Amplify Desmos Math
NEW YORK

Teacher Edition Sampler
Grade 8

Inside you'll find:

- Program overview, scope and sequence, and correlations
- Complete sample lessons from Amplify Desmos Math
- Lesson plans from requested domains, partially designed

For Review Only.
Not Final Format.

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About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify Desmos Math is based on the Illustrative Mathematics (IM) curriculum. IM 6–8 Math was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is © 2017–2019 Open Up Resources. Additional adaptations and updates to IM 6–8 Math are © 2019 Illustrative Mathematics. IM 9–12 Math is © 2019 Illustrative Mathematics. IM 6–8 Math and IM 9–12 are licensed under the Creative Commons Attribution 4.0 International license (CC BY 4.0). Additional modifications contained in Amplify Desmos Math are © 2020 Amplify Education, Inc. and its licensors. Amplify is not affiliated with the Illustrative Mathematics organization.

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55 Washington Street, Suite 800,
Brooklyn, NY 11201
www.amplify.com

Welcome reviewer

Welcome to your Amplify Desmos Math New York Teacher Edition sampler!

Amplify Desmos Math New York is the result of two groundbreaking research and development efforts in K–12 mathematics instruction led by the Amplify and Desmos Classroom teams. Merging the two teams in 2022 enabled us to build a new curriculum around the idea that all students deserve to engage in high-quality grade-level mathematics every day. Based on Illustrative Mathematics'® IM K–12 Math™, Amplify Desmos Math New York combines strong pedagogy, arresting design, and forward-looking collaborative technology to deliver a classroom experience that keeps students engaged and asking productive questions.

Every lesson in the Amplify Desmos Math digital platform has a corresponding lesson in the print teacher and student editions. While we are in the process of finalizing the print materials, we have provided exemplars highlighting the unique design and ease of use of the Amplify Desmos Math print resources. To provide content covering your specific domain requests, in this physical sampler we have included both robust Amplify Desmos Math lesson plans and partially designed lesson plans. However, all of the lessons can be reviewed in their complete forms online.

All Amplify Desmos Math lessons include:

- Easy-to-follow lesson plans, tested in classrooms across the country.
- Clear teaching suggestions and strategies, including math language routines.
- Recommended differentiation moves and practice sets.

Diagnostic, formative, and summative assessments are provided with each unit along with lesson-level checks for understanding.

Amplify and New York City have a long history of partnering to provide equitable, high-quality instruction to our next generation of leaders. We look forward to continuing this partnership with New York City Public Schools in middle school mathematics.

—Jason Zimba and the
Amplify Desmos Math team

Amplify Desmos Math New York

Helping New York City teachers develop and celebrate student thinking

Deep and lasting learning occurs when students are able to make connections to prior thinking and experiences. This requires teachers to deliver math instruction that balances exploration and explanation, and that puts student thinking at the center of classroom instruction.

Amplify Desmos Math students are invited to explore the math that fills their everyday lives, while strengthening their knowledge of math facts, procedural skills, and conceptual knowledge. Using the Amplify Desmos Math print and digital lesson plans, teachers can confidently guide and instruct as they build on students' understandings to help them develop a better grasp of mathematics.

Amplify Desmos Math is **a truly student-centered program** built around three core tenets:

1

A strong foundation in problem-based learning is critical to developing deep conceptual understanding, procedural fluency, and application.

Students are introduced to interesting problems and leverage both their current understandings and problem-solving strategies to develop reasonable answers. The learning experience is an active one that leads students to explore, notice, question, solve, justify, explain, represent, and analyze. Teachers guide the process, supporting synthesis and sensemaking at the end of each lesson.

2

Technology can provide ongoing, enriched feedback that encourages students to persevere in problem solving.

Especially when new ideas are being introduced, Desmos Classroom technology shows students the meaning of their thinking in context, interpreting it mathematically rather than reducing it to a question of right or wrong. This creates a culture of going deep with mathematics and students as doers of mathematics, so that as learning progresses and correctness is the goal, incorrect answers become objects of curiosity rather than embarrassment. This information in response to student ideas is what we call "enriched feedback." Amplify Desmos Math New York offers more enriched feedback than any other math program.

3

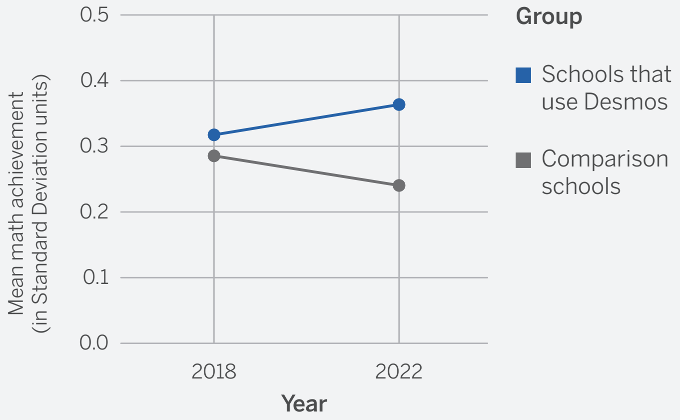
A commitment to access and equity should underpin every development decision.

All students can dive into problems on their own, and activities are designed to honor different approaches. Activities rely on collaboration and lots of hands-on, experiential learning.

And the program works.

Amplify Desmos Math New York expands on the Desmos Math 6–8 curriculum, which was recently proven to increase average math achievement in a study of more than 900 schools in nine states led by WestEd.

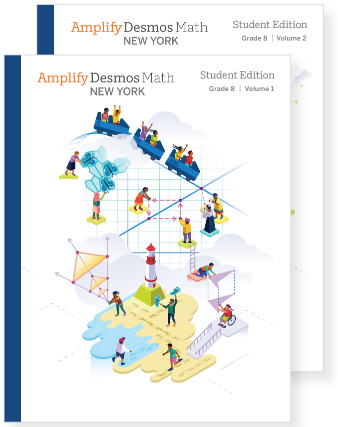
Mean Math Achievement for Desmos Schools and Matched Comparison Schools in 2018 and 2022



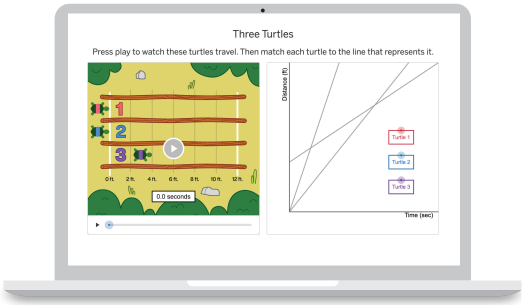
The Effect of Desmos Math Curriculum on Middle School Mathematics Achievement in Nine States. WestEd., (McKinney, D., Strother, S., Walters, K. & Schneider, S., 2023).

Amplify Desmos Math New York program resources

Student bundle includes:



NY Student Edition,
multivolume, consumable



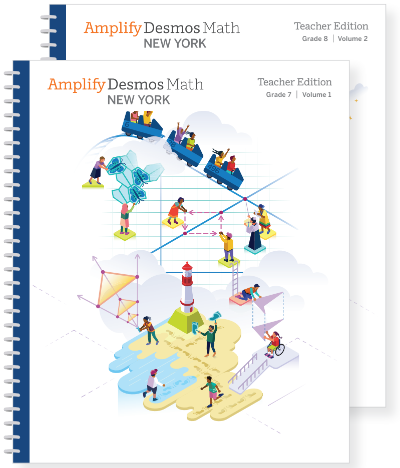
Three Turtles

Press play to watch these turtles travel. Then match each turtle to the line that represents it.

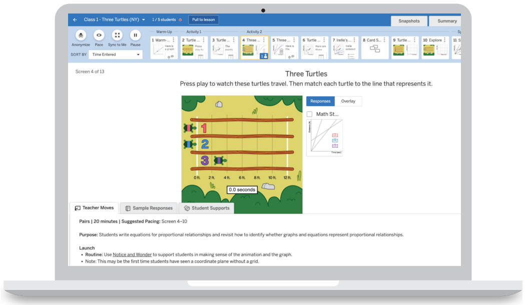
NY Digital Experience (English and Spanish), featuring:

- Interactive Student Activity Screens
- Enriched feedback
- Collaboration tools

Teacher bundle includes:



NY Teacher Edition,
multivolume, spiral-bound



Screen 4 of 13

Three Turtles

Press play to watch these turtles travel. Then match each turtle to the line that represents it.

Pairs | 20 minutes | Suggested Pacing: Screen 4–10

Purpose: Students write equations for proportional relationships and revisit how to identify whether graphs and equations represent proportional relationships.

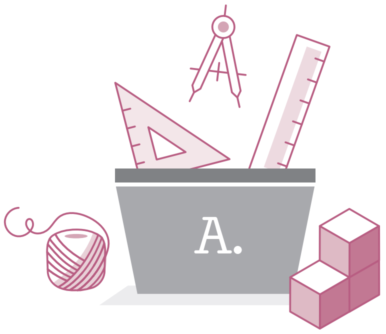
Launch

- **Routine:** Use Notice and Wonder to support students in making sense of the animation and the graph.
- Note: This may be the first time students have seen a coordinate plane without a grid.

NY Digital Experience (English and Spanish), featuring:

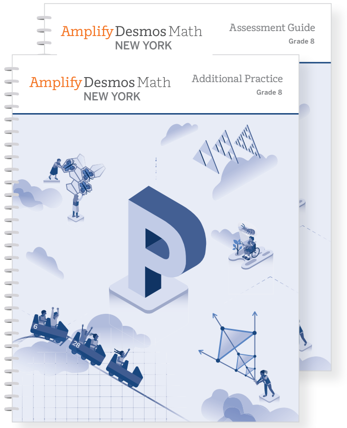
- Facilitation and progress monitoring tools
- Presentation Screens
- Instructional supports
- Assessment

Optional:



Middle School Manipulative Kit (Grades 6–8)

Extra Practice and Assessment Blackline Masters



Additional components and features may roll out over time.

Program architecture

Course

I **mCLASS beginning-of-year diagnostic screener**

I **Interim Assesments**

I		I	I		I		I
UNIT 1	UNIT 2	UNIT 3	UNIT 4	UNIT 5	UNIT 6	UNIT 7	UNIT 8
20 days	16 days	16 days	20 days	22 days	17 days	19 days	20 days

Unit

A **Pre-Unit Check**

A **Sub-Unit Quiz**

End-of-Unit Assessment A

Sub-Unit 1	**Sub-Unit 2** Practice Day	**Sub-Unit 3** Practice Day
1 2 3 4 5 6 7	8 9 10 11 12 PD	13 14 15 16 17 18 19 20 PD

Note: The number of sub-units and lessons vary from unit to unit; this depiction shows the general structure of a unit.

Lesson

Warm-up	Activity 1	Activity 2	Synthesis	Exit Ticket	Practice
5 min	15 min	15 min	5 min	5 min	timing varies
👤 👥 👪 🏫	👤 👥 👪 🏫	👤 👥 👪 🏫	🏫	👤	👤

Note: The number of activities and timing vary from lesson to lesson; this depiction shows the general structure of a lesson.

Key:

👤 Independent	👪 Small Groups
👥 Pairs	🏫 Whole Class

Program Scope and Sequence

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 164 days total	Area and Surface Area 18 Instructional Days 3 Assessment Days 21 days total	Introducing Ratios 17 Instructional Days 3 Assessment Days 20 days total	Unit Rates and Percentages 16 Instructional Days 3 Assessment Days 19 days total	Dividing Fractions 17 Instructional Days 3 Assessment Days 20 days total
Grade 7 158 days total	Scale Drawings 15 Instructional Days 3 Assessment Days 18 days total	Introducing Proportional Relationships 16 Instructional Days 3 Assessment Days 19 days total	Measuring Circles 13 Instructional Days 3 Assessment Days 16 days total	Proportional Relationships and Percentages 17 Instructional Days 3 Assessment Days 20 days total
Grade 8 150 days total	Rigid Transformation and Congruence 16 Instructional Days 4 Assessment Days 20 days total	Dilations, Similarity, and Introducing Slope 12 Instructional Days 4 Assessment Days 16 days total	Proportional and Linear Relationships 13 Instructional Days 3 Assessment Days 16 days total	Linear Equations and Linear Systems 17 Instructional Days 3 Assessment Days 20 days total

Unit 5	Unit 6	Unit 7	Unit 8
Decimal Arithmetic 18 Instructional Days 4 Assessment Days 22 days total	Expressions and Equations 19 Instructional Days 3 Assessment Days 22 days total	Positive and Negative Numbers 15 Instructional Days 3 Assessment Days 18 days total	Describing Data 19 Instructional Days 3 Assessment Days 22 days total
Operations With Positive and Negative Numbers 16 Instructional Days 4 Assessment Days 20 days total	Expressions, Equations, and Inequalities 21 Instructional Days 3 Assessment Days 24 days total	Angles, Triangles, and Prisms 16 Instructional Days 3 Assessment Days 19 days total	Probability and Sampling 19 Instructional Days 3 Assessment Days 22 days total
Functions and Volume 18 Instructional Days 4 Assessment Days 22 days total	**Associations in Data** 15 Instructional Days 2 Assessment Days 17 days total	**Exponents and Scientific Notation** 16 Instructional Days 3 Assessment Days 19 days total	**The Pythagorean Theorem and Irrational Numbers** 17 Instructional Days 3 Assessment Days 20 days total

Unit 1 Rigid Transformations and Congruence

In this unit, students investigate translations, rotations, and reflections, and use these transformations to make informal arguments about congruence. They also explore angle relationships on parallel lines and the triangle sum theorem.

Unit 2 Dilations, Similarity, and Introducing Slope

In this unit, students study dilations and similar figures. They use similar triangles to explain and understand the concept of slope.

Pre-Unit

Sub-Unit 1 Dilations

Sub-Unit 2 Similarity

Sub-Unit 3 Slope

End-Unit

Unit 3 Proportional and Linear Relationships

In this unit, students study linear relationships in both slope-intercept and standard form.

Unit 4 Linear Equations and Linear Systems

In this unit, students solve linear equations with rational coefficients and determine the number of possible solutions. They also solve systems of two linear equations algebraically and by graphing.

Pre-Unit

Sub-Unit 1 Solving Linear Equations

Sub-Unit 2 Systems of Linear Equations

End-Unit

Unit 5 Functions and Volume

In this unit, students learn about functions for the first time, analyze representations of functions, and examine functions in the context of the volume of cylinders, cones, and spheres.

Pre-Unit

Sub-Unit 1 Introduction to Functions

Sub-Unit 2 Representing and Interpreting Functions

Sub-Unit 3 Volume

End-Unit

Unit 6 Associations in Data

Students analyze bivariate data. They use scatter plots and fitted lines to analyze numerical data and two-way tables and use bar graphs and segmented bar graphs to analyze categorical data. This unit builds on students' experience in earlier units with the coordinate plane and linear functions to focus on data in two variables.

Pre-Unit

Sub-Unit 1 Organizing Numerical Data

Sub-Unit 2 Analyzing Numerical Dat

Sub-Unit 3 Categorical Data

End-Unit

Unit 7 Exponents and Scientific Notation

In this unit, students gain fluency with expressions involving exponents, powers of 10, and scientific notation. They also perform operations on numbers written in scientific notation.

Pre-Unit

Sub-Unit 1 Exponent Properties

Sub-Unit 2 Scientific Notation

End-Unit

Unit 8 The Pythagorean Theorem and Irrational Numbers

In this unit, students explore the Pythagorean theorem and different types of numbers (square roots and cube roots, rational and irrational numbers).

New York State Next Generation Mathematics Learning Standards, Grade 8

The following shows the alignment of Amplify Desmos Math to the New York State Next Generation Mathematics Learning Standards for Grade 8 Mathematics.

NY-8.NS	**The Number System**	**Lesson(s)**
Know that there are numbers that are not rational, and approximate them by rational numbers.		
NY-8.NS.1 *CCSS: 8.NS.A.1*	Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion eventually repeats. Know that other numbers that are not rational are called irrational.	8.8.12, 8.8.13
NY-8.NS.2 *CCSS: 8.NS.A.2*	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions.	8.8.01, 8.8.03, 8.8.04, 8.8.05, 8.8 Practice Day 1, 8.8.14
NY-8.EE	**Expressions, Equations, and Inequalities**	
Work with radicals and integer exponents..		
NY-8.EE.1 *CCSS: 8.EE.A.1*	Know and apply the properties of integer exponents to generate equivalent numerical expressions.	8.7.02, 8.7.03, 8.7.04, 8.7.05, 8.7.06, 8.7 Practice Day 1, 8.7.11, 8.7 Practice Day 2
NY-8.EE.2 *CCSS: 8.EE.A.2*	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Know square roots of perfect squares up to 225 and cube roots of perfect cubes up to 125. Know that the square root of a non-perfect square is irrational.	8.8.02, 8.8.03, 8.8.04, 8.8.05, 8.8 Practice Day 1, 8.8.10, 8.8 Practice Day 2, 8.8.14
NY-8.EE.3 *CCSS: 8.EE.A.3*	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.	8.7.07, 8.7.08, 8.7.09, 8.7.11, 8.7.13, 8.7 Practice Day 2
NY-8.EE.4 *CCSS: 8.EE.A.4*	Perform multiplication and division with numbers expressed in scientific notation, including problems where both standard decimal form and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology.	8.7.09, 8.7.10, 8.7.11, 8.7.13, 8.7 Practice Day 2

Understand the connections between proportional relationships, lines, and linear equations.		
NY-8.EE.5 *CCSS: 8.EE.B.5*	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	8.3.02, 8.3.03, 8.3 Practice Day
NY-8.EE.6 *CCSS: 8.EE.B.6*	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	8.2.09, 8.2.10, 8.3.08, 8.3.11
Analyze and solve linear equations and pairs of simultaneous linear equations.		
NY-8.EE.7 *CCSS: 8.EE.C.7*	Solve linear equations in one variable.	8.4.03, 8.4.04, 8.4.05, 8.4.06, 8.4.08, 8.4 Practice Day 1, 8.4 Practice Day 2
NY-8.EE.7a *CCSS: 8.EE.C.7.A*	Recognize when linear equations in one variable have one solution, infinitely many solutions, or no solutions. Give examples and show which of these possibilities is the case by successively transforming the given equation into simpler forms.	8.4.07, 8.4 Practice Day 1
NY-8.EE.7b *CCSS: 8.EE.C.7.B*	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.	8.4.04, 8.4.06, 8.4 Practice Day 1
NY-8.EE.8 *CCSS: 8.EE.C.8*	Analyze and solve pairs of simultaneous linear equations.	8.4.09, 8.4.11, 8.4.12, 8.4.13, 8.4.14, 8.4 Practice Day 2
NY-8.EE.8a *CCSS: 8.EE.C.8.A*	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. Recognize when the system has one solution, no solution, or infinitely many solutions.	8.4.11, 8.4.12, 8.4 Practice Day 2
NY-8.EE.8b *CCSS: 8.EE.C.8.B*	Solve systems of two linear equations in two variables with integer coefficients: graphically, numerically using a table, and algebraically. Solve simple cases by inspection.	8.4.11, 8.4.12, 8.4.13, 8.4.14, 8.4 Practice Day 2
NY-8.EE.8c *CCSS: 8.EE.C.8.C*	Solve real-world and mathematical problems involving systems of two linear equations in two variables with integer coefficients.	8.4.09, 8.4.10, 8.4.11, 8.4.14, 8.4.13, 8.4.14, 8.4 Practice Day 2

New York State Next Generation Mathematics Learning Standards, Grade 8

NY-8.F	**Functions**	**Lesson(s)**
Define, evaluate, and compare functions.		
NY-8.F.1 *CCSS: 8.F.A.1*	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	8.5.02, 8.5.03, 8.5.04, 8.5 Practice Day 1, 8.5 Practice Day 2
NY-8.F.2 *CCSS: 8.F.A.2*	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	8.5.07, 8.5 Practice Day 1, 8.5 Practice Day 2
NY-8.F.3 *CCSS: 8.F.A.2*	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line. Recognize examples of functions that are linear and non-linear.	8.5 Practice Day 1, 8.5.12, 8.5 Practice Day 2
Use functions to model relationships between quantities.		
NY-8.F.4 *CCSS: 8.F.B.4*	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	8.3.05, 8.3 Practice Day, 8.5.05, 8.5.08, 8.5.09, 8.5 Practice Day 1, 8.5 Practice Day 2
NY-8.F.5 *CCSS: 8.F.B.5*	Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described in a real-world context.	8.5.05, 8.5.06, 8.5.09, 8.5 Practice Day 1, 8.5 Practice Day 2
NY-8.G	**Geometry**	
Understand congruence and similarity using physical models, transparencies, or geometry software.		
NY-8.G.1 *CCSS: 8.G.A.1*	Verify experimentally the properties of rotations, reflections, and translations.	8.1.02, 8.1.03, 8.1.04, 8.1.06, 8.1.07, 8.1 Practice Day, 8.1.10, 8.3.06
NY-8.G.1a *CCSS: 8.G.A.1.A*	Verify experimentally lines are mapped to lines, and line segments to line segments of the same length.	8.1.08, 8.1 Practice Day, 8.1.10
NY-8.G.1b *CCSS: 8.G.A.1.B*	Verify experimentally angles are mapped to angles of the same measure.	8.1.08, 8.1 Practice Day, 8.1.10
NY-8.G.1c *CCSS: 8.G.A.1.C*	Verify experimentally parallel lines are mapped to parallel lines.	8.1.10

NY-8.G.2 *CCSS: 8.G.A.2*	Know that a two-dimensional figure is congruent to another if the corresponding angles are congruent and the corresponding sides are congruent. Equivalently, two two-dimensional figures are congruent if one is the image of the other after a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence that maps the congruence between them on the coordinate plane.	8.1.07, 8.1.09, 8.1 Practice Day, 8.2.06 **Alignment note:** NYS adds specific mention of the coordinate plane, which is addressed in 8.1.09.
NY-8.G.3 *CCSS: 8.G.A.3*	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	8.1.05, 8.1.06, 8.1 Practice Day, 8.2.03, 8.2.04
NY-8.G.4 *CCSS: 8.G.A.4*	Know that a two-dimensional figure is similar to another if the corresponding angles are congruent and the corresponding sides are in proportion. Equivalently, two two-dimensional figures are similar if one is the image of the other after a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence that maps the similarity between them on the coordinate plane.	8.2.05, 8.2.06, 8.2.07, 8.2.08, 8.2 Practice Day
NY-8.G.5 *CCSS: 8.G.A.5*	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	8.1.10, 8.1.11, 8.1.12, 8.2.07
Understand and apply the Pythagorean Theorem.		
NY-8.G.6 *CCSS: 8.G.B.6*	Understand a proof of the Pythagorean Theorem and its converse.	8.8.07, 8.8.09
NY-8.G.7 *CCSS: 8.G.B.7*	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	8.8.06, 8.8.07, 8.8.08, 8.8.10, 8.8 Practice Day 2
NY-8.G.8 *CCSS: 8.G.B.8*	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	8.8.11, 8.8 Practice Day 2
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.		
NY-8.G.9 *CCSS: 8.G.C.9*	Given the formulas for the volume of cones, cylinders, and spheres, solve mathematical and real-world problems.	8.5.11, 8.5.12, 8.5.13, 8.5.14, 8.5.15, 8.5 Practice Day 2

New York State Next Generation Mathematics Learning Standards, Grade 8

NY-8.SP	Statistics and Probability	Lesson(s)
Investigate patterns of association in bivariate data.		
NY-8.SP.1 *CCSS: 8.SP.A.1*	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	8.6.01, 8.6.02, 8.6.03, 8.6.04, 8.6 Practice Day 1, 8.6.05, 8.6.06, 8.6.07, 8.6.08, 8.6 Practice Day 2, 8.6 Practice Day 3
NY-8.SP.2 *CCSS: 8.SP.A.2*	Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	8.6.04, 8.6 Practice Day 1, 8.6.05, 8.6.06, 8.6.08, 8.6 Practice Day 2, 8.6 Practice Day 3
NY-8.SP.3 *CCSS: 8.SP.A.3*	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	8.6.06, 8.6 Practice Day 2

The Standards for Mathematical Practice, New York State Next Generation Mathematics Learning Standards, Grade 8

The following shows the alignment of Amplify Desmos Math, Grade 8, to the Standards for Mathematical Practice for the New York State Next Generation Mathematics Learning Standards.

MP1 Make sense of problems and persevere in solving them. ***CCSS: MP1***	**Lesson(s)**
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	8.1.06, 8.2.05, 8.3.11, 8.4.08, 8.5.07, 8.5.09, 8.5.12, 8.7.09, 8.7.12, 8.8.07
MP2 Reason abstractly and quantitatively. ***CCSS: MP2***	
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	8.2.08, 8.3.01, 8.3.05, 8.3.07, 8.3.11, 8.4.02, 8.4.09, 8.4.11, 8.4.12, 8.5.01, 8.5.05, 8.5.07, 8.5.09, 8.6.04, 8.6.06, 8.6.08, 8.7.02, 8.7.11, 8.7.13, 8.8.04, 8.8.10
MP3 Construct viable arguments and critique the reasoning of others. ***CCSS: MP3***	
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	8.1.09, 8.1.10, 8.1.12, 8.2.06, 8.3.02, 8.3.04, 8.4.03, 8.4.04, 8.4.05, 8.4.10, 8.4.14, 8.5.02, 8.5.06, 8.6.05, 8.6.10, 8.6.11, 8.7.03, 8.8.04

The Standards for Mathematical Practice, New York State Next Generation Mathematics Learning Standards, Grade 8

MP4 Model with mathematics. *CSS: MP4*	**Lesson(s)**
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	8.2.08, 8.3.05, 8.4.08, 8.5.06, 8.5.08, 8.5.09, 8.6.04, 8.6.11, 8.7.09, 8.7.13
MP5 Use appropriate tools strategically. *CCSS: MP5*	
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	8.1.01, 8.1.04, 8.1.08, 8.1.09, 8.1.10, 8.5.08, 8.6.02, 8.6.05
MP6 Attend to precision. *CCSS: MP6*	
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	8.1.01, 8.1.02, 8.1.04, 8.1.07, 8.1.09, 8.2.02, 8.2.03, 8.2.04, 8.2.05, 8.2.06, 8.3.02, 8.3.09, 8.4.01, 8.4.10, 8.5.06, 8.6.05, 8.6.06, 8.6.07, 8.7.08, 8.7.09, 8.7.11, 8.7.12, 8.8.02, 8.8.08

MP7 Look for and make use of structure. ***CCSS: MP7***	**Lesson(s)**
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	8.1.03, 8.1.04, 8.1.07, 8.1.08, 8.1.10, 8.1.11, 8.2.03, 8.2.04, 8.3.06, 8.4.04, 8.4.06, 8.4.07, 8.4.13, 8.5.10, 8.5.11, 8.5.14, 8.6.01, 8.6.02, 8.6.04, 8.6.06, 8.6.07, 8.7.02, 8.7.05, 8.7.08, 8.8.03, 8.8.06, 8.8.11, 8.8.12
MP8 Look for and express regularity in repeated reasoning. ***CCSS: MP8***	
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	8.1.05, 8.1.10, 8.2.03, 8.2.07, 8.3.04, 8.3.05, 8.3.08, 8.3.09, 8.4.07, 8.4.12, 8.5.03, 8.5.13, 8.5.14, 8.7.01, 8.7.05

GRADE 8

Amplify Desmos Math
NEW YORK

Teacher Edition Sample Lessons

In this section, two lesson samples showcase the unique design and ease of use of lesson plans found in the Amplify Desmos Math New York Teacher Edition. All Teacher Edition lessons will be created following this structure and design for the 2024-2025 school year.

Contents of this lesson:

Powerful learning experiences with the flexibility you need in the classroom.

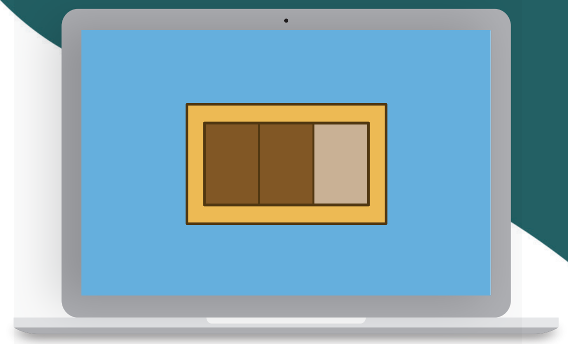
Every lesson in Amplify Desmos Math New York can be taught with students using print while the teacher projects digital Presentation Screens. For lessons that are best taught with students on devices, we make a clear recommendation and provide instructional guidance to support students using digital and on print. The robust collaboration tools, interactive visuals, and enriched feedback of Desmos technology are integral to daily learning as a whole class, pairs, or individuals.

Print and digital resources for every lesson!

Two types of lesson delivery

Digital Lesson RECOMMENDED

Assign Student Activity Screens for this lesson so students can explore visual potting soil models!



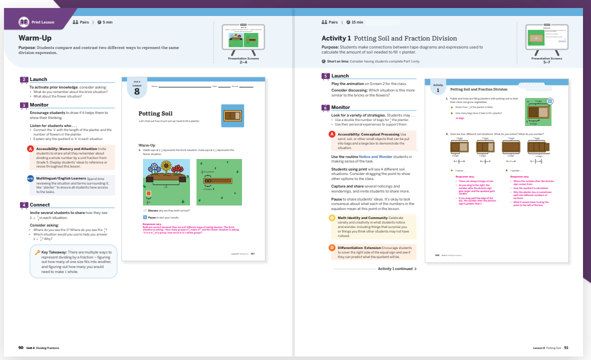
Digital recommended

Lesson goals best learned digitally

- Students use devices and interact with Student Activity Screens.
- Teachers present the Student Activity Screens to facilitate the lesson.
- Closely-aligned student print pages are available for off-line note taking or for students who may need to use print.
- About 75% of lessons

Print Lesson

Assign the Student Edition with Presentation Screens for this lesson.



Print

Lesson goals best learned with pencil-and-paper

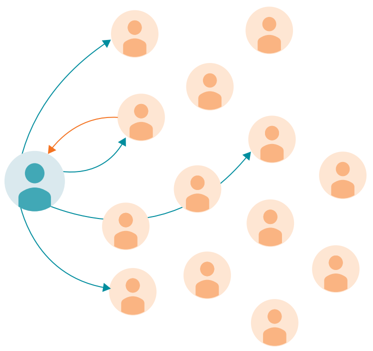
- Students interact with printed Student Edition pages and hands-on manipulatives (when applicable).
- Teachers present Presentation Screens to facilitate the lesson.
- Students must use consumable Student Edition.
- About 25% of lessons

REVIEWER TIP

These print/digital flexibility enhancements can be found in Amplify Desmos Math New York lessons in this section and online, but are not yet available in the partially designed lesson plans.

Activities are designed to provide collaborative learning experiences.

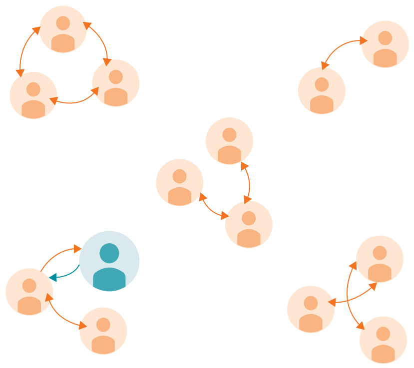
Following a *Warm-Up*, a lesson includes two or more learning activities. All activities in Amplify Desmos Math New York utilize a *Launch, Monitor, Connect* framework to surface student thinking and spark interesting and productive discussions.



1

Launch

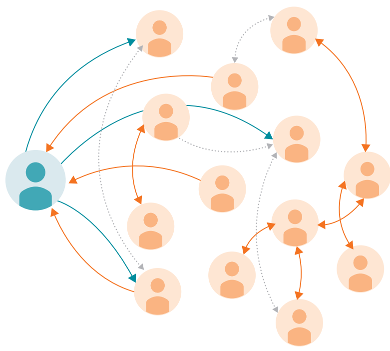
Teachers launch an activity and ensure students understand what's being asked. Launches are designed to ensure all students can access and engage with the problem.



2

Monitor

Students interact with each other to discuss and work out strategies for solving a problem. Teachers can better understand what their students are thinking so that they can choose their next move while students are working.



3

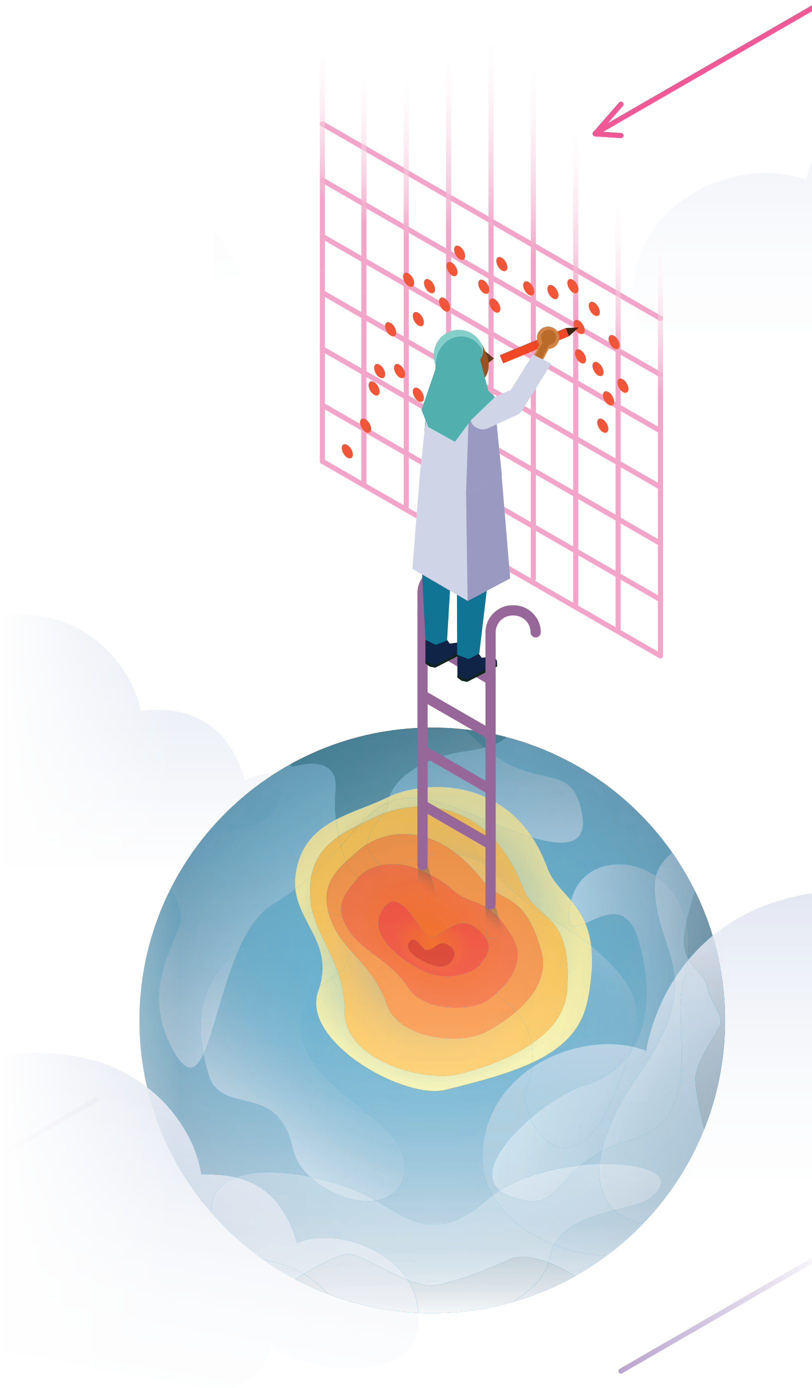
Connect

Students construct viable arguments and critique each other's reasoning.Then, at the end of the activity, they synthesize their learning with the teacher in a moment called the *Key Takeaway*.

Following all activities, each lesson wraps with *Synthesis and Summary* to consolidate thinking and refine strategies across activities. An *Exit Ticket* enables students to share how well they understood the math of the lesson and how they felt about learning that math.

Every moment in the classroom is valuable.

Teachers play an active role as discussion facilitators, monitoring student work in real time, choosing moments to share and discuss, and synthesizing learning.



At Amplify, we want teachers to spend their time focused on their students, rather than preparing instruction and managing materials. Our comprehensive Teacher Edition and intuitive technology is designed with busy educators in mind.

Inside the Amplify Desmos Math New York Teacher Edition, you'll find:

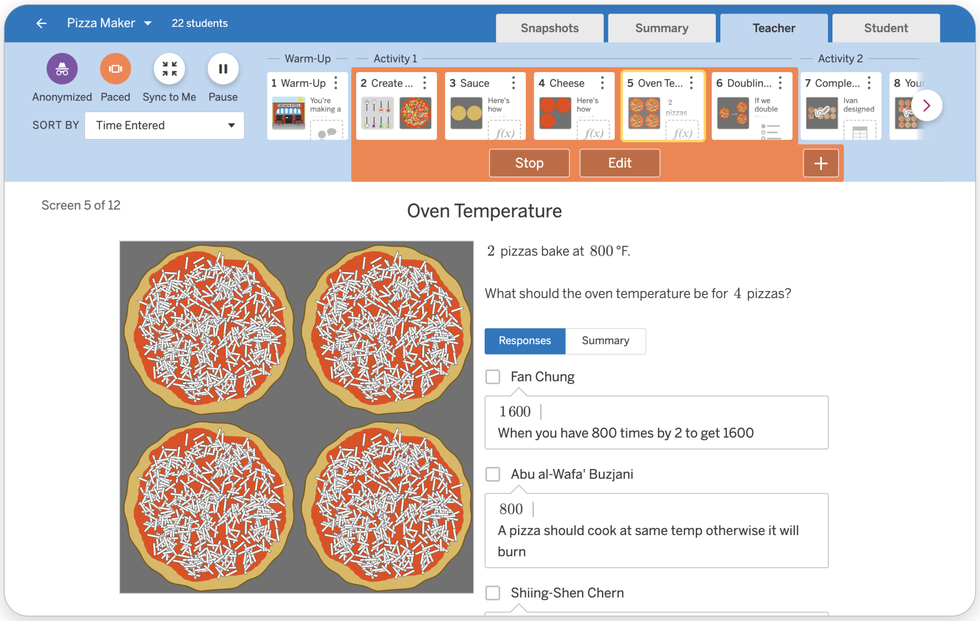
- ***Unit at a Glance*** and ***Lesson at a Glance*** sections to quickly understand what to expect from a unit or lesson.
- ***Focus and Coherence*** information to connect today's goals to prior and future learning.
- A ***Prep Checklist*** to prepare materials for the day's lesson.
- ***Suggested pacing*** to allot the appropriate amount of time for each activity.
- ***Visuals of student pages and screens*** to streamline lesson planning.
- ***Practice problem item analysis*** to easily map learning to Depth of Knowledge (DOK) levels.

REVIEWER TIP

These time-saving enhancements can be found in Amplify Desmos Math New York lessons in this section and online, but are not yet available in the partially designed lesson plans.

Teacher facilitation tools enable dynamic interactions.

The teacher dashboard gives you insight into student thinking in real time, meaning you can select student work to display and discuss quickly and easily, and ask better questions to guide more productive discussions.



Screen 5 of 12

Oven Temperature

2 pizzas bake at 800 °F.

What should the oven temperature be for 4 pizzas?

☐ Fan Chung

1600

When you have 800 times by 2 to get 1600

☐ Abu al-Wafa' Buzjani

800

A pizza should cook at same temp otherwise it will burn

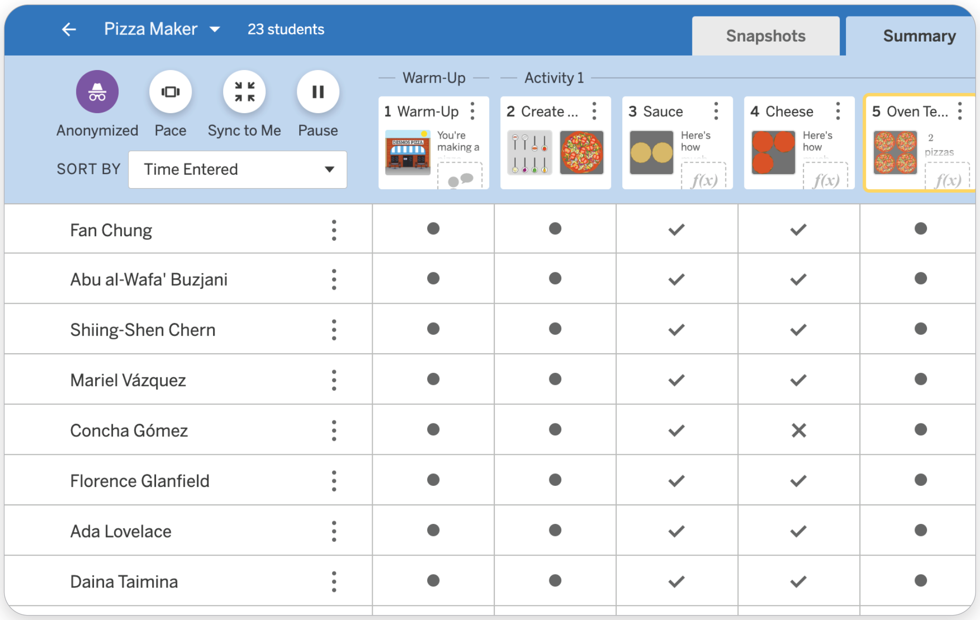
☐ Shiing-Shen Chern

Teacher view and pacing

The Teacher view gives you access to student responses, student-facing content, teacher moves, and sample responses, as well as the ability to pace screens.

Summary view

The Summary view shows you where students are working. If a question is auto-scored it shows how they are doing, and the ability to look at individual student work.



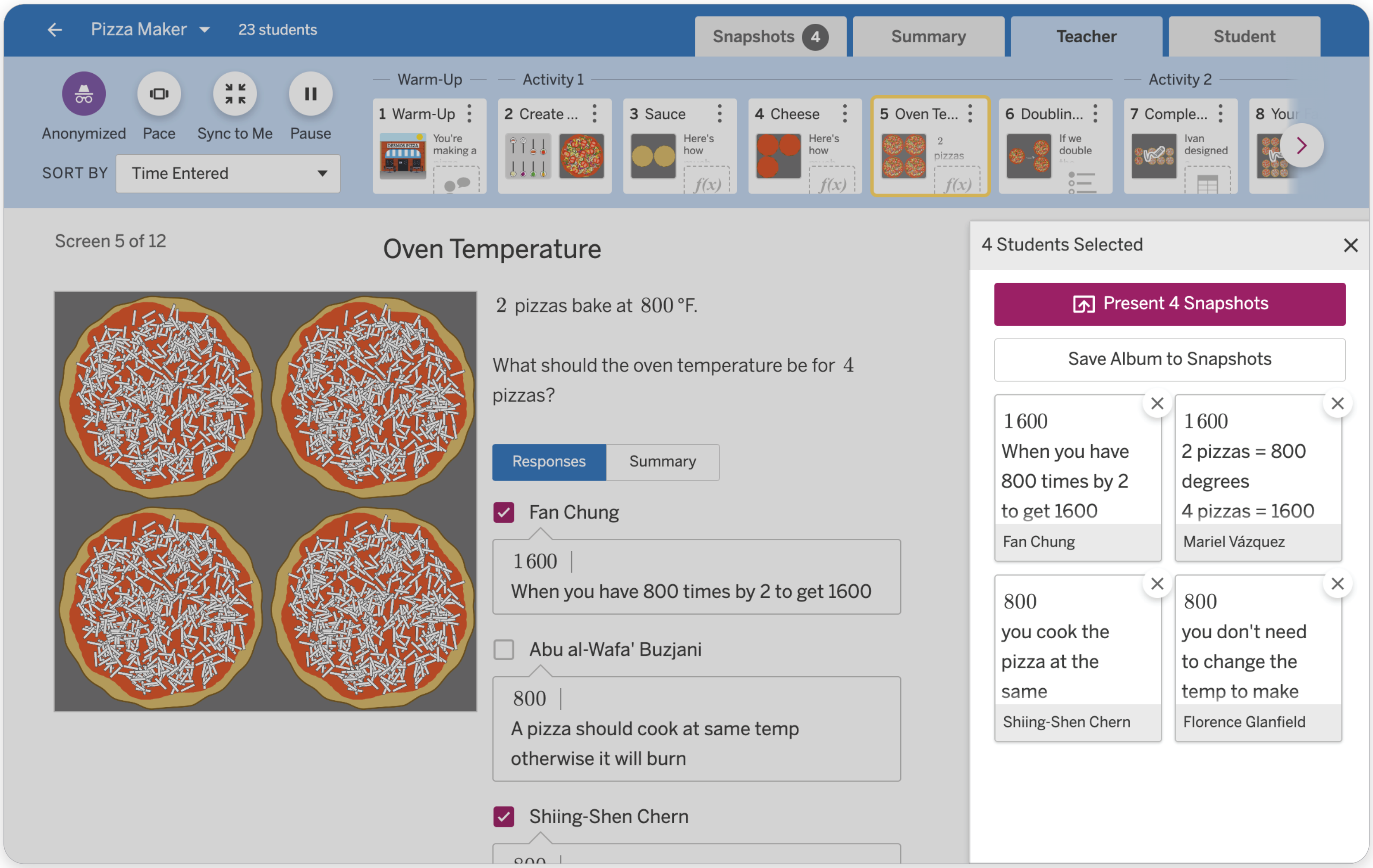
	1 Warm-Up	2 Create ...	3 Sauce	4 Cheese	5 Oven Te...
Fan Chung	●	●	✓	✓	●
Abu al-Wafa' Buzjani	●	●	✓	✓	●
Shiing-Shen Chern	●	●	✓	✓	●
Mariel Vázquez	●	●	✓	✓	●
Concha Gómez	●	●	✓	✕	●
Florence Glanfield	●	●	✓	✓	●
Ada Lovelace	●	●	✓	✓	●
Daina Taimina	●	●	✓	✓	●

TRY IT OUT

Start your review at
amplify.com/math-review-nyc

Snapshots

When you find student work you want to share, you can collect it in your snapshots and then show individual or even groups of students' responses to move the conversation in the direction you want. Names can be anonymized to protect students' identity.



Screen 5 of 12

Oven Temperature

2 pizzas bake at 800 °F.

What should the oven temperature be for 4 pizzas?

☑ Fan Chung

$1\,600$

When you have 800 times by 2 to get 1600

☐ Abu al-Wafa' Buzjani

800

A pizza should cook at same temp otherwise it will burn

☑ Shiing-Shen Chern

4 Students Selected

$1\,600$
When you have 800 times by 2 to get 1600
Fan Chung

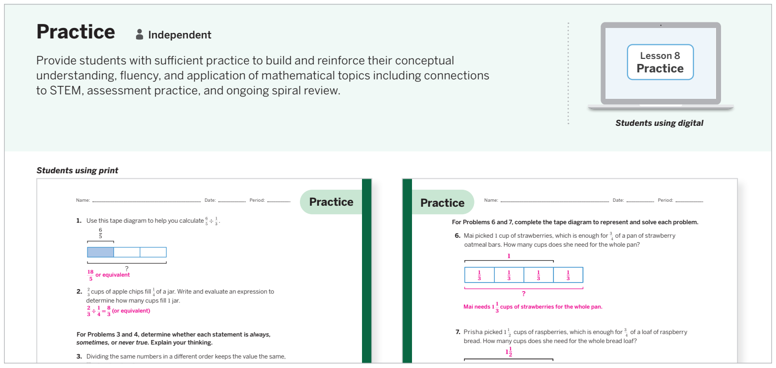
$1\,600$
2 pizzas = 800 degrees
4 pizzas = 1600
Mariel Vázquez

800
you cook the pizza at the same
Shiing-Shen Chern

800
you don't need to change the temp to make
Florence Glanfield

Practice makes progress.

When it comes to cementing new learning into long-term understanding, ample practice opportunities are key. Amplify Desmos Math New York builds practice opportunities into both daily instruction and independent practice.



Practice Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics including connections to STEM, assessment practice, and ongoing spiral review.

Students using digital

Students using print

Name: Date: Period: **Practice**

1. Use this tape diagram to help you calculate $\frac{6}{5} \div \frac{1}{3}$.

$\frac{18}{5}$ or equivalent

2. $\frac{2}{3}$ cups of apple chips fill $\frac{1}{4}$ of a jar. Write and evaluate an expression to determine how many cups fill 1 jar.

$\frac{2}{3} \div \frac{1}{4} = \frac{8}{3}$ (or equivalent)

For Problems 3 and 4, determine whether each statement is *always*, *sometimes*, or *never true*. Explain your thinking.

3. Dividing the same numbers in a different order keeps the value the same.

Practice Name: Date: Period:

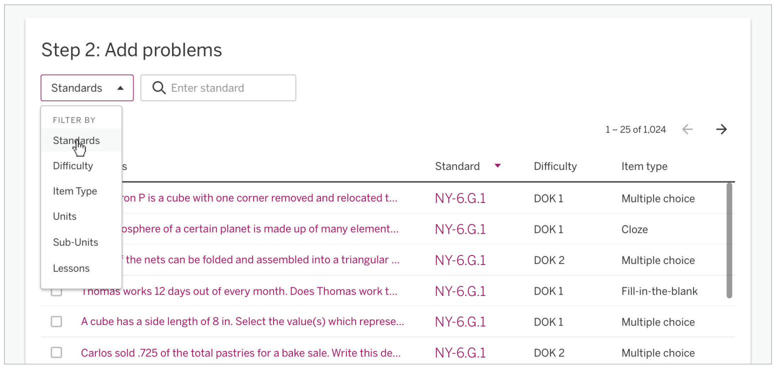
For Problems 6 and 7, complete the tape diagram to represent and solve each problem.

6. Mai picked 1 cup of strawberries, which is enough for $\frac{3}{4}$ of a pan of strawberry oatmeal bars. How many cups does she need for the whole pan?

Mai needs $1\frac{1}{3}$ cups of strawberries for the whole pan.

7. Prisha picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a loaf of raspberry bread. How many cups does she need for the whole bread loaf?

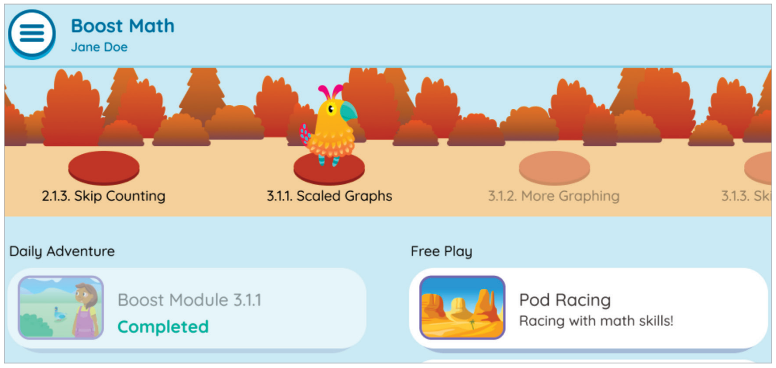
Daily practice problems for the day's lesson are included online and in the Student Edition, including fluency and test practice. This daily practice also includes *Spiral Review* to revisit formerly acquired math learning. A Depth of Knowledge (DOK) table is provided for practice problem item analysis and further insight into how students are doing conceptually.



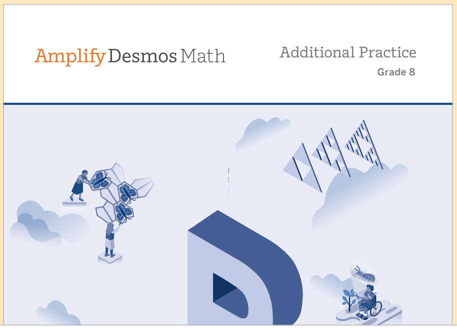
Step 2: Add problems

	s	Standard	Difficulty	Item type
	ron P is a cube with one corner removed and relocated t...	NY-6.G.1	DOK 1	Multiple choice
	osphere of a certain planet is made up of many element...	NY-6.G.1	DOK 1	Cloze
	the nets can be folded and assembled into a triangular ...	NY-6.G.1	DOK 2	Multiple choice
☐	homas works 12 days out of every month. Does Thomas work t...	NY-6.G.1	DOK 1	Fill-in-the-blank
☐	A cube has a side length of 8 in. Select the value(s) which represe...	NY-6.G.1	DOK 1	Multiple choice
☐	Carlos sold .725 of the total pastries for a bake sale. Write this de...	NY-6.G.1	DOK 2	Multiple choice

An **online item bank** contains additional practice sets, or teachers can customize their own based on unit or sub-unit concepts and standards.



Boost Tutored Practice offers engaging, digital independent practice for students that provides access to grade-level math through personalized feedback that responds to student work to support their learning.



Additional Practice Blackline Masters contain additional practice problems to further address fluency, spiral review, and a variety of DOK questions in lesson learning, supporting differentiated practice based on the needs of students.

REVIEWER TIP

These practice enhancements can be found in Amplify Desmos Math New York lessons in this section, online, and in the Student Edition sampler, but are not yet available in the partially designed lesson plans.

In-the-moment instructional supports help teachers meet the needs of every learner.

Embedded instructional supports provide practical guidance for scaffolding or extending learning for all students using an asset-based approach.

D Differentiation

Provides a lens with which to anticipate, view, and guide individual student work, including *Extensions* and *Differentiation Support* guidance. In addition, robust recommendations to *Support, Strengthen, and Stretch* are provided at the unit level.

A Accessibility

Promotes main areas of cognitive functioning, including memory and attention, conceptual processing, visual-spatial processing, executive functioning, fine motor skills, and affective functioning.

ML/EL Multilingual / English Learners

Provides math language development supports to help all students achieve the *Language Goal* of the lesson.

Math identity and Community

Highlights opportunities to recognize and celebrate the brilliance from all students.

Boost mini lessons

Offer teacher-led small group assistance to students who need more direct and explicit support to re-engage with grade-level math.

This just-in-time instruction is informed by assessment data such as pre-unit and sub-unit quizzes.

REVIEWER TIP

In-the-moment instructional supports are included in Amplify Desmos Math New York lessons in this section and online, but most are not yet available in the partially designed lesson plans.

Unit 4

Lesson

4

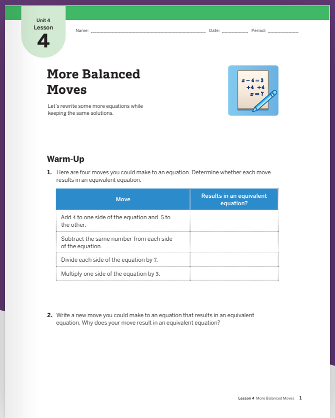
More Balanced Moves

Solving Linear Equations, Part 1

Let’s rewrite some more equations while keeping the same solutions.

Print Lesson

This is a print lesson with Presentation Screens.



Unit 4
Lesson
4

Name: ______ Date: ______ Period: ______

More Balanced Moves

Let’s rewrite some more equations while keeping the same solutions.

Warm-Up

1. Here are four moves you could make to an equation. Determine whether each move results in an equivalent equation.

Move	Results in an equivalent equation?
Add 4 to one side of the equation and 5 to the other.	
Subtract the same number from each side of the equation.	
Divide each side of the equation by 7.	
Multiply one side of the equation by 3.	

2. Write a new move you could make to an equation that results in an equivalent equation. Why does your move result in an equivalent equation?

Lesson 4 More Balanced Moves 1

Focus and Coherence

Today’s Goals

1. **Goal:** Solve a linear equation in one variable.
2. **Language Goal:** Analyze strategies for solving a linear equation in one variable. **(Reading, Writing, Speaking, and Listening)**

Students continue to reinforce the connections of three fundamental ideas: a solution to an equation is a value that makes the equation true, performing the same operation on each side of an equation results in an equivalent equation, and two equations related by such a move have the same **solutions**. Students use the structure of an equation to determine possible next steps as they practice solving linear equations with variables on both sides. **(MP7)**

Prior Learning

In Lessons 2 and 3, students used hanger diagrams to gain a conceptual basis for solving linear equations.

Future Learning

In Lessons 5 and 6, students will continue to practice solving linear equations in one variable. In Lesson 7, they will analyze equations with no solutions or infinitely many solutions.

Rigor and Balance

- Students continue to develop **procedural fluency** in solving equations with variables on both sides.

Vocabulary

New Vocabulary

solution

Standards

Addressing

NY-8.EE.7b

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Also Addressing: **NY-8.EE.7**

Mathematical Practices: MP3, MP7

Building On	Building Toward
NY-7.EE.4	NY-8.EE.7

Amplify Desmos Math NEW YORK
Lesson Sample

Lesson at a Glance

~ 45 min

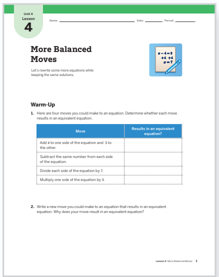
Print Lesson

Standards: NY-8.EE.7, NY-8.EE.7b

Warm-Up

Pairs | 5 min

Students use the Think-Pair-Share routine while identifying moves that result in an equivalent equation to continue their work with solving equations. **(MP3)**



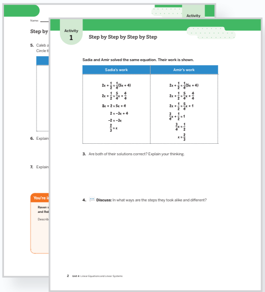
Lesson 4
Presentation Screens

Presentation Screen
1

Activity 1

Pairs | 15 min

Students use MLR7: Compare and Connect when examining the solutions of others and describing potential errors as they build fluency in solving equations. Decide and Defend **(MP3)**



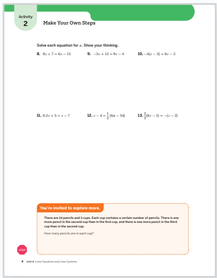
Lesson 4
Presentation Screens

Presentation Screens
2–4

Activity 2

Pairs | 15 min

Students practice solving linear equations with variables on both sides and with rational coefficients to increase fluency in solving linear equations. MLR2: Collect and Display **(MP7)**



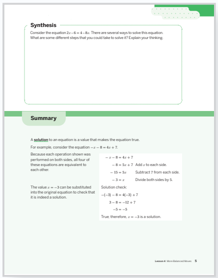
Lesson 4
Presentation Screens

Presentation Screens
5–6

Synthesis

Whole Class | 5 min

There are many ways to approach solving linear equations with variables on both sides, as long as each step results in an equivalent equation. MLR2: Collect and Display



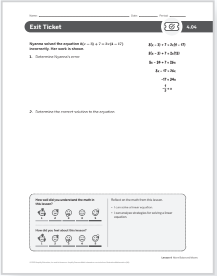
Lesson 4
Presentation Screens

Presentation Screen
7

Exit Ticket

Independent | 5 min

Students demonstrate their understanding by analyzing an incorrect solution to a linear equation and determining the correct solution.



Lesson 4
Presentation Screens

Presentation Screen
8

Prep Checklist

Assign the print lesson and prepare the additional materials. Display the Presentation Screens.

This lesson includes:

Student Edition

Exit Ticket PDF

Coloring tools (as needed)

Warm-Up

Purpose: Students use the Think-Pair-Share routine while identifying moves that result in an equivalent equation to continue their work with solving equations.

Lesson 4 Presentation Screens

Presentation Screen 1

Launch

Use the Think-Pair-Share routine to help students make sense of the task.

Connect

Display each move from the Warm-Up one at a time.

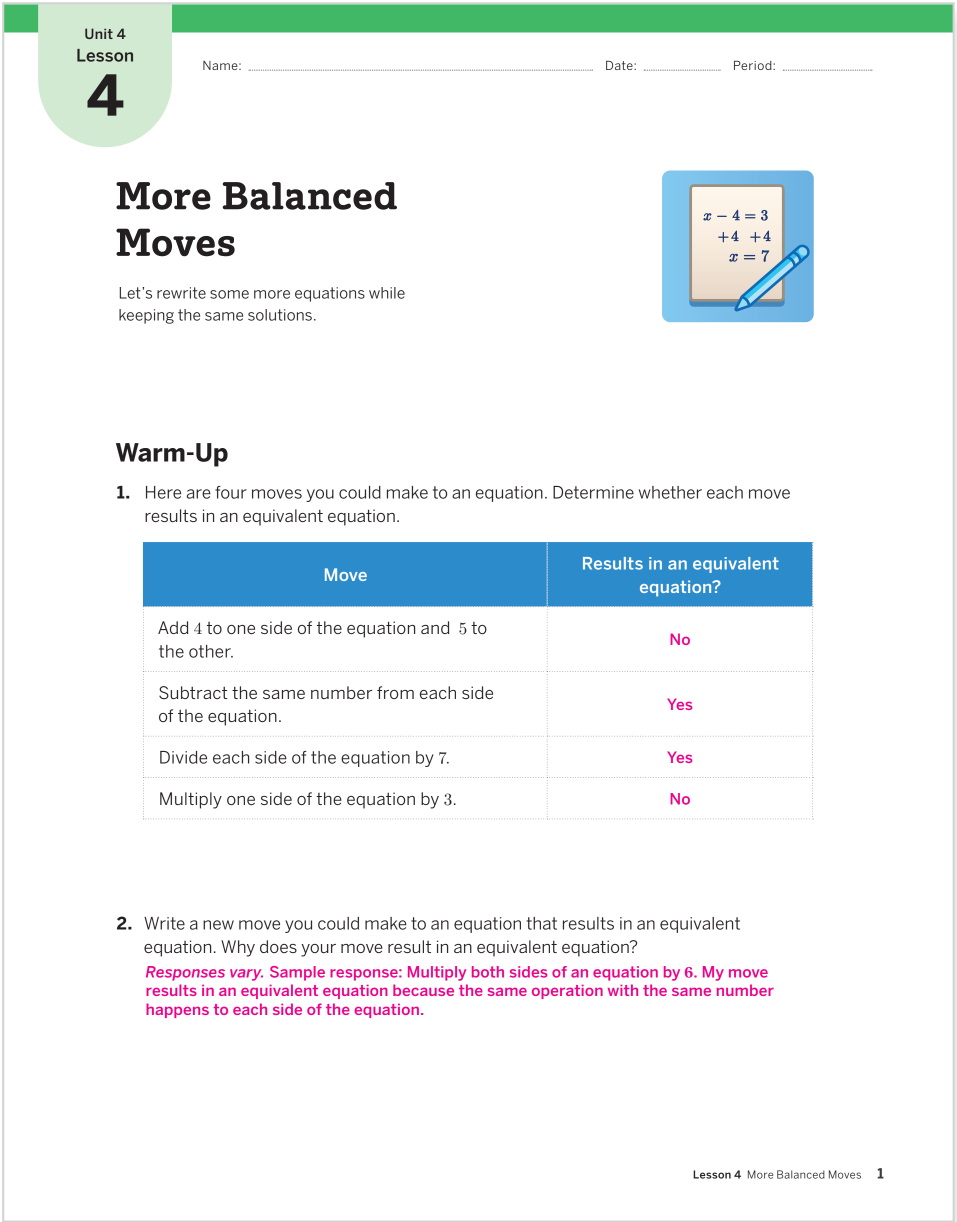
Invite students to share their responses and to critique the reasoning of others. **(MP3)**

Math Identity and Community Invite students to notice and celebrate the variety of mathematical thinking during an activity.

Consider asking:

- "When making a move to an equation, how do you know if it results in an equivalent equation?"
- "Why did the first move in the table not result in an equivalent equation when it involved addition on both sides of the equation?" Amplify responses that recognize the same number was not added to both sides of the equation.

Emphasize that when creating equivalent equations, the operation performed on the left side of the equation must also be performed on the right side.



Unit 4
Lesson
4

Name: ______ Date: ______ Period: ______

More Balanced Moves

Let's rewrite some more equations while keeping the same solutions.

Warm-Up

1. Here are four moves you could make to an equation. Determine whether each move results in an equivalent equation.

Move	Results in an equivalent equation?
Add 4 to one side of the equation and 5 to the other.	No
Subtract the same number from each side of the equation.	Yes
Divide each side of the equation by 7.	Yes
Multiply one side of the equation by 3.	No

2. Write a new move you could make to an equation that results in an equivalent equation. Why does your move result in an equivalent equation?

Responses vary. Sample response: Multiply both sides of an equation by 6. My move results in an equivalent equation because the same operation with the same number happens to each side of the equation.

Lesson 4 More Balanced Moves 1

Pairs | 15 min

Activity 1 Step by Step by Step by Step

Purpose: Students use MLR7: Compare and Connect when examining the solutions of others and describing potential errors as they build fluency in solving equations.

Short on time: Consider completing this activity as a whole class.

Lesson 4
Presentation Screens

Presentation Screens
2–4

2 Launch

Display the equation on the Teacher Presentation Screen and ask students, "What is a first step you might take to solve this equation?"

2-4 Monitor

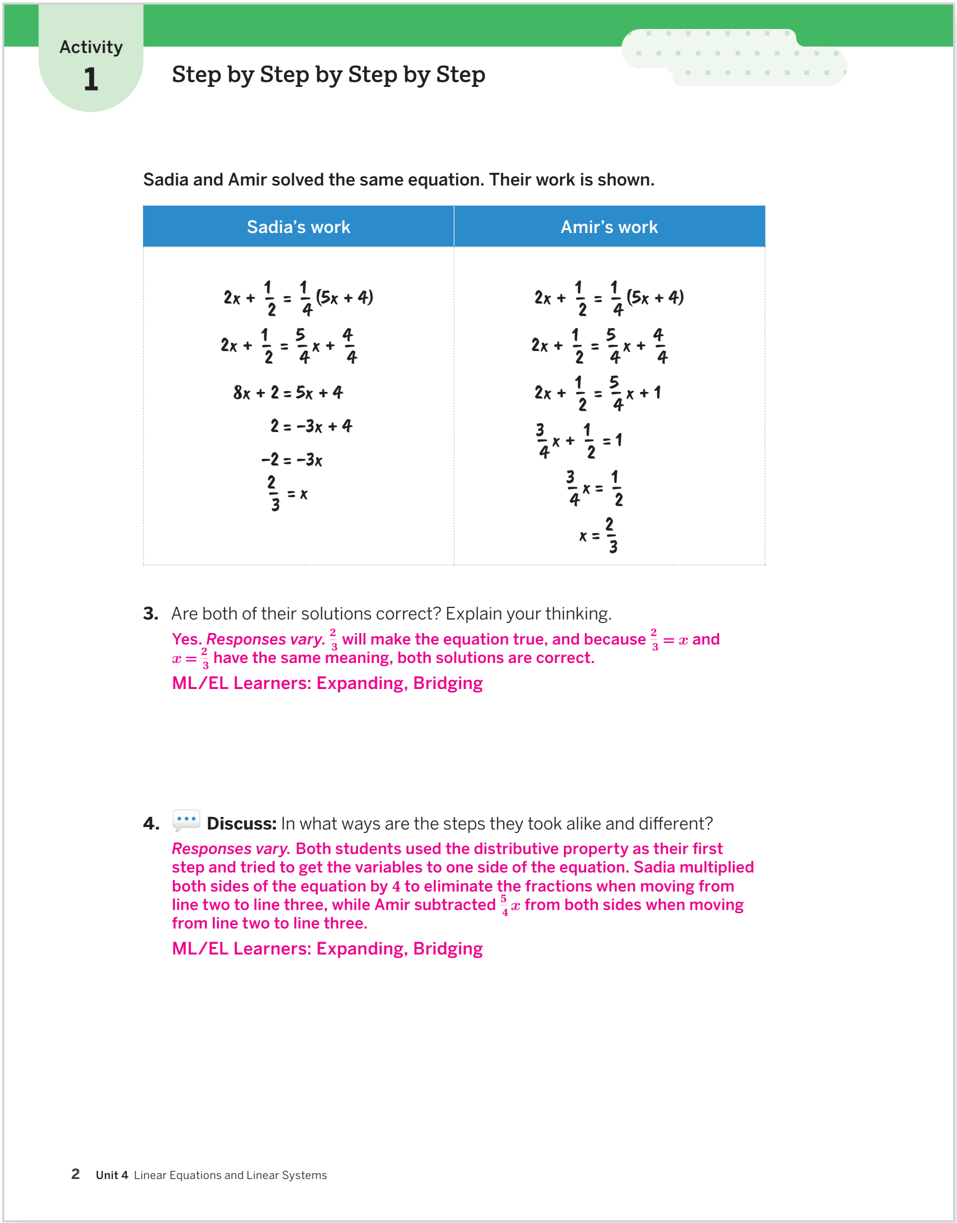
Support getting started by asking, "How can you check whether a solution is correct?"

Pause to remind students that when solving an equation, they can rewrite the solution so that the variable is on the left side or right side, so $\frac{2}{3} = x$ is the same as $x = \frac{2}{3}$.

D Differentiation

Look for students who . . .	Teacher Moves
Think that the second step in Caleb's work is correct because he subtracted $5x$ from each side. *(Screen 4)*	**Support:** Ask "What do the parentheses represent on the right side of the original equation?"
Think that the fourth step in Roberto's work is correct because he added $5x$ to each side. *(Screen 4)*	**Support:** Ask, "If $5x$ is added to the right side, what is $5x + 5x$?"
Substitute $\frac{2}{3}$ into the original equations to determine they are both true. *(Screen 4)*	Invite students to share this strategy during the Connect.
Would benefit from a challenge during this activity.	**Extension:** ***You're invited to explore more.*** Encourage students to share responses with each other in place of a whole class discussion.

Activity 1 continued >



Activity
1

Step by Step by Step by Step

Sadia and Amir solved the same equation. Their work is shown.

Sadia's work	Amir's work
$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$	$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$
$2x + \frac{1}{2} = \frac{5}{4}x + \frac{4}{4}$	$2x + \frac{1}{2} = \frac{5}{4}x + \frac{4}{4}$
$8x + 2 = 5x + 4$	$2x + \frac{1}{2} = \frac{5}{4}x + 1$
$2 = -3x + 4$	$\frac{3}{4}x + \frac{1}{2} = 1$
$-2 = -3x$	$\frac{3}{4}x = \frac{1}{2}$
$\frac{2}{3} = x$	$x = \frac{2}{3}$

3. Are both of their solutions correct? Explain your thinking.

Yes. *Responses vary.* $\frac{2}{3}$ will make the equation true, and because $\frac{2}{3} = x$ and $x = \frac{2}{3}$ have the same meaning, both solutions are correct.

ML/EL Learners: Expanding, Bridging

4. **Discuss:** In what ways are the steps they took alike and different?

Responses vary. Both students used the distributive property as their first step and tried to get the variables to one side of the equation. Sadia multiplied both sides of the equation by 4 to eliminate the fractions when moving from line two to line three, while Amir subtracted $\frac{5}{4}x$ from both sides when moving from line two to line three.

ML/EL Learners: Expanding, Bridging

Print Lesson Pairs | 15 min

Activity 1 Step by Step by Step by Step (continued)

Purpose: Students use MLR7: Compare and Connect when examining the solutions of others and describing potential errors as they build fluency in solving equations.

Short on time: Consider completing this activity as a whole class.

Lesson 4
Presentation Screens

Presentation Screens
2–4

2–4 Monitor

M/EL **Multilingual/English Learners** Encourage students to refer to and use language from the class display to support them as they use precise language when explaining their thinking. **(Reading, Speaking, and Listening)**

A **Accessibility: Conceptual Processing** Provide access to coloring tools and invite students to annotate each step with the reasoning they used. For example, in the second step of Sadia's work, students may write "Distributive Property."

3–4 Connect

Display each problem as it is being discussed.

Invite students to share their responses to each problem. Sequence responses by starting with Sadia's and Amir's solutions first.

Collectively define the term **solution**.

MLR **MLR7: Compare and Connect** Invite students to compare their responses to each of the problems as they are discussed. Consider using these questions to guide their discussion:

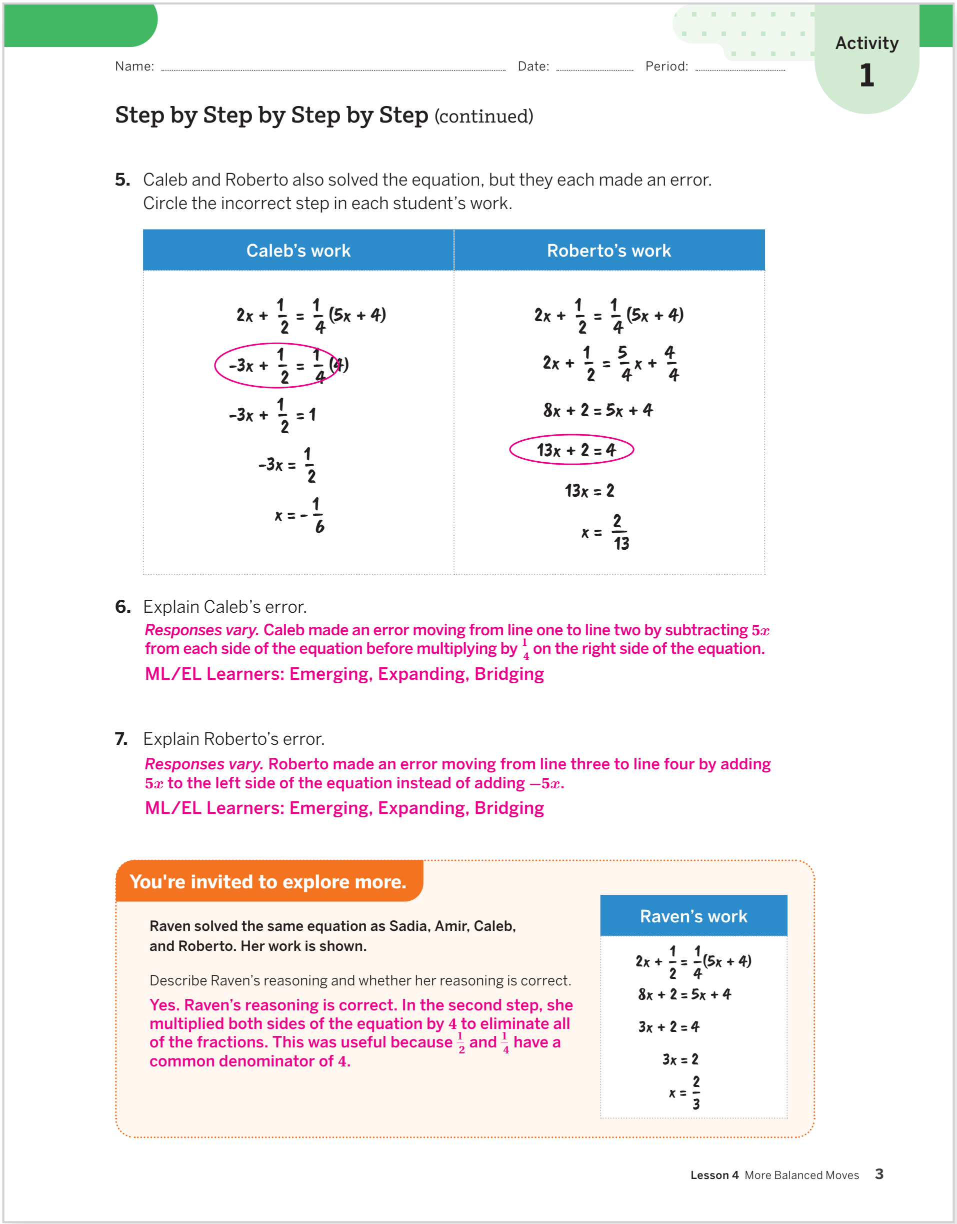
- "How were the ways you and your partner determined whether both Sadia's and Amir's solutions were correct alike? How were they different?"
- "Why does one approach use multiplication and the other does not?"
- "Why did Sadia's and Amir's different strategies lead to the same solution?"

Emphasize that neither solution path is better than the other. There are multiple ways to solve for x.

Use the Decide and Defend **routine** to discuss Caleb's and Roberto's solutions. **(MP3)**

Consider asking, "What advice would you give Caleb and Roberto for checking their work in the future?"

Key Takeaway: A solution is a value that makes an equation true. Although Sadia's and Amir's final steps may look different, their previous steps worked to decrease the total number of terms until only an x term and a number remained on either side of the equal sign.



Name: ________ Date: ________ Period: ________

Activity
1

Step by Step by Step by Step (continued)

5. Caleb and Roberto also solved the equation, but they each made an error. Circle the incorrect step in each student's work.

Caleb's work	Roberto's work
$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$	$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$
$-3x + \frac{1}{2} = \frac{1}{4}(4)$	$2x + \frac{1}{2} = \frac{5}{4}x + \frac{4}{4}$
$-3x + \frac{1}{2} = 1$	$8x + 2 = 5x + 4$
$-3x = \frac{1}{2}$	$13x + 2 = 4$
$x = -\frac{1}{6}$	$13x = 2$
	$x = \frac{2}{13}$

6. Explain Caleb's error.

Responses vary. Caleb made an error moving from line one to line two by subtracting $5x$ from each side of the equation before multiplying by $\frac{1}{4}$ on the right side of the equation.

ML/EL Learners: Emerging, Expanding, Bridging

7. Explain Roberto's error.

Responses vary. Roberto made an error moving from line three to line four by adding $5x$ to the left side of the equation instead of adding $-5x$.

ML/EL Learners: Emerging, Expanding, Bridging

You're invited to explore more.

Raven solved the same equation as Sadia, Amir, Caleb, and Roberto. Her work is shown.

Describe Raven's reasoning and whether her reasoning is correct.

Yes. Raven's reasoning is correct. In the second step, she multiplied both sides of the equation by 4 to eliminate all of the fractions. This was useful because $\frac{1}{2}$ and $\frac{1}{4}$ have a common denominator of 4.

Raven's work
$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$
$8x + 2 = 5x + 4$
$3x + 2 = 4$
$3x = 2$
$x = \frac{2}{3}$

Lesson 4 More Balanced Moves 3

Pairs | 15 min

Activity 2 Make Your Own Steps

Purpose: Students practice solving linear equations with variables on both sides and with rational coefficients to increase fluency in solving linear equations.

Short on time: Have students choose two problems to complete and assign the remaining problems as additional practice.

Lesson 4
Presentation Screens

Presentation Screens
5–6

5 Launch

Demonstrate how to solve Problem 8.

6 Monitor

Support getting started by telling students that although they may take different steps, as long as each step results in an equivalent equation, they will reach the correct solution.

D Differentiation

Look for students who:	Teacher Moves
Multiply by the first number inside the parentheses, but not the second, when distributing.	Ask, "What does it mean for the parentheses to be around both terms?"
Would benefit from a challenge during this activity.	**Extension:** ***You're invited to explore more.*** Encourage students to compare their solutions and strategies.

6 Connect

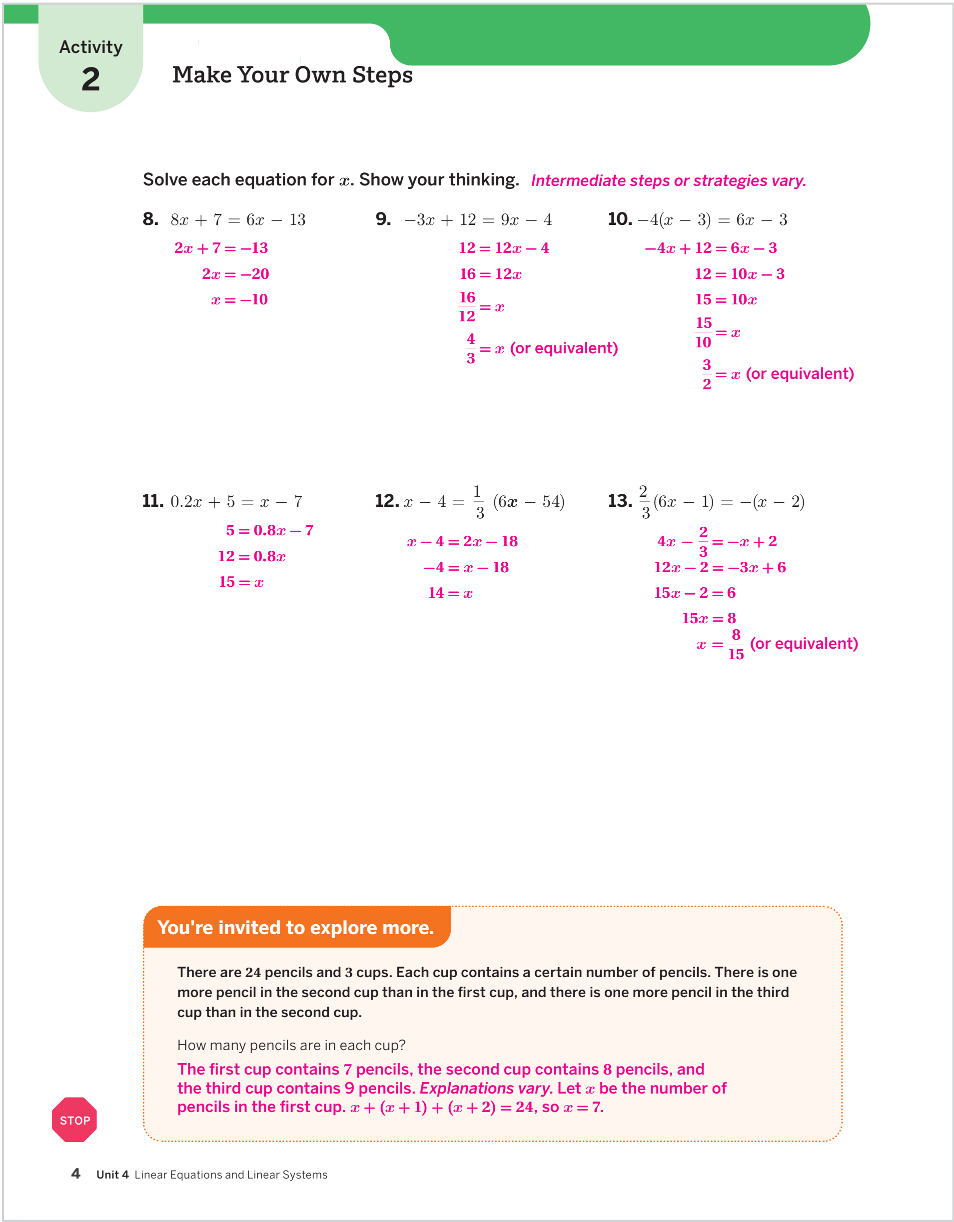
Display Problems 8–10 and invite students to share their solution methods for each problem.

MLR **MLR2: Collect and Display** As students share their solutions:

- Collect the language used to describe their solution methods and record different solution paths that led to the same result. Consider grouping words and phrases used for each step in different areas of the visual display. Amplify phrases, such as: "Multiply first," "Divide first," "Subtract from the right," and "Add from the left."
- Display an example of one equation solved two different ways and annotate each using the language of your students to describe the solution methods. Encourage students to continue using language from the class display during group discussions to support their use of mathematical language while explaining solution pathways.

Emphasize that the structure of an equation can be helpful as students think about different ways to approach solving it. To check the solution, substitute the value of the solution into the original equation. If the resulting equation is true, the solution is correct. **(MP7)**

Consider asking, "In Problem 8, which step do you prefer: subtracting $8x$ or $6x$ from both sides? Why?"



Activity
2

Make Your Own Steps

Solve each equation for x. Show your thinking. *Intermediate steps or strategies vary.*

8. $8x + 7 = 6x - 13$

$2x + 7 = -13$

$2x = -20$

$x = -10$

9. $-3x + 12 = 9x - 4$

$12 = 12x - 4$

$16 = 12x$

$\frac{16}{12} = x$

$\frac{4}{3} = x$ (or equivalent)

10. $-4(x - 3) = 6x - 3$

$-4x + 12 = 6x - 3$

$12 = 10x - 3$

$15 = 10x$

$\frac{15}{10} = x$

$\frac{3}{2} = x$ (or equivalent)

11. $0.2x + 5 = x - 7$

$5 = 0.8x - 7$

$12 = 0.8x$

$15 = x$

12. $x - 4 = \frac{1}{3}(6x - 54)$

$x - 4 = 2x - 18$

$-4 = x - 18$

$14 = x$

13. $\frac{2}{3}(6x - 1) = -(x - 2)$

$4x - \frac{2}{3} = -x + 2$

$12x - 2 = -3x + 6$

$15x - 2 = 6$

$15x = 8$

$x = \frac{8}{15}$ (or equivalent)

You're invited to explore more.

There are 24 pencils and 3 cups. Each cup contains a certain number of pencils. There is one more pencil in the second cup than in the first cup, and there is one more pencil in the third cup than in the second cup.

How many pencils are in each cup?

The first cup contains 7 pencils, the second cup contains 8 pencils, and the third cup contains 9 pencils. *Explanations vary.* Let x be the number of pencils in the first cup. $x + (x + 1) + (x + 2) = 24$, so $x = 7$.

STOP

4 **Unit 4** Linear Equations and Linear Systems

Synthesis

Purpose: There are many ways to approach solving linear equations with variables on both sides, as long as each step results in an equivalent equation.

Lesson 4 Presentation Screens

Presentation Screen 7

7 Synthesis

Invite students to respond independently, and then share their thinking with a partner.

Display several students' responses. Invite students to share the connections they see between responses.

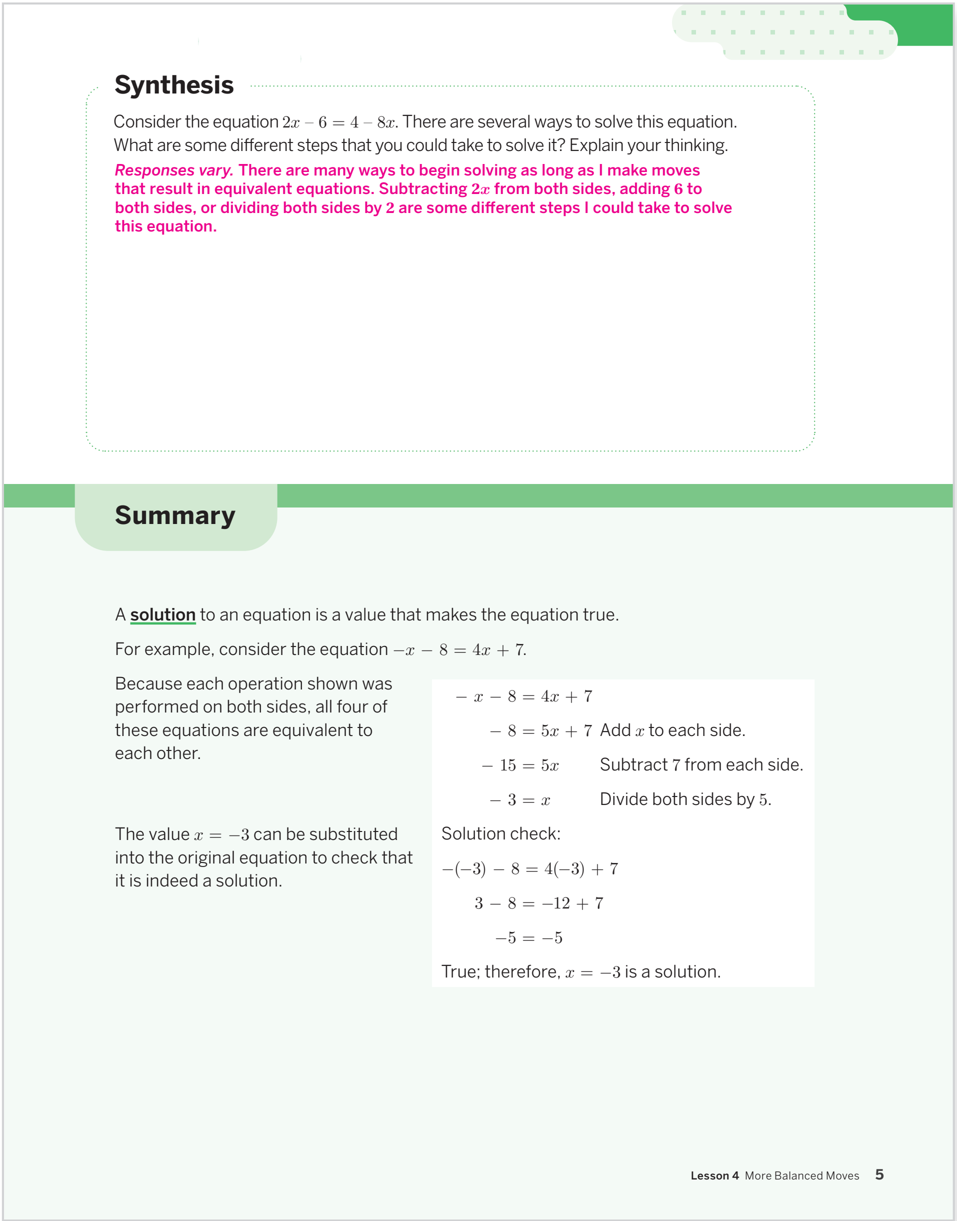
MLR **Use the MLR2: Collect and Display routine** to formalize the definition of **solution** using students' language. Encourage students to refer to the class display for this unit. Add any words, phrases, or diagrams related to the term *solution*.

Emphasize that students have choices to make when solving equations. They can expect to become more fluent with equation solving as they practice and become more familiar with moves they can take that result in equivalent equations.

Lesson Takeaway: Different solution strategies can be used to solve the same equation. Using the structure of an equation can be helpful in determining effective steps to solving the equation.

Summary

Share the Summary. Students can refer back to this throughout the unit and course.



Synthesis

Consider the equation $2x - 6 = 4 - 8x$. There are several ways to solve this equation. What are some different steps that you could take to solve it? Explain your thinking.

Responses vary. **There are many ways to begin solving as long as I make moves that result in equivalent equations. Subtracting $2x$ from both sides, adding 6 to both sides, or dividing both sides by 2 are some different steps I could take to solve this equation.**

Summary

A **solution** to an equation is a value that makes the equation true.

For example, consider the equation $-x - 8 = 4x + 7$.

Because each operation shown was performed on both sides, all four of these equations are equivalent to each other.

$-x - 8 = 4x + 7$

$-8 = 5x + 7$ Add x to each side.

$-15 = 5x$ Subtract 7 from each side.

$-3 = x$ Divide both sides by 5.

The value $x = -3$ can be substituted into the original equation to check that it is indeed a solution.

Solution check:

$-(-3) - 8 = 4(-3) + 7$

$3 - 8 = -12 + 7$

$-5 = -5$

True; therefore, $x = -3$ is a solution.

Lesson 4 More Balanced Moves 5

Independent | 5 min

Exit Ticket

Purpose: Students demonstrate their understanding by analyzing an incorrect solution to a linear equation and determining the correct solution.

Lesson 4
Presentation Screens

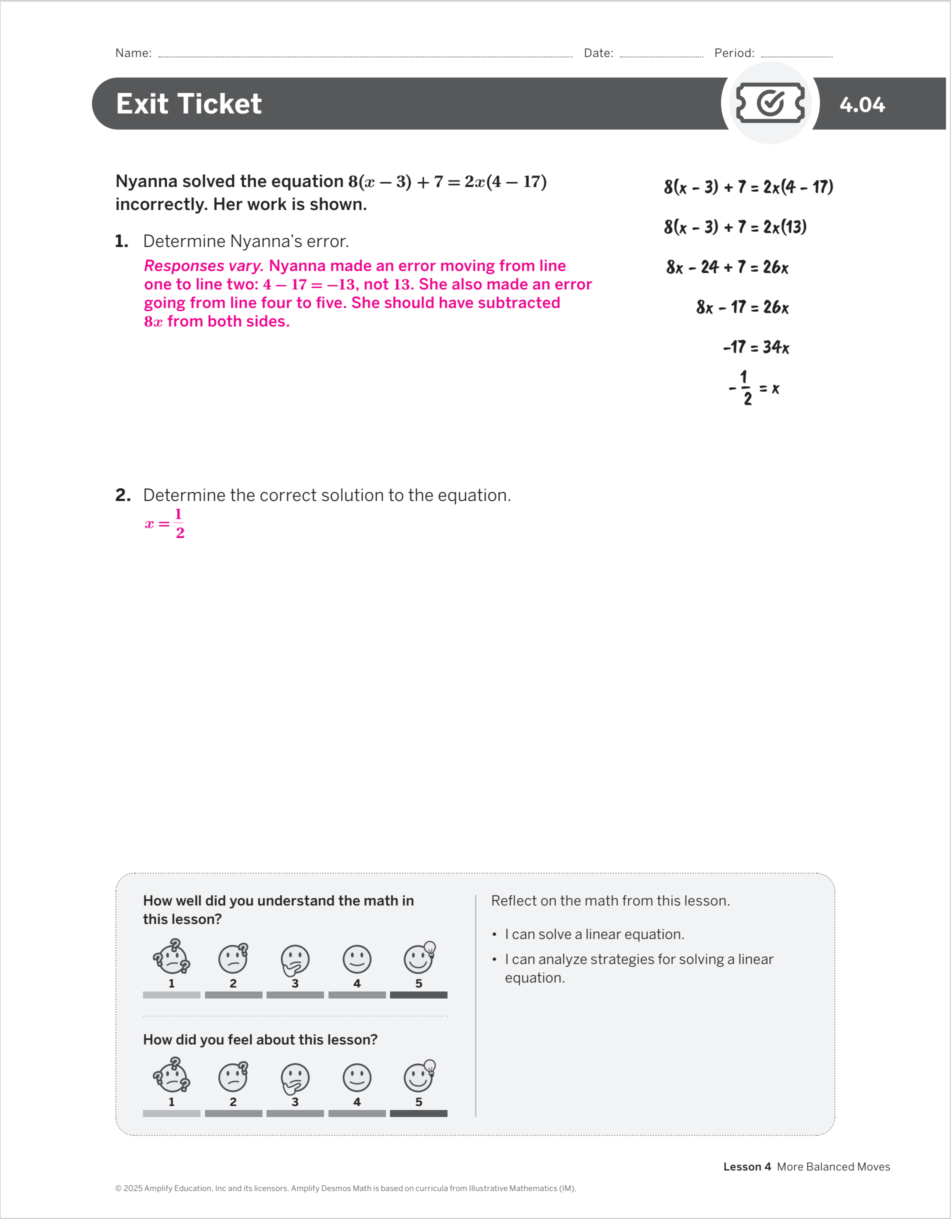
Presentation Screen
8

8 Today's Goals

Goal: Solve a linear equation in one variable.

Language Goal: Analyze strategies for solving a linear equation in one variable. **(Reading, Writing, Speaking, and Listening)**

Support for Future Learning: If students struggle with solving the equation, plan to emphasize this when opportunities arise over the next several lessons. The Exit Ticket in Lesson 5 provides a similar error analysis opportunity.



Name: .... Date: .... Period: ....

Exit Ticket 4.04

Nyanna solved the equation $8(x - 3) + 7 = 2x(4 - 17)$ incorrectly. Her work is shown.

$8(x - 3) + 7 = 2x(4 - 17)$

$8(x - 3) + 7 = 2x(13)$

$8x - 24 + 7 = 26x$

$8x - 17 = 26x$

$-17 = 34x$

$-\frac{1}{2} = x$

1. Determine Nyanna's error.

   Responses vary. Nyanna made an error moving from line one to line two: $4 - 17 = -13$, not 13. She also made an error going from line four to five. She should have subtracted $8x$ from both sides.

2. Determine the correct solution to the equation.

   $x = \frac{1}{2}$

How well did you understand the math in this lesson?

1 2 3 4 5

How did you feel about this lesson?

1 2 3 4 5

Reflect on the math from this lesson.

- I can solve a linear equation.
- I can analyze strategies for solving a linear equation.

Lesson 4 More Balanced Moves

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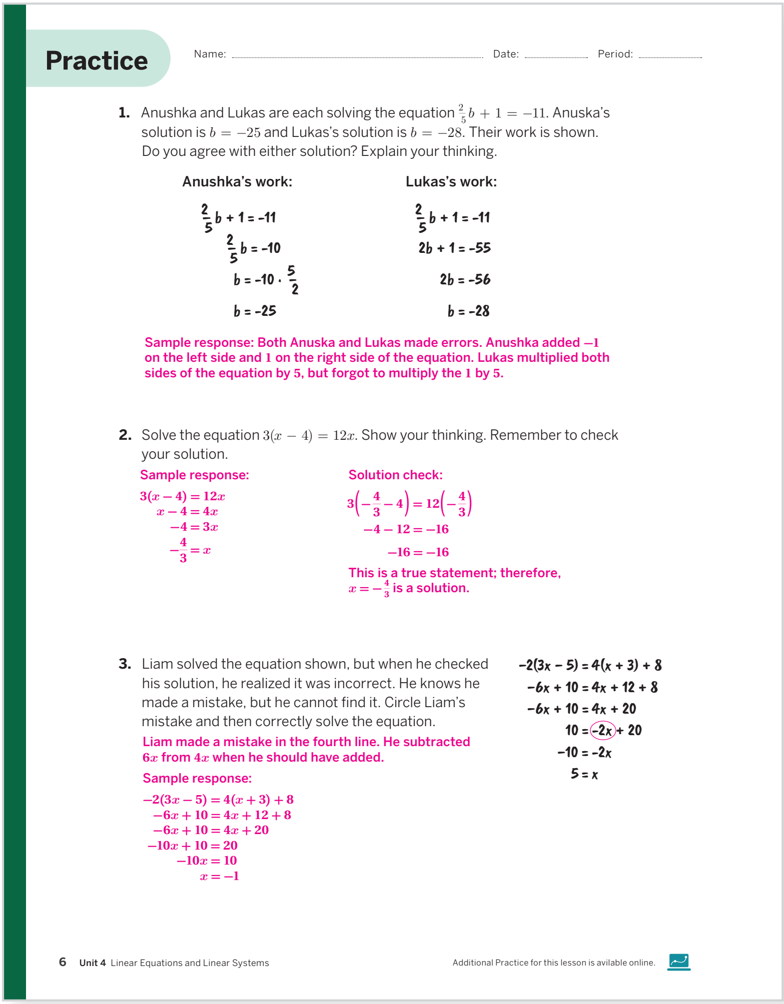
Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Students using digital

Students using print



Practice

Name: ______ Date: ______ Period: ______

1. Anushka and Lukas are each solving the equation $\frac{2}{5}b + 1 = -11$. Anuska's solution is $b = -25$ and Lukas's solution is $b = -28$. Their work is shown. Do you agree with either solution? Explain your thinking.

Anushka's work:

$$\frac{2}{5}b + 1 = -11$$
$$\frac{2}{5}b = -10$$
$$b = -10 \cdot \frac{5}{2}$$
$$b = -25$$

Lukas's work:

$$\frac{2}{5}b + 1 = -11$$
$$2b + 1 = -55$$
$$2b = -56$$
$$b = -28$$

Sample response: Both Anuska and Lukas made errors. Anushka added –1 on the left side and 1 on the right side of the equation. Lukas multiplied both sides of the equation by 5, but forgot to multiply the 1 by 5.

2. Solve the equation $3(x - 4) = 12x$. Show your thinking. Remember to check your solution.

Sample response:

$$3(x-4) = 12x$$
$$x - 4 = 4x$$
$$-4 = 3x$$
$$-\frac{4}{3} = x$$

Solution check:

$$3\left(-\frac{4}{3} - 4\right) = 12\left(-\frac{4}{3}\right)$$
$$-4 - 12 = -16$$
$$-16 = -16$$

This is a true statement; therefore, $x = -\frac{4}{3}$ is a solution.

3. Liam solved the equation shown, but when he checked his solution, he realized it was incorrect. He knows he made a mistake, but he cannot find it. Circle Liam's mistake and then correctly solve the equation.

$$-2(3x - 5) = 4(x + 3) + 8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$10 = \boxed{-2x} + 20$$
$$-10 = -2x$$
$$5 = x$$

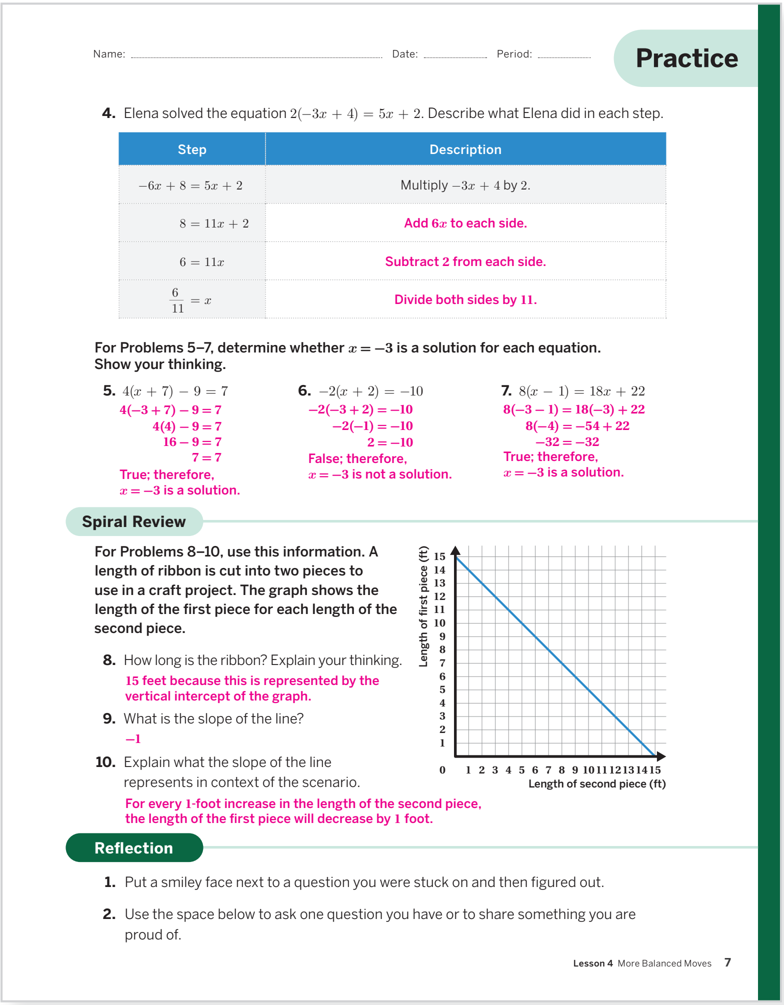
Liam made a mistake in the fourth line. He subtracted $6x$ from $4x$ when he should have added.

Sample response:

$$-2(3x-5) = 4(x+3)+8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$-10x + 10 = 20$$
$$-10x = 10$$
$$x = -1$$

6 **Unit 4** Linear Equations and Linear Systems

Additional Practice for this lesson is avilable online.



Name: ______ Date: ______ Period: ______

Practice

4. Elena solved the equation $2(-3x + 4) = 5x + 2$. Describe what Elena did in each step.

Step	Description
$-6x + 8 = 5x + 2$	Multiply $-3x + 4$ by 2.
$8 = 11x + 2$	**Add $6x$ to each side.**
$6 = 11x$	**Subtract 2 from each side.**
$\frac{6}{11} = x$	**Divide both sides by 11.**

For Problems 5–7, determine whether $x = -3$ is a solution for each equation. Show your thinking.

5. $4(x + 7) - 9 = 7$

$$4(-3+7) - 9 = 7$$
$$4(4) - 9 = 7$$
$$16 - 9 = 7$$
$$7 = 7$$

True; therefore, $x = -3$ is a solution.

6. $-2(x + 2) = -10$

$$-2(-3+2) = -10$$
$$-2(-1) = -10$$
$$2 = -10$$

False; therefore, $x = -3$ is not a solution.

7. $8(x - 1) = 18x + 22$

$$8(-3-1) = 18(-3) + 22$$
$$8(-4) = -54 + 22$$
$$-32 = -32$$

True; therefore, $x = -3$ is a solution.

Spiral Review

For Problems 8–10, use this information. A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length of the first piece for each length of the second piece.

8. How long is the ribbon? Explain your thinking.

15 feet because this is represented by the vertical intercept of the graph.

9. What is the slope of the line?

–1

10. Explain what the slope of the line represents in context of the scenario.

For every 1-foot increase in the length of the second piece, the length of the first piece will decrease by 1 foot.

Reflection

1. Put a smiley face next to a question you were stuck on and then figured out.
2. Use the space below to ask one question you have or to share something you are proud of.

Lesson 4 More Balanced Moves 7

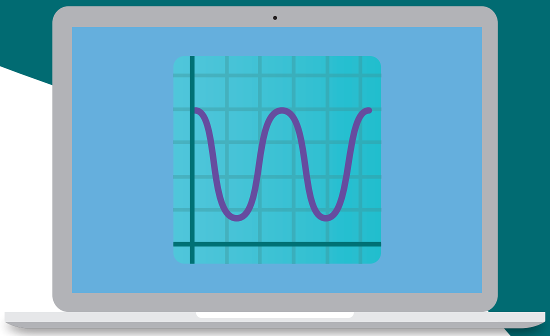
Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1, 3, 4	2	NY-8.EE.7 NY-8.EE.7b
Fluency	2	1	NY-8.EE.7 NY-8.EE.7b
Test Practice	5–7	1	NY-8.EE.7 NY-8.EE.7b
Spiral Review			
	8, 10	2	NY-8.EE.6
	9	1	NY-8.EE.6

Unit 5
Lesson
6

Digital Lesson RECOMMENDED

This is a digital lesson. A print option is also available.



Graphing Stories

Creating Graphs of Functions

Let's make connections between scenarios and the graphs that represent them.

Focus and Coherence

Today's Goals

1. **Goal:** Draw the graph of a function that represents a real-world situation.
2. **Language Goal:** Describe where the graph of a function is increasing, decreasing, linear, or non-linear. **(Reading, Writing, Speaking, and Listening)**

Students draw the graphs of functions based on short videos they watch of real-world situations, learning the important features to consider when modeling a situation with a graph. They consider the qualitative features of a function, such as whether it is increasing, decreasing, linear, or non-linear, and interpret specific points in context. **(MP4)**

Prior Learning

In Lessons 1 and 5, students interpreted graphs in context.

Future Learning

In Lesson 7, students will investigate and make connections between linear functions as represented by graphs, tables, equations of the form $y = mx + b$, and verbal descriptions.

Rigor and Balance

- Students build **conceptual understanding** of the qualitative aspects of the graphs of functions as they use the terms *increasing*, *decreasing*, *linear*, and *non-linear* to describe the features of these graphs.

Standards

Addressing

NY-8.F.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described in a real-world context.

Mathematical Practices: MP3, MP4, MP6

Building On	Building Toward
NY-8.F.1	NY-8.F.2
	NY-8.F.3

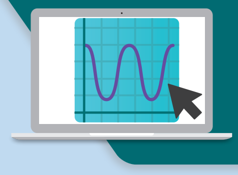
Amplify Desmos Math **NEW YORK**

Lesson Sample

Lesson at a Glance

~ 45 min

Standard: NY-8.F.5



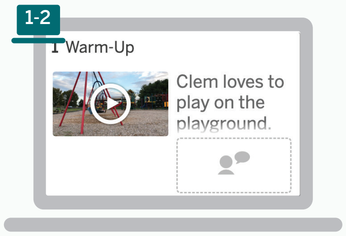
Why digital?

Students compare sample graph sketches overlaid with their own sketches in real time.

Warm-Up

Pairs | 5 min

Students analyze a video showing a real-word situation to determine the changing quantities and compare graphs that could represent it.



1-2

1 Warm-Up

Clem loves to play on the playground.

