lesson (both Activities 1 and 2) can be *better* represented by inequalities than by equations. Invite students to read Bao's situation aloud, discuss what information they believe is important, and work on Problems 1–7.

If students are having difficulty getting started, encourage them to make up an amount and to decide if Bao would have at least $\$25$ left if he took out that amount each month.

When most students have responded to Problem 4, consider pausing the class to discuss students' responses. Poll the class to see whether students believe the solution should be $x \geq$ or $x \leq$. Ask students to justify their choice and build on each other's reasoning (MP3).

While students are working, circulate to observe student strategies for Problems 6–7, listen to small group discussions, and offer help or encouragement where needed. Consider selecting a few students to share their inequalities and solutions. Once most students have responded to Problem 7, consider asking those students to write their responses to Problems 6–7 on the board. Then discuss what each number in the inequality represents in Bao's situation and what the solutions mean.

If it does not come up naturally, consider discussing whether the solution to Problem 7 should be written as $x \geq$ or $x \leq$, and why.

Early Finishers: Encourage students who finish Activity 2 early to create their own story and then trade stories with a classmate. After trading, invite students to write and solve an inequality that represents their classmate's story.

Support for Students With Disabilities
Conceptual Processing: Processing Time
Check in with individual students at each step of the activity to assess their understanding, as needed.

Lesson Synthesis (5 minutes)

Purpose: Students connect parts of an inequality to a situation in context.

Facilitation: Give students two minutes to respond and one minute to share their responses with a partner. Follow with a brief whole-class discussion in which students share how each value and sign is connected to the situation. If it does not come up naturally, consider asking a question like: *Why do you think Tay's inequality has a $\geq$ symbol?*

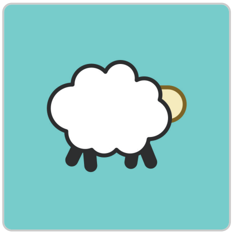
If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Support for Students With Disabilities
Executive Functioning: Visual Aids
Create an anchor chart for public display that connects a word description to an inequality.

Cool-Down (5 minutes)

Support for Future Learning: If students struggle to solve the inequality and interpret the solution, plan to emphasize this when opportunities arise during Lesson 17 and Practice Day 2. For example, consider spending extra time during Activity 1 of Lesson 17 discussing students' strategies for solving the inequality.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Shira the Sheep

Lesson 16: Solving Inequalities With Positive and Negative Numbers

Overview

Students practice solving inequalities with both positive and negative coefficients, and connect the solutions of inequalities to their graphs.

Learning Goals

- Solve inequalities with positive and negative coefficients, and graph their solutions.
- Test values to determine which inequality symbol represents the solutions to an inequality.

Lesson Checklist

- [ ] Complete the lesson using the student preview.
- [ ] Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- [ ] Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- [ ] Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Consider how to use snapshots to select and present student thinking for class discussion.
- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to practice solving inequalities with both positive and negative coefficients, and to connect the solutions of inequalities to their graphs. Students reason about the direction of an inequality when solving by asking the question: *What happens if I make* x *larger or smaller than the boundary?*

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to orient students to the context they will explore throughout the lesson: a fictional sheep who eats grass according to an inequality.

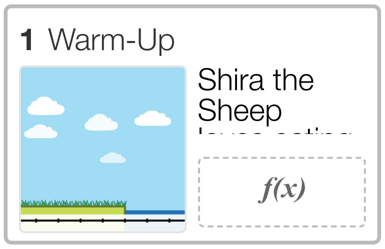
Activity 1: Shira the Sheep (30 minutes)

The purpose of this activity is for students to solve inequalities with both positive and negative coefficients. Students solve inequalities to program how Shira the Sheep moves. This activity explores what happens when an inequality contains a negative coefficient and one strategy for the direction of the inequality.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consolidate and refine their ideas about how to solve and graph the solutions to an inequality.

Cool-Down (5 minutes)



Shira the Sheep loves eating grass.

She does not like water.

Try out different inequalities to see what happens.

Teacher Moves

Lesson Overview: The purpose of this lesson is for students to practice solving inequalities with both positive and negative coefficients, and to connect the solutions of inequalities to their graphs.

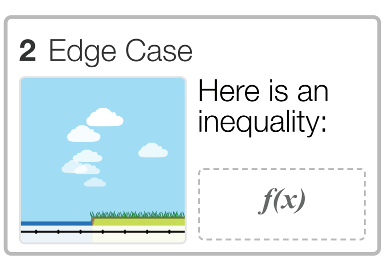
Warm-Up Purpose: Students are oriented to the context they will explore throughout the lesson: a fictional sheep who eats grass according to an inequality.

Facilitation: Consider starting with the activity paused and displaying the dashboard's student view. Press "Try It" and reflect as a class to see if Shira the Sheep moved as students expected.

Then unpause and invite students to try several different inequalities, predict what they think Shira the Sheep will do, and then see what happens. Encourage students to try different types of inequality symbols as well as different values.

After 2–3 minutes, invite students to share inequalities that they thought had an interesting or surprising result. If it does not come up naturally, consider trying out $x < 10$ and $x \leq 10$.

Pacing: Consider using pacing to restrict students to this screen.



Here is an inequality:

$5x > 15$

Solve the inequality to help Shira eat all the grass without falling in the water.

Teacher Moves

Activity 1 Purpose: On Screens 2–8, students solve inequalities with both positive and negative coefficients, and reason about strategies to determine the direction of the inequality when solving.

Facilitation: Consider starting with the activity paused and asking students what they notice is similar and different between the image here and the one in the warm-up. Some students might notice that there is a crumbly brown edge or that the numbers in the number line are different.

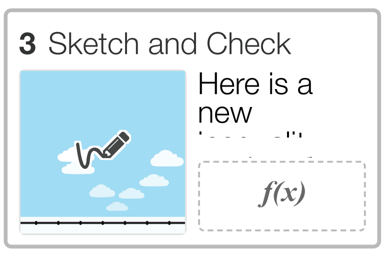
Then unpause and invite students to solve the inequality.