Pacing: Screens 1–2

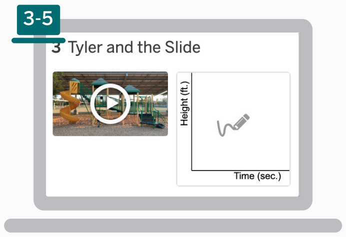
Activity 1

Pairs | 15 min

Students draw the graph of a function that represents a context to focus on its qualitative aspects and how they affect the shape of the graph.

Routine:

- MLR1: Stronger and Clearer Each Time **(MP4)**



3-5

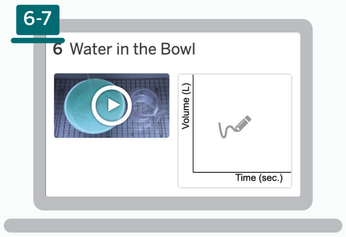
3 Tyler and the Slide

Pacing: Screens 3–5

Activity 2

Pairs | 10 min

Students draw the graph of a function that represents a context to apply strategies learned in the previous activity. **(MP6)**



6-7

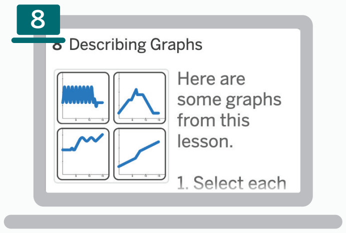
6 Water in the Bowl

Pacing: Screens 6–7

Activity 3

Pairs | 5 min

Students analyze parts of graphs to develop their own meaning of where functions are increasing, decreasing, linear, and non-linear.



8

8 Describing Graphs

Here are some graphs from this lesson.

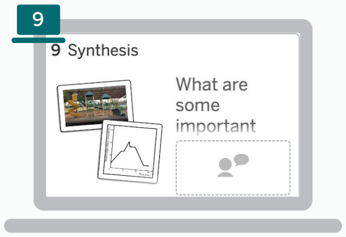
1. Select each

Pacing: Screen 8

Synthesis

Whole Class | 5 min

Students synthesize their understanding of using different variables to describe the same real-world situation, which may produce different graphs.



9

9 Synthesis

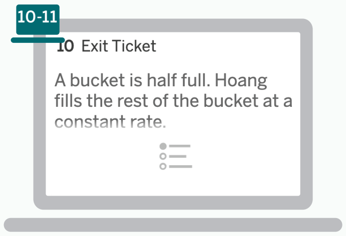
What are some important

Pacing: Screen 9

Exit Ticket

Independent | 5 min

Students demonstrate their understanding by determining which graph could represent the given real-world situation.



10-11

10 Exit Ticket

A bucket is half full. Hoang fills the rest of the bucket at a constant rate.

Pacing: Screens 10–11

Prep Checklist

Assign the digital lesson. A print option is also available.

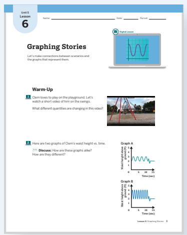
Students using digital:	**Students using print:**
Digital Lesson	Print Option in Student Edition
	Exit Ticket PDF
	Coloring tools (as needed)

Digital Lesson Pairs | 5 min | Pacing: Screens 1–2

Warm-Up

Purpose: Students analyze a video showing a real-word situation to determine the changing quantities and compare graphs that could represent it.

Short on time: Consider omitting the Warm-Up.



Students using print

Students using digital

1 Launch

Play the video showing Clem on the swing. Let students know that the "different quantities that are changing" are the same as the variables of the situation.

2 Connect

Display the graphs or video to aid the class discussion.

A **Accessibility: Visual-Spatial Processing** For students using digital, consider providing access to the Student Edition, which contains printed versions of the two graphs for students to draw on or highlight.

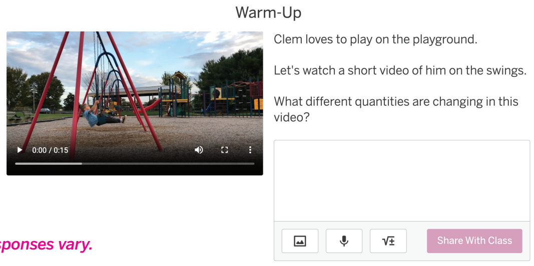
Invite students to share their responses.

Consider asking, "Which graph do you think more accurately represents what happened in the video?"

Math Identity and Community Invite students to notice and celebrate the variety of mathematical thinking during an activity.

Emphasize that a single situation can contain many different quantities or variables. Each of these variables may produce its own unique graph. Students will explore more of this concept in Activity 1.

1



Warm-Up

Clem loves to play on the playground.

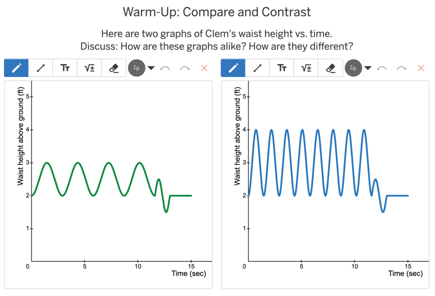
Let's watch a short video of him on the swings.

What different quantities are changing in this video?

Responses vary.

- Time
- Clem's waist height above ground
- Clem's shoe height above ground
- The number of times Clem swings back and forth
- The horizontal distance between Clem and the edge of the video screen

2



Warm-Up: Compare and Contrast

Here are two graphs of Clem's waist height vs. time.
Discuss: How are these graphs alike? How are they different?

Responses vary.

Similarities

- The initial height is the same in each graph (2 feet).
- The final height is the same in each graph (2 feet).
- Each graph has the same basic shape.
- Each graph represents Clem jumping off at the same time.

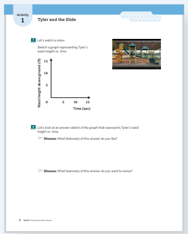
Differences

- The graph on the right indicates a greater number of swings.
- The graph on the left indicates a smaller distance between the highest and lowest waist heights.
- The graph on the left indicates a smaller maximum height above the ground.

Pairs | 15 min | Pacing: Screens 3–5 one at a time

Activity 1 Tyler and the Slide

Purpose: Students draw the graph of a function that represents a context to focus on its qualitative aspects and how they affect the shape of the graph. **(MP4)**



Students using print

3 Launch

Play the video showing Tyler on the slide.

Support getting started by pausing the video at each point and asking, "Where is Tyler's waist at $t = 0$, $t = 5$, and $t = 10$?"

MLR **MLR1: Stronger and Clearer Each Time** Invite students to meet with 1–2 other pairs of students to share their responses. Consider using these sentence frames to support students in giving and receiving feedback and using the feedback to revise their responses:

- I noticed that in the video ______, so I ______.
- I know that this part of the graph is ______, because ______.
- The video matches with this part of the graph because ______.

3 Monitor

D **Differentiation**

Look for students who:	Teacher Moves
Sketch a graph that starts at the origin.	**Support:** Tell students that placing the starting point at $(0, 0)$ means there is no distance between Tyler's waist height and the ground. Ask, "How might Tyler look if there's no distance between his waist and the ground?"
Notice Tyler's waist height must be constant for part of the graph when he reaches the top of the first ladder and when he sits down before sliding down the slide.	Capture this strategy to share during the Connect.

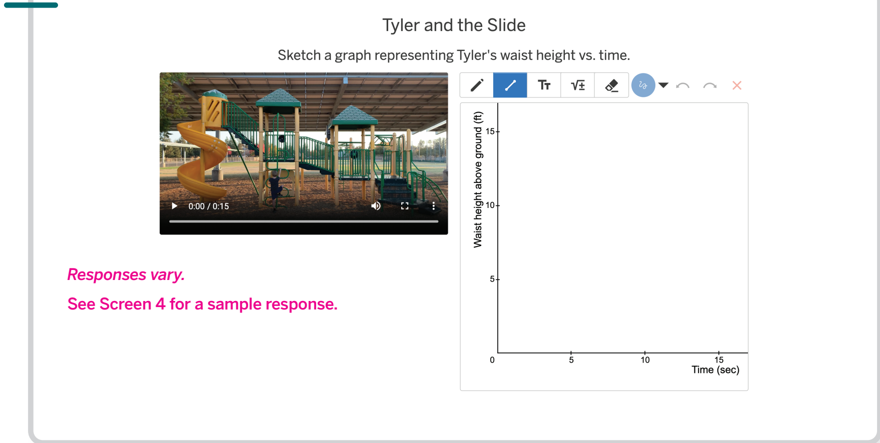
4 **Play** the video showing an answer sketch after students have had time to complete their sketch.

Math Identity and Community Consider highlighting the value of changing one's mind by asking if any students revised their thinking after seeing the sketch of the graph.

Activity 1 continued ›

Students using digital

3



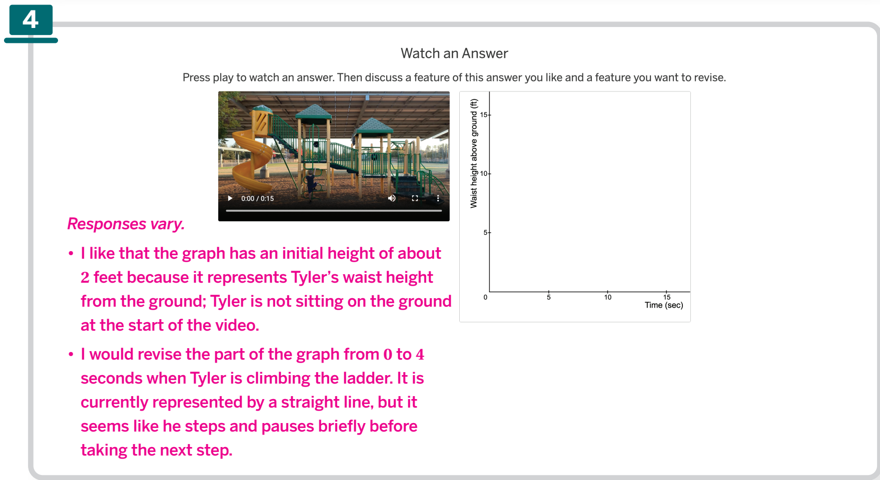
Tyler and the Slide

Sketch a graph representing Tyler's waist height vs. time.

Responses vary.

See Screen 4 for a sample response.

4



Watch an Answer

Press play to watch an answer. Then discuss a feature of this answer you like and a feature you want to revise.

Responses vary.

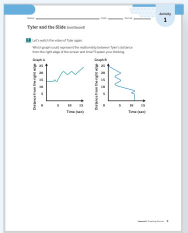
- I like that the graph has an initial height of about 2 feet because it represents Tyler's waist height from the ground; Tyler is not sitting on the ground at the start of the video.
- I would revise the part of the graph from 0 to 4 seconds when Tyler is climbing the ladder. It is currently represented by a straight line, but it seems like he steps and pauses briefly before taking the next step.

Digital Lesson

Pairs | 15 min | Pacing: Screens 3–5 one at a time

Activity 1 Tyler and the Slide (continued)

Purpose: Students draw the graph of a function that represents a context to focus on its qualitative aspects and how they affect the shape of the graph. **(MP4)**



Students using print

5 Monitor

Play the video showing Tyler on the slide again.

A **Accessibility: Visual-Spatial Processing**
For students using digital, consider providing access to the Student Edition, which contains printed versions of the two graphs for students to draw on or highlight.

5 Connect

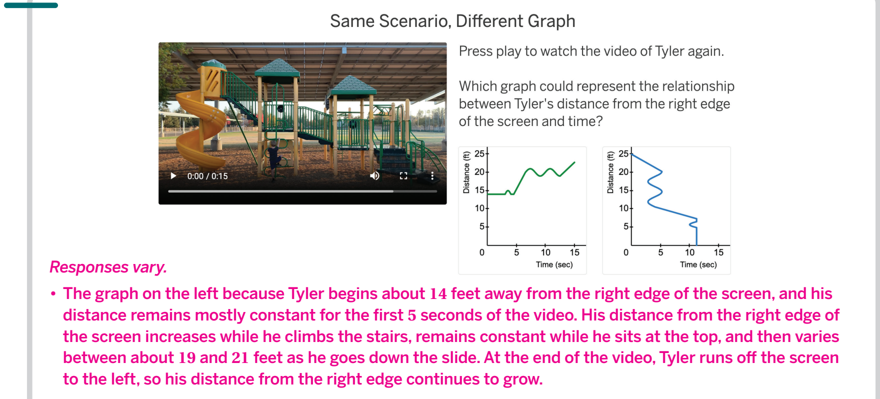
Invite students to share their responses and justify their reasoning in terms of what is happening in the video. **(MP3)**

Emphasize that when sketching a graph from a context, it is important to pay attention to the variables being measured. For example, Tyler's waist height above the ground vs. his distance from the right edge of the screen. Even though the context (video) is the same, using different variables creates different graphs.

Key Takeaway: The purpose of this discussion is to make connections between the features of a situation and the features of a graph, and to make arguments and critique the reasoning of others. **(MP3)**

Students using digital

5



Same Scenario, Different Graph

Press play to watch the video of Tyler again.

Which graph could represent the relationship between Tyler's distance from the right edge of the screen and time?

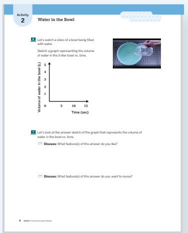
Responses vary.

- The graph on the left because Tyler begins about 14 feet away from the right edge of the screen, and his distance remains mostly constant for the first 5 seconds of the video. His distance from the right edge of the screen increases while he climbs the stairs, remains constant while he sits at the top, and then varies between about 19 and 21 feet as he goes down the slide. At the end of the video, Tyler runs off the screen to the left, so his distance from the right edge continues to grow.
- The graph on the right cannot represent Tyler's distance from the right edge of the screen and time because it would indicate that Tyler is occasionally in more than one place at a time. For example, the graph on the right indicates that at just over 5 seconds Tyler's distance from the right edge of the screen is approximately 10, 15, and 20 feet. This is not possible because Tyler cannot be in more than one place at a time.

Pairs | 10 min | Pacing: Screens 6–7

Activity 2 Water in the Bowl

Purpose: Students draw the graph of a function that represents a context to apply strategies learned in the previous activity. **(MP6)**



Students using print

6 Launch

Play the video showing the volume of water in the bowl over time.

6 Monitor

D Differentiation

Look for students who:	Teacher Moves
Need support getting started.	**Support:** Ask, “What do you notice and wonder about the volume of water in the bowl?”
Think the graph should have a constant slope.	**Support:** Ask, “If the graph was a line, what would that mean about the amount of water being poured into the bowl for the entire video?”
Ask to replay and pause the video to record the volume at different time intervals.	Replay and pause the video as needed.

7 **Play** the video showing an answer sketch.

M/EL **Multilingual/English Learners** Use intentional grouping so students with different English language proficiency levels can interact and have an opportunity to listen to peers with more advanced proficiency. **(Speaking and Listening)**

7 Connect

Display students’ sketches representing the volume of water. Then display the answer sketch.

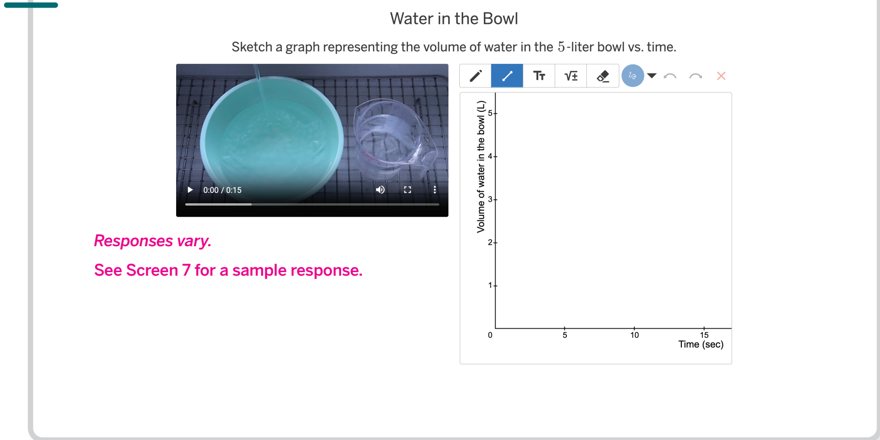
Invite students to share their responses to the prompts. Encourage students to use mathematically precise language, such as *rate of change*, y*-intercept*, and *linear*. **(MP6)**

Consider asking, “Why is the rate of change greater in the middle section of the graph? Why are the slopes of the first and third section the same?”

Emphasize that the water is pouring into the bowl at a constant rate before additional water is poured at the same time, so the graph is steeper before returning to its original steepness.

Students using digital

6



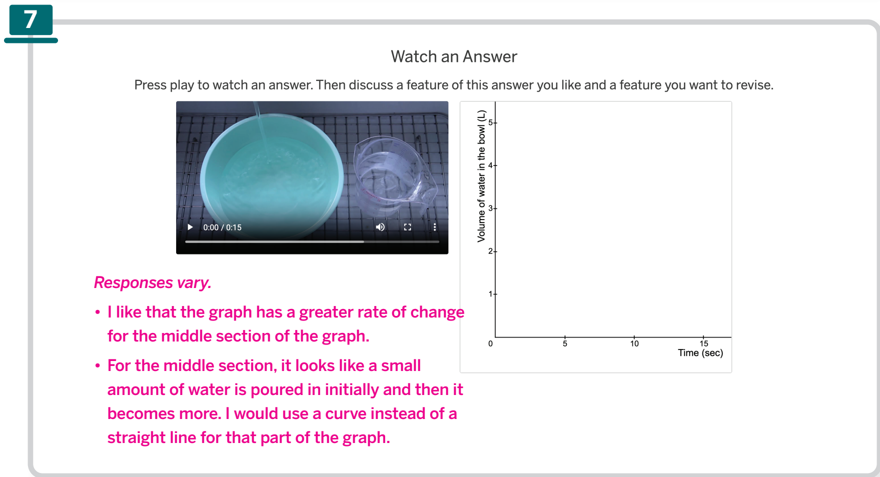
Water in the Bowl

Sketch a graph representing the volume of water in the 5-liter bowl vs. time.

Responses vary.

See Screen 7 for a sample response.

7



Watch an Answer

Press play to watch an answer. Then discuss a feature of this answer you like and a feature you want to revise.

Responses vary.

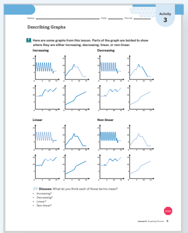
- I like that the graph has a greater rate of change for the middle section of the graph.
- For the middle section, it looks like a small amount of water is poured in initially and then it becomes more. I would use a curve instead of a straight line for that part of the graph.

Digital Lesson | Pairs | 5 min | Pacing: Screen 8

Activity 3 Describing Graphs

Purpose: Students analyze parts of graphs to develop their own meaning of where functions are increasing, decreasing, linear, and non-linear.

Short on time: Consider completing this activity as a whole class.



Students using print

8 Launch

Display the graphs. Select each term to show where on each graph the term applies.

Support getting started by asking, "What do all of the parts of the graphs that are *increasing* have in common?" Repeat for *decreasing*, *linear*, and *non-linear*.

8 Monitor

D Differentiation

Look for students who:	Teacher Moves
Use informal language to describe the graphs such as *going up*, *going down*, *curved*, or *straight*.	Ask, "Which of these descriptions can you use instead?"
Think they need to formally define each description.	Tell students to write the meaning of each description in their own words.

A Accessibility: Visual-Spatial Processing Invite students to use coloring tools to annotate the graphs to support interpreting visual representations.

M/EL Multilingual/English Learners Provide sentence frames to support students as they write and share the meanings of each description. For example, "*Increasing* means the graph ______," "*Linear* means the graph ______." **(Writing and Speaking)**

8 Connect

Display any graphs that will help the class discussion.

Invite students to share their responses.

Consider asking, "What is a real-world situation that could produce a linear part of a graph? Non-linear?"

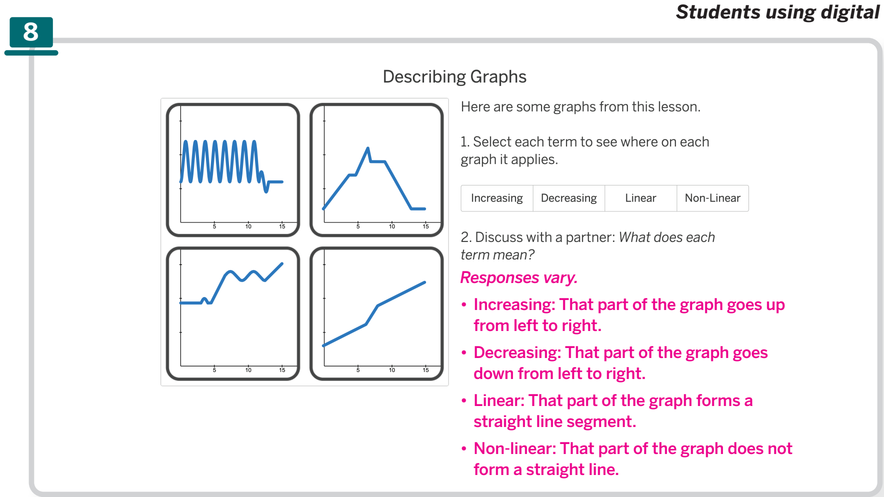
Key Takeaway: When the parts of the graph of a function are . . .

- *Increasing*, the values of the function are increasing.
- *Decreasing*, the values of the function are decreasing.
- *Linear*, these parts are straight line segments.
- *Non-linear*, these parts are not straight line segments.

Some functions that consist of all linear segments are non-linear because the rate of change is not consistent throughout.

Students using digital

8



Describing Graphs

Here are some graphs from this lesson.

1. Select each term to see where on each graph it applies.

Increasing	Decreasing	Linear	Non-Linear

2. Discuss with a partner: *What does each term mean?*

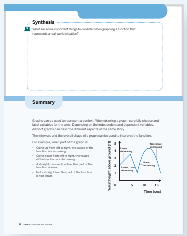
Responses vary.

- **Increasing: That part of the graph goes up from left to right.**
- **Decreasing: That part of the graph goes down from left to right.**
- **Linear: That part of the graph forms a straight line segment.**
- **Non-linear: That part of the graph does not form a straight line.**

Whole Class | 5 min | Pacing: Screen 9

Synthesis

Purpose: Students synthesize their understanding of using different variables to describe the same real-world situation, which may produce different graphs.



Students using print

9 Synthesis

Invite students to respond independently, and then share their thinking with a partner.

Math Identity and Community If time allows, invite students to celebrate other students whose strategies they found most helpful.

MLR **Use the MLR2: Collect and Display routine** to review the terms *increasing, decreasing, linear*, and *non-linear*. Consider showing the graphs to help students visualize each description.

Consider asking, "What strategies or tools did you find helpful today when drawing the graph of a function from a context? How were they helpful?"

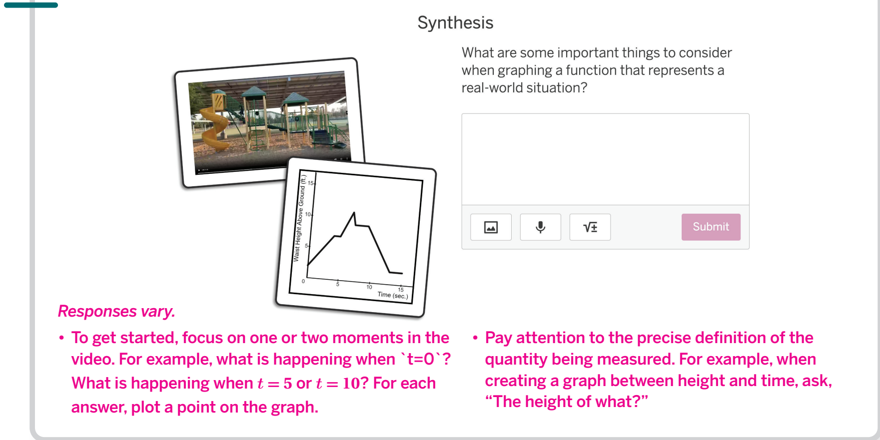
Lesson Takeaway: When drawing the graph of a function from a context, it is important to pay attention to the variables of the situation, the initial amount, if the values are increasing or decreasing, and whether the rate of change is constant.

Summary

PDF **Share** the Summary. Students can refer back to this throughout the unit and course.

Students using digital

9



Synthesis

What are some important things to consider when graphing a function that represents a real-world situation?

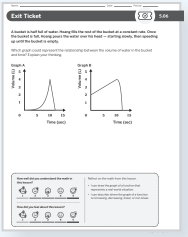
Responses vary.

- To get started, focus on one or two moments in the video. For example, what is happening when `t=0`? What is happening when $t = 5$ or $t = 10$? For each answer, plot a point on the graph.
- Pay attention to the precise definition of the quantity being measured. For example, when creating a graph between height and time, ask, "The height of what?"

Independent | 5 min | Pacing: Screens 10–11

Exit Ticket

Purpose: Students demonstrate their understanding by determining which graph could represent the given real-world situation.



Students using print

Students using digital

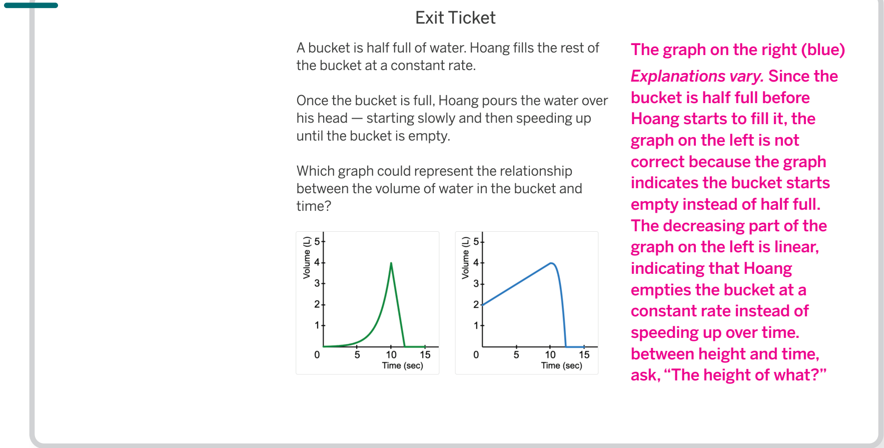
10-11 Today's Goals

Goal: Draw the graph of a function that represents a real-world situation.

Language Goal: Describe where the graph of a function is *increasing, decreasing, linear*, or *non-linear*. **(Reading, Writing, Speaking, and Listening)**

Support for Future Learning: If students struggle to determine which graph represents the scenario, consider reviewing this Exit Ticket as a class before beginning Lesson 7.

10



Exit Ticket

A bucket is half full of water. Hoang fills the rest of the bucket at a constant rate.

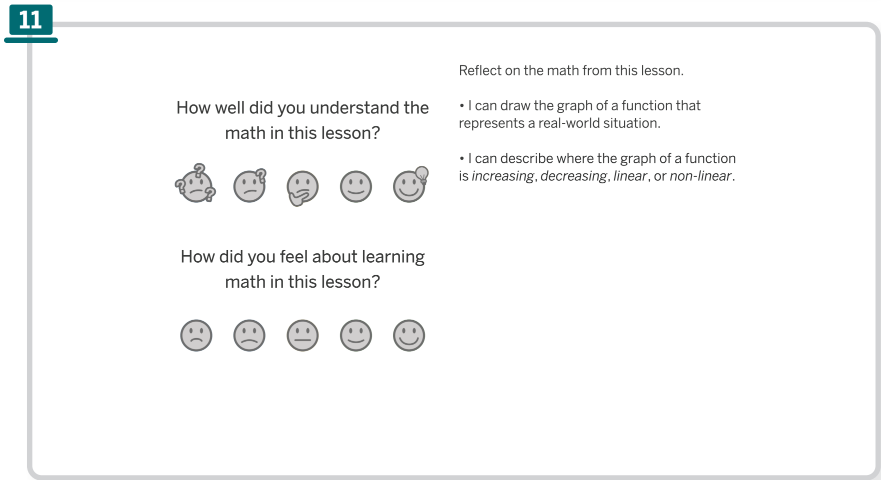
Once the bucket is full, Hoang pours the water over his head — starting slowly and then speeding up until the bucket is empty.

Which graph could represent the relationship between the volume of water in the bucket and time?

The graph on the right (blue)

Explanations vary. Since the bucket is half full before Hoang starts to fill it, the graph on the left is not correct because the graph indicates the bucket starts empty instead of half full. The decreasing part of the graph on the left is linear, indicating that Hoang empties the bucket at a constant rate instead of speeding up over time. between height and time, ask, "The height of what?"

11



How well did you understand the math in this lesson?

How did you feel about learning math in this lesson?

Reflect on the math from this lesson.

- I can draw the graph of a function that represents a real-world situation.
- I can describe where the graph of a function is *increasing*, *decreasing*, *linear*, or *non-linear*.

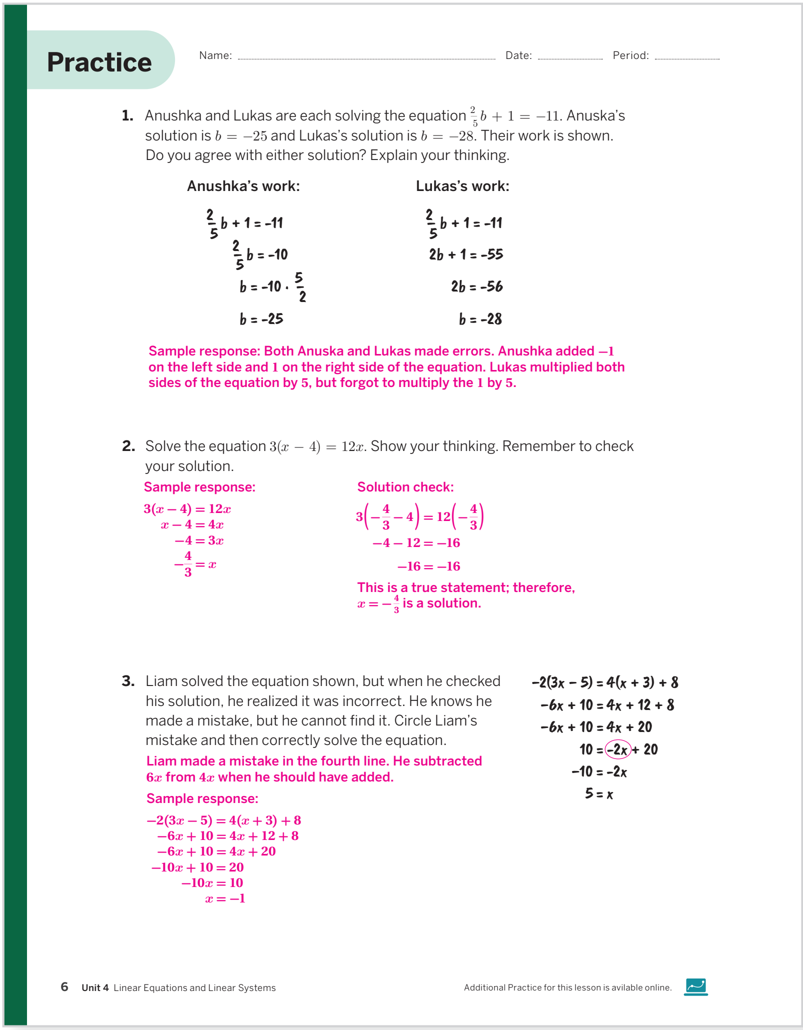
Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Students using digital

Students using print



Practice

Name: ________ Date: ________ Period: ________

1. Anushka and Lukas are each solving the equation $\frac{2}{5}b + 1 = -11$. Anuska's solution is $b = -25$ and Lukas's solution is $b = -28$. Their work is shown. Do you agree with either solution? Explain your thinking.

Anushka's work:	Lukas's work:
$\frac{2}{5}b + 1 = -11$	$\frac{2}{5}b + 1 = -11$
$\frac{2}{5}b = -10$	$2b + 1 = -55$
$b = -10 \cdot \frac{5}{2}$	$2b = -56$
$b = -25$	$b = -28$

Sample response: Both Anuska and Lukas made errors. Anushka added –1 on the left side and 1 on the right side of the equation. Lukas multiplied both sides of the equation by 5, but forgot to multiply the 1 by 5.

2. Solve the equation $3(x - 4) = 12x$. Show your thinking. Remember to check your solution.

Sample response:

$$3(x-4) = 12x$$
$$x - 4 = 4x$$
$$-4 = 3x$$
$$-\frac{4}{3} = x$$

Solution check:

$$3\left(-\frac{4}{3} - 4\right) = 12\left(-\frac{4}{3}\right)$$
$$-4 - 12 = -16$$
$$-16 = -16$$

This is a true statement; therefore, $x = -\frac{4}{3}$ is a solution.

3. Liam solved the equation shown, but when he checked his solution, he realized it was incorrect. He knows he made a mistake, but he cannot find it. Circle Liam's mistake and then correctly solve the equation.

$$-2(3x - 5) = 4(x + 3) + 8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$10 = -2x + 20$$
$$-10 = -2x$$
$$5 = x$$

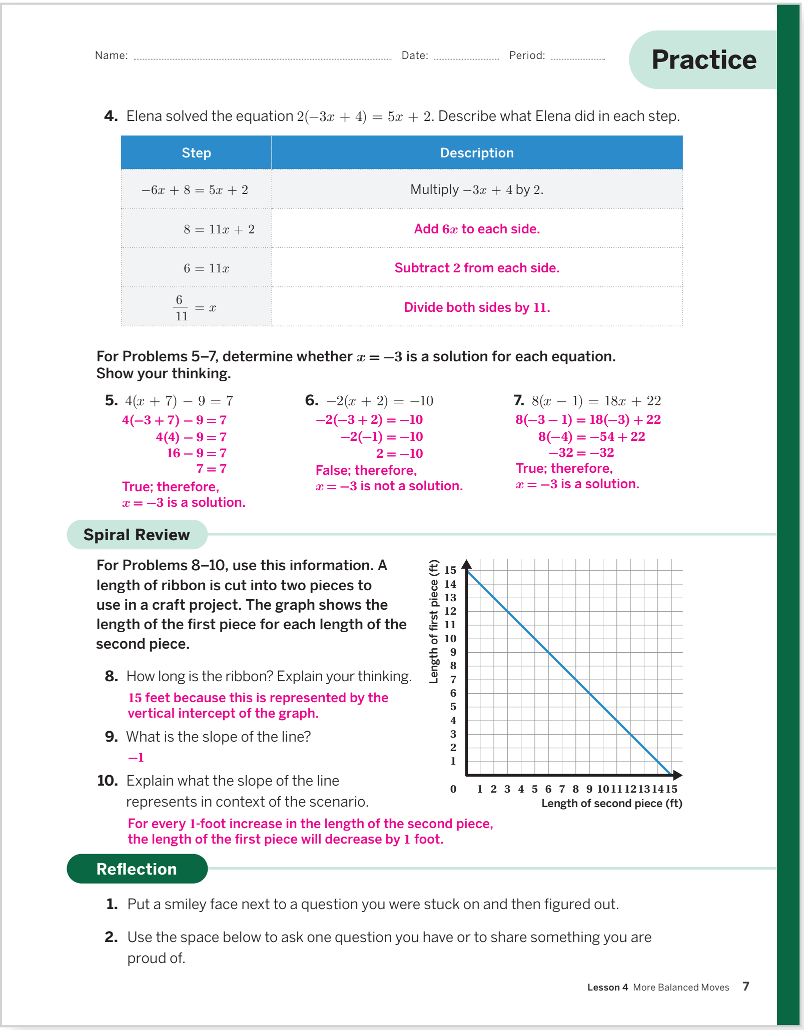
Liam made a mistake in the fourth line. He subtracted $6x$ from $4x$ when he should have added.

Sample response:

$$-2(3x-5) = 4(x+3) + 8$$
$$-6x + 10 = 4x + 12 + 8$$
$$-6x + 10 = 4x + 20$$
$$-10x + 10 = 20$$
$$-10x = 10$$
$$x = -1$$

6 **Unit 4** Linear Equations and Linear Systems

Additional Practice for this lesson is avilable online.



Name: ________ Date: ________ Period: ________

Practice

4. Elena solved the equation $2(-3x + 4) = 5x + 2$. Describe what Elena did in each step.

Step	Description
$-6x + 8 = 5x + 2$	Multiply $-3x + 4$ by 2.
$8 = 11x + 2$	Add $6x$ to each side.
$6 = 11x$	Subtract 2 from each side.
$\frac{6}{11} = x$	Divide both sides by 11.

For Problems 5–7, determine whether $x = -3$ is a solution for each equation. Show your thinking.

5. $4(x + 7) - 9 = 7$

$$4(-3+7) - 9 = 7$$
$$4(4) - 9 = 7$$
$$16 - 9 = 7$$
$$7 = 7$$

True; therefore, $x = -3$ is a solution.

6. $-2(x + 2) = -10$

$$-2(-3+2) = -10$$
$$-2(-1) = -10$$
$$2 = -10$$

False; therefore, $x = -3$ is not a solution.

7. $8(x - 1) = 18x + 22$

$$8(-3-1) = 18(-3) + 22$$
$$8(-4) = -54 + 22$$
$$-32 = -32$$

True; therefore, $x = -3$ is a solution.

Spiral Review

For Problems 8–10, use this information. A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length of the first piece for each length of the second piece.

8. How long is the ribbon? Explain your thinking.

15 feet because this is represented by the vertical intercept of the graph.

9. What is the slope of the line?

–1

10. Explain what the slope of the line represents in context of the scenario.

For every 1-foot increase in the length of the second piece, the length of the first piece will decrease by 1 foot.

Reflection

1. Put a smiley face next to a question you were stuck on and then figured out.
2. Use the space below to ask one question you have or to share something you are proud of.

Lesson 4 More Balanced Moves 7

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1, 3, 4	2	NY-8.EE.7 NY-8.EE.7b
Fluency	2	1	NY-8.EE.7 NY-8.EE.7b
Test Practice	5–7	1	NY-8.EE.7 NY-8.EE.7b
Spiral Review			
	8, 10	2	NY-8.EE.6
	9	1	NY-8.EE.6

GRADE 8

Unit 2
Lesson Plans

Teacher lesson plans from Unit 2 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

NOTE: *We have included only those lessons from Unit 2 that cover the standards in the Expressions, Equations, and Inequalities domain.*

Grade 8 Unit 3

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

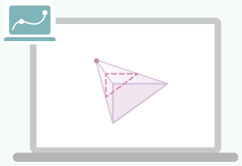
Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

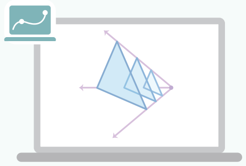
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



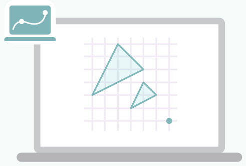
1 Sketchy Dilations

Understand that figures that are scaled copies of one another are similar figures.



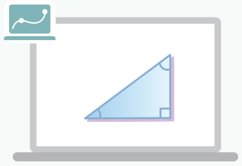
2 Dilation Mini Golf

Identify the center and scale factor used in a dilation.



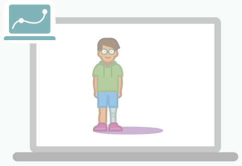
3 Match My Dilation

Apply dilations to figures on a grid.



7 Are Angles Enough?

Understand that for the special case of triangles, two pairs of congruent corresponding angles are sufficient for establishing similarity.



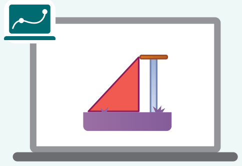
8 Shadows

Determine missing side lengths in pairs of similar triangles using quotient relationships between side lengths.

Assess and Respond

Quiz: Sub-Unit 2

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

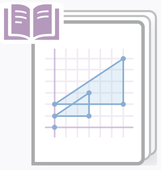


9 Water Slide

Show that all slope triangles on one line are similar and have the same slope.

Pacing: 14 days | **Short on time?** See pacing considerations below.

Pre-Unit Check: (Optional)
10 Lessons: 45 min each
1 Practice Day: 45 min
2 Sub-Unit Quizes: 45 min each
End-of-Unit Assessment: 45 min



4 Dilations on a Plane

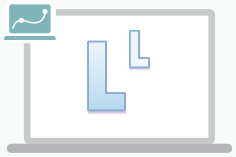
Describe and apply dilations to polygons on a grid given the coordinates of the vertices and the center of dilation.

Assess and Respond

Quiz: Sub-Unit 1

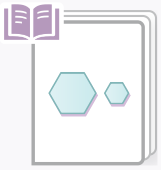
Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2



5 Transformation Golf With Dilations

Apply a sequence of transformations to show that two figures are similar.



6 Social Scavenger Hunt

Understand that similar polygons have congruent corresponding angles and a single scale factor between corresponding sides.

10 Points on a Line

Explain whether a point is on a line by finding quotients of horizontal and vertical distances.

Practice Day

Practice Day 1

Practice the concepts and skills developed during Lessons 1–10 Consider using this time to prepare for the upcoming Quiz.

Summative Assessment

End-of-Unit Assessment

Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing Considerations

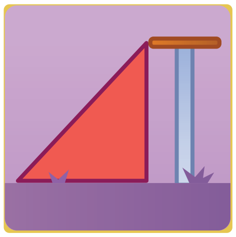
Lesson 1: This lesson introduces students to concepts of dilation and similarity, which will be addressed in more depth in upcoming lessons. If most students demonstrate a strong understanding of dilations and similarity in Problem 7 of the Pre-Unit Check, this lesson may be omitted.

Lesson 4: This lesson supports students in developing fluency with precisely communicating the information needed to perform a dilation on paper. If students show a strong understanding working with dilations in earlier lessons, this lesson may be omitted.If omitted, be sure to discuss how to describe and apply dilations to a polygon on a grid given the coordinates of its vertices and the center of dilation.

Lesson 8: This lesson supports students in connecting similar triangles to slope in the next lesson. If students show a strong understanding working with similar triangles and proportional relationships in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how to determine missing side lengths in pairs of similar triangles using the quotients of their side lengths elsewhere in the unit.

Lesson 10: This lesson extends students' work with slope by asking them to decide whether a point is on a line by finding quotients of horizontal and vertical distances. This lesson prepares students to work with linear equations in Unit 3.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Water Slide

Lesson 9: Slope of Lines

Overview

This lesson establishes the remarkable fact that the quotient of the vertical side length and the horizontal side length does not depend on the triangle: this number is called the slope of the line.

Learning Goals

- Show that all *slope triangles* on one line are similar and have the same slope.
- Determine the slope of a line in a plane.

Materials

- Blank paper

Vocabulary

- slope

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

A slope triangle for a line is a triangle whose longest side lies on the line and whose other two sides are vertical and horizontal. This lesson establishes the remarkable fact that the quotient of the vertical side length and the horizontal side length does not depend on the triangle: this number is called the *slope* of the line. The argument builds on many key ideas developed in this unit:

- The dilation of a slope triangle, with a center of dilation on the line, is a slope triangle for the same line.
- Triangles sharing two common angle measures are similar.
- Quotients of corresponding sides in similar polygons are equal.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to understand that the longest side of certain similar triangles can be arranged to form a line, or in this context, a water slide. Here, students will estimate the height of a triangle so that it will be similar to two triangles. Using estimation helps to establish the usefulness of using the precise height and base measurements of slope triangles to create smooth slides later in the activity.

Activity 1: Water Slide (30 minutes)

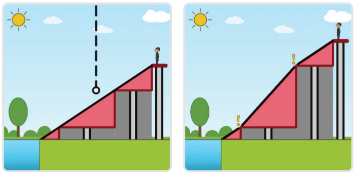
The purpose of this activity is to introduce students to the concept of slope. In this activity, similar triangles and height-to-base ratios help students in making certain slopes, and therefore, smooth rides.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to reflect on their new knowledge about slope.

Cool-Down (5 minutes)

1 Warm-Up



Teacher Moves

Purpose

The purpose of this lesson is to introduce students to the concept of slope. Students will recognize that right triangles with one side along the same line are similar. This fact will be used to define the slope of the line.

Warm-Up Launch

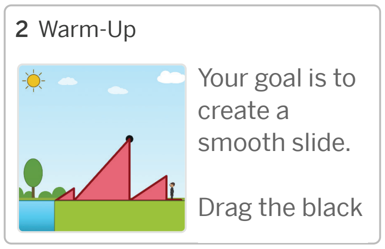
Arrange students into pairs. Tell students that their goal is to determine how to create a smooth waterslide ride.

Readiness Check (Problem 4)

If most students struggled, plan to support this thinking throughout the lesson as students investigate why two triangles sharing one side along the same line are similar.

Pacing

Consider using pacing to restrict students to Screens 1–2.


2 Warm-Up

Your goal is to create a smooth slide.

Drag the black

Your goal is to create a smooth slide.

Drag the black point to adjust the height of the middle ramp.

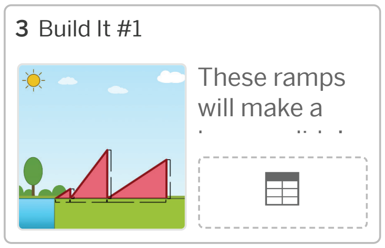
Teacher Moves

Give students two minutes of work time, followed by a whole-class discussion. Invite several students to explain what makes a smooth ride. If it doesn't come up naturally, ask students to share what they remember about the term *similar* and how similarity might be illustrated in this task.

Ask students to share their strategies for creating a smooth ride. Then ask, “What information would be helpful to determine the height of the ramp more precisely?” [The base lengths of all three triangles and the height of the two fixed triangles.] The goal of this discussion is for students to understand that for a smooth ride, the three ramps are similar triangles. They'll calculate base and height ratios later in the lesson.

Sample Responses

Image solution


3 Build It #1

These ramps will make a

These ramps will make a bumpy slide!

Update the height for Ramp 2 to make a smooth slide.

Press "Try It" to check your work.

Teacher Moves

The purpose of this activity is to introduce students to the concept of slope and how the height-to-base ratio allows them to precisely calculate the dimensions of the similar triangles.

Here, and throughout this lesson, encourage students to use scratch paper to help support their thinking.

Encourage students to write their calculations on paper if it helps with their thinking. It may also be helpful to point out to students that numerical expressions (e.g., $\frac{12 \cdot 3}{4}$) can be entered into the cells of the table.

Pacing
Consider using pacing to restrict students to Screens 3-4.

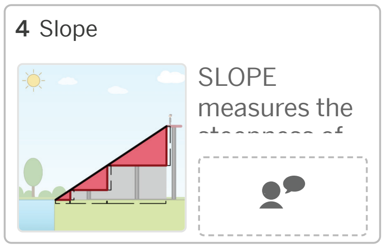
Sample Responses

10 feet

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*
Allow students to use calculators to ensure inclusive participation in the activity.

- *Social-Emotional Functioning: Peer Tutors*
Pair students with their previously identified peer tutors.



4 Slope

SLOPE measures the

SLOPE measures the steepness of a line.

This slide forms a line with a slope of $\frac{2}{3}$.

How do you think slope is calculated?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is for students to understand that slope is measured by calculating the height-to-base ratio of the slope triangle. Consider posting an anchor chart that displays how to calculate slope, so that students may reference this information for the remainder of this unit and Unit 3.

Once students have had time to record their answers on this screen, bring the class together and facilitate a whole-class discussion. Ask a student to explain what steepness means and to give a different context as an example (e.g., staircases, accessibility ramps into buildings, etc.). Then consider using snapshots or the teacher view in the dashboard to display several responses showing different ways students think about how slope is calculated. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

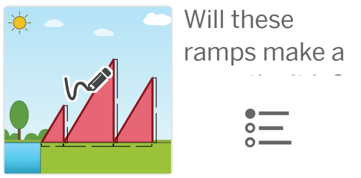
Sample Responses

Responses vary.

You can calculate slope by finding the height-to-base ratio of one of the similar triangles.

---

5 Slope



Will these ramps make a

Will these ramps make a smooth slide?

Teacher Moves

Tell students that they will have a chance to explore slope further on the next several screens.

Pacing
Consider using pacing to restrict students to Screens 5–10.

Sample Responses

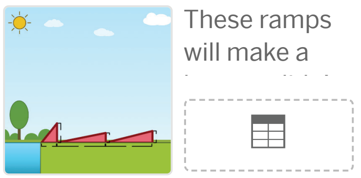
Yes

Responses vary.

These ramps will make a smooth ride because the height-to-base ratio is the same, $\frac{5}{3}$, for all three triangles.

---

6 Build It #2



These ramps will make a

These ramps will make a bumpy slide!

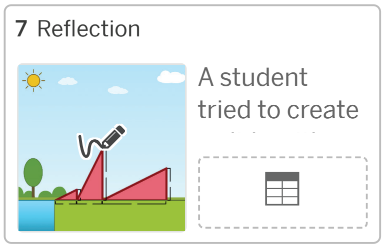
Update the ramps so the slide will have a slope of $\frac{1}{4}$.

Teacher Moves

Use the summary view in the teacher dashboard to identify students who may need additional support.

Sample Responses

- **Ramp 1 Height:** 2 feet
- **Ramp 2 Base:** 20 feet



7 Reflection

A student tried to create

A student tried to create a slide with a slope of $\frac{1}{2}$.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

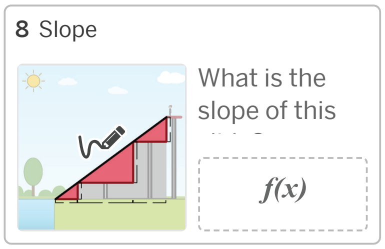
Early Student Thinking
Some students will understand that these ramps will make a bumpy slide because the middle triangle is too steep. Consider asking these students to tell you how steep the triangle is compared to the other triangles. If slope doesn't come up, consider asking students to compare the slope of each triangle.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language.

Sample Responses

Responses vary.

The height-to-base ratio for Ramps 1 and 3 is $\frac{1}{2}$, but for Ramp 2, the height-to-base ratio is 2 or $\frac{2}{1}$. This student seems to have reversed the ratio for Ramp 2. I would tell this student to be sure to put the height over the base when making their ratio.



8 Slope

What is the slope of this

What is the slope of this slide?

Teacher Moves

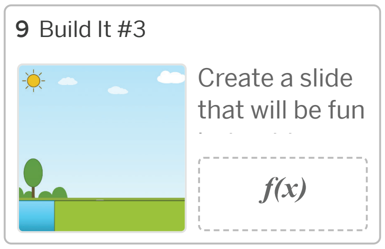
There are lots of interesting ways to express the slope of the slide on this screen. Consider using snapshots or the teacher view of the dashboard to show a variety of responses, calling attention to any conflict or consensus you see.

Sample Responses

$\frac{3}{4}$ (or equivalent)

---

9 Build It #3


Create a slide that will be fun

Create a slide that will be fun but not too scary.

First, enter a slope for your slide.

Then, go to the next screen.

Teacher Moves

On this screen, students create their own water slide. On the next screen, students will create three ramps with a common slope in order to create a smooth ride. The goal for this screen is to create a slide that is fun (a slope greater than 0.1) but not too scary (a slope less than 10).

Sample Responses

Responses vary.

Students entering a slope greater than or equal to 0.1 and less than or equal to 10 will be marked correct for this problem. These slopes create slides that are considered both fun and not too scary.

---

10 Build It #3


Here's the slope of your

Here's the slope of your slide.

Create three possible ramps for your slide.

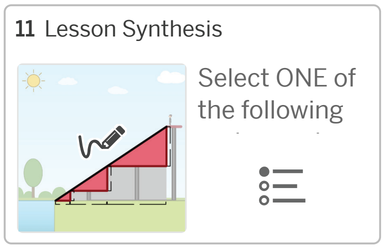
Teacher Moves

If time allows, use snapshots or the teacher view of the dashboard to highlight unique water slides. Invite students to share their strategies for finding the height and base measurements for their ramps.

Sample Responses

Responses vary.

(Students are successful here when the height-to-base ratios of all three ramps are equivalent to the slope entered on the previous screen.)



11 Lesson Synthesis

Select ONE of the following

Select ONE of the following and record your response.

Use the sketch tool if it helps you with your thinking.

Teacher Moves

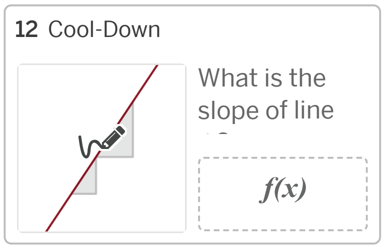
Key Discussion Screen 🔑
The purpose of this discussion is to consolidate what slope is and how to calculate it.

Synthesis Launch
Give students 2–3 minutes to respond to their selected question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

- Slope measures how steep a line is. A large slope means a steep line. The closer the slope is to 0, the less steep the line is.
- You calculate slope by finding the height-to-base ratio of the slope triangle.


12 Cool-Down

What is the slope of line

What is the slope of line k?

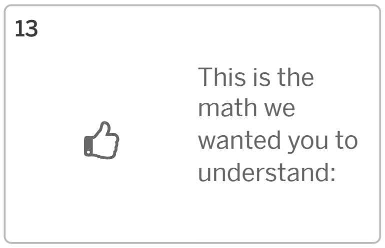
Teacher Moves

Support for Future Learning
If students struggle to identify the slope, plan to revisit this when opportunities arise during Lesson 10. Consider spending extra time during the warm-up discussing the slope of the line.

Pacing
Consider using pacing to restrict students to Screens 12–13.

Sample Responses

$\frac{3}{2}$ (or equivalent)

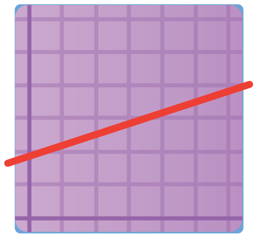

13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can show that all slope triangles on the same line are similar.
- I can figure out the slope of a line using slope triangles.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Points on a Line

Lesson 10: Slope and Coordinates

Overview

Prior to this lesson, students saw that right triangles with a horizontal side, a vertical side, and a long side along the same line are all similar. The height-to-base ratio of these triangles is called slope. This lesson uses these facts to help determine if points lie on a particular line.

Learning Goals

- Explain whether a point is on a line by finding quotients of horizontal and vertical distances.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Prior to this lesson, students saw that right triangles with a horizontal side, a vertical side, and a long side along the same line are all similar. The height-to-base ratio of these triangles is called slope. This lesson uses these facts to help determine if points lie on a particular line.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to recall the work they did with slope triangles in the previous lesson and use this knowledge to help them identify coordinates of points on a line.

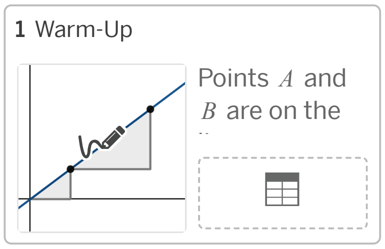
Activity 1: Points on a Line (30 minutes)

The purpose of this activity is for students to develop strategies for determining if a point lies on a line. Students think about coordinates of points on various lines, both in and outside of the graph window. The first few screens of this activity prepare students to create their *own* challenge and solve challenges from their classmates.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to reflect on the strategies they used for finding coordinates of points on a line.

Cool-Down (5 minutes)



1 Warm-Up

Points A and B are on the ..

Points A and B are on the line.

What are the coordinates of each point?

Teacher Moves

Purpose
The purpose of this lesson is for students to use slope as a tool to determine whether a point is on a line.

Warm-Up Launch
Arrange students in groups of two. Give students two minutes of quiet work time to complete Screens 1–2.

Early Student Thinking
Students may reverse their x- and y-coordinate values. For example, students might enter the point $(3, 4)$ in the table rather than $(4, 3)$. In their notes, have students sketch a pair of coordinate axes and plot a point with its x- and y-values labeled. Encourage students to use this diagram for reference.

Pacing
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

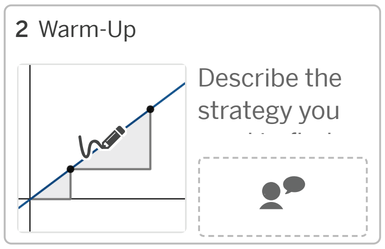
$A = (4, 3)$
$B = (12, 9)$

Student Supports

Students With Disabilities

- *Executive Functioning: Visual Aids*
Create an anchor chart of steps to calculate slope, publicly displaying important definitions, rules, formulas, or concepts for future reference.

- *Conceptual Processing: Eliminate Barriers*
Allow students to use calculators to ensure inclusive participation in the activity.



2 Warm-Up

Describe the strategy you

Describe the strategy you used to find the coordinates of each point.

Use the sketch tool if it helps you with your thinking.

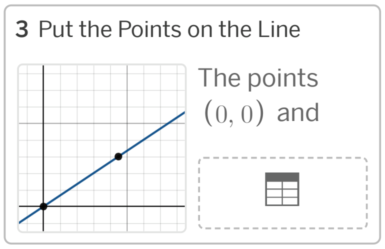
Teacher Moves

After students have had a few minutes to discuss their work with a partner, facilitate a whole-class discussion. Invite several students to share out loud their strategy for finding the coordinates of the points on the line. Address any early student thinking you see in students' responses. Consider using snapshots or the teacher view in the dashboard to highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

I looked at the slope triangle to tell me how far to go over on the x-axis and how far to go up on the y-axis from the origin. To get to point B, you have to add 12 units to the x-value and add 9 units to the y-value of the coordinate $(0, 0)$.



3 Put the Points on the Line

The points $(0, 0)$ and

The points $(0, 0)$ and $(9, 6)$ lie on the line.

Add two more points that lie on the line.

We've started one for you.

Teacher Moves

Activity Launch
Arrange students into pairs. Tell students that in this activity they will use the strategies they discussed in the warm-up to determine the coordinates of points on a line.

Throughout this lesson, encourage students to use blank paper to sketch, to write down calculations, or to capture their thinking.

Early Student Thinking

Students may use a base-to-height ratio for calculating slope. Consider posting an anchor chart that displays how to calculate slope so that students may reference this information for several days.

Pacing
Consider using pacing to restrict students to Screens 3–5.

Sample Responses

$(3, 2)$

Responses vary.

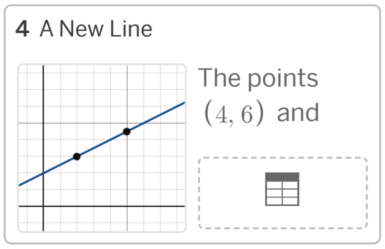
Any point that is a solution to the equation $y = \frac{2}{3}x$.

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*
For students who benefit from extra processing time, provide them the image to review prior to implementation of this activity. Also check in with individual students, as needed, to assess for comprehension during each step of the activity.

- *Memory: Processing Time*
Provide sticky notes or mini whiteboards to aid students with working memory challenges

- *Social-Emotional Functioning: Peer Tutors*
Pair students with their previously identified peer tutors.



4 A New Line

The points $(4, 6)$ and

The points $(4, 6)$ and $(10, 9)$ lie on a line.

Complete the table.

Teacher Moves

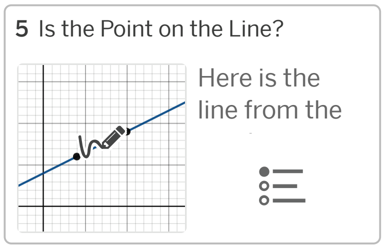
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking

Some students may use estimation based on the graph to find the x-coordinate in the last row of the table. Ask those students: *What is the slope of the line between the two given points on the screen?* They also might find it helpful to sketch a graph with a slope triangle connecting the points $(4, 6)$ and $(x, 20)$.

Sample Responses

$(0, 4)$
$(32, 20)$



5 Is the Point on the Line?

Here is the line from the

Here is the line from the previous screen.

Valeria thinks that the point $(20, 40)$ lies on the line because the slope is $\frac{1}{2}$.

Is Valeria right?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface several strategies for determining whether a point is on a line, including those that use similar triangles.

Consider pausing the class and facilitating a whole-class discussion. Use the teacher view of the dashboard to highlight several answers to show the class. Invite students to justify their strategies and critique each other's reasoning.

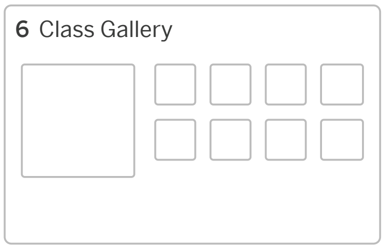
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

No

Responses vary.

The slope of this line is $\frac{1}{2}$, but if you draw a slope triangle between the points $(4, 6)$ and $(20, 40)$, the slope will be $\frac{34}{16}$ or $\frac{17}{8}$. This means that the point $(20, 40)$ is not on this line.


6 Class Gallery

Teacher Moves

Students will create their *own* challenge and solve challenges from their classmates. We recommend students complete Screens 3–5 before creating their challenge. We anticipate this Challenge Creator could take 20 minutes or more.

Encourage students to complete each other's challenges. Use the teacher dashboard to look for unique challenges and unique solutions that may expand your students' understanding of the mathematics, and highlight those for the class to see. Ask students what they've learned from the experience.

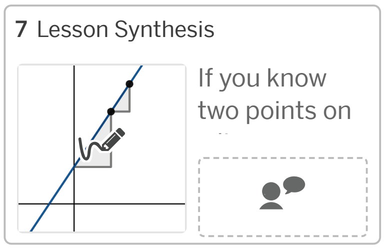
We intend for this to be a social and creative experience for students. We encourage you to emphasize those virtues whenever you see them in your class.

Pacing
Consider using pacing to restrict students to this screen.

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.


7 Lesson Synthesis

If you know two points on a line, describe a strategy you could use to determine another point on that line.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to consolidate and refine strategies for determining an unknown point on a line.

Synthesis Launch

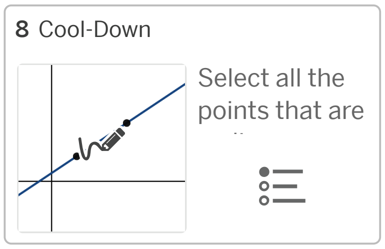
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

If you know two points on a line, you can draw a slope triangle between those two points to calculate the slope of the line. Then you can draw a similar slope triangle to figure out what to add to the x- and y-coordinates of a point you already know.



8 Cool-Down

Select all the points that are

Select all the points that are on line m.

Use the sketch tool if it helps you with your thinking.

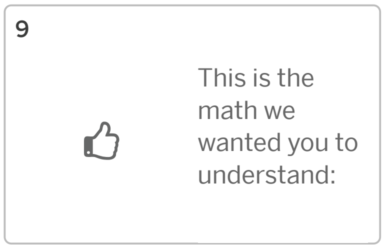
Teacher Moves

Support for Future Learning
If students struggle to identify other points on the line, consider making time to explicitly revisit these ideas before the End-Unit Assessment or spend extra time on Problem 7 of the Practice Day.

Pacing
Consider using pacing to restrict students to Screens 8–9.

Sample Responses

$(6, 5)$
$(12, 9)$
$(18, 13)$



9

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can decide whether a point is on a line by finding quotients of horizontal and vertical distances.

GRADE 8

Unit 3
Lesson Plans

Teacher lesson plans from Unit 3 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 8 Unit 3

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

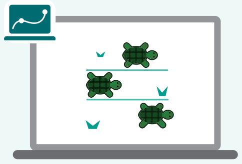
Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

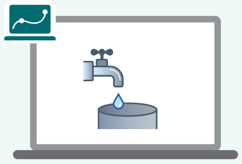
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



1 Turtle Time Trials

Begin to see connections between a context and features of a corresponding graph, equation, and table.

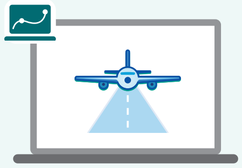


2 Water Tank

Graph a proportional relationship from an equation.

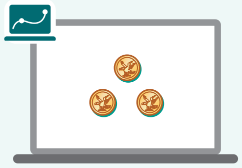
3 Posters

Compare two different proportional relationships given different representations of them.



8 Landing Planes

Generate a method to find slope values given two points on the line.



9 Coin Capture

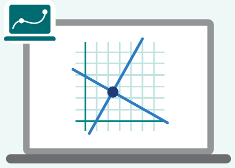
Write equations of horizontal and vertical lines.

Assess and Respond

Quiz: Sub-Unit 1

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2

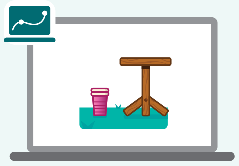


10 Solutions

Understand that linear equations don't always look like y=mx+b.

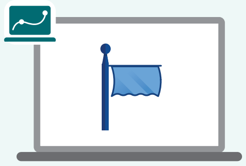
Pacing: 14 days | **Short on time?** See pacing considerations below.

Pre-Unit Check: (Optional)
11 Lessons: 45 min each
1 Practice Day: 45 min
1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



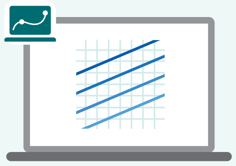
4 Stacking Cups

Understand that there are linear relationships that are not proportional.



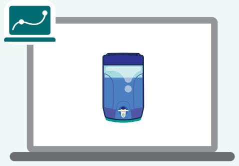
5 Flags

Identify and interpret the positive vertical intercept and slope of the graph of a linear relationship.



6 Translations

Derive y=mx+b by graphing y=mx and the same graph shifted up using the translation that takes (0,0) to (0,b).



7 Water Cooler

Understand the difference in visual appearance between lines with positive slopes and lines with negative slopes.

11 Pennies and Quarters

Describe how real-world constraints on quantities define the limitations of their representations.

Practice Day

Practice Day 1

Practice the concepts and skills developed during Lessons 1–11. Consider using this time to prepare for the upcoming Quiz.

Summative Assessment

End-of-Unit Assessment

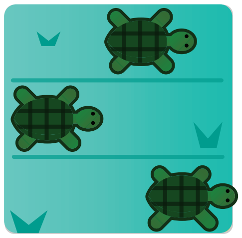
Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing Considerations

Lesson 6: This lesson introduces the idea that any line in a plane can be considered a vertical translation of a line through the origin. If students show a strong understanding of writing equations of lines in the coordinate plane in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how graphs of linear relationships can be interpreted as a translation of a graph of a proportional relationship.

Lesson 11: This lesson gives students an opportunity to solve real-world problems using all of the different representations of linear relationships. There is no new content introduced in this lesson.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Turtle Time Trials

Lesson 1: Understanding Proportional Relationships

Overview

Students explore proportional relationships through the context of racing turtles, with a glimpse at non-proportional relationships. They begin to make connections between the different representations of proportional relationships: number lines, graphs, tables, and equations (MP2). Students will have opportunities throughout the unit to further develop their understanding of these connections and are not expected to master the ideas by the end of this lesson.

Learning Goals

- Remember that a graph representing a proportional relationship is a line through $(0, 0)$ and $(1, k)$.
- Begin to see connections between a context and features of a corresponding graph, equation, and table.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About this Lesson

The purpose of this lesson is to get students thinking about connections among the different representations of proportional relationships, with a glimpse at nonproportional relationships. The lesson centers around a race between turtles of different constant speeds. After first encountering the context with an animation, students analyze and create other representations of the scenario: number lines, graphs, tables, and equations (MP2). Students will have opportunities throughout the unit to further develop their understanding of these connections and are not expected to master the ideas by the end of this lesson.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to orient students to the turtle-racing context. In the activity that follows, students explore this context to develop their understanding of proportional and non-proportional relationships in multiple representations.

Activity 1: Turtle Race (30 minutes)

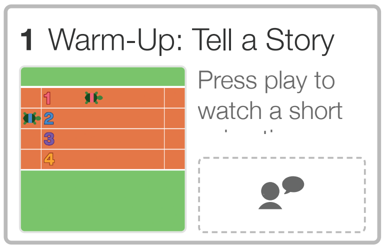
In this activity, students investigate four turtles competing in a race. Students use different mathematical representations of the race to help them represent and analyze the race. After the initial animation, students encounter a table of values. They then make a table of values and analyze the table they make to determine a turtle's speed. They follow a similar progression with graphs and equations: they view examples of the representation, they make the representation, and they analyze the representation to interpret it in context (MP2).

An important aspect of this activity is for students to make connections between these different representations. Each turtle's speed corresponds to the rate of change in its table, the slope of its graph, and the coefficient of the independent variable in its equation. Similarly, each turtle's starting distance corresponds to its value for distance at $t = 0$ in the table, the vertical intercept in its graph, and the constant in its equation. These connections will be developed in depth throughout this unit.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to draw out the connections between the context and different mathematical representations as well as the connections between the representations themselves.

Cool-Down (5 minutes)



1 Warm-Up: Tell a Story

Press play to watch a short

Press play to watch a short animation.

Then write a story about what you see.

Teacher Moves

Purpose
The purpose of this lesson is to get students thinking about connections among the different representations of proportional and linear relationships. Students will have opportunities throughout the unit to further develop their understanding of these connections and are not expected to master the ideas by the end of this lesson.

Warm-Up Launch
Give one minute of quiet think-time. Then ask students to discuss with a partner. Invite several students to share their responses. After each response, consider asking the class if the story makes sense to them, referring back to the animation as needed.

Readiness Check (Problems 1 and 2)
If most students struggled on Problem 1, plan to use this problem or a similar one as an additional warm-up activity. While they are working, listen for and record student language. Note any words or phrases that can be added to a visual display for students to use throughout the unit. If most students struggled on Problem 2, consider using 7.2.01 to practice the concept of generating equivalent ratios before beginning Lesson 1.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

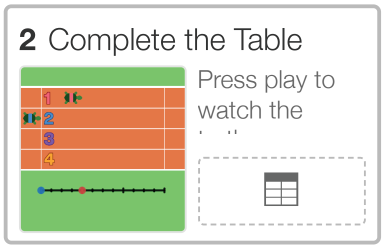
Responses vary.

Two turtles are racing. The red turtle got a head start, but the blue turtle was faster and won the race.

Student Supports

Multilingual Learners
- *Writing, Speaking: MLR 5 Co-Craft Questions*

Use this routine to help students interpret the first animation and to increase awareness of language used to make comparisons about speed. When students share their stories with the class, highlight those that mention distance and time in the animation. This may help students produce the language for different representations of speed.



2 Complete the Table

Press play to watch the

Press play to watch the turtles race.

The table shows each turtle's distance at various times.

Complete the table. Then press "Check My Work."

Teacher Moves

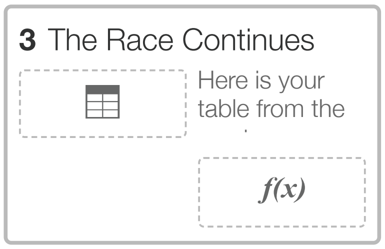
Activity Launch
Arrange students in groups of two. Consider introducing this activity by asking students to notice what is new in this animation compared to the previous screen. These are the same turtles running the same race in the same amount of time, but now a timer and a number line have been added as well as a table in the screen's right-hand column. Additional representations will be introduced in subsequent screens as well.

Representations like these and the new representations forthcoming enable us to understand and communicate more precisely about the race. Before students start working, ensure that they understand that each turtle's position is measured at the front of their head.

Pacing
Consider using pacing to restrict students to Screens 2–5.

Sample Responses

From top to bottom: 0, 3, 6, 9



3 The Race Continues

Here is your table from the

Here is your table from the previous screen.

What is the Lane 2 turtle's speed?

Teacher Moves

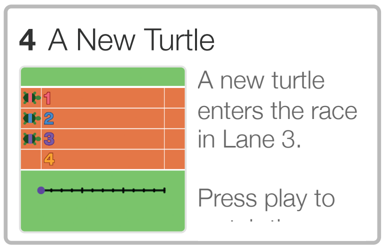
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling. Students can draw upon their work from the previous unit where they investigated slope and rate of change. The previous screen may also be helpful.

Sample Responses

3 ft. per sec.

Responses vary.

By looking at the blue turtle's distance at 0 seconds and 1 second, I can see that the blue turtle's distance increases by 3. The same is true between 1 second and 2 seconds as well as between 2 seconds and 3 seconds.



4 A New Turtle

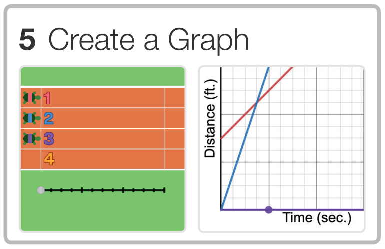
A new turtle enters the race in Lane 3.

Press play to

A new turtle enters the race in Lane 3.

Press play to watch the race.

Then continue to the next screen.



5 Create a Graph

Teacher Moves

Encourage students to watch this animation again and take note of any information about the purple turtle's race performance that seems important. The number-line diagram below the track and the timer will be helpful for precision. For instance, students may notice that the purple turtle does not get a 6-foot head start like the red turtle and that the purple turtle travels at half the speed of the blue turtle.

If students have trouble creating the graph, encourage them to make their best guess—or create anything at all—and then play the animation. The journey that they represent will be incorporated into the animation.

Sample Responses

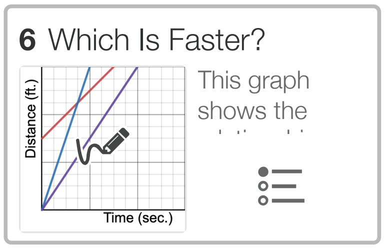
A line that goes through $(8, 12)$.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors. Allow students who struggle with fine motor skills to dictate directions as needed.



6 Which Is Faster?

This graph shows the

This graph shows the relationship between distance and time for the three turtles.

Which is faster: the Lane 1 turtle or the Lane 3 turtle?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to make connections between the steepness, or slope, of a graph and the speed of a turtle.

Teacher Moves
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Some students may draw upon what they witnessed in the animation or on their intuition. Others may take note of the steepness or slope of the graphs and make a connection to turtle speed. Invite several students to share their responses. Consider using the dashboard to show the distribution of responses, calling attention to any conflict or consensus you see.

Pacing
Consider using pacing to restrict students to Screens 6–8.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

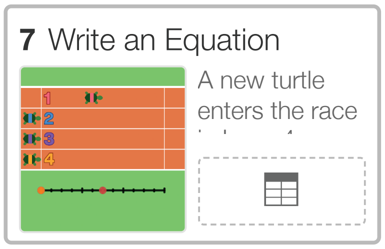
Sample Responses

Lane 3

Responses vary.

- The Lane 1 turtle beats the Lane 3 turtle in the race, but that's because the Lane 1 turtle got a head start. As the animation goes on, the Lane 3 turtle closes the gap, which means the Lane 3 turtle must be faster.

- The Lane 3 line is steeper than the Lane 1 line, which means that the Lane 3 turtle gains distance faster than the Lane 1 turtle does.



7 Write an Equation

A new turtle enters the race

A new turtle enters the race in Lane 4.

Write an equation for the new turtle. Then press play.

(Make the turtle finish in whatever place you want!)

Teacher Moves

Invite students to try different equations and to notice the results, looking for connections between the equation and aspects of the race. An important aspect of this unit is students making connections between contexts and the numbers and variables in equations. For example, some students may reason from the unit rates they can see on their graphs and write equations in the form of $y = kx$, where k is the unit rate (constant of proportionality). Use snapshots or the teacher view of the dashboard to highlight unique equations to show the class. Encourage the class to predict what will happen under different equations.

For students needing an additional challenge, invite them to try various unconventional scenarios. For instance, they could attempt to start the turtle off-screen, far behind the starting line, and have it zoom ahead to finish first, or they can have it start at the finish line and move backwards to the starting line.

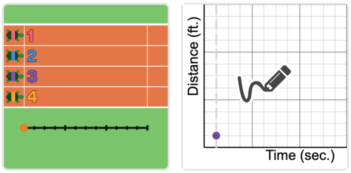
Readiness Check (Problem 3)
If most students struggled, plan to spend extra time on this screen selecting a variety of students' equations and inviting the class to predict the turtle's race results before revealing the animation.

Sample Responses

Responses vary.

- $d = 5t$
- $d = 2t$
- $d = -12 + 6t$
- $d = 6 - \frac{1}{2}t$

8 Create Your Own



Teacher Moves

This screen is intended for students to experiment and play with the turtle race context. Invite students to try various conventional and unconventional scenarios, using proportional and/or non-proportional relationships.

We intend for this to be a creative experience for students. We encourage you to emphasize those virtues whenever you see them in your class.

Early Student Thinking
Some students may wonder why no turtles are visible at the beginning or why some turtles disappear at the end of the animation. Invite them to consider what their sketches on the graph say about the the turtles' distances at 0 seconds and at 12 seconds. If there is no sketch visible at those points on the graph, there will be no turtle visible in the animation.

Sample Responses

Responses vary.

Student Supports

Students With Disabilities
- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors. Allow students who struggle with fine motor skills to dictate sketching their graphs as needed.

---

9 Lesson Synthesis

Discuss the following questions.

Discuss the following questions.

Then select ONE and record your response.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to come to consensus about how to determine the speed of a turtle from an equation, table, and graph.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion, addressing each of the three questions. Help

students see that these three representations—graphs, tables, and equations—each have their own way of communicating information about a context, such as the speed of a turtle.

Time permitting, point out to students that these three representations will be central to this unit, and much of the remainder of the course, and that this unit especially will focus on understanding the connections between them.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

1. How can you tell from a TABLE which turtle is fastest?

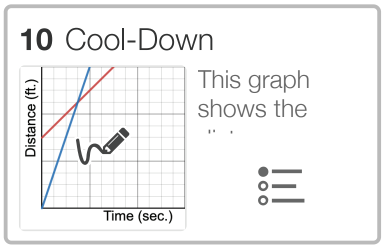
I can calculate the speed of each turtle by seeing how much distance it gains in one second.

2. How can you tell from a GRAPH which turtle is fastest?

I can determine the speed of each turtle by examining the steepness of each turtle's graph. The steepest one, which is the one with the greatest slope, is fastest.

3. How can you tell from an EQUATION which turtle is fastest?

The coefficient (the number that is multiplied by t) determines the speed of the turtle. The greatest coefficient is the fastest turtle.



10 Cool-Down

This graph shows the

This graph shows the distance vs. time relationship for the Lane 1 and Lane 2 turtles. Here are their equations:

Lane 1: $d = 6 + 1t$
Lane 2: $d = 3t$

Select all of the true statements.

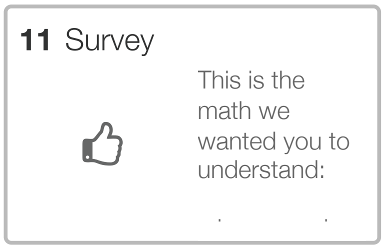
Teacher Moves

Support for Future Learning
Students will have more opportunities to analyze graphs of linear equations, so if students struggle with this cool-down, there is no need to slow down or add additional work to the next lessons.

Pacing
Consider using pacing to restrict students to Screens 10–11.

Sample Responses

- The Lane 2 turtle is faster than the Lane 1 turtle.
- The Lane 1 turtle has a head start.



11 Survey

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can graph a proportional relationship from a story.
- I can use the constant of proportionality to compare the pace of different turtles.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Water Tank

Lesson 2: Graphs of Proportional Relationships

Overview

Students understand that there are many ways to set up and scale axes in order to graph a proportional relationship. Sometimes, we choose specific ranges for the axes in order to see specific information, and those choices can have an impact on how information appears in a graph.

Learning Goals

- Graph a proportional relationship from an equation.
- Identify the same proportional relationship that is graphed using differently scaled axes.

Materials

- Graph paper
- Straightedge

Vocabulary

- rate of change

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to understand that there are many ways to set up and scale axes in order to graph a proportional relationship. Sometimes, we choose specific ranges for the axes in order to see specific information, and those choices can have an impact on how information appears in a graph.

Students predict which of two water tanks will fill up first based on the proportional relationships represented on two differently scaled axes. Then students complete a series of challenges that highlight the importance of paying careful attention to how the axes of each graph are scaled. By looking at the same two relationships graphed at different scales, students see the effect that the scale of the axes has on the questions we can answer. Throughout the lesson, students are encouraged to pay attention to scale and to rely on mathematical definitions of steepness and not just visual features of the graphs (MP6).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to understand why it is important to pay careful attention to graph scale instead of visual features of a graph. In the previous lesson, students compared different proportional relationships on the same set of axes. In this lesson, they compare two proportional relationships on two different sets of axes. For the warm-up, students predict which of two water tanks will fill up first based on the information shown in two graphs. However, there is not enough information because the graph scales are not yet labeled.

Activity 1: Compare the Rates (30 minutes)

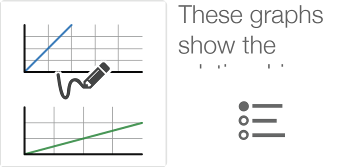
The purpose of this activity is for students to use the scales of graphs to compare rates. The context from the warm-up continues into this activity, but students now have sufficient information to answer the question because the graph scales are included. The scales reveal that the tank with a line that appears *less* steep is actually filling up at a faster rate. Students deepen that initial understanding that the scale of a graph matters by completing a series of graphing challenges, including a card sort where they sort graphs into groups that represent the same proportional relationship.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to recreate the graph of a given proportional relationship—first making it appear steeper and then making it appear less steep—through careful selection of the scale on each axis.

Cool-Down (5 minutes)

1 Warm-Up



These graphs show the

These graphs show the relationship between volume, V, and time, t, for two water tanks filling at a constant rate.

Which tank will fill up first?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Warm-Up Launch
Give students one minute of quiet think-time and a few minutes to discuss their answers with a partner. Invite several students to share their responses. Then consider using the dashboard to show the distribution of responses.

Pacing
Consider using pacing to restrict students to this screen.

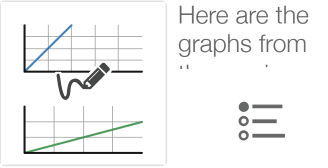
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

- I think Tank A will fill up faster because the line on the Tank A graph is steeper.
- We don't have enough information because the graph axes are not scaled.

---

2 Rates of Change



Here are the graphs from

Here are the graphs from the previous screen but with additional information.

- Each tank can hold up to 60 liters.
- Each vertical axis shows volume (in liters).
- Each horizontal axis shows time (in minutes).

Now, which tank do you think will fill up first?

Teacher Moves

Activity Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. Then consider using the dashboard to show the distribution of responses, calling attention to any conflict or consensus you see.

Introduce the term *rate of change.* The rate of change in a linear relationship is the amount y changes when x increases by 1. The rate of change in a linear relationship is also the slope of its graph. Invite students to identify the rate of change for the tanks. [Tank A fills at a rate of 5 liters per minute, while Tank B fills at a rate of 7.5 liters per minute.]

Early Student Thinking
Some students may still rely on the visual appearance of the lines rather than the information they now have about the axis scales. Consider selecting a point that each line passes through (e.g., $(2, 10)$ for Tank A and $(2, 15)$ for Tank B) and asking students what each point means in context.

Pacing
Consider using pacing to restrict students to Screens 2–3, one screen at a time.

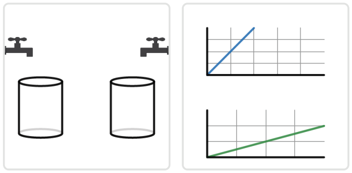
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Responses vary.

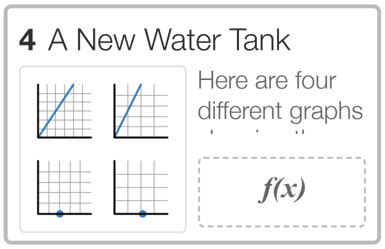
- Tank B will fill up faster. After 4 minutes, Tank B has 30 liters, while Tank A has only 20 liters.
- Tank B will fill up faster. I looked at the graphs to see what the volume of each tank will be after one minute and saw that Tank A fills at a rate of 5 liters per minute, while Tank B fills at a rate of 7.5 liters per minute.

3 Rates of Change



Teacher Moves

To aid students who relied only on visual appearance on Screen 2, consider using this animation to further establish what the graphs communicate about the two water tanks.



4 A New Water Tank

Here are four different graphs

Here are four different graphs showing the same relationship between volume and time for Tank C.

Drag the movable points so that all four graphs show the same proportional relationship.

Then write an equation that represents the relationship between volume, V, and time, t, shown in all four graphs.

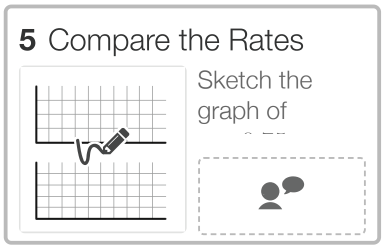
Teacher Moves

Consider using pacing to restrict students to Screens 4–7.

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

- The bottom-left graph should pass through the point $(4, 6)$.
- The bottom-right graph should pass through the point $(8, 12)$.
- $V = \frac{3}{2}t$



5 Compare the Rates

Sketch the graph of

Sketch the graph of $y = 0.75x$ on each set of axes.

Then explain how you decided where to sketch the lines.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to make connections between an equation of a proportional relationship and how it looks on a graph, particularly that the graph will look different depending on how the axes are scaled.

Highlight unique answers for the class. Ask students to justify their responses and critique each other's reasoning (MP3).

Sample Responses

- The top graph should be a line passing through $(0, 0)$ and $(4, 3)$.
- The bottom graph should be a line passing through $(0, 0)$ and $(12, 9)$.

Responses vary.

- I converted 0.75 to $\frac{3}{4}$. Then I tried to find points for my lines to pass through that begin at $(0, 0)$ and have a slope of $\frac{3}{4}$.

---

6 Match each equation …

Teacher Moves

Facilitation
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking
Some students may match only one graph per equation. Encourage students who take this approach to continue working as some equations may have more than one graph, and others may have none.

Sample Responses

Image solution

---

7 Are You Ready for M…

A giant tortoise travels at 0.17 miles per hour, and an arctic hare travels at

A giant tortoise travels at 0.17 miles per hour, and an arctic hare travels at 37 miles per hour.

Complete the following questions on paper:

1. Draw separate graphs showing the relationship between time elapsed, in hours, and distance traveled, in miles, for both the tortoise and the hare.

2. Would it be helpful to put both graphs on the same pair of axes? Why or why not?

3. The tortoise and the hare start out together, and after half an hour, the hare stops to take a rest. How long does it take the tortoise to catch up?

Teacher Moves

⚠ Before students can see this “Are you ready for more?” screen, they'll have to press a button that says, “I'm ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 2–6 ahead of time before the Lesson Synthesis. Because only a subset of your students will complete this screen, we recommend you don't discuss it with the entire class.

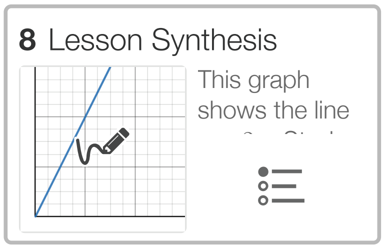
Sample Responses

Responses vary.

1. Each graph should have "time elapsed (hours)" on the horizontal axis and "distance traveled (miles)" on the vertical axis. The scale for the giant tortoise graph is likely much smaller than the scale for the arctic hare graph.

2. Because the scales for each vertical axis are so different, it is very difficult to put both graphs on the same axes without one of the graphs being squashed up very close to an axis. This makes it difficult to read coordinate values from the graph, so it is not very useful.

3. After half an hour, the hare has traveled $0.5 \cdot 37 = 18.5$ miles, and the tortoise has traveled $0.5 \cdot 0.17 = 0.085$ miles, so the hare is $18.5 - 0.085 = 18.415$ miles ahead of the tortoise. Assuming the hare doesn't move, it will take the tortoise $\frac{18.415}{0.17} = 108.32$ hours to catch up (or about 4.5 days).



8 Lesson Synthesis

This graph shows the line

This graph shows the line $y = 2x$. Study the scale used on each axis.

Then complete the following tasks on graph paper:

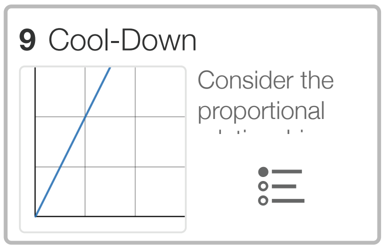
Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface how scaling an axis can make the same relationship look more or less steep.

Lesson Synthesis Launch
Provide access to graph paper and a straightedge. Give students several minutes to create their graphs on paper. Then ask: *What kind of vertical scale did you use to make the line look less steep? What kind of horizontal scale did you use to make the line look more steep?*

Sample Responses

To make $y = 2x$ look *less* steep, increase each interval on the vertical axis.
To make $y = 2x$ look *more* steep, increase each interval on the horizontal axis.



9 Cool-Down

Consider the proportional

Consider the proportional relationship on the left.

Which of the graphs below show the same relationship?

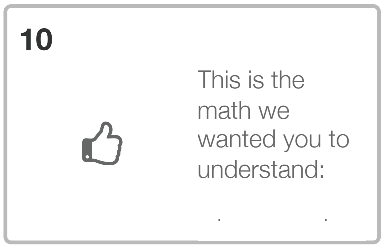
Teacher Moves

Support for Future Learning
If students struggle to identify equivalent scales, plan to revisit this when opportunities arise over the next several lessons. Consider spending extra time during the warm-up of Lesson 3 discussing possible scales for the axes.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

The original graph shows the relationship $y = \frac{1}{2}x$. The top-left and bottom-right graphs show this same relationship.



10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can graph a proportional relationship from an equation.
- I can tell when two graphs are of the same proportional relationship even if the scales are different.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Posters

Lesson 3: Comparing Proportional Relationships

Overview

In this lesson on proportional relationships, students expand on the work of the previous lessons by comparing two situations represented in different ways.

Note: This is a digital lesson that includes an activity where students create visual displays.

Learning Goals

- Compare two different proportional relationships given different representations of them.

Materials

- Tools for creating a visual display

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson on proportional relationships, students expand on the work of the previous lessons by comparing two situations represented in different ways. For example, one situation might specify a constant of proportionality, while the other is represented by a table of values, a graph, or an equation. Students move flexibly between representations and consider how to find the information they need from each type. They respond to context-related questions that compare the two situations and solve problems with the information they've garnered from each representation.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to connect the graph, the equation, and the context of a proportional relationship. They'll exercise their own creativity by inventing the context and specifying the variables in the relationship. This work is preparation for the next activity where students compare proportional relationships represented in different ways.

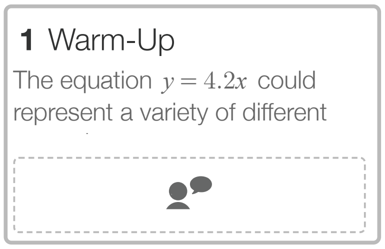
Activity 1: Comparing Two Different Representations (30 minutes)

The purpose of this activity is for students to compare two different proportional relationships represented in different ways using the skills they have worked on over the past two lessons. Working in groups, students compare the relationships and respond to questions about the rates of change, about which rate of change is higher, and about one other situation. Groups make a visual display for their problem set to explain each of their responses and convince others of their accuracy.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to critically consider the work of their classmates and to use their approaches to more effectively interpret proportional relationships represented in a variety of ways.

Cool-Down (5 minutes)



1 Warm-Up

The equation $y = 4.2x$ could represent a variety of different

The equation $y = 4.2x$ could represent a variety of different scenarios.

Write a scenario represented by this equation. Decide what quantities x and y represent in your scenario.

Teacher Moves

Warm-Up Launch
Consider arranging students into pairs. Give them 2–3 minutes of quiet work time, followed by a whole-class discussion.

Pacing
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

Responses vary.

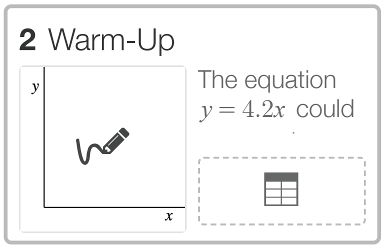
A frog jumps 16.8 feet in 4 seconds. In my scenario, x represents time (in seconds) and y represents distance jumped (in feet).

Student Supports

Students With Disabilities

- *Receptive/Expressive Language: Processing Time*
Students who benefit from extra processing time would also be aided by MLR 1 (Stronger and Clearer Each Time) for written descriptions.

- *Fine Motor Skills: Peer Tutors*
Pair students with their previously identified peer tutors, and allow students who struggle with fine motor skills to dictate graphing as needed.



2 Warm-Up

The equation $y = 4.2x$ could

The equation $y = 4.2x$ could represent a variety of different scenarios.

Make a table with at least three rows, and sketch a graph to represent the scenario.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface how to see information about a proportional relationship in a table, graph, and equation.

Monitor the different ways students label their axes and specify a scale. Consider using snapshots or the teacher view in the dashboard to highlight interesting or helpful sketches.

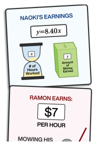
Invite several students to share their situations from Screen 1. Use the teacher dashboard to display their graphs while they share. Consider asking the class: *What does the rate of change represent in this situation?*

Sample Responses

- The graph should have a line that begins at the origin and has a slope of 4.2.
- The table should contain values that reflect the relationship $y = 4.2x$. For example, $(1, 4.2)$, $(2, 8.4)$, and $(5, 21)$.

---

3 Create a visual displa…



Naoki babysits her neighbor's children. Ramon mows his neighbors' lawns.

1. Who makes more money after working 12 hours? How much more do they make? Explain how you know.

2. What is the rate of change for each situation and what does it mean?

3. How long would it take each person to earn $\$150$? Explain or show your reasoning.

Teacher Moves

⚠ Before they can see this screen, students will have to find a group and press a button that says, "We're ready!"

Activity Launch
Remind students that in previous lessons, they identified representations of and created representations for a single proportional relationship. In this activity, they will consider representations of two different proportional relationships and make comparisons between them.

Arrange students in groups of 2–3, and invite groups to choose one of the two scenarios. Tell groups they will make a visual display for their responses to the questions. The display should clearly demonstrate their reasoning and use multiple representations in order to be convincing. Let them know that there will be a gallery walk when they finish for the rest of the class to inspect their solutions' accuracy.

Early Student Thinking

Some students may confuse the values for the rate of change of a situation. For example, Lemonade Recipe 1's equation, $y = 4x$, shows that the rate of change is 4 cups of water per cup of lemonade mix. Students may switch these values and think that the rate of change is 4 cups of lemonade mix per cup of water. Ask students who do this to explain where they see the rate of change in the original representations. Students may need to list a few values or sketch a graph in order to see their mix-up between the two quantities.

Pacing
Consider using pacing to restrict students to Screens 3–4.

Sample Responses

Naoki and Ramon
1. Naoki will earn $\$16.80$ more after working 12 hours. For 12 hours of work, Naoki makes $8.40(12) = 100.8$, and Ramon makes $7(12) = 84$.

2. The rate of change for Naoki is $\$8.40$. The rate of change for Ramon is $\$7$. In each situation, the rate of change is the amount of additional money earned for each additional hour of work.

3. Naoki would have to work about 17.9 hours and Ramon would have to work about 21.4 hours to earn $\$150$.

Ahmed's Lemonade
1. For Recipe 1, Ahmed needs 64 cups of water because $x = 16$, so $y = 4(16) = 64$. For Recipe 2, Ahmed needs 80 cups of water because the table shows that for each cup of mix we add 5 cups of water, and $5(16) = 80$.

2. The rate of change for Recipe 1 is 4 cups of water per cup of mix. The rate of change for Recipe 2 is 5 cups of water per cup of mix. In each situation, the rate of change is the amount of additional water needed for each additional cup of mix.

3. Ahmed should use Recipe 1 since there is 4 times as much water as there is lemonade mix, and Recipe 1 requires 4 cups of water for each cup of mix.

Student Supports

For students who struggle when working in groups, consider one of the following modifications:

- **Option 1 (Independent First):** Assign the task to each student, and allow them to select a scenario and work through the questions on their own first. Then assign groups to combine their thinking into a convincing and unified response.

- **Option 2 (Pairs):** Assign the task to each pair of students, and allow them to select a scenario and work through the questions together. Then assign two pairs to work together as a group of four to combine their thinking into one convincing and unified response.

English Language Learners

- *Lighter Support: MLR 8 (Discussion Support)*

Add these questions to each of the situations in this activity: What strategies did you use to find the rate of change in the _____ (i.e., table, equation, or graph)? What types of wording in the problem statements guided your decision?

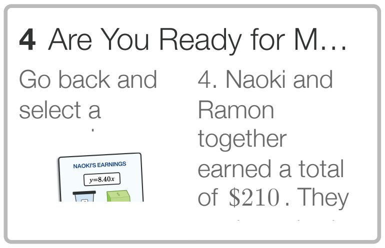
Students With Disabilities

- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., presenting one problem at a time) to aid students who benefit from support with organizational skills in problem solving.

- *Conceptual Processing: Processing Time*

Check in with individual students, as needed, to assess for comprehension during each step of the activity.



4 Are You Ready for M…

Go back and select a

4. Naoki and Ramon together earned a total of $\$210$. They

Go back and select a scenario.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

The questions on this screen depend on the scenario selected on the previous screen.

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screen 3 ahead of time before the class discussion on Screen 5. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Naoki and Ramon

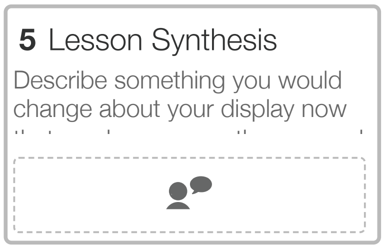
4. If Naoki and Ramon each worked the same number of hours to earn $\$210$, then their combined income is $\$15.40$ per hour. This means they worked about 13.64 hours each since $\frac{210}{15.4} = 13.64$. Since Naoki makes $\$8.40$ per hour, she earned about $\$115$ and Ramon makes $\$7$ per hour, which is about $\$95$. Naoki earned $\$20$ more than Ramon.

5. If Naoki and Ramon each earned the same amount of money and combined it to get $\$315$, then they each made $\frac{\$315}{2} = \$157.50$. Naoki must have worked $\frac{\$157.50}{8.40} = 18.75$ hours, while Ramon worked $\frac{\$157.50}{7} = 22.5$ hours. Ramon worked 3.75 more hours.

Ahmed's Lemonade

4. Ahmed should use 7.5 cups of lemonade mix. Using Recipe 1, with $y = 30,$ then $30 = 4x$, and $x = \frac{30}{4} = 7.5$ cups of lemonade mix.

5. Recipe 2 requires the amount of water to be 5 times the amount of lemonade mix. Ahmed's mistake was that he made the amount of mix to be 5 times the amount of water. Since Ahmed used 20 cups of mix, he should use $20 \cdot 5 = 100$ cups of water. He already added 4 cups of water, so he should add 96 additional cups to fix his mistake.



5 Lesson Synthesis

Describe something you would change about your display now

Describe something you would change about your display now that you have seen other groups' work.

Teacher Moves

Key Discussion Screen

The purpose of this discussion is for students to reflect on what makes a poster of a proportional relationship strong and clear.

Lesson Synthesis Launch

Begin with a gallery walk for students to see how other groups answered the same set of questions they did or how they answered questions for the other context.

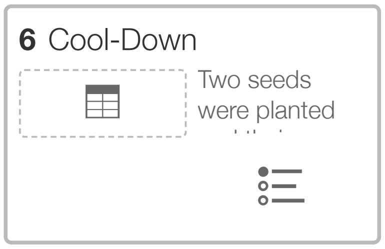
Invite groups to share the strategies they used with the various representations. Consider asking groups the following questions:

- *What representations did you choose to answer the questions? Why did you pick them?*
- *What representation did you not use? Why?*
- *How did you decide what scale to use when you made your graph?*

Consider asking some of the following questions. Ask students to use examples from today's lesson when responding, if possible.

- *What do you need in order to compare two proportional relationships?*
- *What type of wording in a problem statement or description of a situation tells you that you have a rate of change?*
- *How did you decide which representation to use to solve the different types of problems?*

Pacing
Consider using pacing to restrict students to this screen.


6 Cool-Down

Two seeds were planted and their heights were measured each day.

Plant A's data was recorded in a table, while Plant B's data is in a graph.

Which plant grew at a faster rate?

Teacher Moves

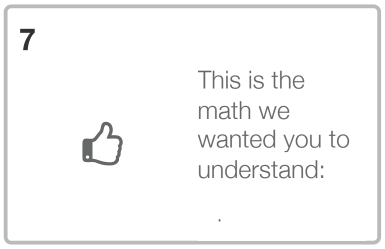
Support for Future Learning
If students struggle to determine the rates of change, plan to revisit this when opportunities arise over the next several lessons. Consider focusing Lesson 4's lesson synthesis discussion on the slope of the line.

Pacing
Consider using pacing to restrict students to Screens 6–7.

Sample Responses

Plant B

Responses vary. The point $(5, 15)$ is on Plant B's graph, so Plant B's growth rate is 3 centimeters per day. From the table, you can calculate the unit rate for Plant A, $3 \div 2 = 1.5$, and see that it is a slower rate than Plant B.



7

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can compare proportional relationships represented in different ways.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Stacking Cups

Lesson 4: Introduction to Linear Relationships

Overview

After revisiting examples of proportional relationships in the previous lessons, this lesson is the first of several lessons that moves from proportional relationships to linear relationships with positive rates of change.

Learning Goals

- Understand that there are linear relationships that are not proportional.
- Recognize that the rate of change of a linear relationship is the same value as the slope of the graph of the relationship.

Vocabulary

- linear relationship

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

After revisiting examples of proportional relationships in the previous lessons, this lesson is the first of several lessons that moves from proportional relationships to linear relationships with positive rates of change. The activities in this lesson are based on a situation where the height of a stack of styrofoam cups is not proportional to the number of cups in the stack. Students use the same tools they use to represent proportional relationships—graphs, tables, and equations—to represent nonproportional linear relationships. They see that each cup increases the height of the stack by the same amount (unit rate becomes rate of change) and that they can use this to answer questions about the height for an unknown number of cups. They investigate and describe similarities and differences between linear relationships and proportional relationships in this context. They make connections between the rate of change of the relationship and the slope of a line representing the relationship. In this lesson, the focus is proportionality vs. linear relationships and rate of change. The meaning of the vertical intercept of the graph comes up briefly but will be revisited more fully in the next lesson.

Interpreting features of a graph or an equation in terms of a real-world context is an important component of mathematical modeling (MP4).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to provide students with an easy entry point (via estimation) for the key question of the lesson: "How many stacked cups are needed to reach the top of the 50 cm table?"

Activity 1: Stacking Cups (30 minutes)

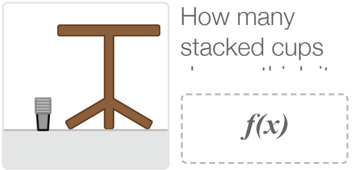
The purpose of this activity is for students to gather data, to look for patterns related to rate of change, and to use those patterns to make a more precise prediction for a nonproportional linear scenario. Students use representations and ideas from previous lessons on proportional relationships. They use tables and graphs to represent the situation, and they make deductions by generalizing from repeated reasoning (MP8), arguing that each additional cup increases the height of the stack by the same amount.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to help students: a) understand that there are linear relationships that are not proportional, b) note that the rate of change of a linear relationship is the same value as the slope of a line representing the relationship, and c) interpret the rate of change in the context of the situation.

Cool-Down (5 minutes)

1 Warm-Up



How many stacked cups do you think it would take to reach the top of the table?

What information would help you make a more precise prediction?

Teacher Moves

Purpose
The purpose of this lesson is for students to a) find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship and b) use what they know about that rate of change to determine the number of cups required to reach a height of 50 cm.

Warm-Up Launch
Ask students to make a prediction about how many cups it would take to build a stack that reaches the top of the table. Tell students that their job in this lesson is to figure out how many cups are needed in order to stack them to a height of 50 centimeters, which is the height of the table.

Sample Responses

Responses vary.

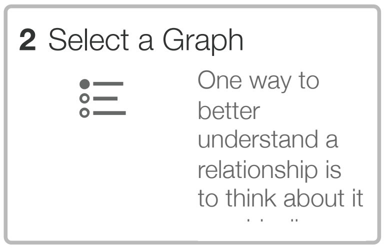
- 10 cups
- 60 cups
- 30 cups (actual answer)

- Knowing the total height of each cup as well as the height of the "lip" of a cup would help.
- If we knew the height of several stacks of cups, each with a different number of cups, we could determine the answer more precisely.

Student Supports

Students With Disabilities
- *Conceptual Processing: Manipulatives*

Begin with realia (i.e., real foam cups and rulers) to provide access for students who benefit from concrete contexts.



2 Select a Graph

One way to better understand a relationship is to think about it

One way to better understand a relationship is to think about it graphically.

Which graph can be used to represent the relationship between stack height and number of cups?

Teacher Moves

Activity Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

Introduce the term *linear relationship*, which is when one quantity changes by a certain amount, the other quantity always changes by a set amount. In a *linear relationship*, one quantity has a constant rate of change with respect to the other.

The relationship is called linear because its graph is a line.

Early Student Thinking
Some students may have difficulty deciding between two of the options (top right and bottom right) since both of these graphs represent a non-proportional, linear relationship. The top-right graph uses a continuous line, while the bottom-right uses discrete points. The bottom-right graph may *technically* be more visually accurate (because the relationship is true for integer numbers of one or more cups and is not valid for non-integer numbers of cups), however the top-right graph is still a common way to represent the scenario, and it could be considered both helpful and valid.

Other students may think that the top-left graph represents the situation because it accurately shows that a stack of 0 cups has a height of 0 centimeters. However, it also implies that the relationship is proportional (in other words, doubling the number of cups would double the stack height), which is not true.

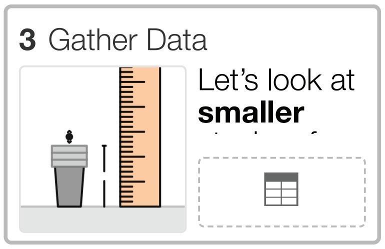
Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

Two responses may be considered valid.

- **Top right:** A non-proportional *linear relationship* with a positive slope, represented as a continuous line.
- **Bottom right:** A non-proportional *linear relationship* with a positive slope, represented as discrete points.



3 Gather Data

Let's look at **smaller**

Let's look at **smaller** stacks of cups to help us make a more accurate prediction about how many stacked cups it would take to reach the top of the table.

Drag the movable point to adjust the number of cups and record the data in the table.

Collect at least two data points before continuing to the next screen.

Teacher Moves

Early Student Thinking
Some students may think that the relationship between stack height and number of cups is proportional. Invite those students to compare the height of 6 cups and the height of 12 cups as an example so they can see that doubling the number of cups does not double the height.

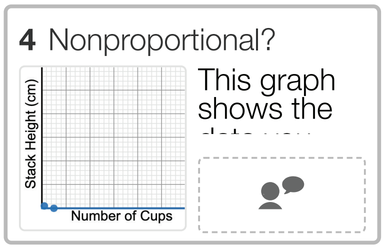
Tell students that even though this relationship is not proportional, it shares things in common with proportional relationships—a topic we'll explore in the next activity.

Pacing
Consider using pacing to restrict students to Screens 3–6 until a majority of students are ready to verify their answer on Screen 7.

Sample Responses

Responses vary.

- $(3, 12.2)$
- $(5, 15)$
- $(12, 24.8)$



4 Nonproportional?

This graph shows the

This graph shows the data you collected (in blue) and the data your classmates collected (in light blue).

Why is a proportional model a bad fit here?

Note: You can drag the movable point to create a proportional model if it helps you with your thinking.

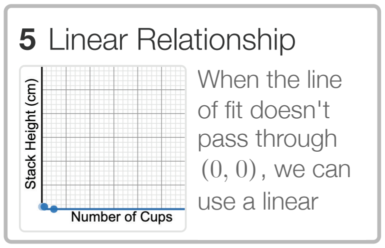
Teacher Moves

Use the teacher view in the teacher dashboard to identify students who may need additional support.

Readiness Check (Problem 4)
If most students struggled, plan to use this screen as an opportunity to strengthen student understanding of how a coordinate plane relates to the context. Consider asking students what each of the points on the graph represents.

Sample Responses

A proportional model won't work here because the line of fit doesn't go through the point $(0, 0)$.



5 Linear Relationship

When the line of fit doesn't pass through $(0, 0)$, we can use a linear

When the line of fit doesn't pass through $(0, 0)$, we can use a linear model.

Drag the movable points to fit the linear model to the data.

Teacher Moves

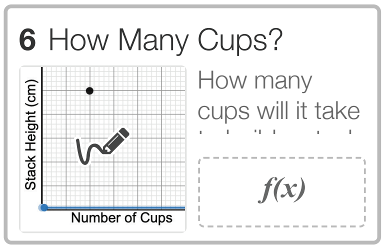
Key Discussion Screen 🔑
The purpose of this discussion is to introduce the term *linear relationship* and surface when linear relationships may be more useful than proportional relationships.

Use the teacher view in the teacher dashboard to identify students who may need additional support.

Students may notice that one of the points always has an x coordinate of 0 and may wonder how it is possible to have 0 cups with a height that is not 0 cm tall. Consider sharing with students that sometimes we can begin a line at a point that does not have meaning in our context. Even though you can't have 0 cups that are 8 cm tall, it still makes sense to begin the line at that point.

Sample Responses

The linear model should go through the point $(0, 8)$ and have an approximate slope of 1.4.



6 How Many Cups?

How many cups will it take

How many cups will it take to build a stack as tall as the top of the table ($50\,\text{cm}$)?

Enter your answer below. Then press "Submit."

Use the sketch tool if it helps you with your thinking.

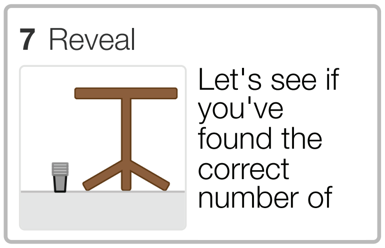
Teacher Moves

Early Student Thinking
Watch for students who use data points such as $(5, 15)$ to compute the ratio $\frac{15}{5} = 3$ and interpret this as the increase of the stack height per cup. Encourage these students to return to Screen 3 to further explore the relationship between number of cups and height of the cup stack.

Sample Responses

Responses vary.

Students will be marked correct for answers between 29 and 31 cups.



7 Reveal

Let's see if you've found the correct number of

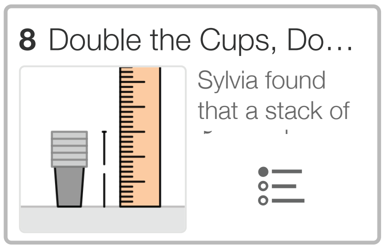
Let's see if you've found the correct number of cups.

Press "Try It" to see if you're right.

Teacher Moves

Invite students to revise their answers on Screens 5–6 based on the feedback they see on this screen. Encourage them to justify any changes they make rather than use the feedback on this screen as a tool for guessing and checking without reflection. Once students are ready, they may continue to Screens 8 for a reflection question and to Screen 9 for an optional challenge.

Pacing
Consider using pacing to restrict students to this screen and then to Screens 3–9.



8 Double the Cups, Do…

Sylvia found that a stack of

Sylvia found that a stack of 5 cups has a height of 15 cm.

She thinks a stack of 10 cups will have a height of 30 cm.

Is she correct?

Teacher Moves

Highlight unique answers for the class. Ask students to justify their responses and critique each other's reasoning.

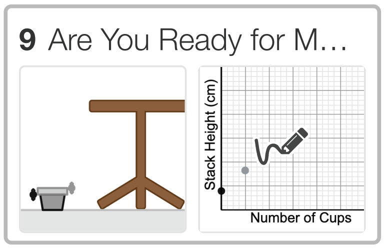
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

No

Responses vary.

- We can’t double the height for double the cups because the relationship isn’t proportional.
- Her calculations say that each cup adds 2.2 centimeters to the height of the stack, which isn’t true. Each cup only adds 1.4 centimeters to the height of the stack.



9 Are You Ready for M…

Teacher Moves

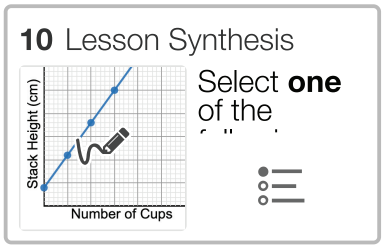
⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 2–8 ahead of time before the class discussion on Screen 10. Because only a subset of your students will complete this screen, we recommend you don’t discuss it with the entire class.

Sample Responses

Responses vary.

Each graph should pass through the point $(5, 50)$.



10 Lesson Synthesis

Select **one** of the

Select **one** of the following questions and record your response.

Use the sketch tool if it helps you to show your thinking.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to make sense of a graph of a linear relationship that is not proportional, including how to determine its slope.

Synthesis Launch
Give students 2–3 minutes of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

Early Student Thinking
If not suggested by students, draw a slope triangle with a horizontal distance of 1 on the graph. Highlight the fact that the rate per 1 cup (1.4) is not the constant of proportionality since this is not a proportional relationship! This value is how much each cup adds to the height of the stack, and it is called the *rate of change*. The rate of change of y in a linear relationship between x and y is the increase in y when x increases by 1. Note that the rate of change of the relationship has the same value as the slope of the line representing the relationship. This means that asking, *What is the slope?* is the same as asking, *How much height does each cup after the first add to the stack?*

Pacing

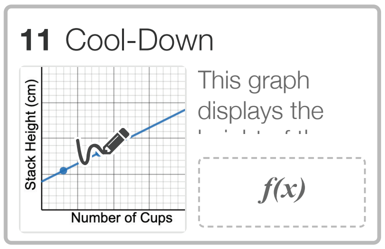
Consider using pacing to restrict students to this screen.

Sample Responses

1. You can tell that the number of cups isn't proportional to the height of the stack because the graph doesn't go through the point $(0, 0)$.

2. I found the slope of the line by drawing a slope triangle to find how much each cup added to the height of the stack.

3. The point $(0, 8)$ tells us that the distance from the bottom to the rim of the first cup is 8 cm.



11 Cool-Down

This graph displays the

$f(x)$

This graph displays the height of the stack in centimeters for different numbers of cups.

How much does each cup after the first add to the height of the stack?

Teacher Moves

Support for Future Learning
If students struggle to determine the slope of the line, plan to revisit this when opportunities arise over the next several lessons. Consider spending extra time during Lesson 5's warm-up discussing the slope of the line representing the purple flag's height.

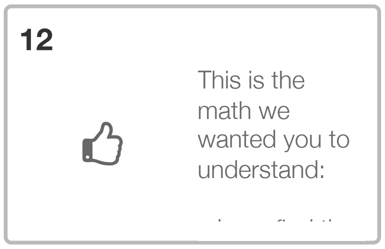
Pacing
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

0.5 cm (or equivalent)

Responses vary.

The line passes through $(3, 5.5)$ and $(8, 8)$, which means that adding 5 cups added 2.5 centimeters to the stack. So each cup adds $\frac{2.5}{5} = 0.5$ centimeters.



12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Flags

Lesson 5: Representations of Linear Relationships

Overview

The previous lesson looked in depth at an example of a linear relationship that was not proportional and then examined an interpretation of the slope as the rate of change for a linear relationship. In this lesson, slope remains important. In addition, students learn the new term vertical intercept or y-intercept for the point where the graph of the linear relationship touches the y-axis.

Learning Goals

- Identify and interpret the positive vertical intercept of the graph of a linear relationship.
- Identify and interpret the slope of the graph of a linear relationship.
- Write an equation for a linear relationship by expressing regularity in repeated reasoning.

Vocabulary

- vertical intercept

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The previous lesson looked in depth at an example of a linear relationship that was not proportional and then examined an interpretation of the slope as the rate of change for a linear relationship. In this lesson, slope remains important. In addition, students learn the new term *vertical intercept* or *y-intercept* for the point where the graph of the linear relationship touches the y-axis.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to the general context in this lesson (the relationship between flag height and time) and to connect visual and graphical representations of one specific flag.

Activity 1: Flags, Part 1 (5 minutes)

The purpose of this activity is for students to relate the starting height and speed of a flag to a graph showing the flag's height over time (MP2).

Activity 2: Flags, Part 2 (15 minutes)

The purpose of this activity is for students to make connections between various representations (including graphs, tables, and expressions) of two flags' height and time (MP4). Students will use repeated reasoning of a flags height at specific times to develop an equation modeling this relationship (MP8).

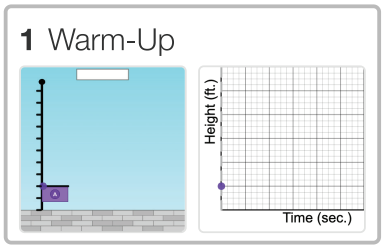
Activity 3: Flags, Part 3 (10 minutes)

The purpose of this activity is for students to strengthen their understanding of how the parameters in a linear equation affect the positive vertical intercept and slope of a graph.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to discuss how to use a graph or an equation to identify the vertical intercept and slope of a given scenario and make sense of them in context.

Cool-Down (5 minutes)



1 Warm-Up

Teacher Moves

Purpose
The purpose of this lesson is for students to identify and interpret the positive vertical intercept and the slope of a linear relationship and to make connections between different representations (including tables, graphs, and equations) of those linear relationships.

Warm-Up Launch
Arrange students into pairs and invite them to watch the animation and the graph. Then pause for a discussion before continuing to the next screens. Ask students to describe 2–3 things they notice about the animation and the graph, including any connections they see between these two representations. Encourage students to replay the animation several times to see if they notice any additional details.

Readiness Check (Problem 6)
If most students struggled, plan to review this problem with them before beginning the lesson. Be sure to amplify terms like "constant of proportionality" and "unit rate" throughout this lesson.

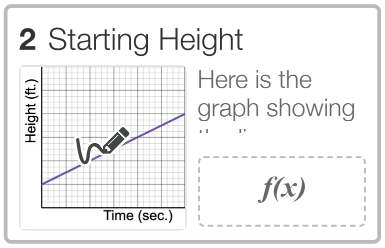
Pacing
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

Responses vary.

- The flag moves up and the line in the graph moves up (as you look from left to right).
- The flag stops at 32 feet, which is 8 feet below the highest mark on the flagpole.
- The top of the flag starts 8 feet above the ground.



2 Starting Height

Here is the graph showing

Here is the graph showing the linear relationship between the height and time of Flag A.

What is the flag's starting height?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to introduce the term *vertical intercept* and make connections between what the vertical intercept is and what it means about the flag.

Activity Launch
Give students one minute of quiet think-time and 1–2 minutes to discuss with a partner. Invite several students to share their responses.

Introduce the term *vertical intercept*. The vertical intercept is the point where the graph of a line crosses the vertical axis.

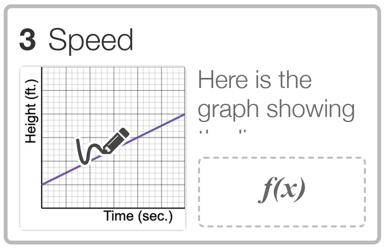
Pacing
Consider using pacing to restrict students to Screens 2–3.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Sample Responses

8 ft.

Responses vary. I looked at the vertical axis and saw that when time is 0 seconds, the height is 8 feet.



3 Speed

Here is the graph showing

Here is the graph showing the linear relationship between the height and time of Flag A.

What is the flag's speed?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Give students one minute of quiet think-time and 1–2 minutes to discuss with a partner. Invite several students to share their responses.

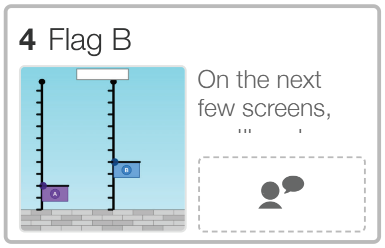
Sample Responses

4 ft. per sec.

Responses vary.

- The line passes through $(0, 8)$ and $(1, 12)$. So the flag is traveling 4 feet per second.

- It took 6 seconds for the flag to move from 8 feet to 32 feet. That is 24 feet in 6 seconds, or 4 feet per second.


4 Flag B

On the next few screens,

On the next few screens, you'll explore the relationship between height and time for a new flag.

Press play to watch a short animation.

Then describe the behavior of Flag B.

Teacher Moves

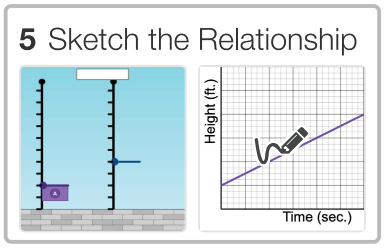
Activity Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. If the starting height or the speed of Flag B does not come up, consider discussing it in comparison to Flag A.

Pacing
Consider using pacing to restrict students to Screens 4–5.

Sample Responses

Responses vary.

- Flag B starts out higher than Flag A, but ends up lower.
- Flag B moves up.
- Flag B begins at 16 feet, and after 6 seconds, ends at 28 feet.


5 Sketch the Relationship

Teacher Moves

Invite students to play the animation before attempting to sketch the relationship between height and time. Let them know that the horizontal segment (which begins at 16 feet) is the “target.”

Encourage students to press play to verify the accuracy of their sketch and revise their sketch for increased accuracy, as appropriate (MP6).

Consider using snapshots or the teacher view of the dashboard to display unique answers to the class. Ask students to justify their responses and critique each other's reasoning.

Early Student Thinking

Some students may sketch a blue line that does not extend all the way to $t = 6$ seconds and may wonder why the blue flag disappears at the end of the animation. Consider asking: *Where in your graph can we see the height of the blue flag at* 6 *seconds?*

Sample Responses

Image solution

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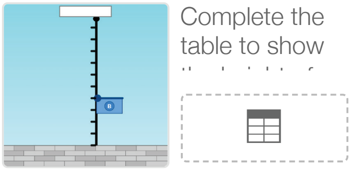
6 Complete the Table



Complete the table to show the height of Flag B over time.

Then press "Check My Work."

Teacher Moves

Give students one minute of quiet think-time. Invite them to share their answers and discuss their strategies with a partner. Then ask several students to describe their strategies to the class.

Early Student Thinking
Some students may think that the height of the flag at 6 seconds will be double the height of the flag at 3 seconds. Invite them to compare the height at 2 seconds compared to the height at 1 second to see that doubling the time does not result in a doubling of the height for nonproportional linear relationships.

Pacing
Consider using pacing to restrict students to Screens 6–7.

Sample Responses

At 3 seconds, the height is 22 feet.
At 6 seconds, the height is 28 feet.

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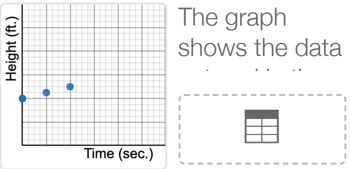
7 Extend the Table



The graph shows the data entered in the table.

Write an expression for the height of the flag after t seconds.

Then press "Check My Work."

Teacher Moves

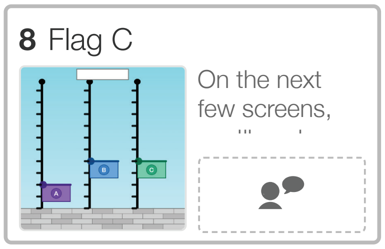
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking
Some students may struggle to generalize the height of the flag. Consider asking: *If the animation kept going (and the flagpole were taller), how could we find the flag's height at* 20 *seconds?* [The flag starts at 16 feet and goes up 2 feet per second, so after 20 seconds it would be at $16 + 2 \cdot 20 = 16 + 40 = 56$ feet.]

Then invite students to use similar reasoning to determine the height at 30 seconds, 40 seconds, and so on. The repeated reasoning employed to find those heights may help students determine the height at t seconds. Alternatively, consider asking students to describe the height of the flag in words (e.g., The flag starts at 16 feet and goes up 2 feet each second.) Then translate that to an expression ($h = 16 + 2t$).

Sample Responses

$16 + 2t$ (or equivalent)



8 Flag C

On the next few screens, ...

On the next few screens, you'll explore the relationship between height and time for a new flag.

Press play to watch a short animation.

Then describe the behavior of Flag C.

Teacher Moves

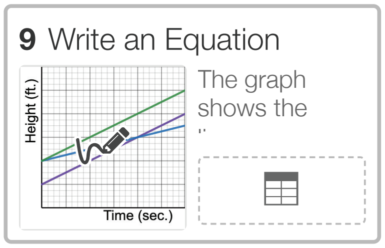
Activity Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. If the starting height or the speed of Flag C does not come up, consider discussing it in comparison to the other flags.

Pacing
Consider using pacing to restrict students to Screens 8–10.

Sample Responses

Responses vary.

- Flag C starts at the same height as Flag B.
- Flag C moves faster than Flag B.
- Flag C begins and ends 8 feet higher than Flag A.



9 Write an Equation

The graph shows the

The graph shows the linear relationship between height and time for each flag.

Write an equation for the height of Flag C.

Then press "Check My Work."

Use the sketch tool if it helps you with your thinking.

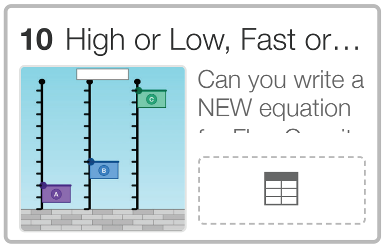
Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking
Some students may struggle to express the height of Flag C with an equation. Invite them to consider, based on the graph, how Flag C is similar to Flag B [starting height] and how Flag C is similar to Flag A [speed]. Then ask: *How could you use that information, along with the equations for Flag A and Flag B, to write an equation for Flag C?*

Sample Responses

$h = 16 + 4t$ (or equivalent)



10 High or Low, Fast or…

Can you write a NEW equation

Can you write a NEW equation for Flag C so it:

Starts high? Starts low? Goes fast? Goes slow?

Experiment with different equations.

Then press play to see what happens.

Teacher Moves

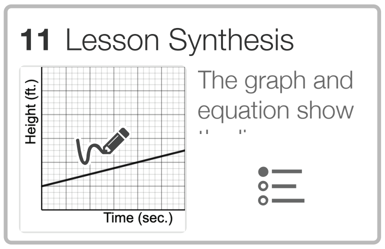
This screen is designed as a free-play experience. Encourage students to experiment with different equations and to reflect on the impact of changing each number in their equation. Which number controls the starting height? Which number controls the speed? How can you make the flag start high or low, or go fast or slow? How can you make the flag go *very* fast?

Early Student Thinking
The table will accept linear equations relating h and t with vertical intercepts between 0 and 40 feet. If students enter an equation that would move the flag below 0 or above 40 *during* the animation, the flag will become semitransparent and stop moving at the bottom of the flagpole (0 feet) or at the top (40 feet).

Rather than offering your own explanation for why the flag turns semitransparent, ask students to offer their own explanation. Invite them to consider what the equation indicates about the flag's height at that time.

Sample Responses

Responses vary.



The graph and equation show the linear relationship between the height and time for a new flag.

Discuss the following questions.

Then select ONE and record your response.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to consolidate how to identify the vertical intercept from a graph and from an equation.

Synthesis Launch
Give students 1–2 minutes to discuss the questions with a partner, followed by 2–3 minutes to record a response to one of the questions. Then use the teacher view of the dashboard to highlight unique answers for the class.

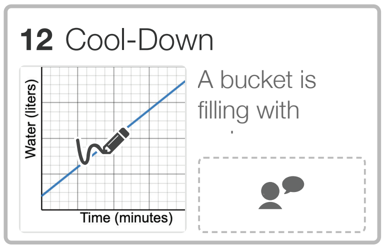
Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

1. You can tell the starting height (8 feet) by looking at the graph to see where the line touches the vertical axis. You can tell the speed by figuring out how much the height increases every 1 second. Between 0 and 4 seconds, the height increases from 8 to 16 feet. That's 8 feet per 4 seconds, or 2 feet per second.

2. You can tell the starting height by looking at the constant in the equation (i.e., the number not being multiplied by a variable). In this case, 8 (feet). You can tell the speed by looking at the number being multiplied by a variable. In this case, that's 2 (feet per second).



12 Cool-Down

A bucket is filling with

A bucket is filling with water.

The graph shows the relationship between water in the bucket, w, and time, t.

What does the 10 in the equation mean in this scenario?

Use the sketch tool if it helps you with your thinking.

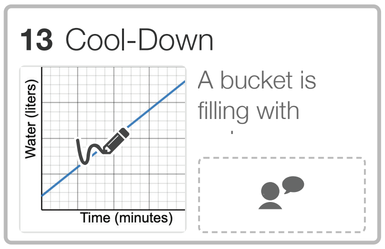
Teacher Moves

Support for Future Learning
If students struggle to describe the meaning of the vertical intercept (Screen 12) or slope (Screen 13) in context, plan to revisit this when opportunities arise over the next several lessons. Consider spending extra time on Screen 6 of Lesson 7 discussing the meaning of 8 in the expression $640 - 8x$.

Pacing
Consider using pacing to restrict students to Screens 12–14.

Sample Responses

Responses vary. 10 is the vertical intercept. When the pouring began, there were 10 liters of water in the bucket.



13 Cool-Down

A bucket is filling with

A bucket is filling with water.

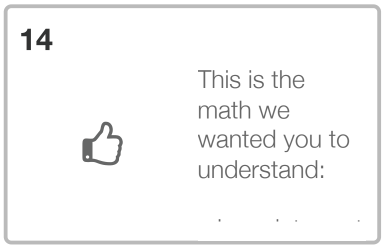
The graph shows the relationship between water in the bucket, w, and time, t.

What does the 2 in the equation mean in this scenario?

Use the sketch tool if it helps you with your thinking.

Sample Responses

Responses vary. 2 is the slope (i.e., speed). Water is pouring into the bucket at 2 liters per minute.



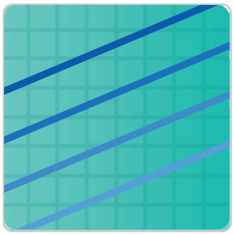
14

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can interpret the vertical intercept of a graph of a real-world situation.
- I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.
- I can use patterns to write a linear equation to represent a situation.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.



Translations

Lesson 6: Translating $y = mx + b$

Overview

This lesson develops a third way to understand an equation of a line in a coordinate plane. In previous lessons, students wrote an equation of a line by generalizing from repeated calculations using their understanding of similar triangles and slope. They also wrote an equation of a linear relationship by reasoning about initial values and rates of change, and they graphed the equation as a line in a coordinate plane. This lesson introduces the idea that any line in a plane can be considered a vertical translation of a line through the origin.

Learning Goals

- Derive $y = mx + b$ by graphing $y = mx$ and the same graph shifted up using the translation that takes $(0, 0)$ to $(0, b)$.
- Connect the equations $y = b + mx$ and $y = mx + b$ to the graph.
- Encounter a graph where the y-intercept is a negative value.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

This lesson develops a third way to understand an equation of a line in a coordinate plane. In previous lessons, students wrote an equation of a line by generalizing from repeated calculations using their understanding of similar triangles and slope (MP8). They also wrote an equation of a linear relationship by reasoning about initial values and rates of change, and they graphed the equation as a line in a coordinate plane. This lesson introduces the idea that any line in a plane can be considered a vertical translation of a line through the origin.

In the previous lesson, the terms in the expression are more likely to be arranged as $b+mx$ because the situation involves a starting amount and the addition of a multiple. In this lesson, $mx+b$ is more likely because the situation involves starting with a relationship that includes $(0, 0)$ and shifting up or down from there. Students continue to only consider lines with positive slopes, but in this lesson, the notion of a negative y-intercept (not in a context) is introduced.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is a) to remind students that the translation of a line is parallel to the original line and b) to plant the seed that a line can be taken to a parallel line by translating it. Students inspect several lines to decide which could be translations of a given line. Then they describe the translations by specifying the number of units and the direction in which the original line should be translated.

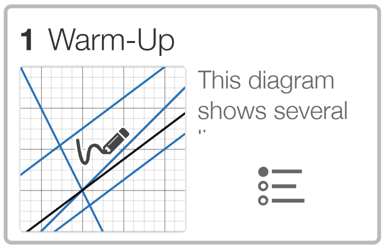
Activity 1: Translating a Line (30 minutes)

The purpose of this activity is for students to encounter and connect graphs and equations of lines to their vertical translations from $(0, 0)$. Students will solve four sets of challenges in which they write equations to match given sets of lines. In the first two challenges, the lines show a proportional relationship. In the third challenge, students consider lines that have a slope of $\frac{3}{2}$ but different y-intercepts. Students will make use of this structure in the equations they write for the two lines and apply that structure (MP7) to a final challenge by writing equations of parallel lines that have been shifted vertically from $(0, 0)$.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to connect the equations of lines with their vertical transformations.

Cool-Down (5 minutes)


1 Warm-Up

This diagram shows several

This diagram shows several lines.

You can only see part of the lines, but they actually continue forever in both directions.

Which lines are images of line f under a translation? Use the sketch tool if it helps you with your thinking.

Teacher Moves

Warm-Up Launch
Arrange students into pairs. Give students two minutes of quiet think-time. Then ask them to share their responses with a partner.

Early Student Thinking
Students may think that lines i and h can't be images of line f because the parts of i and h that we can see are shorter than the part of f that we can see. Tell these students that all of the lines go on indefinitely in both directions.

Readiness Check (Problem 5)
If most students struggled, plan to use this warm-up to review translations. If students need additional practice recalling translations, refer to 8.1.03: Transformation Golf.

Pacing
Consider using pacing to restrict students to Screens 1–2.

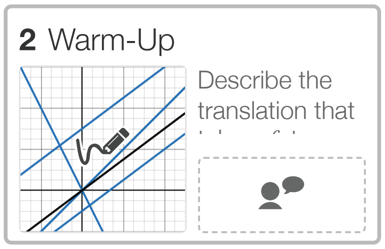
Sample Responses

Lines h and i are images of line f under a translation.

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity.



2 Warm-Up

Describe the translation that

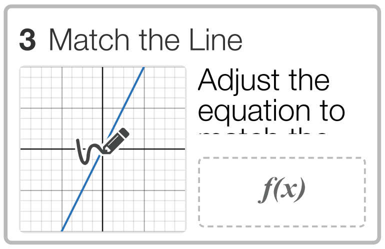
Describe the translation that takes f to each line that is a translation of f .

Teacher Moves

Invite students to share how they know that lines h and i are translations of f . Make sure students see that those lines are parallel to f .

Sample Responses

- Line h has been translated 6 units up from line f .
- Line i has been translated 2 units down from line f .



3 Match the Line

Adjust the equation to

Adjust the equation to match the line.

Then press "Try It."

Use the sketch tool if it helps you with your thinking.

Teacher Moves

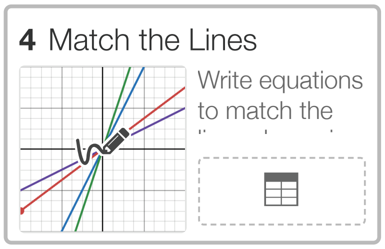
Activity 1 Launch
Ask students to adjust the equation to match the line shown in the graph. Tell students that their task in this activity is to write an equation of a line using the slope of the line and how far it has been translated vertically.

Early Student Thinking
So far in this unit, students have had experience writing equations by finding the constant of proportionality. They've also found the slopes of lines on a coordinate grid. If students aren't sure how to adjust the equation to match the line, ask them to find the slope. Let them know that the slope is the same as the constant of proportionality for a proportional relationship and that they can use that information to find the equation of the line.

Pacing
Consider using pacing to restrict students to Screens 3–9.

Sample Responses

$y = 2x$



4 Match the Lines

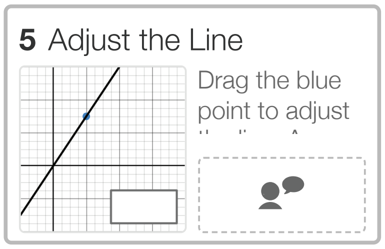
Write equations to match the

Write equations to match the lines shown in the graph. Then press "Check My Work." (One is done for you.)

Use the sketch tool if it helps you with your thinking.

Sample Responses

- Line b: $y = 3x$
- Line c: $y = \frac{1}{2}x$
- Line d: $y = \frac{3}{4}x$



5 Adjust the Line

Drag the blue point to adjust

Drag the blue point to adjust the line. As you drag the point, pay attention to how the equation changes.

What do you notice about the relationship between the vertical intercept of the blue line and its equation?

Teacher Moves

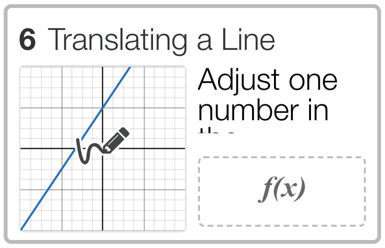
Key Discussion Screen
The purpose of this discussion is to make connections between how a line is translated and the equation of the line.

Use snapshots to highlight several student responses. Ask questions to help students connect concrete and abstract responses as well as formal and informal responses.

Sample Responses

Responses vary.

The arrow shows the vertical distance between the blue line and the black line. In the equation, the number being added to the x-term represents the number of units the blue line has been translated vertically or the vertical intercept of the blue line.



6 Translating a Line

Adjust one number in

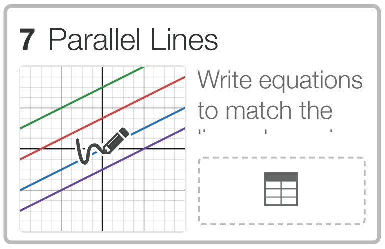
Adjust one number in the equation to match the line.

Then press "Try It."

Use the sketch tool if it helps you with your thinking.

Sample Responses

$$y = \frac{3}{2}x + 4$$


7 Parallel Lines

Write equations to match the

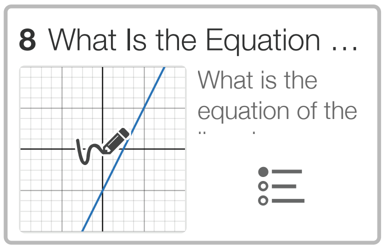
Write equations to match the lines shown in the graph. One is done for you.

Then press "Check My Work."

Use the sketch tool if it helps you with your thinking.

Sample Responses

- Line b: $y = \frac{1}{2}x + 6$
- Line c: $y = \frac{1}{2}x - 2$
- Line d: $y = \frac{1}{2}x + 3$


8 What Is the Equation …

What is the equation of the

What is the equation of the line shown here?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Use the histogram view in the teacher dashboard to see a summary of student responses. Highlight several student responses for the class. Ask questions to help students connect concrete and abstract responses as well as formal and informal responses.

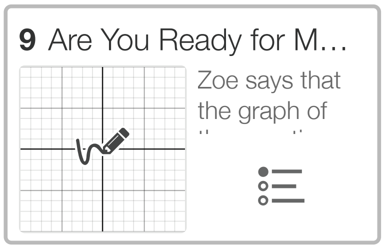
As time allows, ask students what the vertical intercept is for the graph of each linear equation.

Sample Responses

$y = 2x - 4$

Responses vary.

The graph has a slope of 2 and has been shifted down 4 units in a vertical direction.



9 Are You Ready for M…

Zoe says that the graph of

Zoe says that the graph of the equation $y = 3(x + 4)$ is the same as the graph of $y = 3x$, only translated upwards by 4 units.

Is Zoe correct?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

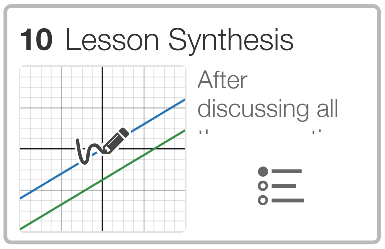
This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 3–8 ahead of time before the class discussion on Screen 10. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

No

Responses vary.

Zoe is not correct. If the graph was $y = 3x + 4$, then it would be the same as the graph of $y = 3x$, shifted 4 units vertically. But, $y = 3(x + 4)$ is actually the same as $y = 3x + 12$, so its graph would be shifted 12 units in a vertical direction.



10 Lesson Synthesis

After discussing all

After discussing all three questions with a classmate, select one question and record your response.

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to consolidate what students know about how equations of lines are connected to translations.

Lesson Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a

whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

1. Line b has been shifted -3 units in a vertical direction from line a.

2. The equation of line a is $y=\frac{3}{5}x$ because the slope of the line is $\frac{3}{5}$

.

3. The equation of line b is similar to the equation of line a because they both have the same slope, $\frac{3}{5}$. These equations are different since you translate vertically to get from the graph of line a, $y=\frac{3}{5}x$, to the graph of line b, $y=\frac{3}{5}x-3$.

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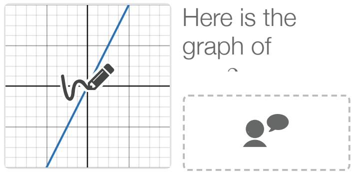
11 Cool-Down



Here is the graph of

Here is the graph of $y=2x$.

How will the graph of $y=2x-7$ look the same and different?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Support for Future Learning
Students will have more opportunities to analyze graphs of linear equations, so if students struggle with this cool-down, there is no need to slow down or add additional work to the next lessons.

Pacing
Consider using pacing to restrict students to screens 11–12.

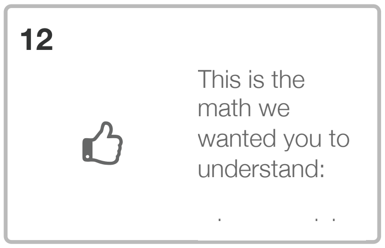
Sample Responses

Responses vary.

- Both graphs are the same because they both have a slope of 2.

- The graphs are different because $y = 2x - 7$ is shifted down 7 units from $y = 2x$.

12

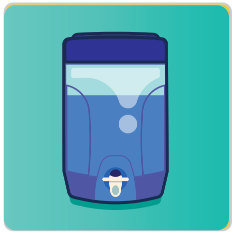


This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain where to find the slope and the vertical intercept in both an equation and its graph.
- I can write equations of lines using $y = mx + b$.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Water Cooler

Lesson 7: Slopes Don't Have to Be Positive

Overview

In previous lessons, students encountered linear relationships with positive rates of change and either positive or negative y-intercepts. The graphs of all of these relationships had an uphill appearance. In this lesson, students get their first glimpse of lines that visually slope downhill. Students reflect on similarities and differences between lines that slope in different directions.

Learning Goals

- Understand the difference in visual appearance between lines with positive slopes and lines with negative slopes.
- Interpret a line with a negative slope that represents a real-world situation.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In previous lessons, students encountered linear relationships with positive rates of change and either positive or negative y-intercepts. The graphs of all of these relationships had an uphill appearance.

In this lesson, students get their first glimpse of lines that visually slope downhill. Students reflect on similarities and differences between lines that slope in different directions. In this lesson, students explore a situation in which one quantity decreases at a constant rate in relation to a second quantity. They use a graph of the situation to reason about the negative slope in terms of the context.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to invite students to explain their reasoning about lines and their properties. Each figure has one obvious reason it does not belong. Encourage students to move past the obvious reason (e.g., line t is dashed) and find reasons based on geometric properties (e.g., the slope triangle of line u is not similar to the slope triangles of the other three lines).

In addition, this warm-up focuses on similarities and differences between lines whose slopes have the same absolute value but opposite signs.

Activity 1: The Water Cooler (25 minutes)

The purpose of this activity is for students to interpret a linear relationship with a negative rate of change using a table, a graph, and an equation (MP2). In previous activities with linear relationships, when x increased, the y value also increased. The slope of the lines that represented these relationships were positive. In this activity, students see negative slopes for the first time and answer questions about a water cooler context.

Activity 2: Card Sort (5 minutes)

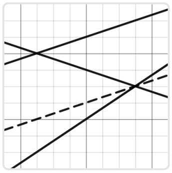
The purpose of this card sort is for students to think more generally about lines with positive and negative slopes.

Lesson Synthesis (5 minutes)

The purpose of this discussion is for students to identify lines with positive and negative slopes given lines in a graph.

Cool-Down (5 minutes)

1 Warm-Up



Which line doesn't

Which line doesn't belong?

Teacher Moves

Purpose
The purpose of this lesson is for students to create and interpret a graph of a line representing a linear relationship with a negative rate of change in context.

Warm-Up Launch
Arrange students into groups of 2–3. Display the image of the four lines, and give students two minutes of quiet think-time and time to share their thinking with their group. Ask students to indicate when they have noticed a line that does not belong and can explain why. Invite each group to offer at least one reason why a particular line doesn't belong.

During the discussion, prompt students to use mathematical terminology (*parallel*, *intersect*, and *slope*) correctly. Also press students on claims without evidence. For example, a student may claim that u does not belong because it has a different slope. Ask how they know for sure that its slope is different from the other lines. Demonstrate by drawing a slope triangle and computing the slope.

Early Student Thinking
Based on the work done up to this point in the unit, students are likely to assume that the slope of v is $\frac{1}{3}$. In the discussion, solicit the idea that there is something fundamentally different about line v compared to the others. You could use informal language like *uphill*, *downhill*, or *tilt direction*. The vocabulary *positive slope* and *negative slope* do not need to be introduced at this time.

Pacing
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Which One Doesn't Belong to support students in noticing the features of each representation.

Sample Responses

Responses vary.

- s doesn't belong because it doesn't pass through the same point like the rest of them do.
- t doesn't belong because it is parallel to s (and it is dashed instead of solid).

- u doesn’t belong because its slope triangle isn’t similar to a triangle whose vertical side has a length of 1 and whose horizontal side has a length of 3.
- v doesn’t belong because it “leans to the left” instead of the right (or slopes down instead of up).

Student Supports

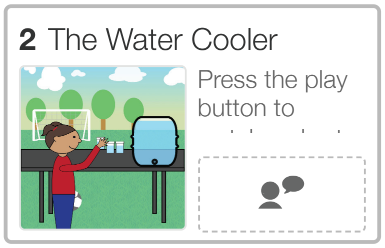
Students With Disabilities

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity.

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., ___________ doesn’t belong because ____________.).



2 The Water Cooler

Press the play button to

Press the play button to watch a short animation.

What do you notice? What do you wonder?

Teacher Moves

Activity Launch

Invite students to play the animation and record at least one thing they notice and at least one thing they wonder about the animation.

Pacing

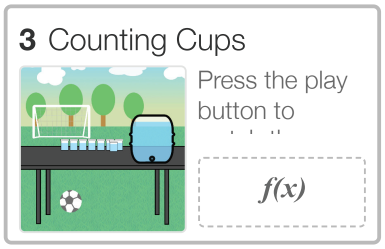
Consider using pacing to restrict students to Screens 2–6.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

Responses vary.

- I notice that water is moving from the cooler to the cups.
- I notice that the water level in the cooler is decreasing.
- I wonder how many cups it will take to empty the water cooler.
- I wonder how much water is poured into each cup and whether it's the same amount of water for each cup.



3 Counting Cups

Press the play button to

Press the play button to watch the animation again.

After how many cups do you think the cooler will run out of water?

Teacher Moves

Emphasize the range of student responses on this screen. It's okay—even desirable—to lack consensus at this stage. The activity will build toward consensus later on.

Do not share the answer on this screen just yet. Students have more work to do before the reveal on Screen 8.

Sample Responses

Responses vary.

- 100 cups
- 50 cups
- 80 cups

4 Sketch the Water in t…

Sketch the relationship between the number of cups filled and

Sketch the relationship between the number of cups filled and the ounces of water remaining in the cooler.

Then discuss how you can see this relationship in your sketch.

Teacher Moves

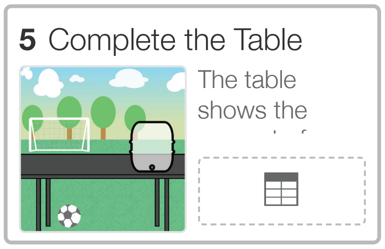
Use the teacher dashboard to share interesting and unique sketches. Ask students to interpret them in words and to ask each other questions.

If time permits, ask students to explain the meaning of the slope of the line in this context: the ounces of water poured from the cooler to fill a cup.

Early Student Thinking
Some students may sketch a line that begins at the origin and increases (from left to right) at a constant rate. Other students may sketch a non-linear graph, either increasing or decreasing. For either of these cases, consider displaying a sample sketch and asking students, "What story does this sketch tell about the water cooler scenario?" Invite students to justify their responses and critique each other's reasoning.

Sample Responses

A sketch of a line with a positive vertical intercept and a negative slope (i.e., a line decreasing from left to right).



5 Complete the Table

The table shows the

The table shows the amount of water remaining in the cooler after 0, 1, and 2 cups have been filled.

Determine the missing values.

Then continue to the next screen.

Teacher Moves

Before giving students quiet work time, consider showing them that it's possible to enter numbers or expressions in the table. Show that when entering an expression, the math input works like a calculator.

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

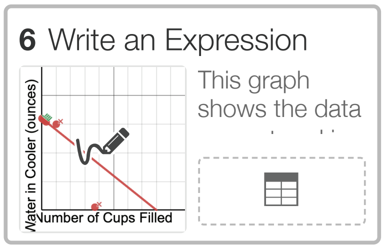
Highlight several student responses for the class. Ask questions to help students use repeated reasoning to develop an efficient method for finding the water level after 37 cups have been filled.

Sample Responses

Responses vary.

- 3 cups filled: $624 - 8 = 616$ ounces; 10 cups filled: $616 - 8 - 8 - 8 - 8 - 8 - 8 - 8 = 560$ ounces; 37 cups filled: 344 ounces
- 3 cups filled: $640 - 8 \cdot 3 = 616$ ounces; 10 cups filled: $640 - 8 \cdot 10 = 560$ ounces; 37 cups filled: $640 - 8 \cdot 37 = 344$ ounces

---



6 Write an Expression

This graph shows the data

This graph shows the data you entered in the table.

Write an expression to represent the amount of water in the cooler after filling x cups. Then press "Graph It."

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to make connections between expressions with negative slopes and the water cooler situation.

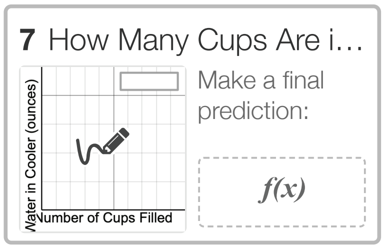
Invite students to revise their expression based on the feedback they see on this screen. Encourage them to justify any changes they make rather than use the feedback on this screen as a tool for guessing and checking without reflection.

Highlight unique answers for the class. Ask questions to help students make connections between repeated reasoning, which offers an efficient way to calculate the 37-cup answer ($640-8-8-...-8=640-8\cdot 37$) and the generalized expression for the water level after x cups have been filled ($640-8x$).

As time permits, ask students what they think the value of the slope is in this scenario and what it means in this context. [-8 because each cup holds 8 ounces of water.]

Sample Responses

$640-8x$



7 How Many Cups Are i…

Make a final prediction:

Make a final prediction:

After how many cups will the cooler run out of water?

Teacher Moves

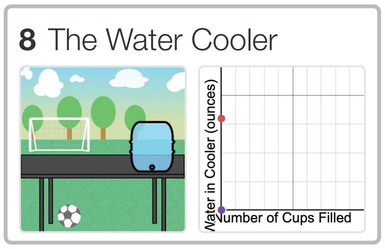
Call attention to the range in estimates on Screen 3 and compare them to the range of estimates on this screen. Invite students to notice that the range of calculations is narrower than the range of estimates from earlier in the activity (if this is true). Math is power, not punishment.

Invite students to explain their reasoning. Consider selecting, sequencing, and connecting a variety of different student approaches.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

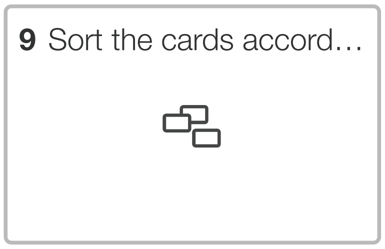
80 cups



8 The Water Cooler

Teacher Moves

Consider using pacing to restrict students to this screen.



9 Sort the cards accord…

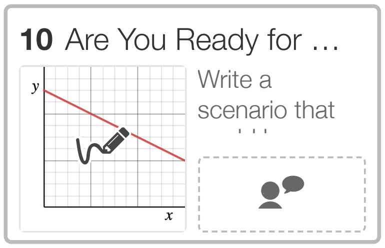
Teacher Moves

Activity Launch
The previous activity focused on a single scenario with a negative slope. The purpose of this card sort is to take a step back from that one scenario and make connections between positive and negative slope as well as the general appearance of lines in the coordinate plane.

Pacing
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

Image solution



10 Are You Ready for …

Write a scenario that could be represented by this graph.

In your description, be sure to mention the meaning of the slope in your scenario.

Teacher Moves

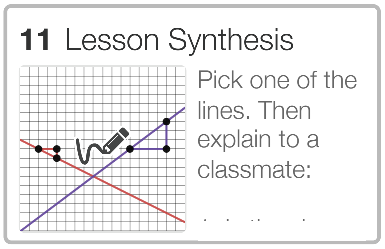
⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screen 9 ahead of time before the class discussion on Screen 11. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.

At the beginning of the month, I have 10 dollars. Each day, I spend $\$0.50$. The graph shows the relationship between how much money I have, y, in dollars and time, x, in days.



11 Lesson Synthesis

Pick one of the lines. Then explain to a classmate:

Pick one of the lines. Then explain to a classmate:

1. Is the slope of the line positive or negative? How do you know?

2. How could you figure out the slope of the line?

Then switch roles and listen to your classmate answer the same questions about another line.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for how to know whether the slope of a line is positive and negative, and how to determine its value.

Synthesis Launch
Arrange students into pairs. Display the graph and ask students to pretend that their partner has been absent from class for a few days. Their job is to explain, verbally or in writing, how someone would figure out if the slope of the line is positive or negative and how to figure out the slope of one of the graphed lines. Then, students should switch roles and listen to their classmate explain how to figure out if the slope of the line is positive or negative and how to figure out the slope of the other line.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

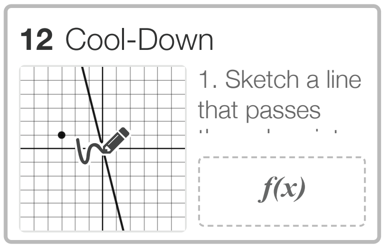
Responses vary.

1. **Red line:** Negative because the line decreases from left to right. The vertical change from A to B is -1, and the horizontal change from A to B is 2. So the slope is $\frac{-1}{2}$, or -0.5.

Purple line: Positive because the line increases from left to right. The vertical change from D to H is 3, and the horizontal change from D to H is 4. So the slope is $\frac{3}{4}$, or 0.75.

2. First, find the amount of vertical change from one point on the line to another. Then, find the amount of horizontal change between those same two points. (Be sure to calculate the change in the same direction. That is, go from point A to point B for both vertical and

horizontal change or from point B to point A for both vertical and horizontal change. Don't calculate vertical change in one direction and horizontal change in another.) Finally, find the ratio of vertical change to horizontal change.



12 Cool-Down

1. Sketch a line that passes

1. Sketch a line that passes through point P and has a slope of -2.

2. What is the slope of line l?

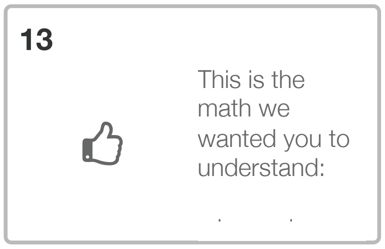
Teacher Moves

Support for Future Learning
If students struggle to sketch a graph with a specific negative slope, plan to revisit this when opportunities arise over the next several lessons. Consider spending extra time on Screen 2's card sort discussing students' strategies for deciding the sign of the slope.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

1. A line through the indicated point with a slope of -2.
2. -4



13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can give an example of a situation that would have a negative slope when graphed.

- I can look at a graph and tell if the slope is positive or negative and explain how I know.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Landing Planes

Lesson 8: Calculating Slope

Overview

Students extend their work with slope triangles to develop a method for finding the slope of any line given the coordinates of two points on the line. They practice calculating slopes this way and use a graph in order to check their answer (especially the sign).

Learning Goals

- Generate a method to find slope values given two points on the line.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Students extend their work with slope triangles to develop a method for finding the slope of any line given the coordinates of two points on the line. They practice calculating slopes this way and use a graph in order to check their answer (especially the sign).

Lesson Summary

Warm-Up: Number Talk (5 minutes)

The purpose of the warm-up is to review operations on positive and negative numbers since students will apply this skill when calculating slope throughout this lesson.

Activity 1: Toward a More General Slope Formula (30 minutes)

The purpose of this activity is for students to calculate the slope of a line given any two points on that line. First, students will practice identifying the sign of a slope given two points. This will help to identify students who understand that lines with positive slopes increase in height as they increase from left to right, while negative slopes decrease in height.

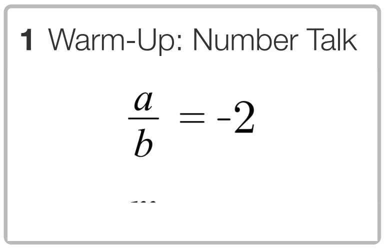
After that, students will calculate the slope of a plane's path. They will land the plane on a runway given the point of the plane and a point at the end of the runway. Those points grow farther and farther apart, offering students the incentive and the opportunity to develop a general and reliable method for calculating slope (MP8).

This lesson supports students in developing their own algorithm for calculating the slope of a line given two points on a line. It isn't necessary to instruct them in a formal algorithm beforehand, although you can help students connect their early strategies for determining slope to a more formal algorithm later in the lesson.

Lesson Synthesis (5 minutes)

The purpose of this discussion is to explore the interplay between the coordinates of points on a line and the slope of that line where the slope could be positive or negative.

Cool-Down (5 minutes)



1 Warm-Up: Number Talk

$$\frac{a}{b} = -2$$

...

Teacher Moves

Warm-Up Launch
Display the problems all at once. Give students one minute of quiet think-time per problem, and ask them to give a signal when they have at least one set of values for each question. Follow with a whole-class discussion.

Facilitate the Number Talk routine. For each problem, ask students to share their values for each variable. Record and display those values for each variable, and include at least one set of values for each problem where a variable (or both) are negative. Ask students how they decided on their values based on the information given in the equation. To involve more students in the conversation, consider asking:

- *Did anyone choose the same values?*
- *Who can restate _____'s reasoning in a different way?*
- *Did anyone choose different values?*
- *Does anyone want to add on to _____'s reasoning?*
- *Do you agree or disagree? Why?*

Early Student Thinking
Students may have forgotten that the quotient of two negative numbers is positive.
Students may have forgotten that getting a negative result when you subtract two numbers means the number you were subtracting must be larger than the number you were subtracting from. Encourage students to think back to their previous work with number lines or other helpful models.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- Any combination of two numbers with different signs where, in absolute value, a is twice b. Examples: $\frac{-6}{3}$; $\frac{6}{-3}$
- Any combination of two numbers with the same sign where, in absolute value, m is twice n. Examples: $\frac{-6}{-3}$; $\frac{6}{3}$
- Any combination of two numbers where r is two more than q. Examples: $6-8$; $-6-(-4)$; $0-2$; $-2-0$; $-1-1$

Student Supports

Students With Disabilities

- *Memory: Processing Time*

Provide sticky notes or mini whiteboards to aid students with working memory challenges.

---

2 Is the slope of the line…

Teacher Moves

Activity Launch

Tell students they'll do more thinking about the slope of different kinds of lines. First, they'll need to decide if a line represented by two coordinates has a positive, negative, or zero slope. Later, they'll calculate slope exactly, but that isn't necessary here.

Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

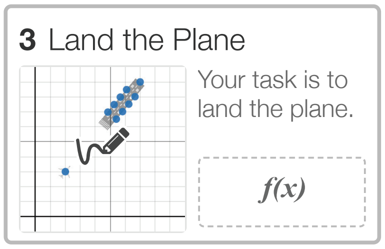
Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Pacing

Consider using pacing to restrict students to Screens 2–7.

Sample Responses

Image solution



3 Land the Plane

Your task is to land the plane.

$f(x)$

Your task is to land the plane.

You'll do that by calculating the slope of the line between $(2, 3)$ and $(7, 9)$.

When you're done, press "Fly the Plane!"

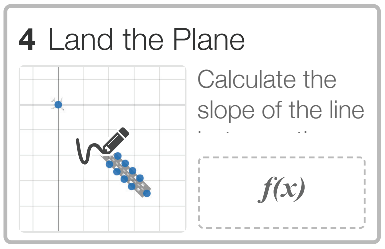
Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to surface strategies for calculating the slope of a line given two points.

Sample Responses

$\frac{6}{5}$ (or equivalent)



4 Land the Plane

Calculate the slope of the line

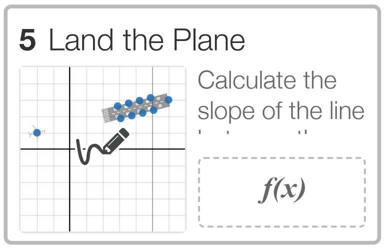
Calculate the slope of the line between these points to land the plane.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

-1 (or equivalent)



5 Land the Plane

Calculate the slope of the line

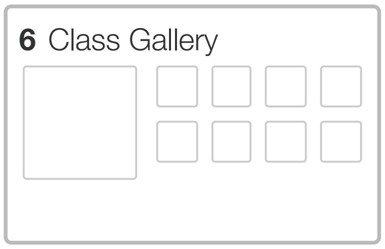
Calculate the slope of the line between these points to land the plane.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

0.5 (or equivalent)



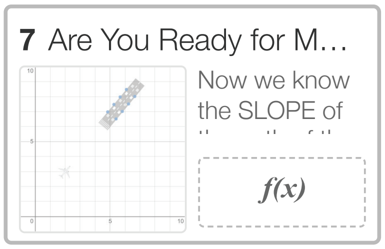
6 Class Gallery

Teacher Moves

Here students will create their *own* challenge and solve challenges from their classmates. We recommend students complete Screens 2–5 before creating their challenge. We anticipate this Challenge Creator will take 15 minutes or more.

Encourage students to complete each other's challenges but also to take some time to review responses to their own. Use the teacher dashboard to look for unique challenges and unique solutions that may expand your students' understanding of the mathematics, and highlight those for the class to see. Ask students what they've learned from the experience.

We intend for this to be a social and creative experience for students. We encourage you to emphasize those virtues whenever you see them in your class.


7 Are You Ready for M…

Now we know the SLOPE of

Now we know the SLOPE of the path of the plane but not its position.

Calculate the missing value to land the plane.

Teacher Moves

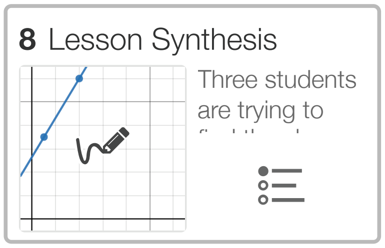
⚠ Before students can see this screen, they will have to press a button that says, "I'm ready!"

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 2–6 ahead of time before the class discussion on Screen 8. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses to the first five challenges are written below.

1. $p = 5$
2. $p = 7$
3. $p = 11$
4. $p = 5$
5. $p = \dfrac{3}{16}$


8 Lesson Synthesis

Three students are trying to

Three students are trying to find the slope of the line between $(4, 12)$ and $(1, 7)$.

Discuss the following question:

Which of their calculations are correct?

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to consolidate what students know about calculating the slope of a line from two points.

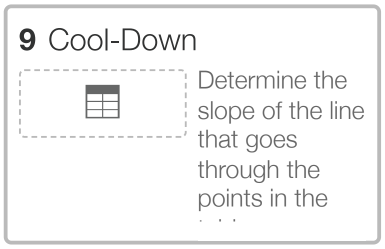
Synthesis Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

As time permits, consider asking students the following questions:
- *If you know the coordinates of two points on a line, how can you tell if it has a positive or a negative slope?* [If the points go down from left to right, then the slope is negative. If they go up, then the slope is positive.]
- *Given any two coordinates on a line, how can you calculate the slope of that line?* [To calculate the slope of any two coordinates, find the vertical change by subtracting the y-coordinates and find the horizontal change by subtracting the x-coordinates (in the same order). Then divide the vertical change by the horizontal change.]

Pacing
Consider using pacing to restrict students to this screen. Use the teacher dashboard to see a distribution of student responses, and highlight unique answers for the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

The first and second options are correct because they result in slopes of $\frac{5}{3}$. The third option is $-\frac{5}{3}$. The line is increasing, so this cannot be the slope.



9 Cool-Down

Determine the slope of the line that goes through the points in the

Determine the slope of the line that goes through the points in the table.

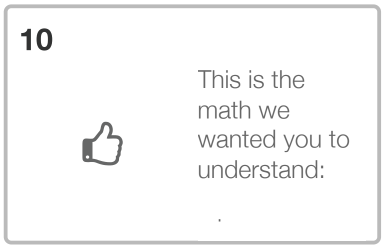
Teacher Moves

Support for Future Learning
If students struggle with calculating the slope between two points, consider making time to explicitly revisit these ideas before the quiz.

Pacing
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

$\frac{3}{8}$



10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can calculate positive and negative slopes given two points on the line.
- I can describe a line precisely enough that another student can draw it.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Coin Capture

Lesson 9: Equations of All Kinds of Lines

Overview

Students experience the need for writing equations of lines with positive and negative slopes.

Learning Goals

- Write equations of horizontal and vertical lines.
- Write equations of lines that have a negative slope.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you’ll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In previous lessons, students studied lines with positive and negative slopes and learned to write equations for them, usually in the form $y = mx + b$. In this lesson, students extend their previous work to include equations for horizontal and vertical lines. Horizontal lines can still be written in the form $y = mx + b$, but because $m = 0$ in this case, the equation simplifies to $y = b$. Students interpret this to mean that for a horizontal line, the y value does not change, but x can take any value. This structure is identical for vertical lines except that now the equation has the form $x = a$, and it is x that is determined, while y can take any value. Note that the equation of a vertical line cannot be written in the form $y = mx + b$. It can, however, be written in the form $Ax + By = C$ (with $B = 0$). This type of linear equation will be studied in greater detail in upcoming lessons.

In this lesson, students experience the need for writing equations of lines with positive and negative slopes. Initially, students are provided with a coordinate grid containing strategically placed coins. Their goal is to capture all of the coins by writing equations of lines that pass through the coins using the fewest possible equations. Later in the lesson, students create and completing their own challenge, placing coins in the plane themselves before capturing them with as few lines as possible.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to make sense of the equations of vertical lines. This warm-up prompts students to move five points to different locations, but with the same x-coordinate. Students will notice that all of these points are on the same vertical line and will connect the equation of this line to the relationship of the coordinates of the points along a vertical line.

Activity 1: All Kinds of Lines (30 minutes)

The purpose of this activity is for students to attend to precision (MP6) in writing equations of horizontal, vertical, and other lines that intersect as many coins as possible. In previous lessons, students studied lines with positive slope and negative slope and wrote equations for those lines. In this activity, they also write equations for horizontal lines (lines of slope 0) and vertical lines, and they graph horizontal and vertical lines from equations.

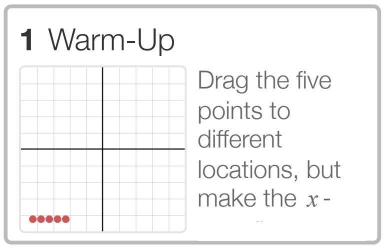
Students will create their own coin challenges by strategically placing coins for their peers to capture on a coordinate grid. Before they can submit their challenge to the class gallery, they’ll have to capture the coins using the linear equations they’ve learned about in this unit.

Throughout this activity, students will practice writing equations of both proportional and non-proportional lines. Through writing equations for horizontal and vertical lines as well as lines with positive and negative slopes, students will employ repeated reasoning (MP8) and make connections between equations of all types of lines.

Lesson Synthesis (5 minutes)

Students have spent considerable time in Grade 7 and Grade 8 solving problems with proportional and non-proportional relationships that can be represented by equations and graphs with positive slopes. Students will notice that for horizontal and vertical lines, the key feature is that one of the two variables does not vary, while the other one can take any value. In the x-y plane, when the variable x can take any value, it is a vertical line, and when the variable y can take any value, it is a horizontal line.

Cool-Down (5 minutes)



1 Warm-Up

Drag the five points to different locations, but make the x-

Drag the five points to different locations, but make the x-coordinate for each location 7.

Think about what all of your class's points would look like if they were all on one graph.

Teacher Moves

Warm-Up Launch
Allow students to work individually or in pairs. Ask students to move each of the five points to different locations but all with an x-coordinate of 7. After everyone has moved their points, consider asking the class to predict what it would look like if every student's points were shown on the same graph.

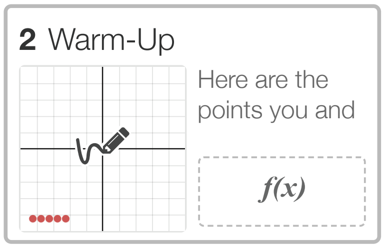
Early Student Thinking
Some students may place the points where $y = 7$, $y = -7$, or $x = -7$. Ask these students to consider the coordinates of their points and whether the x-value is positive 7.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

Correctly placed points will be on the vertical line $x = 7$.



2 Warm-Up

Here are the points you and

$f(x)$

Here are the points you and your classmates graphed.

Your points are in dark red.

Write an equation to represent all of the points with x-coordinate 7.

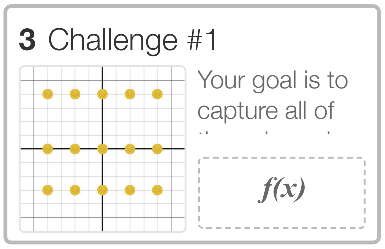
Teacher Moves

Consider using pacing to restrict students to this screen.

Allow students quiet think-time. Instruct them to discuss which equation makes sense and why. Discuss why the equation only contains the variable x and what this means about the relationship between the quantities represented by x and y.

Sample Responses

$x = 7$


3 Challenge #1

Your goal is to capture all of

Your goal is to capture all of the coins using as few equations as possible.

Enter equations one at a time in order to send a line through the coins to "capture" them.

Teacher Moves

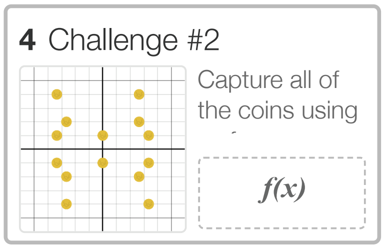
Activity 1 Launch
Arrange students into pairs. Explain to students that their task is to write equations to capture all of the coins on the screen. Allow them a few minutes of quiet think-time.

Pacing
Consider using pacing to restrict students to Screens 3–4.

Sample Responses

Responses vary.

- $y = 4$
- $y = 0$
- $y = -3$


4 Challenge #2

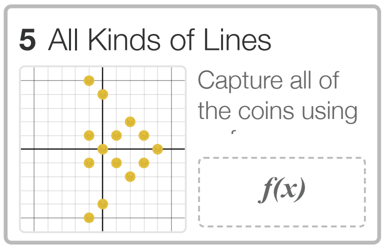
Capture all of the coins using

Capture all of the coins using as few equations as possible.

Sample Responses

Responses vary.

- $x = -3$
- $x = 0$
- $x = 3$



5 All Kinds of Lines

Capture all of the coins using

Capture all of the coins using as few equations as possible.

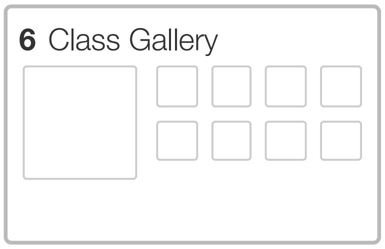
Teacher Moves

Pacing
Consider using pacing to restrict students to Screens 5–6.

Sample Responses

Responses vary.

- $y = x$
- $y = -x$
- $y = x - 4$
- $y = -x + 4$



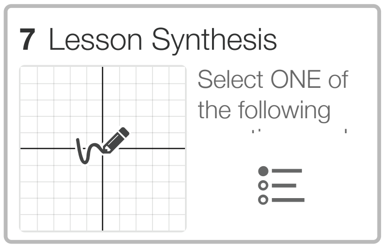
6 Class Gallery

Teacher Moves

Here students will create their *own* challenge and solve challenges from their classmates. We recommend students complete Screens 3–5 before creating their challenge. We anticipate this Challenge Creator could take 20 minutes or more.

Encourage students to complete each other's challenges but also to take some time to review responses to their own. Use the teacher dashboard to look for unique challenges and unique solutions that may expand your students' understanding of the mathematics, and highlight those for the class to see. Ask them what they've learned from the experience.

We intend for this to be a social and creative experience for students. We encourage you to emphasize those virtues whenever you see them in your class.



7 Lesson Synthesis

Select ONE of the following

Select ONE of the following questions and record your response.

Use the sketch tool if it helps you to show your thinking.

How can you tell from a linear equation if its graph will . . .

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to make connections between equations of vertical and horizontal lines and their graphs, particularly that equations of the form $x=\#$ will result in vertical lines and $y=\#$ will result in horizontal lines.

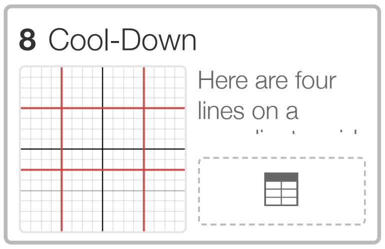
Lesson Synthesis Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- For horizontal and vertical lines, the key feature is that one of the two variables does not vary, while the other one can take any value. In the coordinate plane, when the variable x can take any value, it is a horizontal line, and when the variable y can take any value, it is a vertical line.
- A linear equation will have a graph with a negative slope if one variable increases when the other variable decreases.



8 Cool-Down

Here are four lines on a

Here are four lines on a coordinate grid.

Write an equation for each line.

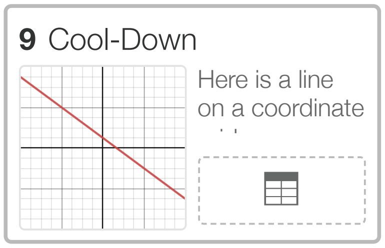
Teacher Moves

Support for Future Learning
If students struggle with writing an equation for a line, consider making time to explicitly revisit these ideas before the quiz.

Pacing
Consider using pacing to restrict students to Screens 8–10.

Sample Responses

- **Line** a**:** $x = -4$
- **Line** b**:** $x = 4$
- **Line** c**:** $y = 4$
- **Line** d**:** $y = -2$



9 Cool-Down

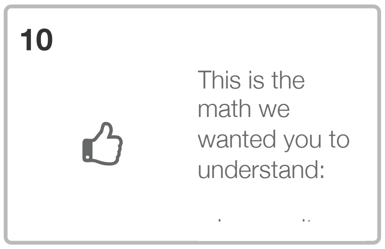
Here is a line on a coordinate

Here is a line on a coordinate grid.

Write an equation for the line.

Sample Responses

Line g**:** $y = -\frac{3}{4}x + 1$ (or equivalent)



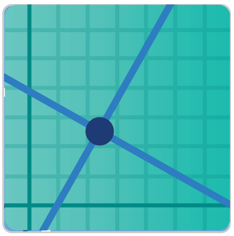
10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can write equations of lines that have a positive or negative slope.
- I can write equations of vertical and horizontal lines.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Solutions

Lesson 10: Solutions to Linear Equations

Overview

Students explore solutions to linear equations. They will also encounter equations where both variables have to satisfy a constraint and learn that a natural way to write the constraint is with an equation of the form $Ax + By = C$.

Learning Goals

- Understand that linear equations don't always look like $y = mx + b$.
- Understand that the graph of an equation is a visual representation of all solutions to an equation.
- Define a solution to an equation in two variables.
- Notice features of equations that can make one variable easier or harder to solve for.

Vocabulary

- solution to a system with two variables

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students explore solutions to linear equations. Prior to this lesson, students have worked with contexts and equations where one variable depends on another, for example, distance depending on time. The linear equation representing such a situation is often written in the form $y = mx + b$. In this lesson, students will encounter equations where both variables have to satisfy a constraint, and a natural way to write the constraint is with an equation of the form $Ax + By = C$.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to enter two pairs of x- and y-values that make a given equation true and to observe patterns in the graph of the points they and their classmates enter (namely that the solutions to a linear equation all lie on a line).

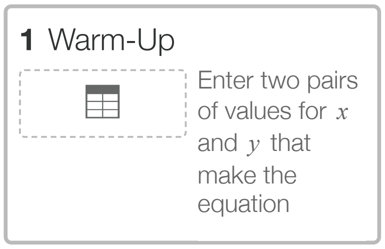
Activity 1: Solutions to Linear Equations (30 minutes)

The purpose of this activity is for students to deepen their understanding of solutions to linear equations algebraically and graphically by practicing two things: 1) finding a solution to a linear equation when given one variable and 2) writing equations of lines that pass through a given point.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to strengthen their understanding of the meaning of a solution to a linear equation and to apply that understanding to questions about solutions in each quadrant, about solutions with non-integer coordinates, and about the number of solutions.

Cool-Down (5 minutes)


1 Warm-Up

Enter two pairs of values for x and y that make the equation

Enter two pairs of values for x and y that make the equation $x + 2y = 10$ true.

Teacher Moves

Purpose
The purpose of this lesson is for students to develop an understanding of solutions to linear equations algebraically and graphically.

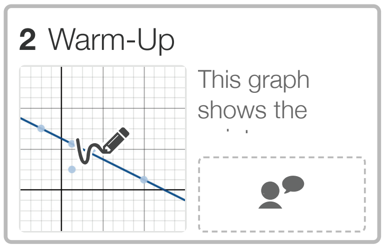
Warm-Up Launch
Give students one minute of quiet think-time. Then invite them to discuss with a partner. Invite several students to share their responses.

Pacing
Consider using pacing to restrict students to Screens 1–2, one screen at a time.

Sample Responses

Responses vary.

- $(-2, 6)$
- $(0, 5)$
- $(4, 3)$
- $(10, 0)$
- $(13, -1.5)$


2 Warm-Up

This graph shows the

This graph shows the points you entered (in dark blue) and your classmates' points (in light blue).

The graph also shows the line $x + 2y = 10$.

How can you tell from the graph if someone chooses a point that is NOT a solution to the equation?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Give students one minute of quiet think-time. Then invite them to discuss with a partner. Invite several students to share their responses.

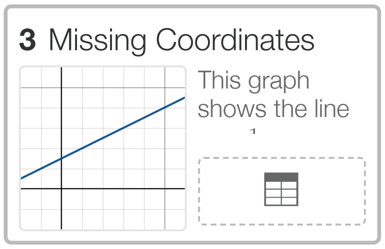
Introduce the term *solution*. A solution to an equation with two variables is a pair of values of the variables that make the equation true.

Point out the range of solutions, and consider inviting students to share how they picked their x- and y-values. Highlight that a solution of an equation in two variables is an ordered pair of numbers and that solutions of an equation lie on the graph of the equation.

Sample Responses

Responses vary.

Solutions to the equation lie on the line. If a point is **not** on the line, that point is **not** a solution.



3 Missing Coordinates

This graph shows the line

This graph shows the line $y = \frac{1}{2}x + 3$.

Complete the table so it includes two solutions to the equation.

Then press "Check My Work."

Teacher Moves

Activity Launch
Arrange students into pairs. Tell students that their task is to find a missing coordinate for each of the two points. Encourage them to use paper and pencil to support their thinking and to use it as a record of their thinking to help them respond to the discussion question on Screen 4.

Pacing
Consider using pacing to restrict students to Screens 3–6.

Sample Responses

- When $x = 4$, $y = 5$.
- When $y = 20$, $x = 34$.

4 Describe Your Strate…

Here is your table from the

Here is your table from the previous screen.

Recall the equation: $y = \frac{1}{2}x + 3$

Describe your strategies for finding the solutions to the equation.

Teacher Moves

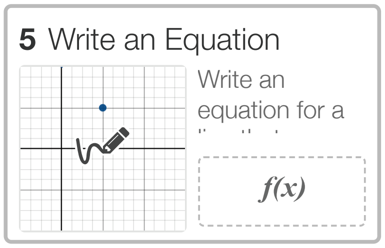
Key Discussion Screen
The purpose of this discussion is to surface strategies for determining solutions to an equation using its graph.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

First, I substituted 4 for x and solved for y. Next, I substituted 20 for y and solved for x.



5 Write an Equation

Write an equation for a

Write an equation for a line that passes through $(4, 4)$.

Then press "Check My Work."

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

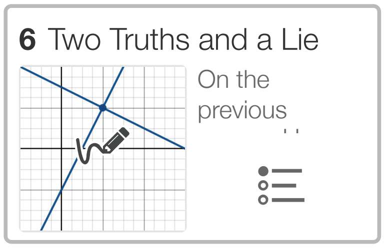
Early Student Thinking
Some students may struggle with the transition from a) determining points for a given equation to b) writing an equation for a given point. Invite students to write down an equation (any equation) and then to ask themselves whether $(4, 4)$ is a solution to that equation (and how they know). From there, invite students to modify their equation so that $(4, 4)$ **is** a solution. For example, students might first write $x + y = 4$ and then realize that $4 + 4 \neq 4$ and revise their equation to $x + y = 8$.

Alternatively, consider inviting students to write a **partial** equation (e.g., $3x + y$) and then to decide how to "finish" the equation so that $(4, 4)$ is a solution. In this case, $3 \cdot 4 + 4 = 16$, so the finished equation would be $3x + y = 16$.

Sample Responses

Responses vary.

- $x + y = 8$
- $x - y = 0$
- $y = \frac{1}{4}x + 3$
- $3x + y = 16$



6 Two Truths and a Lie

On the previous

On the previous screen, Hamza and Neo entered the following equations:

Hamza: $4x - 2y = 8$

Neo: $y = 6 - \frac{1}{2}x$

One of the following statements is a lie. Which is it?

Teacher Moves

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

$(7, 2.5)$ is a solution to $4x - 2y = 8$.

Responses vary.

- The point $(7, 2.5)$ is not on the line $4x - 2y = 8$. (It is actually on the line $y = 6 - \frac{1}{2}x$.)

- If you substitute the values $x = 7$ and $y = 2.5$ into the equation $4x - 2y = 8$ you get $4 \cdot 7 - 2 \cdot 2.5 = 8$ which simplifies to

$28 - 5 = 8$, which is false. Therefore $(7, 2.5)$ is not a solution to $4x - 2y = 8$.

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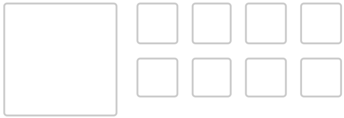
7 Class Gallery



Teacher Moves

Here students will create their *own* challenge and solve challenges from their classmates. We recommend students complete Screens 1–6 before creating their challenge. We anticipate this Challenge Creator could take 20 minutes or more.

Encourage students to complete each other's challenges. Use the teacher dashboard to look for unique challenges and unique solutions that may expand your students' understanding of the mathematics, and highlight those for the class to see. Ask students what they've learned from the experience.

We intend for this to be a social and creative experience for students. We encourage you to emphasize those virtues whenever you see them in your class.

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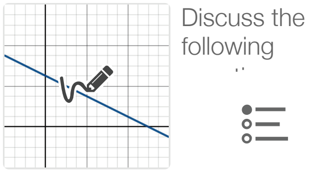
8 Lesson Synthesis



Discuss the following questions about the line $x + 2y = 10$.

Then select ONE question and record your response.

Whichever question you select, explain your thinking.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to make connections between the graph of a linear relationship, what types of solutions, and how many solutions it has.

Synthesis Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

If it does not come up in student work, ask if the equation could be true if $x = -4$ [yes, $y = 7$] or if $y = -2$ [yes, $x = 14$]. What if $y = 4.5$? [Yes, $x = 1$.] Through the discussion questions, bring up that there are an infinite number of solutions to this equation and that, taken collectively and plotted in the plane, they make up the line of all solutions to the equation.

Pacing
Consider using pacing to restrict students to this screen.

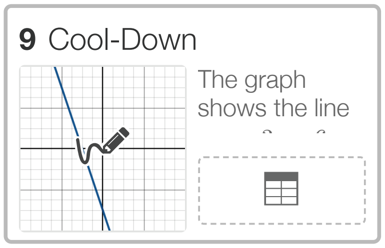
Sample Responses

Responses vary.

1. No. The line $x + 2y = 10$ has no solutions in the bottom-left quadrant (quadrant 3).

2. Yes. Here is one example: $(1, 4.5)$.

3. There are an unlimited number of solutions because the line extends forever in each direction.


9 Cool-Down

The graph shows the line

The graph shows the line $y = -3x - 6$.

Complete the table so it includes two solutions to the equation.

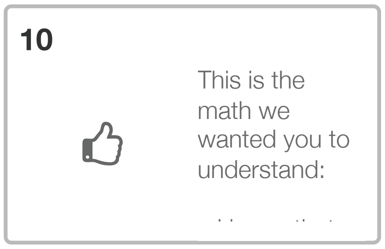
Teacher Moves

Support for Future Learning
If students struggle to identify solutions from a graph, plan to revisit this when opportunities arise over the next lesson. Consider spending extra time identifying other solutions to the equation on Screen 5 of Lesson 11.

Pacing
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

- When $x = 0$, $y = -6$.
- When $y = -15$, $x = 3$.



10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I know that the graph of an equation is a visual representation of all the solutions to the equation.
- I understand what the solution to an equation with two variables is.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Pennies and Quarters

Lesson 11: Using Linear Relationships to Solve Problems

Overview

Students solve real-world problems using all of the different representations of linear relationships they have studied.

Learning Goals

- Describe how real-world constraints on quantities define the limitations of their representations.
- Interpret multiple representations of non-proportional linear relationships in context, including slope, intercept, and solution.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this culminating lesson for the unit, students solve real-world problems using all of the different representations of linear relationships they have studied.
Students consider combinations of pennies and quarters worth a fixed amount. They write an equation that represents all possible combinations and interpret the real-world meaning of points that satisfy (and do not satisfy) that equation, incorporating their knowledge of slope and intercepts.
Students also connect scenarios with different representations of linear relationships. Students begin to consider how some solutions to linear equations may be limited based on the context.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm up is for students to informally consider the context that will support their formal investigation in the next activity where they represent the relationship between pennies, quarters, and total amount of money using equations and graphs.

Activity 1: Pennies and Quarters (20 minutes)

The purpose of this activity is for students to connect a context to its solutions using a table and a graph. Students begin this activity by finding different combinations of pennies and quarters with a total value of $\$62$. They represent this scenario with an equation and use the equation to find solutions. They create a graph and reason abstractly and quantitatively, interpreting the meaning of points on and off the graph in context (MP1).

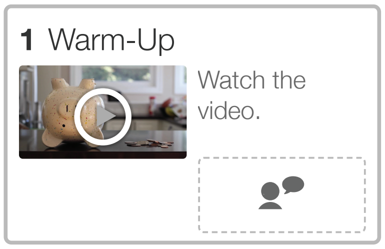
Activity 2: Card Sort (10 minutes)

The purpose of this card sort is for students to connect scenarios with the tables, equations, graphs, and ordered pair solutions that represent them (MP2). Students must also decide if a solution is valid (i.e., “makes sense”) in the scenario. This highlights the importance of context in deciding whether representations are accurate.

Lesson Synthesis (5 minutes)

The purpose of this discussion is to consider whether solutions are valid or invalid in a context. Throughout this discussion, students will consider and critique each other's reasoning.

Cool-Down (5 minutes)



1 Warm-Up

Watch the video.

Watch the video.

What do you notice? What do you wonder?

Teacher Moves

Warm-Up Launch
Ask students if they have ever collected and counted a lot of coins or used a coin machine. Let students know that in this lesson, we will be thinking about coins. Give students two minutes of quiet think-time to record what they notice and wonder about the video.

Invite several students to share their responses, or use snapshots to highlight their written responses.

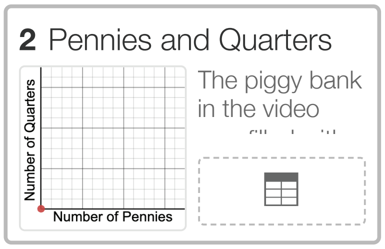
Pacing
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

Responses vary.

- I notice there are lots of pennies and quarters going into the machine.
- I notice there are more pennies than quarters.
- I wonder how many coins there are in total.
- I wonder how much the coins are worth.



2 Pennies and Quarters

The piggy bank in the video

The piggy bank in the video was filled with pennies and quarters worth a total of $\$62.00$.

What are three possible combinations of pennies and quarters that are worth $\$62.00$?

Enter your combinations in the table.

Teacher Moves

Activity Launch
Explain to students that there were only pennies and quarters in the coin-counting machine. Tell them that their goal for this activity is to determine the relationship between all of the combinations of pennies and quarters that total $\$62$.

Before giving students 2–3 minutes of quiet work time, consider showing them that it's possible to enter numbers or expressions in the table. Show that when entering an expression, the table input works like a calculator.

Use the teacher dashboard to monitor student work around the class. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

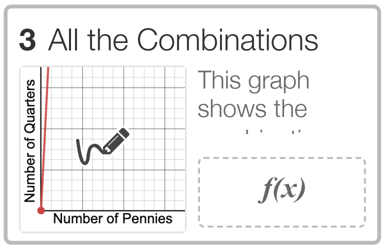
Early Student Thinking
Students may enter in the dollar value of the coins rather than the number of coins. Allow students to submit incorrect values and encourage them to check the animation on this screen. Then consider asking these students why they think their values didn't total $\$62$. Encourage them to revise their answers by thinking about how the number of coins relates to their dollar value.

Pacing
Consider using pacing to restrict students to Screens 2–4 until you're ready for them to see the answer on Screen 5.

Sample Responses

Responses vary.

- $(3000, 128)$
- $(4000, 88)$
- $(5000, 48)$
- $(0, 248)$
- $(6200, 0)$


3 All the Combinations

This graph shows the

This graph shows the combinations you and your classmates entered on the previous screen.

Your points are shown in dark red.

Write an equation that describes ALL the combinations of pennies, p, and quarters, q, that are worth $\$62.00$.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

- $0.01p + 0.25q = 62$
- $1p + 25q = 6\,200$
- $\frac{p}{100} + \frac{q}{4} = 62$

(or equivalent)

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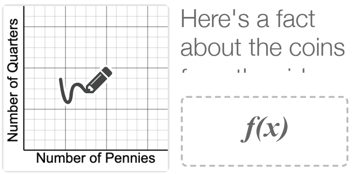
4 Two Hundred Quarters



Here's a fact about the coins from the video: There are 200 quarters.

How many pennies must there be to combine with 200 quarters to equal $\$62.00$?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for using an equation or a graph to solve a problem about a linear relationship in context.

Consider pausing here to facilitate a discussion about students' strategies for determining the number of pennies. This is a great place to use the teacher dashboard or snapshots to highlight and display unique strategies.

Sample Responses

$1\,200$ pennies

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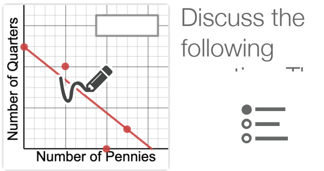
5 Which Are Solutions?



Discuss the following question. Then record your response.

Which points are solutions to $0.01p + 0.25q = 62$?

Teacher Moves

Give students one minute of quiet think-time, followed by 2–3 minutes to discuss in pairs. Invite several students to share their responses.

If time permits, consider asking the class:

- *What is the meaning of each point in terms of pennies and quarters?* [The x-coordinate represents the number of pennies and the y-coordinate indicates the number of quarters.]
- *Are there any points that don't make any sense in this context?* [Point B does not make sense because it is not possible to have 200.8 quarters.]

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

A and D are both solutions because they fall on the graph of $0.01p + 0.25q = 62$. B and C are not solutions because they do not fall on the line.

---

6 Match each scenario …

Teacher Moves

Activity 2 Launch
Tell students that they will match a set of cards describing different scenarios with cards showing representations and coordinates of solutions. Consider completing this card sort yourself, in the role of a student, to anticipate the different questions and ideas your students have.

Arrange students into pairs. Give them at least 8 minutes of group work time, followed by a whole-class discussion.

Teacher Moves
Use the teacher dashboard to monitor student progress and to look for common sorting strategies. Consider using the overview feature in the teacher view to identify and discuss the most controversial groups and cards.

Once most students have completed the card sort, invite them to find a card that would make each equation true but does not make sense in the context. [The point $(0.5, 96)$ assumes that Ichiro spends the same amount of money in each half of each week.]

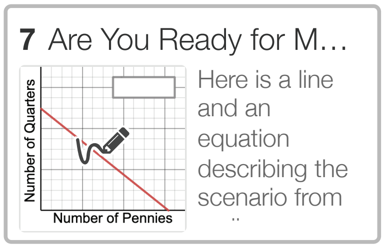
Early Student Thinking
Rather than offering your own explanation of why various solutions do and do not make sense in context, ask students: *What do these numbers represent in the scenario?* Invite them to reason from there whether the solutions make sense in context.

Pacing

Consider using pacing to restrict students to Screens 6–7.

Sample Responses

Image solution



7 Are You Ready for M…

Here is a line and an equation describing the scenario from ..

Here is a line and an equation describing the scenario from earlier (combinations of pennies and quarters that are worth $\$62.00$).

On paper, answer the following questions:

1. What is the slope of the line and what is its meaning in terms of pennies and quarters?

2. Another pile of pennies and quarters is worth $\$30$. Sketch the graph of this scenario either on paper or with the sketch tool. How did you decide on the important features of your graph?

3. Imagine another pile of coins (quarters and nickels) worth $\$30$. How would the graph of this third scenario compare to the first two?

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

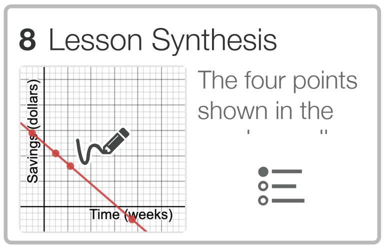
This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 2–6 ahead of time before the class discussion on Screen 8. Because only a subset of your students will complete this screen, we recommend you don’t discuss it with the entire class.

Sample Responses

1. You lose 1 quarter for every 25 pennies you gain.

2. Sketches should show a line parallel to the red line with intercepts at $(0, 120)$ and $(3000, 0)$.

3. Since nickels have a greater value than pennies, the graph of this line will be much steeper.



8 Lesson Synthesis

The four points shown in the

The four points shown in the graph are all solutions to the equation $y = 100 - 8x$, where y represents the savings account balance and x represents the number of weeks.

All four of these points are on the line, but some of them do NOT make sense in the savings account context.

Select a point that does NOT make sense in context.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to highlight that even though points may be solutions to a linear relationship, they may not make sense in context. Students also surface ways in which a solution may not make sense.

Activity Synthesis Launch
Ask students if they or someone they know has a bank account. Consider facilitating a brief discussion about the meaning of the term *account balance*.

Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

The goal for this screen is not to establish consensus on all four points. Some of the points (e.g., C and D) are intentionally ambiguous. Rather, the goal is for students to recognize that not all mathematically valid solutions make sense in context.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- Point A does not make sense because the scenario does not say anything about the balance prior to this year. Therefore, $(-2, 116)$ doesn't make sense in context because it refers to -2 weeks.

- Point C does not make sense because money is not withdrawn continuously but rather once per week. Therefore, $(4.5, 64)$ doesn't make sense in context because it refers to 4.5 weeks.

(Counterargument: The owner of the savings account might withdraw 4 dollars twice each week.)

- Point D does not make sense because most banks do not allow a negative balance for savings accounts. Therefore, $(15, -20)$ doesn't make sense in context because it refers to -20 dollars. (Counterargument: Some accounts can be overdrawn.)

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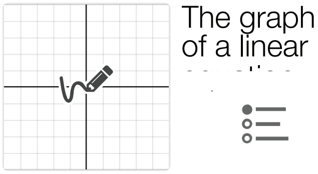
9 Cool-Down



The graph of a linear equation passes through the points $(-2, 0)$ and $(0, 6)$.

Is $3x - y = -6$ an equation for this graph?

Teacher Moves

Support for Future Learning
If students struggle with identifying solutions to an equation not in $y = mx + b$ form, consider making time to explicitly revisit these ideas before the End-Unit Assessment.

Pacing
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

Yes.

Explanations vary.

- The points $(-2, 0)$ and $(0, 6)$ both make the equation $3x - y = -6$ true.
- The graph of $3x - y = -6$ goes through both the points.

10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can find solutions to linear equations given either the x- value or the y-value.
- I can write linear equations to reason about real-world situations.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

8.3 Practice Day 1 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided copy for each student.

Cards

- Print one set of cards for the whole class (option 1) or one set of cards for each group of students (option 2).

Instructions

Option 1: Stations

Print and cut out one set of cards. Place several cards at each station and arrange students into groups of 3–4.

Give each student the student workspace sheet to complete as they solve the task cards at their station. Instruct students to move from station to station after a set amount of time.

Option 2: Task Cards

Arrange students into groups of 2–3. Print and cut out one set of cards for every group of students.

Give each student the student workspace sheet to complete as they work together to solve each of the task cards.

Consider posting the answer key, or walk around with it and provide feedback to students as they work.

GRADE 8

Unit 4
Lesson Plans

Teacher lesson plans from Unit 4 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 8 Unit 4

Teacher Edition Sampler

Unit at a Glance

Key

📖 Print Lessons

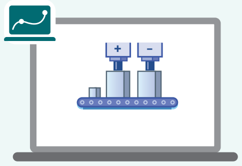
💻 Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

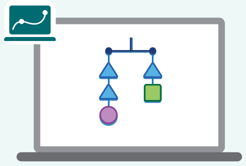
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



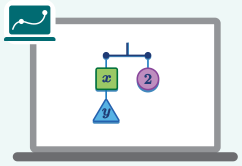
1 Number Machines

Calculate a missing value for a number puzzle that can be represented by a linear equation with one variable.



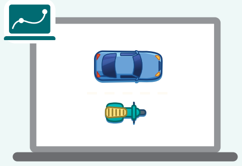
2 Keep It Balanced

Calculate the weight of an unknown object using a hanger diagram, and explain the solution method.



3 Balanced Moves

Correlate changes on hanger diagrams with moves that create equivalent equations.



8 When Are They the Same?

Create an equation with one variable to represent a situation in which two conditions are equal.

Practice Day

Practice Day 1

Practice the concepts and skills developed during Lessons 1–8. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

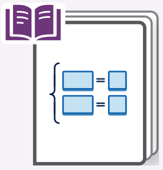
Quiz: Sub-Unit 1

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2

9 On or Off the Line?

Determine a point that satisfies two relationships simultaneously, using tables or graphs.



14 Strategic Solving, Part 2

Solve systems of equations using a variety of strategies.

Practice Day

Practice Day 2

Practice the concepts and skills developed during Lessons 1–14. Consider using this time to prepare for the upcoming Quiz.

Summative Assessment

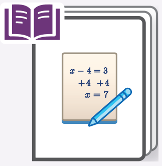
End-of-Unit Assessment

Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing: 18 days | **Short on time?** See pacing considerations below.

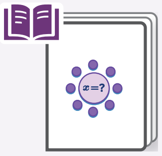
Pre-Unit Check: (Optional)
14 Lessons: 45 min each
2 Practice Days: 45 min each

1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



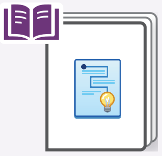
4 More Balanced Moves

Calculate a value that is a solution to a linear equation with one variable, and compare and contrast solution strategies with others.



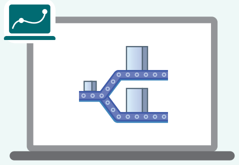
5 Equation Roundtable

Solve equations that involve adding and expanding expressions.



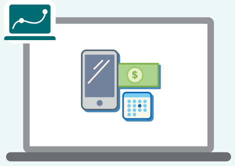
6 Strategic Solving

Categorize linear equations with one variable based on their structure, and solve equations from each category.



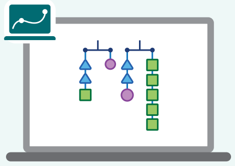
7 All, Some, or None?

Compare and contrast equations that have no solutions or infinitely many solutions.



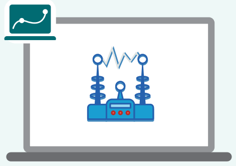
10 On Both Lines

Create a graph that represents two linear relationships in context, and interpret the point of intersection.



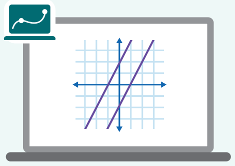
11 Make Them Balance

Understand that solving a system of equations means finding values of the variables that make both equations true at the same time.



12 Line Zapper

Connect the solution of an equation with variables on each side to the solution of a system of two linear equations.



13 All, Some, or None? Part 2

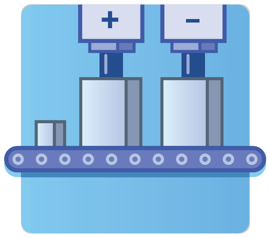
Categorize systems of equations, including systems with infinitely many or no solutions, and calculate the solution using a variety of strategies.

Pacing Considerations

Lesson 1: The purpose of this lesson is for students to create and solve number puzzles in preparation for solving linear equations in upcoming lessons. If students show a strong understanding of solving equations with one variable in Problem 4 of the Pre-Unit Check, this lesson may be omitted.

Lesson 10: This lesson introduces a graph that represents two linear relationships in context, and invites students to make sense of the point of intersection. If students show a strong understanding of the relationship between solutions of a linear equation and its graph in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss the meaning of the point of intersection of two linear relationships.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Number Machines

Lesson 1: Solving Number Puzzles

Overview

Students solve and write number puzzles. In each puzzle, students are given two of the following with the goal of finding the third: a series of operations on a number, a final result, and the original number. These puzzles are good preparation for solving linear equations, in which students have to perform operations on each side of the equation to isolate the variable.

Learning Goals

- Calculate a missing value for a number puzzle that can be represented by a linear equation with one variable, and explain (orally and in writing) the solution method.
- Create a number puzzle that can be represented by a linear equation with one variable.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to solve and write number puzzles. In each puzzle, students are given two of the following with the goal of finding the third: a series of operations on a number, a final result, and the original number. These puzzles are good preparation for solving linear equations, in which students have to perform operations on each side of the equation to isolate the variable.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to how the number machine works. The number machine produces an output by performing a series of operations on a given input.

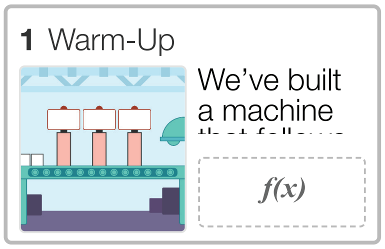
Activity 1: Number Machines (30 minutes)

The purpose of this activity is for students to develop their equation-solving intuition. First, students complete a series of challenges where they work to “undo” a series of operations on a final output to discover the original input. Then, students create their *own* challenge and solve challenges from their classmates.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to analyze a strategy for solving a number puzzle.

Cool-Down (5 minutes)



We've built a machine that follows these instructions:

- Add 4
- Multiply by 2
- Subtract 3

If we put 10 into this machine, what will come out?

Teacher Moves

Purpose
The purpose of this lesson is for students to solve and write number puzzles. In each puzzle, students are given two of these with the goal of finding the third: a series of operations on a number, a final result, and the original number.

Warm-Up Launch
Arrange students into pairs. Consider introducing this activity by telling students they will be working with different number machines throughout this lesson, and their goal for the warm-up is to determine what number will come out of this number machine. Allow students two minutes of quiet work time, followed by a whole-class discussion.

Following the whole-class discussion, consider showcasing that it's possible to enter numbers or expressions in the math input. Show that when entering an expression, the math input works like a calculator.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

25

Student Supports

Support for Students With Disabilities
- *Executive Functioning: Graphic Organizers*

Provide a T-chart for students to record what they notice and wonder prior to being expected to share these ideas with others in the whole-class discussion.

2 Challenge #1


Alina put a number into

Alina put a number into this machine, and 13 came out.

What number did Alina put in?

Teacher Moves

Activity Launch
Arrange students into pairs. Tell students that their task is to determine the number that was placed into the number machine. Encourage students to use paper to help them organize their thinking.

Pacing
Consider using pacing to restrict students to Screens 2–5.

Sample Responses

5

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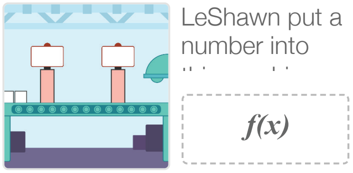
3 Challenge #2


LeShawn put a number into

LeShawn put a number into this machine, and 40 came out.

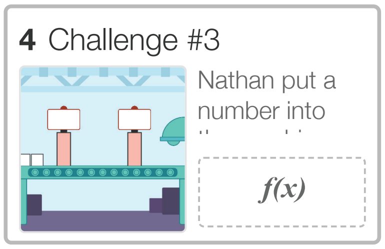
What number did LeShawn put in?

Teacher Moves

Use the teacher view in the dashboard to monitor student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

14


4 Challenge #3

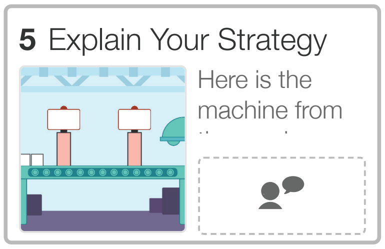
Nathan put a number into

Nathan put a number into the machine, and 15 came out.

What number did Nathan put in?

Sample Responses

17


5 Explain Your Strategy

Here is the machine from

Here is the machine from the previous screen.

If you know what number came out, how can you determine the number that went into this machine?

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to surface strategies for figuring out an unknown input given an output.

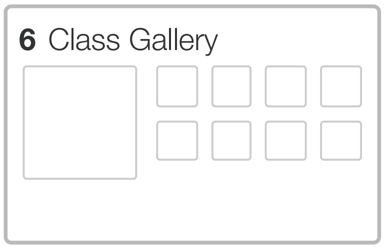
Here students must describe the thinking they've employed on previous screens. Also, without a specific final result to consider, they must think more generally about the strategy they would use to discover the original number (i.e., the input).

Consider pausing here and using snapshots or the teacher view of the dashboard to highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

To find the input, I would multiply the output by 2 and then subtract by 13.

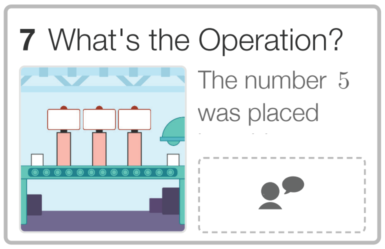

6 Class Gallery

Teacher Moves

Here students will create their *own* challenge and solve challenges from their classmates. We recommend students complete the rest of the activity before creating their challenge. We anticipate this Challenge Creator will add 15 minutes to the duration of the activity.

Encourage students to complete each other's challenges but also to take some time to review responses to their own. Use the teacher dashboard to look for unique challenges and unique solutions that may expand your students' understanding of the mathematics. Highlight those for students and also ask them what they learned from the experience.

We intend for this to be a social and creative experience for students. We encourage you to emphasize those virtues whenever you see them in your class.



7 What's the Operation?

The number 5 was placed

The number 5 was placed into this number machine, and 30 came out.

What could the first step in the number machine be?

Can you think of more than one possibility?

Teacher Moves

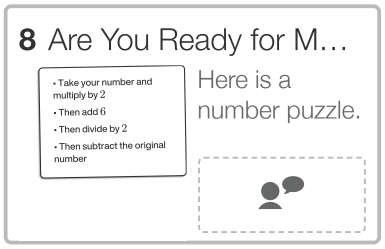
Consider introducing this part of the activity by telling students that since they are experienced at working with number machines, they will now think about the steps for some machines. Give students one minute of quiet think-time. Then invite them to discuss with a partner. Invite several students to share their responses.

Pacing
Consider using pacing to restrict students to Screens 7–8.

Sample Responses

Responses vary.

The first step of this number machine is to add 12. Another possibility would be to multiply by 3.4.



8 Are You Ready for M…

- Take your number and multiply by 2
- Then add 6
- Then divide by 2
- Then subtract the original number

Here is a number puzzle.

Here is a number puzzle.

Start with any number for this puzzle, and the output will ALWAYS be 3.

Try it!

Then design a different number puzzle that will always output 5.

Teacher Moves

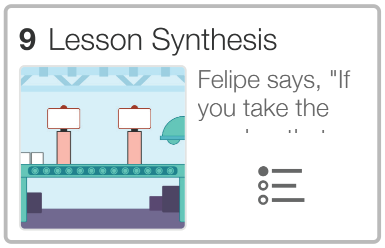
⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screen 7 ahead of time before the class discussion on Screen 9. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.

1. Multiply by 2
2. Add 10
3. Divide by 2
4. Subtract the input



9 Lesson Synthesis

Felipe says, "If you take the

Felipe says, "If you take the number that came out, then divide by 3 and subtract 10, you will get the number that went in."

Do you agree?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to highlight that order matters when determining an unknown input.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to show the distribution of responses and highlight unique answers for the class.

Early Student Thinking
If students struggle to answer this question, encourage them to pick a number and follow the machine to find the output, and then try those same steps on the output to see if they agree with Felipe.

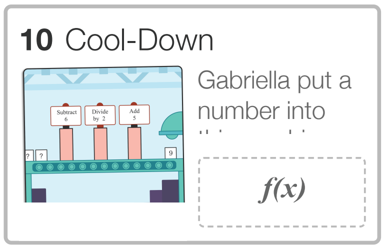
Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

No

Responses vary.

To determine the input, Felipe needs to undo the steps of the number machine in reverse order. Felipe will need to subtract 10 from the output and then divide by 3 to determine the input.



10 Cool-Down

Gabriella put a number into

Gabriella put a number into this machine, and 9 came out.

What number did Gabriella put in?

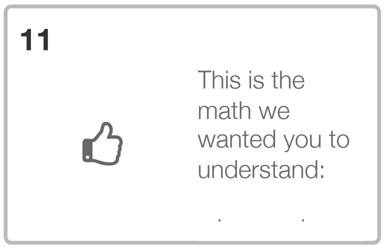
Teacher Moves

Support for Future Learning
Students will have more opportunities to work on multistep equations, so if students struggle with this cool-down, there is no need to slow down or add additional work to the next lessons.

Pacing
Consider using pacing to restrict students to Screens 10–11.

Sample Responses

14



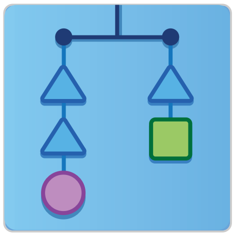
11

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can solve a number puzzle problem.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Keep It Balanced

Lesson 2: Keeping the Equation Balanced

Overview

This lesson is the first of a sequence of eight lessons where students learn to work with equations that have variables on each side. In this lesson, students recall a representation that they have seen in prior grades: the balanced hanger.

Learning Goals

- Calculate the weight of an unknown object using a hanger diagram, and explain the solution method.
- Understand that adding and removing equal items from each side of a hanger diagram or multiplying and dividing items on each side of the hanger by the same amount are moves that keep the hanger balanced.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

This lesson is the first of a sequence of eight lessons where students learn to work with equations that have variables on each side. In this lesson, students recall a representation that they have seen in prior grades: the balanced hanger. The hanger is balanced because the total weight on one side, hanging at the same distance from the center, is equal in measure to the total weight on the other side.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to give students an opportunity to ground their understanding of equality in the context of weight, which is a context that will be used throughout the lesson.

Activity 1: Make It Balance (20 minutes)

The purpose of this activity is to give students an opportunity to notice that adding or removing the same number of each type of shape from each side of a hanger maintains balance. They'll start with solving for an unknown weight and describing strategies before connecting more formally to addition and subtraction, using these operations as balancing moves that keep a hanger balanced.

Activity 2: Hanger Equations (10 minutes)

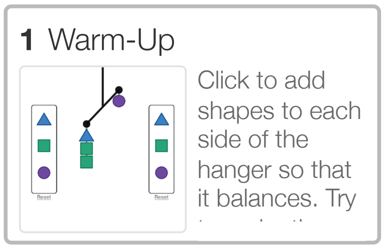
The purpose of this activity is for students to transition their reasoning from solving hangers by maintaining the equality of each side to solving equations using the same logic. In future lessons, students will continue to develop this skill as equations grow more complex, culminating in solving systems of equations at the end of this unit.

Students engage in MP2 as they use concrete quantities to develop their power of abstract reasoning about equations.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to gain a formal understanding of addition and subtraction as balancing moves that keep a hanger in balance.

Cool-Down (5 minutes)



1 Warm-Up

Click to add shapes to each side of the hanger so that it balances. Try

Click to add shapes to each side of the hanger so that it balances. Try to make the sides different!

Discuss what it means for a hanger to be balanced.

Teacher Moves

Purpose
The purpose of this lesson is for students to 1) represent balanced hangers with equations and 2) add or remove blocks from balanced hangers and keep the hangers balanced.

Warm-Up Launch
The purpose of this warm-up is to give students an opportunity to ground their understanding of equality in the context of weight, which is a context that will be used throughout the lesson.

Tell students they will be adding weights to a hanger by clicking on the shape icons on either side of the hanger. Their goal is to keep the hanger balanced. Encourage students to make different combinations of shapes on each side of the hanger.

Arrange students into pairs. Give students two minutes of quiet work time, followed by a partner discussion and a whole-class discussion. Tell students that they will have a chance to practice keeping hangers balanced throughout the lesson. Consider using snapshots to highlight and display unique strategies.

Pacing
Consider using pacing to restrict students to this screen.

Blank paper will be an important tool for this and other screens as students work to maintain balance by adding or removing blocks from each side of the hanger.

Sample Responses

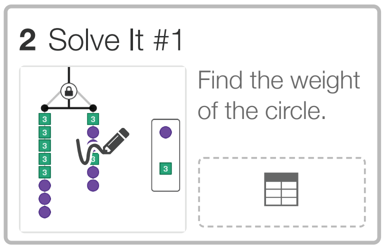
Responses vary.

The hanger is balanced when the total weight on each side of the hanger is the same.

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*
Allow students to use calculators to ensure inclusive participation in the activity.

- *Social-Emotional Functioning: Peer Tutors*
Pair students with their previously identified peer tutors.



2 Solve It #1

Find the weight of the circle.

Find the weight of the circle.

Press "Try It" to see if the hanger is balanced.

Teacher Moves

Activity Launch
Arrange students into pairs. Tell students that their job on this screen and the next is to find the unknown weight of a shape. Let them know that they may use blank paper to help record their thinking and that they will be invited to share their strategies with the class.

For this screen and the next, consider using snapshots to highlight and display unique strategies.

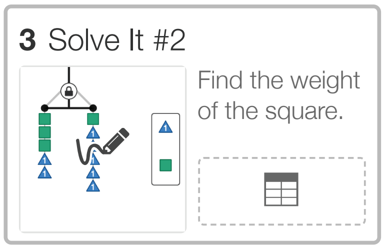
Pacing
Consider using pacing to restrict students to Screens 2–3.

Sample Responses

9 lb.

Responses vary.

I crossed off 3 circles and 2 squares from each side, leaving 9 on the one side and a circle on the other.



3 Solve It #2

Find the weight of the square.

Find the weight of the square.

Press "Try It" to see if the hanger is balanced.

Teacher Moves

Use the teacher view in the dashboard to identify students who may need additional support.

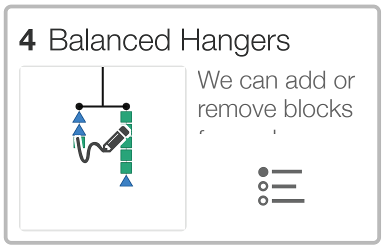
Consider using snapshots to highlight and display unique strategies. Watch for students who are using equations to represent their thinking or those who are using the sketch tool to cross off pairs of shapes from each side of the hanger. Ask these students to share their strategies with the class.