Note: If students type $x \geq 3$, the sheep will fall off the edge into the water.

Pacing: Consider using pacing to restrict students to Screens 2–5.

Sample Responses

$x > 3$ (or equivalent)



Here is a new inequality:

$$11 \geq 2x - 5$$

1. Sketch the solutions to this inequality on the number line.

2. Enter the solutions below to help Shira eat all the grass.

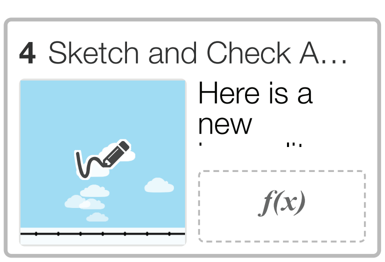
Teacher Moves

Progress Check: This is a great place to check students' progress solving inequalities with positive coefficients and graphing their solutions. Students should be able to see the grass match up with the sketch they created. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

1. Image solution

2. $x \leq 8$ (or equivalent)



Here is a new inequality:

$$10 - 6x < 70$$

Solve the inequality to help Shira eat all the grass.

Use the sketch tool if it helps you with your thinking.

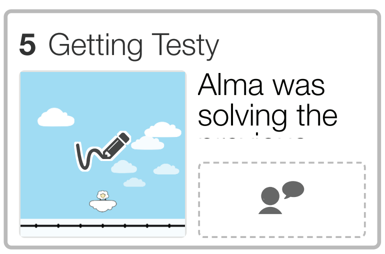
Teacher Moves

Facilitation: This is the first time that students encounter an inequality with a negative coefficient out of context. Invite students to revise their answers based on the feedback they see on this screen. Consider monitoring for students who wrestle with why the inequality is $x > -10$, and surface their strategies in the discussion on Screen 5.

Early Student Thinking: Students may correctly identify the boundary at $x = -10$ and use the same sign from the original inequality in their solution. Consider asking these students if they could find another value that would be a solution to the inequality.

Sample Responses

$x > -10$ (or equivalent)



Alma was solving the previous inequality, $10 - 6x < 70$.

She knew the sheep needed to land at -10 , but didn't know if the grass was to the **right** or **left**.

She wrote $10 - 6(0) < 70$.

How might that help Alma decide where the grass is?

Teacher Moves

This is a key discussion screen.

The purpose of this discussion is to make sense of Alma's strategy for determining which side of the boundary represents the solutions to the inequality.

Facilitation: If students are having difficulty getting started, consider asking students whether they think 0 would be a solution to the original inequality or not, and how they can use this to decide which direction to make the inequality symbol in the solution.

While students are working, select and sequence several student responses using the snapshot tool. Monitor for student responses that mention what the 0 could mean if the grass is to the right or to the left from -10 , or ones that describe a creative or clear strategy.

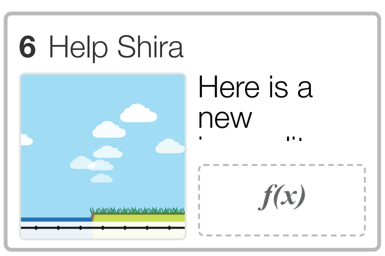
When most students have responded, facilitate a whole-class discussion to examine student responses and explain Alma's strategy. Consider asking questions like:

- *Is* 0 *a solution to the original inequality? Why or why not?*
- *Could she have tried any number or was there something special about* 0*?*
- *What do you think would happen if we tried a number less than* -10*, like* -100 *or* $-1\,000$*?*
- *How does testing numbers help you to decide which way to face the inequality symbol?*

Early Finishers: Encourage students who finish Screens 2–5 early to discuss whether or not Alma's strategy would still work using a number other than 0, and to decide on other numbers that would be helpful to test when solving this inequality or others.

Sample Responses

Responses vary. 0 is a nice number to test to see if all the values to the right of -10 are solutions. The inequality that Alma wrote, $10 - 6(0) < 70$, is true, which means that 0 *is* a solution. This tells her that *all* the numbers to the right of -10 are solutions.



Here is a new inequality:

$$-5x + 2 \leq 12$$

Solve the inequality to help Shira eat all the grass.

Teacher Moves

Progress Check: This is a great place to check students' progress solving inequalities with negative coefficients. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Facilitation: Consider asking students who write $x \leq -2$ to reflect on why they think their solution ended up in the opposite direction of the grass and what strategy they could use to figure out the direction in the future.

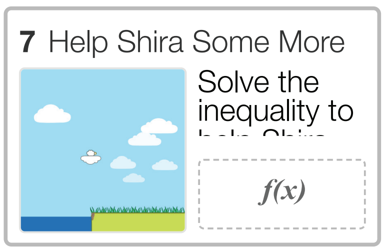
Note: Resist the temptation to offer students a procedure like "Whenever you multiply or divide by a negative, flip the inequality."

(Students will generalize this rule in later grades.) Instead, encourage students to test one or more points to determine the direction of the inequality.

Pacing: Consider using pacing to restrict students to Screens 6–8.

Sample Responses

$x \geq -2$ (or equivalent)



Solve the inequality to help Shira eat all the grass.

Complete as many challenges as you'd like!

$$8 \geq 3x - 13$$

Teacher Moves

Facilitation: Give students 5–10 minutes to complete as many challenges as they can. Consider setting a goal for the total number of correct responses students should get or asking students to set their own goal.

Circulate to observe student strategies, listen to small group discussions, and offer help or encouragement where needed. Consider naming powerful strategies you hear after the students who use them and using those names throughout the rest of the lesson and unit.