Sample Responses

1.5 lb.

Responses vary.

I took a square and two triangles off of each side. That means two squares equals 3 triangles (or two squares equals 3 pounds). That means one square is 1.5 pounds.



4 Balanced Hangers

We can add or remove blocks

We can add or remove blocks from a hanger and keep the hanger balanced.

If Hanger A is balanced, which of the following hangers are also balanced if they are made with the same squares and triangles?

Teacher Moves

Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

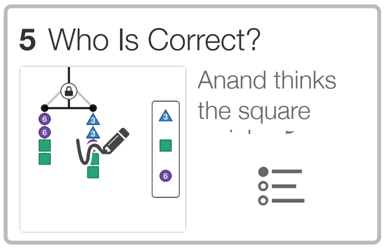
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking
Without knowing any of the weights of the shapes, some students may struggle to reason about this problem. Ask these students to describe to you how the hanger in the first answer choice is different from Hanger A. Then ask them to pick a weight for the square and to consider how the weight of each side of the first hanger must have changed.

Pacing
Consider using pacing to restrict students to Screens 4–6.

Sample Responses

- Top left
- Bottom left
- Bottom right



5 Who Is Correct?

Anand thinks the square

Anand thinks the square weighs 5 pounds.

Darius thinks the square weighs 2 pounds.

Who is correct?

Teacher Moves

Consider using the dashboard to show the distribution of responses, calling attention to any conflict or consensus you see. Use snapshots to highlight and display unique strategies.

Watch for students who substitute the values of the weights into the hangers to check if the hanger is balanced and also for those who realize that any weight will work for the square because of the way this hanger is set up. Both are key understandings that will support work with equations in upcoming lessons.

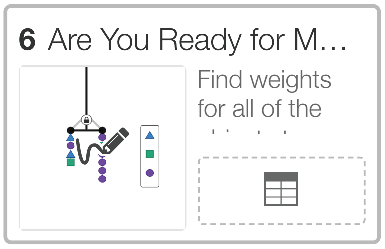
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Both

Responses vary.

They are both correct. The known weights for each side are 12 pounds, and there are an equal number of squares on each side, so the hanger will be balanced regardless of the weight of each square.



6 Are You Ready for M…

Find weights for all of the

Find weights for all of the objects to make the hanger balance.

Press "Try It" to see if the hanger is balanced.

Teacher Moves

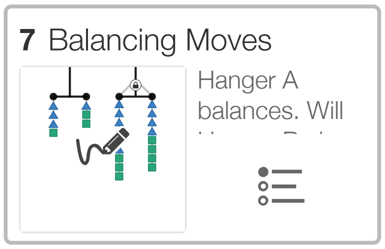
⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 4–5 ahead of time before the next activity begins on Screen 7. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.

The weight of a triangle is double that of a circle, so any answer with this relationship is correct.



7 Balancing Moves

Hanger A balances. Will

Hanger A balances. Will Hanger B also balance?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to highlight that adding or subtracting the same number of items from each side of a balanced hanger keeps it balanced.

Activity Launch
The goal for this activity is for students to connect working with hanger diagrams to working with equations.

Arrange students into pairs. Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. If adding squares and triangles to each side of the hanger does not come up, consider discussing how many of each type of shape was added to each side of Hanger A in order to get Hanger B. This will help students recall that balancing moves include adding or removing shapes from each side of a hanger.

Pacing
Consider using pacing to restrict students to Screens 7–9.

Sample Responses

Yes

Responses vary.

If I add three triangles and two squares to each side of Hanger A, I will get Hanger B. Therefore, Hanger B is also balanced because I added the same number of each type of shape to each side of the hanger.

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.

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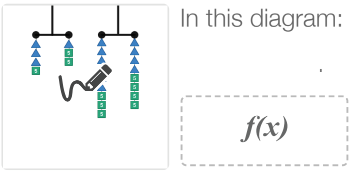
8 Hanger Equations #1



In this diagram:

- x represents the weight of each triangle (in pounds).
- 5 is the weight of each square (in pounds).

Hangers A and B are both balanced.

Write an equation to represent Hanger B.

Teacher Moves

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

$6x + 15 = 4x + 20$ (or equivalent)

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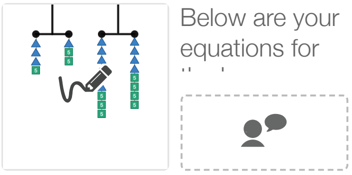
9 Hanger Equations #2



Below are your equations for the hangers.

Hanger A: $3x + 5 = x + 10$
Hanger B: $6x + 15 = 4x + 20$

How are these equations similar?

Teacher Moves

The purpose of this screen is for students to learn that adding or removing the same weight from each side is analogous to adding or subtracting the same amount from each side of an equation.

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

The equations are related because I can get the equation for Hanger B by adding $3x$ and 10 to each side of the equation, just like adding 3 triangles and 10 pounds (2 squares) to each side of the hanger.

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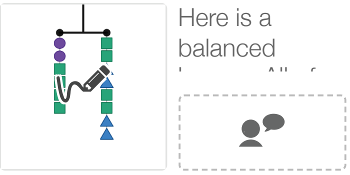
10 Lesson Synthesis



Here is a balanced

Here is a balanced hanger. All of the weights are unknown.

How would you change the number of shapes on it, but keep it in balance?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to consolidate strategies for keeping a hanger balanced while changing the number of shapes on each side.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

I would remove four squares from each side because then I would be removing the same amount of weight from each side, so the hanger would still be balanced. This means one circle weighs the same as two triangles. I could also remove one circle from the left and two triangles from the right.

---

11 Cool-Down

Here is a new balanced

Here is a new balanced hanger.

In this hanger, each square weighs 5 pounds.

What is the weight of a triangle?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

Support for Future Learning
If students struggle with naming moves, consider spending extra time on Lesson 3’s warm-up to reflect on and review the types of hanger diagram moves that keep the hanger balanced.

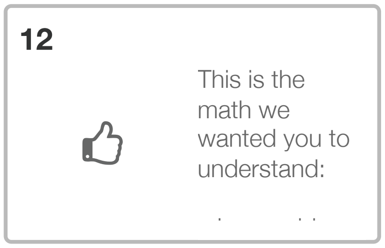
Pacing
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

3

Responses vary.

I took two squares off each side and saw that 15 on the left is balanced with 5 triangles on the right. This means that each triangle must weigh 3 pounds.



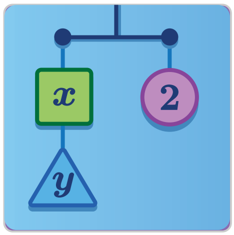
12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can add or remove blocks from a hanger and keep the hanger balanced.
- I can represent balanced hangers with equations.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Balanced Moves

Lesson 3: Balancing Moves and Undoing

Overview

Students move from using hangers to using equations in order to represent a problem.

Learning Goals

- Correlate changes on hanger diagrams with moves that create equivalent equations.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students move from using hangers to using equations in order to represent a problem. They'll start by applying balancing moves to one hanger to produce a new balanced hanger. Then they will apply the same reasoning to equations to keep them balanced. Students engage in MP3 when they reason about why the steps in solving an equation maintain equality.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to recall the balancing moves of addition and subtraction from Lesson 2. Given a balanced hanger, students will create a new balanced hanger using the same shapes. None of the weights of the shapes are known, thereby creating a need to apply balancing moves in order to keep the original hanger balanced.

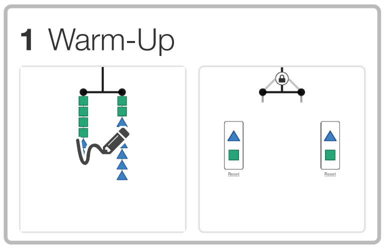
Activity 1: Keep It Balanced (30 minutes)

The purpose of this activity is to introduce multiplication and division as balancing moves and also to begin to apply balancing moves to equations. In the "Keep It Balanced" challenges, students consider balancing moves and equations that represent a hanger after applying the balancing moves. They see how moves that maintain the balance of a hanger correspond to moves that maintain the equality of an equation, such as halving the value of each side or subtracting the same unknown value from each side.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consider different balancing moves that can be applied to each side of an equation to maintain equality.

Cool-Down (5 minutes)



1 Warm-Up

Teacher Moves

Purpose
The purpose of this lesson is for students to 1) move from using hangers to using equations in order to represent a problem and 2) apply balancing moves in order to keep equations balanced.

Warm-Up Launch
Tell students that they will work with hanger diagrams today, and they will apply what they know about balancing moves in order to keep hangers balanced.

Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. Consider using the dashboard to show the variety of responses. Ask participants to share how the new balanced hangers are different from Hanger A.

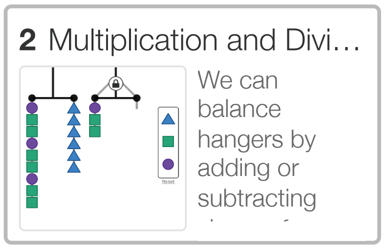
Readiness Check (Problem 1)
If most students struggled, plan to review the distributive property before beginning Activity 1. Consider using hanger diagrams as a context for reviewing the distributive property.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

1 square on the left and 2 triangles on the right will produce a balanced hanger.



2 Multiplication and Divi…

We can balance hangers by adding or subtracting

We can balance hangers by adding or subtracting shapes from each side. We can also balance hangers by multiplying or dividing.

If Hanger A is balanced, build the right side of Hanger B so it also balances.

Press "Try It" to see if Hanger B balances.

Teacher Moves

Activity Launch
In Lesson 2, students practice using the balancing moves of addition and subtraction. Tell students that in this lesson, we will begin to use the

balancing moves of multiplication and division to simplify hangers, and later, to rewrite balanced equations.

Arrange students into pairs. Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their strategy.

Early Student Thinking
If students aren't sure where to start, encourage them to find ways to evenly split up and match the shapes on each side of the hanger. Students should see that for every circle and two squares on the left, we can match with two triangles on the right, and therefore, one circle and two squares is the same as two triangles. This is equivalent to dividing each side of the hanger by 3.

Readiness Check (Problem 5)
If most students struggled, invite them to check the solutions they calculate throughout this lesson and unit by substituting values back into equations. Emphasize that a solution to an equation is a value for the variable that makes the equation true.

Pacing
Consider using pacing to restrict students to this screen.

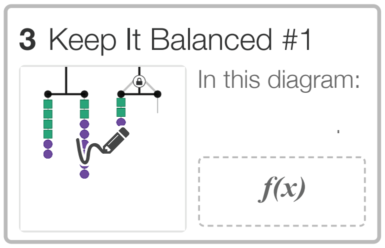
Sample Responses

The right side of the hanger should have 2 triangles.

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.


3 Keep It Balanced #1

In this diagram:

In this diagram:

- x represents the weight of each square.
- y represents the weight of each circle.

Ethan divided each side of Hanger A by 2 and made Hanger B.

Write an EXPRESSION to represent the right side of Hanger B.

Press "Try It" to see if if your expression balances the hanger.

Teacher Moves

The purpose of this screen is for students to connect the balancing move of division to dividing each side of an equation by 2.

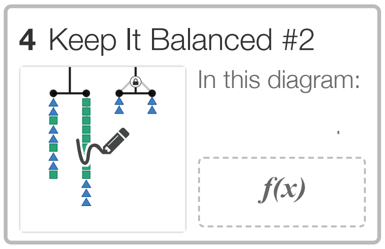
This is a great place to check student progress. Watch for students who are entering the whole equation as opposed to just the expression that represents the right side of the equation. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Pacing
Consider using pacing to restrict students to Screens 3–5.

Sample Responses

Responses vary.

$x + 3y$



4 Keep It Balanced #2

In this diagram:

In this diagram:

- x represents the weight of each triangle.
- y represents the weight of each square.

Jamir changed the number of shapes on each side of Hanger A to make Hanger B.

Write an EQUATION that could represent a balanced Hanger B.

Press "Try It" to see if Hanger B is balanced.

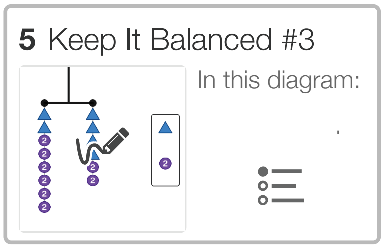
Teacher Moves

Consider using snapshots to highlight and display unique responses. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

$x = 2y$ or other equation equivalent to $3(2x + y) = 9y + 3x$



5 Keep It Balanced #3

In this diagram:

In this diagram:

- x represents the weight of each triangle.
- 2 is the weight of each circle.

Select an equation that could represent a balanced hanger.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface different ways to change an original equation into a new equation without changing the value of the unknown.

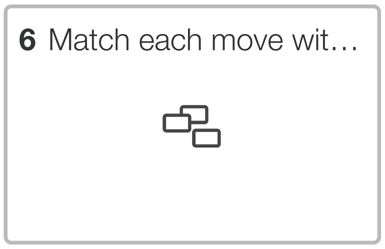
An outcome of the class conversation should be that students know that all of the responses are valid, and they understand the balancing moves that change the original equation into the new equation.

Emphasize the range of student responses on this screen. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

- $x+6=2x+2$: Divide each side of the hanger by 2 .
- $12=2x+4$: Subtract two triangles (or $2x$) from each side of the hanger.
- $8=2x$: Subtract two triangles ($2x$) and two circles (4) from each side of the hanger.
- $2x+8=4x$: Subtract two circles (4) from each side of the hanger.



6 Match each move wit…

Teacher Moves

Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

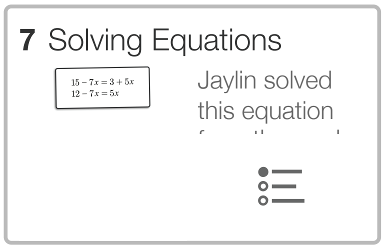
Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Pacing
Consider using pacing to restrict students to Screens 6–8.

Sample Responses

Image solution



7 Solving Equations

Jaylin solved this equation

Jaylin solved this equation from the card sort:

$$\begin{aligned} 15 - 7x &= 3 + 5x \\ 12 - 7x &= 5x \\ 12 &= 12x \\ 1 &= x \end{aligned}$$

Is this correct?

Teacher Moves

Consider using the dashboard to invite students to explain their choice.

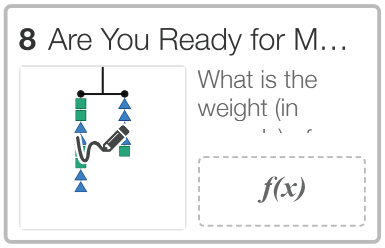
If no one mentions it, make sure students know that one way to check if a solution is correct is to substitute it back into the original equation to see if the resulting numerical expression is true.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Yes

Jaylin is correct. If plugging $x = 1$ into the original equation, we get $15 - 7(1) = 3 + 5(1)$ or $8 = 8$, which is a true statement.



8 Are You Ready for M…

What is the weight (in

What is the weight (in pounds) of a square in this balanced hanger?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

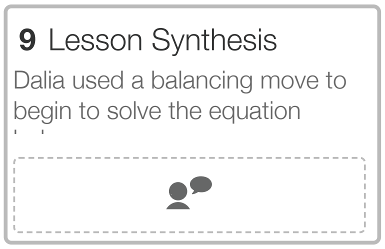
⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 6–7 ahead of time before the class discussion on Screen 9. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.

The weight of the square is 0. I know this is true because if I remove all of the extra squares and triangles from each side, I get three squares on the left and no shapes on the right. The only way this could be true is if the square’s weight is 0 pounds.



9 Lesson Synthesis

Dalia used a balancing move to begin to solve the equation

Dalia used a balancing move to begin to solve the equation below.

$$6x + 12 = 10x - 4$$

The result of Dalia's first step was $12 = 4x - 4$.

Describe the first step Dalia made to solve the equation.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to explain a valid step in solving an equation with variables on both sides.

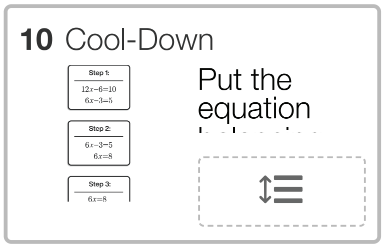
Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

The first step that Dalia made was to subtract $6x$ from each side of the original equation.



10 Cool-Down

Step 1:
$12x-6=10$
$6x-3=5$

Step 2:
$6x-3=5$
$6x=8$

Step 3:
$6x=8$

Put the equation

Put the equation balancing moves in order so that they match up with what was done in each step to solve the equation $12x - 6 = 10$.

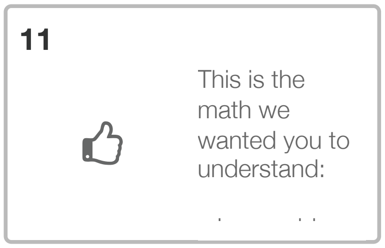
Teacher Moves

Support for Future Learning
Students will have more opportunities to name the moves for keeping equations balanced while moving towards solving, however they won't be prompted as explicitly to name or label the moves. Consider prompting students to name the moves they are using and recognizing in Lesson 4, Activity 1.

Pacing
Consider using pacing to restrict students to Screens 10–11.

Sample Responses

- Divide both sides by 2
- Add 3 to both sides
- Divide both sides by 6



11

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can add, subtract, multiply, or divide each side of an equation by the same expression to get a new equation with the same solution.

Unit 4

Lesson

4

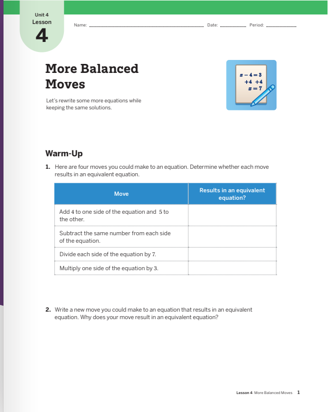
More Balanced Moves

Solving Linear Equations, Part 1

Let’s rewrite some more equations while keeping the same solutions.

Print Lesson

This is a print lesson with Presentation Screens.



Unit 4
Lesson
4

Name: ________ Date: ________ Period: ________

More Balanced Moves

Let’s rewrite some more equations while keeping the same solutions.

Warm-Up

1. Here are four moves you could make to an equation. Determine whether each move results in an equivalent equation.

Move	Results in an equivalent equation?
Add 4 to one side of the equation and 5 to the other.	
Subtract the same number from each side of the equation.	
Divide each side of the equation by 7.	
Multiply one side of the equation by 3.	

2. Write a new move you could make to an equation that results in an equivalent equation. Why does your move result in an equivalent equation?

Lesson 4 More Balanced Moves 1

Focus and Coherence

Today’s Goals

1. **Goal:** Solve a linear equation in one variable.
2. **Language Goal:** Analyze strategies for solving a linear equation in one variable. **(Reading, Writing, Speaking, and Listening)**

Students continue to reinforce the connections of three fundamental ideas: a solution to an equation is a value that makes the equation true, performing the same operation on each side of an equation results in an equivalent equation, and two equations related by such a move have the same **solutions**. Students use the structure of an equation to determine possible next steps as they practice solving linear equations with variables on both sides. **(MP7)**

Prior Learning

In Lessons 2 and 3, students used hanger diagrams to gain a conceptual basis for solving linear equations.

Future Learning

In Lessons 5 and 6, students will continue to practice solving linear equations in one variable. In Lesson 7, they will analyze equations with no solutions or infinitely many solutions.

Rigor and Balance

- Students continue to develop **procedural fluency** in solving equations with variables on both sides.

Vocabulary

New Vocabulary

solution

Standards

Addressing

NY-8.EE.7b

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Also Addressing: **NY-8.EE.7**

Mathematical Practices: MP3, MP7

Building On	Building Toward
NY-7.EE.4	NY-8.EE.7

Amplify Desmos Math NEW YORK
Lesson Sample

Lesson at a Glance

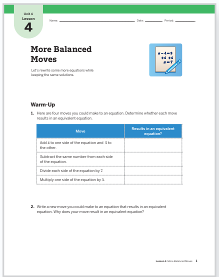
~ 45 min

Standards: NY-8.EE.7, NY-8.EE.7b

Warm-Up

Pairs | 5 min

Students use the Think-Pair-Share routine while identifying moves that result in an equivalent equation to continue their work with solving equations. **(MP3)**

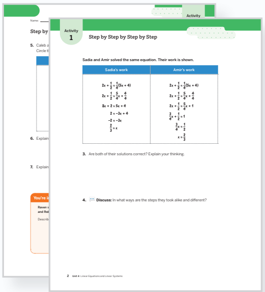


Presentation Screen
1

Activity 1

Pairs | 15 min

Students use MLR7: Compare and Connect when examining the solutions of others and describing potential errors as they build fluency in solving equations. Decide and Defend **(MP3)**

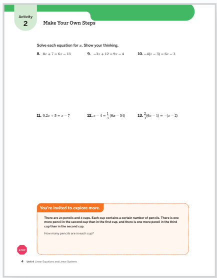


Presentation Screens
2–4

Activity 2

Pairs | 15 min

Students practice solving linear equations with variables on both sides and with rational coefficients to increase fluency in solving linear equations. MLR2: Collect and Display **(MP7)**

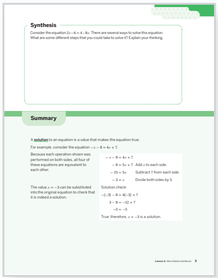


Presentation Screens
5–6

Synthesis

Whole Class | 5 min

There are many ways to approach solving linear equations with variables on both sides, as long as each step results in an equivalent equation. MLR2: Collect and Display

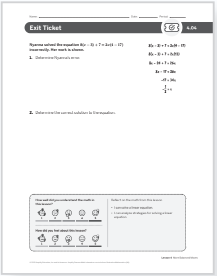


Presentation Screen
7

Exit Ticket

Independent | 5 min

Students demonstrate their understanding by analyzing an incorrect solution to a linear equation and determining the correct solution.



Presentation Screen
8

Prep Checklist

Assign the print lesson and prepare the additional materials. Display the Presentation Screens.

This lesson includes:

- Student Edition
- Exit Ticket PDF
- Coloring tools (as needed)

Print Lesson

Pairs | 5 min

Warm-Up

Purpose: Students use the Think-Pair-Share routine while identifying moves that result in an equivalent equation to continue their work with solving equations.

Lesson 4
Presentation Screens

Presentation Screen
1

1 Launch

Use the Think-Pair-Share routine to help students make sense of the task.

1 Connect

Display each move from the Warm-Up one at a time.

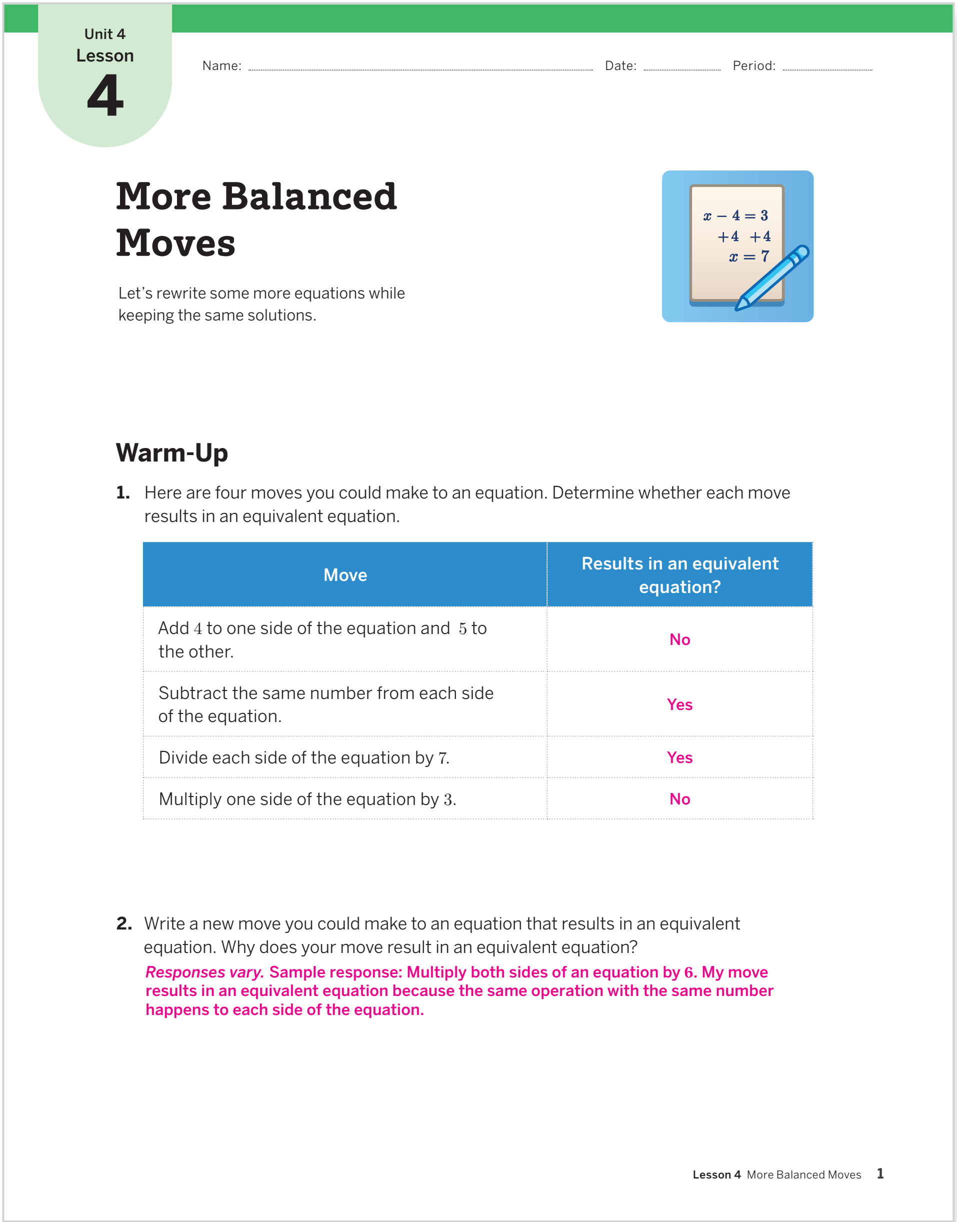
Invite students to share their responses and to critique the reasoning of others. **(MP3)**

> **Math Identity and Community** Invite students to notice and celebrate the variety of mathematical thinking during an activity.

Consider asking:

- "When making a move to an equation, how do you know if it results in an equivalent equation?"
- "Why did the first move in the table not result in an equivalent equation when it involved addition on both sides of the equation?" Amplify responses that recognize the same number was not added to both sides of the equation.

Emphasize that when creating equivalent equations, the operation performed on the left side of the equation must also be performed on the right side.



Unit 4
Lesson
4

Name: ____________ Date: ________ Period: ________

More Balanced Moves

Let's rewrite some more equations while keeping the same solutions.

Warm-Up

1. Here are four moves you could make to an equation. Determine whether each move results in an equivalent equation.

Move	Results in an equivalent equation?
Add 4 to one side of the equation and 5 to the other.	No
Subtract the same number from each side of the equation.	Yes
Divide each side of the equation by 7.	Yes
Multiply one side of the equation by 3.	No

2. Write a new move you could make to an equation that results in an equivalent equation. Why does your move result in an equivalent equation?
Responses vary. Sample response: Multiply both sides of an equation by 6. My move results in an equivalent equation because the same operation with the same number happens to each side of the equation.

Lesson 4 More Balanced Moves 1

Pairs | 15 min

Activity 1 Step by Step by Step by Step

Purpose: Students use MLR7: Compare and Connect when examining the solutions of others and describing potential errors as they build fluency in solving equations.

Short on time: Consider completing this activity as a whole class.

Lesson 4
Presentation Screens

Presentation Screens
2–4

2 Launch

Display the equation on the Teacher Presentation Screen and ask students, "What is a first step you might take to solve this equation?"

2-4 Monitor

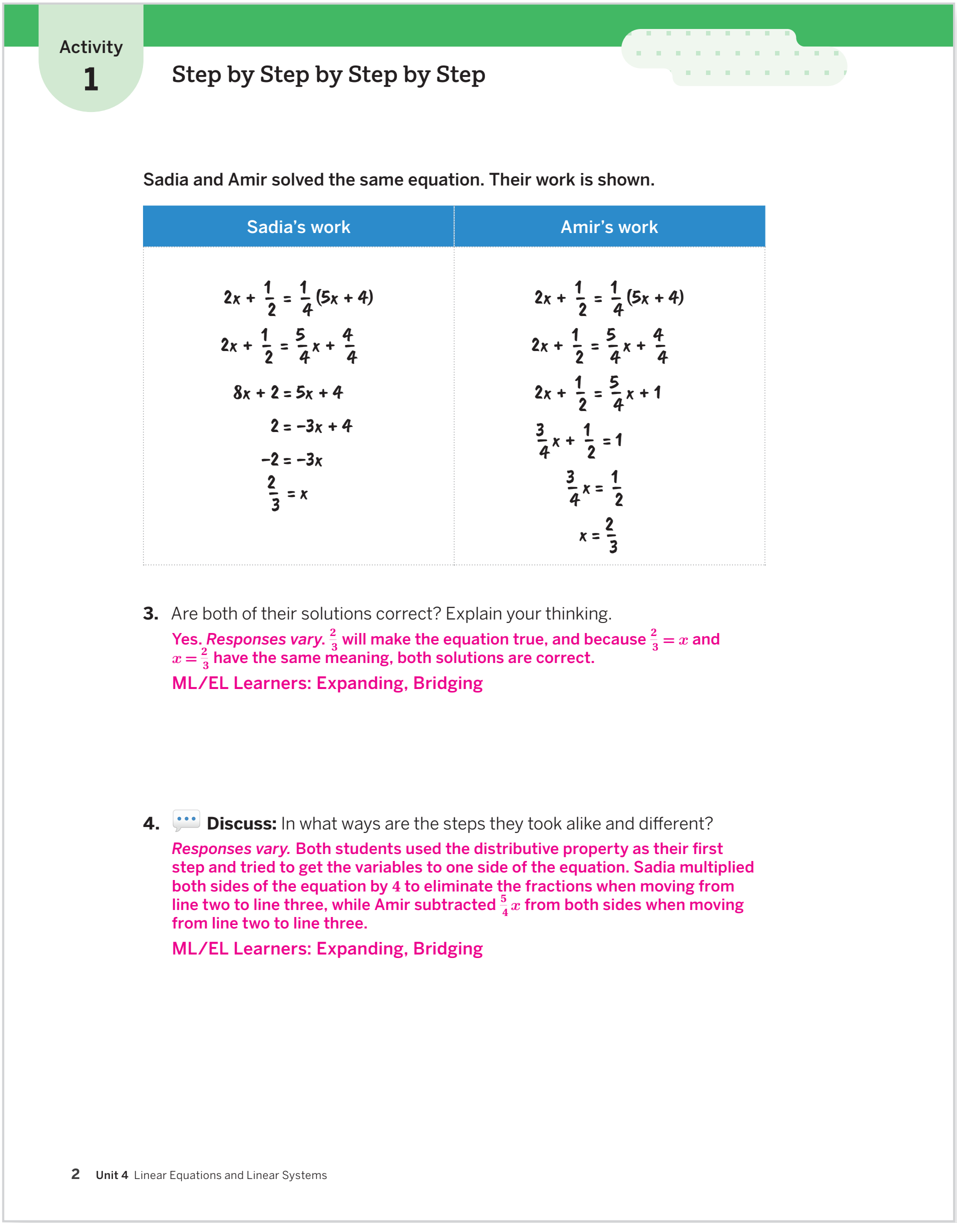
Support getting started by asking, "How can you check whether a solution is correct?"

Pause to remind students that when solving an equation, they can rewrite the solution so that the variable is on the left side or right side, so $\frac{2}{3} = x$ is the same as $x = \frac{2}{3}$.

D Differentiation

Look for students who . . .	Teacher Moves
Think that the second step in Caleb's work is correct because he subtracted $5x$ from each side. *(Screen 4)*	**Support:** Ask "What do the parentheses represent on the right side of the original equation?"
Think that the fourth step in Roberto's work is correct because he added $5x$ to each side. *(Screen 4)*	**Support:** Ask, "If $5x$ is added to the right side, what is $5x + 5x$?"
Substitute $\frac{2}{3}$ into the original equations to determine they are both true. *(Screen 4)*	Invite students to share this strategy during the Connect.
Would benefit from a challenge during this activity.	**Extension:** ***You're invited to explore more.*** Encourage students to share responses with each other in place of a whole class discussion.

Activity 1 continued >



Activity
1

Step by Step by Step by Step

Sadia and Amir solved the same equation. Their work is shown.

Sadia's work	Amir's work
$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$ $2x + \frac{1}{2} = \frac{5}{4}x + \frac{4}{4}$ $8x + 2 = 5x + 4$ $2 = -3x + 4$ $-2 = -3x$ $\frac{2}{3} = x$	$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$ $2x + \frac{1}{2} = \frac{5}{4}x + \frac{4}{4}$ $2x + \frac{1}{2} = \frac{5}{4}x + 1$ $\frac{3}{4}x + \frac{1}{2} = 1$ $\frac{3}{4}x = \frac{1}{2}$ $x = \frac{2}{3}$

3. Are both of their solutions correct? Explain your thinking.

Yes. *Responses vary.* $\frac{2}{3}$ will make the equation true, and because $\frac{2}{3} = x$ and $x = \frac{2}{3}$ have the same meaning, both solutions are correct.

ML/EL Learners: Expanding, Bridging

4. **Discuss:** In what ways are the steps they took alike and different?

Responses vary. Both students used the distributive property as their first step and tried to get the variables to one side of the equation. Sadia multiplied both sides of the equation by 4 to eliminate the fractions when moving from line two to line three, while Amir subtracted $\frac{5}{4}x$ from both sides when moving from line two to line three.

ML/EL Learners: Expanding, Bridging

2 **Unit 4** Linear Equations and Linear Systems

Pairs | 15 min

Activity 1 Step by Step by Step by Step (continued)

Purpose: Students use MLR7: Compare and Connect when examining the solutions of others and describing potential errors as they build fluency in solving equations.

Short on time: Consider completing this activity as a whole class.

Lesson 4
Presentation Screens

Presentation Screens
2–4

2–4 Monitor

M/EL **Multilingual/English Learners** Encourage students to refer to and use language from the class display to support them as they use precise language when explaining their thinking. **(Reading, Speaking, and Listening)**

A **Accessibility: Conceptual Processing** Provide access to coloring tools and invite students to annotate each step with the reasoning they used. For example, in the second step of Sadia's work, students may write "Distributive Property."

3–4 Connect

Display each problem as it is being discussed.

Invite students to share their responses to each problem. Sequence responses by starting with Sadia's and Amir's solutions first.

Collectively define the term **solution**.

MLR **MLR7: Compare and Connect** Invite students to compare their responses to each of the problems as they are discussed. Consider using these questions to guide their discussion:

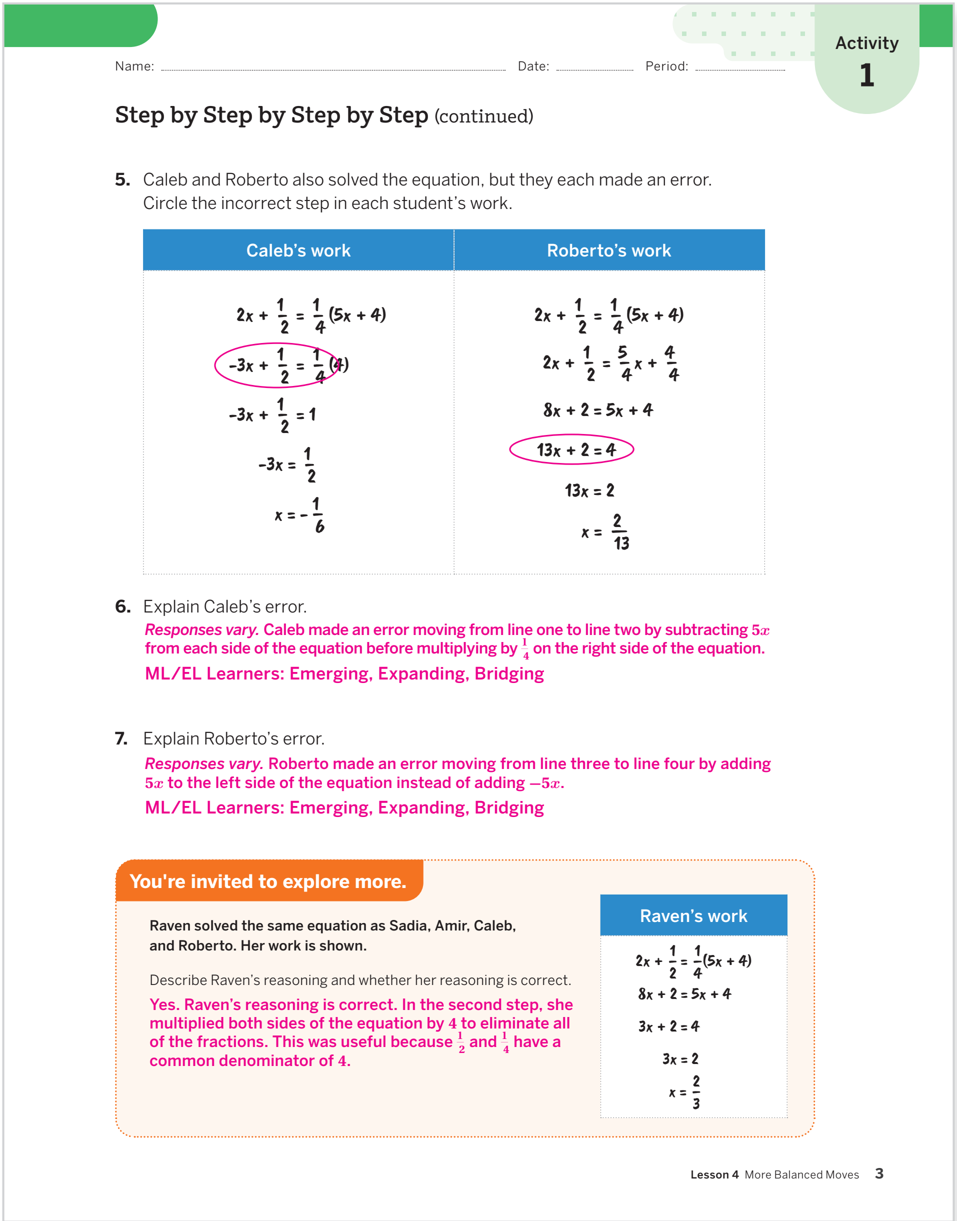
- "How were the ways you and your partner determined whether both Sadia's and Amir's solutions were correct alike? How were they different?"
- "Why does one approach use multiplication and the other does not?"
- "Why did Sadia's and Amir's different strategies lead to the same solution?"

Emphasize that neither solution path is better than the other. There are multiple ways to solve for x.

Use the Decide and Defend routine to discuss Caleb's and Roberto's solutions. **(MP3)**

Consider asking, "What advice would you give Caleb and Roberto for checking their work in the future?"

Key Takeaway: A solution is a value that makes an equation true. Although Sadia's and Amir's final steps may look different, their previous steps worked to decrease the total number of terms until only an x term and a number remained on either side of the equal sign.



Name: ______ Date: ______ Period: ______

Activity 1

Step by Step by Step by Step (continued)

5. Caleb and Roberto also solved the equation, but they each made an error. Circle the incorrect step in each student's work.

Caleb's work	Roberto's work
$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$	$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$
$-3x + \frac{1}{2} = \frac{1}{4}(4)$	$2x + \frac{1}{2} = \frac{5}{4}x + \frac{4}{4}$
$-3x + \frac{1}{2} = 1$	$8x + 2 = 5x + 4$
$-3x = \frac{1}{2}$	$13x + 2 = 4$
$x = -\frac{1}{6}$	$13x = 2$
	$x = \frac{2}{13}$

6. Explain Caleb's error.

Responses vary. Caleb made an error moving from line one to line two by subtracting $5x$ from each side of the equation before multiplying by $\frac{1}{4}$ on the right side of the equation.

ML/EL Learners: Emerging, Expanding, Bridging

7. Explain Roberto's error.

Responses vary. Roberto made an error moving from line three to line four by adding $5x$ to the left side of the equation instead of adding $-5x$.

ML/EL Learners: Emerging, Expanding, Bridging

You're invited to explore more.

Raven solved the same equation as Sadia, Amir, Caleb, and Roberto. Her work is shown.

Raven's work
$2x + \frac{1}{2} = \frac{1}{4}(5x + 4)$
$8x + 2 = 5x + 4$
$3x + 2 = 4$
$3x = 2$
$x = \frac{2}{3}$

Describe Raven's reasoning and whether her reasoning is correct.

Yes. Raven's reasoning is correct. In the second step, she multiplied both sides of the equation by 4 to eliminate all of the fractions. This was useful because $\frac{1}{2}$ and $\frac{1}{4}$ have a common denominator of 4.

Lesson 4 More Balanced Moves 3

Pairs | 15 min

Activity 2 Make Your Own Steps

Purpose: Students practice solving linear equations with variables on both sides and with rational coefficients to increase fluency in solving linear equations.

Short on time: Have students choose two problems to complete and assign the remaining problems as additional practice.

Lesson 4
Presentation Screens

Presentation Screens
5–6

5 Launch

Demonstrate how to solve Problem 8.

6 Monitor

Support getting started by telling students that although they may take different steps, as long as each step results in an equivalent equation, they will reach the correct solution.

D Differentiation

Look for students who:	Teacher Moves
Multiply by the first number inside the parentheses, but not the second, when distributing.	Ask, "What does it mean for the parentheses to be around both terms?"
Would benefit from a challenge during this activity.	**Extension:** ***You're invited to explore more.*** Encourage students to compare their solutions and strategies.

6 Connect

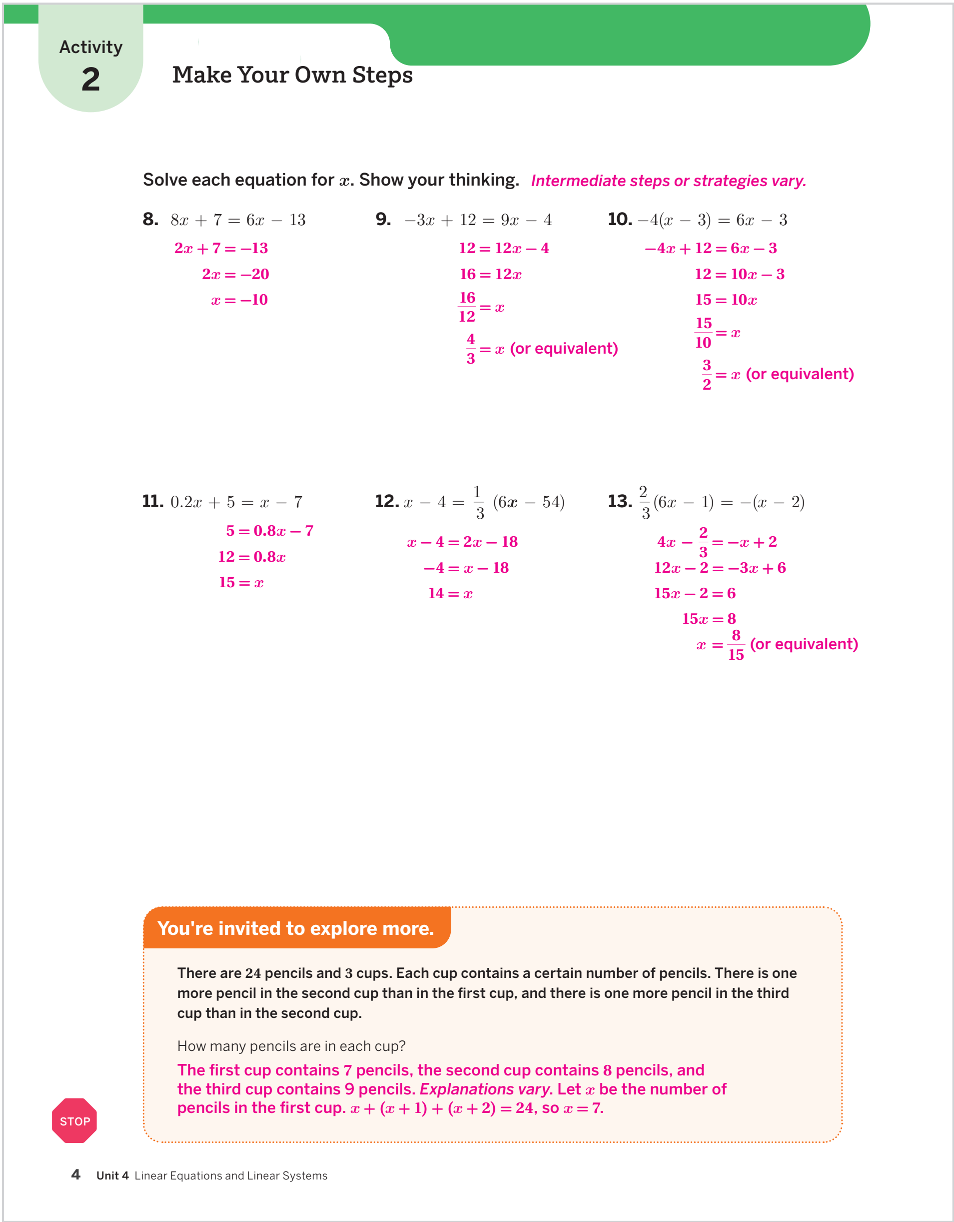
Display Problems 8–10 and invite students to share their solution methods for each problem.

MLR **MLR2: Collect and Display** As students share their solutions:

- Collect the language used to describe their solution methods and record different solution paths that led to the same result. Consider grouping words and phrases used for each step in different areas of the visual display. Amplify phrases, such as: "Multiply first," "Divide first," "Subtract from the right," and "Add from the left."
- Display an example of one equation solved two different ways and annotate each using the language of your students to describe the solution methods. Encourage students to continue using language from the class display during group discussions to support their use of mathematical language while explaining solution pathways.

Emphasize that the structure of an equation can be helpful as students think about different ways to approach solving it. To check the solution, substitute the value of the solution into the original equation. If the resulting equation is true, the solution is correct. **(MP7)**

Consider asking, "In Problem 8, which step do you prefer: subtracting $8x$ or $6x$ from both sides? Why?"



Activity
2

Make Your Own Steps

Solve each equation for x. Show your thinking. *Intermediate steps or strategies vary.*

8. $8x + 7 = 6x - 13$

$2x + 7 = -13$

$2x = -20$

$x = -10$

9. $-3x + 12 = 9x - 4$

$12 = 12x - 4$

$16 = 12x$

$\frac{16}{12} = x$

$\frac{4}{3} = x$ (or equivalent)

10. $-4(x - 3) = 6x - 3$

$-4x + 12 = 6x - 3$

$12 = 10x - 3$

$15 = 10x$

$\frac{15}{10} = x$

$\frac{3}{2} = x$ (or equivalent)

11. $0.2x + 5 = x - 7$

$5 = 0.8x - 7$

$12 = 0.8x$

$15 = x$

12. $x - 4 = \frac{1}{3}(6x - 54)$

$x - 4 = 2x - 18$

$-4 = x - 18$

$14 = x$

13. $\frac{2}{3}(6x - 1) = -(x - 2)$

$4x - \frac{2}{3} = -x + 2$

$12x - 2 = -3x + 6$

$15x - 2 = 6$

$15x = 8$

$x = \frac{8}{15}$ (or equivalent)

You're invited to explore more.

There are 24 pencils and 3 cups. Each cup contains a certain number of pencils. There is one more pencil in the second cup than in the first cup, and there is one more pencil in the third cup than in the second cup.

How many pencils are in each cup?

The first cup contains 7 pencils, the second cup contains 8 pencils, and the third cup contains 9 pencils. *Explanations vary.* Let x be the number of pencils in the first cup. $x + (x + 1) + (x + 2) = 24$, so $x = 7$.

STOP

4 Unit 4 Linear Equations and Linear Systems

Synthesis

Purpose: There are many ways to approach solving linear equations with variables on both sides, as long as each step results in an equivalent equation.

Lesson 4
Presentation Screens

Presentation Screen
7

7 Synthesis

Invite students to respond independently, and then share their thinking with a partner.

Display several students' responses. Invite students to share the connections they see between responses.

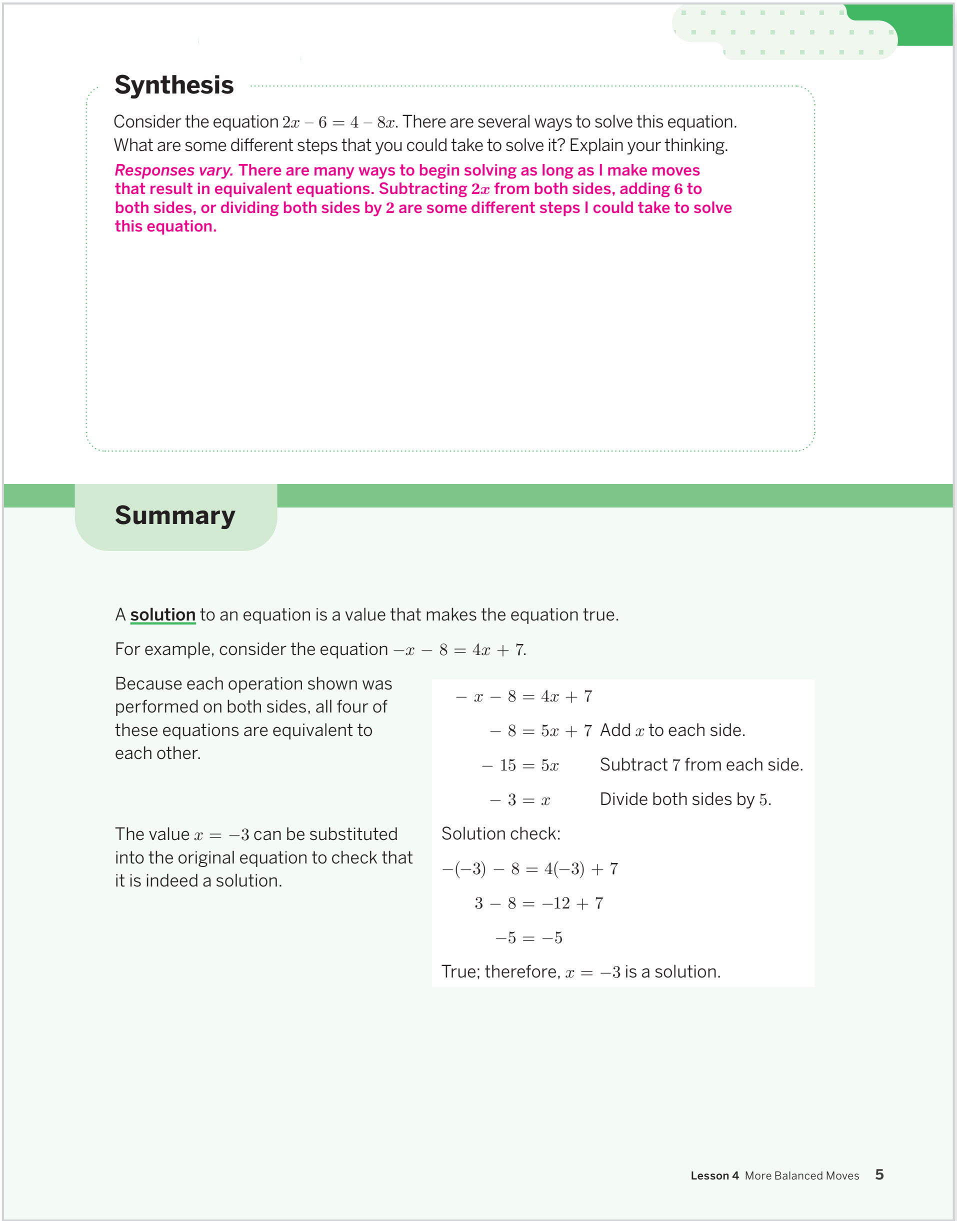
MLR **Use the MLR2: Collect and Display routine** to formalize the definition of **solution** using students' language. Encourage students to refer to the class display for this unit. Add any words, phrases, or diagrams related to the term *solution*.

Emphasize that students have choices to make when solving equations. They can expect to become more fluent with equation solving as they practice and become more familiar with moves they can take that result in equivalent equations.

Lesson Takeaway: Different solution strategies can be used to solve the same equation. Using the structure of an equation can be helpful in determining effective steps to solving the equation.

Summary

Share the Summary. Students can refer back to this throughout the unit and course.



Synthesis

Consider the equation $2x - 6 = 4 - 8x$. There are several ways to solve this equation. What are some different steps that you could take to solve it? Explain your thinking.

Responses vary. **There are many ways to begin solving as long as I make moves that result in equivalent equations. Subtracting $2x$ from both sides, adding 6 to both sides, or dividing both sides by 2 are some different steps I could take to solve this equation.**

Summary

A **solution** to an equation is a value that makes the equation true.

For example, consider the equation $-x - 8 = 4x + 7$.

Because each operation shown was performed on both sides, all four of these equations are equivalent to each other.

$-x - 8 = 4x + 7$
$-8 = 5x + 7$ Add x to each side.
$-15 = 5x$ Subtract 7 from each side.
$-3 = x$ Divide both sides by 5.

The value $x = -3$ can be substituted into the original equation to check that it is indeed a solution.

Solution check:
$-(-3) - 8 = 4(-3) + 7$
$3 - 8 = -12 + 7$
$-5 = -5$
True; therefore, $x = -3$ is a solution.

Lesson 4 More Balanced Moves 5

Independent | 5 min

Exit Ticket

Purpose: Students demonstrate their understanding by analyzing an incorrect solution to a linear equation and determining the correct solution.

Lesson 4
Presentation Screens

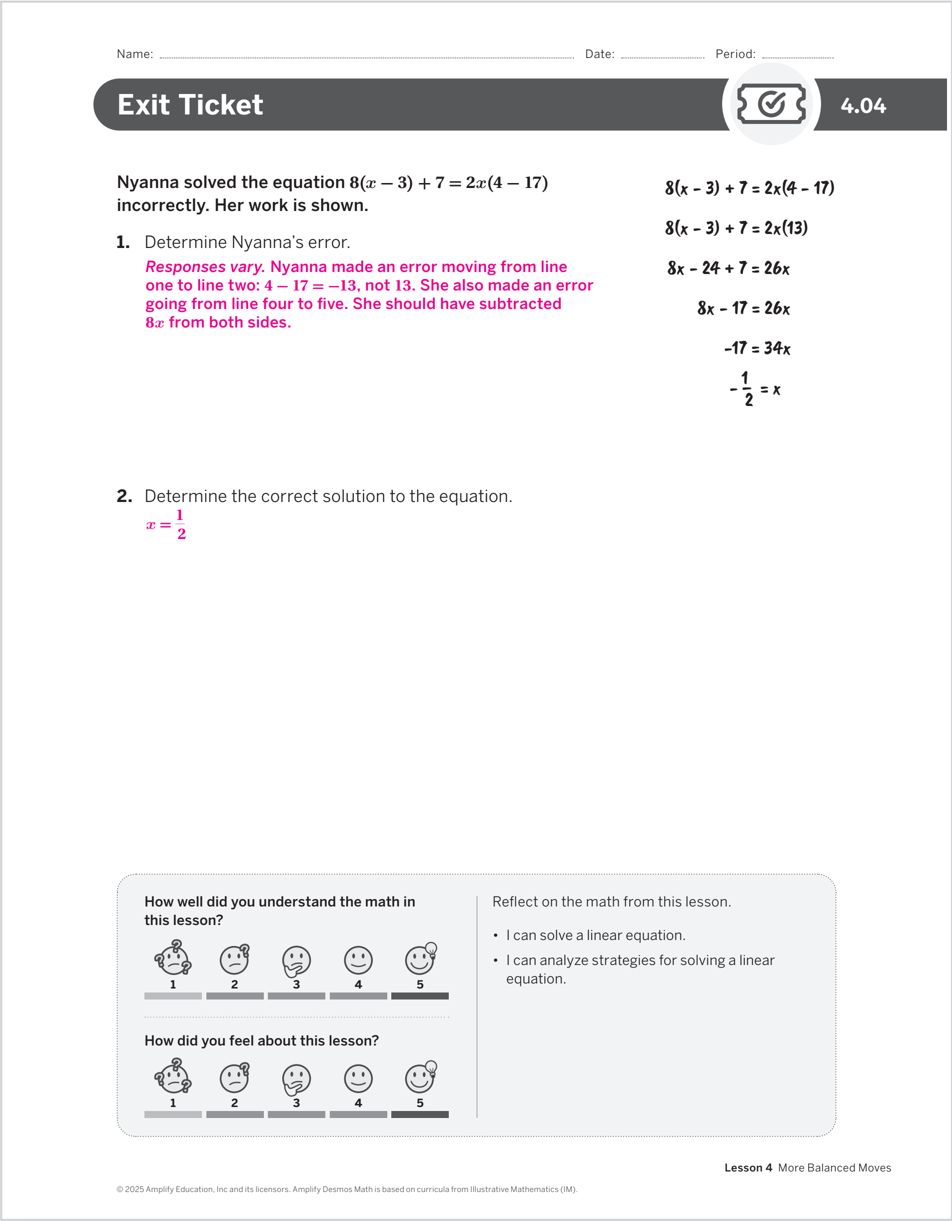
Presentation Screen
8

8 Today's Goals

Goal: Solve a linear equation in one variable.

Language Goal: Analyze strategies for solving a linear equation in one variable. **(Reading, Writing, Speaking, and Listening)**

Support for Future Learning: If students struggle with solving the equation, plan to emphasize this when opportunities arise over the next several lessons. The Exit Ticket in Lesson 5 provides a similar error analysis opportunity.



Name: .... Date: .... Period: ....

Exit Ticket 4.04

Nyanna solved the equation $8(x-3)+7=2x(4-17)$ incorrectly. Her work is shown.

$8(x - 3) + 7 = 2x(4 - 17)$

$8(x - 3) + 7 = 2x(13)$

$8x - 24 + 7 = 26x$

$8x - 17 = 26x$

$-17 = 34x$

$-\frac{1}{2} = x$

1. Determine Nyanna's error.

Responses vary. Nyanna made an error moving from line one to line two: $4-17=-13$, not 13. She also made an error going from line four to five. She should have subtracted $8x$ from both sides.

2. Determine the correct solution to the equation.

$x=\frac{1}{2}$

How well did you understand the math in this lesson?

1 2 3 4 5

How did you feel about this lesson?

1 2 3 4 5

Reflect on the math from this lesson.

- I can solve a linear equation.
- I can analyze strategies for solving a linear equation.

Lesson 4 More Balanced Moves

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Practice

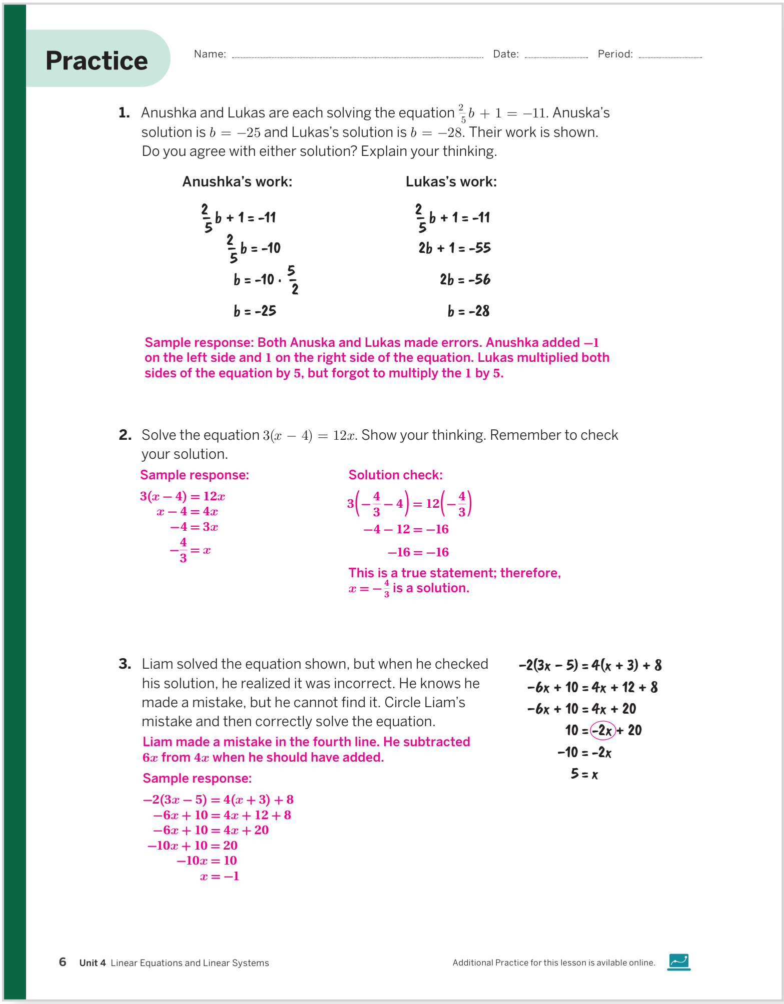
Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.


Lesson 4
Practice

Students using digital

Students using print



Practice

Name: .... Date: .... Period: ....

1. Anushka and Lukas are each solving the equation $\frac{2}{5}b + 1 = -11$. Anuska's solution is $b = -25$ and Lukas's solution is $b = -28$. Their work is shown. Do you agree with either solution? Explain your thinking.

Anushka's work:

$\frac{2}{5}b + 1 = -11$
$\frac{2}{5}b = -10$
$b = -10 \cdot \frac{5}{2}$
$b = -25$

Lukas's work:

$\frac{2}{5}b + 1 = -11$
$2b + 1 = -55$
$2b = -56$
$b = -28$

Sample response: Both Anuska and Lukas made errors. Anushka added -1 on the left side and 1 on the right side of the equation. Lukas multiplied both sides of the equation by 5, but forgot to multiply the 1 by 5.

2. Solve the equation $3(x - 4) = 12x$. Show your thinking. Remember to check your solution.

Sample response:

$3(x-4) = 12x$
$x - 4 = 4x$
$-4 = 3x$
$-\frac{4}{3} = x$

Solution check:

$3\left(-\frac{4}{3} - 4\right) = 12\left(-\frac{4}{3}\right)$
$-4 - 12 = -16$
$-16 = -16$

This is a true statement; therefore, $x = -\frac{4}{3}$ is a solution.

3. Liam solved the equation shown, but when he checked his solution, he realized it was incorrect. He knows he made a mistake, but he cannot find it. Circle Liam's mistake and then correctly solve the equation.

$-2(3x - 5) = 4(x + 3) + 8$
$-6x + 10 = 4x + 12 + 8$
$-6x + 10 = 4x + 20$
$10 = \boxed{-2x} + 20$
$-10 = -2x$
$5 = x$

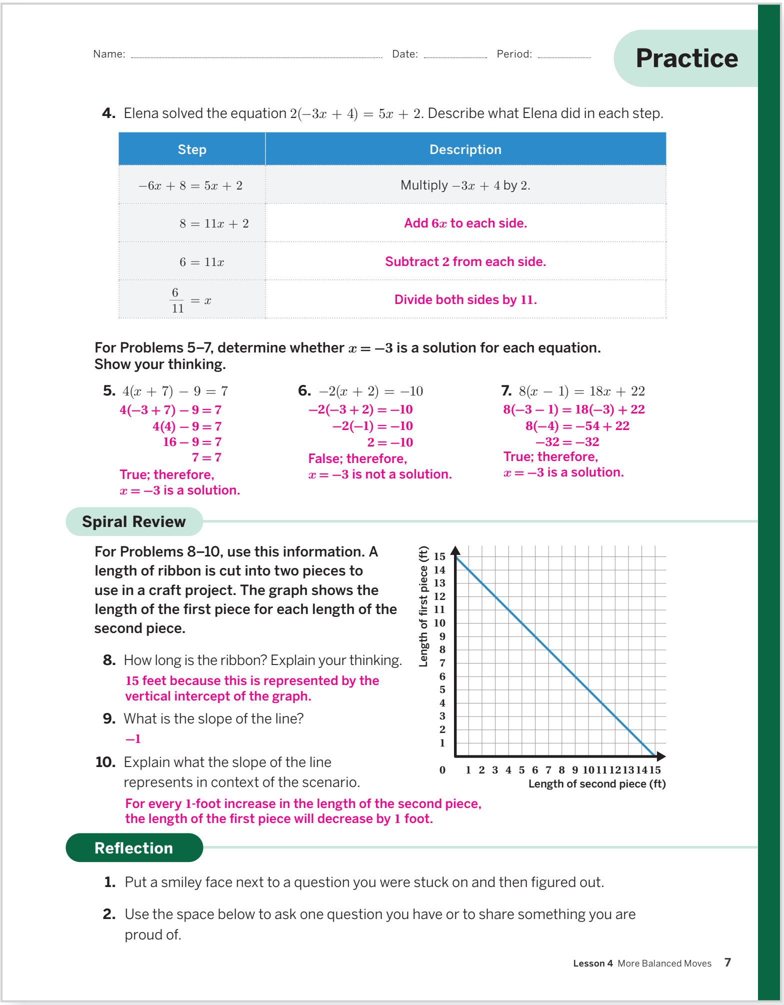
Liam made a mistake in the fourth line. He subtracted $6x$ from $4x$ when he should have added.

Sample response:

$-2(3x-5) = 4(x+3) + 8$
$-6x + 10 = 4x + 12 + 8$
$-6x + 10 = 4x + 20$
$-10x + 10 = 20$
$-10x = 10$
$x = -1$

6 **Unit 4** Linear Equations and Linear Systems

Additional Practice for this lesson is avilable online.



Name: .... Date: .... Period: ....

Practice

4. Elena solved the equation $2(-3x + 4) = 5x + 2$. Describe what Elena did in each step.

Step	Description
$-6x + 8 = 5x + 2$	Multiply $-3x + 4$ by 2.
$8 = 11x + 2$	**Add $6x$ to each side.**
$6 = 11x$	**Subtract 2 from each side.**
$\frac{6}{11} = x$	**Divide both sides by 11.**

For Problems 5–7, determine whether $x = -3$ is a solution for each equation. Show your thinking.

5. $4(x + 7) - 9 = 7$
$4(-3+7) - 9 = 7$
$4(4) - 9 = 7$
$16 - 9 = 7$
$7 = 7$
True; therefore, $x = -3$ is a solution.

6. $-2(x + 2) = -10$
$-2(-3+2) = -10$
$-2(-1) = -10$
$2 = -10$
False; therefore, $x = -3$ is not a solution.

7. $8(x - 1) = 18x + 22$
$8(-3-1) = 18(-3) + 22$
$8(-4) = -54 + 22$
$-32 = -32$
True; therefore, $x = -3$ is a solution.

Spiral Review

For Problems 8–10, use this information. A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length of the first piece for each length of the second piece.

8. How long is the ribbon? Explain your thinking.
15 feet because this is represented by the vertical intercept of the graph.

9. What is the slope of the line?
-1

10. Explain what the slope of the line represents in context of the scenario.
For every 1-foot increase in the length of the second piece, the length of the first piece will decrease by 1 foot.

Reflection

1. Put a smiley face next to a question you were stuck on and then figured out.
2. Use the space below to ask one question you have or to share something you are proud of.

Lesson 4 More Balanced Moves 7

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1, 3, 4	2	NY-8.EE.7 NY-8.EE.7b
Fluency	2	1	NY-8.EE.7 NY-8.EE.7b
Test Practice	5–7	1	NY-8.EE.7 NY-8.EE.7b
Spiral Review			
	8, 10	2	NY-8.EE.6
	9	1	NY-8.EE.6

Equation Roundtable (NYC)

Lesson 5: Solving Linear Equations, Part 2

Purpose

The purpose of this lesson is to move towards general methods for solving linear equations.

Preparation

Worksheet

- *Activity 1*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Warm-Up: Equation Talk (5 minutes)

The purpose of this warm-up is to elicit students' strategies for solving an equation for the value of x. The negative integers and the location of x in each equation are purposeful to spark a discussion about operations of integers and a negative coefficient of a variable.

Activity Launch

Facilitate the Number Talk routine. Tell students that you will show them some equations that they must solve mentally. Use the projection sheets to display each problem one at a time for all to see. Give students 30 seconds of quiet think-time for each equation. Ask students to share their strategies for finding the value of x.

Teacher Moves

Some students may reason about the value of x using logic. For example, in $-3x = 9$, the x must be -3 since $-3 \bullet -3 = 9$. Other students may reason about the value of x by changing the value of each side of the equation equally (e.g., dividing each side of $-3x = 9$ by -3 to get the result $x =-3$). Highlight both of these strategies during the discussion where possible.

To involve more students in the conversation, consider asking these questions as students share their ideas:

- *Can you explain why you chose your strategy?*
- *Can anyone restate* _________ *'s reasoning in a different way?*
- *Did anyone reason about the problem in the same way but can explain it differently?*
- *Did anyone reason about the problem in a different way?*
- *Does anyone want to add on to* __________ *'s strategy?"*
- *Do you agree or disagree? Why?*

Support for Students With Disabilities

Memory: Processing Time

Provide sticky notes or mini whiteboards to aid students with working memory challenges.

Activity 1: Equation Roundtable (25 minutes)

Activity Launch

Arrange students in groups of 2–4. Distribute a double-sided Activity 1 worksheet to each student. Tell the class that you will project one equation at a time and that they will copy the equation onto their sheet and write one step towards solving the equation. Model this process using a problem from the projection sheets to ensure that students only complete one step before passing their papers. These equations were written to allow for a variety of first steps. Remind students that there are many different correct ways to find the value of x.

Tell students that after everyone in their group has written down a first step, they will pass their sheet to the student on their left and receive a new sheet from the student on their right. They will check the work on this new sheet and ask the step's author to justify their thinking. Once both students agree that this step is correct, each student will complete the next step on their classmate's sheet and pass again. Students continue this process until every worksheet in the group has been solved for the value of x. Students then get their original sheet back and check the work on their sheet before repeating the process with the next problem.

To help students understand how they are expected to solve each problem, be sure to provide clear instructions, and support them through each step, particularly with the first problem. Emphasize that the "why" justification should include how their step maintains the equality of the equation. Remind students to push each other to explain how their step guarantees that the equation is still balanced. For example, a student might say they are combining two terms on one side of the equation, which maintains the equality because the value of that side does not change, only the appearance does.

If any groups finish early, make sure they have checked their solutions, and then challenge them to find a solution to one of the problems using fewer steps than their first solution. Conclude with a whole-class discussion.

If time is a concern, give each group two equations rather than all four. Make sure each problem is discussed in a final whole-group discussion. Alternatively, extend the activity by selecting more problems using the practice problems in this unit for students to solve with their groups.

Teacher Moves

To highlight some of the differences in solution paths, ask:

- *Did your partner ever make a move different than the one you expected them to? Describe it.*
- *For Problem 4, could you start by halving the value of each side? Why might you want to do this?*
- *What's an arithmetic error you made but then caught when you checked your work?*

Support for Students With Disabilities

Conceptual Processing: Eliminate Barriers

Allow students to use calculators to ensure inclusive participation in the activity.

Support for Multilingual Learners

Speaking: MLR 8 Discussion Supports

Use this routine to help students produce statements about common errors in problem solving. Provide sentence frames to support the discussion. For example:

- The error this student made was _____.
- I believe this happened because _____.
- A different solution path could be _____.

Restate or revoice student language to demonstrate the correct mathematical language to describe each move (e.g., “distribute the,” “combine like terms,” etc.) and include mathematical reasoning (e.g., “. . . because this maintains the equality”). Clarify explanations that detail differences in problem-solving strategies rather than errors to help students see differences in solution paths. This will help students describe differences in solution paths and justify each step.

Lesson Synthesis (10 minutes)

Arrange students into pairs. Distribute one double-sided half sheet of the lesson synthesis and cool-down to each student.

Give students 2–3 minutes to respond to this question, followed by a few minutes to share their responses with their partner and a whole-class discussion.

After the lesson synthesis discussion, ask students to work individually to complete the cool-down on their worksheet.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Cool-Down (5 minutes)

Students will have more opportunities to develop fluency with solving multistep equations. However, Problem 2 in the Practice provides a similar error analysis to this cool-down.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Strategic Solving (NYC)

Lesson 6: Solving Linear Equations, Part 3

Purpose

The purpose of this lesson is for students to identify, describe, and employ strategies for solving linear equations in one variable with different features or structures.

Preparation

Worksheet

- *Activity 1 and 2:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one double-sided half sheet for each student.

Equation Cards

There are two options for using these equation cards:

- *Option 1:* Print and cut one single-sided copy of the equation cards for each student or pair of students. These cards can be reused if you have multiple classes.
- *Option 2:* Print (and do not cut out) one single-sided copy of the equation cards for each student or pair of students. Laminate or place the sheets in sheet protectors to reuse with multiple classes and/or to allow students to write on them with dry erase markers.

Warm-Up (10 minutes)

Activity Launch

Tell students to choose a number (but not share the number with anyone else). Tell them they will perform a sequence of operations on their number and then tell you their final answer. Say each step of Katie's number puzzle from the teacher projection sheet, giving students time to calculate their new number after each step. Select 5–6 students to share their final number, and after each, tell them their original number as quickly as you can.

Pause here and ask students if they can tell how you are able to figure out their number so fast. If no students notice that each number you say is always 6 more than their final number, you may wish to record and display the pairs of numbers for all students to see, or call on more students so that everyone can hear more pairs of numbers.

Once the class agrees that you are able to figure out their original numbers by adding 6 to their final number, tell them that their task is to figure out how it works. Give students 3–4 minutes of quiet work time to write their ideas. Then follow with a whole-class discussion.

While students work, identify those who use expressions with different structures for their representation of the number puzzle, and invite them to share during the whole-class discussion. Focus the discussion on connecting these representations together with $x - 6$, the outcome of the number puzzle.

Teacher Moves

Select previously identified students to share their representations with the class. Record and display the different ways Katie's number puzzle can be written as an expression. To highlight the connection between the different expressions, ask:

- What do all of these expressions have in common that make the number puzzle work? [All of them are equivalent to $x - 6$.]
- How would you justify that, for example, $\frac{1}{6}((3x - 7) \bullet 2 - 22) = x - 6$? [I would simplify the left side of the equation until it was $x - 6$.]
- What does it mean if we have an equation that says $x - 6 = x - 6$? [The two sides of this equation are always the same.]

Readiness Check (Problem 2)

If most students struggled, plan to revisit this question with them before the lesson begins. Consider inviting students to choose a few values to substitute for x to verify their choice of equivalent expressions. (Note: This method is not sufficient for judging equivalent expressions.)

Support for Students With Disabilities

Conceptual Processing: Eliminate Barriers

Allow students to use calculators to ensure inclusive participation in the activity.

Conceptual Processing: Visual Aids

Display the print statement while verbally sharing each step of Katie's number puzzle.

Support for Multilingual Learners

Representing: MLR 7 Compare and Connect

Use this routine when students write an expression to represent Katie's number puzzle. In pairs or groups, ask students to switch their expressions and compare them. Prompt students to share and explain their strategy and to discuss what is the same or different about their approaches. Monitor student discussion, and then clarify how operations should be correctly arranged and grouped to help produce $x - 6$. Allow time for students to share with the whole class. This will help them make connections between different expressions, using appropriate mathematical language to detail their steps.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Activity 1: Predicting Solutions (10 minutes)

Activity Launch

Arrange students into pairs. Display the equation $5x = 6x$ on the projection sheet for all to see. Ask students, "Without solving the equation, how might we know whether x is a positive number, a negative number, or zero?" [The variables can be combined into one term, but there are no constant terms. That means, eventually, the variable term has to equal 0, so x must be 0.]

Display the next equation on the projection sheet, $5.5x + 3 =- 15$, for all to see and ask the same question: "Without solving the equation, how might we know whether x is a positive number, a negative number, or zero?" [If you subtract 3 from each side, you will be left with a positive amount of x's equal to a negative number, so x must be a negative number.]

Instruct students to inspect each equation carefully and use reasoning to answer the questions in the activity rather than trying to solve each equation for a specific value. Give five minutes of quiet think-time, and then ask students to compare their work with their partner. For any questions they disagree on, students should work to reach an agreement.

Teacher Moves

For each equation, invite groups to share how they decided if the solution was positive, negative, or zero. After each group shares, ask if any other group reasoned about the problem in a different way and invite them to share their reasoning.

The purpose of this discussion is for students to practice talking about equations and the operations within them and to use logical thinking. There is no need to try and formally generalize student thinking for all cases at this time. In later grades, students will continue the work started here looking for structure in equations.

Support for Multilingual Learners

Speaking: MLR 8 Discussion Supports

When partners compare their work, display sentence frames to help students describe reasons for whether an equation has a solution that is positive, negative, or zero:

- "I know that equation _____ will have a positive/negative/zero solution because ____."
- "Some features of equations with a positive/negative/zero solution are ____."
- "When I look at the structure of this equation I notice that ____."

This will help students practice talking about the structure of equations and the operations within them.

Support for Students With Disabilities

Conceptual Processing: Processing Time

Begin with a demonstration of the first equation, which will provide access for students who benefit from clear and explicit instructions.

Activity 2: Least and Most Difficult (5 minutes)

Activity Launch
The purpose of this activity is for students to think about what they see as “least difficult” and “most difficult” when looking at equations. Students also discuss strategies for dealing with “difficult” parts of equations.

Keep students in the same pairs. Give students one minute of quiet think-time to get started and then three minutes to discuss and work with their partner. Follow with a whole-class discussion.

Teacher Moves
Facilitate a discussion for students to talk about strategies for solving different types of equations. Some students may have thought that a certain equation was in the “least difficult” category, while others thought that the same equation was in the “most difficult” category. Remind students that once they feel confident about the strategies for solving an equation, their opinion of the difficulty level may change and that recognizing good strategies takes practice and time.

If time allows, invite students to discuss one problem they thought would be the most difficult to solve. This can be repeated several times.

Consider asking some of the following questions to further the discussion:

- For equation D, what can we do to eliminate the fraction? [Multiply each side by the common denominator of 8. Then all the terms will have integer coefficients.]
- Which other equations can we use this strategy for? [Any equation that has a fraction, such as B, C, and E.]
- What steps do you need to do to solve equation G? Which other equations are like this one?” [There is a lot of distributing and collecting like terms. H requires distributing several times.]

Support for Students With Disabilities
Expressive Language: Eliminate Barriers
Provide sentence frames for students to explain their reasoning (e.g., _____ would be least or most difficult to solve because _____.)

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Activity 3: Solve ’em (10 minutes)

Activity Launch
The purpose of this activity is for students to practice solving equations. If time is short, invite students to solve one or two of the equations instead of three.

Give students 3–5 minutes quiet think-time to get started and then 3–5 minutes to discuss and work with their partner.

Teacher Moves
Consider asking some of the following questions to further the discussion:

- Were there any equations that were more difficult to solve than you expected?
- Were there any that were less difficult to solve than you expected?
- What other strategies or steps did you use in solving the equations?

Support for Students With Disabilities
Conceptual Processing: Eliminate Barriers
Allow students to use calculators to ensure inclusive participation in the activity.

Lesson Synthesis (5 minutes)

Arrange students into pairs. Distribute one double-sided half sheet of the lesson synthesis and cool-down to each student.

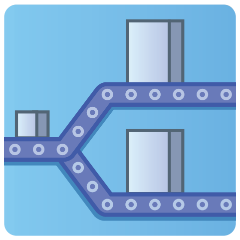
Give students 2–3 minutes to respond to the questions, followed by a few minutes to share their responses with their partner and a whole-class discussion.

After the lesson synthesis discussion, ask students to work individually to complete the Cool-Down on their worksheet.

Cool-Down (5 minutes)

If students struggle, plan to discuss the answer to the first problem in the cool-down as a class so that students can hear how other students recognized that the solution must be positive. If students struggle to operate with negative numbers in the second part of the cool-down, leverage the Practice Problems to provide extra attempts with discussion.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



All, Some, or None?

Lesson 7: Equations With One, Many, or No Solutions

Overview

In previous lessons, students mostly worked with equations that have exactly one solution and solved those equations by a sequence of steps that lead to an equation of the form $x = number$. In this lesson, they encounter equations that have no solutions and equations for which every number is a solution.

Learning Goals

- Compare and contrast equations that have no solutions or infinitely many solutions.
- Using structure, create linear equations with one variable that have either no solutions or infinitely many solutions, and explain the solution method.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In previous lessons, students mostly worked with equations that have exactly one solution and solved those equations by a sequence of steps that lead to an equation of the form $x = number$. In this lesson, they encounter equations that have no solutions and equations for which every number is a solution. In the first case, when students try to solve the equation, they end up with a false statement like $0 = 5$. In the second case, they end up with a statement that is always true, such as $2n = 2n$.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to think about features of pairs of expressions that result in them always or never having equal outputs for a given input.

Activity 1: Always and Never Equal Machines (20 minutes)

The purpose of this activity is for students to think about equality and properties of operations when deciding whether equations are always or never true.

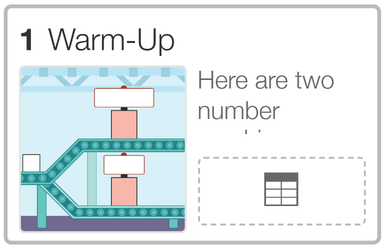
Activity 2: Sorting Solutions (10 minutes)

The purpose of this activity is to encourage students to pause before solving an equation and to build their skill understanding and manipulating the structure of equations (MP7). Students sort a variety of equations into categories based on their number of solutions (MP8).

Lesson Synthesis (5 minutes)

The purpose of this discussion is to consider ways to predict the number of solutions from the structure of an equation. Throughout this discussion, students will make use of structure.

Cool-Down (5 minutes)



1 Warm-Up

Here are two number

Here are two number machines.

Tasia put a number into both machines, and the numbers that came out were the same.

What number did Tasia put in?

Teacher Moves

Purpose
The purpose of this lesson is for students to a) encounter equations that have no solutions and equations for which every number is a solution and b) learn to predict the number of solutions from the structure of an equation.

Warm-Up Launch
Arrange students into pairs. Consider introducing this activity by telling students that the goal here is to notice which inputs make the two machines produce the same outputs. Allow students two minutes of quiet work time, followed by a whole-class discussion.

Pacing
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

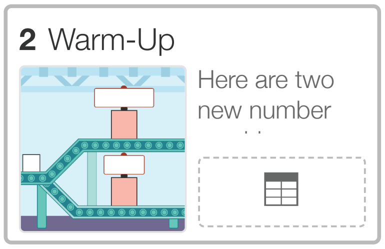
Since the expression $x + 40 + x - 10$ is equivalent to $2(x + 15)$, every number put into both machines will get the same number out.

Student Supports

Students With Disabilities

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



2 Warm-Up

Here are two new number

Here are two new number machines.

Try to find a number to put into both machines to get the same number out.

Teacher Moves

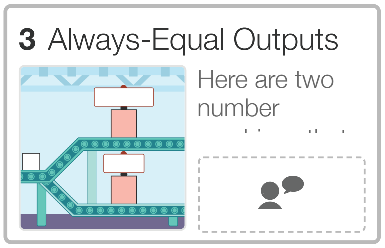
After students have discussed with their partners, invite pairs to share one reason they cannot find a number to put into both machines to get the same number out. Look for students who consider the relationship between the two expressions, and attend to students' explanations to ensure the reasons given are correct.

If no students point out that on the previous screen the two expressions are equivalent, invite them to combine like terms for the top machine and distribute the 2 on the bottom machine, and then share what they notice and how they think this affects the outputs.

This lesson will work towards supporting students in recognizing that they can set the two expressions equal and solve the equation for the unknown. It is not necessary at this point for students to use the machines to write equations.

Sample Responses

Since there is no value for s that will make the expression $s + 10 + 2s$ equal to $3(s + 4)$, there is no number to put into both machines to get the same number out.


3 Always-Equal Outputs

Here are two number

Here are two number machines that ALWAYS give the same number for any input.

Use the machines' expressions to explain why both machines will produce the same outputs for any input.

Teacher Moves

Activity Launch
Arrange students into pairs. Give students two minutes of quiet work time, followed by a partner discussion and a whole-class discussion. Ask students why they think this machine always produces outputs that are the same. Encourage students to use a whiteboard or paper to help them with their thinking.

Highlight unique answers to show the class. Look for responses that consider the equivalence of these expressions. Ask students to justify their responses and critique each other's reasoning.

Pacing
Consider using pacing to restrict students to Screens 3–7.

Sample Responses

Responses vary.

Since both machines' expressions can be simplified to $3t+6$, they are the same. So any input will produce the same value for either machine.

Student Supports

Support for Multilingual Learners

- *Representing: MLR 2 Collect and Display*

On a visual display, record the language students use to justify their thinking. Ask students to name what these equations have in common. Listen for phrases such as "variables with the same coefficient" or "the variable was eliminated." Consider dividing the display into sections labeled "true for all values" and "true for no values," and group words and phrases in the appropriate area. Remind students to borrow language from the display as needed. This will help students use mathematical language to describe their reasoning and increase awareness about what these types of equations look like.

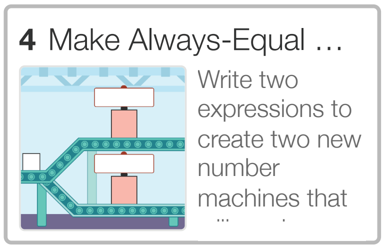
Support for Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation in the activity.

- *Processing Time*

Check in with individual students, as needed, to assess for comprehension during each step of the activity.



4 Make Always-Equal …

Write two expressions to create two new number machines that …

Write two expressions to create two new number machines that will produce the same outputs for any input.

Teacher Moves

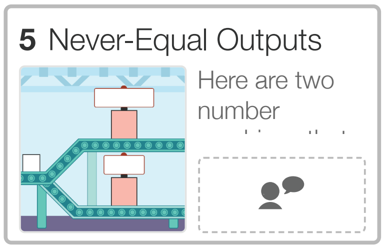
Consider pausing the lesson to highlight unique answers to show the class. Ask students to justify how they know their expressions will always produce equal outputs and to critique each other's reasoning.

Sample Responses

Responses vary.

Any pair of expressions that are equivalent, such as:

- $2x+2$ and $x+x+10-8$
- $5(x+10)$ and $x+x+x+x+x+50$


5 Never-Equal Outputs

Here are two number

Here are two number machines that NEVER give the same number for any input.

Use the machines' expressions to explain why both machines will NEVER produce the same outputs for any input.

Teacher Moves

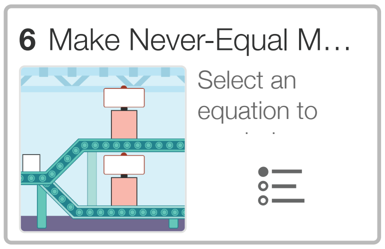
Key Discussion Screen 🔑
The purpose of this discussion is to invite students to reason about why some expressions will have outputs that are never equal. If it was not discussed earlier, pause and discuss when expressions have outputs that are always equal.

Highlight unique answers to show the class. Look for responses that consider the equivalence of these expressions. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

I distributed the 3 in the bottom expression and got $6+3y$. There is no value of y where $10+3y$ will equal $6+3y$, so they will never produce the same value for both machines.


6 Make Never-Equal M…

Select an equation to

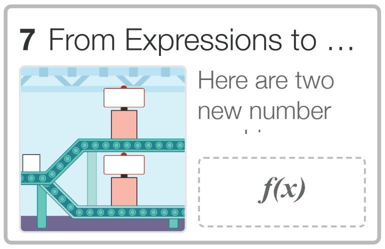
Select an equation to create two new number machines that will NEVER produce the same outputs for any input.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

$2x+3=5+2x$



7 From Expressions to …

Here are two new number

Here are two new number machines.

Write an equation to find an input number that will produce the same outputs for each machine.

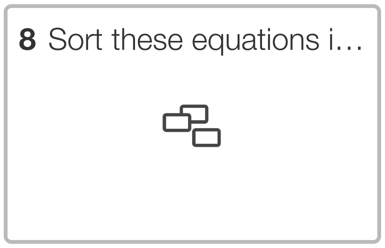
Teacher Moves

The purpose of this screen is for students to connect the thinking they have been doing with number machines to working with equations in the card sort on the next screen. Check student progress and offer individual support where needed. Or lead a whole-class discussion if enough students are struggling.

If time allows, ask students when each machine will produce the same outputs. [They will only produce the same outputs when $n = 1$.] Ask them to use the equation to explain how they know.

Sample Responses

- $6 + 2n = 8n$
- One value of n



8 Sort these equations i…

Teacher Moves

Activity Launch

Pause the lesson and project the cards on this screen. Remind students that they just looked at cases where machines always, sometimes, and never produce equal outputs. Now they will apply this same reasoning to the equations on this screen.

Arrange students into pairs. Give students five minutes of quiet work time, followed by a partner discussion and a whole-class discussion. Encourage students to use a whiteboard or paper to help them with their thinking.

Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

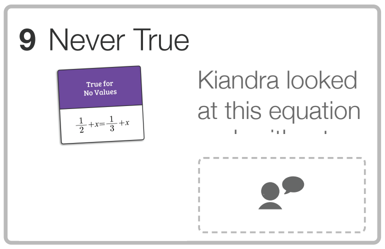
Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Pacing
Consider using pacing to restrict students to Screens 8–9.

Sample Responses

Image solution



9 Never True

True for No Values

$\frac{1}{2}+x=\frac{1}{3}+x$

Kiandra looked at this equation

Kiandra looked at this equation and, without writing anything, said it must never be true.

Explain what she may have noticed to lead to this conclusion.

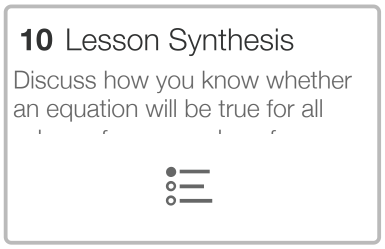
Teacher Moves

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

Kiandra probably noticed that there are different constants being added to x on each side of the equation, so there is no single number that x can be that would make both sides the same.



10 Lesson Synthesis

Discuss how you know whether an equation will be true for all

Discuss how you know whether an equation will be true for all values of x, one value of x, or no values of x.

Then select one choice and record your response.

Write an equation that is true for:

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to summarize the types of equations that will have zero, one, or infinite number of solutions.

Synthesis Launch
Give students 2–3 minutes to respond to this question and to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Ask students: *Describe how you know whether an equation will be true for all values of* x*, one value of* x*, or no values of* x*.* [Equations that are true for any value of x have equivalent expressions on each side. Equations that are true for one value of x have different expressions on each side and can be simplified to $x = number$. Equations that have no solution for any value of x simplify to a statement of two unequal numbers being equal, which is always false.]

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- **No Values of x:** $2 + x = 10 + x$
- **All Values of x:** $2 + x = 2 + x$
- **One Value of x:** $2 + x = 2$

Student Supports

Support for Students With Disabilities

- *Expressive Language: Eliminate Barriers*

Provide sentence starters for students to explain their reasoning (e.g., Equations that are always true for x have _____. Equations that are true for one value of x have _______. Equations that have no solution for any value of x have _____.)

11 Cool-Down

Teacher Moves

Support for Future Learning
If students struggle with this cool-down, plan to review it as a class and invite students to share their strategies for using the structure of the equations to sort the cards. Consider making an explicit connection between this cool-down and the work in Lesson 8, Screens 4–8.

Pacing
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Image solution

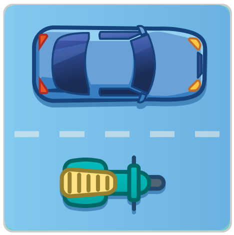
12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can determine whether an equation has no solutions, one solution, or infinitely many solutions.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



When Are They the Same?

Lesson 8: Solving Linear Equations in Context

Overview

Students apply their knowledge of solving equations by considering two vehicles passing on a road. The work in this lesson is a prelude to a simple form of a system of equations, where each equation can be written in the form $y =$ [some expression] (though students do not need to know the term "system of equations" at this point).

Learning Goals

- Create an equation with one variable to represent a situation in which two conditions are equal.
- Interpret the solution to an equation with one variable in context.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to apply their knowledge of solving equations by considering two vehicles passing on a road. Using the speed and the initial position of the vehicles, students are asked to determine when the vehicles will meet. It is the work of the student to recognize that they can set the two expressions equal and solve the equation for the unknown (MP4).

The work in this lesson is a prelude to a simple form of a system of equations, where each equation can be written in the form $y =$ *some expression* (though students do not need to know the term "system of equations" at this point).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to think about the different components in this context, like speed and starting position. This work prepares students for the next activity where they will write expressions to represent the position of different vehicles over time.

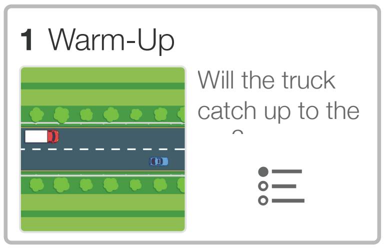
Activity 1: Passing Vehicles (30 minutes)

The purpose of this activity is for students to solve an equation in a real-world context while previewing some future work solving systems of equations. Here, students first make sense of the situation by completing a table of values describing the position of two vehicles over time to help them write expressions for each vehicle (MP1). A key point in this activity is the next step: taking two expressions representing the position of two vehicles for a given time and recognizing that the equation created by setting the two expressions equal to each other has a solution that is the value for time, x, when the vehicles meet (MP4).

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to consider a new context and determine when two changing quantities are equal.

Cool-Down (5 minutes)



1 Warm-Up

Will the truck catch up to the

Will the truck catch up to the car?

Teacher Moves

Purpose
The purpose of this lesson is for students to apply their knowledge of solving equations by considering two vehicles passing on a road.

Warm-Up Launch
Arrange students into pairs. Consider introducing this activity by telling students that in this lesson, they will explore vehicles moving at a constant speed. Allow students two minutes of quiet work time, followed by five minutes of partner and whole-class discussion. Invite several students to share what information they think would be helpful to prove their answer. Ask students to explain how and why the information they identified would be helpful.

Before students start working, ensure that they understand that each vehicle's position is measured at the front of the vehicle.

Pacing
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Yes

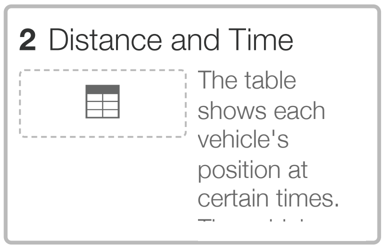
Responses vary.

We would need to know how fast each vehicle is moving.

Student Supports

Support for Students With Disabilities
- *Receptive Language: Processing Time*
Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.

- *Social-Emotional Functioning: Peer Tutors*
Pair students with their previously identified peer tutors.



2 Distance and Time

The table shows each vehicle's position at certain times.

The table shows each vehicle's position at certain times. The vehicles are moving at a constant rate.

Fill in the missing information in the table.

Teacher Moves

Activity Launch
Arrange students into pairs. Explain to students that their task is to determine when the vehicles will meet. Encourage students to use paper to help them with their thinking.

Pacing
Consider using pacing to restrict students to Screens 2–3.

Sample Responses

Truck Position

- At 3 seconds: 45 m
- At 4 seconds: 60 m
- At t seconds: $15t$ m

Car Position

- At 3 seconds: 51 m
- At 4 seconds: 62 m
- At t seconds: $11t + 18$ m

Student Supports

Support for Multilingual Learners

- *Representing, Conversing: Collect and Display*

To begin the whole-class discussion, give students the opportunity to discuss their solutions to the first question in groups of 3–4. Circulate through the groups, and record language students use to describe what is happening in each vehicle. Listen for language related to "rate of change," "differences between rates," "initial starting point," etc. Post the collected language in the front of the room so that students can refer to it throughout the rest of the activity and lesson. This will help students talk about the relationship between the two vehicles prior to being asked to find the time when they are equal.

Support for Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

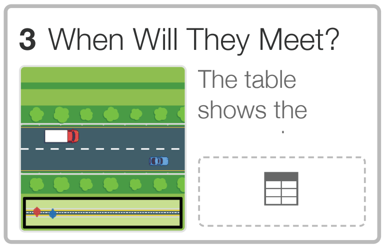
Allow students to use calculators to ensure inclusive participation in the activity.

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the table to review prior to implementation of this activity. Also, check in with individual students, as needed, to assess for comprehension during each step of the activity.

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



3 When Will They Meet?

The table shows the

The table shows the expressions you entered on the previous screen.

Teacher Moves

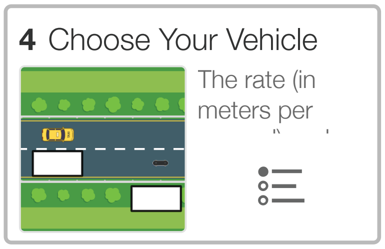
Key Discussion Screen 🔑
The purpose of this discussion is to surface how using equations can be helpful in answering questions about a situation in context (in this case, when the truck will meet the car).

Give students several minutes of work time with their partners, and then follow with a whole-class discussion. Use the teacher view in the dashboard or snapshots to display several student responses. Highlight several answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Invite students to revise their answers based on the feedback they see on this screen. Encourage them to justify any changes they make rather than use the feedback on this screen as a tool for guessing and checking without reflection.

Sample Responses

4.5 seconds



4 Choose Your Vehicle

The rate (in meters per

The rate (in meters per second) and the starting point are displayed for each vehicle.

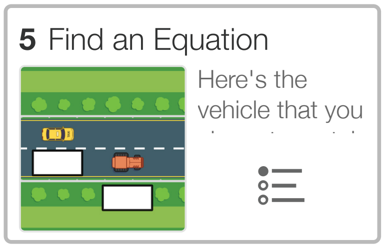
Choose a vehicle to compare with the taxi.

Then go to the next screen.

Teacher Moves

Explain to students that in the next few screens, they will explore several matchups between different vehicles.

Pacing
Consider using pacing to restrict students to Screens 4–10.



5 Find an Equation

Here's the vehicle that you

Here's the vehicle that you chose to match with the taxi.

Which equation could you solve to find out when the two vehicles will meet?

Teacher Moves

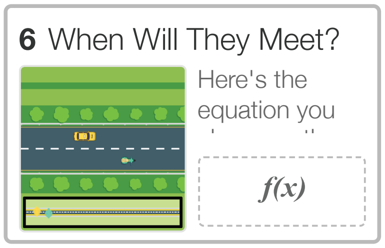
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Responses vary according to the selection made on Screen 4.

- **Taxi vs. scooter:** $6t+8=9t$
- **Taxi vs. skateboard:** $4t+12=9t$
- **Taxi vs. tractor:** $7t+9=9t$

The rate of the taxi is 9 m/s, so the taxi can be represented with the expression $9t$. The skateboard can be represented by the expression $4t+12$. Setting the expressions equal to one another will help us find exactly when the vehicles will meet.



6 When Will They Meet?

Here's the equation you

f(x)

Here's the equation you chose on the previous screen:

When will the two vehicles meet?

Teacher Moves

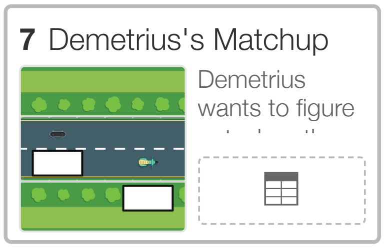
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Responses vary according to the selection made on Screen 5.

- **Taxi vs. scooter:** $\frac{8}{3}$ seconds (or equivalent)

- **Taxi vs. skateboard:** 2.4 seconds
- **Taxi vs. tractor:** 4.5 seconds



7 Demetrius's Matchup

Demetrius wants to figure

Demetrius wants to figure out when these vehicles will meet, so he wrote these expressions:

Teacher Moves

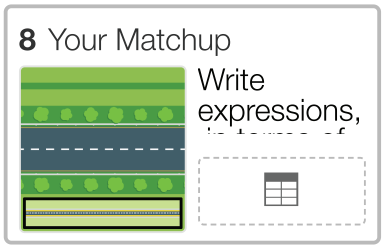
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling. Consider asking students: *If the vehicles never meet, what does that tell us about the solution to the equation* $6t+9=6t+30$*?* [It means there is no solution that will make the equation true.]

Have students recall the work they did on previous lessons with equations that are never true. How does this equation connect to the structure of equations that are never true? [Since there are different constants being added to $6t$ in each expression, there is no single number that t can be that would make both expressions the same.]

Sample Responses

Responses vary.

By looking at the expressions, you can tell that the vehicles are both moving at 6 m/s. Since they are both moving at the same rate, they will never meet.



8 Your Matchup

Write expressions,

Write expressions, in terms of t, that could represent two vehicles traveling at different rates with different starting positions that will eventually meet.

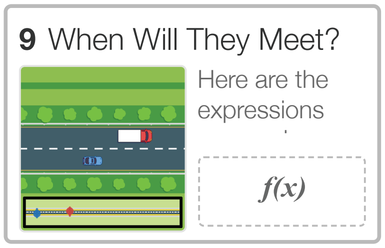
Once you create your match-up, go to the next screen.

Teacher Moves

On this screen, students will create their own expressions to represent two vehicles traveling at different rates.

Sample Responses

Responses vary.



9 When Will They Meet?

Here are the expressions

Here are the expressions you wrote on the previous screen:

Teacher Moves

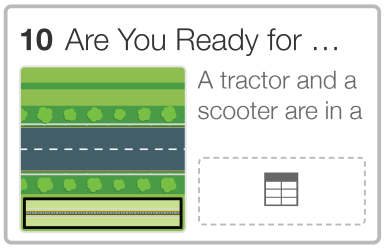
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking
Some students may wonder why their vehicles do not meet. Have these students think about the rate and the starting point of each vehicle. Consider asking: *Look at how the vehicles are positioned. Which is in front? What does the vehicle in the back need to do in order to catch up to the other vehicle? Which vehicle is moving faster?*

Invite students to revise their expressions on the previous screen based on the feedback they see on this screen. Encourage them to justify any changes they make rather than use the feedback on this screen as a tool for guessing and checking without reflection.

Sample Responses

Responses vary.



10 Are You Ready for …

A tractor and a scooter are in a

A tractor and a scooter are in a race.

Write expressions, in terms of t, for each vehicle so that the vehicles start separated and meet at 10 seconds.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

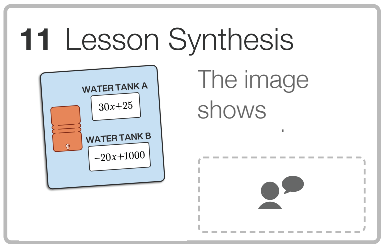
This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 4–9 ahead of time before the class discussion on Screen 11. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.

Tractor Distance Expression: $5t+30$

Scooter Distance Expression: $20t-120$



11 Lesson Synthesis

The image shows

The image shows expressions that represent the amount of water, in liters, in two water tanks.

Let x represent the number of seconds that pass.

How could you determine when the tanks will have the same amount of water?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to describe strategies for using equations to solve a problem in context (in this case, when the tanks will have the same amount of water).

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

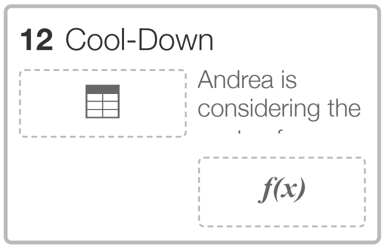
Pacing

Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

Create an equation by setting the two expressions equal to one another. The solution to that equation will be the time it takes the two tanks to have equal amounts of water.



12 Cool-Down

Andrea is considering the

Andrea is considering the costs of printing p pages at home and at a store.

She wrote the following equation: $100 + 0.05p = 0.25p$

Solve Andrea's equation.

Use paper if it helps you with your thinking.

Explain what the solution represents.

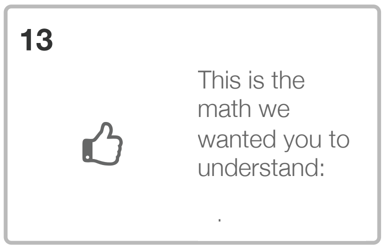
Teacher Moves

Support for Future Learning
If students struggle to solve and interpret a solution in context, plan to revisit this concept before the quiz.

Sample Responses

$p = 500$ (or equivalent)

Responses vary. The solution represents the number of pages for when the cost will be the same for printing at the store and at home.



13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use an expression or an equation to find when two things, like distance, are the same in a real-world situation.

8.4 Practice Day 1 (NYC)

Purpose

The purpose of this lesson is for students to practice solving all kinds of linear equations.

Preparation

Scavenger Hunt Sheets

- Print one single-sided set. If possible, laminate or use sheet protectors to facilitate reuse with other classes. Shuffle the sheets and post them around the classroom, hallway, or outside.

Worksheet

- Print a double-sided worksheet for each student (or pair of students).

Instructions

Students can start at any of the scavenger hunt sheets. Invite students to solve the problem and record their thinking on their worksheet. They will then look for their answer at the top of another Scavenger Hunt sheet and solve the problem on that sheet. If they cannot find their answer, suggest that they check their work. This process continues until the student has solved all 10 problems and is back to their starting point.

There are two problems that require students to solve for a missing weight in a hanger diagram. Let students know they can write on the hanger diagrams at the bottom of their worksheet if it helps them with their thinking.

Once students finish all 10 problems, ask them to write their answers in the order that they found them on the bottom of their worksheets. This will allow you to check that they followed the order of problems correctly.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

On or Off the Line?

Lesson 9: Interpreting Points On or Off the Line

Overview

Students consider pairs of linear equations in various contexts and interpret the meaning of various points on and off the lines, including the point of intersection. This lesson builds upon earlier work with linear equations in two variables in a variety of contexts (MP2).

Learning Goals

- Determine a point that satisfies two relationships simultaneously, using tables or graphs.
- Interpret points that lie on one, both, or neither line(s) of a graph of two simultaneous equations in context.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to consider pairs of linear equations in various contexts and interpret the meaning of various points on and off the lines, including the point of intersection (MP2). This lesson builds upon earlier work with linear equations in two variables in a variety of contexts. In this lesson, students consider pairs of linear equations in each type of context and interpret the meaning of points on the graphs of the equations.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to elicit ways students can describe different characteristics that arise when more than one line is graphed on a coordinate plane.

Activity 1: Two Dollars (15 minutes)

The purpose of this activity is for students to investigate the mathematical structure of two stated facts using graphs representing each fact while reasoning about what must be true.

In this activity, students focus on a context involving coins and think about the context in different ways. In previous lessons, students set two expressions equal to each other to find a common value where both expressions are true (if it exists). A system of two equations asks a similar question: At what common pair of values are both equations true?

Activity 2: When Is It True? (15 minutes)

The purpose of this activity is for students to work with two lines at the same time to determine whether a point lies on one line, both lines, or neither line. In this activity, a system of equations is partially given in words, but key elements are only provided in the graph.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to consolidate students' work with recognizing that a point on a line represents a solution to a relationship.

Cool-Down (5 minutes)

1 Warm-Up

Which one doesn’t belong?

Which one doesn’t belong?

Teacher Moves

Purpose

The purpose of this lesson is for students to consider pairs of linear equations in different contexts and interpret the meaning of various points on and off the lines, including the point of intersection.

Warm-Up Launch

The purpose of the warm-up is to elicit ways students can describe different characteristics that arise when more than one line is graphed on a coordinate plane. Arrange students in groups of 2–4. Give students one minute of quiet think-time and then time to share their thinking with their group. After everyone has conferred in groups, ask the groups to offer at least one reason why each graph doesn’t belong.

Since there is no single correct answer to the question of which graph does not belong, attend to students’ explanations and ensure the reasons given are correct. Encourage students to use concepts and language introduced in previous lessons about lines, such as *slope* and *intercept*. In particular, draw students’ attention to any intersections of the lines.

Pacing

Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Which One Doesn't Belong to support students in noticing the features of each representation.

Sample Responses

Responses vary.

- Top left is the only one with no intersection points (they are all parallel). Or, it is the only one that appears to be the same line translated vertically in two different ways.
- Top right is the only one with an intersection point that has a negative coordinate or the only one with two parallel lines.
- Bottom left is the only one with three lines through a single point.
- Bottom right is the only one with 3 intersection points.

Student Supports

Support for Students With Disabilities

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity.

- *Expressive Language: Eliminate Barriers*
  Provide sentence frames for students to explain their reasoning (e.g., ____________ doesn't belong because _____________.)


2 Two Dollars

I have $\$2$ worth of coins

I have $\$2$ worth of coins in my pocket.

What is a combination of coins that I could have?

Try to think of a combination that no one else in the class will write.

Teacher Moves

Activity Launch
Before looking at the task, tell students: *I have* $\$2$ *in my pocket. What might be in my pocket?* Students will likely guess that you have two $\$1$ bills, but ask what else it might be. Some answers could be 8 quarters; 200 pennies; a $\$2$ bill; or 20 nickels, 2 quarters, and 5 dimes. Explain to students that in this lesson they are going to explore different combinations of coins that total $\$2$.

Allow 1–2 minutes for students to submit a possible combination of coins. Then use the teacher view in the dashboard to highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

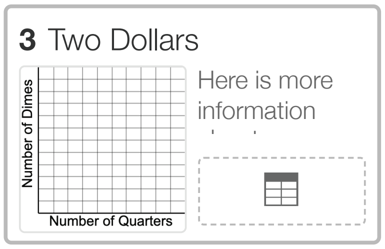
Responses vary.

- 8 quarters
- 2 dimes, 1 nickel, and 7 quarters
- 200 pennies

Student Supports

Support for Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation in the activity.



3 Two Dollars

Here is more information

Here is more information about my coins:

- I only have quarters and dimes.

Fill in at least 3 rows of possible combinations of quarters and dimes that are worth $\$2$.

Teacher Moves

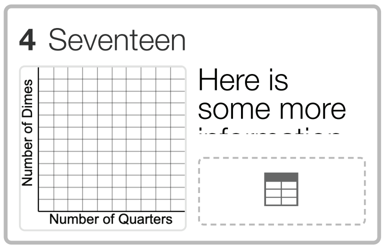
Invite students to revise their answers based on the feedback they see on this screen. Encourage them to justify any changes they make rather than use the feedback on this screen as a tool for guessing and checking without reflection.

Pacing
Consider using pacing to restrict students to Screens 3–5.

Sample Responses

Responses vary.

- $(0, 20)$
- $(2, 15)$
- $(4, 10)$
- $(6, 5)$
- $(8, 0)$



4 Seventeen

Here is some more

Here is some more information about my coins:

- I have a total of 17 coins.

Fill in at least 3 rows of possible combinations of 17 coins.

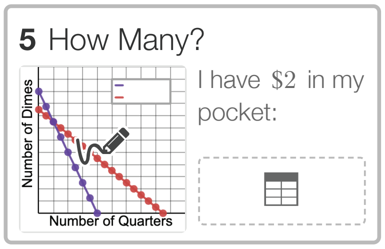
Teacher Moves

Invite students to revise their answers based on the feedback they see on this screen. Encourage them to justify any changes they make rather than use the feedback on this screen as a tool for guessing and checking without reflection.

Sample Responses

Responses vary.

- $(0, 17)$
- $(2, 15)$
- $(4, 13)$
- $(6, 11)$
- $(8, 9)$
- $(10, 7)$
- $(12, 5)$
- $(14, 3)$
- $(16, 1)$



5 How Many?

I have $\$2$ in my pocket:

I have $\$2$ in my pocket:

I only have quarters and dimes, and I have a total of 17 coins.

How many quarters and dimes must I have?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to make connections between the graph and the number of quarters and dimes that meet both conditions.

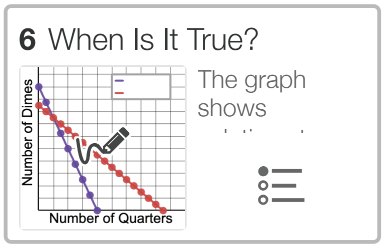
Consider using snapshots or the teacher view of the dashboard to display unique answers to the class.

Sample Responses

Number of Quarters: 2
Number of Dimes: 15

Responses vary.

These numbers are the point of intersection of the two lines in the graph.



6 When Is It True?

The graph shows

The graph shows solutions to two linear equations about the coins.

Select which conditions the statement meets:

- There are 12 quarters and 5 dimes.

Teacher Moves

This screen is designed to facilitate a discussion about how to use the graph to identify and interpret points that satisfy a relationship.

Arrange students into pairs. Consider introducing the task by reminding students of the two conditions we have been considering:

- I have $\$2$ in quarters and dimes.
- I have 17 coins altogether.

Then ask students: *What if I told you someone had* 12 *quarters and* 5 *dimes? What conditions does that combination of coins meet?* Direct their attention to the graph to help them answer the question.

Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

Consider using the dashboard to show the distribution of responses, calling attention to any conflict or consensus you see.

Pacing
Consider using pacing to restrict students to this screen.

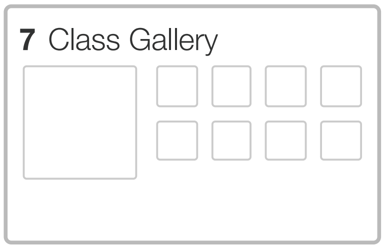
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

I have 17 coins altogether.

Responses vary.

The point $(12, 5)$ is only on the line representing 17 coins combined.



7 Class Gallery

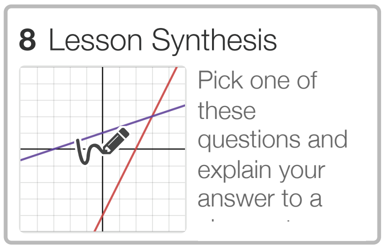
Teacher Moves

Here students will create their *own* challenge and solve challenges from their classmates. We recommend students complete Screen 6 before creating their challenge. We anticipate this Challenge Creator could take 10 minutes or more.

Encourage students to complete each other's challenges but also to take some time to review responses to their own. Use the teacher dashboard to look for unique challenges and unique solutions that may expand your students' understanding of the mathematics.

Highlight those responses for students, and ask them what they learned from the experience.

We intend for this to be a social and creative experience. We encourage you to emphasize those virtues whenever you see them in your class.



8 Lesson Synthesis

Pick one of these questions and explain your answer to a

Pick one of these questions and explain your answer to a classmate.

Then switch roles and listen to your classmate answer the other question.

1. What is a combination of values that makes both relationships true? How do you know?

2. What is a combination of values that makes one relationship true but not the other? How do you know?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to review how to use a graph to determine combinations that make one or more conditions true.

Synthesis Launch
Arrange students into pairs. Display the graph, and ask students to pretend that their partner has been absent from class for a few days. Their task is to answer one of the questions, verbally or in writing, and explain their answer to their partner. Then, students should switch roles and listen to their classmate answer the other question.

Follow with a whole-class discussion. To facilitate the discussion, consider asking students: *When using graphs, where are the points whose coordinates do not make a given relationship true?* [Points that are not on the line (i.e., either above or below it) do not make the given relationship true.]

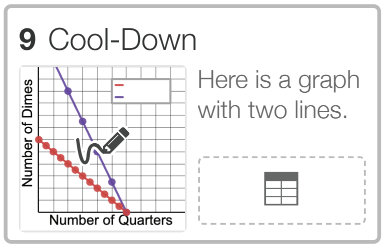
Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

1. Both relationships are true when $x = 6$ and $y = 4$. I know because the point $(6, 4)$ is on both lines.

2. The values $x = 0$ and $y = -8$ make the equation $y = 2x - 8$ true and do not make the equation $y = \frac{1}{3}x + 2$ true. I know because the point $(0, -8)$ is only on the line representing $y = 2x - 8$.



9 Cool-Down

Here is a graph with two lines.

Here is a graph with two lines.

- One line shows combinations of dimes and quarters that are worth $\$3$ altogether.
- The other line shows combinations of dimes and quarters that total to 12 coins.

How many quarters and dimes would you need to have both 12 coins and $\$3$ at the same time?

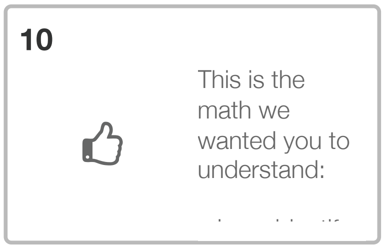
Teacher Moves

Support for Future Learning
Students will have more opportunities to identify solutions on lines, so if students struggle with this cool-down, there is no need to slow down or add additional work to the next lessons.

Pacing
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

Number of Quarters: 12
Number of Dimes: 0



10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can identify and interpret points that satisfy two relationships at the same time using graphs.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

On Both Lines

Lesson 10: Representing Systems of Linear Equations

Overview

For the next several lessons, students will study systems of linear equations where there is an initial value and a rate of change. The equations in the system are in the form $y = mx + b$. Such contexts are useful in thinking about the meaning of the solution to the system. In this lesson, students are introduced to the graphical interpretation of such systems. Keeping the graphs in mind will be useful as students navigate algebraic techniques for solving systems in the lessons to come.

Learning Goals

- Create a graph that represents two linear relationships in context, and interpret the point of intersection.
- Interpret a graph of two equivalent lines in context.

Materials

- Tools for creating a visual display
- Straightedge
- Graph paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

For the next several lessons, students will study systems of linear equations where there is an initial value and a rate of change. The equations in the system are in the form $y = mx + b$. Such contexts are useful in thinking about the meaning of the solution to the system. The purpose of this lesson is to introduce students to the graphical interpretation of such systems. Keeping the graphs in mind will be useful as students navigate algebraic techniques for solving systems in the lessons to come.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to get students to think about 1) the meaning of two linear graphs on one set of axes in a familiar context and 2) about the relationship between the speed, height, and time of each flag. This work will prepare students for the next activity, where students will write equations and graph different linear relationships.

Activity 1: On Both Lines (30 minutes)

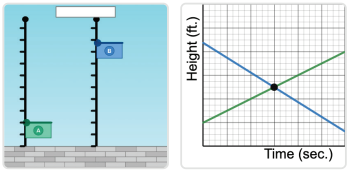
The purpose of this activity is for students to graph and compare two different linear relationships. Working in groups, students compare the relationships and determine when each phone plan is a better deal. Groups make a visual display for their selected phone plans to explain their thinking and convince others of their accuracy (MP3, MP6).

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to critically consider the work of their classmates and to use their approaches to more effectively interpret pairs of linear relationships.

Cool-Down (5 minutes)

1 Warm-Up



Teacher Moves

Purpose
The purpose of this lesson is to introduce students to the graphical interpretation of systems of equations.

Warm-Up Launch
Tell students they will see an animation of two flags for six seconds. Ask students to think of all of the details that they notice about the animation. Give one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

If the meaning of the point $(3, 20)$ or the equations of the lines do not come up, consider asking students about both to prepare them for the upcoming activity. As time allows, ask students the following questions:

- *What is the meaning of the point* $(3, 20)$*?* [At 3 seconds, both flags are 20 feet above the ground.]
- *How fast are the flags moving? How do you know?* [Flag A is moving 8 feet every 2 seconds, and Flag B is moving 10 feet every 2 seconds. I can tell by looking at the slopes in the graph.]
- *Where do you see each flag's starting height in the graph?* [The flags' starting heights are the y-values when x is zero.]
- *What are the equations of the lines representing Flag A and Flag B?* [Flag A's equation is $y = 8 + 4x$. Flag B's equation is $y = 35 - 5x$.]

Pacing
Consider using pacing to restrict students to this screen.

Routine: Use the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

Responses vary.

Things students might notice:

- Flag A is moving up and Flag B is moving down.
- Flag A and Flag B are each moving at a constant speed.
- Flag A is moving 8 feet every 2 seconds, and Flag B is moving 10 feet every 2 seconds.
- At 3 seconds, the flags are at the same height.
- Flag B is moving faster than Flag A.
- Flag A's equation is $y = 8 + 4x$.

- Flag B's equation is $y = 35 - 5x$.

Student Supports

Support for Students With Disabilities

- *Executive Functioning: Graphic Organizers*

Provide paper for students to record what they notice prior to being expected to share those ideas with others.

2 Phone Lines

You are shopping for a

You are shopping for a new cell phone and a plan with unlimited data.

Select TWO different companies, and make a poster comparing the phone plans.

Be sure to include these items in your poster:

Teacher Moves

⚠ Before they can see this screen, students will need to meet with their group and press a button that says, “We're ready!”

Activity Launch

Remind students that in previous lessons, they’ve graphed lines from equations, descriptions, and tables. In this activity, they will consider two different phone plans and make comparisons between them.

Arrange students in groups of 2–3, and invite groups to choose two of the four phone plans. Tell groups they will make a visual display comparing two phone plans. The display should clearly demonstrate their reasoning, and it should use detailed explanations in order to be convincing. Let them know that there will be a gallery walk when they finish for the rest of the class to inspect their work.

For the purposes of this task, tell students to assume that all monthly payments continue indefinitely. If it comes up, invite students to consider how their results would be different if the monthly payments for the new phone end after two years.

For students who finish their posters early, encourage them to click the “Are you ready for more?” button. Students will be prompted to select a third phone provider to add to the poster and explain when this third plan would be a better and worse deal than the two previously selected plans.

Early Student Thinking

Some students may struggle to remember how to set up a graph from a blank set of axes by hand. Remind students to carefully consider which variable to put on each axis to attend to precision in labeling axes by choosing an appropriate scale.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. Each poster should include two of the graphs chosen from the four phone plans, an equation for each of the selected phone plans, and their conclusions about when each plan is a better deal.

Student Supports

Support for Multilingual Learners
- *Speaking, Listening: Collect and Display*

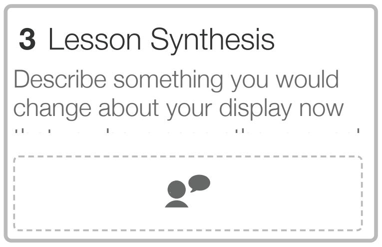
Circulate through the room and record the language students use to talk about each situation. Listen for words or phrases such as “rate of change,” “ordered pair,” “increasing/decreasing,” and “initial value.” Display grouped strategies for students to reference throughout the lesson. For example, group strategies that reference the intersection point and strategies that involve working from the given information to construct a graph. This will help students solidify their understanding about constructing a graph using a description.

Students With Disabilities
- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., presenting one task at a time) to aid students who benefit from support with organizational skills in problem solving.

- *Conceptual Processing: Processing Time*

Check in with individual students, as needed, to assess for comprehension during each step of the activity.



3 Lesson Synthesis

Describe something you would change about your display now

Describe something you would change about your display now that you have seen other groups' work.

Teacher Moves

Lesson Synthesis Launch
Begin with a gallery walk for students to see how other groups created their posters and interpreted their graphs.

Invite groups to share the strategies they used with the various representations. Consider asking groups the following questions:

- What representations did you use to determine when each phone plan was the better deal? Why did you pick them?
- What representation did you not use? Why?
- How did you decide what scale to use when you made your graph?

If it has not already come up, point out the phone plans for Company #3 and #4. These two pricing structures are the same. Connect this to the concept of “infinitely many solutions” encountered in earlier lessons. Graphically, students see that there are situations where two lines align “on top of each other.” We can interpret each point on the line as representing a time where the cost is the same. In this way, there are infinitely many points that are solutions to both equations at the same time.

Once students have completed the gallery walk, invite them to think about what they would like to change about their poster. Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their group. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

As time allows, consider asking some of the following questions. Ask students to use examples from today’s lesson when responding, if possible.

- What do you need in order to compare two linear relationships?
- What type of wording in a problem statement or description of a situation tells you about the rate of change? About the starting value?
- What is the meaning of the point where the lines intersect?

Pacing

Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary. Now that I have seen the other displays, I would change my display by carefully labeling my graph so that it's easier to interpret when each plan is a better deal.

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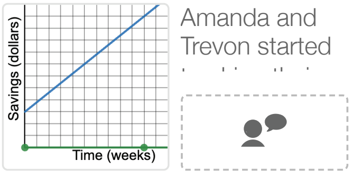
4 Cool-Down



Amanda and Trevon started tracking their savings at the same time.

- Trevon starts with $\$15$ and deposits $\$4$ per week. The graph of Trevon's savings is given, and his equation is $y = 4x + 15$, where x is the number of weeks and y is his savings.
- Amanda starts with $\$10$ and deposits $\$5$ per week.

1. Drag the points to graph Amanda's savings.

2. Explain what the intersection point of the graphs means in this situation.

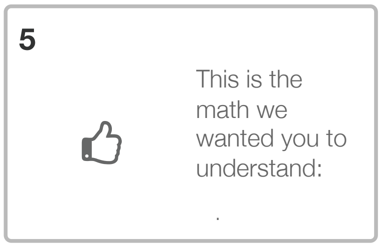
Teacher Moves

Support for Future Learning
If students struggle to graph and interpret the relationship correctly, consider reviewing this cool-down as a class before Lesson 11 or offering individual support where needed during the next lesson.

Pacing
Consider using pacing to restrict students to Screens 4–5.

Sample Responses

Responses vary. In this situation, the intersection at $(5, 35)$ means that after 5 weeks, Trevon and Amanda each have $\$35$.



5

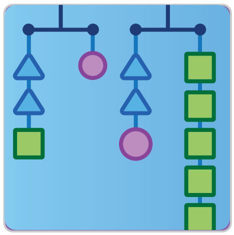
This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use graphs to find an ordered pair that two real-world situations have in common.

Digital Lesson

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Make Them Balance

Lesson 11: Graphing Systems of Linear Equations

Overview

This lesson formally introduces the concept of system of equations using the familiar context of balanced hangers. Students will recognize that in the past few lessons, they have found solutions to systems of equations by examining the intersection of graphed lines. This lesson builds on that work, adding in the concept of a solution as the set of values that makes two equations true. Students are introduced to a system where they connect lines with no intersection point to their representations with no common solution.

Learning Goals

- Understand that solving a system of equations means finding values of the variables that make both equations true at the same time.
- Connect graphs of parallel lines and a system of equations that has no solutions.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

This lesson formally introduces the concept of system of equations using the familiar context of balanced hangers. Students will recognize that in the past few lessons, they have found solutions to systems of equations by examining the intersection of graphed lines. This lesson builds on that work, adding in the concept of a solution as the set of values that makes two equations true. Students are introduced to a system where they connect lines with no intersection point to their representations with no common solution.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to use the familiar context of hanger diagrams to practice thinking about what it means for a value to be a solution to an equation in two variables. This discussion will prepare students to think about what it means to have a solution to a system of equations in two variables.

Activity 1: Make it Balance (10 minutes)

The purpose of this activity is to connect the coordinates of points on a line to pairs of weights that balance hangers. This activity is a culmination of students' work writing and graphing equations in Unit 3, along with the thinking they have done on what it means for an equation to be true in Lessons 1–8 (MP2). From this foundation, students are ready to understand solving systems of equations from an algebraic standpoint in Lessons 12–14. Fluently solving systems algebraically is not expected at this time.

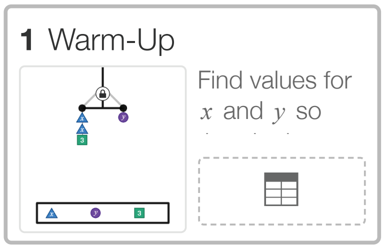
Activity 2: Hanger Solutions (20 minutes)

The purpose of this activity is to formally introduce systems of equations. Students explore systems of equations with one solution, and they consider a system with no solutions in the familiar context of hanger diagrams. The context reinforces a discussion about what it means for a system of equations to have no solutions, both in terms of the hangers and in terms of the equations. Over the next few lessons, the concept of one solution, no solutions, and infinitely many solutions will be abstracted to problems without context.

Lesson Synthesis (5 minutes)

The purpose of this discussion is to strengthen the connection between solving systems of equations and explaining the solution in the context of balanced hangers.

Cool-Down (5 minutes)



1 Warm-Up

Find values for x and y so

Find values for x and y so that the hanger balances.

Press "Try It" to see if the hanger balances.

Teacher Moves

Purpose
The purpose of this lesson is for students to learn what a system of equations is and for them to be able to explain the solution to a system of equations in the context of balanced hanger diagrams.

Warm-Up Launch
Arrange students into pairs. Tell students that their task in this warm-up is to balance hangers, similar to the work they did back in Lessons 2 and 3. Let them know that they may use blank paper to express their thinking during this activity.

Highlight unique answers to show the class. In particular, highlight a range of responses. Some students may think of the solution to the equation in terms of the weights in the diagram, while others may make sense of a solution as being the set of values that makes the equation true. Ask students to justify their responses and critique each other's reasoning.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

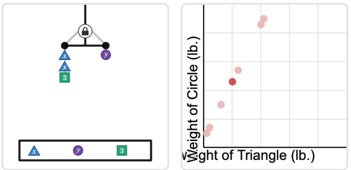
Any set of positive numbers where the y-value is double the x-value plus 3 will make the hanger balance.

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation in the activity.

2 Notice and Wonder



Teacher Moves

Activity Launch
Arrange students into pairs. Consider introducing this activity by telling students that there are many different pairs of weights for the triangle and circle that will balance the hanger and that some of them are plotted in this graph.

Give students one minute of quiet think-time. Then invite them to discuss with a partner. Invite several students to share their responses.

Pacing
Consider using pacing to restrict students to Screens 2–3.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

Responses vary.

I notice that all of the points are on a line. I wonder if any set of numbers that make the hanger balance will be on that line?

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.

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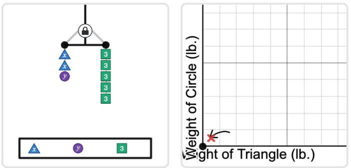
3 Make It Balance



Teacher Moves

Use the overlay in the teacher view of the dashboard to show the distribution of responses. If time permits, consider asking students the following questions in order to help them prepare for the upcoming activity challenges:

- *What do all of our red x's represent?* How do you know? [These points represent x - and y - values that don't make the hanger balance. When I plug in the coordinates from these points, they do not make a true equation.]
- *What do all of our blue points represent? How do you know?* [These points represent x - and y -values that do make the hanger balance. When I plug in the coordinates from these points, they make a true equation.]

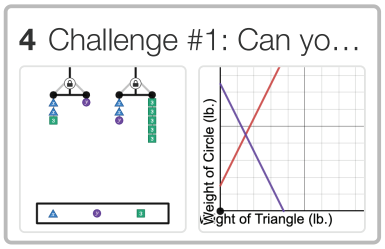
- *Is it possible to find a point NOT on the line that also makes the hanger balance?* [No. If we pick a point to the right of the line, that makes the weight of the triangle heavier for a given y-value. That means the hanger will tilt to the left. Likewise, if we pick a point to the left of the line, that makes the weight of the triangle lighter for a given y-value, which will make the hanger tilt right.]

Sample Responses

Responses vary.

- $(1, 13)$
- $(2, 11)$
- $(3, 9)$
- $(4, 7)$
- $(5, 5)$

Points that are on the line will make the hanger balance. Points to the left of the line will make the hanger tilt right, and points to the right of the line will make the hanger tilt left since there will be more weight on the left side of the hanger.



4 Challenge #1: Can yo…

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to make connections between solutions on a hanger and solutions on a graph, particularly the fact that values that make two hangers balance are also the intersection of two lines on a graph.

Activity Launch
Arrange students into pairs. Tell students that on the next few screens, their job is to solve a series of challenges in which they aim to make one of the hangers balance, both of them balance, or neither of them balance. They will try various solutions and discuss the results with their partner.

Let them know that they may solve the challenges using any strategies and tools that make sense to them. An algebraic approach is not important at this time as it will be the focus of the next several lessons.

Once students have had several minutes to work through the challenges, consider pausing the class for a discussion about the relationship between the graphs and the hangers. Several questions you might ask include:

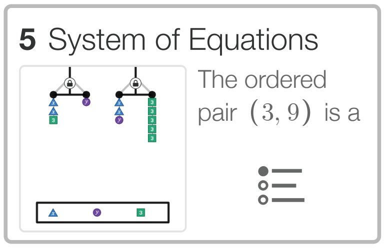
- *What will happen to the hangers when I place the point above the intersection point?* [Hanger A tilts right and Hanger B tilts left.]
- *What will happen to the hangers when I place the point to the right of the intersection point?* [Both hangers will tilt left because we've increased the x-values for any given y-value.]
- *How can we know if a point will make the hangers balance by looking at the graph?* [A point on the line makes the hanger balance, so if we want both hangers to balance, we should choose a point on both lines.]
- *How can we know if a point will make the hangers balance by looking at the equations?* [Plug the x- and y-values into each equation and check to see if both equations are true.]

Pacing
Consider using pacing to restrict students to Screens 4–8.

Sample Responses

Responses vary.

Both hangers balance when the point is on both of the lines. One hanger balances when the point is on one of the lines. Neither hanger balances when the point is off both lines.



5 System of Equations

The ordered pair $(3, 9)$ is a

The ordered pair $(3, 9)$ is a solution to the system of equations below because the values make both equations true.

$$2x + 3 = y$$
$$2x + y = 15$$

Is $(2, 7)$ also a solution to this system of equations?

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Early Student Thinking
Students may think that $(2, 7)$ is a solution to the system of equations because it makes the first equation true. Ask these students what happens if we replace the weights for each hanger with these values.

This may help students realize that the solution to the system of equations is the set of values that makes both equations true.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

No

Responses vary.

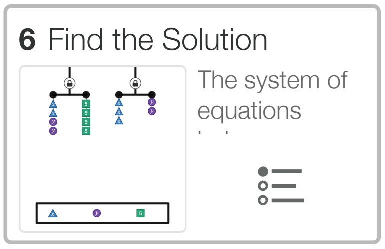
$(2, 7)$ isn't a solution to the system of equations because if I plug the values into the equation for Hanger B, I get $2(2) + (7) = 15$, which isn't a true statement. These values do not make Hanger B balance.

Student Supports

Students With Disabilities

- *Executive Functioning: Visual Aids*

Create an anchor chart for public display that includes a description of what a system of equations is and what a solution to the system is along with important definitions, rules, formulas, or concepts for future reference.



6 Find the Solution

The system of equations

The system of equations below represents the hangers:

$$2x + 2y = 20$$
$$3x = 2y$$

Which of the following points is a solution to this system?

Teacher Moves

The purpose of this screen is to transition to using coordinates as a way to represent the solution to a system of equations, and for students to recognize substitution as a strategy for checking the validity of a solution.

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking

If students aren't sure how to get started on this screen, ask them how we would check whether $x = 1, y = 9$ is a solution to the system.

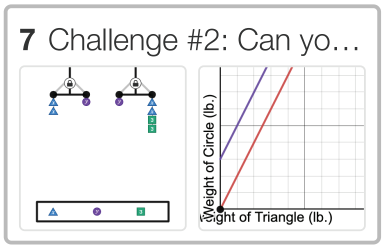
Then point out that $(1, 9)$ represents a solution of $x = 1, y = 9$ using an ordered pair, which is a way we will represent solutions to systems using graphs later in the unit.

Sample Responses

$(4, 6)$

Responses vary.

I know that $(4, 6)$ is the solution to the system of equations because that is the point where the lines intersect. I can plug $x = 4$ and $y = 6$ into each equation and the values make the equations true.


7 Challenge #2: Can yo…

Teacher Moves

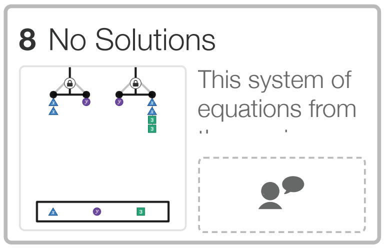
The purpose of this screen is to give students an opportunity to work informally with a system of equations that has no solutions. Over the next few lessons, the concept of one solution, no solutions, and infinitely many solutions will be abstracted to problems without context. In those situations, it may be useful to refer back to this screen as a way to guide students towards abstraction.

Students should try various coordinate pairs in order to solve the challenges and discuss the results with their partner.

Sample Responses

Responses vary.

Any point that is on one of the lines will make that hanger balance. Any point that is not on one of the lines will make neither hanger balance. It is not possible to make both hangers balance for this challenge.


8 No Solutions

This system of equations from

This system of equations from the previous screen has no solutions:

$$2x = y$$
$$y = 2x + 6$$

How can you tell that this system of equations has no solutions by looking at the hangers, the graphs, or the equations?

Teacher Moves

Tell students that this is the same set of hangers and the same graph from the previous screen. Give students one minute of quiet think-time and a few minutes to discuss with a partner. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

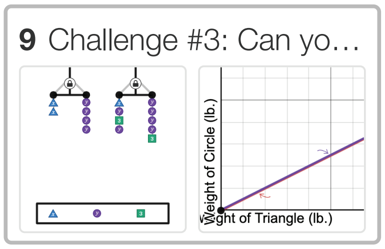
Pacing

Consider using pacing to restrict students this screen.

Sample Responses

Responses vary.

If I look at the graph, I can tell that the system has no solution because the lines are parallel and will never intersect. If I set the equations equal to each other and subtract $2x$ from each side, I get $0 = 6$, which is a false statement. This means that there is no solution to the equation $2x = 2x + 6$, and therefore, there are no solutions to the system of equations given by $2x = y$ and $y = 2x + 6$.



Teacher Moves

The purpose of this screen is to give students an opportunity to work informally with a system of equations that has infinite solutions. Over the next few lessons, the concept of one solution, no solutions, and infinitely many solutions will be abstracted to problems without context. In those situations, it may be useful to refer back to this screen as a way to guide students towards abstraction.

Students should try various coordinate pairs in order to solve the challenges and discuss the results with their partner.

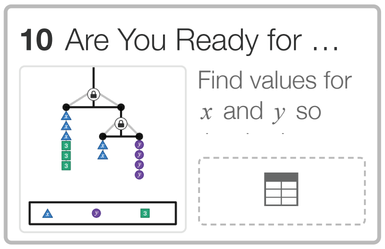
Pacing

Consider using pacing to restrict students to Screens 9–10.

Sample Responses

Responses vary.

Any response where the x-value is double the y-value is a solution to the system of equations represented by the balanced hangers, and it will make both hangers balance. Any point that is not on the line will make neither hanger balance. For this challenge, it is not possible to make one hanger balance and not the other.



10 Are You Ready for …

Find values for x and y so

Find values for x and y so that both hangers balance.

Press "Try It" to see if the hangers balance.

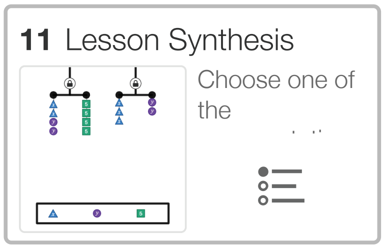
Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screen 9 ahead of time before the class discussion on Screen 11. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

$x = 9, y = 4.5$



11 Lesson Synthesis

Choose one of the

Choose one of the representations and use it to discuss the following questions.

Then select ONE question and record your response.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to revisit what a system of equations is and what it means to find solutions using a graph or a set of hangers.

Synthesis Launch
If time permits, allow students to share their responses from Challenges 1–3 before discussing these questions. Tell students that in each challenge, there was a system of equations. Ask students to share what the solution to each system of equations is. If no one brings it up, note that it’s possible for a system to have one solution, no solutions, or infinitely many solutions.

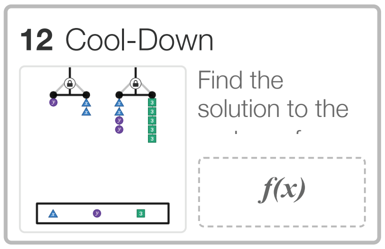
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

1. A system of equations is a set of two or more equations. Each equation contains two or more variables.
2. Finding a solution to the system of equations means that you have found the value of the variables that makes both equations true.


12 Cool-Down

Find the solution to the

Find the solution to the system of equations.

$$y = 2x$$
$$2x + 2y = 15$$

Teacher Moves

Support for Future Learning
If students struggle to solve and interpret the solution of the system, consider reviewing this cool-down as a class before Lesson 12 or offering individual support where needed during the next lesson.

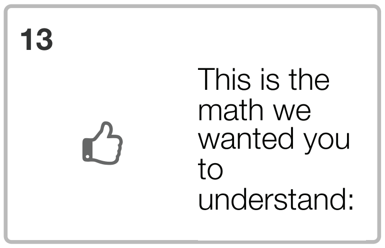
Pacing
Consider using pacing to restrict students to Screens 12–13.

Sample Responses

$(2.5, 5)$

Responses vary.

The solution $(2.5, 5)$ means that when the weight of the triangle is 2.5 and the weight of the circle is 5, both hangers will be balanced. This makes sense because plugging in those values makes each side of Hanger A 5 pounds and each side of Hanger B 15 pounds.

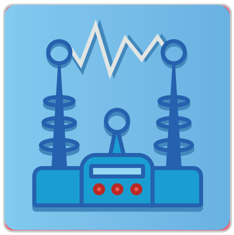

13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I understand that solving a system of equations means finding values of the variables that make both equations true at the same time.
- I know what the graph of a system of equations that has no solutions looks like.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Line Zapper

Lesson 12: Solving Systems of Linear Equations

Overview

Students continue to explore systems where the equations are both of the form $y = mx + b$.

Learning Goals

- Connect the solution of an equation with variables on each side to the solution of a system of two linear equations.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students continue to explore systems where the equations are of the form $y = mx + b$. They experience the need for solving systems of equations algebraically in order to determine the coordinates of intersection points on a graph. Initially, students are provided with a coordinate grid containing two lines and their equations. Their goal is to capture both of the lines by entering coordinates of points on the lines. Later in the lesson, students reason abstractly and quantitatively by using equations to find the coordinates of intersection points of lines without a graph (MP2). It is the work of the student to develop an algebraic method for finding the coordinates of intersection points, developing their conceptual understanding of what it means to solve a system algebraically and how it relates to the graph of the lines (MP8). Students will work towards fluency solving systems of equations algebraically in Lessons 13 and 14.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to reason about the x- and y-values of points that are solutions to linear equations by considering their graphs. While some students may guess and check by looking at the graph to find points on the line, encourage all students to show why their answer is correct based on the equations during the whole-class discussion.

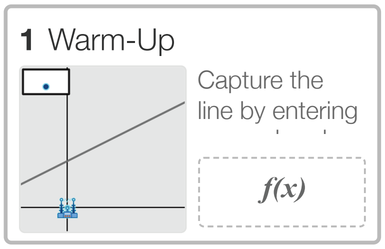
Activity 1: Line Zapper (30 minutes)

The purpose of this activity is for students to solve systems of equations using algebraic methods for the first time. Students begin by finding the x- and y-values of points on lines given their equations and a graph without labeled axes. They have a limited number of “zaps” to capture the lines, motivating the need to find the coordinates of points that are on both lines. Later in this activity, students find the coordinates of intersection points of lines in order to capture them without the added aid of a graph. The purpose of capturing the lines is for students to have a way to check that their algebraic solutions are correct, but not to shortcut the algebraic process since the graphs themselves do not include enough detail to accurately guess the coordinates of the solution.

Lesson Synthesis (5 minutes)

The purpose of this discussion is to deliberately connect the topic of systems of equations to the previous topic of solving equations with variables on both sides in Lessons 2–8.

Cool-Down (5 minutes)



Capture the line by entering one ordered pair for a point on the line.

Then press "Zap" to see if you captured the line.

Teacher Moves

Purpose
The purpose of this lesson is for students to solve systems of equations by using algebra and graphs.

Warm-Up Launch
Arrange students into pairs. Give students one minute of quiet work time and then one minute to discuss their responses with a partner. Follow with a whole-class discussion.

Ask students to explain how the zapper works and to describe how they captured the line. Use snapshots or the teacher view in the dashboard to highlight the variety of points that will capture the line. If it does not come up, ask students to explain how they can use the equation to know if a point will capture the line.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- $(0, 2)$
- $(1.5, 2.75)$
- $(2, 3)$

Student Supports

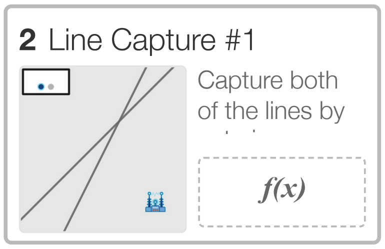
Support for Students With Disabilities

- *Memory: Processing Time*

Provide sticky notes or mini whiteboards to aid students with working memory challenges.

- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation in the activity.



2 Line Capture #1

Capture both of the lines by

Capture both of the lines by entering ordered pairs for points on these lines:

$$y = 2x + 7$$
$$y = 3x - 3$$

Use no more than two zaps.

Use paper to help you with your thinking.

Teacher Moves

Activity Launch
Keep students in the same pairs they were in for the warm-up. Consider introducing this activity by explaining to students that their goal is to capture both lines, but they only have two zaps.

It is important that students have paper ready to help them find and use algebraic methods to determine x - and y-values for points where two (or more) lines intersect.

Give 1–2 minutes of quiet work time, and then consider pausing the class to challenge students to find a way to capture both lines with one zap. Encourage students to use blank paper to help them find the coordinates of a point that is on both lines, but be careful not to be too helpful. To support student thinking, consider asking if there is a value for x that would give the same value for y in both equations. Allow students to develop their own methods of finding the coordinates of the point of intersection.

Invite students who find the coordinates of the point of intersection to explain their strategies. Celebrate and discuss the advantages and disadvantages of each approach, as time allows.

It's okay if all students do not yet solve this system using algebraic methods. The next few screens will build towards this strategy.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

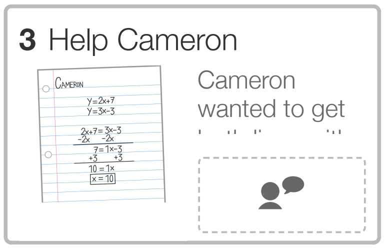
Responses vary.

The point $(10, 27)$ is on both lines.

Student Supports

Support for Students With Disabilities

- *Conceptual Processing: Processing Time*
Check in with individual students, as needed, to assess for comprehension during each step of the activity.



3 Help Cameron

Cameron wanted to get

Cameron wanted to get both lines with one zap.

They solved an equation to find the intersection point and figured out that $x=10$.

How could Cameron find the y-value of the point of intersection?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to review strategies for determining one half of a solution to a system of equations and surface strategies for determining the value of the other variable.

Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. If substituting 10 for x into either equation does not come up, consider discussing this strategy.

Consider using snapshots or the teacher view of the dashboard to display unique answers to the class.

Pacing
Consider using pacing to restrict students to this screen.

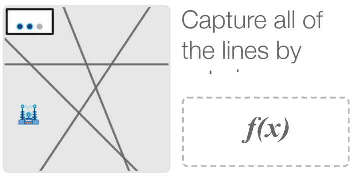
Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language.

Sample Responses

Responses vary.

Cameron could substitute the value of x into either equation and then simplify to find the value of y. Using either equation, they will determine that y must be 27.

4 Line Capture #2


Capture all of the lines by . . .

Capture all of the lines by entering ordered pairs for points on the lines. Use no more than two zaps.

Teacher Moves

Encourage students to be strategic in deciding which intersection points they choose to find, and encourage them to use blank paper to help them find solutions algebraically. Use the teacher view in the teacher dashboard to identify students who may need additional support.

As time allows, consider pausing here and asking students:

- *Do you need to see the graphs of the equations in a system in order to find the x- and y-values of a point of intersection?* [No, but the graphs make me feel more confident that my answer is correct.]
- *What are other ways you can know that your solution to a system is correct?* [I can substitute the values for x and y, and see if they make both equations true.]

Pacing

Consider using pacing to restrict students to Screens 4–8.

Sample Responses

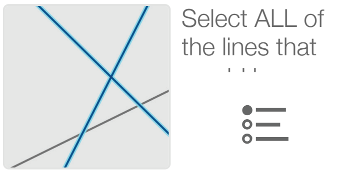
Responses vary.

Here are the coordinates of the intersection points:

- $(-11, 8)$
- $(4, 8)$
- $\left(\frac{32}{3}, 8\right)$
- $(6.5, 1.75)$
- $(2, -5)$
- $(14, -17)$

---

5 Reflection


Select ALL of the lines that . . .

Select ALL of the lines that would be captured if the point $(2, 4)$ was zapped.

Teacher Moves

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

- $y = 2x$
- $y = -x + 6$

Responses vary.

I know the point $(2, 4)$ is on these lines because if I substitute 2 for x in these equations, I get $y = 4$.

---

6 Line Capture #3


There are two lines hidden in

There are two lines hidden in the graph. Here are their equations:

$$y = -x + 10$$
$$y = 2x + 4$$

Capture both of the lines that are hidden in the graph by entering an ordered pair for a point on these lines.

You only have one zap!

Teacher Moves

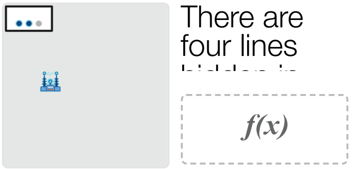
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

$(2, 8)$

---

7 Line Capture #4


There are four lines

There are four lines hidden in the graph. Here are their equations:

$$y = -2x + 9$$
$$y = -2x - 6$$
$$y = 3x - 1$$
$$y = \frac{1}{2}x - 1$$

Capture all of the lines by entering ordered pairs for points on the lines. Use no more than three zaps.

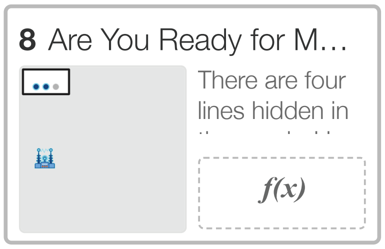
Teacher Moves

This screen is optional and can provide students with practice finding intersection points algebraically. If time is short, consider skipping this screen and pacing students to the lesson synthesis.

Sample Responses

Responses vary.

Here are the coordinates of the intersection points:
- $(2, 5)$
- $(4, 1)$
- $(0, -1)$
- $(-2, -2)$
- $(-1, -4)$



8 Are You Ready for M…

There are four lines hidden in

There are four lines hidden in the graph. Here are their equations:

$$y = -\frac{2}{3}x + 45$$

$$y = \frac{4}{3}x - 51$$

$$y = -2x - 36$$

$$-2x + y = 85$$

Capture all of the lines by entering ordered pairs for points on the lines. Use no more than three zaps.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

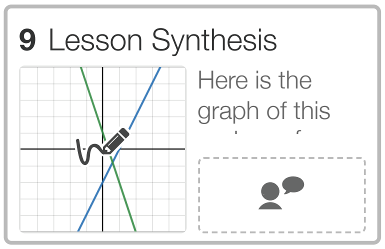
This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 4–7 ahead of time before the class discussion on Screen 9. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.

Here are the coordinates of the intersection points:
- $(-15, 55)$
- $(-30.25, 24.5)$

- $(48, 13)$
- $(4.5, -45)$



9 Lesson Synthesis

Here is the graph of this

Here is the graph of this system of equations:

$y = -3x + 2$
$y = 2x - 4$

How can you determine the exact solution to this system of equations?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is for students to consolidate strategies for determining the solution to a system of equations when a graph is not precise enough.

Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

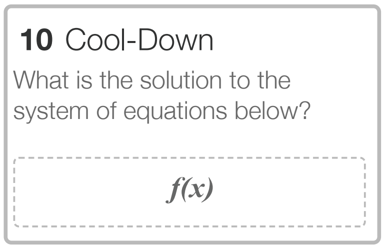
As time allows, consider asking students: *When you solved equations with variables on both sides, some had one solution, some had no solutions, and some had infinite solutions. Do you think systems of equations can have no solutions or infinite solutions?* [Yes. We have seen some graphs of parallel lines where there were no solutions and some graphs of lines that were on top of each other where there were infinite solutions.]

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

I can find the solution to this system of equations by finding the x-value when the y-values are equal. I would write $-3x + 2 = 2x - 4$ and then solve for x. Once I find x, I can substitute the value in either equation to calculate the value for y. The solution to this system of equations is $(1.2, -1.6)$.



10 Cool-Down

What is the solution to the system of equations below?

What is the solution to the system of equations below?

$y = 2x$
$y = 3x - 10$

Enter your solution as an ordered pair.

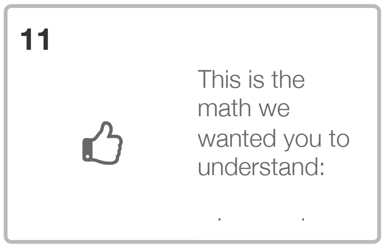
Teacher Moves

Support for Future Learning
Students will have more opportunities to solve a system of equations algebraically, so if students struggle with this cool-down, there is no need to slow down or add additional work to the next lessons.

Pacing
Consider using pacing to restrict students to Screens 10–11.

Sample Responses

$(10, 20)$



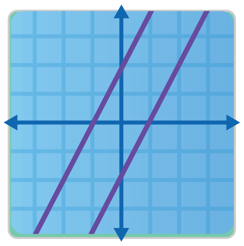
11

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can solve systems of equations using algebra.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



All, Some, or None? Part 2

Lesson 13: Systems of Equations With One, Many, or No Solutions

Overview

Students connect algebraic and graphical representations of systems. Students see how to determine the number of solutions from both the graphical and the algebraic representations.

Note: Provide students with the System of Equations cards to use on Screen 6.

Learning Goals

- Justify that a particular system of equations has no solutions using the structure of the equations.
- Categorize systems of equations, including systems with infinitely many or no solutions, and calculate the solution for a system using a variety of strategies.
- Calculate values that are solutions to a system of equations, and explain the solution method.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students connect algebraic and graphical representations of systems. Students see how to determine the number of solutions from both the graphical and the algebraic representations. In the graphical representation, the number of solutions is equal to the number of points where the graphs intersect. In the algebraic representation, two equations with the same rate of change can have 0 or infinitely many solutions, depending on whether the initial values (intercepts) are the same or not. If the rates of change are different, then there is a single solution, which can be interpreted as the point at which two quantities changing at different rates become equal.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to get students to reason about systems of equations by looking at their structure and considering their graphs.

Activity 1: Connecting Equations and Graphs (10 minutes)

The purpose of this activity is for students to connect the features of the graphs of the equations in a system to the number of solutions in a system of equations (MP7). While students have encountered equations with different numbers of solutions in earlier activities, this is the first activity where students connect systems of equations with their previous thinking about equations that have no solutions, one solution, or infinitely many solutions.

Activity 2: Classifying and Solving Systems (20 minutes)

The purpose of this activity is for students to draw conclusions about the relationship between the structure of the equations in a system and the number of solutions the system of equations has. Students will use the structure of the systems of equations to sort them by the number of solutions and then check their work by graphing the systems. While students are not asked to solve the systems of equations, they may choose to rewrite the equations in equivalent forms as they consider the graphs of the lines.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to highlight the connection between the number of solutions to a system of equations and features of its graph and equations.

Cool-Down (5 minutes)

1 Warm-Up

Select the graph that could represent this system of equations:

Select the graph that could represent this system of equations:

$$y = 2x + 4$$
$$y = -x + 10$$

Teacher Moves

Purpose
The purpose of this lesson is for students to a) encounter systems of equations that have infinitely many and no solutions and b) learn to predict the number of solutions from the structure of the equations in a system.

Warm-Up Launch
Arrange students into pairs. Explain to students that their work in this lesson is to notice the different features of equations in systems of equations and to see what these equations can tell us about the graphs and solutions in the system. Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

Top left

Responses vary.

If you look at the equations, one line has a negative slope and the other line has a positive slope. The top-left graph is the only graph that has lines with a negative and positive slope.

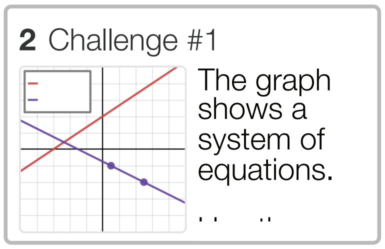
Student Supports

Support for Students With Disabilities
- *Processing Time*

For students who benefit from extra processing time, provide them the graphs to review prior to implementation of this activity. Also, check in with individual students, as needed, to assess for comprehension during each step of the activity.

Support for Multilingual Learners
- *Representing, Conversing: Collect and Display*

Give students the opportunity to discuss their responses in pairs. Circulate through the groups, and record language students use to describe each graph and equation. Listen for language related to slopes and intercepts. Post the collected language in the front of the room so that students can refer to it throughout the rest of the activity and lesson. This will help students talk about the relationship between equations and graphs.


2 Challenge #1

The graph shows a system of equations.

The graph shows a system of equations.

Use the movable points to create a system of equations that has no solutions.

Teacher Moves

Activity Launch

Arrange students into pairs. Consider introducing this activity by telling students that in previous lessons, they used algebra to explore systems with different numbers of solutions. Today, they are going to connect that learning to the graphs of systems that have different numbers of solutions. Allow groups several minutes of quiet work time.

Invite students to revise their work based on the feedback they see on this screen. Encourage them to justify any changes they make rather than use the feedback on this screen as a tool for guessing and checking without reflection.

Pacing

Consider using pacing to restrict students to Screens 2–5.

Sample Responses

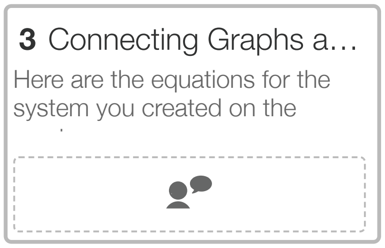
Any vertical translation of line $y = \frac{2}{3}x + 4$.

Student Supports

Support for Students With Disabilities

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.


3 Connecting Graphs a…

Here are the equations for the system you created on the

Here are the equations for the system you created on the previous screen:

$$y = \frac{2}{3}x + 4$$

$$y = \frac{2}{3}x - 5$$

How can you determine from the equations that the system will have no solutions?

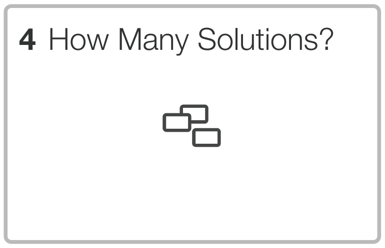
Teacher Moves

After students have had time to work on this screen, consider pausing the lesson and facilitating a whole-class discussion. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

If the equations have the same slope and different y-intercepts, then the system will have no solutions.



4 How Many Solutions?

Teacher Moves

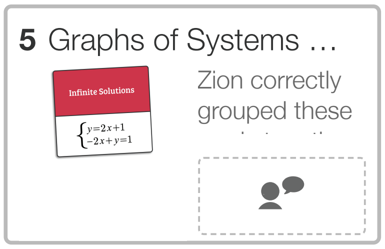
- Use the teacher dashboard to monitor student progress and to look for common sorting strategies.
- To help students with their thinking, encourage them to use scratch paper to rearrange the equations.
- If time allows, consider asking pairs to compare their card sorts, justify their card placement, and make revisions based on their conversation.

Early Student Thinking

Some students may try to match the graphs with the equations. Remind them that the cards represent five *different* systems of questions, some shown as graphs and some as equations.

Sample Responses

Image solution



5 Graphs of Systems …

Infinite Solutions

$\begin{cases} y=2x+1 \\ -2x+y=1 \end{cases}$

Zion correctly grouped these

Zion correctly grouped these cards together.

What might the graph of a system of equations with infinite solutions look like?

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to draw conclusions about the relationship between the number of solutions a system of equations has and the graphs of the equations in the system.

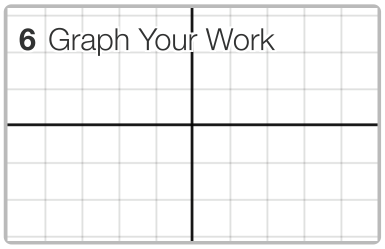
After students have had time to work on this screen, consider pausing the lesson and facilitating a whole-class discussion.

If no students mention it, bring in slope language and how inspecting the slopes of the equations before graphing or solving can give clues to the number of solutions the system has. In particular, students should notice that systems with lines that have different slopes have a single solution; lines that have the same slope and different y-intercepts have no solution; and lines that have the same slope and y-intercepts will have infinitely many solutions.

Sample Responses

Responses vary.

Both equations have the same slope and y-intercept. The graph will be one line on top of the other or a single line.



Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “We’re ready!”

Arrange students into groups of 2–3. Distribute one set of system cards for every group of students. Consider pausing the activity so that students do not engage with the graphing calculator on this screen before you are ready for them to check their work.

Tell students that they will practice what they’ve learned about systems of equations that have different numbers of solutions. First, invite students to sort the cards into three different groups based on the number of solutions: no solutions, one solution, infinite solutions. Then, depending on the instructional time available, you may wish to alter the activity and ask students to solve one or more of the systems of equations with one solution algebraically.

Once students have had time to work out their solutions, encourage them to check their work by graphing their systems in the calculator on this screen.

Pacing

Consider using pacing to restrict students to Screens 6–7.

Sample Responses

Infinite solutions: B, C, and G
One solution: A ($x = 11$ and $y = 7$), D ($x = -5$ and $y = 1$), and E ($x = 2$ and $y = 8$)
No solutions: F and H

Student Supports

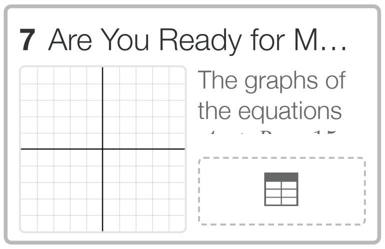
Support for Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation in the activity.

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



7 Are You Ready for M…

The graphs of the equations

The graphs of the equations $Ax + By = 15$ and $Ax - By = 9$ intersect at $(2, 1)$.

Find A and B.

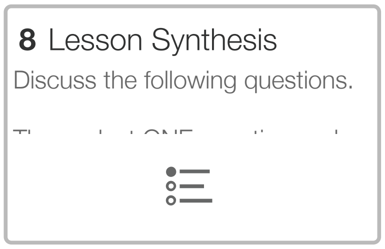
Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish sorting and solving the systems of equation cards ahead of time before the class discussion on Screen 8. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

$A = 6$ and $B = 3$


8 Lesson Synthesis

Discuss the following questions.

Discuss the following questions.

Then select ONE question and record your response.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to consolidate how to use graphs and equations to determine if a system of equations has zero, one, or infinite solutions.

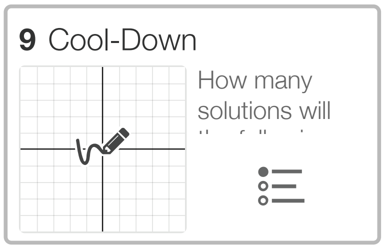
Synthesis Launch
Arrange students into pairs. Give students two minutes of quiet work time, followed by a partner discussion and a whole-class discussion.

Pacing
Consider using pacing to restrict students to this screen.

Sample Responses

- If the two equations have different slopes, there is one solution. If the two equations have the same slope and different y-intercepts, there are no solutions. If the two equations have the same slope and the same y-intercept, there are infinitely many solutions.

- If the two lines intersect at a point, there is one solution. If the two lines are parallel and do not intersect, there are no solutions. If the two lines are drawn through the same points, there are infinitely many solutions.


9 Cool-Down

How many solutions will

How many solutions will the following system have?

$$4x + y = 13$$
$$\frac{1}{2}y = -2x + 5$$

Use the sketch tool or paper if that helps you with your thinking.

Teacher Moves

Support for Future Learning
If students struggle to identify the number of solutions a system of equations has, plan to have two different students share their strategies during the following lesson.

Pacing
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

No solutions

Responses vary.

If you rewrite each equation in the form $y =$, both equations will have a slope of -4 . Since they have the same slope but different y -intercepts, this system will have no solutions.

10

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can determine whether a system of equations has no solutions, one solution, or infinitely many solutions.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Strategic Solving Part 2 (NYC)

Lesson 14: Solving More Systems of Equations

Purpose

The purpose of this lesson is for students to identify, describe, and employ strategies for solving linear systems of equations with different features or structures.

Preparation

Worksheet

- *Activities 1–3:* Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down:* Print one double-sided half sheet for each student.

Equation Cards

There are two options for using these equation cards:

- *Option 1:* Print and cut one single-sided copy of the equation cards for each student or pair of students. These cards can be reused if you have multiple classes.
- *Option 2:* Print (and do not cut out) one single-sided copy of the equation cards for each student or pair of students. Laminate or place the sheets in sheet protectors to reuse with multiple classes and/or to allow students to write on them with dry erase markers.

Warm-Up (10 minutes)

Activity Launch

Facilitate the Number Talk routine. Tell students that their goal is to solve the systems of equations without writing anything down. Display each problem on the projection sheet, one at a time. Give students 30 seconds of quiet think-time followed by a whole-class discussion. Leave each problem displayed throughout the discussion.

Teacher Moves

After each problem, ask students to share their solutions. Record and display their responses for all to see. As students share their strategies, highlight the term *substitution* as a strategy to solve an equation. For the final question, ask students why they could not use the same strategy as they did in the earlier questions.

Support for Students With Disabilities

Memory: Processing Time

Provide sticky notes or mini whiteboards to aid students with working memory challenges.

Support for Multilingual Learners

Expressive Language: Eliminate Barriers

Provide sentence frames for students to explain their strategies (e.g., I noticed that _____ or First, I _____ because _____.) When students share their answers with a partner, prompt them to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to clarify their thinking.

Activity 1: Least and Most Difficult (10 minutes)

Activity Launch

Tell students that their goal for this activity is to think about what they see as “least difficult” and “most difficult” when beginning to solve systems of equations. Their focus will be on identifying the different structures of the systems of equations, which will prepare them for the next activity where they will solve these systems of equations.

Arrange students into pairs. Distribute the student worksheet and systems cards. Give students one minute of quiet think-time to get started and then two minutes to discuss and work with their partner. Follow with a whole-class discussion. The goal of the class discussion is for students to discuss strategies for dealing with “difficult” parts of systems of equations. Students will put these strategies to use when they begin solving the systems in the next activity.

Teacher Moves

Facilitate a discussion for students to talk about strategies for solving different types of systems of equations. Some students may have thought that a certain system was in the “least difficult” category, while others thought that the same system of equations was in the “most difficult” category. Remind students that once they feel confident about the strategies for solving a system of equations, their opinion of the difficulty level may change and that recognizing good strategies takes practice and time.

If time allows, invite students to discuss one problem they thought would be the most difficult to solve. This can be repeated several times.

Support for Multilingual Learners

Collect and Display

As students discuss which systems they thought would be easiest to solve and which would be hardest, create a two-column table with the headings “least difficult” and “most difficult.” Circulate through the groups and record student language in the appropriate column. Look for phrases such as “different variables on the same side,” “variables already isolated,” and “various terms.” Invite students to share strategies they can use to address the features that make these systems of equations more difficult to solve. This will help students begin to generalize and make sense of the structures of equations for substitution.

Support for Students With Disabilities

Expressive Language: Eliminate Barriers

Provide sentence frames for students to explain their reasoning (e.g., _____ would be least or most difficult to solve because _____.)

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Activity 2: Solve 'em (15 minutes)

Activity Launch
The purpose of this activity is for students to practice using substitution to solve systems of equations. Give students 3–5 minutes quiet think-time to get started and then 3–5 minutes to discuss and work with their partner. Follow with a whole-class discussion.

Teacher Moves
This discussion has a two main takeaways. The first is to formalize the idea of substitution in a system of equations. Another is to recognize that systems where both equations are written with one variable isolated are actually special cases of substitution.

Invite students to share which systems they thought were the easiest to solve and which were the hardest. To involve more students in the conversation, consider asking:

- Did you change your mind about any of the systems being more or less difficult after you solved them?
- What was similar in these problems? What was different? [The systems vary slightly in how they are presented, but all of the problems can be solved by replacing a variable with an expression it is equal to.]
- Will your strategy work for the other systems on this list?" [Yes, substitution works in all of the given problems.]

Tell students that the key underlying concept for all of these problems is that it is often helpful to replace a variable with the expression it is equal to, and that this "replacing" is called "substitution." Point out that, in problems F and D, even setting the expressions for y equal to each other is really substituting y in one equation with the expression it is equal to as given by the other equation. It may be helpful for students to hear language like, "Since y is equal to $-\ 2x$, wherever I see y, I can substitute in $-\ 2x$."

Consider asking some of the following questions to further the discussion:

- Were there any systems of equations that were more difficult to solve than you expected?
- Were there any that were less difficult to solve than you expected?
- What other strategies or steps did you use in solving the systems of equations?

Early Student Thinking
Some students may only solve for one variable. Remind these students that a solution to a system of equations contains an x- and y-value.

Some students may have trouble transitioning from systems where both equations are given with one variable isolated to systems where an equation is given in standard form. Ask these students to look at a system where one of the variables is given as a constant. For example, ask them to look at equation A:

$$\begin{cases} y=4 \\ x=-5y+6 \end{cases}$$

Ask: *If y is equal to 4, then what is $-\ 5y$ equal to?* If a student continues to struggle, refer them back to this example, and then ask: *In this new problem, do we know what expression y (or x) is equal to? Then whenever we see y (or x), what can we replace it with instead?*

Support for Students With Disabilities
Conceptual Processing: Eliminate Barriers
Allow students to use calculators to ensure inclusive participation in the activity.

Activity 3: Thinking About Solutions (Optional)

Activity Launch
The purpose of this activity is for students to continue reasoning about the structure of a system of equations. Give students 2–3 minutes of quiet think-time to read the problem and decide if they agree or disagree with Martina. Use the remaining time for a whole-class discussion.

The goal of this discussion is to look at one way to reason about the structure of a system of equations in order to determine the solution. Then have students make their own reasoning about a different, but similar, system of equations.

Invite students from each side to explain their reasoning. As students explain, it should come out that Martina is correct. If no student brings up the idea, make sure to point out that we can also visualize this by graphing the equations in the system and noting that the lines look parallel and will never cross.

In the previous activity, students noticed that if they knew what one variable was equal to, they could substitute that value or expression into another equation in the same problem. Point out that in this problem, the expression $(x\ +\ y)$ is equal to 5 in the first equation. If, in the second equation, we replace $(x\ +\ y)$ with 5, the resulting equation is $5\ =\ 7$, which cannot be true regardless of the choice of x and y.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Lesson Synthesis (5 minutes)

Arrange students into pairs. Distribute one double-sided half sheet of the lesson synthesis and cool-down to each student.

Give students 2–3 minutes to respond to the questions, followed by a few minutes to share their responses with their partner and a whole-class discussion.

After the lesson synthesis discussion, ask students to work individually to complete the cool-down on their worksheet.

Cool-Down (5 minutes)

If students struggle to solve a system of equations, consider making time to explicitly revisit these ideas before the End-Unit Assessment.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

8.4 Practice Day 2 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided sheet for each student.

Task Cards

- *Option 1 (Task Cards)*: Print one set of cards for each group of 2–3 students.
- *Option 2 (Level Up)*: Print and cut one set of single-sided task cards for each pair of students. Make a pile for each task card in the front of the room for students to drop off and pick up.

Instructions

Option 1: Task Cards

Arrange students into groups of 2–3. Print and cut out one set of cards for every group of students.

Give each student the Student Workspace Sheet to complete as they work together to solve each of the task cards.

When groups have completed all five tasks, invite them to complete the "Are You Ready for More?" tasks.

Consider posting the answer key, or walk around with it and provide feedback to students as they work.

Option 2: Level Up

Invite students to complete this activity in groups of 2–3.

Tell students that there are five tasks in this activity. Distribute one copy of Task 1 to each pair.

Once they complete Task 1, review their thinking and highlight any incorrect solutions for students to try again. Once students have successfully completed Task 1, invite them to pick up a copy of Task 2 to complete.

Continue this process until students have completed all five tasks. If students finish early, invite them to complete the "Are You Ready for More?" tasks.

GRADE 8

Unit 5
Lesson Plans

Teacher lesson plans from Unit 5 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Functions** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

NOTE: *We have included only those lessons from Unit 5 that cover the standards in the Functions domain.*

Grade 8 Unit 5

Teacher Edition Sampler

Unit at a Glance

Key

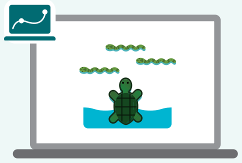
- Print Lessons
- Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

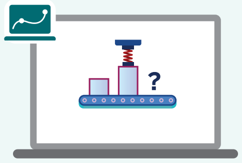
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



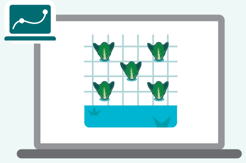
1 Turtle Crossing

Make connections between scenarios and the graphs that represent them.



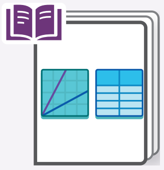
2 Guess My Rule

Write rules for producing outputs from inputs.



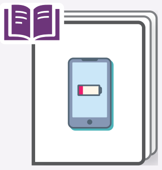
3 Function or Not?

Determine whether or not a graph represents a function, and explain the reasoning.



7 Feel the Burn

Describe the strengths and weaknesses of different representations of functions.



8 Charge!

Use data points to model a linear function.

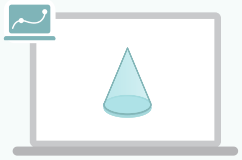
9 Piecing It Together

Approximate non-linear functions with piecewise linear functions.

Practice Day

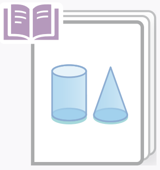
Practice Day 1

Practice the concepts and skills developed during Lessons 1–9. Consider using this time to prepare for the upcoming Quiz.



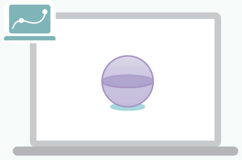
13 Cones

Describe the strengths and weaknesses of different representations of functions.



14 Missing Dimensions

Solve systems of equations using a variety of strategies.



15 Spheres

Reason about the volume of a hemisphere using the volume of the circumscribed cylinder and inscribed cone.

Practice Day

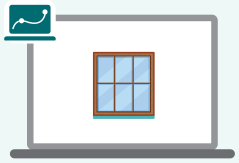
Practice Day 2

Practice the concepts and skills developed during Lessons 1–15. Consider using this time to prepare for the upcoming Quiz.

Pacing: 20 days | **Short on time?** See pacing considerations below.

Pre-Unit Check: (Optional)
15 Lessons: 45 min each
2 Practice Day: 45 min each

2 Sub-Unit Quizes: 45 min each
End-of-Unit Assessment: 45 min



4 Window Frames

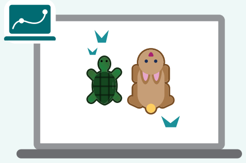
Represent a function with an equation.

Assess and Respond

Quiz: Sub-Unit 1

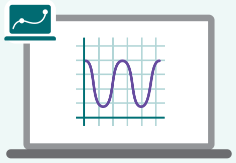
Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2



5 The Tortoise and the Hare

Interpret the graph of a function in context without an equation.



6 Graphing Stories

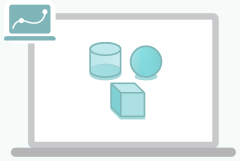
Draw the graph of a function that represents a context.

Assess and Respond

Quiz: Sub-Unit 2

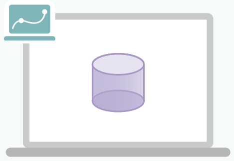
Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 3



10 Volume Lab

Estimate the volumes of various containers and explain the reasoning.



11 Cylinders

Recognize that the volume of a cylinder is the area of the base times the height.

12 Scaling Cylinders

Use representations of functions to analyze the relationship between one of a cylinder's dimensions and its volume.

Summative Assessment

End-of-Unit Assessment

Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing Considerations

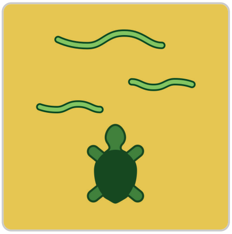
Lesson 1: This lesson supports students in making connections between scenarios and the graphs that represent them, which will be addressed in more depth in upcoming lessons, starting in Lesson 5. If this lesson is omitted, provide extra support for students as they interpret qualitative features of a function and interpret specific points in context in Lesson 5.

Lesson 8: This lesson supports students in modeling a linear function, which will be addressed in more depth in Unit 6. This lesson could be omitted if students show a strong understanding of functions in earlier lessons and time is spent in other lessons considering when it is reasonable to use a linear model for a set of data.

Lesson 12: This lesson supports students in using functions to explore how changing a cylinder's radius or height impacts its volume. If students show a strong understanding working with volumes of cylinders in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how changing one dimension of a cylinder would impact its volume as part of Lesson 11.

Lesson 12: This lesson supports students in developing fluency with calculating unknown dimensions of different solids. If students show a strong understanding of volume in earlier lessons and of solving in Unit 8.4, this lesson may be omitted. If omitted, be sure to discuss how solving problems about the volume of a cone is similar to and different from solving problems about the volume of a cylinder elsewhere in the unit.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Turtle Crossing

Lesson 1: Making Sense of Graphs

Overview

Students begin to make connections between scenarios and graphs that represent them (MP2). Later in Unit 5, they will explore more formal and precise connections between scenarios and graphs that represent them (as well as other representations like tables and equations).

Learning Goals

- Make connections between scenarios and the graphs that represent them.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to begin making connections between scenarios and graphs that represent them (MP2). Later in Unit 5, they will explore more formal and precise connections between scenarios and graphs that represent them (as well as other representations like tables and equations).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to the turtle-crossing context and to give students a chance to explore the relationship between the sketch and the animation.

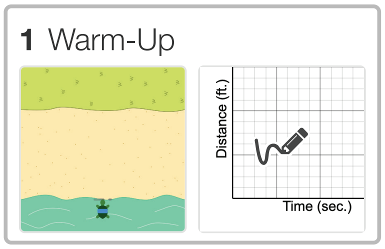
Activity 1: Turtle Crossing (30 minutes)

In this activity, students sketch and interpret graphs representing a turtle's journey across the sand. Their sketches are connected dynamically to the animation of the turtle. The dynamic link between graph and scenario provides students with an opportunity to discover connections between these two representations as they sketch, watch, and revise. Students are asked to both create their own graphs and interpret the meaning of points in others' graphs.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to plant seeds for future discussions about graphical representations of functions and non-functions by inviting students to notice multiple output values for a single input value.

Cool-Down (5 minutes)



Teacher Moves

Purpose
The purpose of this lesson is for students to make connections between scenarios and graphs that represent them.

Warm-Up Launch
Tell students that their task is to explore how the graph and the animation are connected. Give students 2–3 minutes to experiment with different sketches. Highlight unique answers to show the class. For several answers, ask students: *What story does the graph tell about the turtle's journey?*

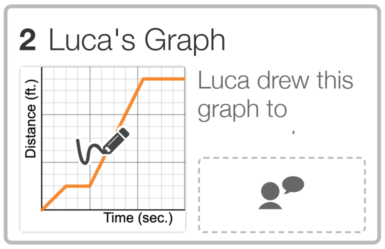
To help students get started, invite them to sketch anything and then press play to see what happens. Encourage students to use this "sketch, watch, revise" process here and throughout the lesson as they find helpful.

Note: When students place their cursor over the graph, a dashed line segment appears in the animation. This segment shows the location of the turtle (specifically, the turtle's nose) that corresponds to that point in the graph. The same feature appears on Screens 8 and 9.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.



Luca drew this graph to represent a new turtle.

What story does the graph tell about the turtle's journey?

Teacher Moves

Activity Launch: Give one minute of quiet think-time. Then ask students to discuss with a partner. Highlight unique answers to show the class. Ask students to explain how their stories connect to the graph and to critique each other's stories and reasoning.

Consider asking students: *What is the turtle doing between* 2 *and* 4 *seconds? How do you know?* [The turtle is standing still because its distance from the water is not changing.]

Facilitation: Consider using pacing to restrict students to Screens 2–4.

Sample Responses

Responses vary.

- The turtle walks, stops, and then walks again.

- The turtle begins with its nose at the edge of the water and walks towards the grass for 2 seconds at a speed of 1 foot per second. Then the turtle stops for 2 seconds. Then the turtle walks towards the grass for 4.5 seconds at a speed of 2 feet per second. After a total of 8.5 seconds, it stops on the grass, 11 feet away from the water.

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them with the graph to review prior to discussing. Students can also sketch directly on the graph.

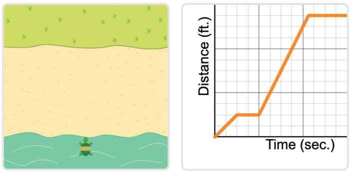
Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., The story of the turtle's journey is ________ because ________.).

---

3 Reveal



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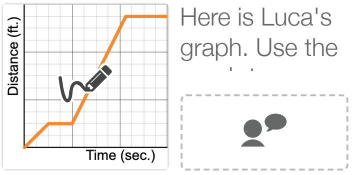
4 Interpret the Graph



Here is Luca's graph. Use the

Here is Luca's graph. Use the graph to answer the following questions:

1. At 8 seconds, how far is the turtle from the water?

2. When is the turtle 4 feet away from the water?

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to surface connections between graphs and information about a story.

Give one minute of quiet think-time. Then ask students to discuss with a partner. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Consider displaying Luca's graph while discussing these responses.

Sample Responses

1. 10 feet
2. 5 seconds

Student Supports

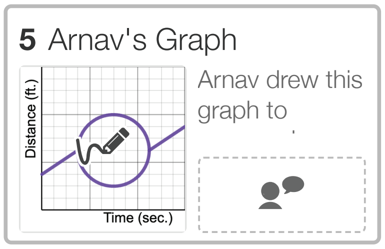
Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the graph for students to draw on or highlight. Students can also sketch directly on the graph.

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.



5 Arnav's Graph

Arnav drew this graph to

Arnav drew this graph to represent a new turtle.

What story does the graph tell about the turtle's journey?

Teacher Moves

Give one minute of quiet think-time. Then ask students to discuss with a partner. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

It's okay to lack consensus during this discussion on Screen 5. The reveal on Screen 6 will build towards consensus.

Facilitation: Consider using pacing to restrict students to Screens 5–6.

Sample Responses

Responses vary.

- The turtle walks forward, then splits into two turtles with one walking forward and one walking backward, then merges back into one turtle that continues walking forward.

- The turtle begins 3 feet away from the water, walks forward 2 feet over 3 seconds, and splits into two turtles, which move in opposite directions for 3 seconds before they move toward each other for 3 seconds. The turtles then merge back into a single turtle that walks from 5 feet to 7 feet over 3 seconds.

Student Supports

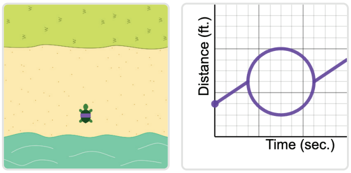
Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., The story of the turtle's journey is ________ because ________.).

---

6 Reveal

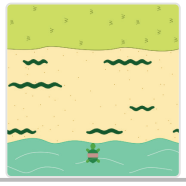


Teacher Moves

Consider asking: *Does this graph represent a possible journey for the turtle?* [No, it does not. It shows that the turtle is in two places at once for half of the journey.]

---

7 A Dangerous Crossing



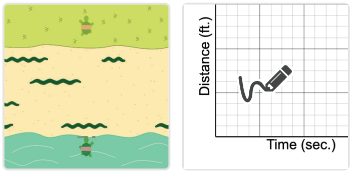
Teacher Moves

Tell students that on this screen they will watch a short animation, and on the next screen, their task is to draw a graph that matches the turtle's journey across the sand.

Facilitation: Consider using pacing to restrict students to Screens 7–9.

---

8 Draw a Graph

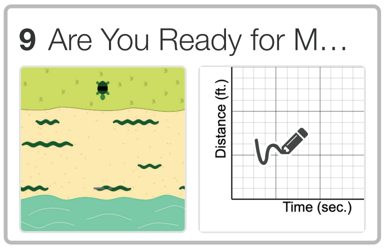


Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Image solution



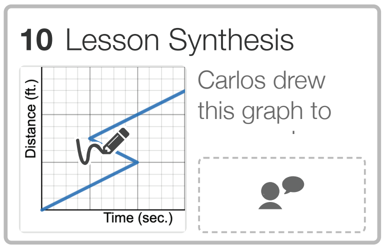
Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 7–8 ahead of time before the class discussion on Screen 10. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.



Carlos drew this graph to

Carlos drew this graph to represent a new turtle.

What does the graph say is happening at 6 seconds?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface what it means for a graph to have more than one output for a specific input (in this case, that there is more than one turtle on the screen).

Synthesis Launch: Give students 2–3 minutes to respond to this question and 1 minute to share their responses with a classmate. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Early Student Thinking: Some students may not recognize that the graph indicates the turtle is at three different positions at 6 seconds. Consider using the student view of the dashboard to sketch a point at $(6, 3)$. Ask students: *What does this point tell us about the turtle?* Then sketch points at $(6, 5)$ and $(6, 7)$ and ask: *What do these three points tell us about the turtle?* Finally, consider asking if there are any other times when the turtle appears at more than one distance from the water.

Facilitation: Consider using pacing to restrict students to this screen.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

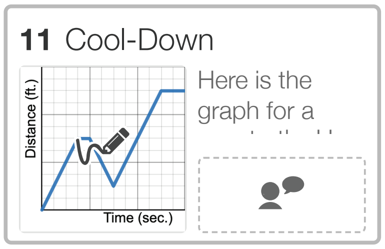
There are three turtles (or three copies of one turtle) that are 3 feet, 5 feet, and 7 feet away from the water. The first and the third turtle are walking forward. The second turtle is walking backward.

Student Supports

Students With Disabilities

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., At 6 seconds, ____________ because _____________.).



11 Cool-Down

Here is the graph for a

Here is the graph for a new turtle. Use the graph to answer the following questions:

1. At 3 seconds, how far is the turtle from the water?

2. When is the turtle 4 feet away from the water?

Teacher Moves

Support for Future Learning: Students will have more chances to connect scenarios and graphs in the upcoming lessons, particularly in Lesson 3 and 5.

Facilitation: Consider using pacing to restrict students to Screens 11–12.

Sample Responses

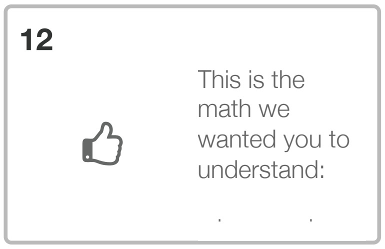
1. 6 feet
2. 2 seconds, 5 seconds, and 7 seconds

Student Supports

Multilingual Learners

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.



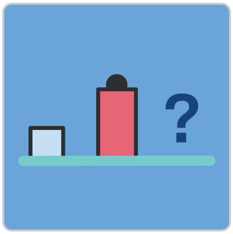
12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can make connections between scenarios and the graphs that represent them.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Guess My Rule

Lesson 2: Introduction to Functions

Overview

Students write rules based on input-output pairs represented in tables and are introduced to the concept of function through these rules.

Note: A paper version of this lesson is also available.

Learning Goals

- Write rules for producing outputs from inputs.
- Understand that a function has one and *only one* output for each allowable input.
- Identify rules that do and do not represent functions.

Vocabulary

- function

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to write rules based on input-output pairs represented in tables and to be introduced to the concept of function through these rules.

This is the first of three lessons introducing students to functions. In this lesson, they learn a function as a rule that only produces one possible output for a given input. In future lessons, students will expand on this definition as they work with representations of functions.

There are two options for facilitating this lesson: a digital lesson or a paper lesson. In the digital lesson, students guess a series of mystery rules shown as rule machines and use those machines to develop and refine the definition of a function. In the paper lesson, students work in pairs to guess each other's mystery rules and then do a structured activity to develop and refine the definition of a function.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce students to a rule machine, give examples of the types of rules used in the lesson, and reinforce the idea that you often need more than one input-output pair to determine what the rule is.

Activity 1: Guess My Rule (20 minutes)

The purpose of this activity is for students to write rules when they know input-output pairs. It also suggests the idea that a rule can have either one possible output or many different possible outputs. The word *function* is introduced in Activity 2.

In the digital version, students test inputs in a table and use the outputs generated by the machine to guess the rule. In the paper version, partners switch back and forth between the roles of rule holder and rule guesser. The rule guesser tests inputs and uses the outputs generated by the rule holder to guess the rule.

Activity 2: What Is a Function? (10 minutes)

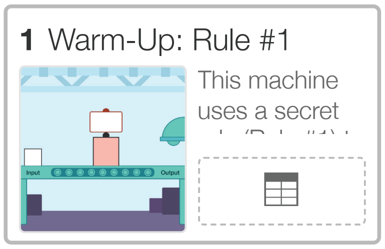
The purpose of this activity is for students to create their own definition of what makes something a function or not.

In the digital version, students revise their definitions based on reading other students' responses. In the paper version, students use a "stronger and clearer" protocol to refine their definitions (MP3).

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to provide students with a more formal definition of a function and have them apply the definition.

Cool-Down (5 minutes)



1 Warm-Up: Rule #1

This machine uses a secret

This machine uses a secret rule (Rule #1) to turn inputs into outputs.

Rule #1 allows ALL INTEGERS as inputs.

Click "Try It" to watch this machine at work.

Teacher Moves

Purpose
The purpose of this lesson is for students to write rules based on input-output pairs represented in tables and to be introduced to the concept of function through these rules.

Warm-Up Launch
The purpose of this warm-up is to introduce the idea of *input-output rules*. Tell students that their task is to figure out what Rule #1 could be based on the information they have about one input-output pair.

Teacher Moves
Consider pausing the activity and viewing the animation as a group. Consider asking students if more than one of the rules could be Rule #1 and what they might do to narrow down the choices. [Three of the four rules could be Rule #1 based on the information on this screen.]

Facilitation
Consider using pacing to restrict students to Screens 1–2.

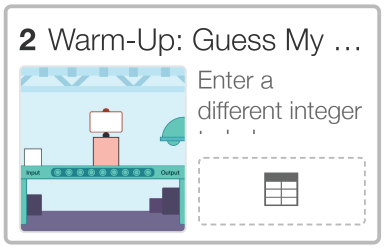
Sample Responses

- Divide by 3
- Subtract 10
- Take the ones digit

Student Supports

Multilingual Learners
- *Receptive Language: Processing Time*

Read all possible rules aloud. Students who both listen to and read the information will benefit from extra processing time. This may include reading the information in the table and in the graph.



2 Warm-Up: Guess My …

Enter a different integer

Enter a different integer to help you decide what Rule #1 is.

Teacher Moves

Give students 2–3 minutes to test another input and answer the question on this screen. Ask students to justify their choice(s) for Rule #1 and critique each other's reasoning. Consider asking students how many input-output pairs they think they need in order to be confident in the rule they chose and why.

Sample Responses

Subtract 10.

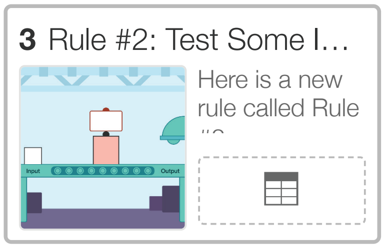
Note: If a student chooses a value greater or equal to 10 but less than 20, both "Subtract 10" and "take the ones digit" are possible.

Student Supports

Students With Disabilities

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



3 Rule #2: Test Some I…

Here is a new rule called Rule

Here is a new rule called Rule #2.
Rule #2 allows all numbers as inputs.

Test several inputs to see how Rule #2 works.

When you think you know the rule, go to the next screen.

Teacher Moves

Activity Launch

The purpose of this activity is for students to experience rules where only one output is possible for each input and rules where multiple outputs are possible for each input. The word *function* is introduced in Activity 2.

Arrange students into pairs. Tell students that their job in this activity is to guess each mystery rule by testing different inputs and looking for patterns.

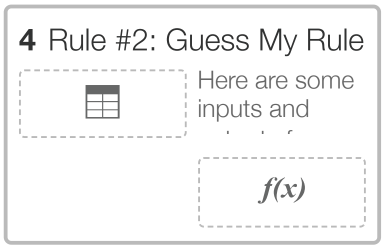
Let them know that each rule has different allowable inputs and that it may take a different amount of inputs to be confident about each rule.

Facilitation
Consider using pacing to restrict students to Screens 3–8.

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*
Check in with individual students, as needed, to assess for comprehension of how to test different inputs into the machine.



4 Rule #2: Guess My Rule

Here are some inputs and

Here are some inputs and outputs from your classmates.

If you input 6, what do you think the output will be?

Teacher Moves

Use the teacher view in the teacher dashboard to identify students who may need additional support. Consider asking students how they think the machine created the outputs.

Sample Responses

7

Responses vary.

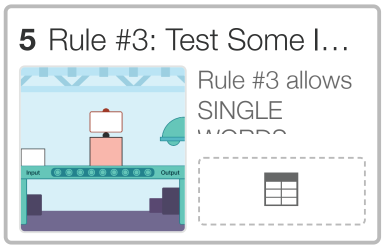
If you input 6, the output should be 7 because this rule outputs the number 7 no matter what the input is.

Student Supports

Multilingual Learners

- *Expressive Language: Visual Aids*
Create or review an anchor chart that publicly displays important mathematical definitions and phrases (input, output, etc.)

- *Expressive Language: Eliminate Barriers*
Provide sentence frames to help students explain their reasoning (e.g., I think Rule #2 is ____________ because _____________.).



5 Rule #3: Test Some I…

Rule #3 allows SINGLE

Rule #3 allows SINGLE WORDS as inputs.

Test several inputs to see how Rule #3 works.

When you think you know the rule, discuss it with someone you are not currently working with.

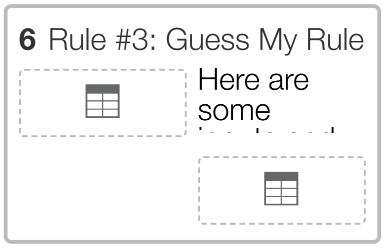
Teacher Moves

Screens 5 and 6 are similar to Screens 3 and 4 with a more challenging rule. This rule takes words (technically any string of letters and spaces) as inputs and returns a single letter—the next letter in the alphabet of the last letter of the word.

For example, the word "hello" becomes "P" because "o" is the last letter of the word, and the next letter in the alphabet is "P."

For students struggling to find a rule, consider encouraging them to try more inputs. You might suggest that they enter a letter, like "a" or "I."

Remind students that when they think they know the rule, they should find a classmate who they are not currently working with and discuss their rules together before moving on to the next screen.



6 Rule #3: Guess My Rule

Here are some

Here are some inputs and outputs from your classmates.

Put your rule to the test with some new words below.

Then press "Try It" to see what the machine will do.

Teacher Moves

Give students one minute of quiet think-time. Then invite them to discuss their output predictions with a partner. Consider holding a whole-class discussion and displaying the student view of the dashboard. Ask students to justify their output predictions for the four words and critique each other's reasoning.

In the second table, the word "friend" appears twice as an input. Consider highlighting that for this machine, when you use the same input, you expect the same output.

Sample Responses

- friend → E
- wallet → U

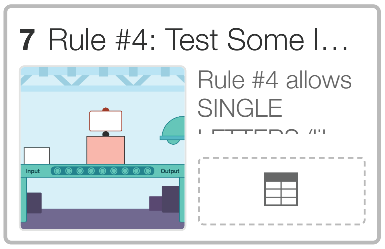
- friend → E
- party → Z

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., I think the output should be _____________ because ______________.).


7 Rule #4: Test Some I…

Rule #4 allows SINGLE

Rule #4 allows SINGLE LETTERS (like "A") as inputs.

Test several inputs to see how Rule #4 works.

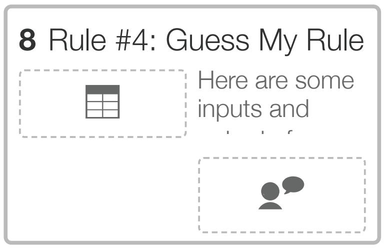
Then go to the next screen.

Teacher Moves

Screens 7 and 8 explore a rule that is not a function. The purpose of this screen is for students to notice that sometimes the same input will lead to different outputs.

Rule #4 takes single letters as inputs and returns a name beginning with that letter. For example, the letter "J" might output "Jada" or "Jacy" or "Jaleel."

Consider encouraging students to test the same input more than once and to focus less on the rule and more on any patterns they notice (e.g., that different outputs are created from the same input).


8 Rule #4: Guess My Rule

Here are some inputs and

Here are some inputs and outputs from your classmates.

If you input A into this rule, explain what you think the output will be and why.

Then compare your response with your classmates'.

Teacher Moves

Give students one minute of quiet think-time. Then invite them to discuss with a partner how their response was similar to other students' responses and why they believe that occurred. Highlight unique answers to show the class.

Consider asking students: *How is this rule different from the other rules we have seen today?*

Early Student Thinking
Students may believe that because there is more than one possible output for each input, this means that there is no rule at all. Consider asking students what the outputs with the same input have in common. This highlights that there is a rule, but that this rule is not deterministic (i.e., we cannot know for sure what name will be the output).

Sample Responses

Responses vary.

- If you input "A," the output should be some name starting with "A," but I'm not sure what.
- The output will be "Alan" or "Adrian" if the input is "A" because I noticed that the first letter of the name is always the input.

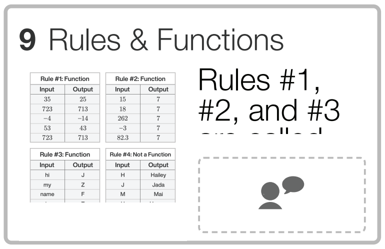
Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., If the input is ____, the output should be ____________ because ______________.).

Students With Disabilities
- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them with a hand-created table of this rule to review prior to discussing as a class.



9 Rules & Functions

Rule #1: Function	
Input	Output
35	25
723	713
−4	−14
53	43
723	713

Rule #2: Function	
Input	Output
15	7
18	7
262	7
−3	7
82.3	7

Rule #3: Function	
Input	Output
hi	J
my	Z
name	F

Rule #4: Not a Function	
Input	Output
H	Hailey
J	Jada
M	Mai

Rules #1, #2, and #3

Rules #1, #2, and #3 are called FUNCTIONS.
Rule #4 is NOT a function.

What do you think makes a rule a function?

Write and submit your response.

Then read the responses from your classmates. Edit your response to make it stronger and clearer.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to develop a common definition for a function.

Activity Launch
Tell students that in this activity, they will try to come up with a definition of a new word, *function*, based on the rules they have discovered in class today. They will then make their rule stronger and clearer by reading other students' rules. A formal definition of *function* appears in the lesson synthesis.

Give students two minutes of quiet work time to write and submit their own definition. Follow with a whole-class discussion. Ask students to justify their definitions and critique each other's responses to make them stronger and clearer.

Then give students one minute to revise their response based on other students' responses and the discussion.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Sample Responses

Responses vary.

- A rule is a function if every number on the input side goes to exactly one number on the output side.
- A function is a rule where, if you know the input, you know what the output is going to be.

Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the image for students to draw on or highlight.

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., A rule is a function if_______________.).

---

10 Lesson Synthesis

Definition: A function is a

Definition: A function is a rule that assigns EXACTLY ONE output to each possible input.

This table DOES NOT represent a function.

Change at least one number in the table so that it DOES represent a function.

Explain why you made each change.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to consider a formal definition of a function and come to consensus about how to apply that definition to a table of values.

Synthesis Launch
Consider launching the synthesis by having a student read the prompt aloud. Then change one number in the table and ask students whether or not the table is now a function.

Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion.

In the discussion, consider stating that there are several different ways to be correct, and name different students' strategies.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

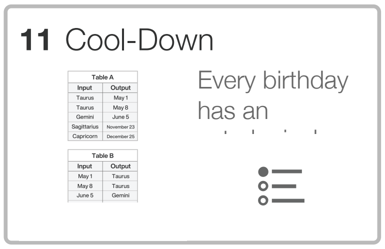
Responses vary.

- I changed the input in the fourth row from 2 to 4 and the input in the fifth row from 1 to 5 so that each number on the input side is different.
- I changed the output in the fourth row from 20 to 10 and the output in the fifth row from 24 to 5 so that each input always goes to the same output.

Student Supports

Students With Disabilities
- *Executive Functioning: Visual Aids*

Create an anchor chart for public display that describes what a function is and includes examples for future reference.



11 Cool-Down

Table A	
Input	Output
Taurus	May 1
Taurus	May 8
Gemini	June 5
Sagittarius	November 23
Capricorn	December 25

Table B	
Input	Output
May 1	Taurus
May 8	Taurus
June 5	Gemini

Every birthday has an

Every birthday has an astrological sign, like Gemini or Scorpio. Both tables show a relationship between birthday and astrological sign.

Which table(s) represent a function?

Teacher Moves

Support for Future Learning
If students struggle to determine from a table whether a relationship is a function, plan to emphasize this when opportunities arise over the next several lessons. For example, consider spending extra time discussing Lesson 3's warm-up.

Facilitation
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Table B

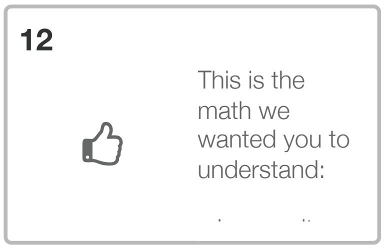
Responses vary.

- Table B is a function because for any date (input), there is only one possible astrological sign (output).
- Table A is a not function because each astrological sign (input) has many different outputs.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., __________ is a function because ______________.).



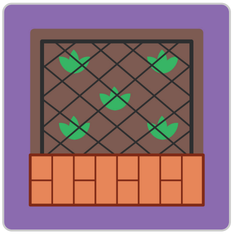
12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can write rules when I know input-output pairs.
- I know that a function is a rule with exactly one output for each allowable input.
- I can identify rules that do and do not represent functions.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Function or Not?

Lesson 3: Graphs of Functions and Non-Functions

Overview

Students determine whether or not a graph represents a function and explain their reasoning.

Learning Goals

- Determine whether or not a graph represents a function, and explain the reasoning.
- Describe a context using function language (e.g., "The *output* is a function of the *input*" or "the *input* determines the *output*").

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to determine whether or not a graph represents a function and explain their reasoning.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to re-engage with the definition of a function. Specifically, students enter two missing values into a table to create a relationship that does not represent a function.

Activity 1: Rectangular Pen (10 minutes)

In this activity, students build a rectangular pen and then explore whether the relationship between amount of fencing and area is a function. Students also consider how a graph might make it easier to quickly determine whether a given relationship represents a function.

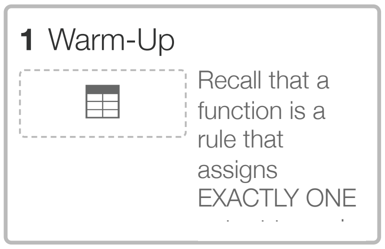
Activity 2: Turtle Crossing (20 minutes)

In this activity, students revisit the turtle-crossing context (from Lesson 1) and decide whether two new relationships (distance vs. steps and steps vs. distance) represent functions. Through this repeated reasoning, students generalize how to determine whether or not a graph represents a function (MP8). Students then complete a card sort using their understanding of how to determine whether a graph is a function.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to determine whether a given relationship in a new scenario represents a function, and explain their reasoning.

Cool-Down (5 minutes)



1 Warm-Up

Recall that a function is a rule that assigns EXACTLY ONE

Recall that a function is a rule that assigns EXACTLY ONE output to each possible input.

In other words, each input determines a SINGLE output.

Complete the table so that y is NOT a function of x.

Teacher Moves

Purpose
The purpose of this lesson is for students to determine whether or not a graph represents a function and explain their reasoning.

Warm-Up Launch
The purpose of this warm-up is for students to re-engage with the definition of a function.

Tell students that their task is to enter two missing values in the table to create a relationship that is not a function. Give students 1–2 minutes of quiet think-time. Then highlight unique answers to show the class. Ask students to explain how they know whether each table is a function or not, and to critique each other's reasoning.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- **Row 1 output:** 1, **Row 3 input:** 3
- **Row 1 output:** 1, **Row 3 input:** 5

Student Supports

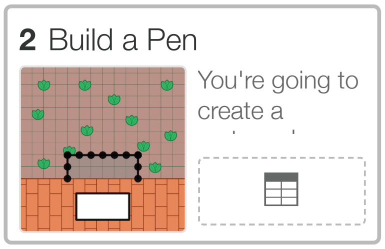
Multilingual Learners

- *Expressive Language: Visual Aids*

Create or review an anchor chart that publicly displays the definition of a function to aid in explanations and reasoning.

- *Receptive Language: Processing Time*

Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.



2 Build a Pen

You're going to create a

You're going to create a rectangular pen by building three sides against a brick wall.

Drag the movable points to observe the relationship between the area and the amount of fencing.

Then enter the amount of fencing and the area for one pen.

Teacher Moves

Activity Launch
Tell students that their task on this screen is to create a three-sided rectangular pen by dragging the movable points. Consider offering a brief demonstration of how the interaction works. Then encourage students to continue to the next screen when they are ready.

Facilitation
Consider using pacing to restrict students to Screens 2–4.

Sample Responses

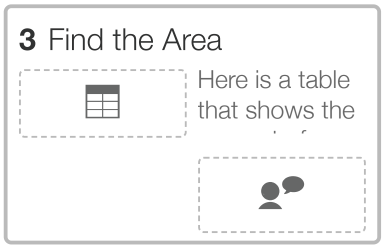
Responses vary.

- **Fencing:** 10 m, **Area:** 12 sq. m
- **Fencing:** 15 m, **Area:** 28 sq. m
- **Fencing:** 16 m, **Area:** 24 sq. m

Student Supports

Students With Disabilities
- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors, and allow students who struggle with fine motor skills to dictate the movement of the fence.



3 Find the Area

Here is a table that shows the

Here is a table that shows the amount of fencing and the area for several pens, including yours (at the top).

Emma used 12 meters of fencing to create her pen.

What is the area of Emma's pen?

Teacher Moves

Highlight unique answers to show the class.

Help students connect this lesson to the previous one by asking: *What does this say about whether area is a function of the amount of fencing?* [Area is *not* a function of the amount of fencing. Some input values correspond to more than one output value.]

Sample Responses

Responses vary.

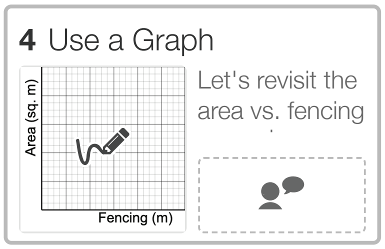
It is not possible to determine the area of Emma's pen because there are two rows in the table with 12 meters as the input (amount of fencing).

Student Supports

Students With Disabilities

- *Executive Functioning: Graphic Organizers*

Provide a Venn diagram so students can compare the similarities and differences between functions and non-functions. Consider adding student learning from this screen to a graphic organizer.



4 Use a Graph

Let's revisit the area vs. fencing

Let's revisit the area vs. fencing scenario once more—this time with a graph.

How could you use the graph to quickly determine that area is not a function of amount of fencing?

Use the sketch tool if it helps you to show your thinking.

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to surface how to decide if a relationship is a function from a graph.

Highlight unique answers to show the class.

Consider asking: *When determining whether a relationship is a function, would you rather use a table or a graph?*

Sample Responses

Responses vary.

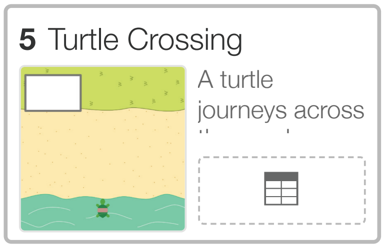
The graph shows that for one amount of fencing (12 meters) there is more than one possible area (10 square meters and 16 square meters). Therefore, area is not a function of amount of fencing.

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review before they write their thinking.



5 Turtle Crossing

A turtle journeys across

A turtle journeys across the sand.

Press play or drag the slider to observe the relationship between the turtle's distance from the water and the number of steps.

Then enter the turtle's number of steps and distance from the water for one point in time.

Teacher Moves

Activity Launch

Tell students that their task on this screen is to briefly explore the relationship between the turtle's number of steps and its distance from the water, and then to enter one pair of values in the table. Consider offering a brief demonstration of how the interaction works. Then encourage students to continue to the next screen when they are ready.

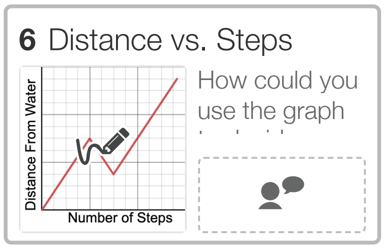
Facilitation

Consider using pacing to restrict students to Screens 5–10.

Sample Responses

Responses vary.

- 20 steps, 6 feet
- 30 steps, 3 feet
- 40 steps, 6 feet



6 Distance vs. Steps

How could you use the graph

How could you use the graph to decide whether distance from the water is a function of number of steps?

(Your point from Screen 5 is shown on the graph.)

Teacher Moves

Highlight unique answers to show the class.

Consider asking: *What is the turtle's distance after* 20 *steps? After* 30 *steps? After* 40 *steps? What does this indicate about whether distance from the water is a function of number of steps?*

Early Student Thinking
Some students may think that distance from the water is *not* a function of number of steps because some distances from the water (i.e., some outputs) correspond to multiple numbers of steps (i.e., multiple inputs). Consider reminding them that a function is a rule where each input has exactly one output. It is possible for a function to have multiple inputs with the same output.

Sample Responses

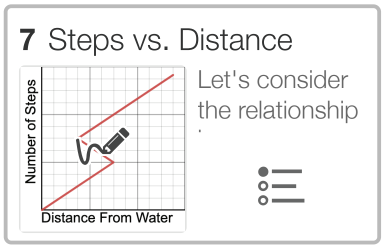
Responses vary.

Look along the horizontal axis at all of the possible input values. For each input value, look up to see how many output values there are. If every possible input value corresponds to exactly one output value, then the relationship is a function.

Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the graphs on this screen and the next for students to draw on or highlight.



7 Steps vs. Distance

Let's consider the relationship

Let's consider the relationship in reverse.

Is number of steps a function of distance?

Use the graph to explain how you know.

Teacher Moves

This is an important place to check student progress. Use the teacher view of the dashboard to show the distribution of responses, calling attention to any conflict or consensus that you see.

Consider asking: *When the turtle's distance from the water is* 4 *feet, how many steps has it taken? What does this indicate about whether number of steps is a function of distance from the water?*

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

No

Responses vary.

There are some distances that correspond with multiple numbers of steps. For example, the turtle is 6 feet from the water at 20 steps and also at 40 steps.

---

8 Card Sort

Teacher Moves

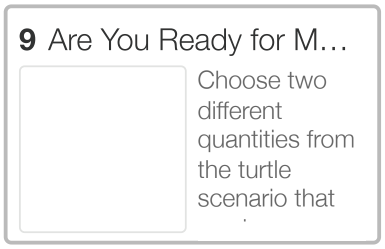
Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Sample Responses

Image solution



9 Are You Ready for M…

Choose two different quantities from the turtle scenario that

Choose two different quantities from the turtle scenario that produce a graph that is surprising in some way.

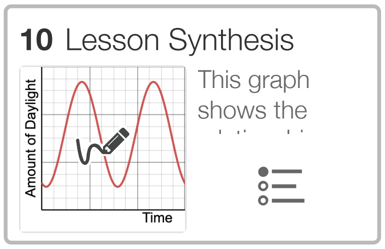
On paper, state whether the graph represents a function, and describe what the surprising part means in context.

Quantity for the x-axis:

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 5–8 ahead of time before the class discussion on Screen 10. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.



10 Lesson Synthesis

This graph shows the

This graph shows the relationship between the time of year and the amount of daylight for Fairbanks, Alaska.

Is the amount of daylight a function of time?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to consolidate and refine students ideas about how to determine if a relationship is a function from a graph.

Synthesis Launch
Give students 2–3 minutes to respond to this question and one minute to share their responses with a classmate. Highlight unique answers to show the class. Ask students to explain how they decided whether the graph represents a function.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Yes

Responses vary.

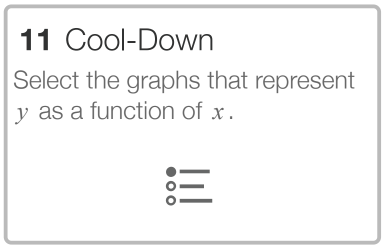
For each possible input (time in months), there is exactly one output (amount of daylight in hours).

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the graphs on this screen for students to draw on or highlight.



11 Cool-Down

Select the graphs that represent y as a function of x.

Select the graphs that represent y as a function of x.

Teacher Moves

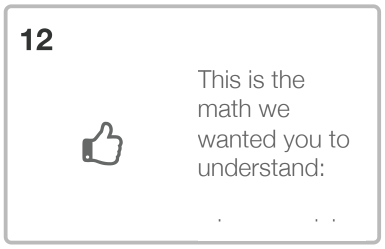
Support for Future Learning
If students struggle to determine whether a given relationship represents a function, consider making time to explicitly revisit these ideas. A strong understanding of identifying a function will support students in the upcoming quiz.

Readiness Check (Problem 6)
If students struggled, consider inviting them to revisit this question and revise their response after finishing Lesson 3.

Facilitation
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Image solution



12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain why a graph does or does not represent a function.
- I can use precise language to describe functions (e.g., “is a function of” or “determines”).

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Window Frames

Lesson 4: Functions and Equations

Overview

Students explore the relationship between equations and functions, and understand that an equation may look different depending on how the independent and dependent variables are defined. This lesson also exposes students to new types of equations as they generate equations from a complex visual diagram. Students are not expected to master these types of equations within this lesson or unit.

Learning Goals

- Represent a function with an equation.
- Identify independent and dependent variables.

Vocabulary

- dependent variable
- independent variable

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to explore the relationship between equations and functions, and to understand that an equation may look different depending on how the independent and dependent variables are defined. This lesson also exposes students to new types of equations as they generate equations from a complex visual diagram. Students are not expected to master these types of equations within this lesson or unit.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to be introduced to the scenario they will explore in Activity 1. The warm-up may also surface strategies that will be useful when analyzing equations later in the activity.

Activity 1: Window Frames (20 minutes)

Students explore when equations are useful for representing functions. Students are motivated to use an equation by first calculating window frame heights for various widths with a fixed amount of wood trim. Then students analyze four equivalent equations and select which equation is most useful to answer different questions about the scenario.

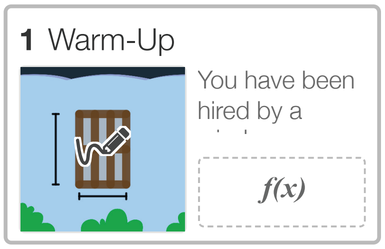
Activity 2: Independent and Dependent Variables (10 minutes)

Students revisit the phrases *independent variable* and *dependent variable* (which they may have seen in math 6) as a way to describe more precisely when different equations might be useful. Students connect these phrases to the window frame scenario.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to solidify the connection between the features of equations of functions and independent and dependent variables. Students do so with a card sort related to the cup stacking scenario from Unit 3.

Cool-Down (5 minutes)



1 Warm-Up

You have been hired by a

You have been hired by a window framing company.

Teacher Moves

Purpose

The purpose of this lesson is for students to explore the relationship between equations and functions, and to understand that an equation may look different depending on how the independent and dependent variables are defined.