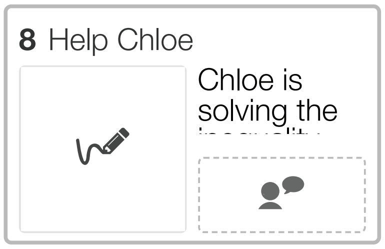
Consider pausing the activity to celebrate students who persisted through struggle (e.g., “I saw a student struggling on the first few screens, and because they kept at it, they’re crushing it now!”).

Note: Check the dashboard's teacher view to see how many challenges each student has completed correctly.

Sample Responses

This screen contains many challenges. Here are the first two answers:

- $x \leq 7$
- $x < -3$



Chloe is solving the inequality $25 - 4x < 1$.

Chloe says the solutions to the inequality are $x < 6$.

Convince her that her response is not correct.

Teacher Moves

This is a possible discussion screen.

Facilitation: The purpose of this screen is for students to apply what they have practiced on Screens 4–7 to construct an argument (MP3). Consider snapshotting imprecise or unfinished explanations. During the discussion, highlight the strengths of these explanations by asking students to identify what parts of each explanation they found to be valuable.

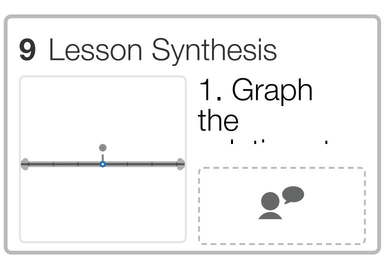
If time allows, give students one minute to make their response stronger and clearer.

If time runs short, consider using this screen instead of Screen 9 as the lesson synthesis.

Routine (optional): Consider using the mathematical language routine Critique, Correct, Clarify.

Sample Responses

Responses vary. If $x < 6$ represents the solutions, then $x = 0$ should make the inequality true. But $25 - 4(0) < 1$ is not true.



1. Graph the solutions to $4 - 3x \leq 19$.

2. Explain how you can determine the graph of the solutions to any inequality.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students consolidate and refine their ideas about how to solve and graph the solutions to an inequality.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Sample Responses

1. Image solution

2. *Responses vary*. First, figure out the boundary point, which is when both sides are equal. The circle is filled in if there is an $\leq$ or $\geq$ symbol. Then pick a number for x and find out if it is a solution or not. If it is, graph toward that number. If it isn't, graph away from that number.

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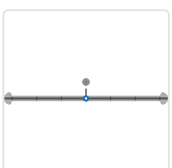
10 Cool-Down



Graph the solutions to $-3 > -2x +$.

Graph the solutions to $-3 > -2x + 9$.

Teacher Moves

Support for Future Learning: If students struggle to solve and graph the solutions to the inequality, consider reviewing this screen as a class before beginning Lesson 17 or offering individual support where needed during Lesson 17 and Practice Day 2.

Pacing: Consider using pacing to restrict students to Screens 10–11.

Sample Responses

Image solution

11

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can solve an inequality with positive and negative numbers and graph the solutions.
- I can test values to decide which inequality symbol makes sense.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Write Them and Solve Them (NYC)

Lesson 17: Modeling With Inequalities

Purpose

The purpose of this lesson is for students to practice writing and solving inequalities and to critically examine what the solutions to those inequalities mean in context. Students determine whether or not it is appropriate to round their calculations in order to answer the question posed in a story.

Preparation

Student Worksheet

- *Activity 1 and 2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Problem and Support Cards

- *Option 1*: Print and cut one double-sided set of problem and support cards (4 total cards) for each pair of students
- *Option 2*: Print and cut one single-sided set of problem and support cards (8 total cards) for each pair of students. See Activity 2's Facilitation for more details.

Warm-Up (5 minutes)

Purpose: Students reason about the solutions to an inequality that make sense in a context.

Facilitation: Display Sheet 1 of the Teacher Projection Sheets. Consider reading the story aloud as a class and asking students what connections they make to the story. Ask them what the variable x represents in Jamal's inequality. Then give students one minute to think quietly and another minute to share their reasoning about which statements are true with a partner.

Then, facilitate a brief whole-class discussion about connecting the solutions to the original question (MP4). Consider asking: *How do we use Jamal's solution to help him figure out how many sandwiches he can buy?*

Note: Students may select different statements depending on their personal experience. For example, some stores allow you to purchase 9.5 sandwiches and others do not.

Activity 1: Orange Juice and Donuts (10 minutes)

Purpose: Students write and solve an inequality and then interpret their solution. This is intended to prepare students for the work they will do in Activity 2.

Facilitation: Arrange students into pairs. Consider reading the situation aloud and then answering the first question on Sheet 2 of the Teacher Projection Sheets as a class.

Invite students to work together to share their reasoning and reach an agreement about how to answer each question. Circulate to observe student strategies, and offer help or encouragement where needed. Consider asking students questions from the displayed Support Card as they are working.

Then facilitate a whole-class discussion to surface strategies students used to write their inequalities, and invite them to explain what their solutions mean in Kiandra's situation. Consider asking students questions from the displayed Support Card to generate discussion.

Early Finishers: Encourage students who finish Activity 1 early to research the cost of orange juice and donuts in their community and answer Problems 1–3 using the costs they find.

Support for Students With Disabilities
Receptive Language: Processing Time
Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.

Support for Multilingual Learners
Expressive Language: Visual Aids
Create or review an anchor chart that publicly displays common words or phrases used in situations that involve inequalities to aid in explanations and reasoning.

Activity 2: Solve It! (20 minutes)

Purpose: Students develop fluency writing and solving inequalities and interpreting their solutions. Students support each other in making sense of the problem and its solutions.

Facilitation: Share with students that they are going to take turns being the problem solver and the coach, and that each role is crucial to their success as a pair. Review the instructions that are written at the beginning of Activity 2 on the Student Worksheet. Invite students to decide who will be the first problem solver and who will be the first coach.

There are several options for facilitating this activity:

- **Option 1:** Print and cut one double-sided set of Problem and Support cards for each pair of students. The coach takes the card with Problem #1 on one side and Support #1 on the other side. The coach reads the problem aloud and the problem solver notes what is important. The coach asks the first set of questions on their support card. Then the problem solver completes Step 2 (writing an inequality). Once both students agree that this step is correct, the coach reads the next set of questions and the problem solver continues to the next step. Once the problem solver has completed all three steps and both students agree, the students switch roles and continue to the next problem.

- **Option 2:** Print and cut one single-sided set of Problem and Support Cards for each pair of students. The problem solver takes Problem Card #1. The coach takes Support Card #1. The problem solver reads the card aloud. The coach asks the first set of questions on their support card. Then the problem solver completes Step 2 (writing an inequality). Once both students agree that this step is correct, the coach reads the next set of questions and the problem solver continues to the next step. Once the problem solver has completed all three steps and both students agree, the students switch roles and continue to the next problem.

If time is running short, consider asking students to complete only Problems 1 and 2 or as many as they can in the allotted time.

Then facilitate a brief whole-class discussion to surface what students learned as they worked together and to shout out actions that were helpful. Consider asking questions like: *What was it like to support another student as they solved a problem? What did your partner do that was helpful?*

Early Finishers: Invite students who finish early to reflect on what advice they would give themselves or other students when writing and solving inequalities. This is the final lesson before the End-Unit Assessment.

Support for Students With Disabilities
Conceptual Processing: Eliminate Barriers
Demonstrate the steps for the activity by having a group act as an example pair while the rest of the class observes.

Lesson Synthesis (5 minutes)

Purpose: Students use another example to describe how to write an inequality for a situation, focusing particularly on the appropriate inequality symbol.

Facilitation: Give students 2–3 minutes to respond and a minute to share their responses with a partner. Follow with a brief whole-class discussion in which students share how they decided which direction the inequality would be. If time allows, consider asking students to revise their response to be stronger and clearer.

Cool-Down (5 minutes)

Support for Future Learning: If students struggle to identify and correct the error, consider checking in with individual students as they solve inequalities during Practice Day 2 or reviewing this problem as a class before beginning Practice Day 2.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

7.6 Practice Day 2 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided copy for each student.

Cards

- Print one set of cards for each group of students (option 1), or 2–3 sets of cards for the entire class (option 2).

Instructions

Option 1: Group Questions

This structure supports student collaboration and focuses students' attention on one problem at a time. Arrange students into groups of 2–3. Print and cut out one set of cards for each group. Give each student the Student Workspace Sheet to complete as they work together to solve each of the task cards. Consider posting the answer key, or walking around with it and providing feedback to students as they work.

Option 2: Solve and Swap

This structure supports student collaboration with many different partners and allows for movement around the classroom. Students are positioned as experts as they discuss each problem and support one another.

Print and cut out two or three sets of cards for the entire class. Be sure there is one card for each student. Students circulate the class with their card and then pair up with a classmate. Each student in the pair solves the problem on their partner's card, collaborating as needed, and records their thinking on their workspace sheets. If a pair of students wind up with a problem they've already solved, they should compare strategies and solutions with their partner.

When both students have completed their problems, they swap cards, then stand up with a hand up, and find another classmate to pair up with. Repeat the process.

GRADE 7

Unit 7 Lesson Plans

Teacher lesson plans from Unit 7 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

NOTE: *We have included only those lessons from Unit 7 that cover the standards in the Expressions, Equations, and Inequalities domain.*

Grade 7 Unit 7

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

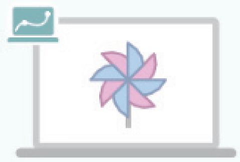
Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

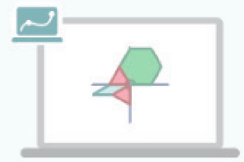
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



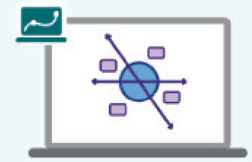
1 Pinwheels

Use reasoning to determine angle measures around a vertex.



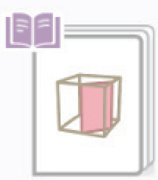
2 Friendly Angles

Describe what it means for angles to be complementary or supplementary.



3 Angle Diagrams

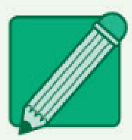
Describe what it means for two angles to be vertical.



8 Can You Draw It?

Use a protractor and ruler to draw triangles given three measurements.

Practice Day



Practice Day 1

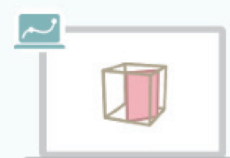
Practice the concepts and skills developed during Lessons 1–5. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

Quiz: Sub-Unit 1

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

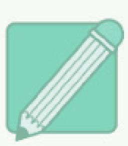
Sub-Unit 2



9 Slicing Solids

Describe possible cross sections of a solid.

Practice Day



Practice Day 2

Practice the concepts and skills developed during Lessons 1–12. Consider using this time to prepare for the upcoming Quiz.

Summative Assessment

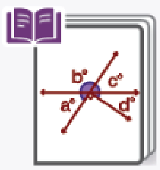
End-of-Unit Assessment

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing: 17 days | **Short on time?** See pacing considerations below.

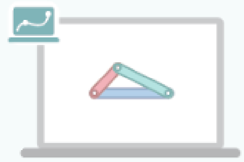
Pre-Unit Check: (Optional)
13 Lessons: 45 min each
2 Practice Days: 45 min each

1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



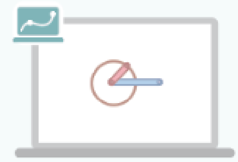
4 Missing Measures

Solve multistep problems involving complementary, supplementary, and vertical angles.



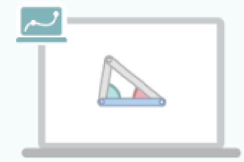
5 Can You Build It?