Warm-Up Launch

The purpose of the warm-up is for students to be introduced to the scenario they will explore in Activity 1. Arrange students in groups of two. Give students two minutes of quiet work time, followed by a whole-class discussion.

Consider asking the question: *What strategy did you use to calculate the total amount of wood trim?* It is crucial that students leave with an understanding of how the window frame is built (using 3 horizontal segments and 4 vertical segments).

Early Student Thinking

Students may believe that there are 7 feet of wood trim because that is the number of pieces of wood trim that make up the window frame. Consider asking students: *How long is each piece of wood trim?* or *Are all of the pieces of wood trim the same length?*

Readiness Check (Problems 1 and 4)

If most students struggled, plan to review these problems as part of this warm-up. Consider using tape diagrams to support students with connecting equations and situations.

Facilitation

Consider using pacing to restrict students to this screen.

Sample Responses

$3 \cdot 4 + 4 \cdot 6 = 36$ feet of wood trim

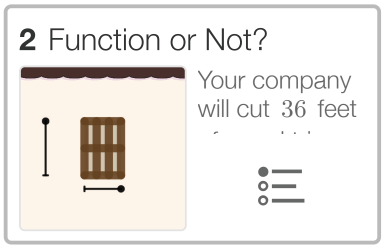
Responses vary.

The total wood trim is equal to 3 times the width plus 4 times the height, so $3 \cdot 4 + 4 \cdot 6 = 36$ feet of wood.

Student Supports

Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*
Allow students to use calculators throughout this lesson to ensure inclusive participation.



2 Function or Not?

Your company will cut 36 feet

Your company will cut 36 feet of wood trim into any size pieces for a flat fee.

Drag the sliders to explore windows that use exactly 36 feet of wood trim.

Teacher Moves

Activity Launch
Tell students that their task in this activity is to explore the relationship between the frame height and frame width of windows that use 36 feet of wood trim.

Teacher Moves
Arrange students into pairs. Encourage students to share their reasoning with a partner and work to reach an agreement during the task.

This is a great place to check student progress about deciding whether or not relationships are functions, which they learned in previous lessons. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Facilitation
Consider using pacing to restrict students to this screen.

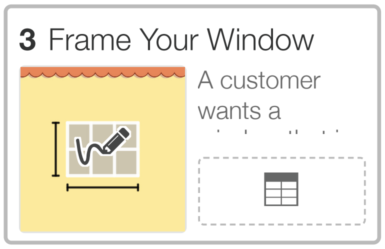
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Both

Student Supports

Multilingual Learners
- *Expressive Language: Visual Aids*
Review an anchor chart that publicly displays the definition of a function to aid in explanations and reasoning.



3 Frame Your Window

A customer wants a

A customer wants a window that is 6 feet wide.

How tall should the window frame be?

Enter expressions like $2 + 2$, or use the sketch tool if it helps you with your thinking.

Teacher Moves

Teacher Moves
Arrange students into pairs. Encourage students to share their reasoning with a partner and work to reach an agreement during the task.

As students are working, walk around and ask them to explain their strategy for finding the height. If they're completely stuck or using a guess-and-check strategy, consider asking them: *How much wood trim is needed for all of the horizontal pieces of the frame? How much wood is left for all of the vertical pieces?*

Facilitation
Consider using pacing to restrict students to Screens 3–5.

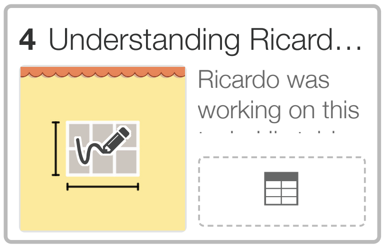
Sample Responses

- 4.5 feet
- $\frac{36 - 3 \cdot 6}{4} = 4.5$ feet

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors. This activity may be challenging for students.



4 Understanding Ricard…

Ricardo was working on this

Ricardo was working on this task. His table is below.

Teacher Moves

Give students 2–3 minutes to respond to this question. Encourage students to share their reasoning with a partner and work to reach an agreement.

Use the teacher view of the dashboard to highlight unique and precise explanations of Ricardo’s expression. Understanding this expression may support students to be successful on Screens 5 and 6.

Sample Responses

Yes

Responses vary.

The 3 represents the number of horizontal bars of wood trim there are. The 6 represents how wide the window is. You multiply the two numbers and subtract from 36 to find out how much wood trim is left for the vertical bars. The 4 represents how many vertical bars are needed for the window.

Student Supports

Students With Disabilities

- *Executive Functioning: Graphic Organizers*

Provide students a T-chart to record what they notice and wonder before they are expected to share their ideas with others.

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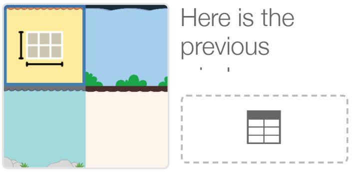
5 More Windows



Here is the previous

Here is the previous window. Ricardo's expression has been copied into the top row.

Click on each row of the table to see three new windows.

Calculate how tall each new window should be.

Enter expressions like $2+2$ if it helps with your thinking.

Teacher Moves

Allow students two minutes of quiet work time, followed by five minutes of partner work time.

Use the teacher view in the teacher dashboard to identify students who may need additional support. Consider inviting students who struggle to analyze Ricardo’s expression in the top row. Students can copy and paste Ricardo's expression into the other rows.

Use snapshots to highlight a student sample that uses the same expression (with a different width) in each row. If no students use this strategy, consider leading a whole-class discussion by copy-andpasting Ricardo's expression into each row of the table. Ask students: *What would we need to change to use Ricardo's expression here?*

Sample Responses

- $\frac{36-3\cdot 8}{4}=3$ feet
- $\frac{36-3\cdot 5}{4}=5.25$ feet
- $\frac{36-3\cdot 7.5}{4}=3.375$ feet

Student Supports

Students With Disabilities

- *Memory: Processing Time*

Provide sticky notes, scratch paper, or mini whiteboards to aid students with working memory challenges.

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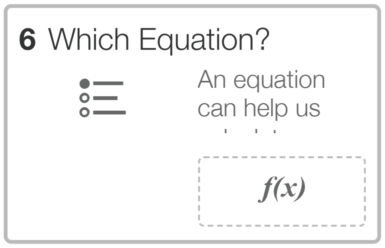
6 Which Equation?



An equation can help us calculate values more efficiently.

Here are four equations that represent this situation.

Which equation would best help answer this question:

What is the height, h, for a frame whose width is 7.5 feet?

Use the equation you selected to answer the question.

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to expose students to new types of equations and to highlight the connection between equations and independent/dependent variables. Students are not expected to master these types of equations within this lesson or unit.

Consider beginning the conversation on this screen with the questions, *What do you notice? What do you wonder?* so that students can attend to the different features of the equations.

Then give students one minute of quiet think-time to select the equation that will best answer the question. Then invite them to discuss with a partner.

This is an important place to develop whole-class understanding. Use the teacher view of the dashboard to show the distribution of responses, calling attention to any conflict or consensus that you see. Ask students to justify their choice of equation and critique each other's reasoning.

If the form of each equation does not come up, consider discussing the advantages of writing an equation solved for one variable (e.g., $h =$).

Facilitation
Consider using pacing to restrict students to Screens 6–7.

Sample Responses

- $h = \dfrac{36 - 3w}{4}$

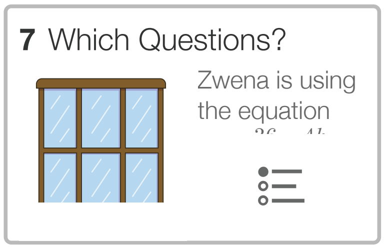
When $w = 7.5$, $h = \dfrac{36 - 3(7.5)}{4} = 3.375$ feet

Student Supports

Students With Disabilities

- *Executive Functioning: Graphic Organizers*

Provide students a T-chart to record what they notice and wonder before they are expected to share their ideas with others.


7 Which Questions?

Zwena is using the equation

Zwena is using the equation $w = \dfrac{36 - 4h}{3}$.

Which question would her equation answer most easily?

Teacher Moves

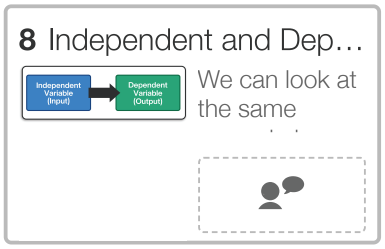
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

What is the width of the frame if the height is 5.5 feet?

Responses vary.

- The equation is solved for w, so it is easiest to find the width of the frame.
- If you input the height into Zwena's equation, you do not need to rearrange the equation to find the width.


8 Independent and Dep…

We can look at the same

We can look at the same scenario in different ways based on what are called independent and dependent variables.

Consider Zwena's equation: $w = \dfrac{36 - 4h}{3}$.

The independent variable is the window height, h.
The dependent variable is the width, w.

Why do you think the dependent variable is named that?

Teacher Moves

Activity Launch
Tell students that the purpose of this activity is for them to revisit the phrases they may have seen before: *independent variable* and *dependent variable*. These phrases help describe more precisely when different equations might be useful.

Teacher Moves
Consider analyzing the diagram on this screen together as a class to introduce the terms independent variable and dependent variable. Give

students two minutes of quiet work time, followed by a partner discussion. Encourage students to attend to precision in their response.

Facilitation
Consider using pacing to restrict students to Screens 8–9.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Sample Responses

Responses vary.

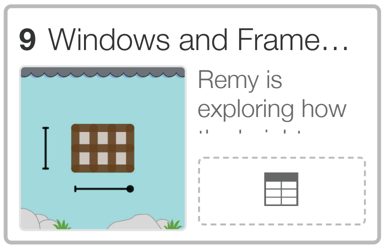
The dependent variable is the result or output. Its value *depends* on the input (i.e., the independent variable), so we call it the *dependent* variable.

Student Supports

Students With Disabilities

- *Executive Functioning: Visual Aids*

Create an anchor chart for public display that describes independent and dependent variables, and includes important definitions, rules, formulas, or concepts for future reference.



9 Windows and Frame…

Remy is exploring how

Remy is exploring how the height depends on the width of the window frame.

Remy uses the equation $h = \dfrac{36 - 3w}{4}$.

In this situation, which quantity is the independent variable and which is the dependent variable?

Teacher Moves

Consider using snapshots to highlight and display unique responses from the previous screen while students work.

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

- **Independent variable:** Window frame width
- **Dependent variable:** Window frame height

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.

---

10 Lesson Synthesis

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to solidify the connection between equations, questions, and choice of independent variable.

Synthesis Launch

Tell students that in this task, they will be applying what they learned about independent and dependent variables to a familiar context: stacking cups.

Teacher Moves

Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort. Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Conduct a whole-class discussion about the placement of specific cards. Consider asking students their strategies for sorting each of the question and equation cards. Ask students to justify their responses and critique each other's reasoning.

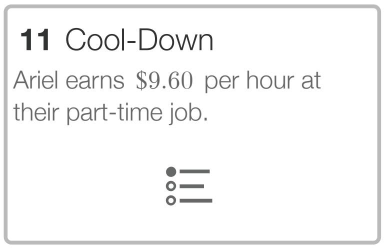
Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Readiness Check (Problem 3)

If most students struggled, plan to spend extra time on this screen discussing the relationship between the two equations. Students worked with this content in 7.2.06: Two and Two. It may be helpful to revisit this lesson and the practice problems.

Sample Responses

Image solution



11 Cool-Down

Ariel earns $\$9.60$ per hour at their part-time job.

Ariel earns $\$9.60$ per hour at their part-time job.

Ariel wrote the equation $y = 9.60x$.

Which variable is independent based on the equation given?

Teacher Moves

Support for Future Learning
If students struggle to decide which is the independent variable, plan to emphasize this when opportunities arise over the next several lessons. For example, consider asking students which is the independent variable in the tortoise-hare scenario in Lesson 5.

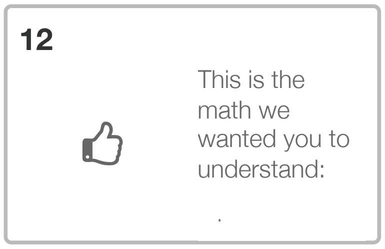
Facilitation
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Number of hours worked

Responses vary.

The independent variable is the input or x-value. If you multiply the number of hours worked by $\$9.60$, you will find the amount of money earned.



12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can represent a function with an equation.
- I can name the independent and dependent variables for a function.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

The Tortoise and the Hare

Lesson 5: Interpreting Graphs of Functions

Overview

Students further develop the work they began in Lesson 1 (making connections between graphs and the functional relationships they represent) in the context of an animal race. In addition to asking students to interpret qualitative features of a function and interpret specific points in context (MP2), this lesson asks students to consider the rate of change (increasing, decreasing, and constant) over given intervals.

Learning Goals

- Interpret the graph of a function in context without an equation.
- Interpret points on the graph of a function.
- Understand that if the rate of change is positive over an interval, the function is increasing over that interval. If it is negative over an interval, the function is decreasing over that interval.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to further develop the work they began in Lesson 1 (making connections between graphs and the functional relationships they represent) in the context of an animal race. In addition to asking students to interpret qualitative features of a function and interpret specific points in context (MP2), Lesson 5 asks students to consider the rate of change (increasing, decreasing, and constant) over given intervals.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is orient students to the scenario. It Invites them to make observations about an informal representation (an animation) that will help prepare them to make observations about a more formal representation (a graph) throughout the rest of the lesson.

Activity 1: The Tortoise, the Hare, and the Fox (15 minutes)

In the first activity, students make connections between the animal-racing scenario and the graphs of the distance vs. time relationship for each animal. In addition to interpreting specific points of the graph in context, students will need to reason about rates of change (e.g., by using the graph to determine which animal is traveling the fastest at a given point in time).

Activity 2: Tortoise vs. Dog (15 minutes)

In the second activity, students create their own graph to represent the distance vs. time relationship for a new animal, a dog. The graph must meet three conditions, including a condition in which the dog's distance is decreasing.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to strengthen their understanding of graphs and the scenarios these graphs represent. Students will do this by making observations about a new related scenario (a snail race).

Cool-Down (5 minutes)

1 Warm-Up: The Tortoi…

Press play and watch the

Press play and watch the animation.

Then tell a story based on what you see.

Teacher Moves

Purpose
The purpose of this lesson is for students to make connections between graphs and the functional relationships they represent.

Warm-Up Launch
Ask students to watch the animation and then tell a story based on what they see. Invite students to use the slider at the bottom of the animation to adjust the time and investigate specific moments of the race more closely.

Facilitation
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

Responses vary.

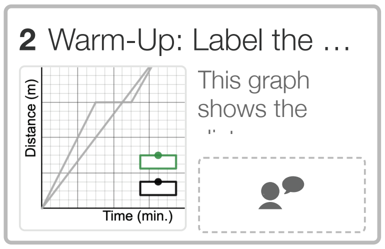
Once upon a time, there was a race between a tortoise and a hare. The hare was faster than the tortoise but lost the race because it took a nap after 6 minutes. By the time the hare woke up from its nap (4 minutes later), the tortoise was too far ahead, and the hare could not catch up.

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors and allow students who struggle with fine motor skills to dictate their story as needed.



2 Warm-Up: Label the …

This graph shows the

This graph shows the distance vs. time relationships for the tortoise and the hare.

Which animal does each graph represent?

Drag the labels to show your answer.

Then explain your thinking.

Teacher Moves

Give students one minute of quiet think-time. Then ask students to discuss their labeling decisions with a partner.

Consider asking students to identify or interpret one or two specific points on the graph. For example: *When does the tortoise catch up with the hare? How do you know?* [After 9 minutes, at $1\,200$ meters.]

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Sample Responses

Responses vary.

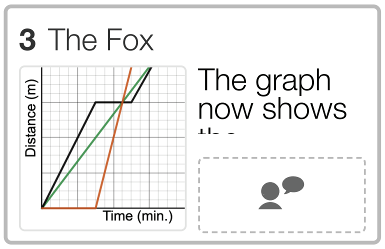
- The linear graph represents the tortoise because the tortoise's distance changed at a constant rate.
- The graph with a horizontal section from 6 to 10 minutes represents the hare because the hare took a four-minute nap during the race.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., The hare's graph is ________ because ________.).



The graph now shows the distance vs. time relationship for a third animal, the fox.

Tell a story about the fox's journey during the race.

Include specific details about time and distance.

Teacher Moves

Activity Launch

Arrange students into pairs. Give students two minutes of quiet work time, followed by a partner discussion.

Highlight several stories to show the class. Consider drawing attention to stories that include references to time, distance, and/or speed.

Facilitation

Consider using pacing to restrict students to Screens 3–5, one screen at a time.

Sample Responses

Responses vary.

The fox begins at the starting line and stays there (possibly napping) for the first 6 minutes of the race. The fox then races ahead, faster than the tortoise *and* the hare, and finishes the race first after a total of 10 minutes (including only 4 minutes of running).

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

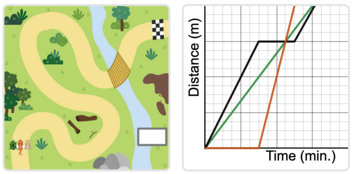
Pair students with their previously identified peer tutors and allow students who struggle with fine motor skills to dictate their story as needed.

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the graph for students to draw on or highlight. Students can also sketch directly on the graph.

---

4 Reveal

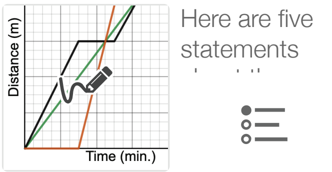


Teacher Moves

Invite students to watch the reveal to see how their stories from Screen 3 compare to the actual race.

---

5 Five Statements



Here are five statements

Here are five statements about the race.

Select all of the true statements.

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to come to consensus about how to interpret information about a story shown on a graph.

Give students 2–3 minutes of quiet think-time to make their selections. Then encourage them to discuss their answers with a partner.

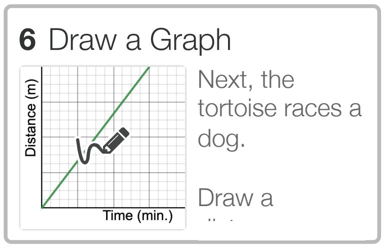
This is a great place to check student progress. Rather than offering individual support where needed, consider leading a whole-class discussion asking students to defend or reject each statement. Ask students to justify their responses and critique each other's reasoning.

If you are running short on time, consider focusing the class discussion on one choice: "At 9 minutes, the fox is traveling faster than the other animals." The cool-down at the end of the lesson asks students to use a graph to determine which object has the greatest rate of change.

Early Student Thinking
Some students may think that the fox's distance is always increasing because the fox is never moving backwards. Consider asking: *During the first* 6 *minutes of the race, does the fox's distance increase, decrease, or neither?* [Neither.]

Sample Responses

- Between 4 and 8 minutes, the fox is in last place.
- At 9 minutes, the fox is traveling faster than the other animals.
- When the hare reaches 800 meters, the fox is still at the starting line.



6 Draw a Graph

Next, the tortoise races a dog.

Draw a

Next, the tortoise races a dog.

Draw a distance vs. time graph for the dog that makes ALL of these statements true.

1. The dog gets a head start but loses the race.
2. The dog and tortoise are tied at 800 meters.
3. The dog's distance is decreasing for 3 minutes.

When you're ready, continue to the next screen.

Teacher Moves

Activity Launch
Tell students that their task in this activity is to create a graph to represent distance vs. time for a new animal, a dog. Later, they will critique a graph drawn by a fictional student.

Give students 2–3 minutes of quiet think-time. Then ask students to share and discuss their graph with a partner. Highlight unique sketches to show the class. Ask students to justify their responses and critique each other's reasoning. Consider displaying two graphs and asking: *How are these similar? How are they different? Do they both meet all three criteria? How do you know?*

Facilitation
Consider using pacing to restrict students to Screens 6–8.

Sample Responses

Responses vary.

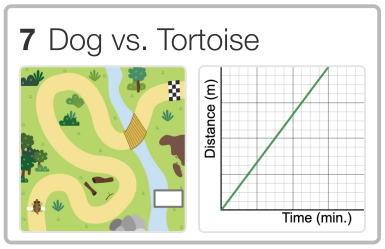
Image solution

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*
Pair students with their previously identified peer tutors and allow students who struggle with fine motor skills to dictate physical manipulation of graphing as needed.

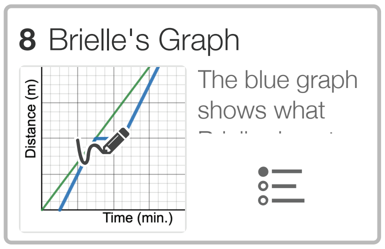
- *Conceptual Processing: Processing Time*
Begin with a demonstration of the drawing tool to provide access to students who benefit from clear and explicit instructions. Check in with individual students, as needed, to assess for comprehension.



7 Dog vs. Tortoise

Teacher Moves

Consider displaying Screen 6 in the dashboard's student view while students view the tortoise vs. dog race on Screen 7. This will help students keep an eye on the three conditions from Screen 6 while they watch the tortoise and dog in action on Screen 7.



8 Brielle's Graph

The blue graph shows what

The blue graph shows what Brielle drew to represent the dog's distance vs. time.

At least one of the following statements is false.

Select a false statement.

Teacher Moves

Give 1–2 minutes of quiet think-time. Then ask students to discuss their choice (and rationale) with a partner. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

If no students select the third choice, consider asking: *What does it mean for the dog's distance to be decreasing? How would that be represented in the graph?* [The dog would move back toward the starting line. The graph will decrease—i.e., go down—from left to right.]

Sample Responses

Two statements are false.

Responses vary.

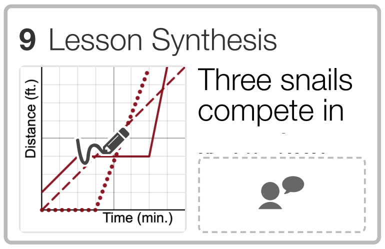
- **The first statement is false.** The dog does not get a head start. Rather, it starts the race 2 minutes late.
- **The third statement is false.** The dog’s distance *stops increasing* for 3 minutes, but it is never decreasing. During those 3 minutes, the dog is standing (or sitting) still.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., This statement is false because ________.).



9 Lesson Synthesis

Three snails compete in

Three snails compete in a 16-foot race.

The graph shows their distance vs. time relationships.

Describe everything you can about what is happening at 6 minutes.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to surface information you can know about a graph at a particular value for x, including information about speeds and distances in this case.

Synthesis Launch
Give students 2–3 minutes to respond to this question and 1 minute to share their responses with a classmate. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

At 6 minutes . . .

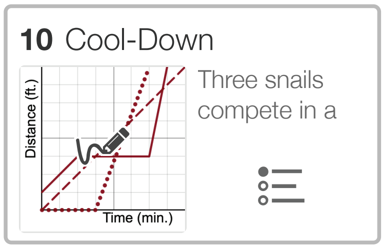
- Snail 1 is in last place. It is ending its rest and is about to race ahead (at a faster pace than any other snail).
- Snail 2 is in second place, continuing its steady pace of 2 feet per minute.
- Snail 3 just reached the finish line (16 feet away from the starting line).

Student Supports

Students With Disabilities

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., At 6 minutes: Snail 1 is ______________; Snail 2 is ______________; Snail 3 is ______________.).



10 Cool-Down

Three snails compete in a

Three snails compete in a race.

The graph shows their distance vs. time relationships.

Which snail is traveling the fastest at 4 minutes?

Teacher Moves

Support for Future Learning

If students struggle with interpreting graphs in context, plan to emphasize this when opportunities arise over the next several lessons. For example, consider pausing on Screen 2 of Lesson 6 to make connections between the graphs and the video on Screen 1.

Facilitation

Consider using pacing to restrict students to Screens 10–11.

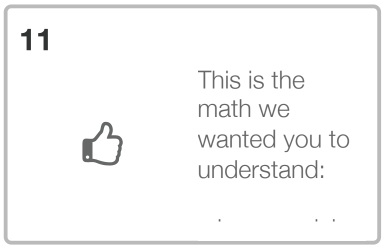
Sample Responses

Snail 3

Responses vary.

- At 4 minutes, Snail 3 has the steepest line.

- At 4 minutes, Snail 1 is standing still, Snail 2 is moving at 2 feet per minute, and Snail 3 is moving at $\frac{16}{3}$ (or about 5.3) feet per minute.



11

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain the story told by the graph of a function.
- I can find and interpret points on the graph of a function.
- I can determine whether a function is increasing or decreasing based on whether its rate of change is positive or negative.

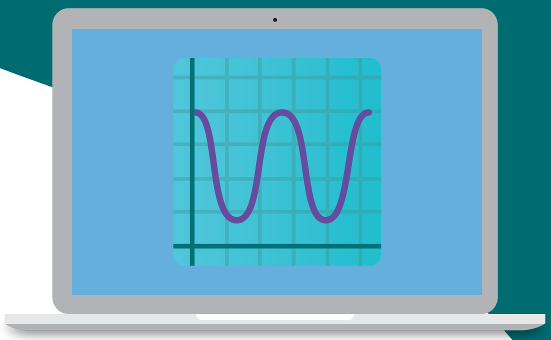
Unit 5

Lesson

6

Digital Lesson RECOMMENDED

This is a digital lesson. A print option is also available.



Graphing Stories

Creating Graphs of Functions

Let's make connections between scenarios and the graphs that represent them.

Focus and Coherence

Today's Goals

1. **Goal:** Draw the graph of a function that represents a real-world situation.
2. **Language Goal:** Describe where the graph of a function is increasing, decreasing, linear, or non-linear. **(Reading, Writing, Speaking, and Listening)**

Students draw the graphs of functions based on short videos they watch of real-world situations, learning the important features to consider when modeling a situation with a graph. They consider the qualitative features of a function, such as whether it is increasing, decreasing, linear, or non-linear, and interpret specific points in context. **(MP4)**

Prior Learning

In Lessons 1 and 5, students interpreted graphs in context.

Future Learning

In Lesson 7, students will investigate and make connections between linear functions as represented by graphs, tables, equations of the form $y = mx + b$, and verbal descriptions.

Rigor and Balance

- Students build **conceptual understanding** of the qualitative aspects of the graphs of functions as they use the terms *increasing*, *decreasing*, *linear*, and *non-linear* to describe the features of these graphs.

Standards

Addressing

NY-8.F.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described in a real-world context.

Mathematical Practices: MP3, MP4, MP6

Building On	Building Toward
NY-8.F.1	**NY-8.F.2** **NY-8.F.3**

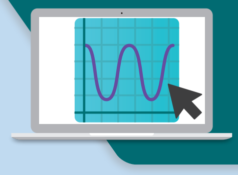
Amplify Desmos Math **NEW YORK**

Lesson Sample

Lesson at a Glance

~ 45 min

Standard: NY-8.F.5

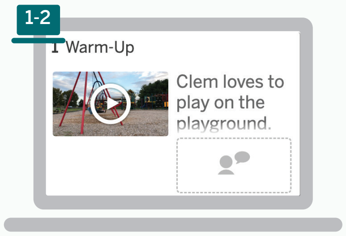


Why digital?
Students compare sample graph sketches overlaid with their own sketches in real time.

Warm-Up

Pairs | 5 min

Students analyze a video showing a real-word situation to determine the changing quantities and compare graphs that could represent it.


1-2
1 Warm-Up
Clem loves to play on the playground.

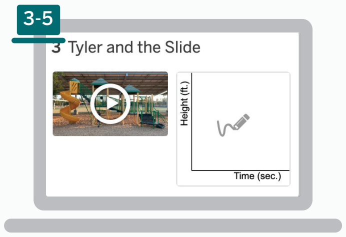
Pacing: Screens 1–2

Activity 1

Pairs | 15 min

Students draw the graph of a function that represents a context to focus on its qualitative aspects and how they affect the shape of the graph.

Routine:
- MLR1: Stronger and Clearer Each Time **(MP4)**

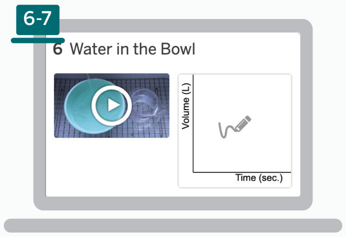

3-5
3 Tyler and the Slide

Pacing: Screens 3–5

Activity 2

Pairs | 10 min

Students draw the graph of a function that represents a context to apply strategies learned in the previous activity. **(MP6)**

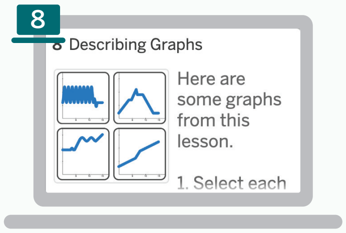

6-7
6 Water in the Bowl

Pacing: Screens 6–7

Activity 3

Pairs | 5 min

Students analyze parts of graphs to develop their own meaning of where functions are increasing, decreasing, linear, and non-linear.

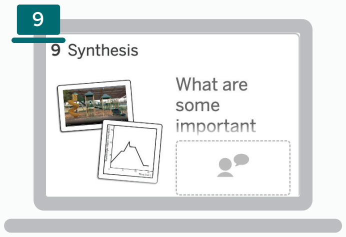

8
8 Describing Graphs
Here are some graphs from this lesson.
1. Select each

Pacing: Screen 8

Synthesis

Whole Class | 5 min

Students synthesize their understanding of using different variables to describe the same real-world situation, which may produce different graphs.

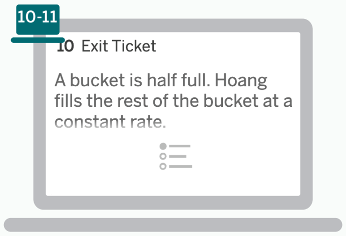

9
9 Synthesis
What are some important

Pacing: Screen 9

Exit Ticket

Independent | 5 min

Students demonstrate their understanding by determining which graph could represent the given real-world situation.


10-11
10 Exit Ticket
A bucket is half full. Hoang fills the rest of the bucket at a constant rate.

Pacing: Screens 10–11

Prep Checklist

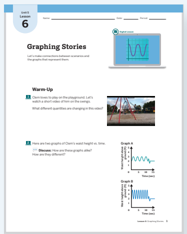
Assign the digital lesson.
A print option is also available.

Students using digital:
- Digital Lesson

Students using print:
- Print Option in Student Edition
- Exit Ticket PDF
- Coloring tools (as needed)

Digital Lesson

Pairs | 5 min | Pacing: Screens 1–2



Students using print

Warm-Up

Purpose: Students analyze a video showing a real-word situation to determine the changing quantities and compare graphs that could represent it.

Short on time: Consider omitting the Warm-Up.

1 Launch

Play the video showing Clem on the swing. Let students know that the "different quantities that are changing" are the same as the variables of the situation.

2 Connect

Display the graphs or video to aid the class discussion.

A **Accessibility: Visual-Spatial Processing** For students using digital, consider providing access to the Student Edition, which contains printed versions of the two graphs for students to draw on or highlight.

Invite students to share their responses.

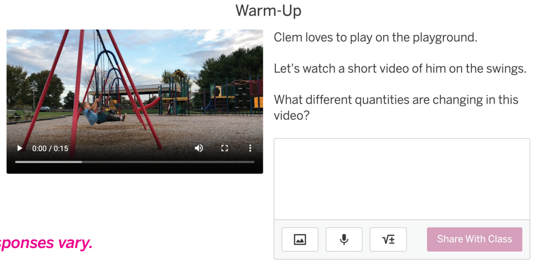
Consider asking, "Which graph do you think more accurately represents what happened in the video?"

Math Identity and Community Invite students to notice and celebrate the variety of mathematical thinking during an activity.

Emphasize that a single situation can contain many different quantities or variables. Each of these variables may produce its own unique graph. Students will explore more of this concept in Activity 1.

Students using digital

1



Warm-Up

Clem loves to play on the playground.

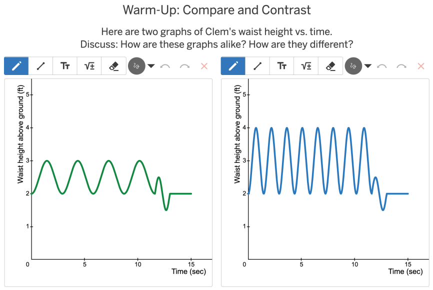
Let's watch a short video of him on the swings.

What different quantities are changing in this video?

Responses vary.

- Time
- Clem's waist height above ground
- Clem's shoe height above ground
- The number of times Clem swings back and forth
- The horizontal distance between Clem and the edge of the video screen

2



Warm-Up: Compare and Contrast

Here are two graphs of Clem's waist height vs. time.
Discuss: How are these graphs alike? How are they different?

Responses vary.

Similarities

- The initial height is the same in each graph (2 feet).
- The final height is the same in each graph (2 feet).
- Each graph has the same basic shape.
- Each graph represents Clem jumping off at the same time.

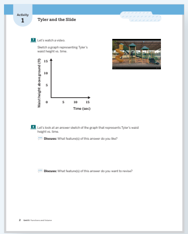
Differences

- The graph on the right indicates a greater number of swings.
- The graph on the left indicates a smaller distance between the highest and lowest waist heights.
- The graph on the left indicates a smaller maximum height above the ground.

Pairs | 15 min | Pacing: Screens 3–5 one at a time

Activity 1 Tyler and the Slide

Purpose: Students draw the graph of a function that represents a context to focus on its qualitative aspects and how they affect the shape of the graph. **(MP4)**



Students using print

3 Launch

Play the video showing Tyler on the slide.

Support getting started by pausing the video at each point and asking, "Where is Tyler's waist at $t = 0$, $t = 5$, and $t = 10$?"

MLR **MLR1: Stronger and Clearer Each Time** Invite students to meet with 1–2 other pairs of students to share their responses. Consider using these sentence frames to support students in giving and receiving feedback and using the feedback to revise their responses:

- I noticed that in the video ______, so I ______.
- I know that this part of the graph is ______, because ______.
- The video matches with this part of the graph because ______.

3 Monitor

D **Differentiation**

Look for students who:	Teacher Moves
Sketch a graph that starts at the origin.	**Support:** Tell students that placing the starting point at $(0, 0)$ means there is no distance between Tyler's waist height and the ground. Ask, "How might Tyler look if there's no distance between his waist and the ground?"
Notice Tyler's waist height must be constant for part of the graph when he reaches the top of the first ladder and when he sits down before sliding down the slide.	Capture this strategy to share during the Connect.

4

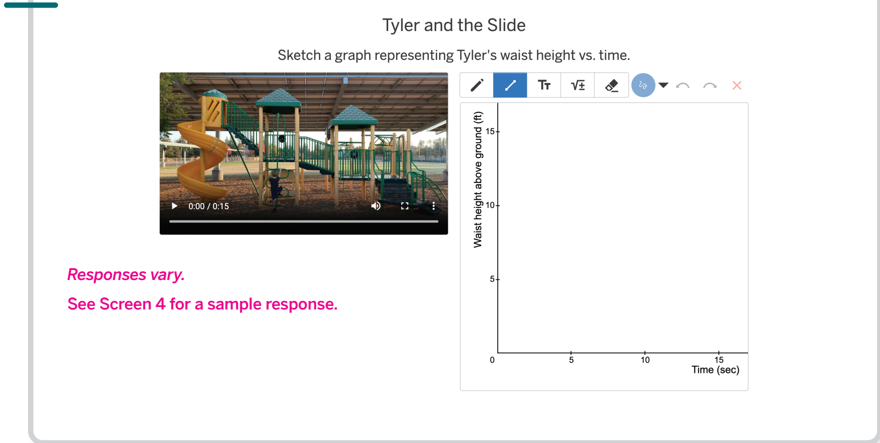
Play the video showing an answer sketch after students have had time to complete their sketch.

Math Identity and Community Consider highlighting the value of changing one's mind by asking if any students revised their thinking after seeing the sketch of the graph.

Activity 1 continued >

Students using digital

3



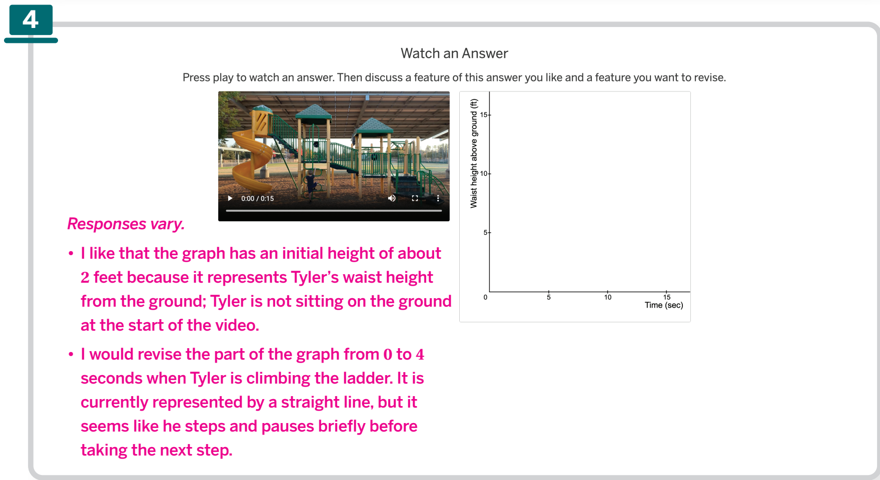
Tyler and the Slide

Sketch a graph representing Tyler's waist height vs. time.

Responses vary.

See Screen 4 for a sample response.

4



Watch an Answer

Press play to watch an answer. Then discuss a feature of this answer you like and a feature you want to revise.

Responses vary.

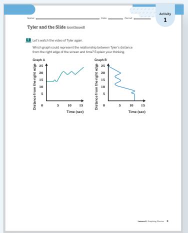
- I like that the graph has an initial height of about 2 feet because it represents Tyler's waist height from the ground; Tyler is not sitting on the ground at the start of the video.
- I would revise the part of the graph from 0 to 4 seconds when Tyler is climbing the ladder. It is currently represented by a straight line, but it seems like he steps and pauses briefly before taking the next step.

Digital Lesson

Pairs | 15 min | Pacing: Screens 3–5 one at a time

Activity 1 Tyler and the Slide (continued)

Purpose: Students draw the graph of a function that represents a context to focus on its qualitative aspects and how they affect the shape of the graph. **(MP4)**



Students using print

5 Monitor

Play the video showing Tyler on the slide again.

A **Accessibility: Visual-Spatial Processing**
For students using digital, consider providing access to the Student Edition, which contains printed versions of the two graphs for students to draw on or highlight.

5 Connect

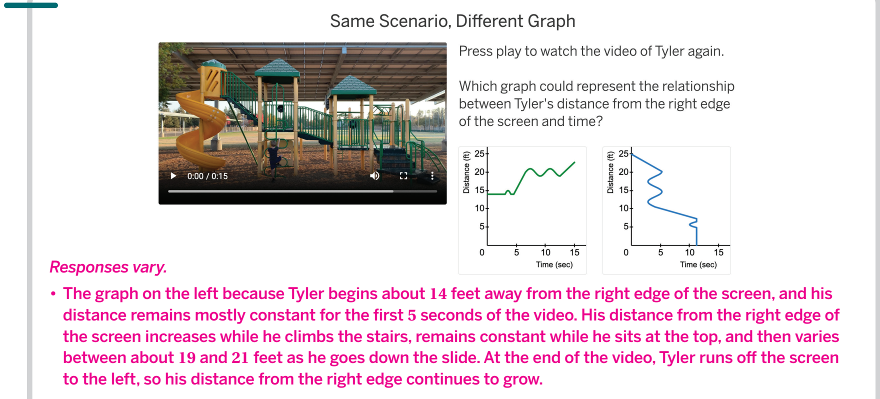
Invite students to share their responses and justify their reasoning in terms of what is happening in the video. **(MP3)**

Emphasize that when sketching a graph from a context, it is important to pay attention to the variables being measured. For example, Tyler's waist height above the ground vs. his distance from the right edge of the screen. Even though the context (video) is the same, using different variables creates different graphs.

Key Takeaway: The purpose of this discussion is to make connections between the features of a situation and the features of a graph, and to make arguments and critique the reasoning of others. **(MP3)**

Students using digital

5



Same Scenario, Different Graph

Press play to watch the video of Tyler again.

Which graph could represent the relationship between Tyler's distance from the right edge of the screen and time?

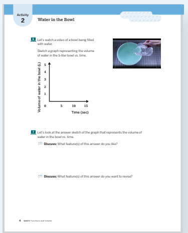
Responses vary.

- The graph on the left because Tyler begins about 14 feet away from the right edge of the screen, and his distance remains mostly constant for the first 5 seconds of the video. His distance from the right edge of the screen increases while he climbs the stairs, remains constant while he sits at the top, and then varies between about 19 and 21 feet as he goes down the slide. At the end of the video, Tyler runs off the screen to the left, so his distance from the right edge continues to grow.
- The graph on the right cannot represent Tyler's distance from the right edge of the screen and time because it would indicate that Tyler is occasionally in more than one place at a time. For example, the graph on the right indicates that at just over 5 seconds Tyler's distance from the right edge of the screen is approximately 10, 15, and 20 feet. This is not possible because Tyler cannot be in more than one place at a time.

Pairs | 10 min | Pacing: Screens 6–7

Activity 2 Water in the Bowl

Purpose: Students draw the graph of a function that represents a context to apply strategies learned in the previous activity. **(MP6)**



Students using print

6 Launch

Play the video showing the volume of water in the bowl over time.

6 Monitor

D Differentiation

Look for students who:	Teacher Moves
Need support getting started.	**Support:** Ask, "What do you notice and wonder about the volume of water in the bowl?"
Think the graph should have a constant slope.	**Support:** Ask, "If the graph was a line, what would that mean about the amount of water being poured into the bowl for the entire video?"
Ask to replay and pause the video to record the volume at different time intervals.	Replay and pause the video as needed.

7 **Play** the video showing an answer sketch.

M/EL **Multilingual/English Learners** Use intentional grouping so students with different English language proficiency levels can interact and have an opportunity to listen to peers with more advanced proficiency. **(Speaking and Listening)**

7 Connect

Display students' sketches representing the volume of water. Then display the answer sketch.

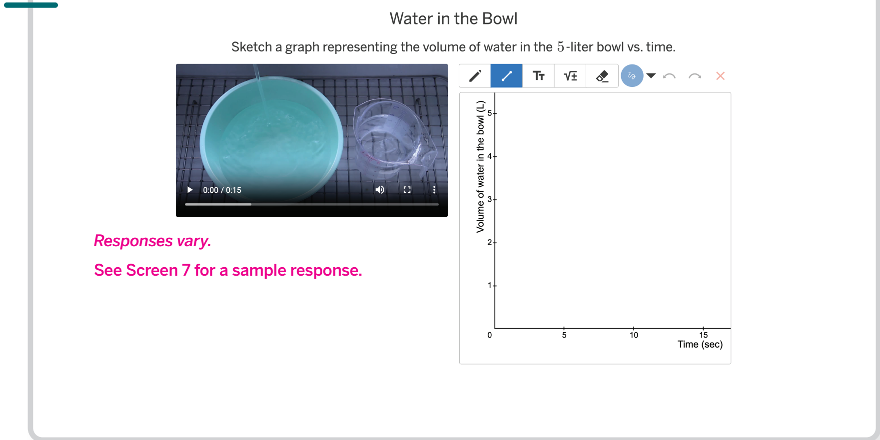
Invite students to share their responses to the prompts. Encourage students to use mathematically precise language, such as *rate of change*, y*-intercept*, and *linear*. **(MP6)**

Consider asking, "Why is the rate of change greater in the middle section of the graph? Why are the slopes of the first and third section the same?"

Emphasize that the water is pouring into the bowl at a constant rate before additional water is poured at the same time, so the graph is steeper before returning to its original steepness.

Students using digital

6



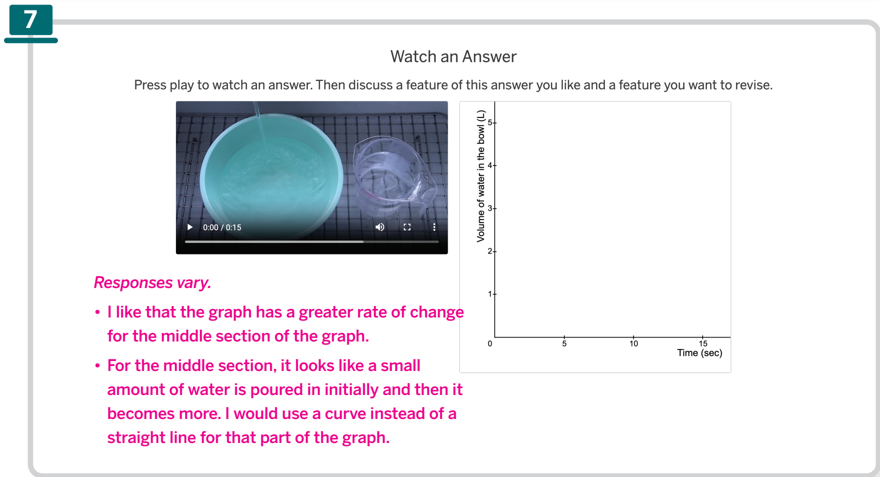
Water in the Bowl

Sketch a graph representing the volume of water in the 5-liter bowl vs. time.

Responses vary.

See Screen 7 for a sample response.

7



Watch an Answer

Press play to watch an answer. Then discuss a feature of this answer you like and a feature you want to revise.

Responses vary.

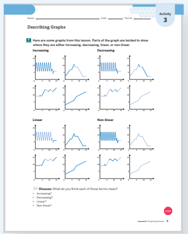
- I like that the graph has a greater rate of change for the middle section of the graph.
- For the middle section, it looks like a small amount of water is poured in initially and then it becomes more. I would use a curve instead of a straight line for that part of the graph.

Digital Lesson Pairs | 5 min | Pacing: Screen 8

Activity 3 Describing Graphs

Purpose: Students analyze parts of graphs to develop their own meaning of where functions are increasing, decreasing, linear, and non-linear.

Short on time: Consider completing this activity as a whole class.



Students using print

8 Launch

Display the graphs. Select each term to show where on each graph the term applies.

Support getting started by asking, "What do all of the parts of the graphs that are *increasing* have in common?" Repeat for *decreasing*, *linear*, and *non-linear*.

8 Monitor

D Differentiation

Look for students who:	Teacher Moves
Use informal language to describe the graphs such as *going up*, *going down*, *curved*, or *straight*.	Ask, "Which of these descriptions can you use instead?"
Think they need to formally define each description.	Tell students to write the meaning of each description in their own words.

A Accessibility: Visual-Spatial Processing Invite students to use coloring tools to annotate the graphs to support interpreting visual representations.

M/EL Multilingual/English Learners Provide sentence frames to support students as they write and share the meanings of each description. For example, "*Increasing* means the graph ______," "*Linear* means the graph ______." **(Writing and Speaking)**

8 Connect

Display any graphs that will help the class discussion.

Invite students to share their responses.

Consider asking, "What is a real-world situation that could produce a linear part of a graph? Non-linear?"

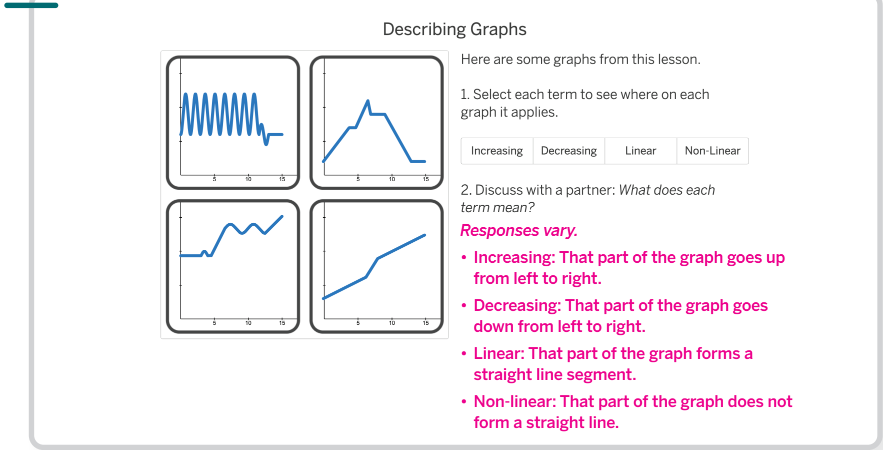
Key Takeaway: When the parts of the graph of a function are . . .

- *Increasing*, the values of the function are increasing.
- *Decreasing*, the values of the function are decreasing.
- *Linear*, these parts are straight line segments.
- *Non-linear*, these parts are not straight line segments.

Some functions that consist of all linear segments are non-linear because the rate of change is not consistent throughout.

Students using digital

8



Describing Graphs

Here are some graphs from this lesson.

1. Select each term to see where on each graph it applies.

2. Discuss with a partner: *What does each term mean?*

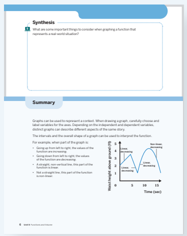
Responses vary.

- **Increasing: That part of the graph goes up from left to right.**
- **Decreasing: That part of the graph goes down from left to right.**
- **Linear: That part of the graph forms a straight line segment.**
- **Non-linear: That part of the graph does not form a straight line.**

Whole Class | 5 min | Pacing: Screen 9

Synthesis

Purpose: Students synthesize their understanding of using different variables to describe the same real-world situation, which may produce different graphs.



Students using print

9 Synthesis

Invite students to respond independently, and then share their thinking with a partner.

Math Identity and Community If time allows, invite students to celebrate other students whose strategies they found most helpful.

Use the MLR2: Collect and Display routine to review the terms *increasing*, *decreasing*, *linear*, and *non-linear.* Consider showing the graphs to help students visualize each description.

Consider asking, "What strategies or tools did you find helpful today when drawing the graph of a function from a context? How were they helpful?"

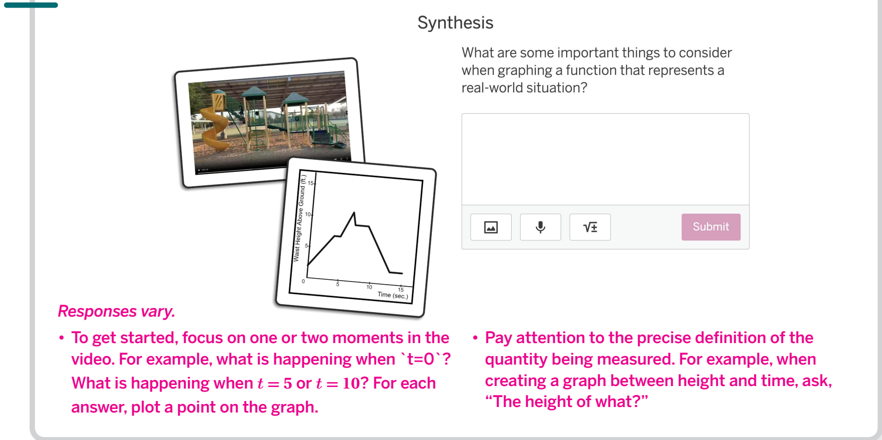
Lesson Takeaway: When drawing the graph of a function from a context, it is important to pay attention to the variables of the situation, the initial amount, if the values are increasing or decreasing, and whether the rate of change is constant.

Summary

Share the Summary. Students can refer back to this throughout the unit and course.

Students using digital

9



Synthesis

What are some important things to consider when graphing a function that represents a real-world situation?

Responses vary.

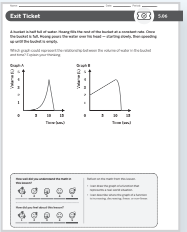
- To get started, focus on one or two moments in the video. For example, what is happening when `t=0`? What is happening when $t = 5$ or $t = 10$? For each answer, plot a point on the graph.
- Pay attention to the precise definition of the quantity being measured. For example, when creating a graph between height and time, ask, "The height of what?"

Digital Lesson

Independent | 5 min | Pacing: Screens 10–11

Exit Ticket

Purpose: Students demonstrate their understanding by determining which graph could represent the given real-world situation.



Students using print

10-11 Today's Goals

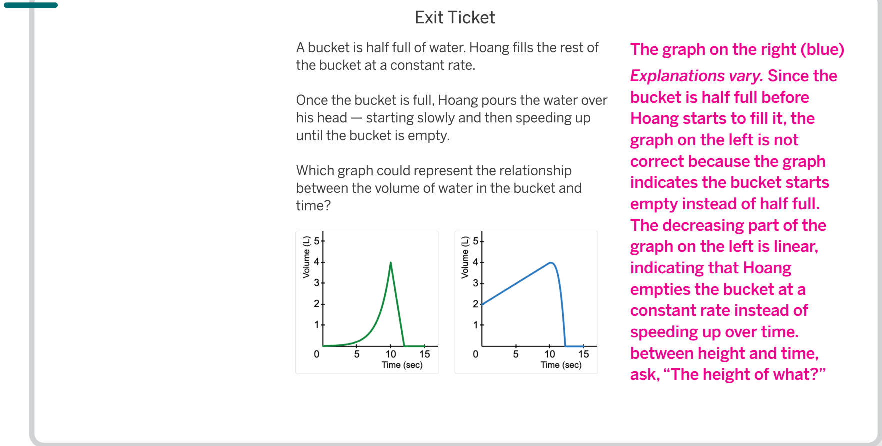
Goal: Draw the graph of a function that represents a real-world situation.

Language Goal: Describe where the graph of a function is *increasing, decreasing, linear*, or *non-linear*. **(Reading, Writing, Speaking, and Listening)**

Support for Future Learning: If students struggle to determine which graph represents the scenario, consider reviewing this Exit Ticket as a class before beginning Lesson 7.

Students using digital

10



Exit Ticket

A bucket is half full of water. Hoang fills the rest of the bucket at a constant rate.

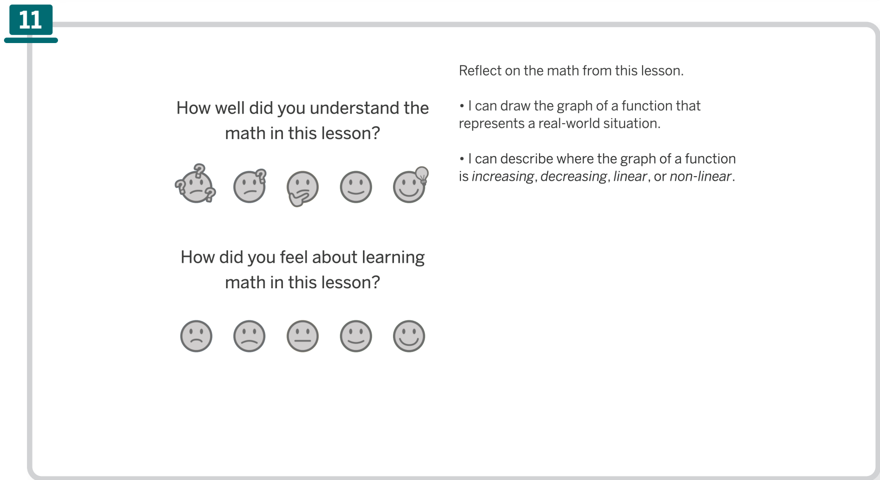
Once the bucket is full, Hoang pours the water over his head — starting slowly and then speeding up until the bucket is empty.

Which graph could represent the relationship between the volume of water in the bucket and time?

The graph on the right (blue)

Explanations vary. Since the bucket is half full before Hoang starts to fill it, the graph on the left is not correct because the graph indicates the bucket starts empty instead of half full. The decreasing part of the graph on the left is linear, indicating that Hoang empties the bucket at a constant rate instead of speeding up over time. between height and time, ask, "The height of what?"

11



How well did you understand the math in this lesson?

How did you feel about learning math in this lesson?

Reflect on the math from this lesson.

- I can draw the graph of a function that represents a real-world situation.
- I can describe where the graph of a function is *increasing*, *decreasing*, *linear*, or *non-linear*.

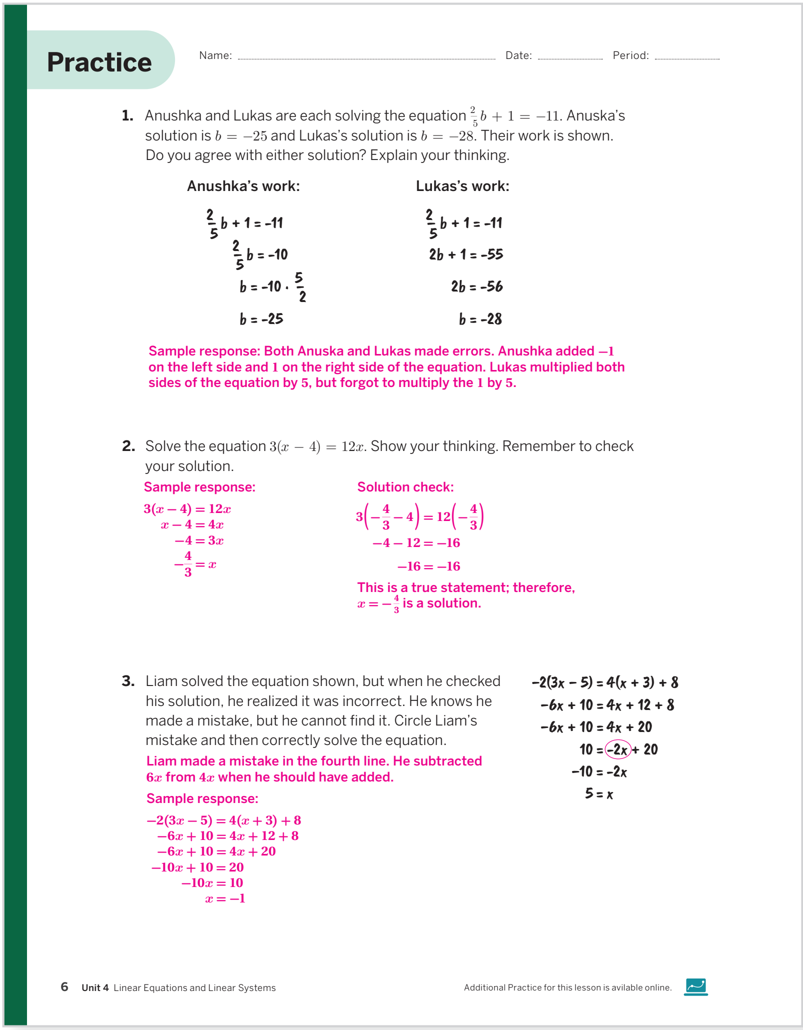
Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Students using digital

Students using print



Practice

Name: ______ Date: ______ Period: ______

1. Anushka and Lukas are each solving the equation $\frac{2}{5}b + 1 = -11$. Anuska's solution is $b = -25$ and Lukas's solution is $b = -28$. Their work is shown. Do you agree with either solution? Explain your thinking.

Anushka's work:

$$\frac{2}{5}b + 1 = -11$$
$$\frac{2}{5}b = -10$$
$$b = -10 \cdot \frac{5}{2}$$
$$b = -25$$

Lukas's work:

$$\frac{2}{5}b + 1 = -11$$
$$2b + 1 = -55$$
$$2b = -56$$
$$b = -28$$

Sample response: Both Anuska and Lukas made errors. Anushka added -1 on the left side and 1 on the right side of the equation. Lukas multiplied both sides of the equation by 5, but forgot to multiply the 1 by 5.

2. Solve the equation $3(x - 4) = 12x$. Show your thinking. Remember to check your solution.

Sample response:

$$\begin{aligned} 3(x-4) &= 12x \\ x - 4 &= 4x \\ -4 &= 3x \\ -\frac{4}{3} &= x \end{aligned}$$

Solution check:

$$\begin{aligned} 3\left(-\frac{4}{3} - 4\right) &= 12\left(-\frac{4}{3}\right) \\ -4 - 12 &= -16 \\ -16 &= -16 \end{aligned}$$

This is a true statement; therefore, $x = -\frac{4}{3}$ is a solution.

3. Liam solved the equation shown, but when he checked his solution, he realized it was incorrect. He knows he made a mistake, but he cannot find it. Circle Liam's mistake and then correctly solve the equation.

$$\begin{aligned} -2(3x - 5) &= 4(x + 3) + 8 \\ -6x + 10 &= 4x + 12 + 8 \\ -6x + 10 &= 4x + 20 \\ 10 &= \boxed{-2x} + 20 \\ -10 &= -2x \\ 5 &= x \end{aligned}$$

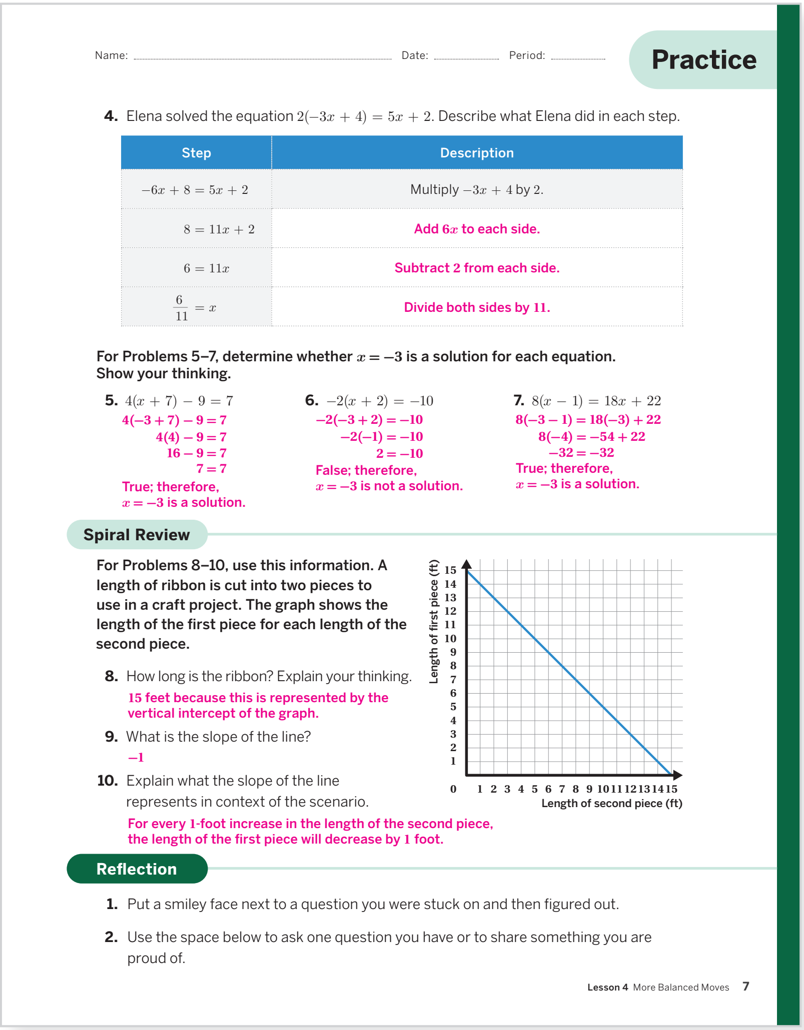
Liam made a mistake in the fourth line. He subtracted $6x$ from $4x$ when he should have added.

Sample response:

$$\begin{aligned} -2(3x-5) &= 4(x+3) + 8 \\ -6x + 10 &= 4x + 12 + 8 \\ -6x + 10 &= 4x + 20 \\ -10x + 10 &= 20 \\ -10x &= 10 \\ x &= -1 \end{aligned}$$

6 **Unit 4** Linear Equations and Linear Systems

Additional Practice for this lesson is avilable online.



Name: ______ Date: ______ Period: ______

Practice

4. Elena solved the equation $2(-3x + 4) = 5x + 2$. Describe what Elena did in each step.

Step	Description
$-6x + 8 = 5x + 2$	Multiply $-3x + 4$ by 2.
$8 = 11x + 2$	**Add $6x$ to each side.**
$6 = 11x$	**Subtract 2 from each side.**
$\frac{6}{11} = x$	**Divide both sides by 11.**

For Problems 5–7, determine whether $x = -3$ is a solution for each equation. Show your thinking.

5. $4(x + 7) - 9 = 7$

$$\begin{aligned} 4(-3+7) - 9 &= 7 \\ 4(4) - 9 &= 7 \\ 16 - 9 &= 7 \\ 7 &= 7 \end{aligned}$$

True; therefore, $x = -3$ is a solution.

6. $-2(x + 2) = -10$

$$\begin{aligned} -2(-3+2) &= -10 \\ -2(-1) &= -10 \\ 2 &= -10 \end{aligned}$$

False; therefore, $x = -3$ is not a solution.

7. $8(x - 1) = 18x + 22$

$$\begin{aligned} 8(-3-1) &= 18(-3) + 22 \\ 8(-4) &= -54 + 22 \\ -32 &= -32 \end{aligned}$$

True; therefore, $x = -3$ is a solution.

Spiral Review

For Problems 8–10, use this information. A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length of the first piece for each length of the second piece.

8. How long is the ribbon? Explain your thinking.

15 feet because this is represented by the vertical intercept of the graph.

9. What is the slope of the line?

-1

10. Explain what the slope of the line represents in context of the scenario.

For every 1-foot increase in the length of the second piece, the length of the first piece will decrease by 1 foot.

Reflection

1. Put a smiley face next to a question you were stuck on and then figured out.
2. Use the space below to ask one question you have or to share something you are proud of.

Lesson 4 More Balanced Moves 7

Practice Problem Item Analysis

	Problem(s)	DOK	Standard(s)
On-Lesson			
	1, 3, 4	2	NY-8.EE.7 NY-8.EE.7b
Fluency	2	1	NY-8.EE.7 NY-8.EE.7b
Test Practice	5–7	1	NY-8.EE.7 NY-8.EE.7b
Spiral Review			
	8, 10	2	NY-8.EE.6
	9	1	NY-8.EE.6

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Feel the Burn (NYC)

Lesson 7: Comparing Representations of Functions

Purpose

The purpose of this lesson is for students to compare and contrast three calorie-burning scenarios represented in different ways.

Preparation

Worksheet

- *Warm-Up and Activity 1*: Print one single-sided sheet for each group of 2–3 students.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Context Cards

- Print and cut out one set of cards for each group of 2–3 students.

Poster

- Provide each group of 2–3 students with one piece of blank paper to create their poster. (Poster-size paper is ideal, but 8.5-by-11 inch will work.)

Warm-Up (5 minutes)

The purpose of this warm-up is for students to begin making sense of the representation on their context card.

Launch

Arrange students in groups of 2–3. Provide each group with one set of the context cards. (For groups of 3, each student gets one context card. For groups of 2, one student gets two context cards.) Give students 1–2 minutes of quiet think-time to answer the question that appears on their card. Then invite them to share their answers with (and explain their thinking to) the members of their group.

Teacher Moves

This is a great place to check students’ progress before they begin Activity 1. Offer support to individual groups where needed, or lead a whole-class discussion if enough students are struggling.

Readiness Check (Problem 2)

If most students struggled, plan to revisit the definition of proportional relationships during this warm-up. Students worked with these relationships in Unit 3: Linear Relationships, so it may be helpful to revisit practice problems from that unit.

Activity 1: Awards (30 minutes)

Launch
This activity has two parts: answer the questions, then create a visual display.

Tell students to continue working in their groups of 2–3. Throughout Activity 1, students will need to work together to answer the questions, as each representation holds a “piece of the puzzle.”

Teacher Moves
Circulate through the room as students work, offering help as needed.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to look at their classmates’ posters, discuss what features of the posters helped them understand their classmates’ thinking, and describe something they would change about their own display now that they have seen other groups’ work.

As each group completes Activity 1, ask them to display their poster in a designated place in the classroom (e.g., a wall, a table, or a collection of desks). Encourage groups to begin the Lesson Synthesis as soon as they are done with Activity 1 (rather than waiting for the entire class to be ready).

Cool-Down (5 minutes)

If students struggle with comparing data represented in tables, equations, and graphs, plan to emphasize this when opportunities arise over the next several lessons, particularly in Lesson 8.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Charge! (NYC)

Lesson 8: Modeling With Linear Functions

Purpose

In this lesson, students use linear functions to model a real-world situation (MP4). They are given data for a relationship that is *almost* linear, and from that, they develop a linear model. They use their model to make predictions and to determine whether a linear approximation is reasonable and for which values it would be reasonable. Students begin to see both the value and the limitations of linear models. One such limitation—that some data sets cannot be modeled well by a single line—leads into the next lesson where students model non-linear scenarios with piecewise linear functions.

Preparation

Worksheet

- *Warm-Up* and *Activity 1*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Warm-Up (5 minutes)

The purpose of this warm-up is to orient students to the problem they will work on in this lesson: How long will it take for a phone to fully charge?

Screen 1 of Teacher Presentation Screens
Display Screen 1 on the projector. Provide each student with the worksheet. Give students one minute of quiet time to think and write. Then invite them to discuss with a partner.

Invite a few students to share their responses. It's okay—even desirable—to hear stories that go in different directions at this stage. Ideally, at least one response includes the fact that it's the evening and the phone battery is low.

Activity 1: Charge! (30 minutes)

The purpose of this activity is for students to use data points to develop a linear model and assess the reasonableness of their model. This activity continues the context introduced in the warm-up.

Screen 2 of Teacher Presentation Screens
Tell students that the question we'll focus on today is: When will the phone be fully charged? Invite students to write this question on their worksheet. Then ask students to write down their estimate(s) for how long it will take the phone to charge. Consider quickly polling the class on their estimates to develop a class range. If you record this range on the board, you can revisit it later in the lesson to show how the class's thinking evolves.

Screen 3 of Teacher Presentation Screens
Ask students what information they know and what additional information would be helpful (MP4). Give them 1–2 minutes of quiet think-time, and then invite several students to share. Consider writing all the responses on the board so that students may refer back to them later.

Provide the additional information that students ask for if you know it. Most information you are unable to provide, such as the type of phone or the age of the battery. It is often the case in real-world situations that only a limited amount of information is known. However, the next screens offer information that might lend insight into some of the additional information that students want to know.

Screens 4–6 of the Teacher Presentation Screens
These screens are meant to give students more data to make their model. Consider revealing these screens gradually (about one minute per screen). For example, after students are shown the charge percentage at $9{:}02$ and $9{:}10$, have them estimate the charge percentage at $9{:}14$, and then reveal it. Repeat that for $9{:}22$. Now that students have four data points, give them substantial time to work on the main question of the lesson: When will the phone be fully charged?

As students are working, encourage them to use the tools they deem appropriate to solve the problem (MP5), such as the provided blank paper, the Desmos calculator, or any other tools that would be helpful. If students are having difficulty getting started, ask them how they might represent the information they have mathematically, such as in a table.

When most students have a prediction that they are satisfied with, hold a class discussion about the approaches students took and the conclusions that they drew. Ask students if they found any patterns as they were looking at the data and how they used those patterns to make their prediction. Different student approaches may lead to interesting discussions about variables (for instance, whether the independent variable should be "the time" or "minutes since $9{:}02$"), or about the advantages of representing the problem in different ways. For the latter, it may be useful for students to see how each of the following representations could be used to make a prediction: a table, plotted points with a fitted line, and a linear equation. A document camera and/or the supporting Desmos graph may be useful.

When students share their work, ask them about their assumptions as they made their predictions. For instance, did they consider whether or not a linear model is a good fit for predicting the charge percentage?

Screen 7 of the Teacher Presentation Screens
Display Screen 7 and reveal the answer. Students may be surprised that a linear model isn't very helpful for predicting when the phone will be fully charged. Lead a short discussion to help students reflect on what happened. Students likely assumed—with what appeared at first to be good reason—that the rate of change would remain constant. In fact, due to the chemistry of how

batteries work, it is common to see a big drop-off in the charging speed as the battery gets close to fully charged. Invite them to respond to the reflection prompt on their worksheet.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Lesson Synthesis (5 minutes)

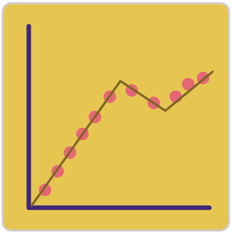
The purpose of the lesson synthesis is to reinforce students' understanding of linear relationships, linear models, and functions. Specifically, this problem contained data that was a function, but the function was not a linear relationship. The data points suggested that the relationship could be *modeled* with a linear function, and modeling always comes with some uncertainty and imprecision. Once more data points were available, we could see that a single linear function only worked well as a model until the phone reached about 85% charge.

Give students one minute of quiet think-time to consider the three prompts and 1–2 minutes to discuss with a partner. Then lead a class discussion about each question.

Cool-Down (5 minutes)

Students will have more chances to develop their understanding of modeling nonlinear functions in the upcoming lessons, particularly in Lesson 9.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.



Piecing It Together

Lesson 9: Modeling With Piecewise Linear Functions

Overview

This lesson builds on Lesson 8 where students learned when a single linear model is or is not appropriate to model data. The purpose of this lesson is to model with piecewise linear functions (MP4). Students analyze different real-world data sets presented as graphs. The focus of this lesson is not necessarily to find equations for the piecewise linear functions (though students may choose to do so), but rather, to analyze graphs qualitatively and to compute and compare rates of change.

Learning Goals

- Approximate non-linear functions with piecewise linear functions.
- Calculate different rates of change of a piecewise linear function using a graph, and interpret rates of change in context.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

This lesson builds on Lesson 8 where students learned when a single linear model is or is not appropriate to model data. The purpose of this lesson is to model with piecewise linear functions (MP4). Students analyze different real-world data sets presented as graphs. The focus of this lesson is not necessarily to find equations for the piecewise linear functions (though students may choose to do so), but rather, to analyze graphs qualitatively and to compute and compare rates of change (MP2).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to introduce the use of multiple linear segments as a model for data using a familiar context: phone charging.

Activity 1: Recycling (5 minutes)

The purpose of this activity is for students to build upon and analyze an existing piecewise linear model. Students also calculate and interpret the rate of change of one of the function's intervals (MP1).

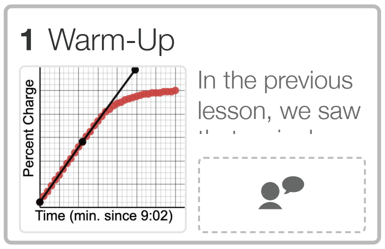
Activity 2: Modeling Data (25 minutes)

The purpose of this activity is for students to build and analyze a piecewise linear model. Students choose a data set, build a graphical model, and then analyze and use it to make predictions. At the end of the activity, they share what they've learned with peers who have chosen other data sets.

Lesson Synthesis (5 minutes)

The purpose of the discussion is to consider the advantages and disadvantages of different modeling choices, particularly the number of segments used.

Cool-Down (5 minutes)



1 Warm-Up

In the previous lesson, we saw

In the previous lesson, we saw that a single linear model wasn't appropriate for modeling a phone charging over time.

Here is the data from that lesson. Adjust the points to make a better model. Then answer the question:

When was the phone charging slowest? Explain your thinking using the graph.

Teacher Moves

Purpose
The purpose of this lesson is for students to approximate nonlinear functions with piecewise linear functions and calculate rates of change of piecewise functions and interpret them in context.

Warm-Up Launch
The purpose of this warm-up is to introduce the use of multiple linear segments as a model for data using a familiar context: phone charging.

Tell students that we are revisiting the phone-charging data from the previous lesson. Their first task is to make a model, now with a major restriction lifted: they are no longer limited to using a *single* linear model. Tell them that this lesson will focus on making models by combining multiple linear segments rather than just a single line.

Arrange students into groups of two. Give students 1–2 minutes of quiet think-time. Then highlight unique answers to show the class. Push the class to make connections between the rate at which the phone charges and the graph. Consider asking:

- *If we only looked at the line segments and not the points, how could we tell when the phone was charging the slowest?* [The slopes of the segments represent the rates of change.]
- *Could we compare the rates of change numerically? What are they?* [We can calculate the slope of each segment. The slope of the left-most segment is approximately 1. The slope of the right-most segment is approximately $\frac{1}{6}$.]

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- Once the phone reached around 85% charge, it began to charge more slowly until it was fully charged. You can see this in the graph

because the points start increasing at a slower rate.
- The phone was charging the slowest between about 10:20 and when it was fully charged, around 11:22. This aligns with my model, where my segment from about 10:20 to 11:20 has a very small slope.

Student Supports

Students With Disabilities
- *Fine Motor Skills: Peer Tutors*

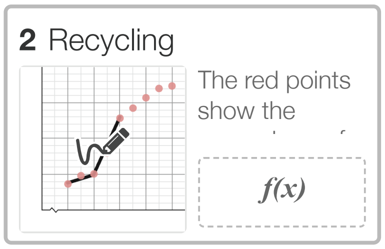
Pair students with their previously identified peer tutors, and allow students who struggle with fine motor skills to dictate how to create the model.

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time. This may include reading the information in the graph.

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g.,The phone was charging slowest between minutes ___ and ___ because _________.).



2 Recycling

The red points show the

The red points show the percentage of waste produced in the United States that gets recycled over time.

A student started to model the data with linear segments.

Sketch one more linear segment to complete the model.

Then approximate the slope of the segment you created.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for determining the slope of a part of a piecewise linear function.

Activity Launch
The purpose of this activity is for students to build upon and analyze an existing piecewise linear model.

Consider launching this activity with a prompt that invites students to make sense of the data. For instance:

- Ask students what they notice and wonder about the graph.
- Ask students to describe what's happening in the graph.

- Ask students to tell a brief story about the graph.

Then arrange students into pairs. Tell students that their job is to finish the incomplete model by adding a segment, and then answer questions about that segment. Give students 2–3 minutes, plus a couple minutes to share their responses with their partner. Then follow with a whole-class discussion. Consider asking: *How do the slopes of the three segments compare?* Follow with: *What does the slope in this interval mean about recycling?*

Early Student Thinking
Students may calculate the slope by counting the horizontal and vertical change in *grid* units (e.g., up 4, over 4) rather than associating those grid units with their quantities. Encourage students to sketch the horizontal and vertical change of their segment and to identify what exactly is changing [about 20 years and about 10 percentage points]. Consider calculating the slope of one of the given two segments as a class.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

About 0.5 percentage points/year.

Responses vary.

The slope is the rate of change. It means that between 1995–2015, the percentage of waste recycled grew by 0.5 percentage points per year.

Student Supports

Multilingual Learners
- *Expressive Language: Visual Aids*

Review an anchor chart that publicly displays the definition and strategies for calculating slope in order to aid in explanations and reasoning.

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the representations for students to draw on or highlight.

3 Four Data Sets

Teacher Moves

Activity Launch
The purpose of this activity is for students to build and analyze a piecewise linear model. Students choose a data set, build a graphical model, and then analyze and use it to make predictions.

The activity begins with a card sort where students are introduced to four new data sets. Consider launching the activity by showing the four graphs and four quantities using the student view in the teacher dashboard. With help from the class, explain the general meaning of each quantity. Then tell students to match each quantity with the graph they think is most likely to represent it. Tell students that after this screen, they will become an expert on one of these four graphs and then share what they learned with their peers.

Teacher Moves
Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort. Use the teacher dashboard to monitor student progress.

Facilitation
Consider using pacing to restrict students to Screens 3–8.

Sample Responses

Image solution

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4 Choose a Data Set

Here are four data sets. Pick

Cost of College:

Teacher Moves

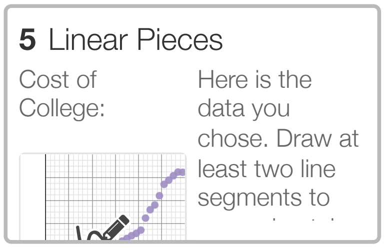
On this screen, students select the data set that they will become the expert on for the rest of this activity. Consider allowing each student to choose a data set and having them move to a specific part of the room to be around other students who chose the same data set. Students can work individually with the support of those seated nearby.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

- I notice that the cost of college has increased quite a bit in the last thirty years. I wonder what the cost of college will be when I am old enough to enroll. I wonder if there is a connection between college cost and government budgets for colleges.

- I notice that life expectancy was flat and then has been trending upwards. I wonder what that little dip in life expectancy is between 1940–1945.
- I notice that the birth rate in the United States has been decreasing consistently, with a few exceptions, since 1800. I wonder why the birth rate has declined and if it will continue to do so.
- I notice that the global temperature doesn't just change in one direction—it has gone up and down a lot in the past 150 years, but does appear to be trending upwards. I wonder what kind of impact temperature changes of this amount have on ecosystems and the environment.



5 Linear Pieces

Cost of College:

Here is the data you chose. Draw at least two line segments to

Cost of College:

Teacher Moves

Use the teacher dashboard to monitor student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

A reasonable piecewise model will be critical for later screens. To this end, consider using snapshots to highlight and display students' unique piecewise linear models.

Sample Responses

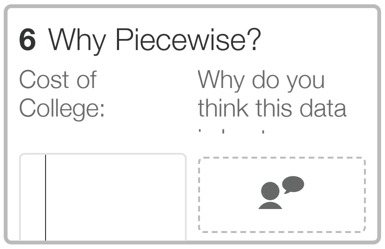
Responses vary.

Student Supports

Students With Disabilities

- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the graphs for students to draw on or highlight.



6 Why Piecewise?

Cost of College:

Why do you think this data

Cost of College:

Teacher Moves

As students begin working on this screen, consider offering some examples of outside knowledge or ideas that might be relevant (e.g., the 20th century saw modern medicine spread throughout the world, helping more children survive into adulthood. The first half of the 20th century also involved two world wars). Consider asking: *How might these affect life expectancy or the birth rate?* (Note that these questions are not critical to the goals of the lesson. Allocate time accordingly.)

Give 3–4 minutes of quiet think-time. Highlight unique answers to show the class, such as those that reference specific years or intervals and those that bring in relevant outside knowledge about the context.

Sample Responses

Responses vary.

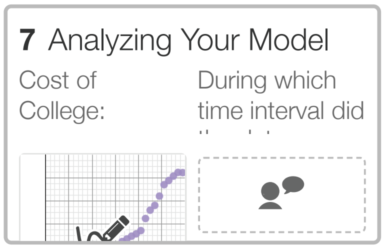
- **Cost of college** makes sense to model with a piecewise function because the cost was fairly constant for a while before it started to increase in the early 1980s, and then its rate of increase seemed to change in the early 2000s.
- **Life expectancy** makes sense to model with a piecewise function because it has changed at different rates over the years. For instance, there was a 70-year period where life expectancy didn't seem to change at all. During one 5-year period, it seemed to decrease. At almost all other times, life expectancy increased. A piecewise function also makes sense because life expectancy will likely level off at some point. I don't think life expectancy will reach 100 for quite a while.
- **Birth rate** has had a steady decline since 1800, so a single linear model might not be terrible, but a piecewise linear model is better because the birth rate has not decreased at a constant rate, and in fact, it increased during some periods of time. Also, I think the birth rate might be reaching a steady period: families used to be big. Now they are small, and they're probably not going to get much smaller.
- The **global temperature** data shows a steady upward trend, but it makes sense to model with a piecewise linear function rather than a single linear function because global temperature has periods where it increased and periods where it decreased.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.



7 Analyzing Your Model

Cost of College:

During which time interval did

Cost of College:

Teacher Moves

Highlight unique responses to show the class, such as those that reference specific years or intervals.

To highlight the use and interpretation of piecewise linear models, consider putting a piecewise model up on the projector and asking:

- *What do the slopes of the different lines mean?* [The slopes of the lines tell us the rate of change for the specific intervals of time.]
- *Can we use this model to predict information in the future?* [It depends. If the data continues in accordance to the most recent interval, yes. If the data does not continue its current rate of change, our model may not make very good predictions.]

The latter question provides a useful transition to the next screen.

Sample Responses

Responses vary.

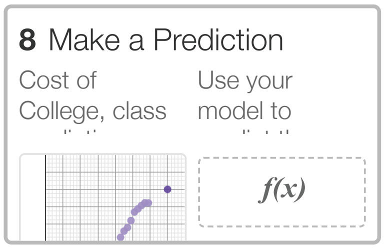
- **Cost of college** seems to have changed the most in the decade between 2000 and 2010, from about $\$5\,500$ to about $\$9\,000$.
- **Life expectancy** increased considerably between 1920 and 1970, from about 35 to 60. I can see from the graph that the segment modeling that period of time has the steepest slope of any segment in my model.
- **The birth rate** changed considerably between 1925 and 1975. There was a period of a steep decline, followed by a steep increase, followed by another steep decline.
- **Global temperature** seems to have changed the most in the period between 1965 and the present day. That's where I have a line segment on my piecewise model that has the greatest slope.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., ____________ seems to change the most because ______________.).


8 Make a Prediction

Cost of College, class

Use your model to

Cost of College, class predictions:

Teacher Moves

Consider using the teacher view of the dashboard to show the distribution of predictions for each data set, calling attention to any conflict or consensus that you see. It's okay—even desirable—to lack consensus, since many different predictions are justifiable.

Rather than hold a discussion, encourage students to prepare to share their findings with classmates, which they are invited to do on the next screen.

Sample Responses

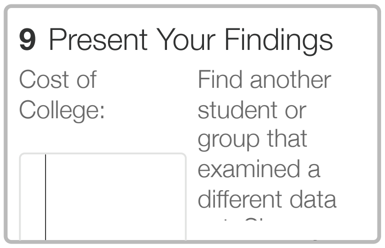
2030 prediction: *Responses vary.*
- Cost of college: $\$11\,500$
- Life expectancy: 75 years
- Birth rate: 11 births per thousand people
- Global temperature: $59^\circ F$

2050 prediction: *Responses vary.*
The trend in the graph has changed rates and even directions many times over the years. I don't think I can make a prediction for 2050 that's very precise or accurate.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., ____ is/is not a reasonable model to predict 2050 because ____________.).


9 Present Your Findings

Cost of College:

Find another student or group that examined a different data

Cost of College:

Teacher Moves

On this screen, students present to another student or group of students who analyzed a different data set. If time allows, consider having students prepare for their presentation by first meeting with one or more students who analyzed *the same* data set. Then have students move so that they are with one or more students who analyzed a different context.

Time permitting, consider having a brief whole-class discussion. Ask students: *What was one surprising thing you learned today about one or more of the data sets?*

Student Supports

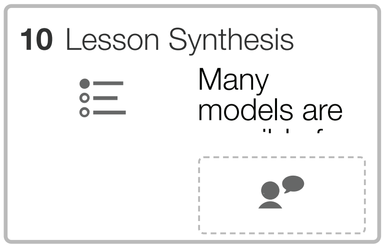
Students With Disabilities

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the questions to review as they complete Screens 4–9.

- *Executive Functioning: Graphic Organizers*

Provide students a graphic organizer to record what they observed before they are expected to share their ideas with others.



10 Lesson Synthesis

Many models are

Many models are possible for a data set.

Look at each of the three models on the left. Then select one model.

Describe one advantage and one disadvantage of the model you chose.

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to compare and contrast linear and piecewise linear models to represent a set of data.

Synthesis Launch

Give students 2–3 minutes to respond to this question and one minute to share their responses with a classmate.

Consider using the teacher view of the dashboard to display a distribution of representations selected. Ask students: *What are the benefits of having fewer segments in the piecewise linear function? What are the benefits of having more segments?* [Fewer segments are easier to write equations for and help show long-term trends. More segments give a more accurate model of the data.]

Facilitation

Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- One advantage of Model A is that it is very accurate. It is more likely than the other models to produce a good prediction for the near future.

One disadvantage is that it does not represent long-term trends very well.
- One advantage of Model B is that it models the data reasonably well without having too many segments. One disadvantage is that it may lead to an overly rosy and inaccurate short-term prediction.
- One advantage of Model C is that it shows the long-term trend. One disadvantage is that it will not be very useful for predicting global temperature in the near future, such as 2030. Also, a single linear model may not factor in relevant circumstances that have changed since 1875, such as the amount of carbon in the air.

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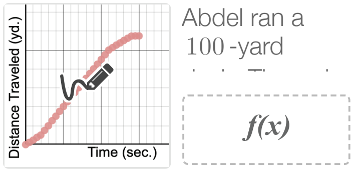
11 Cool-Down



Abdel ran a 100-yard

Abdel ran a 100-yard dash. The red points show his distance every half-second.

Draw line segments to approximately model the data.

Then answer this question:

When Abdel was running his fastest, approximately how fast was he running?

Teacher Moves

Support for Future Learning
If students struggle with analyzing data from a graph, consider making time to explicitly revisit these ideas. A strong understanding of calculating different rates of change from a graph will support students in the upcoming end assessment.

Facilitation
Consider using pacing to restrict students to Screens 11–12.

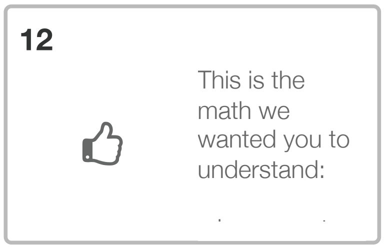
Sample Responses

- Image solution
- Approximately 10 yards per second

Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the representations for students to draw on or highlight.



12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can create graphs of non-linear functions with pieces of linear functions.
- I can calculate and interpret rates of change in context.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

8.5 Practice Day 1 (NYC)

Preparation

Student Workspace Sheet

- Print one double-sided sheet for each student.

Task Cards

- *Option 1 (Stations)*: Print two sets of cards (8 cards total) for the whole class.
- *Option 2 (Task Cards)*: Print one set of cards for each group of 2–3 students.

Resource Card

- Print enough cards for the option you choose below.

Instructions

Option 1: Stations

Print and cut out two sets of cards to create eight stations (one card per station).

Arrange students into groups of 3–4. Give each student the student workspace sheet to complete as they solve the task card at each station.

Options for student movement:

- As students finish a station, instruct students to move from station to station.
- After a set amount of time, instruct students to move as a group from station to station.
- After a set amount of time, instruct students to move to a new station such that no one from their previous group is in their new group.

Option 2: Task Cards

Arrange students into groups of 2–3. Print and cut out one set of cards for every group of students.

Give each student the student workspace sheet to complete as they work together to solve each of the task cards.

Consider posting the answer key, or walk around with it and provide feedback to students as they work.

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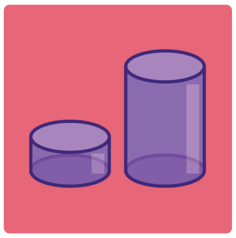


Scaling Cylinders

Lesson 12: Scaling Cylinders Using Functions

Overview

Students use functions to explore how changing a cylinder's radius or height impacts its volume. They also see examples of linear and nonlinear functions that arise in geometry.

Learning Goals

- Use representations of functions to analyze the relationship between one of a cylinder's dimensions and its volume
- Explain why the relationship between height and volume is linear but the relationship between radius and volume is non-linear.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The main purpose of this lesson is for students to use functions to explore how changing a cylinder's radius or height impacts its volume. A secondary purpose is to see examples of linear and nonlinear functions that arise in geometry.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to remind students about properties of functions they learned earlier in the unit. This lesson will investigate the relationships between a cylinder's radius or height and its volume by seeing those quantities as independent and dependent variables.

Activity 1: Changing the Height (10 minutes)

The purpose of this activity is for students to understand what happens to a cylinder's volume when you hold the radius constant and change the height (MP1). Viewing that relationship as a function helps to unearth interesting patterns.

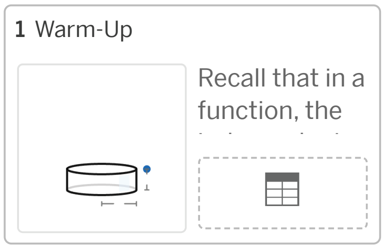
Activity 2: Changing the Radius (20 minutes)

The purpose of this activity is for students to understand what happens to a cylinder's volume when you hold the height constant and change the radius (MP1). As in the previous activity, students use functions to investigate this relationship. They come to see the radius-volume relationship in a cylinder as an example of a *nonlinear* function.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to compare and contrast the two relationships studied in this lesson. Students probe deeper to explain why the relationship between height and volume is linear [because only one dimension is changing], and why the relationship between radius and volume is not linear [because two dimensions are changing].

Cool-Down (5 minutes)



1 Warm-Up

Recall that in a function, the

Recall that in a function, the independent variable represents the input and the dependent variable represents the output.

In this situation, what are the independent variable and dependent variable?

Teacher Moves

Purpose
The purpose of this lesson is for students to use functions to explore how changing a cylinder's radius or height impacts its volume.

Warm-Up Launch
Tell students that their task in this warm-up is to apply what they've learned about independent and dependent variables to a situation involving cylinders. Consider demonstrating on the projector that students are able to drag the movable point, and invite them to notice how the cylinder changes as a result.

If students struggle to recall the difference between the independent and dependent variable, ask students: *What quantity can you change using the movable point? What quantity is changing as a result?*

Facilitation
Consider using pacing to restrict students to this screen.

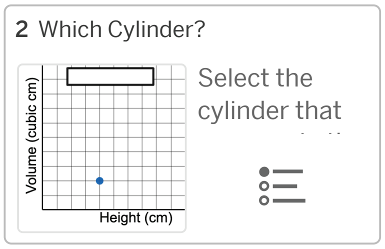
Sample Responses

Independent variable: Height of the cylinder
Dependent variable: Volume of the cylinder

Student Supports

Multilingual Learners
- *Expressive Language: Visual Aids*

Review an anchor chart that publicly displays the definition of independent and dependent variable to aid in explanations and reasoning.



2 Which Cylinder?

Select the cylinder that

Select the cylinder that represents the plotted point. Explain to a classmate how you chose.

Teacher Moves

Activity Launch
Tell students that this lesson is about making sense of the relationships between the radius, height, and volume of a cylinder. One way to investigate these relationships is to hold one quantity constant and investigate the relationship between the other two. In this activity, we will hold the radius constant. All of these cylinders have a radius of 5 centimeters.

Arrange students into pairs. Give students 2–3 minutes to choose a cylinder and explain their thinking. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Facilitation
Consider using pacing to restrict students to Screens 2–4.

Sample Responses

The upper-right cylinder, which has a radius of 5 centimeters and a height of 4 centimeters.

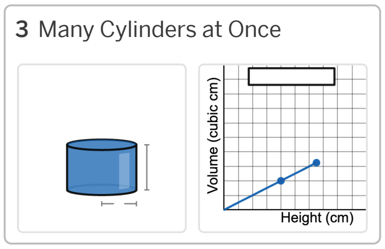
Student Supports

Students With Disabilities
- *Visual-Spatial Processing: Visual Aids*

Provide printed copies of the graph for students to draw on or highlight.

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., I chose this cylinder because ____________.).



3 Many Cylinders at Once

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to surface why the relationship between height and volume is linear.

Invite several students to share what they notice and wonder about the animation and graph. If it does not come up naturally, consider asking: *Why do you think that changing the height creates a linear function?*

If time allows, consider asking: *How can we use the equation for the volume of a cylinder to see the linear relationship?* [When a value is substituted in for r in the formula, what remains is the equation of a proportional relationship.]

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

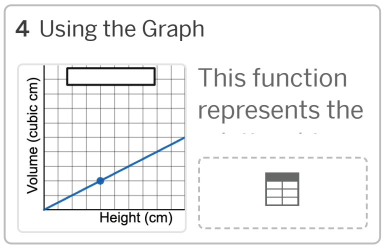
- I notice that as the height of the cylinder increases, the volume increases linearly.
- I notice that for every 2 centimeters of height, the cylinder adds 50π cubic centimeters of volume.
- I wonder if all cylinders have this linear pattern or just some.

Student Supports

Multilingual Learners

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., I notice ____________. I wonder _____________.).



4 Using the Graph

This function represents the

This function represents the relationship between the height and volume for cylinders with a radius of 5 centimeters.

Use the movable point and the table to help you find the volume of each of the four cylinders.

Express each volume in terms of π.

Teacher Moves

Teacher Moves
With the class paused, use the projector to show the student screen. Click on the movable point and drag it around. Ask the class what each point represents. [Each point represents the height and volume of one cylinder with a radius of 5 centimeters.]

Give students 3–4 minutes to complete the table and a couple minutes to share their responses with their partner. If students are struggling, consider asking them how they might use the graph or the patterns they notice in the table. For instance, ask students: *If we double the height, will the volume also double?* [Yes, because the relationship is proportional.]

Early Student Thinking
Students may have difficulty finding the volume for cylinder D, which has a height that's not visible in the graphing window. Consider asking: *How can we use the fact that this relationship is linear to help us find new volumes?* Also remind students that they could calculate the volume using methods they learned in the previous lesson.

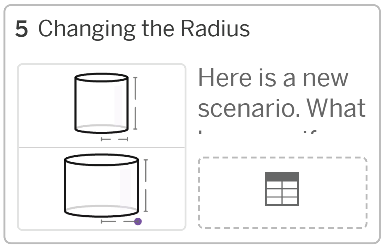
Sample Responses

Cylinder A: 50π
Cylinder B: 100π
Cylinder C: 200π
Cylinder D: 400π

Student Supports

Students With Disabilities
- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors, and allow students who struggle with fine motor skills to dictate where to trace on the graph.



5 Changing the Radius

Here is a new scenario. What

Here is a new scenario. What happens if we keep the height constant but change the radius?

Move the purple points to make two different cylinders that have a height of 10 centimeters.

Then calculate the volume for each. Express each volume in terms of π.

Teacher Moves

Activity Launch
Tell students that we will now change which quantity we hold constant and which we investigate. Here, we will explore the relationship between the radius and volume for all cylinders with a height of 10 centimeters.

Tell students to begin by choosing any value for each radius and then calculate the volume. Consider displaying the formula for the volume of a cylinder if students are struggling to recall it.

Facilitation
Consider using pacing to restrict students to Screens 5–9.

Sample Responses

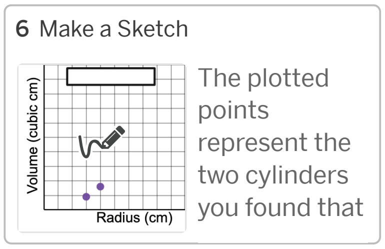
Responses vary.

- When radius is 4, volume is 160π.
- When radius is 7, volume is 490π.

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators throughout the lesson to ensure inclusive participation.



6 Make a Sketch

The plotted points represent the two cylinders you found that

The plotted points represent the two cylinders you found that have a height of 10 centimeters.

Make a sketch of what you think the graph looks like for ALL cylinders with a height of 10 centimeters.

Teacher Moves

Emphasize the range of student responses on this screen. It's okay—even desirable—to lack consensus at this stage. The next screen will reveal the graph of the relationship.

Early Student Thinking
Some students may think that the relationship between radius and volume is linear because the previous relationship was linear or because a line can be drawn through the two points they see.

Consider asking: *What would the volume be if the radius were 0 centimeters? Where would we plot that point on the graph?* [A cylinder with a radius of 0 centimeters would have a volume of 0 cubic centimeters and could be represented by the point $(0, 0)$. It isn't possible to draw a straight line through $(0, 0)$ and the other two points.]

Sample Responses

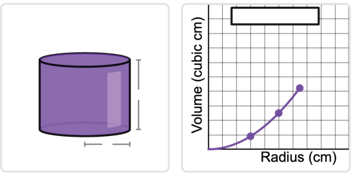
Responses vary.

Student Supports

Students With Disabilities
- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors, and allow students who struggle with fine motor skills to dictate how to create the sketch.

7 Many Cylinders at Once



Teacher Moves

Invite several students to share what they notice and wonder about the animation and graph. If it does not come up naturally, consider asking: *Why do you think that changing the radius creates a nonlinear function?*

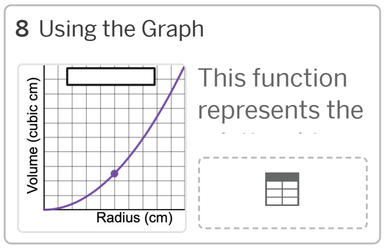
If time allows, consider asking: *How can we use the equation for the volume of a cylinder to see that the function is not linear?* [When a value is substituted in for h in the formula, what remains is *not* the equation of a linear function because the independent variable is squared.]

Note that if we substitute 10 for h in the cylinder volume formula, the result is $V = 10\pi r^2$. In later grades, students will learn that this is a quadratic function and that the curve is called a parabola.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

- I notice that the relationship between the radius and volume of the cylinder is not linear. I wonder why that is.
- I notice that the volume is 10π cubic centimeters when the radius is 1 centimeter, the volume is 40π cubic centimeters when the radius is 2 centimeters, and the volume is 90π cubic centimeters when the radius is 3 centimeters. I wonder how to predict the volume for radius values after that.



This function represents the relationship between the radius and volume for cylinders with a height of 10 centimeters.

Use the movable point and the table to help you find the volume of each of the four cylinders.

Express each volume in terms of π.

Teacher Moves

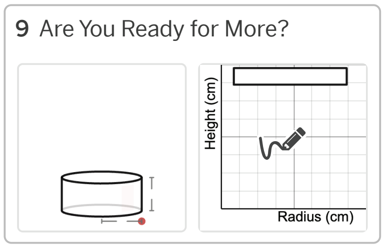
Give students 3–4 minutes to complete the table and a minute to share their responses with their partner. Encourage students to use the movable point to reveal coordinates.

Then follow with a whole-class discussion. If students are struggling, consider asking them how they might use the graph or the patterns they notice in the table. For instance, ask students: *If we double the radius, will the volume also double?* [No, the volume actually quadruples. This is because the radius is squared in the cylinder volume formula, so multiplying the radius by 2 multiplies the volume by 4, multiplying the radius by 3 multiplies the volume by 9, etc.]

Sample Responses

Cylinder E: 40π
Cylinder F: 160π
Cylinder G: 640π

Cylinder H: 2560π



9 Are You Ready for More?

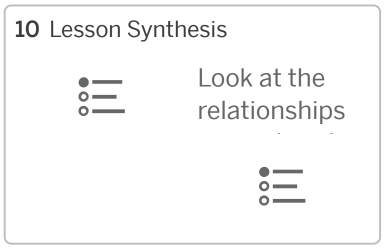
Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 5–8 ahead of time before the class discussion on Screen 10. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

- I notice that the relationship between the radius and height of a cylinder is not linear. I wonder why that is.
- I wonder if this graph will ever intersect the axes, or if not, how close it gets.
- I wonder if there is an equation that could create a graph of this relationship. I wonder how the graph would change if the fixed volume increased or decreased.



10 Lesson Synthesis

Look at the relationships

Look at the relationships we explored today.

Discuss the following questions.

Then select ONE of the questions and record your response.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface why one relationship is linear and the other is not.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with a partner. Consider displaying the formula for the volume of a cylinder for students to refer to.

Facilitate a whole-class discussion. Consider drawing connections to the formula (the radius is squared, but the height is not) or considering the situation visually. When a cylinder's height changes, only one dimension is changing; when a cylinder's radius changes, two dimensions are changing, and so the change to volume is greater.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

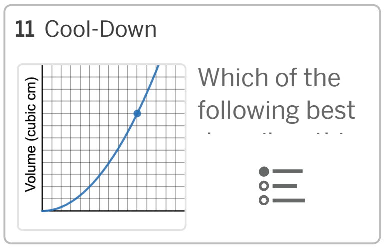
1. The relationship between height and volume is linear because when r is fixed, the formula to find the volume involves just multiplying a constant number by h. Also, if you double the height, it is almost as if there's a second cylinder on top of the original, so it makes sense that the volume would double.

2. The relationship between radius and volume is not linear because the volume formula involves squaring r. Relatedly, increasing the radius changes two dimensions, similar to the "length" and "width" in a rectangular prism. Because of this, doubling the radius ends up quadrupling the volume.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., ____________ is linear/nonlinear because ____________.).


11 Cool-Down

Which of the following best

Which of the following best describes this graph?

Teacher Moves

Support for Future Learning
If students struggle to analyze a graph of the relationship between two dimensions of a 3-D solid, consider making time to explicitly revisit these ideas. A strong understanding of which relationships are linear or nonlinear will support students on the end assessment.

Facilitation
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Radiuses and volumes of cylinders with a 8-cm height.

Responses vary.

The relationship between height and volume is linear, and this graph is not linear, so I knew it had to be a relationship between radius and volume. I noticed that the point $(10, 800\pi)$ was on the curve. That could represent a cylinder that has a radius of 10 and a height of 8, so I concluded that the graph was all cylinders with a height of 8 centimeters.

12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can analyze the relationship between the height or radius of a cylinder and its volume.
- I can explain why the relationship between height and volume is linear but the relationship between radius and volume is not.

GRADE 8

Unit 7
Lesson Plans

Teacher lesson plans from Unit 7 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

Grade 8 Unit 7

Teacher Edition Sampler

Unit at a Glance

Key

📖 **Print Lessons**

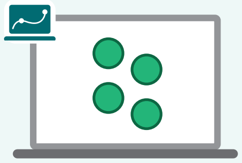
Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

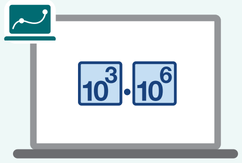
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



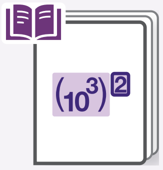
1 Circles

Write an exponential expression to describe repeated multiplication.



2 Combining Exponents

Identify equivalent exponential expressions (e.g. $6^3 \cdot 6^5$, $(6^4)^2$, and $2^8 \cdot 3^8$).



3 Power Pairs

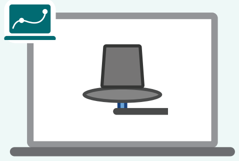
Justify that exponential expressions involving powers of powers and products of powers are equivalent.

Assess and Respond

Quiz: Sub-Unit 1

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.

Sub-Unit 2



7 Scales and Weights

Begin to represent large and small numbers using powers of 10.

8 Point Zapper

Represent large and small numbers as multiples of powers of 10 using number lines.

9 Use Your Powers

Apply powers of 10 and exponent rules to solve problems in context.

Practice Day

Practice Day 2

Practice the concepts and skills developed during Lessons 1–13. Consider using this time to prepare for the upcoming Quiz.

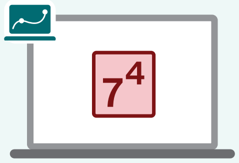
Summative Assessment

End-of-Unit Assessment

Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

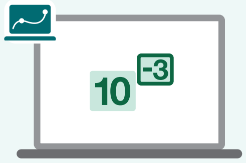
Pacing: 20 days | **Short on time?** See pacing considerations below.

Pre-Unit Check: (Optional)
15 Lessons: 45 min each
2 Practice Day: 45 min each
2 Sub-Unit Quizes: 45 min each
End-of-Unit Assessment: 45 min



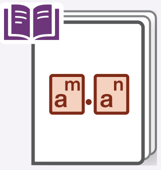
4 Rewriting Powers

Divide expressions involving exponents that have the same base.



5 Zero and Negative Exponents

Determine if two expressions involving positive, zero, and negative exponents are equivalent.



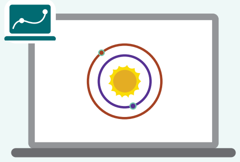
6 Write a Rule

Look for and generalize properties of exponents, including products of powers, quotients of powers, powers of powers, zero exponents, and negative exponents.

Practice Day

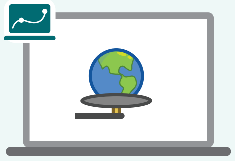
Practice Day 1

Practice the concepts and skills developed during Lessons 1–6. Consider using this time to prepare for the upcoming Quiz.



10 Solar System

Distinguish expressions written in scientific notation from expressions that are not written in scientific notation.

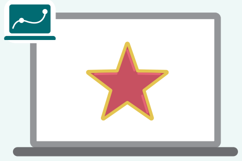


11 Balance the Scale

Compare very large or very small numbers using scientific notation.