Determine whether or not three side lengths will make a triangle.



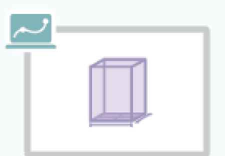
6 Is It Enough?

Explain that shapes are identical copies if they match up exactly when placed on top of each other.



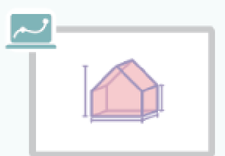
7 More Than One?

Use technology to build triangles given three measurements.



10 Simple Prisms

Explain the relationship between the volume of a prism, the area of its base, and its height.

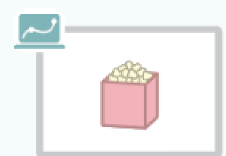


11 More Complicated Prisms

Calculate the volume of a prism with a complicated base using a variety of area strategies.

12 Surface Area Strategies

Compare and contrast different strategies for calculating the surface area of a prism.



13 Popcorn Possibilities

Apply surface area and volume to answer questions about a real-world situation.

Pacing Considerations

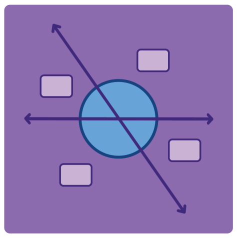
Lesson 1: This lesson surfaces what students know about angles and their measures, setting up ideas for the rest of the unit. These ideas will be addressed in more depth in upcoming lessons. If most students demonstrate a strong understanding of angles and their measures in Problems 1 and 2 of the Pre-Unit Check, this lesson may be omitted

Lesson 4: This lesson gives students an opportunity to practice solving multistep problems to determine angle measures. If students show a strong understanding of writing and solving equations to determine unknown angle measures in earlier lessons, this lesson may be omitted.

Lesson 11: This lesson supports students in developing fluency calculating the volume of more complicated prisms. If students show a strong understanding of calculating volume in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how their understanding of volume of simple prisms applies to more complicated prisms elsewhere in the unit.

Lesson 13: This lesson gives students an opportunity to apply their knowledge of surface area and volume to solve a complex problem. There is no new content introduced in this lesson.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Angle Diagrams

Lesson 3: Vertical Angles and Equations

Overview

Students apply what they've learned so far to understand the relationship between vertical angles. They write their own equations to represent complex diagrams and use those equations to determine unknown angle measures (MP2).

Learning Goals

- Describe what it means for two angles to be vertical.
- Write and use equations to determine unknown angle measures.

Vocabulary

- vertical angles

Lesson Checklist

- ☐ Complete the lesson using the student preview.
- ☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.
- ☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.
- ☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?
- ☐ Anticipate screens where students will struggle, then plan your response.
- ☐ Consider how to use snapshots to select and present student thinking for class discussion.

- [ ] Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Lesson 2 introduced students to two angle relationships: complementary and supplementary angles. In this lesson, students apply what they've learned so far to understand the relationship between *vertical angles*. Students also learned in Lesson 2 that you can use equations to represent angle diagrams. In this lesson, students write their own equations to represent complex diagrams and use those equations to determine unknown angle measures (MP2).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to surface what students know about angle relationships and angle measurements. Students also orient themselves to the diagram they will explore at the beginning of Activity 1.

Activity 1: Vertical Angles (20 minutes)

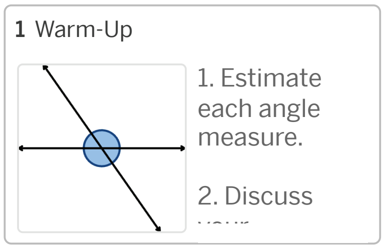
The purpose of this activity is for students to explore and then explain why the measures of vertical angles are equal. Students use an angle puzzle context (selecting one angle to reveal and then figuring out the other measures) to reason about vertical angles.

Activity 2: Writing and Using Equations (10 minutes)

The purpose of this activity is for students to strengthen their understanding of the connections between equations and angle diagrams. Students write equations that represent angle diagrams, and also make sense of other students' equations. This ties back to what students learned in Unit 6 about writing and solving equations in order to solve problems.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to describe how to identify vertical angles and to explain that vertical angle measures are equal.



1. Estimate each angle measure.

2. Discuss your estimates with a classmate.

Teacher Moves

Lesson Overview: In this lesson, students use what they've learned about complementary and supplementary angles as well as the relationship between measures of vertical angles to make sense of complex angle diagrams. Additionally, students write their own equations and use those to determine unknown angle measures.

Warm-Up Purpose: The purpose of the warm-up is to surface what students know about angle relationships and angle measurements. Students also orient themselves to the diagram they will explore at the beginning of Activity 1.

Facilitation: Consider starting with the activity paused and asking students what they remember from the previous lesson about relationships between angles in a diagram. Then unpause and invite students to estimate each angle measure and discuss their estimates with a classmate. While students are working, select and sequence several reasonable estimates and at least one unreasonable estimate using the snapshot tool. Facilitate a whole-class discussion around students' estimates. Consider asking questions like: *Are these estimates reasonable and why? How might you change these estimates and why?* Encourage students to justify their reasoning.

Early Student Thinking: Students may make reasonable estimates for individual angle measurements that are not reasonable when considered together. Highlight these during the discussion in order to celebrate the individual estimates. Then ask students whether the pair of estimates could *both* be correct.

Pacing: Consider using pacing to restrict students to this screen.

Sample Responses

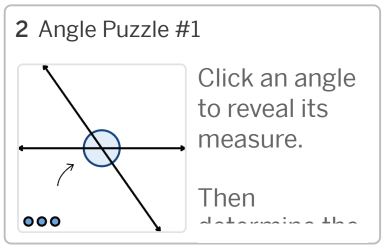
Responses vary.

Angle measures (clockwise from top right): $130°$, $50°$, $130°$, $50°$

Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the representations throughout this lesson for students to draw on or highlight.



Click an angle to reveal its measure.

Then determine the rest of the angle measures using as few reveals as you can.

Teacher Moves

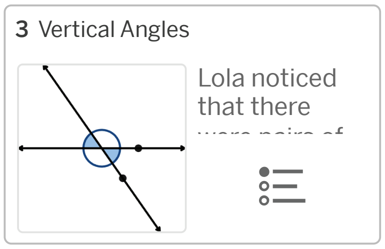
Activity 1 Purpose: On Screens 2–6, students explore vertical angles and then explain why the measures of vertical angles are equal.

Facilitation: Arrange students into pairs. Consider starting with the activity paused and sharing with students that their task will be to figure out the exact measure of each angle in the diagram. Then unpause and invite pairs to discuss which angle measure they want to reveal.

Pacing: Consider using pacing to restrict students to Screens 2–6.

Sample Responses

Angle measures (clockwise from top right): $125°$, $55°$, $125°$, $55°$



Lola noticed that there were pairs of angles that had the same measure.

These angles (angles opposite each other where two lines cross) are called *vertical angles*.

Are the measures of vertical angles always, sometimes, or never the same?

Teacher Moves

Facilitation: Circulate to observe student strategies and listen to pair discussions, particularly the language they use to describe vertical angles. After most students have completed this screen, consider displaying the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see.

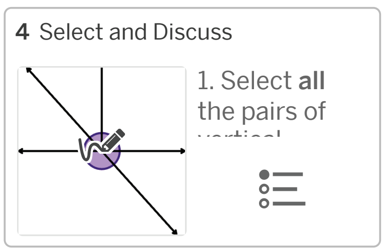
Sample Responses

Always

Explanations vary. No matter how I drag the lines, the shaded angles always look the same size as each other, and the unshaded angles always look the same size as each other.

Student Supports

Multilingual Learners
• *Expressive Language: Eliminate Barriers*
Provide sentence frames to help students explain their reasoning (e.g., Vertical angles are always/sometimes/never the same size because ________.).



1. Select **all** the pairs of vertical angles.

Teacher Moves

This is a possible discussion screen.

Facilitation: When most students have completed this screen, display the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Discuss each pair of angles one by one. Consider asking a student who selected that pair to explain their reasoning. Then discuss whether or not the angles definitely have the same angle measures or not. Consider recording any ideas that emerge for all students to reference throughout the unit.

Consider highlighting the value of changing one's mind by asking if any students revised their thinking based on the discussion.

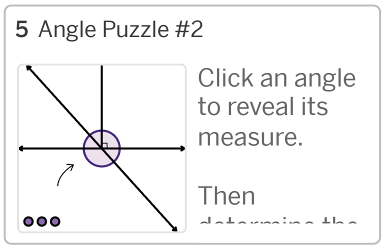
Question to push students' thinking: *If you rotated the diagonal line, what would happen to angles B, C, and F? How can this help you decide if B and F or C and F are vertical angles? Or if B and F or C and F are vertical angles?*

Sample Responses

B and F

Student Supports

Multilingual Learners
• *Expressive Language: Eliminate Barriers*
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.



Click an angle to reveal its measure.

Then determine the rest of the angle measures using as few reveals as you can.

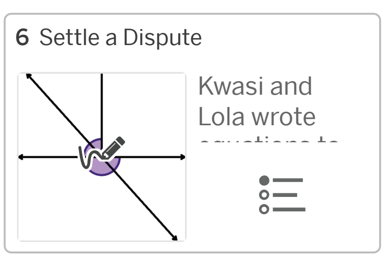
Teacher Moves

Progress Check: This is a great place to check students' progress using angle relationships. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Consider asking students to share with a classmate which angle(s) they chose to reveal and why (MP7).

Sample Responses

$A = 132°$, $B = 48°$, $C = 42°$, $D = 90°$, $F = 48°$



Kwasi and Lola wrote equations to help them solve the previous angle puzzle.

Kwasi's equation: $b + 132 = 180$
Lola's equation: $132 + f = 180$

Who wrote a true equation?

Teacher Moves

🔑 *This is a key discussion screen.* 🔑

The purpose of this discussion is to surface how equations might explain why vertical angles always have the same measure.

Display the distribution of responses using the dashboard's teacher view, calling attention to any conflict or consensus you see. Consider selecting and sequencing 1–2 responses for each of the most popular choices to display, and invite those students to share their reasoning in order to ensure that students understand why both equations are true (MP3).

Consider asking: *How can we use these equations to convince someone that b and f are equal?*

Question to push students' thinking: *Do these equations also show that c and f are equal? Why or why not?*

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Early Finishers: Encourage students who finish Screens 2–6 early to use equations to explain why c and f are not always equal.

Sample Responses

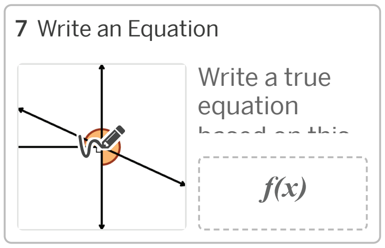
Both

Explanations vary. $b°$ and $132°$ make a line, so they are supplementary. So do $f°$ and $132°$.

Student Supports

Multilingual Learners
• *Expressive Language: Eliminate Barriers*
Provide sentence frames to help students explain their reasoning (e.g., _________'s equation is true because ________.).

Students With Disabilities
• *Fine Motor Skills: Peer Tutors*
Allow students who struggle with fine motor skills to dictate use of the sketch tool as needed throughout the lesson.



Write a true equation based on this diagram.

Try to write an equation none of your classmates will.

Teacher Moves

Activity 2 Purpose: On Screens 7–11, students strengthen their understanding of the connections between equations and angle diagrams. Students write equations that represent angle diagrams and also make sense of other students' equations.