12 City Lights

Add and subtract numbers in scientific notation to answer questions in context.



13 Star Power

Use adding, subtracting, multiplying, and dividing with scientific notation to compare quantities and answer questions in context.

Pacing Considerations

Lesson 1: The purpose of this lesson is for students to recall using whole number exponents to represent repeated multiplication in preparation for upcoming lessons. If students show a strong understanding of using exponents in Problems 3 and 6 of the Pre-Unit Check, this lesson may be omitted.

Lesson 3: This lesson supports students in developing fluency with identifying equivalent expressions using positive exponents. If students show a strong understanding identifying equivalent expressions with exponents in earlier lessons, this lesson may be omitted. If omitted, be sure to support students in justifying how they know expressions are equivalent elsewhere in the unit.

Lesson 9: This lesson gives students an opportunity to apply the concepts they learned about exponents to analyze a context in the world. If students show a strong understanding of working with powers of 10 in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how representing and working with numbers written in powers of 10 can empower us to better understand our world throughout the unit.

Lesson 13: This lesson gives students an opportunity to apply what they've learned about exponents and scientific notation to analyze and compare the net worth of different celebrities. There is no new content introduced in this lesson.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Circles

Lesson 1: Exponent Review

Overview

Students review the concepts of whole number exponents that they worked with in Grade 6. This work anticipates the extended study of exponent rules and scientific notation that continues throughout the unit.

Learning Goals

- Review exponential notation.
- Write an exponential expression to describe repeated multiplication.

Vocabulary

- exponent

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

This lesson reviews the concepts of whole number exponents that students worked with in Grade 6. This work anticipates the extended study of exponent rules and scientific notation that continues throughout the unit.

In the first part of the lesson, exponents are introduced to an animation depicting repeated doubling (MP8). The work here supports students in understanding 2^5 as the result of doubling five times or as *five factors of two*.

Later in the lesson, students think about large-numbered stages of tripling and reason about how they can use exponential notation to compare the relative sizes of numbers.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to do some repeated multiplication and recall how this process can be represented using exponents.

Activity 1: Doubling (20 minutes)

The purpose of this activity is to remind students of the need for—and utility of—exponential notation. In the early stages of the context, students are able to count the number of circles. But as the stages begin to increase, students are encouraged to look for patterns to help them find the number of circles for Stages 5 and 10. As the stages continue to increase, students are faced with the need for exponents as an efficient way to represent large numbers.

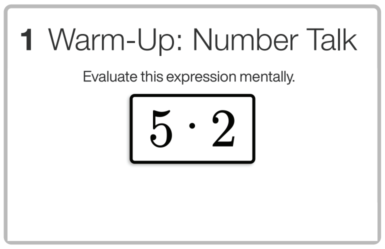
Activity 2: Tripling (10 minutes)

The purpose of this activity is for students to extend and generalize their use of exponential notation by applying exponents to a different pattern (MP8). Students will apply what they know about exponents to use the number of circles in Stage 5 to determine how many circles will be in Stage 8.

Lesson Synthesis (5 minutes)

The purpose of the synthesis discussion is for students to use their understanding that exponents indicate repeated multiplication and to use exponents to reason about a situation that involves repeated multiplication.

Cool-Down (5 minutes)



1 Warm-Up: Number Talk

Evaluate this expression mentally.

$5 \cdot 2$

Teacher Moves

Purpose
The purpose of this lesson is for students to review whole number exponent concepts.

Warm-Up Launch
Facilitate the Number Talk routine. Explain to students that you are going to display several expressions, and their task is to evaluate each expression mentally. Tell students that they will go over their solutions as a class after the final expression, and they should be prepared to share their strategy for evaluating each problem.

Give students a few moments of quiet think-time on each of Screens 1 through 4, and then follow with a whole-class discussion. Consider having students place a thumbs up on their chest to show you they have an answer and are ready for the next problem.

Facilitation
Consider using pacing to restrict students to Screens 1–4, one at a time.

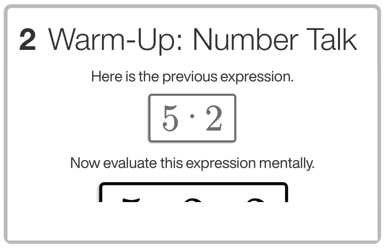
Sample Responses

10

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them with expressions to review prior to implementation of this activity.



2 Warm-Up: Number Talk

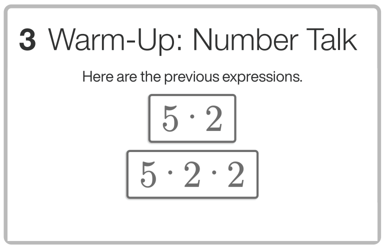
Here is the previous expression.

$5 \cdot 2$

Now evaluate this expression mentally.

Sample Responses

20



3 Warm-Up: Number Talk

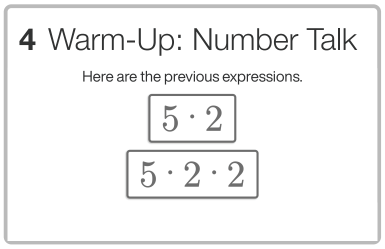
Here are the previous expressions.

$5 \cdot 2$

$5 \cdot 2 \cdot 2$

Sample Responses

40



4 Warm-Up: Number Talk

Here are the previous expressions.

$5 \cdot 2$

$5 \cdot 2 \cdot 2$

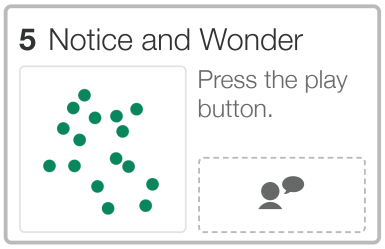
Teacher Moves

Once students have had enough time to evaluate the four expressions, facilitate a whole-class discussion. Ask students to share their solution for each expression. Record and display student solutions for all to see. Once all the solutions are displayed, invite students to explain their strategies for finding the solution. Elicit as many strategies as you can in the time you have.

During the discussion, listen for important ideas and terminology that will be helpful in upcoming work for the unit. When discussing the fourth problem, ask students if they recognize the notation and to explain what it means.

Sample Responses

80



5 Notice and Wonder

Press the play button.

Press the play button.

What do you notice is happening in the display? What do you wonder?

Teacher Moves

Activity Launch
Arrange students into pairs. Tell students that they are going to explore patterns with circles. Give them a few minutes of quiet time to think and write their responses. Encourage students to play the animation as many times as they need to understand the context.

Once students have recorded their responses, use snapshots to highlight and discuss several things that students noticed and wondered.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Notice and Wonder to support students in making sense of the task.

Sample Responses

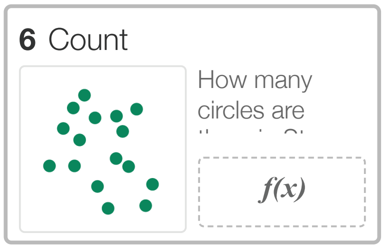
- I notice that the number of circles doubles with each stage.
- I wonder how many circles there will be at Stage 100.

Student Supports

Students With Disabilities

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



6 Count

How many circles are

How many circles are there in Stage 4?

Teacher Moves

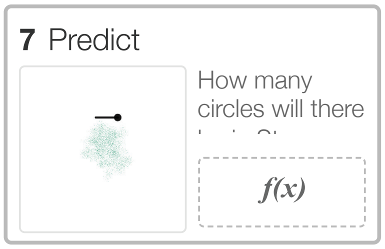
As time allows, invite students to describe how they see this pattern growing at each stage.

Facilitation

Consider using pacing to restrict students to Screens 6–8.

Sample Responses

16 circles



7 Predict

How many circles will there

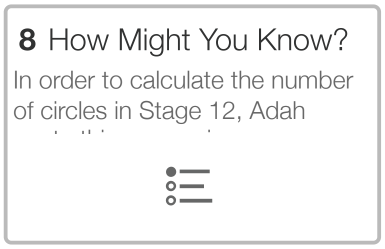
How many circles will there be in Stage 12?

Teacher Moves

Before giving students 2–3 minutes of quiet work time, consider showcasing that it's possible to enter numbers or expressions in the math input. Show that when entering an expression, the math input works like a calculator.

Sample Responses

4096 circles



8 How Might You Know?

In order to calculate the number of circles in Stage 12, Adah

In order to calculate the number of circles in Stage 12, Adah wrote this expression:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

Jamal wrote this expression:

$$2^{12}$$

Who wrote a correct expression?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to make a connection between repeated multiplication and expressions with exponents.

Highlight unique answers to show the class. Use snapshots to connect student responses on this screen with the expressions they entered on the previous screen. Ask students to justify their responses and critique each other's reasoning.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Both

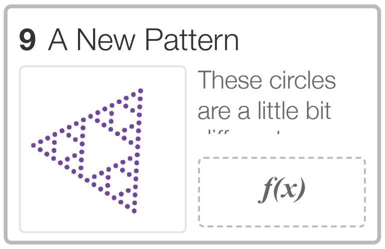
Explanations vary.

Both expressions are correct because $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ has the same value as 2^{12}.

Student Supports

Students With Disabilities
- *Receptive Language: Processing Time*

Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.



9 A New Pattern

These circles are a little bit

These circles are a little bit different.

How many circles are there in Stage 4?

Teacher Moves

Activity Launch
Arrange students into pairs. Explain to students that they are going to explore a new pattern. Give them a few minutes of quiet time to think and write their responses. Encourage students to play the animation as many times as they need to understand the context.

Consider inviting students to describe to a partner what they think Stage 5 will look like to help them think about how this pattern changes and to deepen their understanding of this pattern.

Once students have recorded their responses, use snapshots to highlight and discuss several things that students noticed and wondered.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

81 circles

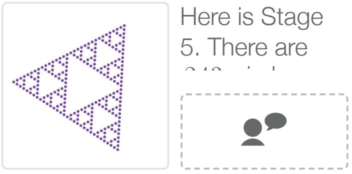
Responses vary.

I know because each stage has the number of circles in the previous stage multiplied by 3. Stage 1 has 3 circles, Stage 2 has 9 circles, Stage 3 has 27 circles, and Stage 4 has 81 circles.

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*
Pair students with their previously identified peer tutors.

10 Extend the Pattern


Here is Stage 5. There are

Here is Stage 5. There are 243 circles.

How can you use this fact to figure out how many circles there will be in Stage 7?

Teacher Moves

Use snapshots to highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

As time allows, consider asking, “How many times as large as Stage 5 is Stage 4?” [3 times as large.]

Facilitation
Consider using pacing to restrict students to Screens 10–11.

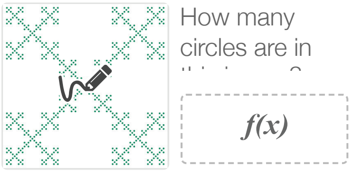
Sample Responses

Responses vary.

I can take the number of circles in Stage 5 and then multiply it by 3 two times since there are two stages between Stage 5 and Stage 7. So there are $3^2 = 9$ times more circles in Stage 7 compared to Stage 5.

---

11 Are You Ready for …


How many circles are in

How many circles are in this image?

Use the sketch tool if it helps you with your thinking.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

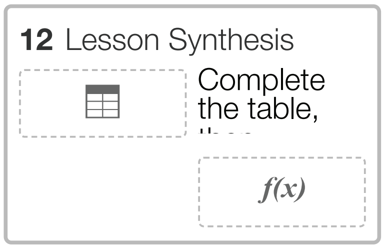
This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screen 10 ahead of time before the class discussion on Screen 12. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

625 circles

Responses vary.

I know because there are 25 circles in one “x” shape, and there are five of those in each of the bigger “x” shapes, so each of the five bigger “x” shapes has $25 \cdot 5 = 125$ circles. Since there are five of these in the image, there are $125 \cdot 5 = 625$ circles total.



12 Lesson Synthesis

Complete the table,

Complete the table, then answer this question:

2^6 is how many times as large as 2^4?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for determining how many times larger one expression is than another when they have the same base but different exponents.

Synthesis Launch
Give students 2–3 minutes to fill in the table, respond to the question, and share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

As time allows, ask students, “How does the expanded form in the table help you see how many times as large 2^6 is than 2^4?”

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

4

Table:

Row 1: $2 \cdot 2$, 2^2, 4

Row 2: $2 \cdot 2 \cdot 2 \cdot 2$, 2^4, 16

Row 3: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, 2^6, 64

Student Supports

English Language Learners

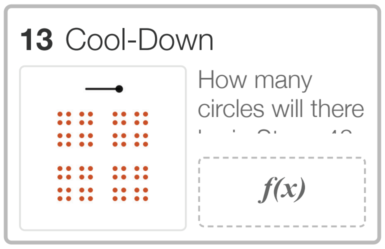
- *Expressive Language: Visual Aids*

Create or review an anchor chart that publicly displays the table on this screen with the conclusion that 2^6 is 4 times as large as 2^4 for students to use to aid in explanations and reasoning throughout the unit.

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time. This may include reading the information in the table and in the note.



13 Cool-Down

How many circles will there

f(x)

How many circles will there be in Stage 4?

Teacher Moves

Support for Future Learning

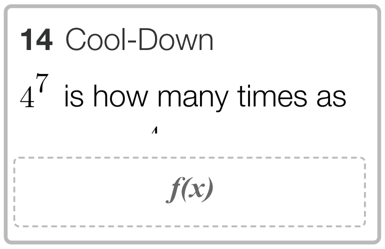
Students will have more chances to develop their understanding of situations that involve repeated multiplication in the upcoming lessons, particularly in Lesson 2–4.

Facilitation

Consider using pacing to restrict students to Screens 13–15.

Sample Responses

256 (or equivalent)



14 Cool-Down

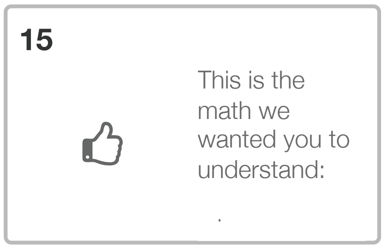
4^7 is how many times as

f(x)

4^7 is how many times as large as 4^4?

Sample Responses

64 times as large (or equivalent)



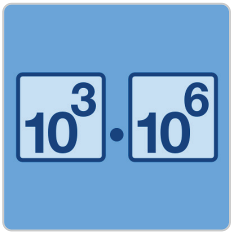
15

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use exponents to describe repeated multiplication.
- I can explain the meaning of an expression with an exponent.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.



Combining Exponents

Lesson 2: Equivalent Expressions With Exponents

Overview

Students look for and make use of structure to discover ways to write equivalent exponential expressions involving the product of powers and powers of powers (MP7). At this time, students only work with positive exponents. In subsequent lessons, students will extend the exponent properties to cases where the exponents are zero or negative.

Learning Goals

- Identify equivalent exponential expressions (e.g., $6^3 \cdot 6^5$, $(6^4)^2$, and $2^8 \cdot 3^8$).
- Write equivalent expressions involving the product of powers and powers of powers.

Materials

- Blank paper

Vocabulary

- base of an exponent
- power of ten

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students look for and make use of structure to discover ways to write equivalent exponential expressions involving the product of powers and powers of powers (MP7). At this time, students only work with positive exponents. In subsequent lessons, students will extend the exponent properties to cases where the exponents are zero or negative.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to notice and describe similarities and differences between equivalent expressions written in several forms.

Activity 1: Combining Exponents (15 minutes)

The purpose of this activity is for students to consider different ways they can work with exponents when using products of powers and powers of powers.

Activity 2: Odd One Out (15 minutes)

The purpose of this activity is for students to reason about expressions with exponents (MP2). While some students may find the numeric value of each expression in order to make a comparison, it is not always necessary. This activity establishes the need for a more efficient way to compare expressions with exponents.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to summarize their understanding of a way to combine exponents.

Cool-Down (5 minutes)

1 Warm-Up

Which one doesn't belong?

Which one doesn't belong?

Teacher Moves

Purpose
The purpose of this lesson is for students to identify and write equivalent expressions involving the product of powers and powers of powers.

Warm-Up Launch
Ask students to select one expression they think does not belong and explain why. Give students one minute of quiet think-time. Once they have made a selection and explained their thinking, consider inviting students to find one or more reasons why *each* expression does not belong.

Teacher Moves
For each expression, select one student to explain their reasoning. Draw out reasons for each, attending to appropriate vocabulary and precise use of language (MP6). During the discussion, if the words *power*, *exponent*, and *base* do not come up naturally, consider reviewing those terms with students.

After the discussion, ask students which expression they think has the greatest value. Students may notice that the expressions are all equivalent.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Which One Doesn't Belong to support students in noticing the features of each representation.

Sample Responses

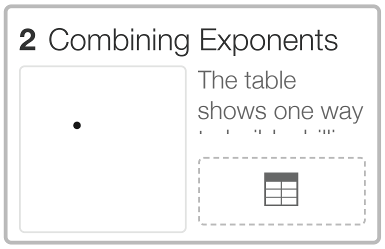
Responses vary.

- **Upper left:** The only expression with parentheses.
- **Upper right:** The only expression that contains numbers with and without exponents.
- **Lower left:** The only expression that is the product of two numbers.
- **Lower right:** The longest expression.

Student Supports

Multilingual Learners
- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., ____________ doesn't belong because _____________.).



2 Combining Exponents

The table shows one way

The table shows one way to build a billion (10^9): by multiplying two powers of 10.

Drag the movable point to find other ways.

Then write your expressions in the table.

Teacher Moves

Activity Launch
Ask students to drag the movable point and observe what happens in the diagram. What changes and what stays the same? What do they notice? What do they wonder?

Give students 2–3 minutes of quiet work time. Highlight several unique student responses for the class. If no one notices that the red and blue exponents always add up to nine, draw attention to this fact.

Facilitation
Consider using pacing to restrict students to Screens 2–4, one screen at a time.

Sample Responses

Responses vary.

- $10^7 \cdot 10^2$
- $10^4 \cdot 10^5$
- $10^0 \cdot 10^9$

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time* and *Visual-Spatial Processing: Visual Aids*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity and allow them to draw on or highlight the images during the activity.

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.

3 Card Sort

Teacher Moves

Arrange students into pairs. Ask them to take turns sorting the cards by putting one card into a category and then explaining their reasoning. After both partners have agreed on the placement of that card, the other partner should repeat these steps. Repeat this process until all cards are sorted.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies. Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions and ideas your students will have.

Early Student Thinking
Students may struggle to determine the placement of some of the cards. Remind students about the meaning of exponents from the previous lesson and from earlier screens in this lesson.

Sample Responses

Image solution

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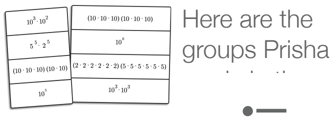
4 Reflection



Here are the groups Prisha made in the card sort.

Prisha has a new card with the expression $\left(10^3\right)^2$.

In which group should they place this card?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for determining if $\left(10^3\right)^2$ is equal to 10^5 or 10^6.

Highlight unique answers for the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Right group

Responses vary.

- $\left(10^3\right)^2$ means there are two groups of three 10 s, which is the same as six 10 s.

---

5 Challenge #1

Two of these expressions are equivalent.

Two of these expressions are equivalent.

Which expression is not equivalent to the others?

Teacher Moves

Activity Launch
Arrange students into pairs. Provide each student with scratch paper. Explain to the class that on each of the next three screens, there will be three exponential expressions. The goal is to identify the expression that is not equivalent to the others. Encourage students to discuss their strategy with a partner and to show their thinking on paper.

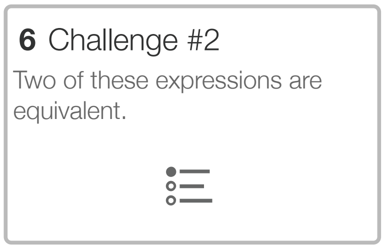
Facilitation
Consider using pacing to restrict students to Screens 5–9.

Sample Responses

$3+3+3+3+3$

Responses vary.

If I change the pluses (+) to dots ($\cdot$), the expressions would all have the same value.



6 Challenge #2

Two of these expressions are equivalent.

Two of these expressions are equivalent.

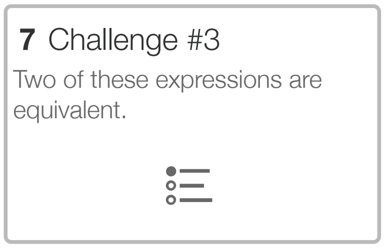
Which expression is not equivalent to the others?

Sample Responses

$(5 \cdot 3) \cdot (4 \cdot 3)$

Responses vary.

If I change the expression to $\left(5^3\right) \cdot \left(4^3\right)$, the expressions would all have the same value.



7 Challenge #3

Two of these expressions are equivalent.

Two of these expressions are equivalent.

Which expression is not equivalent to the others?

Sample Responses

6^2

Responses vary.

If I change the expression to 2^6, the expressions would all have the same value.

8 One More Set

Here are the equivalent expressions from the challenges,

Here are the equivalent expressions from the challenges, plus one more.

For each row, write one more expression that has the same value.

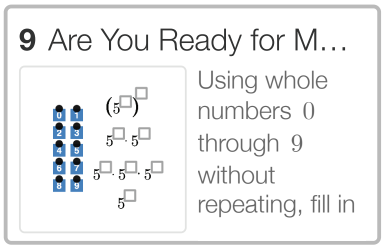
Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Responses vary.

- $3^2 \cdot 3^3$
- $5^3 \cdot 4^3$
- 2^6
- $(5^2)^9$



9 Are You Ready for M…

Using whole numbers 0 through 9 without repeating, fill in

Using whole numbers 0 through 9 without repeating, fill in the blanks to create four equivalent expressions.

Teacher Moves

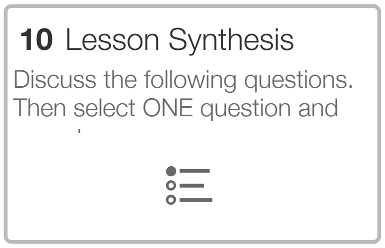
⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 5–8 ahead of time before the class discussion on Screen 10. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.

$(5^2)^4$
$5^1 \cdot 5^7$
$5^3 \cdot 5^5 \cdot 5^0$
5^8



10 Lesson Synthesis

Discuss the following questions. Then select ONE question and

Discuss the following questions. Then select ONE question and record your response.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to summarize three properties of exponents. Students will investigate these properties more over the coming lessons.

Synthesis Launch

Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- I can tell that $3^4 \cdot 3^2 = 3^6$ because each side has six factors of 3.
- I can tell that $3^5 \cdot 4^5 = 12^5$ because I can rearrange the factors of $3^5 \cdot 4^5$ and have five pairs of $3 \cdot 4$, which is the same as 12^5.
- I can tell that $\left(3^2\right)^4 = 3^8$ because four groups of two 3s is the same as eight 3s.

---

11 Cool-Down
Which expressions are equivalent to 8^6?

Which expressions are equivalent to 8^6?

Teacher Moves

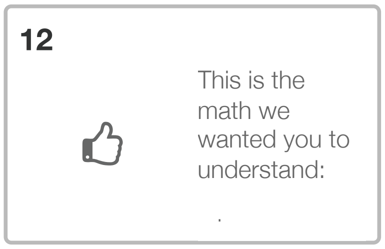
Support for Future Learning
If students struggle with writing equivalent exponential expressions, plan to emphasize this when opportunities arise over the next several lessons. For example, plan to spend extra time discussing the relationship between the exponential expressions used in Lesson 3's warm-up.

Readiness Check (Problem 3)
If most students struggled, consider giving them time to review and revise their response after this lesson or as part of the cool-down.

Facilitation
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

- $(8^3)^2$
- $2^6 \cdot 4^6$



12

This is the math we wanted you to understand:

.

This is the math we wanted you to understand:

- I can describe what it means for two expressions with exponents to be equivalent.
- I can create equivalent expressions with exponents.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Power Pairs (NYC)

Lesson 3: Multiplying Powers and Powers of Powers

Purpose

The purpose of this lesson is for students to develop fluency writing and identifying equivalent expressions written as a single power, as a power to a power, and as the product of powers.

Preparation

Worksheet

- *Power Pairs Score Sheet*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Cards

- Print and cut out one set of cards for each group of 2–4 students.

Warm-Up (5 minutes)

The purpose of the warm-up is for students to recall what they've learned about exponent properties so that they can employ these properties in the upcoming activity.

Warm-Up Launch
Facilitate the Number Talk routine. Explain to students that you are going to display several expressions, and their task is to evaluate each expression mentally. Tell students that they will go over their solutions as a class after the final expression, and they should be prepared to share their strategy for evaluating each problem.

Give students a few moments of quiet think-time for each expression, then follow with a whole-class discussion. Record and display student explanations for all to see. Elicit as many strategies as you can in the time you have.

Facilitation
Display Screens 1–4 of the teacher projection sheets, one sheet at a time.

Teacher Moves
Ask students to explain their strategies for evaluating each problem. Consider asking some of the following questions to focus the conversation on the common exponents:

- What connections do you see between the expressions?
- Is there a way to tell just by looking at the expressions that they would be equal? How?

Early Student Thinking
Some students may not recall that exponents represent repeated multiplication. Remind students that the base represents the number that is multiplied repeatedly, and the exponent represents the number of factors.

Readiness Check (Problem 7)
If most students struggled, consider making connections between writing equivalent fractions and writing equivalent expressions involving exponents as it comes up throughout this lesson.

Support for Students With Disabilities
Memory: Processing Time
Provide sticky notes or mini whiteboards to aid students with working memory challenges.

Activity: Power Pairs (30 minutes)

Activity Launch
Arrange students into groups of 2–4. Distribute one set of 20 cards per group, and provide each student with a Power Pairs Score Sheet—tell students to keep the set of cards face down. Explain to students that in this activity, they are going to play a card game where they match up equivalent expressions. Display Page 5 of the teacher projection sheets as you explain the game.

Here's how the game works:

- Students shuffle the cards, then lay 12 cards face up in a 3x4 grid. The remaining cards should stay face down in a pile.
- Students take turns going clockwise. One student picks two face-up cards that have the same value. When they identify a pair of matching cards, they must show the cards to the group and convince them that the cards are equivalent.
- The group has to agree that the pair of cards has the same value in order to complete each turn. If the cards do not have the same value, the student should put the cards back down and try again.
- Each student writes the pair of cards down on their score sheet.
- Remove the paired cards and replace them with new cards from the pile.
- Repeat the same steps with each turn.

Ending the game:

- When no more equivalent pairs can be made, each student takes one card and writes their own equivalent expression for it on their score sheet.

Teacher Moves
As time allows, students can play additional rounds of the game, using a new score sheet each time.

Remind students to get consensus from the group that their selected pair does contain equivalent expressions before continuing to the next player. Emphasize that the goal is accuracy for the whole group, not finishing quickly.
Circulate as groups play the game. To involve more students in the conversation, consider asking these questions as students share their justifications:

- Can you explain why you chose your strategy?

- Can anyone restate _________ 's reasoning in a different way?
- Did anyone reason about the problem in the same way but can explain it differently?
- Did anyone reason about the problem in a different way?
- Does anyone want to add on to __________ 's strategy?
- Do you agree or disagree? Why?

Support for Students With Disabilities

Conceptual Processing: Eliminate Barriers

Demonstrate the steps for the activity or game by having a group of students and staff play an example round while the rest of the class observes.

Memory: Processing Time

Provide sticky notes or mini whiteboards to aid students with working memory challenges.

Lesson Synthesis (5 minutes)

Arrange students into pairs. Distribute one double-sided half sheet of the lesson synthesis and cool-down to each student.

Give students 2–3 minutes to respond to the lesson synthesis question, followed by a few minutes to share their responses with their partner and a whole-class discussion.

After the discussion, ask students to complete the cool-down individually on their worksheet.

Cool-Down (5 minutes)

Students will have more chances to develop their understanding of equivalent exponential expressions that use powers of powers and products of powers in the upcoming lessons, particularly in Lesson 4 and 6.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Rewriting Powers

Lesson 4: Rewriting Exponential Expressions as a Single Power

Overview

In previous lessons, students worked with multiplication of powers and powers of powers. In this lesson, students will continue working with those kinds of expressions as well as with a new kind of expression: division of powers. The emphasis of this lesson is not on identifying and then memorizing rules, but rather on making sense of the structure of expressions, identifying parts of those expressions that can be written in a simpler form (e.g., writing $\frac{5}{5}$ as 1).

Learning Goals

- Divide expressions involving exponents that have the same base.
- Rewrite products of powers, quotients of powers, and powers of powers as single powers.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop strategies for rewriting exponential expressions as a single power. In previous lessons, students worked with multiplication of powers and powers of powers. In this lesson, students will continue working with those kinds of expressions as well as with a new kind of expression: division of powers. The emphasis of this lesson is not on identifying and then memorizing rules, but rather on making sense of the structure of expressions, identifying parts of those expressions that can be written in a simpler form (e.g., writing $\frac{5}{5}$ as 1).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to invite students to begin thinking about the structure and relative simplicity of several related expressions involving exponents.

Activity 1: Rewriting as a Single Power (10 minutes)

In this activity, students develop and practice strategies for rewriting exponential expressions as a single power. They then put those strategies to use in a card sort to determine which expressions can be rewritten as a specific single power and which cannot.

Activity 2: Writing Equivalent Expressions (20 minutes)

In this activity, students practice writing expressions that can be rewritten as a specific single power. Students will see a target expression, and they must write a more complicated, yet equivalent, expression involving exponents.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to summarize the general strategies students developed and used throughout the lesson to rewrite expressions as single powers.

Cool-Down (5 minutes)


1 Warm-Up

Sort the expressions based on what you think is simplest to

Sort the expressions based on what you think is simplest to most complicated.

Teacher Moves

Purpose
The purpose of this lesson is for students to develop strategies for rewriting exponential expressions as a single power.

Warm-Up Launch
Tell students that they will be working with equivalent expressions written in several different ways, and that you are curious which ones they think are simplest and which ones they think are most complicated.

Give students 1–2 minutes of quiet think-time. Then use the teacher view of the dashboard to display several unique answers to show the class. Invite students to explain their thinking.

Teacher Moves

Students will likely come to consensus on the “simplest” expression (7^4), but will likely **not** come to consensus on much or all of the remaining order. That is okay. The goal for this screen is not to formally define “simplest” or “most complicated” but to invite students to think about the structure of several related expressions involving exponents.

Consider asking:
- Why might someone want a simpler expression?" [To be able to write it more quickly. To be able to compare it more easily to other expressions and numbers.]
- Why might someone want a more complicated expression? [To see the structure of the expression.]

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

2.
Cc
a
cla
thɛ
7^5

Convince a classmate that $\frac{7^5 \cdot 7^2}{7^3}$ can be rewritten as the single power 7^4.

Use paper and pencil if that helps you with your thinking.

Teacher Moves

Activity Launch
The purpose of this activity is for students to develop and practice strategies for rewriting exponential expressions as a single power.

Tell students that their task on this screen is to convince a classmate that one of the more complicated expressions from the previous screen, $\frac{7^5 \cdot 7^2}{7^3}$, can be rewritten as a single power, 7^4.

Arrange students into pairs. Give students 2–3 minutes to create an argument. Encourage them to use paper and pencil to support their thinking and explanation.

Use the teacher view of the dashboard (or your own observations as you circulate through the classroom) to identify and share several unique responses.

Early Student Thinking

Some students may struggle to construct a viable argument for why $\frac{7^5 \cdot 7^2}{7^3}$ can be rewritten as the single power 7^4. If needed, consider pausing the activity in order to work as a class to rewrite a similar expression (e.g., $\frac{6^4 \cdot 6^3}{6^5}$) as a single power. Then unpause the activity and invite students to complete the task on Screen 2.

Facilitation
Consider using pacing to restrict students to Screens 2–5.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary.

I wrote out all the factors of 7 and looked for things that can be rewritten as 1. Then I rewrote the remaining factors using exponential notation.

$$\frac{7^5 \cdot 7^2}{7^3} = \frac{(7\cdot7\cdot7\cdot7\cdot7)\cdot(7\cdot7)}{7\cdot7\cdot7} = \frac{7}{7}\cdot\frac{7}{7}\cdot\frac{7}{7}\cdot7\cdot7\cdot7\cdot7 = 1\cdot1\cdot1\cdot7\cdot7\cdot7\cdot7 = 7^4$$

Student Supports

Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Assist students in recognizing the connections between new problems and prior work. Students may benefit from a review of different representations to activate prior knowledge.

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the expressions to review prior to implementation of this activity.

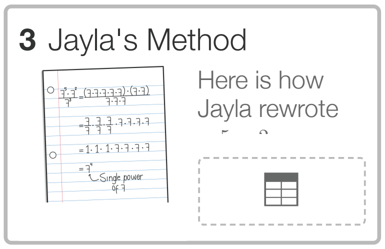
Multilingual Learners

- *Collect and Display*

Circulate and listen to students talk during pair work or group work. Jot down notes about common or important words and phrases (e.g., *exponent*, *base*, *power*, *factor*, *equivalent*), together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.

- *Expressive Language: Eliminate Barriers*

Provide sentence frames to help students explain their reasoning (e.g., $\frac{7^5 \cdot 7^2}{7^3}$ can be rewritten as 7^4 because _____________.).



Here is how Jayla rewrote $\frac{7^5 \cdot 7^2}{7^3}$ as a single power.

Complete the table by rewriting $\frac{4^5}{4^2}$ as a single power.

Then press "Check My Work."

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to make sense of one strategy for dividing exponent expressions with the same base.

Early Student Thinking

On Screen 3 and Screen 4, some students may ask about shortcuts or express a desire for a more efficient method. Encourage students to look for patterns, but *refrain from giving them any of your own shortcuts* for rewriting these expressions as single powers.

For students who have trouble getting started, encourage them to write out all the factors and look for things that simplify, either because they are equal to 1 (e.g., $\frac{7}{7}$) or because they can be rewritten as another single, simple number (e.g., $2 \cdot 3 = 6$ and $\frac{6}{2} = 3$).

Sample Responses

4^3

---

4 Complete the Table

The table shows your work from the previous screen.

The table shows your work from the previous screen.

Complete the table. Then press "Check My Work."

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

- 2^6
- 6^4
- 3^7

---

5 Card Sort

Teacher Moves

Arrange students into pairs. Give students five minutes of quiet work time, followed by a partner discussion and then a whole-class discussion focused on the strategies students used to sort the cards. Encourage students to use a whiteboard or paper to help them with their thinking.

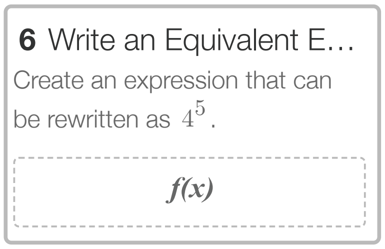
Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Sample Responses

Image solution



6 Write an Equivalent E…

Create an expression that can be rewritten as 4^5.

Create an expression that can be rewritten as 4^5.

Write something unique and as complicated as you want!

Teacher Moves

Activity Launch
Tell students that their task on this screen is to create an expression that can be rewritten as a specific single power. (In other words, they must write an equivalent expression.) Encourage students to write something as unique and as complicated as they can.

On the upcoming Challenge Creator, students will create their own single-power targets and challenge their classmates to write expressions that can be rewritten as those single powers.

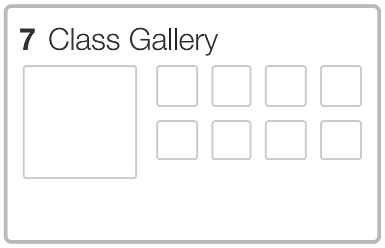
Teacher Moves
Use the teacher view of the dashboard to snapshot several unique answers. Consider snapshotting two expressions that are correct and two that are not. Add those snapshots to a single collection, display the collection to the class, and ask, "Which of these expressions can be rewritten as 4^5? Explain how you know." Encourage students to construct arguments and critique the reasoning of others. Invite students to write on the board if that helps them express their thinking to the class.

Facilitation
Consider using pacing to restrict students to Screens 6–7, one screen at a time.

Sample Responses

Responses vary.

- $\frac{4^8}{4^3}$
- $4^2 \cdot 4^3$
- $\frac{\left(4^3\right)^2}{4^1}$
- $2^5 \cdot 2^5$



7 Class Gallery

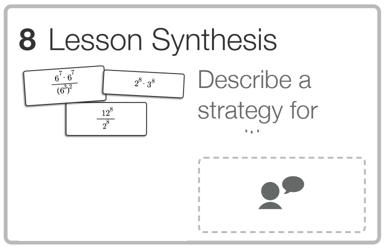
Teacher Moves

Here students will create their own challenge and solve challenges from their classmates. We recommend students complete the previous screen before creating their challenge. We anticipate this Challenge Creator could take 20 minutes or more.

Encourage students to complete each other's challenges but also to take some time to review responses to their own. Use the teacher dashboard to look for unique challenges and solutions that may expand your students' understanding of the mathematics.

Highlight those responses for students, and ask them what they learned from the experience.

We intend for this to be a social and creative experience. We encourage you to emphasize those virtues whenever you see them in your class.



8 Lesson Synthesis

Describe a strategy for ...

Describe a strategy for rewriting an expression as a single power.

Refer to one of the expressions on the left if it helps you explain your thinking.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to summarize the general strategies that students developed and used throughout the lesson to rewrite expressions as single powers.

Synthesis Launch
Give students 2–3 minutes to respond to this question and one minute to share their strategies with a classmate. Highlight unique strategies to

show the class. Consider asking students to identify similarities and differences between the different strategies.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

First, write out all the exponents using repeated multiplication. Then look for things that can be combined (e.g., $2 \cdot 3 = 6$) or rewritten as 1 (e.g., $\frac{12}{12} = 1$). At the end, rewrite everything you can using exponents again.

---

9 Cool-Down
Select each expression that can be rewritten as 8^8.

Select each expression that can be rewritten as 8^8.

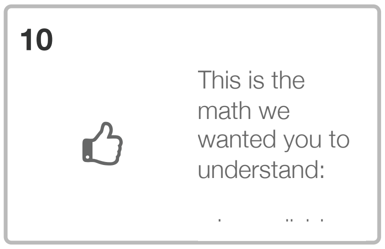
Teacher Moves

Support for Future Learning
Students will have more chances to develop their understanding of equivalent exponential expressions that use division of powers in the upcoming lessons, particularly in Lesson 6 and Practice Day 1.

Facilitation
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

- $2^8 \cdot 4^8$
- $\frac{8^{10}}{8^2}$
- $\frac{8^2 \cdot 8^3 \cdot 8^4}{8^1}$



10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can divide expressions with exponents that have the same base.
- I can rewrite expressions with positive exponents as a single power.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Zero and Negative Exponents

Lesson 5: Using Patterns to Understand Zero and Negative Exponents

Overview

Students develop an understanding of the meaning of zero and negative exponents.

Learning Goals

- Understand and justify that $a^0 = 1$ and that $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$.
- Determine if two expressions involving positive, zero, and negative exponents are equivalent.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is for students to develop an understanding of the meaning of zero and negative exponents. In this lesson, students will use their current understanding of positive exponents to rewrite several exponential expressions in expanded form. They will then identify and use patterns—organized in a table—to reason about zero and negative exponents (MP8).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to reason about the relative value of several different expressions involving powers of 2 and quotients of powers of 2. Teachers and students may find it helpful to refer back to these original expressions as they consider the structure of and look for patterns within the expressions they encounter later in the lesson.

Activity 1: Zero and Negative Exponents (30 minutes)

In this activity, students explore patterns with exponential expressions by considering their exponent form, expanded form, and value (MP7). Students begin by considering powers of 10, and then apply their understanding to similar expressions involving powers of 3.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to express an argument that a non-zero base raised to the power of zero (e.g., 6^0) has a value of 1.

Cool-Down (5 minutes)

1 Warm-Up

Order the expressions by value from least to greatest.

Order the expressions by value from least to greatest.

Teacher Moves

Purpose

The purpose of this lesson is for students to develop an understanding of the meaning of zero and negative exponents.

Warm-Up Launch

Tell students that their task is to order the expressions from least to greatest. They do not need to determine the value of any individual expression (though they are welcome to if they find that helpful). Instead, encourage them to consider the value of each expression relative to the rest.

Give students 2–3 minutes to reorder the expressions. Then use the teacher view of the dashboard to show the distribution of responses and ask students how they decided where to place each expression. Invite students to critique and respond to those reasons.

If it doesn't come up naturally, consider encouraging students to think about the value of $\frac{2^5}{2^5}$ and how they know whether the other expressions are greater than or less than $\frac{2^5}{2^5}$.

Facilitation

Consider using pacing to restrict students to this screen.

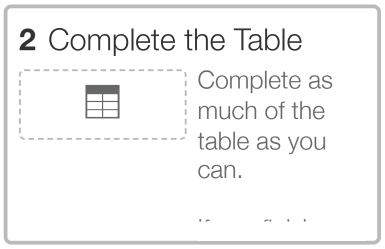
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

From least to greatest:

- 0
- $\frac{2^3}{2^4}$
- $\frac{2^5}{2^5}$

- $\frac{2^6}{2^3}$
- 2^6



2 Complete the Table

Complete as much of the table as you can.

Complete as much of the table as you can.

If you finish early, discuss your answers with a classmate.

Teacher Moves

Activity Launch
Tell students that their task in this activity is to explore the meaning of zero and negative exponents by considering patterns in tables.

Teacher Moves
Here is the recommended flow for Screens 2–3:

- Invite students to complete as much of the table on Screen 2 as they can.
- Ask students to describe any patterns they see on Screen 3.
- Use the teacher view of the dashboard to facilitate a class discussion around their Screen 3 responses.
- After the class discussion, consider inviting students to return to Screen 2 to update their table based on what they learned during the discussion.

Facilitation
Consider using pacing to restrict students to Screens 2–3.

Sample Responses

Responses vary.

Row 1: $10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10\,000$

Row 2: $10^3 = 10 \cdot 10 \cdot 10 = 1\,000$

Row 3: $10^2 = 10 \cdot 10 = 100$

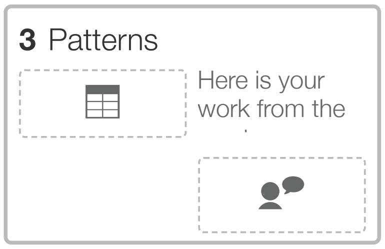
Row 4: $10^1 = 10 = 10$

Row 5: $10^0 = 1 = 1$

Row 6: $10^{-1} = \frac{1}{10} = 0.1$

Row 7: $10^{-2} = \frac{1}{10 \cdot 10} = 0.01$ or $10^{-1} = \frac{1}{10} \cdot \frac{1}{10} = 0.01$

The two options for 10^{-2} are not meant to be an exhaustive list of correct answers, but to highlight the fact that different expressions can be considered correct.



3 Patterns

Here is your work from the

Here is your work from the previous screen.

What patterns do you see in the table?

Describe as many as you can.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to surface properties of zero exponents and negative exponents.

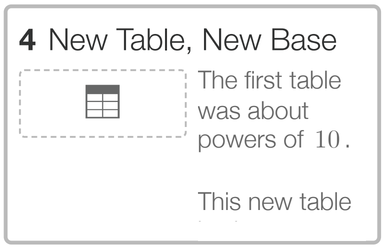
Teacher Moves
Here is the recommended flow for Screens 2–3:

- Invite students to complete as much of the table on Screen 2 as they can.
- Ask students to describe any patterns they see on Screen 3.
- Use the teacher view of the dashboard to facilitate a class discussion around their Screen 3 responses.
- After the class discussion, consider inviting students to return to Screen 2 to update their table based on what they learned during the discussion.

Sample Responses

Responses vary.

- As you move down the table, the exponents decrease by 1 for each row.
- Positive exponents describe the number of factors of 10.
- Negative exponents describe the number of factors of 10 in the denominator.
- Negative exponents describe the number of factors of $\frac{1}{10}$.
- As you move down the table, the values get closer and closer to 0.
- There is a form of mirror symmetry in each column of the table.



4 New Table, New Base

The first table was about powers of 10.

This new table

The first table was about powers of 10.

This new table is about powers of 3.

Complete the table. Then press "Check My Work."

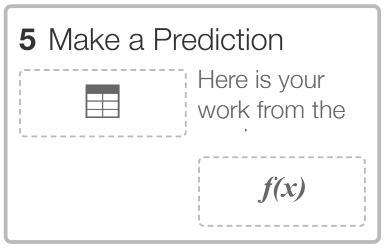
Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Facilitation
Consider using pacing to restrict students to Screens 4–7.

Sample Responses

Row 2: 3^3
Row 5: 1 (or equivalent)
Row 6: 3^{-1}
Row 7: $\frac{1}{3 \cdot 3}$ or $\frac{1}{3} \cdot \frac{1}{3}$
Row 8: $\frac{1}{3 \cdot 3 \cdot 3}$ or $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$ (or equivalent)
Row 9: $\frac{1}{3 \cdot 3 \cdot 3 \cdot 3}$ or $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$ (or equivalent)



5 Make a Prediction

Here is your work from the

Here is your work from the previous screen.

The value of $3^6 = 729$.

Predict what 3^{-6} equals.

Teacher Moves

Use snapshots to select and display several unique answers for the class. Ask, “Which of these expressions are equivalent to 3^{-6}?” Ask students to justify their responses and critique each other's reasoning.

Early Student Thinking

Some students may claim that $3^{-6} = -729$. Consider asking, "Which values in the table are greater than zero and which are less than zero?" [All the values are greater than zero.] Also ask, "Based on what you see in the table, do you expect that 3^{-6} will be positive or negative?" [Positive.]

Sample Responses

Responses vary.

- $3^{-6} = \frac{1}{729}$
- $3^{-6} = \frac{1}{3^6}$
- $3^{-6} = \left(\frac{1}{3}\right)^6$
- $3^{-6} = 0.001371742$

---

6 Card Sort

Teacher Moves

Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Sample Responses

Image solution

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7 Are You Ready for M…

On paper, write as many different expressions that are equivalent to 2^{-4} as you can.

On paper, write as many different expressions that are equivalent to 2^{-4} as you can.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 4–6 ahead of time before the class discussion on Screen 8. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Early Student Thinking

If students ask whether a particular expression is equivalent to 2^{-4}, encourage them to enter 2^{-4} and the expression in question into a scientific calculator, and then to compare the results. Since $2^{-4} = 0.0625$ and $\frac{1}{2^4} = 0.0625$, we can conclude that $2^{-4} = \frac{1}{2^4}$.

Sample Responses

Responses vary.

Here is a non-exhaustive list:

- $\frac{1}{2^4}$
- $\frac{1}{2 \cdot 2 \cdot 2 \cdot 2}$
- $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
- $\left(\frac{1}{2}\right)^4$

---

8 Lesson Synthesis

How could you convince someone that $6^0 = 1$?

How could you convince someone that $6^0 = 1$?

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is for students to express an argument that a non-zero base raised to the power of zero has a value of 1.

Synthesis Launch

Give students 2–3 minutes to respond to this question and one minute to share their strategies with a classmate. Highlight unique arguments to show the class.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

- $\frac{6^3}{6^3} = 6^0$ and $\frac{6^3}{6^3} = 1$. Therefore, $6^0 = 1$.
- $6^4 \cdot 6^0 = 6^{(4+0)} = 6^4$. Since multiplying by 6^0 didn't change the value, $6^0 = 1$.
- $\frac{6^5}{6^0} = 6^{(5-0)} = 6^5$. Since dividing by 6^0 didn't change the value, $6^0 = 1$.
- $6^3 = 216$, $6^2 = 36$, and $6^1 = 6$. Each time the exponent decreases by 1 , the value is divided by 6 . Based on this pattern, and the fact that $6^1 = 6$, we know that $6^0 = 1$.

---

9 Cool-Down

Select each expression that is equivalent to 10^{-6}.

Select each expression that is equivalent to 10^{-6}.

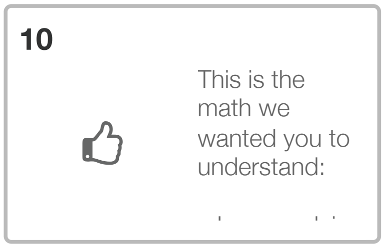
Teacher Moves

Support for Future Learning
If students struggle to develop an understanding of zero and negative exponents, consider making time to explicitly revisit these ideas. Some opportunities include revisiting Screens 2 and 4 of this lesson. A strong understanding of zero and negative exponents will support students in the upcoming practice day and quiz assessment.

Facilitation
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

- $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$
- $\frac{10^3}{10^9}$



10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain what it means for a number to be raised to a zero or a negative exponent.
- I can determine if two expressions with positive, zero, and negative exponents are equivalent.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Write a Rule (NYC)

Lesson 6: Generalizing Exponent Properties

Purpose

The purpose of this lesson is for students to develop rules for rewriting exponential expressions.

Preparation

Worksheet

- *Activity*: Print one double-sided sheet for each student.
- *Lesson Synthesis and Cool-Down*: Print one double-sided half sheet for each student.

Cards

- Print and cut out one single-sided copy of the cards for every group of students.

Warm-Up: Card Sort (15 minutes)

Warm-Up Launch
Arrange students into groups of 3–4. Tell students that their goal is to sort the cards so that each grouping of cards shares a similar characteristic. Give students 6–8 minutes to complete the card sort, and follow with a whole-class discussion.

Teacher Moves
Invite students to share one of their groupings with the class. Ask them to justify their responses and critique each other's reasoning. Consider using the student preview of the teacher projection screen to project the card sort and to help you facilitate the class discussion.

Before moving on to the next activity, students should be able to articulate their reasoning for each grouping and justify each card's placement.

Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Early Student Thinking
At first, some students may over generalize the cards (i.e., group all cards with a common base or all cards with negative exponents together). Invite students to think about ways to form smaller groupings by identifying differences and commonalities other than a common base. Consider asking, “How are the cards within the grouping different from one another?” [Some cards have multiplication and other cards have division.]

Support for Students With Disabilities

Social-Emotional Functioning: Peer Tutors

Pair students with their previously identified peer tutors.

Conceptual Processing: Processing Time

For students who benefit from extra processing time, provide them the card sort images to review prior to implementation of this activity.

Activity 1: Write a Rule (25 minutes)

Activity Launch

Distribute a double-sided worksheet to each student. Tell students that their goal for this activity is to write their own rule for each of the groupings from the card sort.

For each grouping of cards, students will write the example(s) from the cards. Then they will create their own example, write a rule, and explain or show how they know their rule will always work.

Teacher Moves

Use successive pair shares to provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. Students will improve their writing when they are given multiple opportunities to clarify their explanations through conversation.

Invite students to choose one grouping of cards and individually draft a description of the rule. Then ask students to share their responses with a person in their group.

For each round, give students 2–3 minutes to share their response with a partner. Students should first check to see if they agree with each other. Then, they will spend a few minutes revising their response before moving to the next partner within their group.

If time is short, consider using a jigsaw approach for writing the rules: Assign each group of students 1 or 2 groupings of cards to write a rule for. Then, invite each group of students to share their rule(s) with the class.

Early Student Thinking

If students have difficulty writing productive rules, consider displaying one grouping of equations from the card sort for the class to see. Ask students, "How does one side of the equation relate to the other? If we didn't want to expand the expressions, what might be a rule that would help us simplify exponent expressions like these?"

Some students may find that they are unable to describe a grouping with a single rule that expresses the patterns present, or they are unable to determine which grouping a new example might fall under. Invite these students to revise their groupings. Ask students to identify one pattern. Then have them articulate a rule just for that pattern.

Support for Students With Disabilities
Conceptual Processing: Eliminate Barriers
Demonstrate the steps for the activity by having a group of students and staff play an example round while the rest of the class observes.

Executive Functioning: Visual Aids
Create an anchor chart publicly displaying the exponential rules for future reference.

Lesson Synthesis (5 minutes)

Synthesis Launch
The purpose of the lesson synthesis is for students to solidify the learning goals of this lesson and reflect on strategies used when working with exponential expressions.

Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion.

Cool-Down (5 minutes)

If students struggle to generalize properties of exponents, consider reviewing these concepts as a class before the practice day, or offering individual support where needed during the practice day.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

8.7 Practice Day 1 (NYC)

Preparation

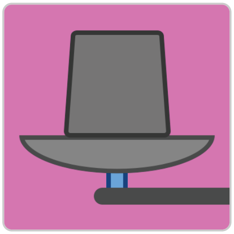
Practice Worksheet

- Print one double-sided sheet for each student.

Instructions

Invite students to complete this activity individually or in pairs. Tell students that there are four parts to this activity. As they complete each part, students should get their work checked. Consider posting the answer key, or walking around with it to provide feedback. To help keep track of which parts you have reviewed, consider using a stamp to indicate that you've checked each part of students' worksheets.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Scales and Weights

Lesson 7: Describing Large and Small Numbers Using Powers of 10

Overview

Students specify how many numbers of various weights are needed to balance objects on a scale.

Learning Goals

- Begin to represent large and small numbers using powers of 10 (e.g., $3\,500 = 3 \cdot 10^3 + 5 \cdot 10^2$).

Materials

- Blank or graph paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students specify how many numbers of various weights are needed to balance objects on a scale. Each weight is a number of kilograms that is a power of 10 (e.g., 10^3 kg or 10^{-4} kg). Students are encouraged to consider various ways in which each mass can be decomposed in order to understand the idea that any number can be expressed as a sum of multiples of powers of 10. Later in the unit, scientific notation will be introduced as a special case of this idea, where only one power of 10 is used and the multiple is a value of at least 1 but less than 10.

Lesson Summary

Warm-Up (10 minutes)

The purpose of the warm-up is to orient students to the context of the scale. On this screen, the weights are written in familiar standard notation, but on later screens, they will be expressed as powers of 10.

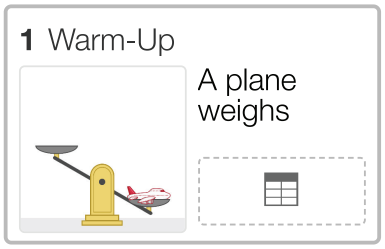
Activity 1: Scales and Weights (25 minutes)

The purpose of this activity is to introduce the phrase and the concept of "multiple of a power of 10," which is essential for the introduction of scientific notation later in the unit. In this activity, students practice various ways of expressing and decomposing numbers using multiples of powers of 10.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to solidify the idea of writing numbers using multiples of powers of 10, and specifically, what it means to write numbers using a single multiple of a power of 10.

Cool-Down (5 minutes)



1 Warm-Up

A plane weighs

A plane weighs $320\,000$ kilograms.

Update the weights in the table to balance the scale.

Teacher Moves

Purpose
The purpose of this lesson is for students to practice expressing and decomposing numbers using multiples of powers of 10, which is an essential skill for the introduction of scientific notation later in the unit.

Warm-Up Launch
Tell students that in this activity they will add weights of different sizes to a scale in order to balance an object. If students aren't sure where to start, encourage them to change one of the numbers in the table and watch the animation.

Readiness Check (Problem 1)
If most students struggled, plan to review this question after this lesson. Consider inviting students to share how writing numbers using powers of 10 might be helpful.

Facilitation
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

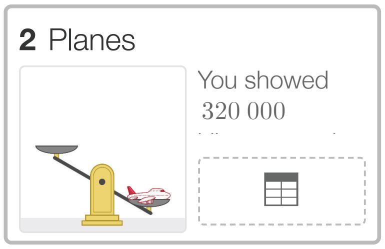
Responses vary.

- 3 weights of size $100\,000$ kg and 2 weights of size $10\,000$ kg
- 3 weights of size $100\,000$ kg and 20 weights of size $1\,000$ kg
- 320 weights of size $1\,000$ kg

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Begin with a demonstration of how the scale works to provide access to students who benefit from clear and explicit instructions. Check in with individual students, as needed, to assess for comprehension during each step of the activity.



2 Planes

You showed $320\ 000$

You showed $320\ 000$ kilograms using 3 hundred thousand kilogram weights and 2 ten thousand kilogram weights.

Now show $320\ 000$ kilograms using a different combination of weights.

Teacher Moves

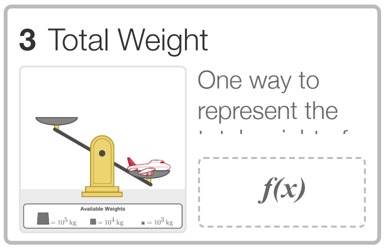
Tell students that there are many combinations of weights that can be used to balance the scales in this activity. Encourage students to try multiple combinations and to describe their strategy for finding new combinations.

If there are combinations of weights that students haven't tried, consider using the student view of the dashboard to display some of these possibilities and asking "Do you think this combination will work?" In particular, be sure to show students that it is not necessary to use whole numbers of weights.

Sample Responses

Responses vary.

- 3 weights of size $100\,000$ kg and 2 weights of size $10\,000$ kg
- 3 weights of size $100\,000$ kg and 20 weights of size $1\,000$ kg
- 3.2 weights of size $100\,000$ kg



3 Total Weight

One way to represent the

One way to represent the total weight of the plane is by using multiples of powers of 10, as shown below:

$$3 \cdot 10^5 + 2 \cdot 10^4$$

Enter the total weight of the plane ($320\,000$ kilograms) using a different combination of the weights shown in the diagram.

Write your answer using multiples of powers of 10.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface several ways of using powers of 10 to express the same value and to come to consensus about how to determine the value of expressions written in this form.

Activity Launch
The purpose of this activity is to transition students to expressing numbers using a single multiple of a power of ten.

Tell students that we can express the total weight of an object by multiplying the number of each type of weight by its weight, and then by combining the totals for each type of weight. Each time we break the weight down in this way, we are using multiples of powers of 10 to express the weight. For example, if we use 3 weights of size 10^3 lbs., we can write the total weight as $3 \cdot 10^3$ lbs.

Highlight unique answers for the class. Ask students to justify their responses and critique each other's reasoning. Consider taking snapshots of several student responses and asking the class:
- “How many weights of each size does this represent?”
- "Do you think this combination of weights will work?"
- "Can we use part of a weight or do we have to use a whole number of each type of weight?"

Facilitation
Consider using pacing to restrict students to Screens 3–6.

Sample Responses

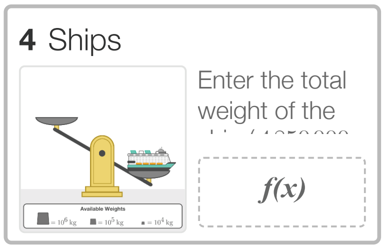
Responses vary.

- $32 \cdot 10^4$ kg
- $320 \cdot 10^3$ kg

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*
Pair students with their previously identified peer tutors.

- *Conceptual Processing- Eliminate Barriers*
Allow students to use calculators to ensure inclusive participation in the activity.



4 Ships

Enter the total weight of the

Enter the total weight of the ship ($4\,850\,000$ kilograms) using the weights shown in the diagram.

Write your answer using multiples of powers of 10.

Teacher Moves

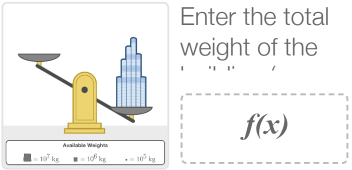
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Responses vary.

- $4 \cdot 10^6 + 8 \cdot 10^5 + 5 \cdot 10^4$ kg
- $485 \cdot 10^4$ kg
- $48.5 \cdot 10^5$ kg

5 Buildings


Enter the total weight of the

Enter the total weight of the building ($450\,000\,000$ kilograms) using ONLY ONE KIND of the weights shown in the diagram.

Write your answer using a single multiple of a power of 10.

Teacher Moves

Use the teacher view in the teacher dashboard to identify students who may need additional support.

Consider using snapshots to highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

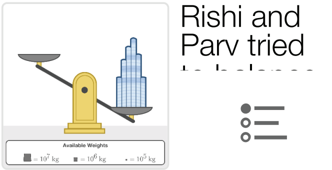
Sample Responses

Responses vary.

- $45 \cdot 10^7$ kg
- $450 \cdot 10^6$ kg
- $4\,500 \cdot 10^5$ kg

---

6 Reflect


Rishi and Parv tried

Rishi and Parv tried to balance the scale using different sizes of weights.

Rishi wrote the total weight of the building as $45 \cdot 10^7$.

Parv wrote the total weight of the building as $4\,500 \cdot 10^5$.

Who will balance the scale?

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

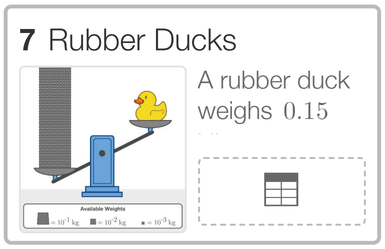
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Rishi and Parv are both correct. Note: Students will be marked correct on this screen for selecting either Rishi or Parv, or for selecting the "Both" option.

Responses vary.

I know that Rishi balanced the scale because $45 \cdot 10^7 = 45 \cdot 10\,000\,000 = 450\,000\,000$, which equals the weight of the building. I know Parv balanced the scale because $4\,500 \cdot 10^5 = 4\,500 \cdot 100\,000 = 450\,000\,000$, which also equals the weight of the building.



7 Rubber Ducks

A rubber duck weighs 0.15

A rubber duck weighs 0.15 kilograms.

Enter the number of each available weight in the table to balance the scale.

Teacher Moves

Tell students that now they are going to explore much smaller weights. This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking

Students may need some support with interpreting the negative exponents in the context of this problem. If this is the case, help students make the connection between negative exponents and place value by reminding them that 10^{-1} is the same as $\frac{1}{10}$ or 0.1, etc.

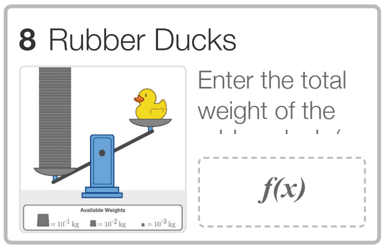
Facilitation

Consider using pacing to restrict students to Screens 7–9.

Sample Responses

Responses vary.

- 15 weights of size 10^{-2} kg
- 1 weight of size 10^{-1} kg and 5 weights of size 10^{-2} kg
- 150 weights of size 10^{-3} kg


8 Rubber Ducks

Enter the total weight of the

Enter the total weight of the rubber duck (0.15 kilograms) using ONLY ONE KIND of the weights shown in the diagram.

Write your answer using a single multiple of a power of 10.

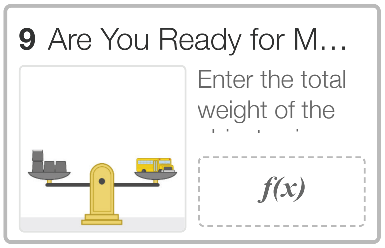
Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Response vary.

- $1.5 \cdot 10^{-1}$ kg
- $15 \cdot 10^{-2}$ kg
- $150 \cdot 10^{-3}$ kg


9 Are You Ready for M…

Enter the total weight of the

Enter the total weight of the object using ONLY ONE KIND of the weights shown in the diagram.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving additional practice problems to students who finish Screens 7–8 ahead of time before the class discussion on Screen 10. Students can access additional practice problems by pressing "Next Problem" after getting a problem correct. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

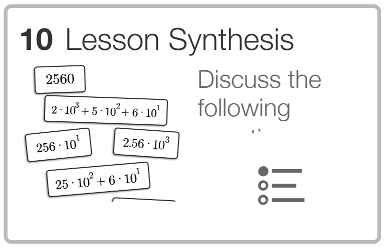
Responses vary.

Student Supports

Students With Disabilities

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



10 Lesson Synthesis

Discuss the following

2560

$2 \cdot 10^3 + 5 \cdot 10^2 + 6 \cdot 10^1$

$256 \cdot 10^1$

$2.56 \cdot 10^3$

$25 \cdot 10^2 + 6 \cdot 10^1$

Discuss the following questions. Then select ONE question and record your response.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to solidify the idea of writing numbers using multiples of powers of 10, and specifically what it means to write a number using a single multiple of a power of 10.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Facilitation
Consider using pacing to restrict students to this screen.

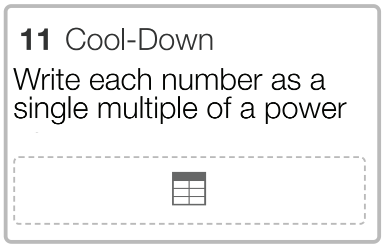
Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

1. Writing a number as multiples of powers of 10 means that multiply one or more numbers by a power of ten and add the results together so that the result is the original number.

2. Writing a number as a single multiple of a power of ten means that I multiply one number by a power of ten so that the result is the original number.



11 Cool-Down

Write each number as a single multiple of a power

Write each number as a single multiple of a power of 10.

Teacher Moves

Support for Future Learning
If students struggle with writing numbers as a single power of 10, consider spending extra time during the warm-up of the next lesson

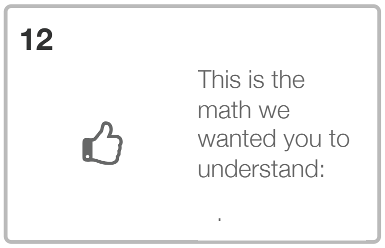
discussing how students decided to plot their point on the number line, or reviewing this cool-down as a class.

Facilitation
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Responses vary.

- $123 \cdot 10^{-6}$
- $123 \cdot 10^{6}$



12

This is the math we wanted you to understand:

.

This is the math we wanted you to understand:

- I can represent large and small numbers as multiples of powers of 10.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

Point Zapper

Lesson 8: Representing Large and Small Numbers on the Number Line

Overview

Students use number lines to represent large and small numbers as multiples of powers of 10. The skills students develop over the course of this lesson—especially writing a given number as a multiple of a power of 10 in multiple ways—will prepare them for formalizing and applying scientific notation (MP7).

Learning Goals

- Represent large and small numbers as multiples of powers of 10 using number lines.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students use number lines to represent large and small numbers as multiples of powers of 10. The skills students develop over the course of this lesson—especially writing a given number as a multiple of a power of 10 in multiple ways—will prepare them for formalizing and applying scientific notation (MP7).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to re-engage with number lines. In particular, students will experience a lack of precision on one number line and the benefit of a zoomed-in number line for plotting a large number more accurately (MP6).

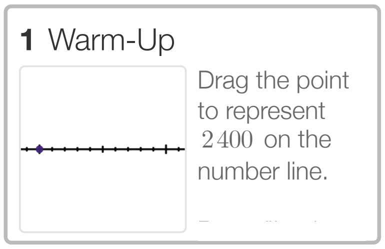
Activity 1: Point Zapper (30 minutes)

In this activity, students complete a series of challenges where they are shown a point representing very large or very small number on a zoomed-in number line, and they must enter its value. The endpoints on the number lines are expressed as single-digit multiples of powers of 10, which invite (but do not require) students to enter their responses as multiples of powers of 10.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to strengthen their understanding of the zoomed-in number lines used throughout this lesson by identifying one or more labeling mistakes on a given number line.

Cool-Down (5 minutes)



1 Warm-Up

Drag the point to represent 2400 on the number line.

Drag the point to represent 2400 on the number line.

Press "Lock Point" when you're ready.

Teacher Moves

Purpose
The purpose of this lesson is for students to use number lines to represent large and small numbers as multiples of powers of 10.

Warm-Up Launch
Tell students that their task on this screen is to plot 2400 on the number line. After they press “Lock Point,” they will be able to see their classmates’ points.

Emphasize the range of student responses on this screen. Let students know that it's okay to lack consensus at this stage.

Teacher Moves
Give students one minute to plot their point and view their classmates’ points. Then use pacing to advance everyone to Screen 2.

Readiness Check (Problems 5 and 6)
If most students struggled, consider reviewing these problems here. Invite students to share how they know where each number belongs on the number line. If it does not come up naturally, discuss the size of the intervals on the number line in Problem 5.

Facilitation
Consider using pacing to restrict students to Screens 1–2.

Student Supports

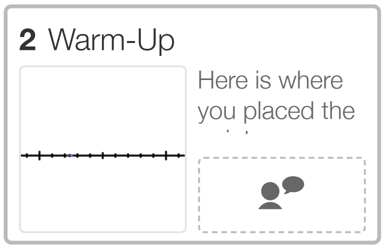
Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation.

- *Receptive Language: Processing Time*

Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time.



2 Warm-Up

Here is where you placed the

Here is where you placed the point on Screen 1.

1. Press "Zoom."

2. Adjust the point so it represents 2400.

3. Describe how the two number lines are connected.

Teacher Moves

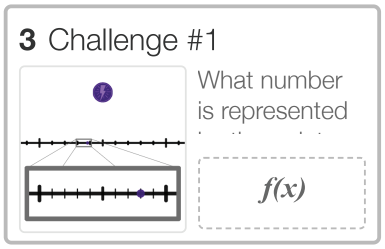
Give one minute of quiet think-time. Then ask students to discuss with a partner. Invite several students to share their responses.

Sample Responses

Image solution

Responses vary.

The purple points represent the same number. The zoomed-in number line just shows more detail.



3 Challenge #1

What number is represented

What number is represented by the point on the number line?

Teacher Moves

Activity Launch
Tell students that their task in this activity is to look at a zoomed-in number line and determine what number is represented by the point.

Teacher Moves
Consider using the student view in the dashboard to show students the type of feedback they'll receive when they submit an answer. Challenge students to get the correct answer in as few tries as possible by reflecting carefully on the feedback they receive after each attempt.

Facilitation
Consider using pacing to restrict students to Screens 3–8.

Sample Responses

$3.8 \cdot 10^7$ (or equivalent)

Student Supports

Students With Disabilities

- *Fine Motor Skills: Peer Tutors*

Pair students with their previously identified peer tutors, and allow students who struggle with fine motor skills to dictate physical manipulation of points as needed.

Multilingual Learners

- *Expressive Language: Visual Aids*

Circulate and listen to students talk during pair work or group work. Jot down notes about common or important words and phrases, together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.

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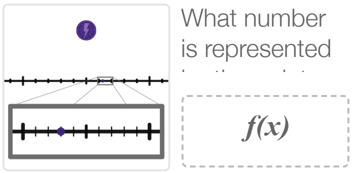
4 Challenge #2



What number is represented

What number is represented by the point on the number line?

Teacher Moves

Before students begin working on this second challenge, consider asking, "Is the number on this number line very large or very small? How do you know?" [The number is very small because it is between 0 and 10^{-3}, which is equal to $\frac{1}{10 \cdot 10 \cdot 10} = 0.001$.]

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

$6.3 \cdot 10^{-4}$ (or equivalent)

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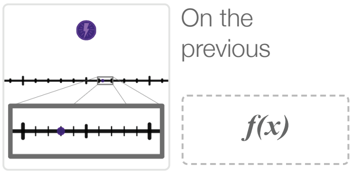
5 Challenge #2 Revisited



On the previous

On the previous screen, you wrote $6.3 \cdot 10^{-4}$ to represent the point shown in the diagram.

What is another way to write this number?

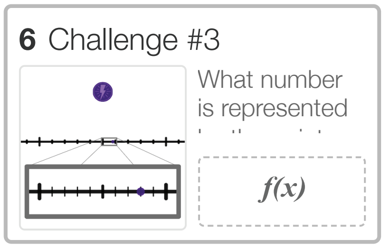
Teacher Moves

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

- $63 \cdot 10^{-5}$
- $630 \cdot 10^{-6}$
- $0.00063 \cdot 10^{0}$
- 0.00063



6 Challenge #3

What number is represented

What number is represented by the point on the number line?

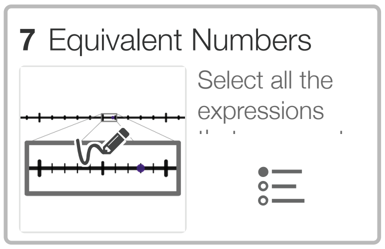
Teacher Moves

This screen presents a slightly different challenge compared to earlier screens: the zoomed-in number line does not have any labels. Consider inviting students to make a quick sketch on paper—with labels on the zoomed-in number line.

Early Student Thinking
Some students may struggle to identify the correct endpoint labels for the zoomed-in number line. Consider offering the following hint: “The labels 0 and 10^8 on the top number line can be rewritten as $0 \cdot 10^7$ and $10 \cdot 10^7$. What does that tell you about the other tick marks on the top number line?” [The other tick marks can be labeled as $1 \cdot 10^7$, $2 \cdot 10^7$, $3 \cdot 10^7$, and so on.]

Sample Responses

$5.8 \cdot 10^7$ (or equivalent)



7 Equivalent Numbers

Select all the expressions

Select all the expressions that represent the number shown on the number line diagram.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to come to consensus about how to represent the same number using different powers of 10.

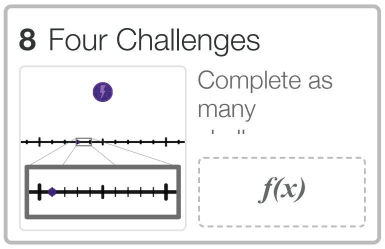
Once most students have completed this screen, consider pausing the class to facilitate a discussion. Use the teacher view of the dashboard to show the distribution of responses. Ask students to justify their responses and critique each other's reasoning.

Spend adequate time here coming to consensus. A strong understanding of equivalent ways to represent a number using powers of 10 will serve students well later in the lesson and throughout the rest of the unit.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

- $5.8 \cdot 10^7$
- $58 \cdot 10^6$
- $580 \cdot 10^5$



8 Four Challenges

Complete as many

Complete as many challenges as you can!

Enter the number shown on the number line diagram :

1. As a multiple of 10^{-5}
2. As a multiple of 10^{-7}
3. As a multiple of 10^{-2}
4. As a multiple of 10^{0}

Teacher Moves

Invite students to work on their own or with a partner. Encourage them to complete as many of the four challenges as they can.

When most students have completed one or more of the challenges, consider pausing the class to facilitate a discussion. Ask, “What strategies did use to complete the challenges? What patterns did you notice?”

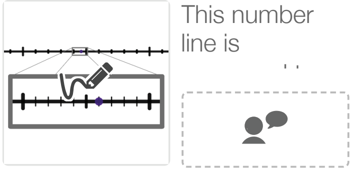
Sample Responses

1. $3.1 \cdot 10^{-5}$

2. $310 \cdot 10^{-7}$
3. $0.0031 \cdot 10^{-2}$
4. $0.000031 \cdot 10^{0}$ or 0.000031

---

9 Lesson Synthesis



This number line is

This number line is supposed to represent $4.6 \cdot 10^{-3}$, but two of the labels include mistakes.

Identify at least one mistake and describe how to fix it.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to solidify how to write a point on a number line as a power of 10.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Facilitation
Consider using pacing to restrict students to this screen.

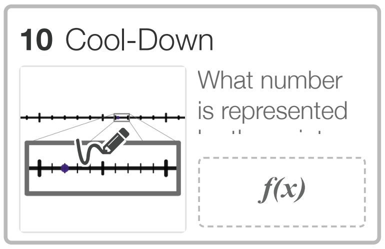
Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

- The label on the right end of the top number line is incorrect. Instead of 10^{-3}, it should be 10^{-2} (because $10^{-2} = 0.01$ is larger than $10^{-3} = 0.001$).

- The label on the right end of the zoomed-in number line is incorrect. Instead of $6 \cdot 10^{-3}$, it should be $5 \cdot 10^{-3}$ because the left and right ends of the zoomed-in number line correspond to the 4th and 5th tick marks on the top number line.



10 Cool-Down

What number is represented

What number is represented by the point on the number line?

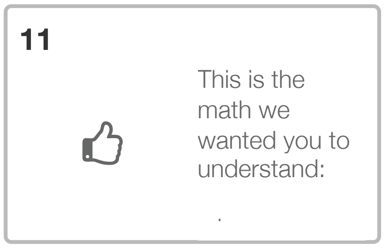
Teacher Moves

Support for Future Learning
Students will have more chances to develop their understanding of how to represent multiples of powers of 10 in the upcoming lessons, particularly Lesson 9.

Facilitation
Consider using pacing to restrict students to Screens 10–11.

Sample Responses

$6.2 \cdot 10^{-8}$ (or equivalent)



11

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can represent large and small numbers as multiples of powers of 10 using number lines.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Use Your Powers

Lesson 9: Applications of Arithmetic With Powers of 10

Overview

Students apply what they've learned about working with exponents (especially powers of 10) to answer challenging questions in context.

Note: This is a digital lesson that includes an activity on Screen 2 where students create visual displays.

Learning Goals

- Apply powers of 10 and exponent rules to solve problems in context.

Materials

- Tools for creating a visual display
- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Students apply what they've learned about working with exponents (especially powers of 10) to answer challenging questions in context. The style of questioning requires students to identify essential features of the problem and to persevere in order to calculate and interpret the solutions in context (MP1). Students must attend to precision when considering appropriate units of measurement (MP6).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to remind students of the exponent rules derived earlier in the unit, which they will apply in the next activity.

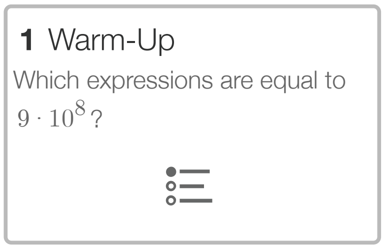
Activity 1: Select a Scenario (30 minutes)

The purpose of this activity is for students to work with and interpret very large quantities in context using powers of 10. The large quantities involved in these questions lend themselves to arithmetic with powers of 10, giving students the opportunity to develop fluency working with powers of 10 before scientific notation is formally introduced. This activity was designed so students could practice modeling skills, such as identifying essential features of the problem and gathering the required information (MP4). Students use powers of 10 and the number line as tools to make it easier to calculate and interpret results.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to solidify the learning goals of this lesson and reflect on strategies used when working with powers of 10.

Cool-Down (5 minutes)



1 Warm-Up

Which expressions are equal to $9 \cdot 10^8$?

Which expressions are equal to $9 \cdot 10^8$?

Teacher Moves

Purpose
The purpose of this lesson is for students to apply what they've learned about working with exponents to solve rich problems in context.

Warm-Up Launch
Arrange students into pairs. Give them one minute of quiet think-time and one minute to share their responses with a partner. Follow with a whole-class discussion.

Use the teacher dashboard to monitor student progress and point out common selections as well as any disagreements. Ask students to justify their responses with reasoning and critique each other's reasoning (MP3).

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

A, C, E, F

2

Answer the question you selected.

Teacher Moves

⚠ Before they can see this screen, students will have to find a partner and press a button that says, "I'm ready!"

Activity Launch
Arrange students in groups of 2–3. Distribute scratch paper and tools for making a visual display. Assign or ask groups to choose one of the given questions, and then use the relevant data provided on the following screen to help answer the selected questions.

Tell groups that they will use what they've learned about exponent rules and powers of 10 to answer questions. They will also make a visual display to illustrate their responses and demonstrate their reasoning. If time allows, ask groups to answer another question. Not all of the information provided will be used. It's important for students to consider only the essential information required to solve problems (MP4).

Notice the ways in which students use relevant information to answer the questions, and identify those who can explain why they are calculating with one operation rather than another. Speed is not as important as carefully thinking through each problem. As students work, look for those who use powers of 10 and the number line as tools to make it easier to calculate and interpret their results.

Facilitation
Consider using pacing to restrict students to Screens 2–3.

Early Student Thinking
Students may have trouble determining which data provided is relevant to their question. Ask them to think about what each question is asking and what information they need to answer the question. Encourage students to consider how reasonable their answer is to help confirm their thinking.

Sample Responses

Responses vary.

Burj Khalifa:

- The stack of cash is $1.5 \cdot 10^3$, or $1\,500$, meters tall (about twice as tall as the Burj Khalifa). This is because $\frac{1.5 \cdot 10^9}{10^6} = 1.5 \cdot 10^3$.

Student Debt:

- The net worth of the five richest people in the U.S. is $4.5 \cdot 10^{11}$. All of the student debt in the U.S. is $1.2 \cdot 10^{12}$. Student debt is approximately 2.7 times larger because $\frac{1.2 \cdot 10^{12}}{4.5 \cdot 10^{11}}$ is approximately 2.7.

Food Waste:

- $4.5 \cdot 10^8$ dollars worth of food is wasted in the U.S. each day. Eating 3 pounds per day, $1 \cdot 10^8$ additional people could survive on the food that is thrown away because $\frac{3 \cdot 10^8}{3}$ is $1 \cdot 10^8$.

Meter Sticks to the Moon:

- $3.5 \cdot 10^{23}$ meter sticks would be needed to equal the mass of the moon. Placed end to end, the meter sticks would reach from Earth to the Moon about 10^{15} times because $4 \cdot 10^{23}$ is 10^{15} times as much as $4 \cdot 10^{8}$.

Student Supports

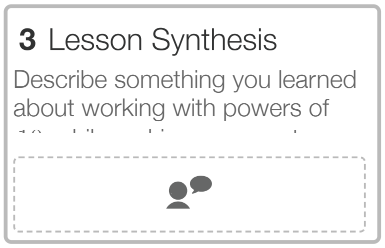
Students With Disabilities

- *Conceptual Processing: Processing Time*

Review an image or video of the contexts in order to activate prior knowledge of the problem.

- *Receptive/Expressive Language: Processing Time*

Read all statements or problems aloud. Students who both listen to and read the information will benefit from extra processing time. Students who benefit from extra processing time would also be aided by MLR 5 (Co-Craft Questions and Problems).



3 Lesson Synthesis

Describe something you learned about working with powers of

Describe something you learned about working with powers of 10 while making your poster.

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to surface important things to remember when solving problems involving quantities written as powers of 10.

Lesson Synthesis Launch

Begin with a gallery walk for students to see how other groups created their posters and interpreted their graphs.

Invite groups to share the strategies they used. Consider asking groups the following questions:

- What operations did your classmates use in their posters?
- What operations did your classmates not use in their posters?
- What features of your classmates’ posters helped you understand their thinking?
- Now that you have seen other groups’ posters, what would you have done differently if you had more time?

Once students have completed the gallery walk, invite them to think about what they learned while making their poster. Give students 2–3 minutes to respond to this question and a few minutes to share their responses with a partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

Give students one minute of quiet think-time. Then invite them to discuss with a partner.

If time allows, select students to share how they organized relevant information in their posters and how they planned to use the information to answer the questions. The important idea students should walk away with is that powers of 10 are a great tool to tackle challenging real-world problems that involve very large numbers.

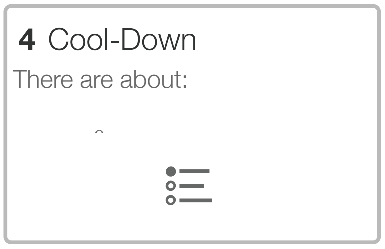
Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary.

I learned that when working with powers of 10, it is helpful to use scientific notation to compare coefficients and exponents.



4 Cool-Down

There are about:

There are about:

- $8 \cdot 10^9$ grains of sand in one cubic meter
- $1 \cdot 10^{11}$ stars in the galaxy
- $7.5 \cdot 10^9$ cubic meters of sand on Earth
- $1 \cdot 10^{10}$ galaxies in the universe

Which is larger?

Teacher Moves

Support for Future Learning
If students struggle with multiplying numbers expressed in scientific notation, plan to emphasize this when opportunities arise over the next several lessons. For example, consider pausing on Screen 2 of Lesson 11 to guide students through the problem as needed.

Facilitation
Consider using pacing to restrict students to Screens 4–5.

Sample Responses

The number of stars in the universe is larger.

Responses vary.

$(1 \cdot 10^{11}) \cdot (1 \cdot 10^{10})$ is $1 \cdot 10^{21}$ stars in the universe. There are only $60 \cdot 10^{18}$ grains of sand because $(8 \cdot 10^{9}) \cdot (7.5 \cdot 10^{9})$ is $60 \cdot 10^{18}$.

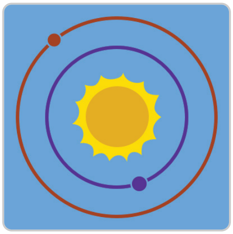
5

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can apply what I learned about powers of 10 to answer questions about real-world situations.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Solar System

Lesson 10: Definition of Scientific Notation

Overview

In the first part of this lesson, students use scientific notation to express large numbers: the diameters of planetary orbits. In the second part of the lesson, students continue to use scientific notation, but they use it to express very small numbers.

Learning Goals

- Distinguish expressions written in scientific notation from expressions that are not written in scientific notation.
- Rewrite expressions using scientific notation.

Materials

- Blank paper

Vocabulary

- scientific notation

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In the first part of this lesson, students use scientific notation to express large numbers: the diameters of planetary orbits. In the second part of the lesson, students continue to use scientific notation, but they use it to express very small numbers.

Lesson Summary

Warm-Up (10 minutes)

The purpose of the warm-up is help students notice that scientific notation makes it easier to compare large numbers. Both sorting sets consist of the same numbers, but one of the sets consists of mixed forms, while the numbers in the other are all in scientific notation.

Activity 1: Solar System and Test Tubes (25 minutes)

The purpose of this activity is to give students practice expressing both large and small numbers in scientific notation. The definition of scientific notation is a refining of the earlier work in this unit with multiples of powers of 10, where scientific notation is a special case that uses a single power of 10 and a first factor that is at least 1 and less than 10.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to make explicit the form and the purpose of scientific notation for large and/or small numbers.

Cool-Down (5 minutes)

The purpose of the cool-down is to check on student proficiency with scientific notation.

1 Warm-Up

Teacher Moves

Purpose
The purpose of this lesson is for students to a) distinguish expressions written in scientific notation from expressions that are not written in scientific notation and b) to write expressions in scientific notation.

Warm-Up Launch
The purpose of the warm-up on Screens 1–3 is to help students get an idea for why writing numbers in scientific notation is useful. Tell students that they will be putting two lists of numbers in order from least to greatest. After completing this task, they will decide which list was easier to put in order and explain why.

Watch for students who are multiplying out each number or changing the numbers to have a common base. Ask these students to describe their strategies to the class.

Facilitation
Consider using pacing to restrict students to Screens 1–3.

Sample Responses

From least to greatest:

- $5 \cdot 10^5$
- $4\,000\,000$
- $0.6 \cdot 10^7$
- $75 \cdot 10^5$

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.

- *Conceptual Processing- Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation in the activity.

2 Warm-Up

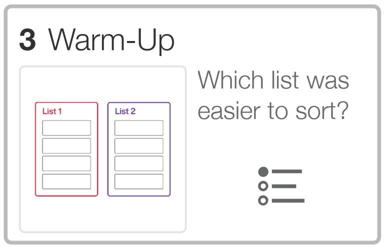
Teacher Moves

Early Student Thinking
Some students may want to rewrite each expression in standard form. While this is a useful practice to help get a sense of the size of each number, encourage students to pay close attention to the steps they are taking to rewrite each number. In particular, rewriting $5 \cdot 10^6$ as $5 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ and $5 \cdot 10^5$ as $5 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ may allow for an easy comparison and may enable students to make the connection that comparing exponents is possible when the first factor is a number greater than or equal to 1 and less than 10.

Sample Responses

From least to greatest:

- $5 \cdot 10^5$
- $4 \cdot 10^6$
- $6 \cdot 10^6$
- $7.5 \cdot 10^6$



3 Warm-Up

Which list was easier to sort?

Which list was easier to sort?

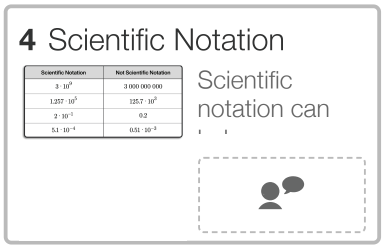
Teacher Moves

Highlight several student responses for the class. Ask questions to help students connect concrete and abstract responses as well as formal and informal responses.

Sample Responses

Responses vary.

- **List 1:** I rewrote the numbers in standard form in order to compare them. List 1 already had a number in standard form, so it was easier to sort.
- **List 2:** It was easier to sort because I could look at the numbers and compare the exponents. This was possible because the first factors were all less than 10.



4 Scientific Notation

Scientific notation can

Scientific notation can help us compare very large and very small numbers.

Some of the numbers in the table are written in scientific notation, and some are not.

What do you think it means for a number to be written in scientific notation?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to come to consensus on what the term *scientific notation* means.

Activity Launch
Tell students that in this activity they will practice identifying and writing numbers using scientific notation. The purpose of this screen is to give students a chance to develop their own understanding of what it means for a number to be written in scientific notation before presenting the definition.

Give students one minute of quiet think-time and one minute to discuss with a partner. Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning. After discussing several responses, tell students the definition of scientific notation:

“A number is written in scientific notation if it is written as a product of two factors: The first factor is a number greater than or equal to 1, but less than 10; the second factor is an integer power of 10.”

For simplicity, this definition does not include negative numbers. For future use, consider asking students to record the definition on the guided note-taking sheet for this lesson, or write it on the board for easy reference during the activity.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Sample Responses

Responses vary.

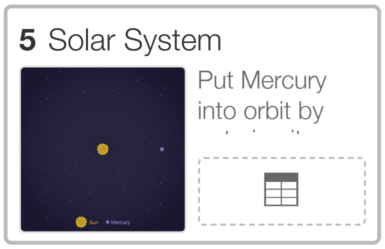
A number is written in scientific notation if it is written as a product of two factors: The first factor is a number greater than or equal to 1, but less than 10; the second factor is an integer power of 10.

Note: For simplicity, this definition does not include negative numbers.

Student Supports

Students With Disabilities
- *Executive Functioning: Visual Aids*

Create an anchor chart for the definition of scientific notation that publicly displays important definitions, rules, formulas, or concepts for future reference.



5 Solar System

Put Mercury into orbit by

Put Mercury into orbit by entering its distance from the Sun using scientific notation.

Mercury's distance from the Sun in scientific notation is $3.6 \cdot 10^7$ miles.

Enter this number below. Then press "Try It."

Teacher Moves

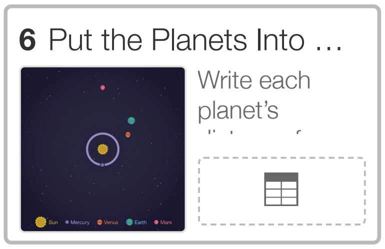
Tell students that for the next several screens, they will practice writing very large and very small numbers using scientific notation, and they’ll be able to check their work by pressing “Try It” to play an animation.

Consider pausing the class for a brief conversation about the definition of scientific notation. Ask students to explain why $3.6 \cdot 10^7$ is written in scientific notation. If time permits, ask students to explain why $36 \cdot 10^6$ *isn't* written in scientific notation.

Facilitation
Consider using pacing to restrict students to Screens 5–10.

Sample Responses

$3.6 \cdot 10^7$



6 Put the Planets Into …

Write each planet's

Write each planet's distance from the Sun using scientific notation.

Then press "Try It."

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

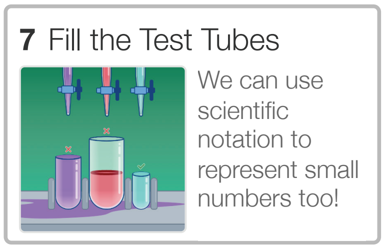
Sample Responses

- **Venus:** $6.7 \cdot 10^7$ miles
- **Earth:** $9.296 \cdot 10^7$ miles
- **Mars:** $1.417 \cdot 10^8$ miles

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Begin with a demonstration of the first problem, which will provide access for students who benefit from clear and explicit instructions. Also check in with individual students, as needed, to assess for comprehension during each step of the activity.



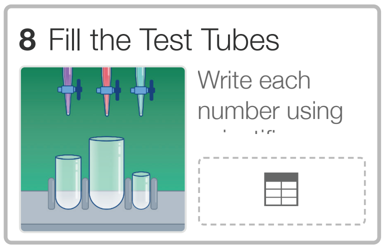
7 Fill the Test Tubes

We can use scientific notation to represent small numbers too!

We can use scientific notation to represent small numbers too!

On the next screen, you'll fill the test tubes by entering the total volume in liters using scientific notation.

Press "Try It" to see what we mean by filling the test tubes.



8 Fill the Test Tubes

Write each number using

Write each number using scientific notation.

Then press "Try It" to fill the test tubes with the amount of liquid you entered.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

- **Purple**: $1.25 \cdot 10^{-4}$
- **Red**: $2 \cdot 10^{-4}$
- **Blue**: $3.25 \cdot 10^{-6}$

9 Group cards together…

Teacher Moves

Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Sample Responses

Image solution

---

10 Are You Ready for …
Answer one or more of the problems on paper.

Answer one or more of the problems on paper.

Teacher Moves

⚠ Before students can see this “Are You Ready for More?” screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving students who finish Screens 5–9 ahead of time another challenge before the lesson synthesis. Because only a subset of your students will complete this screen, we recommend you don’t discuss it with the entire class.

Sample Responses

1. $0.99, \frac{99}{100}$
2. $0.9999, \frac{9999}{10000}$
3. 10^{-6}

11 Lesson Synthesis

Abdullah wrote $10\,000\,000$ as $10 \cdot 10^6$.

Abdullah wrote $10\,000\,000$ as $10 \cdot 10^6$.

Did he correctly use scientific notation?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to consolidate what it means to correctly write a number in scientific notation, particularly that the first factor must be a number greater than or equal to 1, but *less than* 10.

Synthesis Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

No

Responses vary.

Abdullah is not correct. He wrote $10\,000\,000$ using powers of 10. The number isn't written in scientific notation though because the first factor must be a number greater than or equal to 1, but *less than* 10.

---

12 Cool-Down

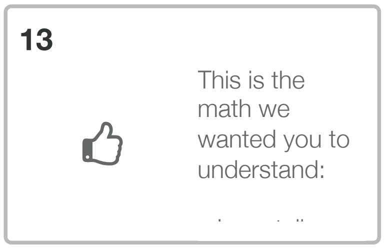
Teacher Moves

Support for Future Learning
If students struggle to use scientific notation to express large and small numbers, consider reviewing this cool-down as a class before Lesson 11, or offering individual support where needed during Lesson 11.

Facilitation
Consider using pacing to restrict students to Screens 12–13.

Sample Responses

- $4.82 \cdot 10^4$
- $9.9 \cdot 10^{-4}$
- $3.6 \cdot 10^6$



13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can tell whether or not a number is written in scientific notation.
- I can rewrite a large or small number using scientific notation.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Balance the Scale

Lesson 11: Multiplying, Dividing, and Estimating With Scientific Notation

Overview

Students perform operations with numbers expressed in scientific notation to estimate very large and very small quantities, and to express how many times as much one quantity is as the other. Students interpret their results in context (MP2) and consider the degree of precision that is appropriate for particular contextualized problems (MP6).

Learning Goals

- Compare very large or very small numbers using scientific notation.
- Multiply and divide numbers given in scientific notation to answer questions in context.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Students perform operations with numbers expressed in scientific notation to estimate very large and very small quantities, and to express how many times as much one quantity is as the other. Students interpret their results in context (MP2) and consider the degree of precision that is appropriate for particular contextualized problems (MP6).

Lesson Summary

Warm-Up (10 minutes)

The purpose of the warm-up is to begin developing students' understanding of how to estimate and use operations when comparing quantities in scientific notation. The warm-up helps students understand the appropriate degree of precision for such problems and orients them to the scale tool that they will use throughout the lesson.

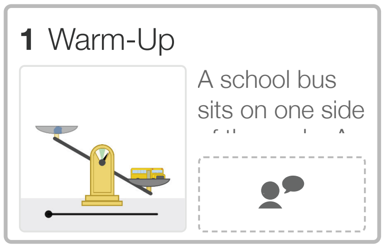
Activity 1: Balance the Scale (25 minutes)

The purpose of this activity is to hone students' strategies for describing quantities, making estimates, and making comparisons with very large numbers. Students will apply multiplication and division strategies and round numbers, as appropriate, to express an approximation of how much one object weighs relative to another.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to generalize their approaches to comparing quantities in scientific notation by applying them to a new context.

Cool-Down (5 minutes)



1 Warm-Up

A school bus sits on one side

A school bus sits on one side of the scale. A big pile of jelly beans sits on the other.

Adjust the slider until the needle moves to the green area.

What do you think it means for the needle to be in the green area?

Teacher Moves

Purpose
The purpose of this lesson is to multiply, divide, and estimate with numbers in scientific notation to answer questions in context.

Warm-Up Launch
Before students begin the warm-up, consider asking them what is the largest amount of jelly beans they have seen at once. Then ask how many jelly beans it would take to weigh as much as a school bus. Encourage students to offer up a guess or a range. Use the projector to show that the first screen of the activity is about answering this very question.

Arrange students into pairs. Give them one minute of quiet think-time, followed by one minute to share their responses with a partner.

Teacher Moves
Use the teacher dashboard to identify student responses that offer a helpful description of what it means for the needle to be in the green area. Pause and share these descriptions, as necessary, since understanding the scale is critical for the remainder of the lesson.

Early Student Thinking
Some students may feel the need to balance the scale exactly. Explain to these students that because we are working with very large quantities, the scale used throughout this lesson allows for more leeway.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

It means the two trays are holding about the same amount of weight.

Student Supports

Students With Disabilities
- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.

- *Conceptual Processing: Processing Time*
Start with the activity paused, show this screen on the projector, and ask students what they notice and wonder prior to beginning the work to activate prior knowledge of the context of the problem.

---

2 Warm-Up


Let's try something

Let's try something bigger. Here are the masses of the two pictured objects:

- Jelly bean: $1.1 \cdot 10^{-3}$ kilograms
- Egyptian pyramid: $5.3 \cdot 10^{7}$ kilograms

How many jelly beans weigh about as much as one Egyptian pyramid?

Adjust the numbers below to balance the scale.

Teacher Moves

Teacher Moves
Consider pausing the class once most students have spent time with this screen. Explain that large numbers, such as the weight of the pyramid or the number of jelly beans, are often estimated using scientific notation. A pyramid weighs about $5.3 \cdot 10^{7}$ kilograms and a jelly bean weighs about $1.1 \cdot 10^{-3}$ kilograms, so we can find the estimated amount of jelly beans that weighs as much as the pyramid by finding the value of $\frac{5.3 \cdot 10^{7}}{1.1 \cdot 10^{-3}}$.

Tell students that estimation will help answer these questions much more easily, so if they get stuck computing, they should try to make reasonable estimates. For example, estimating the number of jelly beans needed to balance the pyramid could have looked like $\frac{5.3 \cdot 10^{7}}{1.1 \cdot 10^{-3}} \approx \frac{5 \cdot 10^{7}}{1 \cdot 10^{-3}} = 5 \cdot 10^{7-(-3)} = 5 \cdot 10^{10}$. Guide students through the problem as needed. Consider recording the steps of the problem on the board for students to refer to throughout the activity.

Readiness Check (Problem 2)
If most students struggled, plan to review this question before Activity 1. Invite students to think about how writing each number in scientific notation might be helpful.

Facilitation

Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- $5 \cdot 10^{10}$ jelly beans
- $4.82 \cdot 10^{10}$ jelly beans

Student Supports

Students With Disabilities

- *Conceptual Processing- Processing Time*

Begin with a demonstration of this problem to provide access to students who benefit from clear and explicit instructions. Check in with individual students, as needed, to assess for comprehension during each step of the activity.

---

3 Describe a Strategy

Describe a strategy for

Describe a strategy for solving problems like the one on the previous screen.

For reference, the masses of the objects were:

- Jelly bean: $1.1 \cdot 10^{-3}$ kilograms
- Egyptian pyramid: $5.3 \cdot 10^{7}$ kilograms

Teacher Moves

Key Discussion Screen

The purpose of this discussion is to surface strategies for determining how many times larger one number written in scientific notation is than another.

Activity Launch

This activity allows students to continue working with the scale as a tool to estimate, multiply, and divide numbers in scientific notation to answer questions in context.

Arrange students into pairs. Give them a few minutes of quiet time to work, followed by 1–2 minutes to discuss their work with their partner.

Encourage students to use precision when describing which operations they used and which object's weight they are referring to. For example, students who say they divide should specify which number is divided by which. This distinction becomes especially important on subsequent screens.

This is a great place to use snapshots to select several unique student responses to display and discuss with the class. Ask students to justify their responses and critique each other's reasoning (MP3).

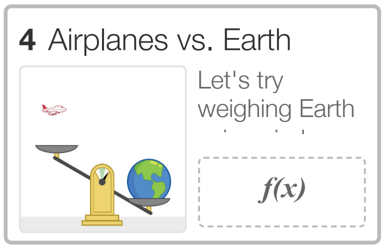
Facilitation
Consider using pacing to restrict students to Screens 3–6.

Sample Responses

Responses vary.

- First, round the first factors to the nearest whole number. Then divide the larger quantity by the smaller quantity. This will result in dividing the first factors and subtracting the exponents.

- Start with the lighter weight. Multiply its first factor so that it reaches the bigger weight's first factor (e.g., $1 \cdot 5 = 5$). Then multiply by the necessary power of 10 to reach the bigger weight's power of 10 (e.g., $10^{-3} \cdot 10^{10} = 10^7$).



4 Airplanes vs. Earth

Let's try weighing Earth

Let's try weighing Earth using airplanes. Here are the masses of the objects:

- Airplane: $2.1 \cdot 10^5$ kilograms
- Earth: $5.972 \cdot 10^{24}$ kilograms

How many airplanes weigh about as much as Earth?

Use scientific notation.

Teacher Moves

Teacher Moves
Tell students they will try more of these problems—with a slight twist. They will weigh some extremely large objects—like planet Earth—using other objects besides jelly beans. Recall that reasonable approximations are acceptable.

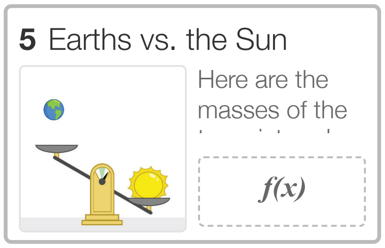
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Early Student Thinking
Review the definition of scientific notation with students as needed.

Sample Responses

Responses vary.

- $3 \cdot 10^{19}$ airplanes
- $2.8 \cdot 10^{19}$ airplanes


5 Earths vs. the Sun

Here are the masses of the

Here are the masses of the two pictured objects:

- Earth: $5.972 \cdot 10^{24}$ kilograms
- Sun: $1.989 \cdot 10^{30}$ kilograms

How many Earths weigh about as much as the Sun?

Use scientific notation.

Teacher Moves

Teacher Moves
Consider pausing the class once most students have developed initial ideas about this screen. Call attention to the fact that this screen has an added challenge because the first factor of the smaller object is greater than that of the larger object. Elicit strategies from students, and mention the following strategies if students do not bring them up:

- Dividing the first factors and powers of 10 still works for this problem. 2 can be divided by 6 even though the result is not a whole number. Since the result is less than 1, the number will have to be converted to scientific notation.
- Alternatively, the Sun's mass can be rewritten with an equivalent expression, such as $1.989 \cdot 10 \cdot 10^{29}$. This means that dividing the first factors will yield a number between 1 and 10, and thus, no conversion to scientific notation will be needed after dividing.

Early Student Thinking

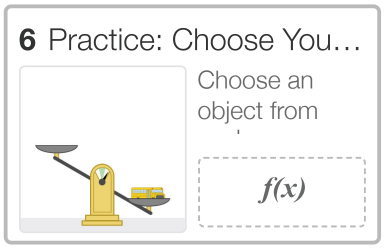
- Students may use 10^6 in their answer because that is the result of 10^{30} divided by 10^{24}. Consider inviting these students to check how reasonable this answer is by multiplying the mass of Earth by 10^6. The scale animation will also help students see that 10^6 is too many Earths.

- Students may use 3 as the first factor of their answer because after rounding, $\frac{6}{2} = 3$. Ask these students which mass they are dividing into which and to support their reasoning that $\frac{2}{6}$ is about 0.33.

Sample Responses

Responses vary.

- $3.1 \cdot 10^5$ Earths
- $3.3 \cdot 10^5$ Earths



6 Practice: Choose You…

Choose an object from

Choose an object from each row below to compare.

How many of the smaller object weigh about as much as the larger object?

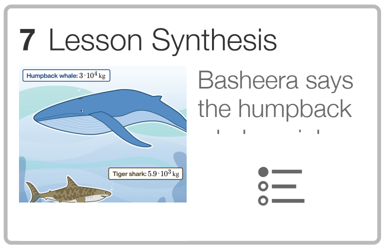
Use scientific notation.

Teacher Moves

Tell students that their task on this screen is to compare the weights of two objects at a time. In particular, students select one object from the top row and one object from the bottom row, and then they will find the number of the small objects needed to balance the larger object. Encourage students to try as many combinations as time permits in order to get additional practice with the lesson objectives.

Sample Responses

Responses vary.



7 Lesson Synthesis

Basheera says the humpback

Basheera says the humpback whale weighs about 20 times as much as the tiger shark.

Elena says the humpback whale weighs about 5 times as much as the tiger shark.

Who is correct?

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to solidify strategies for determining how many times larger one number written in scientific notation is than another.

Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Then follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Early Student Thinking

Many students will likely be tempted by Basheera's answer (20) because 6 divided by 3 is 2, and one power of 10 separates the weights. Consider inviting these students to check how reasonable this answer is by finding the weight of 20 tiger sharks.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Elena is correct.

Responses vary.

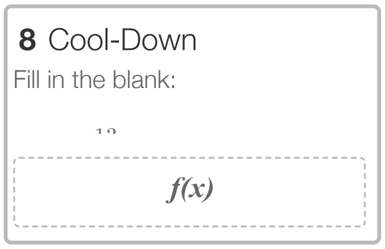
- I rounded 5.9 to 6. When I multiplied $6 \cdot 10^3$ by 5, I got $30 \cdot 10^3$, which is the same as $3 \cdot 10^4$, the whale's weight.
- 10 tiger sharks weigh about $6 \cdot 10^4$ kilograms, so 5 tiger sharks must weigh about $3 \cdot 10^4$ kilograms.

Student Supports

Students With Disabilities

- *Receptive/Expressive Language: Processing Time*

Read the situation aloud. Students who both listen to and read the information will benefit from extra processing time.


8 Cool-Down

Fill in the blank:

f(x)

Fill in the blank:

$6.1 \cdot 10^{13}$ is about ______ times as large as $2.1 \cdot 10^2$.

Use scientific notation.

Teacher Moves

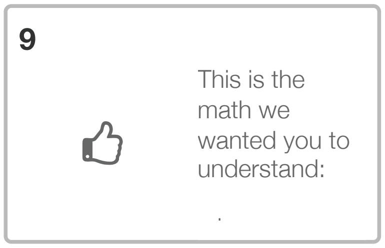
Support for Future Learning
If students struggle with performing operations with numbers expressed in scientific notation, consider reviewing this cool-down as a class before Lesson 13, or offering individual support where needed during Lesson 13.

Facilitation
Consider using pacing to restrict students to Screens 8–9.

Sample Responses

Responses vary.

$3 \cdot 10^{11}$


9

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use scientific notation and estimation to compare very large or very small numbers.
- I can multiply and divide numbers given in scientific notation.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

City Lights

Lesson 12: Adding and Subtracting With Scientific Notation

Overview

Students add and subtract numbers expressed in scientific notation and express the resulting sums and differences in scientific notation.

Learning Goals

- Add and subtract numbers in scientific notation to answer questions in context.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

Students add and subtract numbers expressed in scientific notation, and express the resulting sums and differences in scientific notation. The work throughout this lesson is set in the context of determining the proper amount of electricity to produce and distribute to pairs of cities.
Students will need to make sense of the problems and reason about appropriate units (MP1, MP6).

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to give students an opportunity to reason about adding two numbers in scientific notation before formally introducing the concept and strategies later in the activity.