Facilitation: Consider starting with the activity paused and sharing with students that there are many different equations that represent this diagram. Then unpause and invite students to write an equation. If students have difficulty getting started, consider asking them to describe the relationship between two angles in words (e.g., $a°$ and $b°$ make a line). Then work together to translate their words into an equation.

Sample Responses

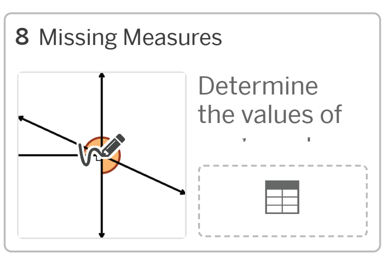
Responses vary.

- $a + 35 = 90$
- $a + b = 180$
- $b + c = 180$
- $35 + 90 + c = 180$
- $a + b + c + 35 + 90 = 360$

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Assist students in recognizing the connections between new problems and prior work. Consider revisiting the equations from Lesson 2 with angles inside of shapes.



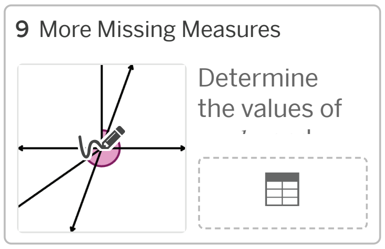
Determine the values of a, b, and c.

Teacher Moves

Progress Check: This is a great place to check students' progress determining unknown angles. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

$a = 55$, $b = 125$, $c = 55$



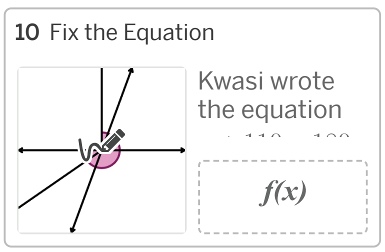
Determine the values of a, b, and c.

Teacher Moves

Facilitation: Consider celebrating students who persisted through struggle (e.g., "I saw a student struggling on the first diagram, and because they kept at it, they're crushing it now!").

Sample Responses

$a = 20$, $b = 70$, $c = 35$



Kwasi wrote the equation $c + 110 = 180$.

Change Kwasi's equation to make it true.

Teacher Moves

This is a possible discussion screen.

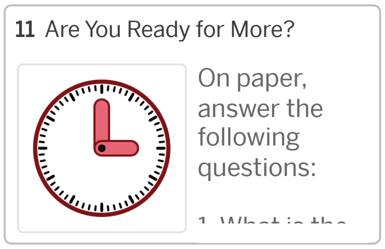
Facilitation: While students are working, select and sequence several correct and incorrect student equations using the snapshot tool. Consider highlighting unique or creative equations.

When most students have responded, facilitate a whole-class discussion around students' equations. Consider asking questions like: *Does this equation represent the diagram? How many different true equations could we write?* Encourage students to justify their reasoning.

Sample Responses

Responses and explanations vary.

- $2c + 110 = 180$
- $b + 110 = 180$
- $c + 110 = 145$
- $b = 2c$



On paper, answer the following questions:

1. What is the angle between the hour and the minute hand of a clock at 3:00?

2. Calculate the angle between the clock hands at 2:20. (Hint: It is not $60°$.)

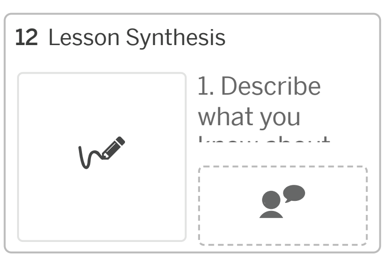
3. Determine at least one time where the hour and the minute hand are $40°$ apart.

Teacher Moves

Facilitation: This screen is designed as an extra challenge for students who finish Screens 7–10 before the class discussion on Screen 12. Consider inviting these students to share responses with each other in place of a whole-class discussion.

Sample Responses

1. $90°$
2. $50°$
3. 5:20 and 6:40



1. Describe what you know about vertical angles.

2. Draw a diagram that includes at least one pair of vertical angles. Mark the vertical angles.

Teacher Moves

This is a key discussion screen.

Lesson Synthesis Purpose: Students describe and show what they know about vertical angles, including how to identify them and that their measures are equal.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Select and sequence several student responses to display using the dashboard's teacher view or snapshot tool. Follow with a brief whole-class discussion in which students share connections they see between responses. If time allows, give students one minute to make their response stronger and clearer based on the conversation.

Pacing: Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the mathematical language routine Collect and Display.

Sample Responses

Responses and drawings vary.

Vertical angles are on opposite sides of intersecting lines. Their angles are the same size.

Student Supports

Multilingual Learners
• *Expressive Language: Eliminate Barriers*
Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

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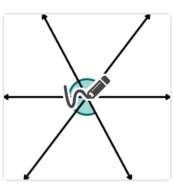
13 Cool-Down



Determine the values of

Determine the values of a and b.

Teacher Moves

Support for Future Learning: If students struggle with determining the missing angle measures, plan to emphasize this during Lesson 4, where students will practice determining unknown angles in diagrams.

Pacing: Consider using pacing to restrict students to Screens 13–14.

Sample Responses

$a = 53$, $b = 65$

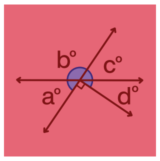
14

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can describe what vertical angles are.
- I can write and use equations to determine unknown angles.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Seeing Structure (NYC)

Lesson 4: Practice With Tape Diagrams and Equations

Purpose

In Lessons 1–3, students learned about the importance of angle relationships, like complementary angles, supplementary angles, and vertical angles. In this lesson, students apply what they've learned to solve multistep problems and determine missing angles measures. Students also create their own challenges to trade with classmates.

Preparation

Worksheet

- *Activity 1 and 2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one single-sided sheet or one double-sided half sheet for each student.

Challenge and Support Cards

- Print and cut one double-sided set of challenge and support cards (6 total cards) for each pair of students.