Activity 1: City Lights (30 minutes)

The purpose of this activity is to extend students' prior knowledge of units, in general, and of place value, in particular, to the more abstract context of large numbers expressed in scientific notation. The units in the introductory screen (gigawatts) are expressed differently on later screens (10^9 watts), but the relationships remain the same. To add or subtract numbers in scientific notation, students must attend to precision by aligning place value (MP6).

Lesson Synthesis (5 minutes)

The purpose of the lesson synthesis is to ensure that students understand addition and subtraction with numbers in scientific notation as well as the reasons why exponents for addition and subtraction are treated differently than they are for multiplication and division.

Cool-Down (5 minutes)

1 Warm-Up

Is the statement below true or false?

Is the statement below true or false?

$$2 \cdot 10^2 + 3 \cdot 10^3 = 5 \cdot 10^5$$

Teacher Moves

Purpose

The purpose of this lesson is for students to practice adding and subtracting numbers expressed in scientific notation.

Warm-Up Launch

The purpose of this warm-up is to give students an opportunity to reason about adding two numbers in scientific notation before formally introducing the concept and strategies later in the activity.

Before students begin the warm-up, consider telling them that they have been working with multiplication and division using scientific notation, and today, we are going to consider how to add and subtract using scientific notation. Give students 2–3 minutes of quiet work time to make a selection and explain their thinking.

Teacher Moves

After students have had time to make their selections, pause the class for a brief discussion. Invite several students to share their responses. Consider using snapshots to highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Early Student Thinking

Students may be unsure of what to do for this problem, or they may misapply one of the many rules about exponents that they learned earlier in the unit. Consider encouraging students to rely on their intuition rather than on rules. For instance, point out that 10^3 is one thousand, so $3 \cdot 10^3$ is three thousand. Encourage students to write the numbers using standard notation before adding or subtracting, which may help them understand the place values involved.

Facilitation

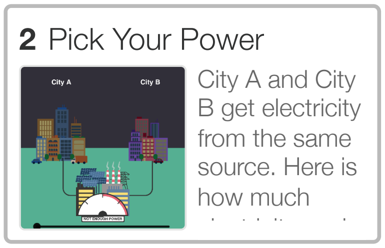
Consider using pacing to restrict students to this screen.

Sample Responses

False

Responses vary.

This statement is false because $2 \cdot 10^2 + 3 \cdot 10^3 = 2 \cdot 100 + 3 \cdot 1000 = 200 + 3000 = 3200$, which is not equal to $5 \cdot 10^5$.



2 Pick Your Power

City A and City B get electricity from the same source. Here is how much

City A and City B get electricity from the same source. Here is how much electricity each city needs:

- City A: 5 gigawatts
- City B: 3 gigawatts

You can control how much electricity is produced. Adjust the slider so that the dial says, "Success!"

Discuss what you think "success" means in this case.

Teacher Moves

Activity Launch
In this activity, students continue to learn about adding and subtracting numbers in scientific notation by applying what they know in context.

Consider taking a brief moment to introduce the term *gigawatt*. Ask students if they have heard of the prefix *giga-* or the unit *watt* before. All they need to know to begin the activity is that watts are a measure of power/electricity with prefixes they may have heard of before, like *kilo-* and *mega-*. A gigawatt is a very large amount of watts and is an appropriate unit for the amount of power that cities use.

Arrange students into pairs. Tell students that for today's lesson, they are in charge of a power plant that provides electricity for two cities. Display the student view of this screen for the class, and move the slider back and forth. Ask students to discuss with their partners what they think "success" means in this context. Then invite a student to share their response with the class.

Facilitation
Consider using pacing to restrict students to Screens 2–5.

Sample Responses

Responses vary.

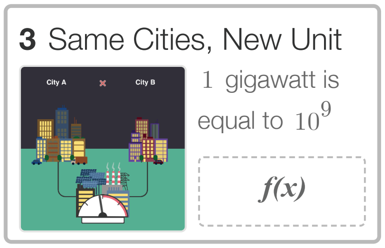
- "Success" means you have sent exactly the right amount of electricity to the two cities.
- "Success" means that everyone who wants power has power, and there is no excess.

Student Supports

Support for Students With Disabilities

- *Social-Emotional Functioning: Peer Tutors*

Pair students with their previously identified peer tutors.



3 Same Cities, New Unit

1 gigawatt is equal to 10^9

f(x)

1 gigawatt is equal to 10^9 watts.

Here are the electricity needs for City A and City B written in watts:

- City A: $5 \cdot 10^9$ watts
- City B: $3 \cdot 10^9$ watts

How many total watts of electricity are needed to power both cities?

Teacher Moves

Teacher Moves

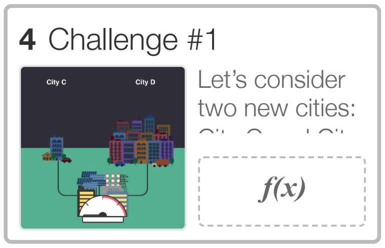
Encourage students to use scratch paper to answer the questions on this screen and throughout the lesson.

Early Student Thinking

Now that the numbers to add are in scientific notation, some students may be unsure of what to do. Help them understand why the exponents stay *unchanged* with addition. It may be helpful to draw on work from the warm-up or from the previous screen where it was easier to see that if you have 5 of one thing and 3 of the same thing, you get 8 of that same thing.

Sample Responses

$8 \cdot 10^9$ watts



4 Challenge #1

Let's consider two new cities:

f(x)

Let's consider two new cities: City C and City D.

Here is how much electricity each city needs:

- City C: $6 \cdot 10^8$ watts
- City D: $3.7 \cdot 10^9$ watts

How many total watts of electricity are needed to power both cities?

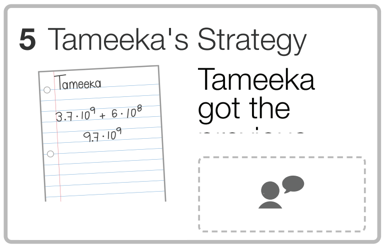
Use scientific notation.

Teacher Moves

Early Student Thinking
Some students may be tempted to enter 9.7 as the first factor and possibly multiply that by 10^8, 10^9, or $10^8.5$. The power animation will give feedback that may help students adjust their answer. Consider supporting them further by asking them to think about how this problem is different from the one on the previous screen.

Sample Responses

$4.3 \cdot 10^9$ watts



5 Tameeka's Strategy

Tameeka

$3.7 \cdot 10^9 + 6 \cdot 10^8$

$9.7 \cdot 10^9$

Tameeka got the

Tameeka got the previous challenge incorrect.

What advice would you give Tameeka?

Teacher Moves

Teacher Moves
Consider pausing the class once most students have spent time with this screen. Use the teacher view of the dashboard to identify unique answers to share with the class. This is a great place to use the snapshots tool to select and display 2–4 responses at the same time. Consider asking, “Why can’t we add 3.7 to 6 right away?” [Because those numbers don’t represent quantities of the same unit.] Then complete the rest of the problem together as a class.

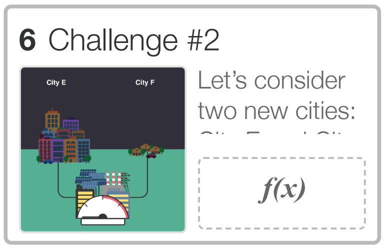
Facilitation
Consider using pacing to restrict students to Screens 5–9.

Routine (optional): Consider using the routine Critique, Correct, Clarify to help students communicate about errors and ambiguities in math ideas and language.

Sample Responses

Responses vary.

Tameeka used the smaller power of 10 as the common power for both terms. My advice for Tameeka is to rewrite $3.7 \cdot 10^9$ as $3.7 \cdot 10 \cdot 10^8$, which equals $37 \cdot 10^8$. From there we can add $37 \cdot 10^8$ and $6 \cdot 10^8$ because they are each multiplied by the same power of 10. Then $37 \cdot 10^8 + 6 \cdot 10^8 = 43 \cdot 10^8$, or $4.3 \cdot 10^9$.



6 Challenge #2

Let's consider two new cities:

Let's consider two new cities: City E and City F.

Here is how much electricity each city needs:

- City E: $1.5 \cdot 10^9$ watts
- City F: $8.3 \cdot 10^7$ watts

How many total watts of electricity are needed to power both cities?

Use scientific notation.

Teacher Moves

This is a great place to use the teacher dashboard to monitor student progress. Offer individual support where needed, or if enough students are struggling, prepare to lead a whole-class discussion using responses from the next screen.

Sample Responses

$1.583 \cdot 10^9$ watts

7 Describe Your Strategy

Describe your strategy for

Describe your strategy for solving the challenge on the previous screen.

For reference, here is the amount of electricity needed to power each city:

- City E: $1.5 \cdot 10^9$ watts
- City F: $8.3 \cdot 10^7$ watts

Your answer was: $1.583 \cdot 10^9$ watts

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for adding two numbers written in scientific notation with different powers of 10.

Use snapshots to select several unique student responses to display and discuss as a class. Ask students to justify their responses and critique each other's reasoning.

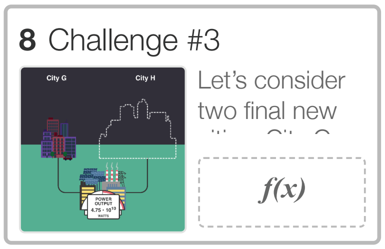
Emphasize the importance of place value and units for solving problems like this. The 1.5 in City E's number corresponds to billions (10^9) of watts, while the 8.3 in City F's number corresponds to tens of millions (10^7) of watts.

Their relative sizes are easier to see when the numbers are multiplied by the same power of 10 and/or when the numbers are converted to standard notation and displayed one above the other. The latter approach can be thought of as using 10^0 rather than 10^7 or 10^9 as the common power of 10.

Sample Responses

Responses vary.

I knew the problem was about adding the numbers, and the numbers must be in the same unit (power of 10) in order to be added. I decided to write both numbers with 10^9, so $8.3 \cdot 10^7$ became $0.83 \cdot 10^8$ and then $0.083 \cdot 10^9$. Then $(1.5 + 0.083) \cdot 10^9 = 1.583 \cdot 10^9$.


8 Challenge #3

Let's consider two final new

Let's consider two final new cities: City G and City H.

- The power plant provides exactly $4.75 \cdot 10^{10}$ watts.
- City G uses $3 \cdot 10^9$ watts.

How many watts does City H use?

Use scientific notation.

Teacher Moves

This screen asks students to solve a variant of the context they have seen so far. This scenario has a fixed amount of power, and one city's power usage is known. Therefore, students must subtract to find the power usage of the other city. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

$4.45 \cdot 10^{10}$ watts

---

9 Match each scenario …

Teacher Moves

This card sort provides students with practice adding and subtracting numbers in scientific notation. If time is short, consider skipping this screen and pacing students to the lesson synthesis.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies. The dashboard may also help you pair students together for conversations about how they sorted the cards.

Sample Responses

Image solution

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10 Lesson Synthesis

Here are two units of measure:

$f(x)$

Here are two units of measure:

1 gigawatt = 10^9 watts
1 megawatt = 10^6 watts

How many watts is 2.5 gigawatts plus 130 megawatts?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to revisit the importance of writing numbers in scientific notation in the same units before adding or subtracting.

Synthesis Launch
While paused, begin the synthesis by asking students what strategies they’ve learned for adding numbers in scientific notation. If students do not bring it up, mention that one key takeaway from today was that adding requires numbers that are of the same unit. This is akin to problems introduced in younger grades where students might be asked

to find the sum of 6 eggs and 4 dozen eggs. We must convert one or both numbers so that the units are the same.

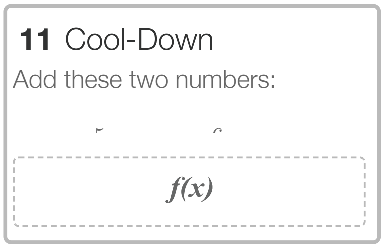
Ask students which of the strategies they mentioned also apply to subtraction. [Subtraction and addition essentially behave the same way. Like addition, subtraction requires numbers that are of the same unit.]

Teacher moves
Unpause and give students 2–3 minutes to respond to the question on this screen, followed by a few minutes to share their responses with their partner. If time permits, follow up with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

$2.63 \cdot 10^9$ watts



11 Cool-Down

Add these two numbers:

Add these two numbers:

$$2.3 \cdot 10^5 + 3.6 \cdot 10^6$$

Write your answer in scientific notation.

Teacher Moves

Support for Future Learning
If students struggle with adding and subtracting numbers expressed in scientific notation, consider reviewing this screen as a class before Lesson 13, or offering individual support where needed during Lesson 13.

Facilitation
Consider using pacing to restrict students to Screens 11–12.

Sample Responses

$3.83 \cdot 10^6$ watts

12

👍 This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can add and subtract numbers given in scientific notation.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Star Power

Lesson 13: Let’s Put It to Work

Overview

In this unit-culminating lesson, students use scientific notation as a tool for comparing, combining, and operating on the net worth of different celebrities. Students identify the essential features of the questions and reason quantitatively and abstractly in order to answer them in context (MP2, MP4).

Note: Provide students the supplement to use on Screen 6.

Learning Goals

- Use adding, subtracting, multiplying, and dividing with scientific notation to compare quantities and answer questions in context.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you’ll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this unit-culminating lesson, students use scientific notation as a tool for comparing, combining, and operating on the net worth of different celebrities. Students identify the essential features of the questions and reason quantitatively and abstractly in order to answer them in context (MP2, MP4).

Lesson Summary

Warm-Up (10 minutes)

The purpose of the warm-up is for students to reason about how to solve problems that require computations with numbers expressed in scientific notation. These problems include both very large and very small numbers.

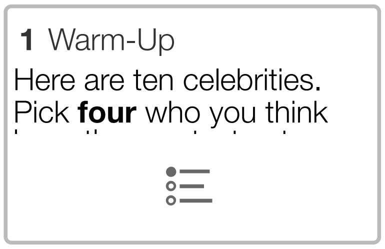
Activity 1: Star Power (25 minutes)

The purpose of this activity is for students to use operations with numbers written in scientific notation to answer questions about real-world situations. Students use scientific notation as a tool to understand the relative scale of different powers of 10 (MP2). They practice modeling skills by identifying essential elements of the problems and gathering relevant information before computing (MP4).

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to discuss how scientific notation helps when making comparisons.

Cool-Down (5 minutes)



1 Warm-Up
Here are ten celebrities.
Pick **four** who you think

Here are ten celebrities. Pick **four** who you think have the greatest net worth.

Teacher Moves

Purpose
The purpose of this lesson is for students to use scientific notation as a tool for comparing, combining, and operating on quantities in context.

Activity Launch
Tell students that they will compare the net worth of celebrities as of 2019 throughout this lesson. Consider asking students which celebrity in the gallery they think has the greatest or the smallest net worth. Ask students if they know what net worth means, and if needed, explain that net worth is everything a person owns minus what they owe. It is important that students understand the context before they work through this lesson.

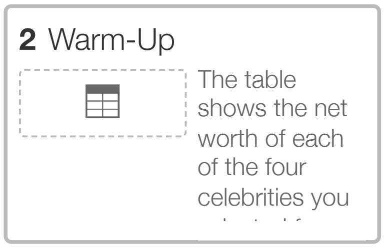
Allow students a minute to select four celebrities. Then invite them to continue to the next screen.

Facilitation
Consider using pacing to restrict students to Screens 1–3.

Note: All net worths in this activity are from 2019.

Sample Responses

Responses vary.



2 Warm-Up
The table shows the net worth of each of the four celebrities you

The table shows the net worth of each of the four celebrities you selected from the gallery.

Write each celebrity's net worth in scientific notation to help you compare them.

Teacher Moves

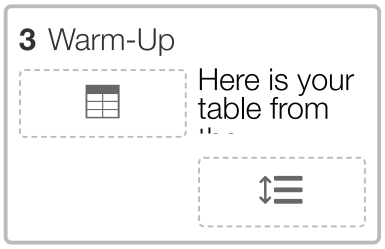
The correct answers on Screens 2–3 will be different depending on the student's selections from Screen 1. This is designed to encourage students to collaborate around general strategies rather than specific answers.

Explain to students that they will receive feedback on this screen, and that they will have an opportunity to revise their work. Encourage them to continue working until all of the celebrities' net worths are written correctly in scientific notation.

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Student results for this screen depend on their selections on Screen 1 and are available in the teacher view of the dashboard.



3 Warm-Up

Here is your table from

Here is your table from the previous screen.

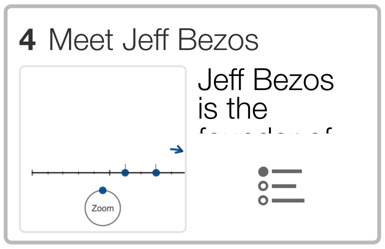
Order the celebrities by their 2019 net worth from smallest to greatest.

Teacher Moves

Invite students to describe their strategies for sorting the celebrities they chose. An important concept to emphasize is that powers of 10 can be used to make comparisons.

Sample Responses

Student results for this screen depend on their selections on Screen 1 and are available in the teacher view of the dashboard.



4 Meet Jeff Bezos

Jeff Bezos is the

Jeff Bezos is the founder of Amazon.com.

His net worth as of 2019 is plotted on the number line, along with the net worths of 10 celebrities.

Who do you think has more money?

Teacher Moves

Activity Launch
Consider pausing here and asking students if they know who Jeff Bezos is. Explain that he is the founder of Amazon.

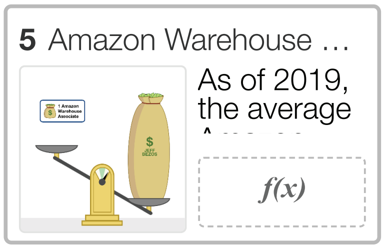
Ask students to estimate his net worth. Do they think he has more or less money than Beyonce? Invite students to zoom out on the number line to test their predictions and to respond to the question on this screen.

Facilitation
Consider using pacing to restrict students to Screens 4–5, one screen at a time.

Sample Responses

Responses vary.

- Jeff Bezos. The distance between his point and zero on the number line is greater than all of the other celebrities' distances combined.
- All 10 celebrities combined. Each of these celebrities have so much money that it doesn't make sense that there is another person who has more than all ten of them combined.
- I don't know. When I look at this number line, it looks like Jeff Bezos has more money than all ten celebrities combined, but I'd need to add their net worths to know for sure.



5 Amazon Warehouse ...

As of 2019, the average

As of 2019, the average Amazon warehouse associate made around $\$30\,000$ per year. At that time, Jeff Bezos's net worth was about $\$120\,000\,000\,000$.

How many years would an Amazon warehouse associate need to work to earn the equivalent of Jeff Bezos's net worth?

Express your answer using scientific notation.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for comparing two large numbers not written in scientific notation, including the strategy of writing the numbers in scientific notation.

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Encourage students to use paper or dry-erase bards to help them with their thinking.

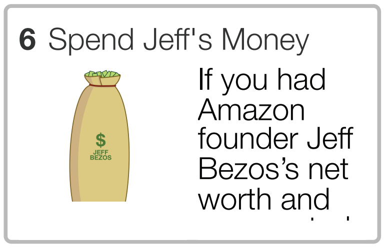
As time allows, consider facilitating a discussion about the distribution of wealth in the U.S. Here are some questions you could use to spark the discussion:

- Do you think it is reasonable for one person to have to work for four million years to have as much money as another person?

- Why do some people have so much more money than other people?
- Does it matter if some people have much more money than others?

Sample Responses

$4 \cdot 10^6$ years



6 Spend Jeff's Money

If you had Amazon founder Jeff Bezos's net worth and

If you had Amazon founder Jeff Bezos's net worth and you wanted to spend it all, what would you buy?

Follow the directions on the worksheet.

Teacher Moves

Activity Launch
⚠ Before students can see this screen, they will have to press a button that says, "You need a worksheet and a partner for this activity."

Arrange students into pairs. Provide each student with their own worksheet and scratch paper or dry erase boards for their calculations. Tell students that their task is to spend as much of Jeff Bezos's money as they can (*without going over*) by purchasing *at least four* different items from the options listed on the worksheet.

As a modification, you can also allow students to add additional items by researching their values and adding them to their worksheet.

Teacher Moves
Circulate to monitor student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Facilitation
Consider using pacing to restrict students to Screens 6–7.

Sample Responses

Responses vary.

Student Supports

English Language Learners
- *Expressive Language: Visual Aids*

Create or review an anchor chart that publicly displays operations using scientific notation to aid in explanations and reasoning.

Students With Disabilities

- *Conceptual Processing: Processing Time*
For students who benefit from extra processing time, provide them the worksheet to review prior to implementation of this activity.

- *Conceptual Processing: Processing Time*
Begin with a demonstration of purchasing one item to provide access to students who benefit from clear and explicit instructions. Check in with individual students as needed to assess for comprehension during each step of the activity.

- *Memory: Processing Time*
Provide sticky notes or mini whiteboards to aid students with working memory challenges.

7 Are You Ready for M…

des NEWS

Amazon Donates $690,000 to Australian Wildfire Relief

In early 2020, Jeff

In early 2020, Jeff Bezos announced that Amazon would donate $\$690\,000$ in assistance to wildfire recovery after Australia's devastating bushfires.

Some people said that this donation was very generous, while other people thought it was not enough.

What do you think?

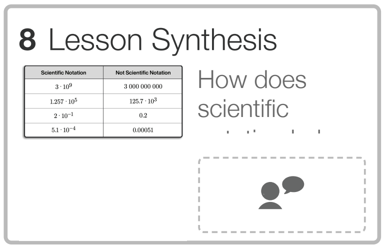
Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, "I'm ready!"

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screen 6 ahead of time before the class discussion on Screen 8. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.



8 Lesson Synthesis

How does scientific

How does scientific notation help when working with very large or very small numbers?

Teacher Moves

Key Discussion Screen 🔑

The purpose of this discussion is to surface the advantages of scientific notation when solving problems involving very large or very small numbers.

Synthesis Launch
The purpose of the lesson synthesis is to discuss how scientific notation helps when making comparisons.

Give students one minute of quiet think-time, and then invite them to discuss with a partner. Invite several students to share their responses.

In a whole-class discussion, ask students what they might have found surprising or interesting when comparing the net worth of different celebrities.

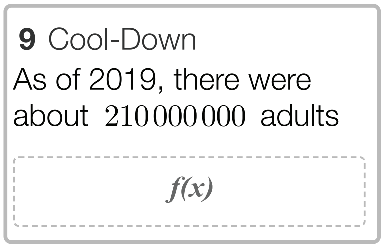
Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

Scientific notation helps because it is more efficient to compare the powers of 10 than it is to count zeros in very large or very small numbers.



9 Cool-Down
As of 2019, there were about $210\,000\,000$ adults

As of 2019, there were about $210\,000\,000$ adults in the United States.

On average, they each purchased 60 clothing items per year.

About how many clothing items did all of the adults in the United States purchase in 2019?

Express your answer using scientific notation.

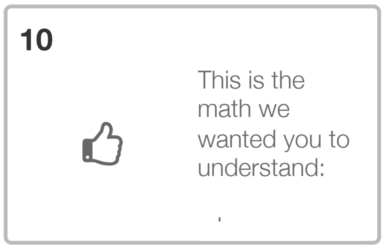
Teacher Moves

Support for Future Learning
If students struggle with applying operations with numbers expressed in scientific notation, consider making time to explicitly revisit these ideas. A strong understanding of comparing, combining, and operating with numbers expressed in scientific notation will support students on the upcoming end assessment.

Facilitation
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

$1.26 \cdot 10^{10}$ items



10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use scientific notation to compare different quantities and answer questions about real-world situations.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

8.7 Practice Day 2 (NYC)

Preparation

Option 1: Speed Dating

- Arrange students and desks in two long rows facing each other. Print and cut the double-sided problem cards with the problems on the front and the solutions on the back.

Option 2: Task Cards

- Print one set of cards for each group of students. You may choose to print the cards single-sided or double-sided (with the solution on the back).

Instructions

Option 1: Speed Dating

Distribute one problem card to each student. Give students 5–6 minutes to solve and become an expert on their problem. Encourage students to check their answer with the solution on the back of the card. Then invite students to trade their problem with the person across from them. Give students several minutes of quiet-think time to solve their new problem. If they have a question about the problem, encourage students to consult the problem's expert.

When the class is ready, students get their original problem back and move one desk to the right. Now, with a new partner, students trade problems. Repeat as many times as possible in the time you have.

Note that there are 24 problem cards. If you have more than 24 students, or prefer to use a subset of the cards, consider separating the class into two groups and printing a set of cards for each group.

Option 2: Task Cards

Arrange students into groups of 2–3. Distribute one set of cards to each group of students. If cards are single sided, consider posting the solution cards, or walking around with them and providing feedback to students as they work.

GRADE 8

Unit 8
Lesson Plans

Teacher lesson plans from Unit 8 are included here to provide NYC reviewers with access to the specific lessons in Amplify Desmos Math New York that demonstrate coverage of the **Expressions, Equations, and Inequalities** domain.

These lessons are partially designed and will be updated to match the exemplar Teacher Edition lessons included earlier in this sampler.

NOTE: *We have included only those lessons from Unit 8 that cover the standards in the Expressions, Equations, and Inequalities domain.*

Grade 8 Unit 8

Teacher Edition Sampler

Unit at a Glance

Key

Print Lessons

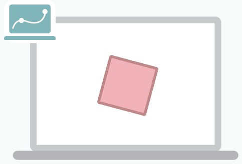
Digital Lessons

Assess and Respond

Pre-Unit Check
(Optional)

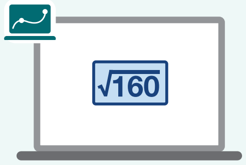
Use student performance to provide support and strengthen student understanding with targeted prerequisites concepts.

Sub-Unit 1



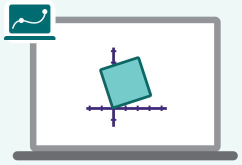
1 Tilted Squares

Calculate the area of a square with vertices at the intersection of grid lines using various strategies.



2 From Squares to Roots

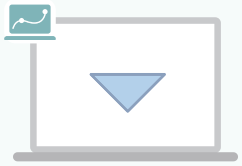
Understand that the square root of a means the side length of a square whose area is a square units.



3 Between Squares

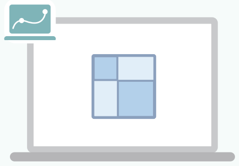
Approximate the value of a square root (e.g., by determining the two integer values it lies between or by drawing a square).

Sub-Unit 2



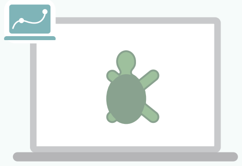
6 The Pythagorean Theorem

Describe patterns in the relationship between the squares of side lengths of a right triangle, called the Pythagorean theorem.



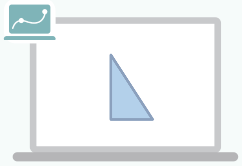
7 Pictures to Prove It

Understand one proof of the Pythagorean theorem.



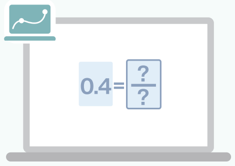
8 Triangle-Tracing Turtle

Calculate unknown side lengths of a right triangle by using the Pythagorean theorem.



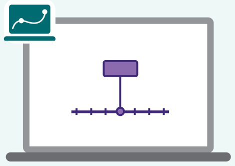
9 Make It Right

Determine whether a triangle with given side lengths is a right triangle using the converse of the Pythagorean theorem.



13 Decimals to Fractions

Express a repeating decimal as a fraction.



14 Hit the Target

Understand that rational numbers are defined as numbers that can be written as a fraction of two integers.

Summative Assessment

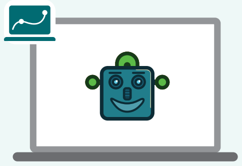
End-of-Unit Assessment

Use student performance to provide support. strengthen student understanding, and offer stretch opportunities to extend student learning.

Pacing: 18 days | **Short on time?** See pacing considerations below.

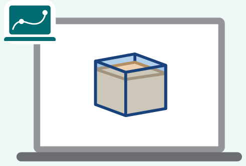
Pre-Unit Check: (Optional)
14 Lessons: 45 min each
2 Practice Days: 45 min each

1 Sub-Unit Quiz: 45 min
End-of-Unit Assessment: 45 min



4 Root Down

Represent a square root as a point on a number line.



5 Filling Cubes

Understand that the cube root of a (or $\sqrt[3]{a}$) means the side length of a cube whose volume is a cubic units.

Practice Day

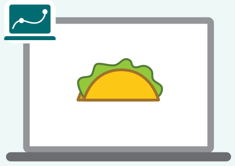
Practice Day 1

Practice the concepts and skills developed during Lessons 1–5. Consider using this time to prepare for the upcoming Quiz.

Assess and Respond

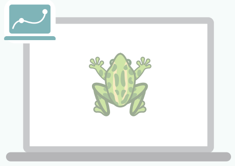
Quiz: Sub-Unit 1

Use student performance to provide support, strengthen student understanding, and offer stretch opportunities to extend student learning.



10 Taco Truck

Use the Pythagorean theorem to solve problems within a context.



11 Pond Hopper

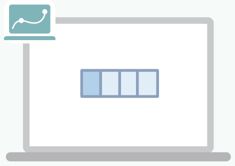
Calculate the distance between two points in the coordinate plane by using the Pythagorean theorem.

Practice Day

Practice Day 2

Practice the concepts and skills developed during Lessons 1–11. Consider using this time to prepare for the upcoming Quiz.

Sub-Unit 3



12 Fractions to Decimals

Express a fraction as either a repeating or a terminating decimal.

Pacing Considerations

Lesson 3: This lesson supports students in making connections between $\sqrt{a}$ as a side length of a square and $\sqrt{a}$ as a value on a number line, which will be addressed in more depth in upcoming lessons. If this lesson is omitted, provide extra support for students as they reason about the values of square roots in Lesson 4.

Lesson 5: This lesson extends students' work with the side lengths of squares as $\sqrt{a}$ to the side lengths of cubes as $\sqrt[3]{a}$. If time is tight, this lesson may be omitted.

Lesson 9: This lesson supports students in applying the Pythagorean theorem to identify right triangles. If students show a strong understanding working with the Pythagorean theorem in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how to use the Pythagorean theorem to determine whether a triangle includes a right angle elsewhere in the unit.

Lesson 11: This lesson gives students an opportunity to apply the concepts they learned in this unit to calculate lengths in the coordinate plane. There is no new content introduced in this lesson.

Lessons 12-13: These lessons support students with connecting unit fractions with their decimal representations. These lessons could be consolidated into one class period if students show a strong understanding of angle relationships in earlier lessons and in Problems 4 and 5 of the Pre-Unit Check.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

From Squares to Roots

Lesson 2: Side Lengths and Areas

Overview

Students learn that the square root of a can be thought of as the side length of a square whose area is a square units.

Learning Goals

- Understand that the square root of a (or $\sqrt{a}$) means the side length of a square whose area is a square units.
- Use square root notation to represent the side length of a square given its area.

Vocabulary

- square root

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students learn about square roots. The warm-up helps them think about placing an irrational side length on the number line. Once students locate the side length of the square as a point on the number line, they are formally introduced to square roots and square root notation:

$\sqrt{a}$ is the length of a side of a square whose area is a square units.

Then students consider square roots as lengths and use these lengths—with square root notation—to direct a turtle along a square's side.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to get students thinking about the side lengths of squares—especially non-integer lengths. Students begin with estimating these lengths before learning to precisely notate them later in the lesson.

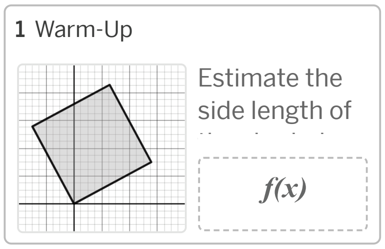
Activity 1: Square Roots (30 minutes)

The purpose of this activity is to introduce students to square root notation and to the idea of a square root as a precise number that can be approximated with a decimal (MP6). The key idea is that if we know the area of the square, then we can find its side length by taking the square root of its area.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to have students explain the relationship between a square's area and its side length and the role of the square root symbol in describing that relationship.

Cool Down (5 minutes)



1 Warm-Up

Estimate the side length of

Estimate the side length of the shaded square.

Teacher Moves

Purpose
The purpose of this lesson is for students to understand that the side length of a square is equal to the square root of its area.

Warm-Up Launch
Tell students that in this activity, they are going to find the lengths of sides of squares. They'll start by estimating the side lengths of squares before moving on to more precise methods.

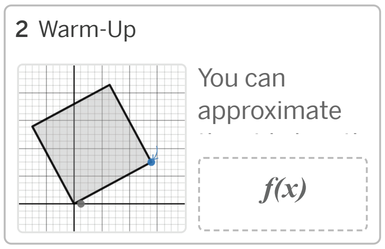
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. Then consider using the dashboard to show the range of student responses on this screen. It's okay—even desirable—to lack consensus at this stage. The activity will build towards consensus later on as more precise methods are introduced.

Facilitation
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

Responses vary.

12 units



2 Warm-Up

You can approximate

You can approximate the side length of the square by rotating it onto an axis and counting squares.

Drag the blue point down to the x-axis to see the side length of the shaded square.

Then enter a new estimate for the side length of the square.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

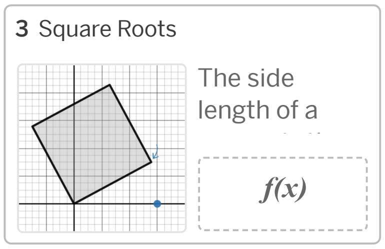
Students should understand that by dragging the blue point to the x-axis, they can use the x-axis as a number line to estimate the side length of the square. When we rotate the square about the origin, the side lengths are preserved because a rotation is a rigid transformation.

Consider using the teacher view in the dashboard to show how the range of student responses has changed compared to Screen 1. Tell students that the goal of this activity is to learn a strategy for finding the exact side lengths of squares. This strategy will be introduced on Screen 3.

Sample Responses

Responses vary.

12.6 units


3 Square Roots

The side length of a

The side length of a square is the square root of its area.

You can write the side length of this square as $\sqrt{160}$.

Enter $\sqrt{160}$ in the space below. (Hint: Type $sqrt$.)

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to surface connections between the area of a square, its side length, and the square root symbol. In particular, if we know the area of the square, then we can find its side length by taking the square root of its area.

Teacher Moves
In order to successfully complete this activity, students will need to know how to type the square root symbol. Encourage students to check out the “Learn how to type math” link located below the math input, or tell them that they can type $sqrt$ followed by the number that they are taking the square root of.

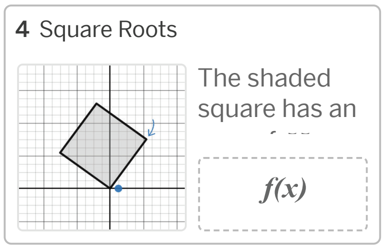
Use responses mode in the teacher dashboard to identify students who may need additional support.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Sample Responses

$\sqrt{160}$



4 Square Roots

The shaded square has an

The shaded square has an area of 55 square units.

Enter the exact value of the side length of the square and press "Try It."

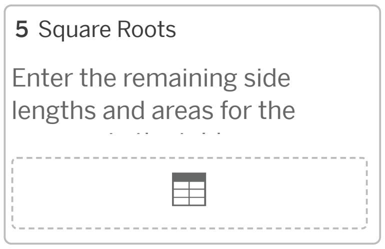
Teacher Moves

Teacher Moves
Use the teacher view in the teacher dashboard to identify students who may need additional support.

Facilitation
Consider using pacing to restrict students to Screens 4–5.

Sample Responses

$\sqrt{55}$



5 Square Roots

Enter the remaining side lengths and areas for the

Enter the remaining side lengths and areas for the squares in the table.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

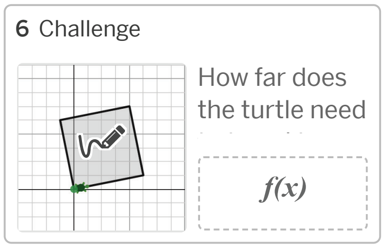
Side length of Square B: $\sqrt{81}$ or 9
Area of Square C: 6.25
Side length of Square D: $\sqrt{14}$
Area of Square E: 44

Side length of Square F: $\sqrt{32}$ or $4\sqrt{2}$

Student Supports

Students With Disabilities
- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation in the activity.


6 Challenge

How far does the turtle need

How far does the turtle need to travel to trace one edge of the shaded square?

Use the sketch tool if it helps you to show your thinking.

Teacher Moves

Teacher Moves
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Facilitation
Consider using pacing to restrict students to this screen.

Early Student Thinking
If students are estimating the side lengths of squares, encourage them to instead find the area of the square using one of the methods from Lesson 1, such as “decompose and rearrange” or “surround and subtract.” Once they know the area of the square, they can take the square root of the area to find the exact side length of the square.

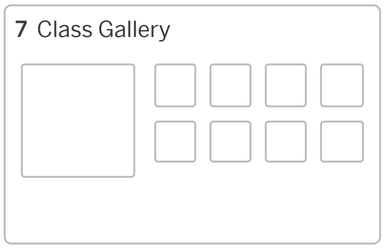
Sample Responses

$\sqrt{26}$

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

The challenges on Screen 7 are similar to the challenge on this screen. Consider beginning with a demonstration of this first problem, which will provide access for students who benefit from clear and explicit instructions. Also check in with individual students, as needed, to assess for comprehension during each step of the activity.



7 Class Gallery

Teacher Moves

Teacher Moves
Here students will create their *own* challenges and solve challenges from their classmates. We recommend students complete Screens 1–6 before creating their challenge. We anticipate this Challenge Creator could take 20 minutes or more.

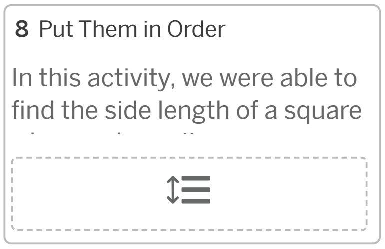
Encourage students to complete each other's challenges but also to take some time to review responses to their own. Use the teacher dashboard to look for unique challenges and solutions that may expand your students' understanding of the mathematics. Highlight those for students and also ask them what they learned from the experience.

We intend for this to be a social and creative experience for students. We encourage you to emphasize those virtues whenever you see them in your class.

Facilitation
Consider using pacing to restrict students to Screens 7–9.

Sample Responses

Responses vary.



8 Put Them in Order

In this activity, we were able to find the side length of a square

In this activity, we were able to find the side length of a square when we knew its area.

Use this knowledge to help you order the squares by size.

Teacher Moves

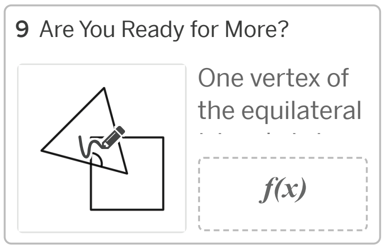
Encourage students to share their reasoning with a partner and work to reach an agreement during the task.

Use the teacher view in the teacher dashboard to identify students who may need additional support.

Sample Responses

From smallest square to largest square:

Area: 50 square units
Side length: $\sqrt{55}$ units
Side length: 8 units
Side length: $\sqrt{81}$ units



9 Are You Ready for More?

One vertex of the equilateral

One vertex of the equilateral triangle is in the center of the square, and one vertex of the square is in the center of the equilateral triangle.

What is the value of x?

Consider using paper if that helps to support your thinking.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 6–8 ahead of time before the class discussion on Screen 10. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

105°

Responses vary.

Draw a segment to connect the center of the square to the center of the equilateral triangle. This segment cuts the 90° angle at the center of the equilateral triangle in half because a line from a vertex of an equilateral triangle through its center is a line of symmetry. In the same way, the segment cuts the 60° angle at the center of the square in half. This segment creates a new triangle with angles 45°, x°, and 30°. Since the angles in a triangle must sum to 180°, x must be equal to 105°.

10 Lesson Synthesis

Discuss the following

Discuss the following questions.

Then select ONE question and record your response.

Use the sketch tool if it helps to support your thinking.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to consolidate the relationship between the side length of a square and its area, particularly that the square root of the area of a square gives its side length.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers for the class.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

1. $\sqrt{100} = 10$ means that the side length of a square with an area of 100 square units is 10 units.
2. If $\sqrt{17}$ is the side length of a square, that means its area is 17 square units.

---

11 Cool-Down

Complete the table for each square.

Complete the table for each square.

Teacher Moves

Support for Future Learning
If students struggle to understand the relationship between the area of a square and its side length, consider reviewing this cool-down as a class before Lesson 3, or offering individual support where needed during Lesson 3.

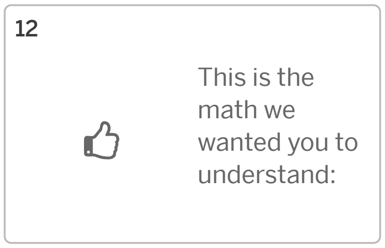
Readiness Check (Problem 7)
If most students struggled, plan to revisit the problem after this lesson. If needed, provide a list or visual display of perfect squares and cubes, and highlight the values of perfect squares and cubes as they arise throughout the unit.

Facilitation

Consider using pacing to restrict students to Screens 11–12.

Sample Responses

Side length of Square A: $\sqrt{100}$ or 10
Side length of Square B: $\sqrt{95}$
Area of Square C: 36
Side length of Square C: $\sqrt{30}$



12

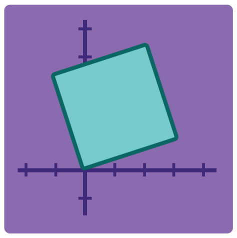
This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain the meaning of square roots in terms of side length and area of a square.

- I can write the side length or the area of a square using square root notation (like $\sqrt{3}$).

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.



Between Squares

Lesson 3: Approximating Square Roots

Overview

Students begin to shift from a focus on $\sqrt{a}$ as the side length of a square with area a to a more abstract understanding of $\sqrt{a}$ as a number on a number line and a solution to the equation of the form $x^2 = a$.

Learning Goals

- Approximate the value of a square root (e.g., by determining the two integer values it lies between or by drawing a square).

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you’ll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students begin to shift from a focus on $\sqrt{a}$ as the side length of a square with area a to a more abstract understanding of $\sqrt{a}$ as a number on a number line and a solution to the equation of the form $x^2 = a$. In the first activity, students still find $\sqrt{10}$ by relating it to the side length of a square of area 10 square units, but they are asked to approximate the value of $\sqrt{10}$ to the nearest tenth. In the second activity, students find a decimal approximation for $\sqrt{5}$ by looking at areas and by computing the squares of numbers. In the next lesson, they will focus primarily on square roots as numbers.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to transition students from work in previous lessons and prepare them to locate square roots on a number line in this lesson.

Activity 1: Squaring Lines (10 minutes)

The purpose of this activity is for students to determine the length of a *diagonal* line segment on a grid. Students can give an exact value for the length of the line segment by finding the area of a square and writing the side length using square root notation. The goal of this activity is for students to connect values expressed in square root notation with values expressed in decimal form. Students use the structure of the circle to relate the length of the segment to a point on the number line (MP7).

Activity 2: The Square Root of 5 (20 minutes)

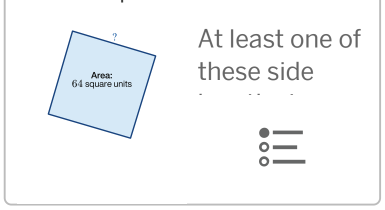
The purpose of this activity is for students to start with a square root of an integer and use a square to verify that a given approximation of the square root is reasonable. In previous activities and lessons, students found the exact area of a square in order to find an approximation for the square root of an integer. This is the first time students see or draw squares that do not have vertices at the intersection of grid lines, so it may take them a few minutes to make sense of the new orientation.

Lesson Synthesis (5 minutes)

The purpose of the discussion is to check that students know how to approximate square roots to the nearest tenth.

Cool-Down (5 minutes)

1 Warm-Up



At least one of these side

At least one of these side lengths is correct.

Select one correct expression for the side length of this square and explain how you know it's correct.

Teacher Moves

Purpose
The purpose of this lesson is for students to begin to shift from a focus on $\sqrt{a}$ as the side length of a square with area a to a more abstract understanding of $\sqrt{a}$ as a number on a number line and a solution to the equation of the form $x^2 = a$.

Warm-Up Launch
Give students two minutes to respond to this question and a few minutes to share their responses with their partner. Follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

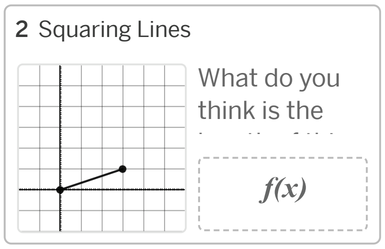
Readiness Check (Problem 2.1)
If most students struggled, revisit the problem before this warm-up. Encourage students to use the Desmos scientific calculator to help them calculate the values of square and cube roots as needed throughout the unit.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

- $\sqrt{64}$. The side length of a square is the square root of the area.
- 8. The area is found by squaring the side length, and 8^2 is 64 square units.



2 Squaring Lines

What do you think is the

What do you think is the length of this segment?

Use any tools you'd like to help you with your thinking.

Teacher Moves

Activity Launch
For this activity, it is best if students do not have access to a calculator with a square root button. If student calculators do have a square root button that students are familiar with, tell students that their explanations about their answers need to dig deeper than pressing a button. In later lessons, however, they will be able to use it.

Begin by displaying the diagram for all to see. Ask students how this diagram is similar and how it is different from the square in the warm-up. Show students the tools available above the graph to help them determine the side length of the square. Then a few minutes after students begin working, pause the class and select previously identified students who drew a square on the grid to share what they did and why. Give them a few minutes to finish the problems and follow with a whole-class discussion.

Consider using snapshots to highlight and display unique strategies. Select students to present their thinking in this sequence:

- Someone who drew a square and used the area to find the exact side length.
- Someone who used the ruler. Ask students what this tells us about the exact value we found by drawing a square.
- Someone who used a circle to find the approximate side length. This is a more formal geometric construction, but it is another way to use the number line as a ruler.

Facilitation
Consider using pacing to restrict students to Screens 2–3, one screen at a time.

Sample Responses

Responses vary.

- $\sqrt{10}$ units
- 3.2 units

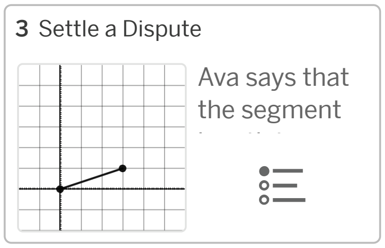
Student Supports

English Language Learners
- *MLR 2 (Collect and Display)*

Circulate and listen to students talk during pair work or group work, and jot notes about common or important words and phrases, together with helpful sketches or diagrams. Record students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson.

Students With Disabilities
- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity.



3 Settle a Dispute

Ava says that the segment

Ava says that the segment length is $\sqrt{10}$ because the area of a square with its side on the segment is 10 square units.

Rebecca says the segment length is about 3.2 units because when you use the segment as the radius of a circle, that circle's radius is about 3.2 units.

Who is correct?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface the difference between exact values and approximations of square roots.

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

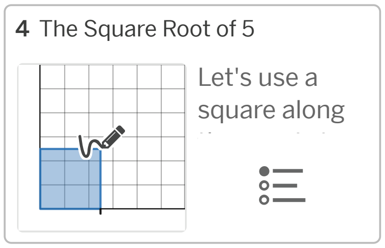
If it does not come up, remind students about the difference between an exact length and an approximation. For example, a square with an area of 17 square units has a side length of exactly $\sqrt{17}$ units, which is a little larger than 4 because $4^2 = 16$.

Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Responses vary.

- Ava. Their answer is more precise than Rebecca's.
- Rebecca. The ruler showed that the segment length was 3.2 units.
- Both. They are both correct, but $\sqrt{10}$ is the exact length, while 3.2 is an approximation.



4 The Square Root of 5

Let's use a square along

Let's use a square along the x-axis to estimate square roots.

Use this square to decide whether $\sqrt{5}$ is greater or less than 2.5.

Teacher Moves

Activity Launch
Display the diagram for all to see. Ask students what is the same and what is different about this diagram compared to diagrams they have seen in earlier activities. Tell students that for this activity, they will try to approximate $\sqrt{5}$. Arrange students into pairs. Give students 2–3 minutes of quiet work time and a few minutes to share their responses with their partner. Follow with a whole-class discussion.

Early Student Thinking
Some students may incorrectly think that $\sqrt{5}$ is the same as dividing 5 by 2. Be sure to ask students to find the area of the square when the side length is 2.5 units. Use the equation $2.5^2 = 6.25$ to connect the fact that the area of a square whose side length is 2.5 units is 6.25 square units.

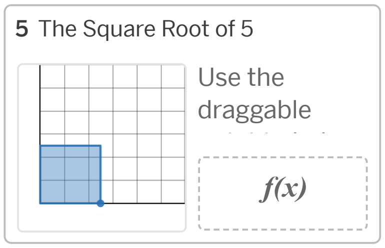
Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

$\sqrt{5}$ is less than 2.5.

Responses vary.

When the side length is 2.5 units, the square's area is 6.25 square units. For the area to be 5 square units, the side length must be less than 2.5 units.



5 The Square Root of 5

Use the draggable

Use the draggable point to help you estimate $\sqrt{5}$. Then enter your estimate below.

Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

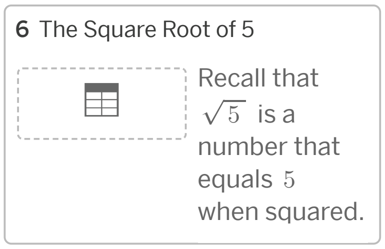
Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

2.3

Responses vary.

Values between 2 and 2.5 are considered correct in the teacher dashboard.



6 The Square Root of 5

Recall that $\sqrt{5}$ is a number that equals 5 when squared.

Recall that $\sqrt{5}$ is a number that equals 5 when squared.

Enter decimals in the table to approximate $\sqrt{5}$ as close as you can.

You have 10 tries remaining.

Teacher Moves

Invite one or two students to share their squares. Then consider telling students, "The square of a point on the number line can be visualized as the area of a literal square. This can help us estimate the value of a square root. We can simply square the number as well. Let’s check the squares of some numbers that are potential approximations of $\sqrt{5}$."

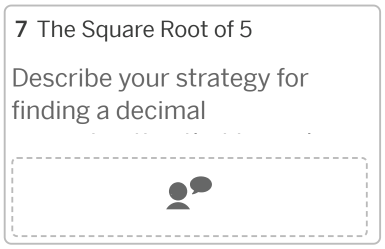
Ask students to suggest decimal approximations and to check their answers by finding their squares. Students should be using each guess to make their next guess better. For example, if they try 2 , then the square is 4, which is too low. This suggests trying a larger number next. We know that 2.5 is too big because $2.5^2 = 6.25$, so the decimal should be somewhere in between 2 and 2.5.

Facilitation
Consider using pacing to restrict students to Screens 6–9.

Sample Responses

Responses vary.

Students should input values in the table approaching 2.236.



7 The Square Root of 5

Describe your strategy for finding a decimal

Describe your strategy for finding a decimal approximation that is as close as possible to $\sqrt{5}$.

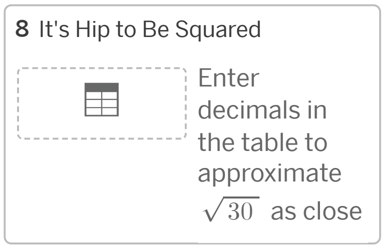
Teacher Moves

Highlight unique answers to show the class. Ask students to justify their responses and critique each other's reasoning.

Sample Responses

Responses vary.

I started with 2.1^2 and noticed it was too low, so I tried 2.4^2, but that was too high. I used each approximation to make a better next approximation.



8 It's Hip to Be Squared

Enter decimals in the table to approximate $\sqrt{30}$ as close

Enter decimals in the table to approximate $\sqrt{30}$ as close as you can.

You have 10 tries remaining.

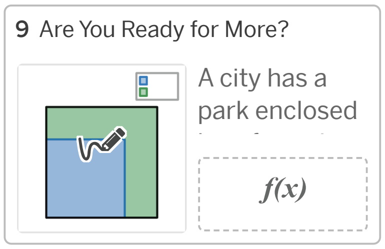
Teacher Moves

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Sample Responses

Responses vary.

Students should input values in the table approaching 5.477.


9 Are You Ready for More?

A city has a park enclosed

A city has a park enclosed by a fence in the shape of a square with 4-meter side lengths.

The city would like to build a pool by making a smaller square filled with water, as shown in the figure.

What should the side length of the smaller square be so that half of the area is grass and half is water?

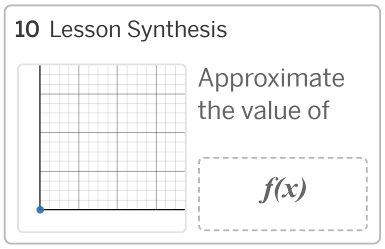
Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 6–8 ahead of time before the class discussion on Screen 10. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

The area enclosed by the fence is 16 square meters, so we want the area of both the grassy region and the water region to be 8 square meters. For the blue square in the figure to have an area of 8 square meters, the side length needs to be $\sqrt{8}$ meters, or about 2.8 meters.



10 Lesson Synthesis

Approximate the value of

Press "Return" to calculate and add a row.

You have 10 tries.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for approximating the value of a square root.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

Ask questions to help students connect concrete responses using the square and abstract responses using the table, as well as formal and informal responses. The goal of this discussion is to check that students know how to approximate square roots and relate squaring numbers to the area of a square.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

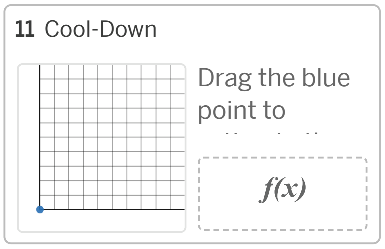
Sample Responses

11.4

Responses vary.

Values between 11 and 12 are considered correct in the teacher dashboard.

I knew $\sqrt{130}$ had to be between 11 and 12 because $11^2 = 121$ and $12^2 = 144$, so I made a square and tried to make the area inside the square equal to 130 square units.



Drag the blue point to

Drag the blue point to estimate the location of $\sqrt{18}$ on the x-axis.

Then approximate $\sqrt{18}$ as a decimal.

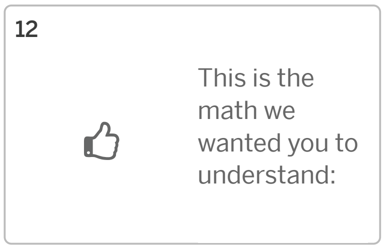
Teacher Moves

Support for Future Learning
Students will have more chances in Lesson 4 to develop their understanding of representing the $\sqrt{a}$ as a number on the number line.

Facilitation
Consider using pacing to restrict students to Screens 10–11.

Sample Responses

The point and the decimal response should be between 4.1 and 4.4.

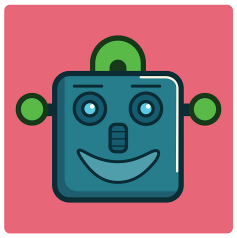


This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can approximate a square root as a decimal.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Root Down

Lesson 4: Reasoning About Square Roots

Overview

Students are encouraged to reason about square roots and to reinforce the idea that square roots are numbers on a number line. This lesson continues students' progression from geometric to algebraic characterizations of square roots.

Learning Goals

- Represent a square root as a point on a number line.
- Identify the two whole number values that a square root is between and explain the reasoning.

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

The purpose of this lesson is to encourage students to reason about square roots and to reinforce the idea that square roots are numbers on a number line. This lesson continues the student's progression from geometric to algebraic characterizations of square roots.
In this lesson, students think about square roots in relation to the two whole number values they are closest to. Students are encouraged to use numerical approaches rather than less efficient geometric methods, especially the fact that $\sqrt{a}$ is a solution to the equation $x^2 = a$. Students can use a number line if it helps them reason about the magnitude of the given square roots, but this is not required. No matter what strategy students provide, they must construct viable arguments (MP3) to justify their ordering of numbers and square roots.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to think about square roots in relation to the two whole number values they are closest to. Students will use the number line and values of familiar square roots as reference points when determining the placement of $\sqrt{50}$.

Activity 1: Card Sort (10 minutes)

The purpose of this activity is for students to think about square roots in relation to the two whole number values they are closest to and to develop a strategy to reason about square roots.

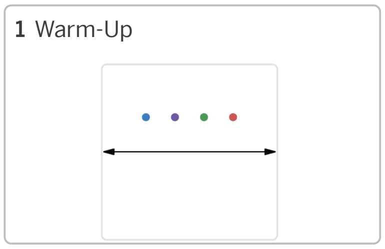
Activity 2: Order These Values (20 minutes)

The purpose of this activity is for students to reason abstractly and quantitatively (MP2), and develop fluency in determining the approximate value of square roots. In the activity, students create and engage in challenges where they order sets of five numbers written as values and as square roots.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to solidify the learning goals of this lesson and reinforce the definition of a square root as a solution to the equation of the form $x^2 = a$.

Cool-Down (5 minutes)



Place these values at the correct locations on the number line.

Teacher Moves

Purpose
The purpose of this lesson is to encourage students to reason about square roots and to reinforce the idea that square roots are numbers on a number line. Calculators should not be used for this lesson.

Warm-Up Launch
The purpose of this warm-up is for students to think about square roots in relation to the two whole number values they are closest to. Tell students to use the number line and the values of familiar square roots as reference points when determining the placement of $\sqrt{50}$.

Arrange students into pairs. Allow two minutes of quiet work time, followed by a whole-class discussion.

Readiness Check (Problem 3)
If most students struggled, plan to spend extra time on this warm-up. Consider inviting students to discuss where 3^2 and 0.5^3 would go on the number line.

Facilitation
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

- $\sqrt{49} = 7$
- $\sqrt{64} = 8$
- $\sqrt{9} = 3$
- $\sqrt{25} = 5$

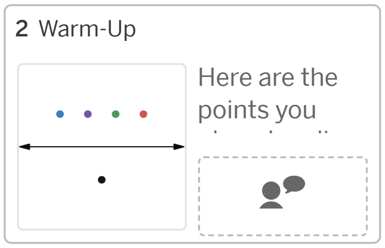
Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Check in with individual students, as needed, to assess for comprehension during each step of the activity.

- *Memory: Processing Time*

Provide students with a poster or a reference of the values of perfect square numbers.


2 Warm-Up

Here are the points you

Here are the points you placed earlier.

Estimate the correct location on the number line for $\sqrt{50}$.

What strategy could you use to figure out where to place any square root on the number line?

Teacher Moves

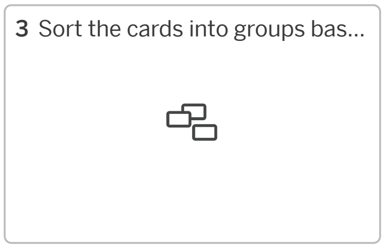
Invite several students to share their responses. Then consider using the overlay view in the teacher dashboard to show the distribution of student responses. Call attention to any conflict or consensus you see.

Sample Responses

$\sqrt{50}$ should be slightly to the right of 7 on the number line.

Responses vary.

To figure out where to place a square root on a number line, find the two perfect squares a number is between.


3 Sort the cards into groups bas...

Teacher Moves

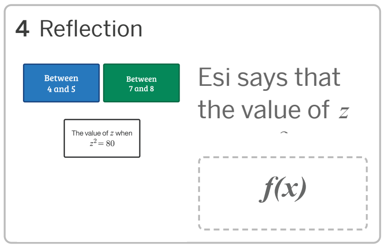
Activity Launch
Tell students that they will sort square root expressions with the pair of consecutive integers that they fall between. There may be some cards that do not fit in either category.

Arrange students into pairs. Give five minutes of group work time, followed by a whole-class discussion. Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Facilitation
Consider using pacing to restrict students to Screens 3–4.

Sample Responses

Image solution



4 Reflection

Esi says that the value of z when $z^2 = 80$ does not belong in either of these categories since z must be greater than 8.

What whole number would z be closest to?

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to surface strategies for estimating the value of a square root.

Use snapshots or the teacher view of the dashboard to display unique answers to the class. Ask students to justify their responses and critique each other's reasoning.

If time allows, consider asking students whether z is greater than 9 or less than 9, and to explain how they know.

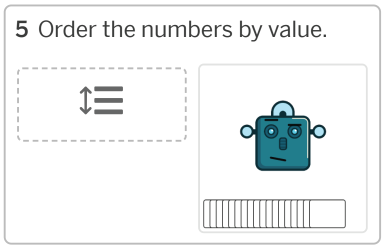
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

9

Responses vary.

Since $9^2 = 81$, $\sqrt{80}$ would be slightly less than 9.



5 Order the numbers by value.

Teacher Moves

Activity Launch
Tell students that they will use the robot to help them order the numbers from least to greatest in as few attempts as possible.

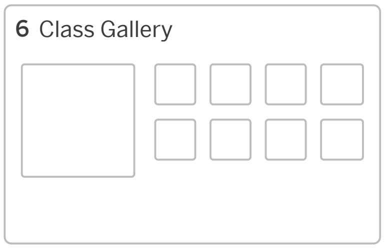
Arrange students into pairs. Encourage students to share their reasoning with a partner and work to reach an agreement during the task.

Facilitation
Consider using pacing to restrict students to Screens 5–6 one at a time.

Sample Responses

Least to greatest:

- $\sqrt{75}$
- 9
- 9.5
- $\sqrt{99}$
- 10



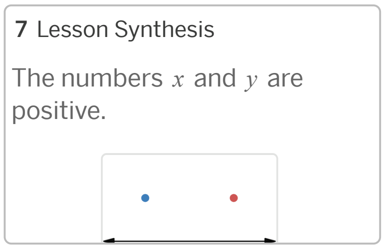
6 Class Gallery

Teacher Moves

Here students will create their *own* challenge and solve challenges from their classmates. We recommend students complete Screen 5 before creating their challenge. We anticipate this Challenge Creator could take 15 minutes or more.

Encourage students to complete each other's challenges but also to take some time to review responses to their own challenge. Use the teacher dashboard to look for unique challenges and solutions that may expand your students' understanding of the mathematics. Highlight those for students and also ask them what they learned from the experience.

We intend for this to be a social and creative experience for students. We encourage you to emphasize those virtues whenever you see them in your class.



7 Lesson Synthesis

The numbers x and y are positive.

The numbers x and y are positive.

$x^2 = 3$ and $y^2 = 35$.

Plot x and y on the number line.

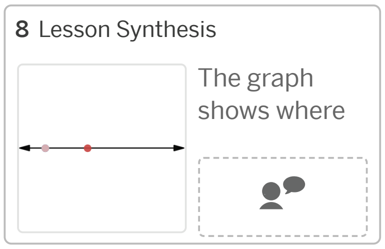
Teacher Moves

Synthesis Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses.

Facilitation
Consider using pacing to restrict students to Screens 7–8.

Sample Responses

- x should be placed between 1.5 and 2.
- y should be placed between 5.5 and 6.



8 Lesson Synthesis

The graph shows where

The graph shows where your classmates placed x and y. $x^2 = 3$ and $y^2 = 35$.

Explain how you decided where to place those values on the number line.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to summarize how to determine where the value of a square root belongs on a number line.

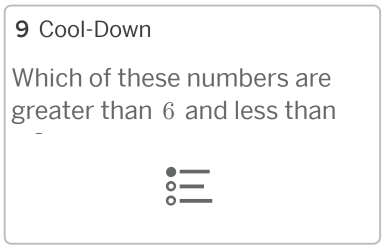
Display the student view of the number line for all to see. Then select groups to share how they chose to place values on the number line. Use snapshots or the teacher dashboard to highlight unique student reasoning.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

I know the square root of 4 is 2, and since 3 is one less than 4, x should be slightly less than to 2. I know the square root of 36 is 6, and since 35 is one less than 36, y should be slightly less than 6.


9 Cool-Down

Which of these numbers are greater than 6 and less than

Which of these numbers are greater than 6 and less than 8?

Teacher Moves

Support for Future Learning

If students struggle to represent the $\sqrt{a}$ as a number on the number line, consider making time to explicitly revisit this idea. Some opportunities include reviewing Problems 1, 4, and 8 on Practice Day 1. A strong understanding of representing the $\sqrt{a}$ as a number on the number line will support students on the upcoming quiz.

Facilitation

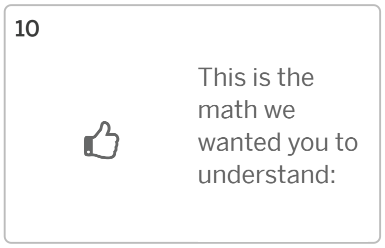
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

- $\sqrt{47}$
- $\sqrt{60}$

Responses vary.

Since $6^2 = 36$ and $8^2 = 64$, the square roots of values between 36 and 64 will evaluate to be between 6 and 8.


10

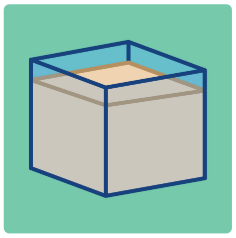
This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can plot square roots on a number line.

- I can identify the two whole numbers a square root is between and explain why.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.



Filling Cubes

Lesson 5: Edge Lengths, Volumes, and Cube Roots

Overview

Students explore the relationship between the edge length and the volume of a cube and develop an efficient method for determining an unknown edge length from a known volume.

Learning Goals

- Understand that the cube root of a (or $\sqrt[3]{a}$) means the side length of a cube whose volume is a cubic units.
- Approximate the value of a cube root (e.g., by determining the two integer values it lies between or by drawing a cube).

Materials

- Blank paper

Vocabulary

- cube root

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students explore the relationship between the edge length and the volume of a cube and develop an efficient method for determining an unknown edge length from a known volume.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to begin reasoning about cube roots by ordering the solutions of two quadratic equations and two cubic equations.

Activity 1: Filling Cubes (15 minutes)

The purpose of this activity is for students to explore the relationship between the edge length and the volume of a cube in the context of filling cubes with sand. First, students determine the volume of a cube when given the edge length. Next, students consider four cases (two with unknown volumes and two with unknown edge lengths). In each case, an exact answer can be found without using the cube root symbol. Then, students consider a case where an exact answer cannot be found using only decimals, which provides motivation for the introduction of the cube root symbol. Students then apply their new skills and notation to four new filling-cube cases.

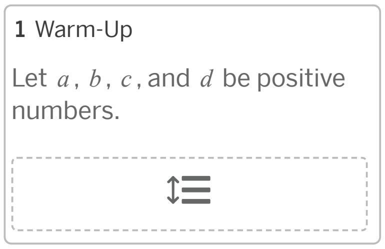
Activity 2: The Number Line (15 minutes)

The purpose of this activity is for students to approximate the value of a cube root and place cube roots of different values on a number line. First, students reason about the value of x when $x^3 = 30$. Next, students complete a card sort where they match each number (square root and cube root) with a corresponding figure (square or cube) or with a number line representation.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is for students to plot a cube root on the number line and to reason about the work of a student who placed that cube root incorrectly.

Cool-Down (5 minutes)



1 Warm-Up

Let a, b, c, and d be positive numbers.

Let a, b, c, and d be positive numbers.

Order the following by value.

Teacher Moves

Purpose
The purpose of this lesson is for students to a) explore the relationship between the edge length and the volume of a cube, b) to learn how to use the cube root symbol to determine an unknown edge length, and c) to reason about which two whole numbers a cube root is between.

Warm-Up Launch
Arrange students into pairs. Encourage students to share their reasoning with a partner and work to reach an agreement during the task.

Readiness Check (Problem 2.2)
If most students struggled, revisit the problem before this warm-up. Encourage students to use the Desmos scientific calculator to help them calculate the values of square and cube roots as needed throughout the unit.

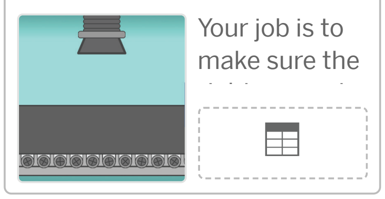
Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

From least to greatest:

- b when $b^3 = 8$
- d when $d^3 = 9$
- c when $c^2 = 8$
- a when $a^2 = 9$

2 Fill It Up


Your job is to make sure the

Your job is to make sure the right amount of sand ends up in each cube.

Each cube has an edge length of 6 inches.

How much sand will it take to fill each one?

Teacher Moves

Activity Launch
Tell students that in this activity they will explore the relationship between edge length and volume in the context of filling cubes with sand in a warehouse.

Consider using picture snapshots to highlight and display unique strategies. For example, some students may compute the answer (216 cubic inches) and enter it directly, while others may enter an expression (6^3 cubic inches).

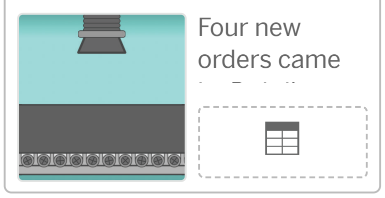
Facilitation
Consider using pacing to restrict students to Screens 2–4.

Sample Responses

216 cubic inches

---

3 Four Boxes


Four new orders came

Four new orders came in. Details are shown in the table.

Complete the table. Then press "Check My Work."

Teacher Moves

For more challenging calculations (e.g., 2.1^3), encourage students to use the calculator functionality built into each table cell. Consider inviting students to type 3^3 into the first row to see this functionality in action.

This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

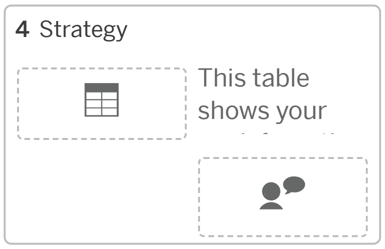
Sample Responses

- **Row 1:** 27 cubic inches
- **Row 2:** 9.261 cubic inches
- **Row 3:** 4 inches
- **Row 4:** 5 inches

Student Supports

Students With Disabilities
- *Executive Functioning: Eliminate Barriers*

Chunk this activity into more manageable parts (e.g., presenting one problem at a time) to aid students who benefit from support with organizational skills.



4 Strategy

This table shows your

This table shows your work from the previous two screens.

Describe the strategy you used to find the unknown edge lengths.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to solidify the relationship between edge length of a cube and its volume, and to surface strategies for determining an unknown edge length.

Consider using snapshots or the teacher view of the dashboard to display unique strategies. Ask students to justify their responses and critique each other's reasoning.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary.

To determine the edge length, take the cube root of the amount of sand.

Student Supports

Students With Disabilities

- *Receptive/Expressive Language: Peer Tutors*
Pair students with their previously identified peer tutors to aid in comprehension and expression of understanding.

5 Impossible Edge?

A customer wants a box with 100 cubic inches of sand.

A customer wants a box with 100 cubic inches of sand.

Can you find the exact side length?

To begin, enter a number and press "Calculate."

You have 10 tries remaining.

Teacher Moves

Tell students that the goal for this screen is to enter an edge length and get as close to 100 cubic inches of sand as possible. Give students 1-2 minutes of quiet think-time and a few minutes to share their results with a partner or the entire class. Invite students to describe their strategies.

After a discussion of strategies, tell students that the next screen will introduce a new symbol that makes it easy to find the *exact* value of the edge length for *any* amount of sand.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

The exact edge length is between 4.641 and 4.642 inches.

6 The Cube Root Symbol

We can write an equation to help us find the edge length of

f(x)

We can write an equation to help us find the edge length of a cube that holds 100 cubic inches of sand:

$$x^3 = 100$$

We can use a cube root to solve that equation.

Here is the exact solution: $x = \sqrt[3]{100}$

Here is an approximate solution: $x \approx 4.64$

Enter $\sqrt[3]{100}$ in the space below. (Hint: Type $cbrt$.)

Teacher Moves

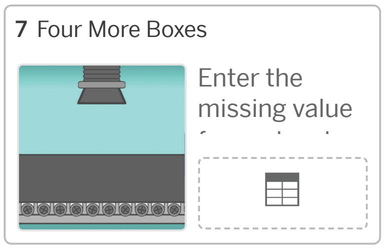
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Facilitation
Consider using pacing to restrict students to Screens 6–8.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Sample Responses

$\sqrt[3]{100}$



7 Four More Boxes

Enter the missing value

Enter the missing value for each cube.

Then press "Check My Work."

Teacher Moves

Encourage students to use $cbrt$ to calculate the exact edge lengths.

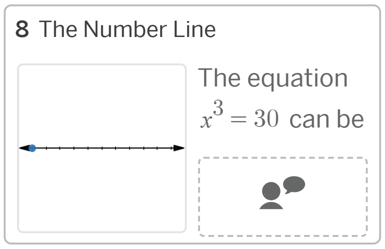
This is a great place to check student progress. Offer individual support where needed, or lead a whole-class discussion if enough students are struggling.

Consider using snapshots or the teacher view of the dashboard to display unique answers to the class. In particular, highlight answers expressed in exact form (using the cube root symbol).

Sample Responses

- **Row 1:** $\sqrt[3]{200}$ inches

- **Row 2:** $\sqrt[3]{150}$ inches
- **Row 3:** $\sqrt[3]{91.125}$ or 4.5 inches
- **Row 4:** 42 cubic inches



8 The Number Line

The equation $x^3 = 30$ can be

The equation $x^3 = 30$ can be used to find the edge length of a cube with a volume of 30 cubic inches.

Drag the point to show an approximate location for x when $x^3 = 30$.

Then describe your thinking.

Teacher Moves

Activity Launch
Tell students that in this activity, they will develop connections between cube roots and the number line.

Give students 1–2 minutes of quiet think-time and a few minutes to discuss with a partner. Use the overlay in the teacher view of the dashboard to display the distribution of points on the number line. Ask students to justify their responses and critique each other's reasoning.

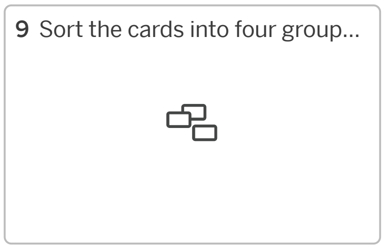
Facilitation
Consider using pacing to restrict students to Screens 8–10.

Sample Responses

The point should be slightly to the right of 3.

Responses vary.

I know that $3^3 = 27$ and $4^3 = 64$. If $x^3 = 30$, then x must be between 3 and 4. Since 30 is only slightly more than 27, x must be only slightly more than 3.



9 Sort the cards into four group...

Teacher Moves

Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Sample Responses

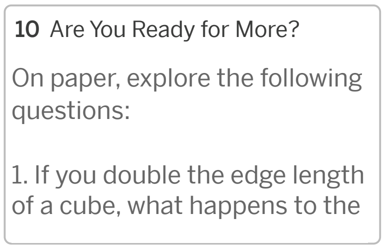
Image solution

Student Supports

Students With Disabilities

- *Conceptual Processing: Eliminate Barriers*

Allow students to use calculators to ensure inclusive participation in the activity.



10 Are You Ready for More?

On paper, explore the following questions:

1. If you double the edge length of a cube, what happens to the

On paper, explore the following questions:

1. If you double the edge length of a cube, what happens to the volume?

2. If you double the volume of a cube, what happens to the edge length?

Teacher Moves

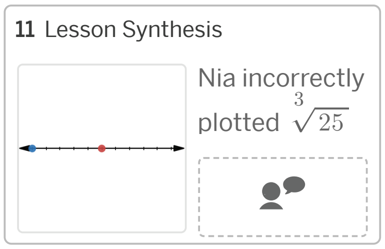
⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 5–9 ahead of time before the class discussion on Screen 11. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

Responses vary.

1. If you double the edge length, the volume is multiplied by 2^3, or 8.
2. If you double the volume, the edge length is multiplied by $\sqrt[3]{2}$.



11 Lesson Synthesis

Nia incorrectly plotted $\sqrt[3]{25}$

Nia incorrectly plotted $\sqrt[3]{25}$ as shown by the red point.

Drag the blue point to the correct approximate location of $\sqrt[3]{25}$.

Then describe what Nia might have been thinking when she misplaced the point.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to consolidate what students know about the value of a cube root.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

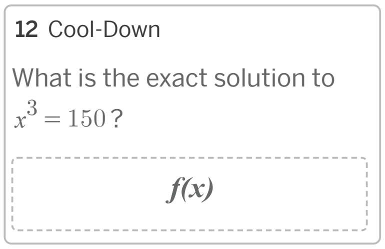
Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

The point should be slightly to the left of 3.

Responses vary.

$\sqrt[3]{8} = 2$ and $\sqrt[3]{27} = 3$, so $\sqrt[3]{25}$ must be between 2 and 3 (and relatively close to 3). Nia might have been thinking about $\sqrt{25}$, which is equal to 5.



12 Cool-Down

What is the exact solution to $x^3 = 150$?

What is the exact solution to $x^3 = 150$?

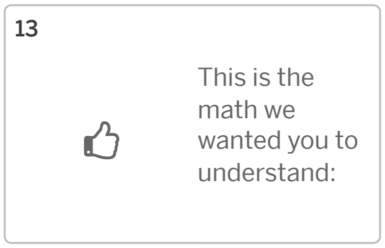
Teacher Moves

Support for Future Learning
If students struggle to approximate the value of a cube root, consider reviewing this cool-down as a class before Practice Day 1, or offering individual support where needed during the practice day. A strong understanding of approximating the value of a cube root will support students on the upcoming quiz assessment.

Facilitation
Consider using pacing to restrict students to Screens 12–13.

Sample Responses

$x = \sqrt[3]{150}$, which is between 5 and 6, because 150 is between $5^3 = 125$ and $6^3 = 216$.



13

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can explain the meaning of a cube root, like $\sqrt[3]{5}$, in terms of its edge length and volume.

- I can identify the two whole numbers a cube root is between and explain why.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

8.8 Practice Day 1 (NYC)

Preparation

Practice Worksheet

- Print one double-sided worksheet for each student.

Instructions

The practice worksheet includes two sets (Set A and Set B). The problems in each set are different, but the answers have the same numerical value (with the exception of different units in Problems 4, 7, and 12). Consider the following two options:

Option 1: Pairs
Arrange students into pairs. Give one student in the pair Set A and the other student Set B. Each student works to solve the problems in their set. When both students are done, encourage them to check their answers with each other. If students get different answers, they should work together to find their mistakes.

Option 2: Jigsaw
Split the class in half, giving Set A to one half of the class and Set B to the other half. Encourage students to work in pairs to complete their set. Once students are ready, invite them to stand up and pair with a student from the other half of the class. These new pairs will compare answers with each other. If students get different answers, they should work together to find their mistakes.

This lesson is still being upgraded to the Amplify Desmos Math design style for the 2024–25 school year.

Taco Truck

Lesson 10: Applications of the Pythagorean Theorem

Overview

Students use the Pythagorean theorem as a tool to solve problems involving diagonal distances.

Learning Goals

- Use the Pythagorean theorem to solve problems within a context.

Materials

- Blank paper

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students use the Pythagorean theorem as a tool to solve problems involving diagonal distances.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is to activate prior knowledge about the Pythagorean theorem and rates using a situation that students may encounter in their daily lives: walking diagonally to save time.

Activity 1: Taco Truck (15 minutes)

The purpose of this activity is to use the Pythagorean theorem to reason about distances and speeds in order to figure out who will win a race. Students encounter two people who want to get to a taco truck as quickly as possible and are asked to calculate the time it takes for each of them.

Activity 2: Zoe’s Route (15 minutes)

The purpose of this activity is to extend the previous context, giving students more opportunity to use the Pythagorean theorem as they reason about distances and speeds (MP2). In this activity, students choose a new route to the taco truck and calculate the time their chosen route would take. The activity culminates in a classwide race in which all the routes chosen by students go head to head.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to help students see structure in expressions that involve the Pythagorean theorem.

Cool-Down (5 minutes)

1 Warm-Up

Alma is going to walk

f(x)

Alma is going to walk through the park from point A to point B.

What distance will she walk?

Teacher Moves

Purpose
The purpose of this lesson is for students to use the Pythagorean theorem to solve problems.

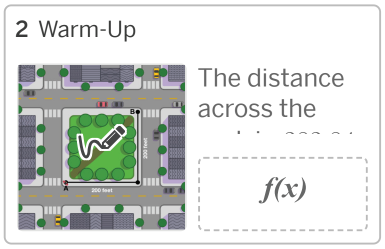
Warm-Up Launch
Invite students to look at the image when you put it up on the projector. Before giving students 2–3 minutes of quiet work time, consider showcasing that it's possible to enter either numbers or expressions in the math input. Show that when entering an expression, the math input works like a calculator.

Teacher Moves
Direct student attention to the calculator button near the top of the screen. Encourage them to use it whenever they find it helpful throughout the lesson.

Facilitation
Consider using pacing to restrict students to Screens 1–2.

Sample Responses

282.8 ft. or the equivalent, such as $\sqrt{200^2 + 200^2}$.



2 Warm-Up

The distance across the

f(x)

The distance across the park is 282.84 feet.

Alma walks 5 feet per second.

How long will it take for her to walk across the park?

Teacher Moves

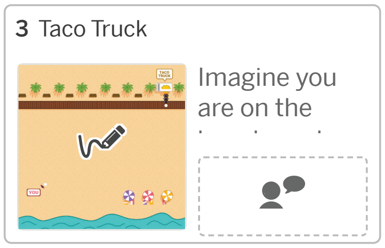
After students have some time to think about the questions on Screens 1–2, pause the class for a brief discussion. Consider using the snapshot tool to show the different expressions and numbers that students entered for both screens.

In addition to understanding the Pythagorean theorem, understanding the relationship between distance, rate, and time is important for this lesson. Students may not immediately recall how to calculate the time given a distance and a rate. Consider using a quick example with simple numbers to remind students that distance equals rate multiplied by time. If distance and rate are known, as is the case here, the time can be found by dividing the distance by the rate.

Once students have completed this screen, consider pausing to ask, "Is it faster to take the hypotenuse? Is that always the case?" This lesson may challenge students' assumptions on those questions.

Sample Responses

56.6 sec. or the equivalent, such as $\frac{282.84}{5}$.


3 Taco Truck

Imagine you are on the

Imagine you are on the beach, and you're getting hungry.

Use the sketch tool to show the route you would take to the taco truck.

Explain the reasoning behind your sketch.

Teacher Moves

Activity Launch
This activity provides students the opportunity to apply the Pythagorean theorem to a contextualized problem.

Display the student view of this screen for the class, and ask a volunteer to read the prompt. Ensure that students understand what their sketch is supposed to show.

Teacher Moves
Use the snapshot tool to showcase interesting and unique sketches as well as students' reasonings.

In particular, consider showcasing one or more sketches that “take the hypotenuse” (similar to the warm-up). Ask students if they think that route will be the fastest. It's okay—even desirable—to lack consensus at this stage. Encourage participation from students who think that the difficulty of walking on sand is a factor worth taking into account.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Responses vary.

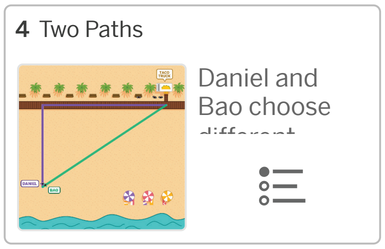
- The quickest way to get from one point to another is to follow a straight line between them.
- I hate walking on sand, so I would walk to the boardwalk as quickly as possible and walk the rest of the way on the boardwalk.

Student Supports

Students With Disabilities

- *Conceptual Processing: Processing Time*

For students who benefit from extra processing time, provide them the images to review prior to implementation of this activity.



4 Two Paths

Daniel and Bao choose

Daniel and Bao choose different routes to get to the taco truck.

Who do you think will reach the taco truck first?

Teacher Moves

Tell students that the first part of this activity is about exploring two different routes to the taco truck. After giving 1–2 minutes of silent think-time, arrange students into pairs to discuss, focusing especially on what information would be helpful to determine the first question.

Highlight a few unique responses for the class. Ask students to explain why they requested certain information and to attend to precision. For example, if a student asks, "How slow does sand make you go?" or "What are the speeds of Bao and Daniel?" ask them for the units they are interested in.

Facilitation
Consider using pacing to restrict students to this screen.

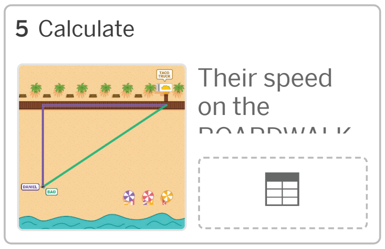
Routine (optional): Consider using the routine Decide and Defend to support students in strengthening their ability to make arguments and to critique the reasoning of others (MP3).

Sample Responses

Responses and explanations vary.

I would want to know:

- Both Daniel's and Bao's speeds.
- The distance to the taco truck.
- The distance to the boardwalk from the starting point and the distance along the boardwalk to the taco truck.



5 Calculate

Their speed on the

Their speed on the BOARDWALK is 5 feet per second.

Their speed on the SAND is 3 feet per second.

The dimensions are shown in the image.

Determine the amount of time it will take for Daniel and for Bao to get to the taco truck.

Use paper and the calculator tool as necessary.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to surface strategies for calculating the total amount of time each path will take, including how to use the Pythagorean theorem.

Tell students that this screen provides them with some of the information they requested from the previous screen to help them know whether Bao or Daniel will arrive first. Give students 2–4 minutes of quiet think-time. Then arrange them into pairs or small groups to share their initial thinking and solve the problem together.

This is a great place to use the teacher dashboard to monitor student progress. Offer support where needed. If enough students are struggling, prepare to lead a whole-class discussion about strategies.

Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

Bao will reach the taco truck first.

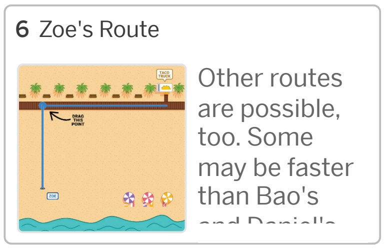
- Daniel's time can be found by using the expression $\frac{327.6}{3}+\frac{489}{5}$, which is 207 seconds.
- Bao's time can be found by using the expression $\frac{\sqrt{327.6^2+489^2}}{3}$, which is about 196.2 seconds.

Student Supports

Students With Disabilities

- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.



6 Zoe's Route

Other routes are possible, too. Some may be faster than Bao's

Other routes are possible, too. Some may be faster than Bao's and Daniel's.

Use the movable point to choose a route for Zoe that you think will be faster than the routes that Bao and Daniel used.

When you're done, go to the next screen.

Teacher Moves

Activity Launch
Refer back to students' sketches from the beginning of this activity to remind students that Bao's and Daniel's routes were just two of many different possible routes. Encourage students to think about whether there might be a route that is faster than those two. Call attention to how the movable point on this screen can help students explore different routes.

Teacher Moves
Use the overlay feature on the dashboard to show the range of responses on this screen. Tell students that later in the activity, all of these routes will race against one another.

Facilitation
Consider using pacing to restrict students to Screens 6–9.

Sample Responses

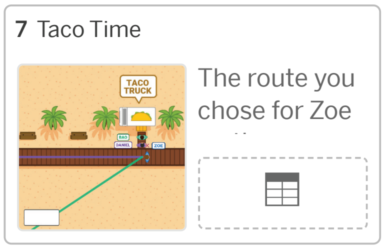
Responses vary.

Any route that aims Zoe *in between* where Bao and Daniel reach the boardwalk will be faster than those two routes.

Student Supports

Students With Disabilities
- *Conceptual Processing: Processing Time*

Check in with individual students as needed to assess for comprehension during each step of the activity.


7 Taco Time

The route you chose for Zoe

The route you chose for Zoe on the previous screen is shown. Determine the amount of time this route will take.

Recall:

- The speed on the BOARDWALK is 5 feet per second.
- The speed on the SAND is 3 feet per second.

Use paper and the calculator tool as necessary.

Teacher Moves

Give students 2–4 minutes of quiet think-time. Then arrange them back into pairs or small groups to share their initial thinking and decide how they'll solve the problem together.

Use the snapshot tool to show students' chosen routes and the accompanying expressions or calculations. Ask students to explain their responses and to critique each other's reasoning (MP3).

Use the dashboard to monitor progress. Students who successfully calculate the timing for their route may go back one screen to seek a faster route. Another worthwhile challenge you could suggest is to find different routes for Zoe that will land her in 1st, 2nd, and 3rd place.

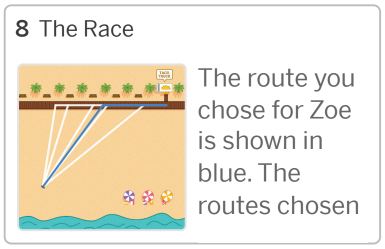
Sample Responses

Responses vary.

Student correctness for this screen is available in the teacher view of the dashboard.

- If the movable point is moved 100 ft. to the right, Zoe's time is about 192 sec.

- If the movable point is moved 300 ft. to the right, Zoe's time is about 186 sec.



8 The Race

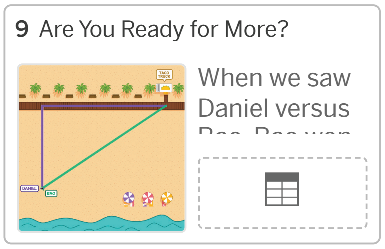
The route you chose for Zoe is shown in blue. The routes chosen

The route you chose for Zoe is shown in blue. The routes chosen by your classmates are also shown.

Press "Race" below to see how your classmates' routes compare to yours.

Teacher Moves

This screen will animate each student's route compared to those of their classmates. Note that students will not be able to identify who they are in the race unless they correctly predict the time of their route.



9 Are You Ready for More?

When we saw Daniel versus

When we saw Daniel versus Bao, Bao won.

Determine the speed on the boardwalk that would make Daniel and Bao arrive at the same time.

Use paper and the calculator tool as necessary.

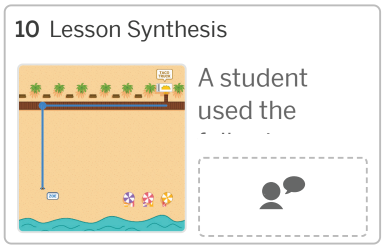
Teacher Moves

⚠ Before students can see this “Are You Ready for More?” screen, they will have to press a button that says, “I'm ready!”

This screen is designed to help differentiate the lesson by giving an extra challenge to students who finish Screens 2–8 ahead of time before the lesson synthesis. Because only a subset of your students will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

About 5.6 feet per second



10 Lesson Synthesis

A student used the

A student used the following expression to calculate the time for Zoe's route:

$$\frac{\sqrt{327.6^2+100^2}}{3}+\frac{389}{5}$$

First, move the draggable point to show the path that matches the expression.

Then, explain what $\sqrt{327.6^2+100^2}$ and 3 represent in the taco truck scenario.

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to make sense of complex expressions that involve the Pythagorean theorem.

Synthesis Launch
With the activity paused, show the class some expressions from Screen 7, either from students or from you. Here are some examples:

- $\frac{\sqrt{327.6^2+200^2}}{3}+\frac{289}{5}$
- $\sqrt{50^2+327.6^2}\,3+\frac{439}{5}$

Ask students to look back at their work on this problem and consider what similarities and differences they see between all of the work shown (their own included). Note that leaving problems like this written as expressions makes it hard to see the answer, but makes it much easier to analyze the different parts of a problem. Then unpause and direct students to complete the synthesis question.

Teacher Moves
Use the teacher view of the dashboard to highlight unique answers for the class. Support students in seeing structure, calling attention to the fact that expressions like $\sqrt{327.6^2+100^2}$ often signify a calculation of distance using the Pythagorean theorem.

If time permits, lead the class in resolving the question of the fastest possible route. One approach to this is to write a generalized expression for Zoe’s time, where the movable point has been moved x feet to the right. In this scenario, her time can be represented with a function: $\frac{\sqrt{327.6^2+x^2}}{3}+\frac{489-x}{5}$. The optimal point can be spotted when that function is graphed in an appropriate window. Here is a link to such a graph.

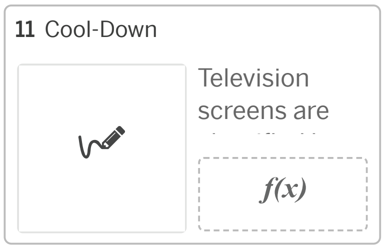
Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Compare and Connect to support students in making sense of multiple strategies and connecting those strategies to their own.

Sample Responses

Responses vary.

This student aims Zoe by dragging the movable point 100 ft. to the right. $\sqrt{327.6^2+100^2}$ represents the diagonal distance traveled on sand, which gets divided by the speed on sand, 3. 389 is the remaining distance to walk on the boardwalk, which gets divided by the speed on the boardwalk, 5.


11 Cool-Down

Television screens are

Television screens are classified by the length of their diagonal.

The television screen shown here is 22.5 inches tall and 40 inches wide.

What is the length of its diagonal?

Use paper and the calculator tool as necessary.

Teacher Moves

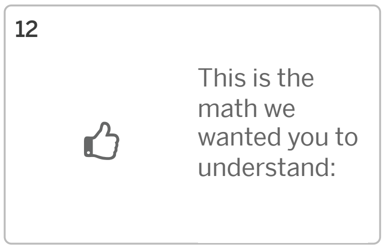
Support for Future Learning
Students will have more chances to develop their understanding of the Pythagorean theorem in the upcoming lessons, particularly in Lesson 11 and Practice Day 2.

Facilitation

Consider using pacing to restrict students to Screens 11–12.

Sample Responses

45.9 in. or equivalent, such as $\sqrt{40^2 + 22.5^2}$.



12

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I can use the Pythagorean theorem to solve problems.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.

8.8 Practice Day 2 (NYC)

Preparation

Practice Worksheet

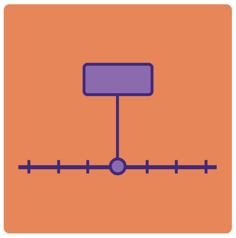
- Print one double-sided worksheet for each student.

Instructions

Option 1: Projector
Using the projector, show one problem at a time to the whole class. Allow 3–5 minutes per problem. Invite students to work in pairs or small groups and to use whiteboards. At the end of the allocated time, all groups show their whiteboards to compare solutions and get feedback. Hold a short discussion, as needed.

Option 2: Worksheets
Print one double-sided copy of the practice worksheet for each student (or pair of students). Students can work through the problems at their own pace. You can post the answer key, or walk around with it, providing feedback to students as they work.

This lesson is still being upgraded
to the Amplify Desmos Math design
style for the 2024–25 school year.



Hit the Target

Lesson 14: Rational and Irrational Numbers

Overview

Students build on their work with square roots, fractions, and decimal representations to learn about a new mathematical idea: irrational numbers.

Learning Goals

- Understand that rational numbers are defined as numbers that can be written as a fraction of two integers.
- Understand that numbers that are not rational are called irrational, and that $\sqrt{2}$ is an example of an irrational number.

Materials

- Blank paper

Vocabulary

- rational number
- irrational number

Lesson Checklist

☐ Complete the lesson using the student preview.

☐ Identify how this lesson extends the learning from previous lessons, and how it prepares students for future lessons.

☐ Think about how you will introduce each new section within the lesson to engage students in the task and maintain focus on the learning goals.

☐ Determine the screens where you'll use Pacing and Pause to bring the class together. What questions will you ask on those screens?

☐ Anticipate screens where students will struggle, then plan your response.

☐ Consider how to use snapshots to select and present student thinking for class discussion.

☐ Think about how you will use the results of previous Cool-Downs and student surveys to inform your approach to this lesson.

About This Lesson

In this lesson, students build on their work with square roots, fractions, and decimal representations to learn about a new mathematical idea: irrational numbers.

Lesson Summary

Warm-Up (5 minutes)

The purpose of the warm-up is for students to think about the relative size of two familiar irrational numbers (which they do not yet know are irrational) in the context of a number line.

Activity 1: Hit the Target (5 minutes)

The purpose of this activity is for students to explore irrational numbers. Students will attempt to use rational numbers to get as close as possible to the target irrational number.

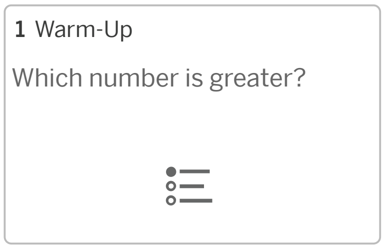
Activity 2: Irrational Numbers (35 minutes)

The purpose of this activity is to introduce the term *irrational* for numbers that cannot be written as a fraction using integers for the numerator and denominator. After a brief introduction, students must test their understanding by selecting all irrational numbers from a small group. They then reinforce their understanding by completing a card sort.

Lesson Synthesis (5 minutes)

The purpose of the synthesis is to ensure that students are able to offer examples of rational and irrational numbers, and describe in their own words what an irrational number is.

Cool-Down (5 minutes)



1 Warm-Up

Which number is greater?

Which number is greater?

Teacher Moves

Purpose
The purpose of this lesson is for students to build on their work with square roots, fractions, and decimal representations to learn about a new mathematical idea: irrational numbers.

Warm-Up Launch
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. Consider using the dashboard to show the distribution of responses, calling attention to any conflict or consensus you see.

Emphasize the range of student responses on this screen. It's okay—even desirable—to lack consensus at this stage. The activity will build towards consensus later on.

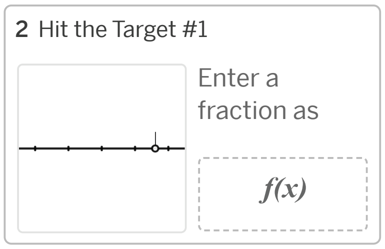
Facilitation
Consider using pacing to restrict students to this screen.

Sample Responses

$\sqrt{13}$

Responses vary.

- π because the decimal representation goes on forever.
- $\sqrt{13}$ because it is closer to $\sqrt{16}=4$ than $\sqrt{9}=3$ (i.e., 13 is closer to 16 than 9).



2 Hit the Target #1

Enter a fraction as

Enter a fraction as close to $\sqrt{13}$ as you can (without using the square root symbol).

Then press "Check My Work."

Teacher Moves

Activity Launch
Tell students that their goal for this activity is to enter a fraction as close to the target number as possible.

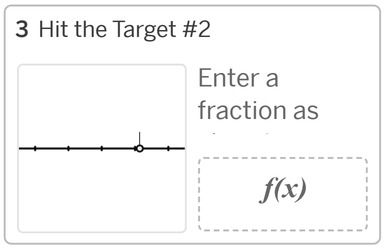
Consider using snapshots or the teacher view of the dashboard to display unique answers to the class.

Facilitation
Consider using pacing to restrict students to Screens 2–3.

Sample Responses

Responses vary.

- $\frac{18}{5}$
- $\frac{361}{100}$
- $\frac{360\,555}{100\,000}$



3 Hit the Target #2

Enter a fraction as

Enter a fraction as close to π as you can (without using the π symbol).

Then press "Check My Work."

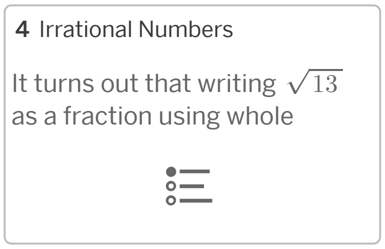
Teacher Moves

Consider using snapshots or the teacher view of the dashboard to display unique answers to the class.

Sample Responses

Responses vary.

- $\frac{314}{100}$
- $\frac{22}{7}$
- $\frac{31\,415\,926}{10\,000\,000}$


4 Irrational Numbers

It turns out that writing $\sqrt{13}$ as a fraction using whole

It turns out that writing $\sqrt{13}$ as a fraction using whole numbers for the numerator and the denominator is impossible.

Such numbers are called IRRATIONAL NUMBERS.

π and $\sqrt{2}$ are both examples of irrational numbers.

Is $\sqrt{\frac{9}{4}}$ rational or irrational?

Teacher Moves

Key Discussion Screen
The purpose of this discussion is to make sense of what irrational numbers are.

Activity Launch
In this activity, students formalize their knowledge of irrational and rational numbers.

Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. Then consider using the dashboard to show the distribution of responses, calling attention to any conflict or consensus you see.

To prepare students for success on Screen 5, resolve any conflict you see in the responses on Screen 4.

Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using the routine Collect and Display to gather students' ideas and create a class definition.

Sample Responses

Rational

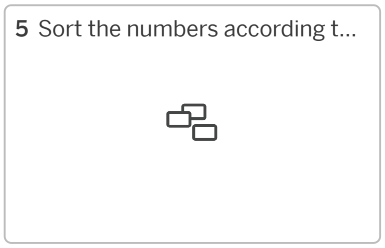
Responses vary.

At first I thought $\sqrt{\frac{9}{4}}$ was irrational because of the square root symbol. But then I realized that $\sqrt{\frac{9}{4}} = \frac{3}{2}$, which means it is rational.

Student Supports

Students With Disabilities
- *Receptive Language: Processing Time*

Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.



5 Sort the numbers according t...

Teacher Moves

Consider anonymizing the class and displaying the teacher dashboard for this screen so that groups can see when they have correctly completed the sort.

Use the teacher dashboard to monitor student progress and to look for common sorting strategies.

Make sure you complete this card sort yourself, in the role of a student, to anticipate the different questions your students will ask and the possible conceptions they'll have.

Facilitation
Consider using pacing to restrict students to Screens 5–7.

Sample Responses

Responses vary.

The answer key is intentionally disabled for this card sort so that students can place cards in the “I'm Not Sure” category without being marked incorrect. While students **do** have the tools to identify with certainty that some numbers are *rational*, they **do not** yet have the tools to identify with certainty whether arbitrary numbers are *irrational*.

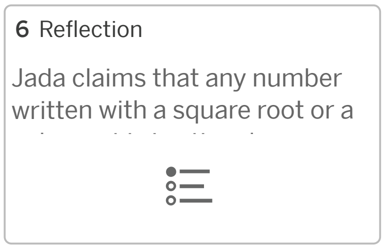
With that in mind, here is a list of the rational numbers in the card sort:

- $\sqrt{\frac{1}{4}}$ because $\sqrt{\frac{1}{4}} = \frac{1}{2}$
- 1.73 because $1.73 = \frac{173}{100}$
- $1.\overline{73}$ because $1.\overline{73} = \frac{172}{99}$

- $\frac{8}{4}$
- $\sqrt[3]{8}$ because $\sqrt[3]{8} = 2$
- $2\frac{3}{20}$ because $2\frac{3}{20} = \frac{43}{20}$

To help students construct an informal argument that $\sqrt{18}$ is irrational, invite them to compare the hypotenuse of a right triangle with side lengths 3, 3, and $\sqrt{18}$ with the hypotenuse of a right triangle with side lengths 1, 1, and $\sqrt{2}$. On Screen 4, students learned (without proof) that $\sqrt{2}$ is irrational. It follows that $3\sqrt{2}$ is also irrational, which means that $\sqrt{18}$ is irrational as well (because $\sqrt{18} = 3\sqrt{2}$).

Note: The "Are You Ready for More" task on Screen 7 introduces students to the basic flow of a proof that $\sqrt{2}$ is irrational.



6 Reflection

Jada claims that any number written with a square root or a

Jada claims that any number written with a square root or a cube root is irrational.

Is Jada correct?

Teacher Moves

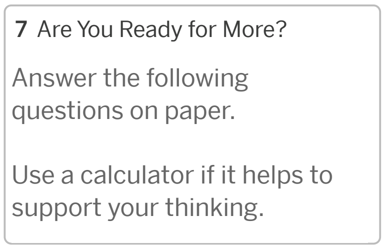
Give students one minute of quiet think-time and a few minutes to discuss with a partner. Invite several students to share their responses. Consider using snapshots or the teacher view of the dashboard to display unique answers to the class.

Sample Responses

No

Responses vary.

Many numbers written with a square root or a cube root are rational. For example, $\sqrt{16} = \frac{4}{1}$ is rational, and so is $\sqrt[3]{\frac{8}{125}} = \frac{2}{5}$.



7 Are You Ready for More?

Answer the following questions on paper.

Use a calculator if it helps to support your thinking.

Answer the following questions on paper.

Use a calculator if it helps to support your thinking.

1. $\left(\frac{577}{408}\right)^2$ is very close to 2. Is it exactly equal to 2?

2. If $\left(\frac{577}{408}\right)^2 = 2$, then $408^2 \cdot 2 = 577^2$. Diya says they know that's not true even though they haven't computed any of these numbers. How can they know that?

3. How does this show that $\frac{577}{408} \neq \sqrt{2}$?

4. Is $\frac{1\,414\,213\,562\,375}{1\,000\,000\,000\,000} = \sqrt{2}$? Explain how you know.

Teacher Moves

⚠ Before students can see this screen, they will have to press a button that says, “I’m ready!”

This screen is designed to help differentiate the lesson by giving students who finish Screens 4–6 ahead of time another challenge before the class discussion on Screen 8. Because only a subset of your class will complete this screen, we recommend you don't discuss it with the entire class.

Sample Responses

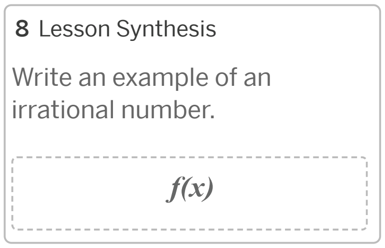
Responses vary.

1. While some simpler calculators display 2 as the result for $\left(\frac{577}{408}\right)^2$, more accurate tools show that $\left(\frac{577}{408}\right)^2 > 2$.

2. $408^2 \cdot 2$ is even. 577^2 is odd. Therefore, $408^2 \cdot 2$ cannot be equal to 577^2.

3. $\sqrt{2}$ is the solution to the equation $x^2 = 2$. Since $\frac{577}{408}$ is not a solution to that equation, $\frac{577}{408} \neq \sqrt{2}$.

4. No, by a similar argument to the one above.



8 Lesson Synthesis

Write an example of an irrational number.

Write an example of an irrational number.

Teacher Moves

Key Discussion Screen 🔑
The purpose of this discussion is to consolidate what students know about irrational numbers.

Synthesis Launch
Give students 2–3 minutes to respond to this question and a few minutes to share their responses with their partner. Follow with a whole-class discussion. Use the teacher view of the dashboard to highlight unique answers to show the class.

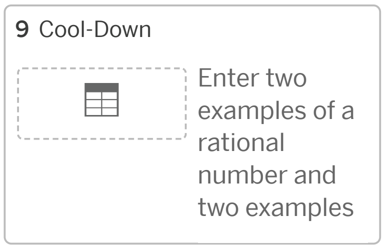
Facilitation
Consider using pacing to restrict students to this screen.

Routine (optional): Consider using one or more rounds of the routine Stronger and Clearer Each Time to help students develop their ideas and language.

Sample Responses

Responses vary.

$\sqrt{5}$. An irrational number is a number that is *not* rational. In other words, an irrational number cannot be written as a fraction using integers for the numerator and denominator.



9 Cool-Down

Enter two examples of a rational number and two examples

Enter two examples of a rational number and two examples of an irrational number in the table.

Teacher Moves

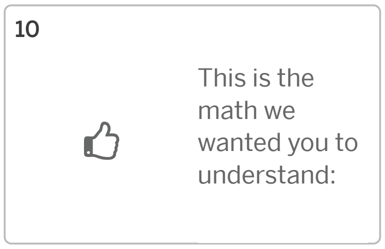
Support for Future Learning
If students struggle to identify examples of rational and irrational numbers, consider reviewing this cool-down as a class before the end assessment, or offering individual support where needed.

Facilitation
Consider using pacing to restrict students to Screens 9–10.

Sample Responses

Responses vary.

- **Rational:** 3 , 0.7
- **Irrational:** $\sqrt{101}$, $\sqrt[3]{2}$



10

This is the math we wanted you to understand:

This is the math we wanted you to understand:

- I know what a rational number is and can give an example.
- I know what an irrational number is and can give an example.

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