Materials

- Blank paper
- Rulers
- Protractors

Warm-Up (5 minutes)

Purpose: Students surface what they know about complementary, supplementary, and vertical angles, and practice the types of thinking they will need during Activity 1.

Facilitation: Arrange students into pairs and display Sheet 1 of the Teacher Projection Sheets. Give students one minute to share what they notice and wonder about the diagram with a partner, then another minute of time to think quietly about the four prompts.

Facilitate a brief discussion to surface pairs of complementary angles, supplementary angles, and vertical angles, and equations. To push students' thinking and to value multiple methods, consider asking a question like: *Is this the only pair of _____? How do you know?*

Consider writing students' equations on the board along with the angle relationship they represent for students to refer back to throughout the lesson.

Support for Students With Disabilities
Visual-Spatial Processing: Visual Aids
Provide printed copies of the diagram for students to draw on or highlight.

Activity 1: Solving Challenges (15 minutes)

Purpose: Students use equations and reasoning to determine missing angle measures involving complementary, supplementary, and vertical angles.

Facilitation: Consider sharing with students that in this activity, partners will support each other to solve several challenges. Pair students up and ask them to decide who will be Partner A and who will be Partner B. Display Sheet 2 of the Teacher Projection Sheets. Demonstrate the steps for the activity by having one student act as the challenge solver and you act as the coach while the rest of the class observes. Emphasize that the challenge solver should estimate the angle measures and write an equation **before** asking for the missing measure from their partner. This develops students' ability to make sense of complex diagrams.

There are several options for facilitating this activity:

- **Option 1:** The coach takes the card with Challenge A1 on one side and Support A1 on the other side, and shows the diagram to the challenge solver. The challenge solver estimates the measures and writes an equation, sharing their thinking aloud. Once both students agree, the coach reads the missing measurement and the challenge solver determines the rest of the measures while the coach asks the support questions. Once the challenge solver has completed all three steps and both students agree, they continue with Challenge B1, switching roles.
- **Option 2:** Each student takes their first challenge card (Challenge A1 and Challenge B1, respectively) and places it on the table with the diagram facing up and the support information on the back. Both students estimate and write equations for their diagrams and then share their estimates and equations with each other. Both students must agree that their estimates and equations for both diagrams are reasonable before they look at the back of their cards for the missing measurements. Once both students have completed all three steps and they both agree on the values of the missing measurements, they will continue with Challenges A2 and B2.

Give students 5–7 minutes to complete as many rounds of challenges as they can. If time is running short, consider asking students to complete a subset of the three challenges.

Then facilitate a brief whole-class discussion to surface what students learned as they worked together and to share any actions that were helpful. Consider asking questions like: *What was it like to support another student as they solved a problem? What did your partner do that was helpful?*

Early Finishers: For pairs who finish all three challenges early, display the "Are You Ready for More?" prompt on Sheet 3 of the Teacher Projection Sheets.

Support for Students With Disabilities
Conceptual Processing: Processing Time
Check in with individual students to assess for understanding as needed at each step of the activity.

Activity 2: Trading Challenges (15 minutes)

Purpose: Students use their creativity to create their own challenges, then solve the challenges of their classmates. This structure supports student collaboration with many different partners and allows for movement around the classroom. Students are positioned as experts as they discuss each challenge and support one another.

Facilitation: Distribute blank paper and protractors to each student. Share with students that they will create their own challenge similar to the ones from the previous activity and then solve challenges created by their classmates. It may be helpful to demonstrate the first part of the activity (drawing a diagram, using a protractor to measure and label one angle, and then labeling the rest of the angles with variables).

Give students five minutes to create their own challenge on a blank sheet of paper and solve their challenge on their worksheet. It is important that they do not write their thinking on the blank paper as that will be what students share as they trade challenges.

After time is up, arrange students into pairs. Each student in the pair solves the challenge created by their partner, collaborating as needed, and records their thinking on their worksheet. When both students have completed their challenge, they swap cards, then stand up with a hand up, and find another classmate to pair up with. Repeat the process.

Support for Students With Disabilities
Fine Motor Skills: Peer Tutors
Allow students who struggle with fine motor skills to dictate physical manipulation of the protractor as needed.

Executive Functioning: Visual Aids
Refer to an anchor chart that describes complementary, supplementary, and vertical angles.

Lesson Synthesis (5 minutes)

Purpose: Students use what they have learned about angle relationships to give advice to other students.

Facilitation: Give students 2–3 minutes to respond and one minute to share their responses with a partner. Follow with a brief whole-class discussion in which students share the advice they wrote.

Consider asking students to reflect on their own or with a partner about which advice they think would be most useful for themselves in the future.

Routine (optional): Consider using the mathematical language routine Collect and Display.

Cool-Down (5 minutes)

Support for Future Learning: If students struggle with writing equations or determining the missing measures, consider reviewing this as a class before Practice Day 1 or offering individual support where needed during the practice day.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

7.7 Practice Day 1 (NYC)

Preparation

Student Worksheet

- Print one double-sided sheet for each student.

Instructions

Option 1: Stations

This structure allows students to collaborate and helps them manage their time by breaking the task into smaller, more manageable chunks. It also increases engagement by giving students opportunities to move around the classroom.

Arrange students into groups of 3–4. Create stations around the room for each activity on the Student Worksheet. At each station, students will work on the designated activity and record their answers on their worksheet. Encourage students to use scratch paper to help with their calculations and thinking.

Options for student movement:

- As students finish the problems for a station, instruct them to move from station to station.
- After a set amount of time, instruct students to move as a group from station to station.
- After a set amount of time, instruct students to move to a new station such that no one from their previous group is in their new group.

Option 2: Worksheet

This structure allows students to work through the problems at their own pace. Arrange students into groups of 2–3 and instruct them to work on each activity together, one activity at a time. Encourage students to use scrap paper to help with their calculations and thinking. You can post the answer key or walk around with it and provide feedback to students as they work.