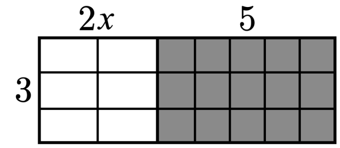
Expressions and Equations
Student Guide

Math 6 Unit 4 Accelerated

<table>
<tr><td>product</td><td>A product describes two or more quantities that are being multiplied together.

For example, the area of this rectangle is the product of 3 and $2x + 5$ or $3(2x + 5)$.</td><td>

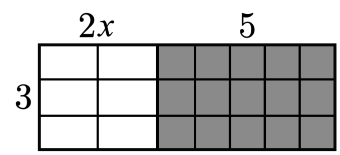
Area as a Product
$3(2x + 5)$</td></tr>
<tr><td>solution to an equation</td><td>A solution to an equation is a value of a variable that makes the equation true.

For example, 5 is a solution to the equation $3x = 15$ because $3(5) = 15$.

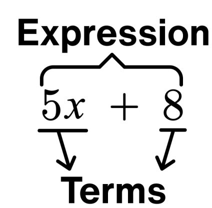
6 is not a solution to the equation $3x = 15$ because $3(6) = 15$ is not true.</td><td>$3x = 15$

$x = 5$

$3(5) = 15$</td></tr>
<tr><td>sum</td><td>A sum describes two or more quantities that are being added together.

For example, the area of this rectangle is the sum of $6x$ and 15 or $6x + 15$.</td><td>

Area as a Sum
$6x + 15$</td></tr>
<tr><td>term</td><td>A term is a part of an expression that involves addition. It can be a single number, a variable, or a variable and a number multiplied together.

For example, the expression $5x + 8$ has two terms. The first term is $5x$ and the second term is 8.</td><td>
</td></tr>
<tr><td>variable</td><td>A variable is a letter or symbol that represents a number. You can choose different numbers for the value of the variable.

In the expression $10 - x$, the variable is x.

If $x = 3$, then $10 - x = 7$.

If $10 - x = 4$, then $x = 6$.</td><td></td></tr>
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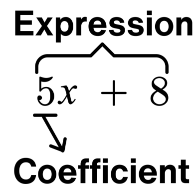
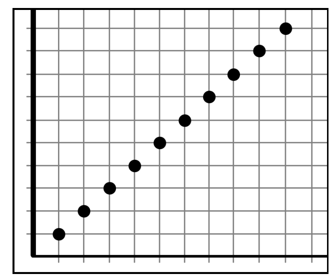
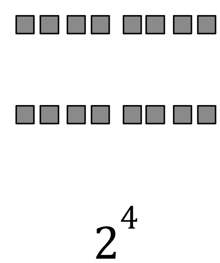
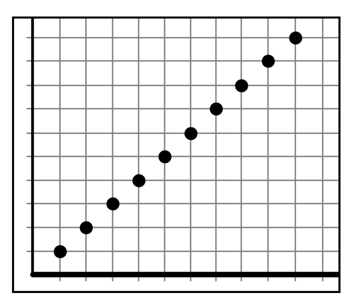
Glossary

Term	Definition
coefficient	A coefficient is a number multiplied by a variable, usually without a symbol in between the number and the variable. In the expression $5x + 8$, the coefficient of x is 5. 
dependent variable	The dependent variable is the variable in a relationship that is the effect or result. For example, if we are exploring the distance a boat can travel in different amounts of time, the dependent variable is the distance traveled, d. The dependent variable is typically on the vertical axis of a graph and the right-hand column of a table. 
equivalent expressions	Equivalent expressions are different ways of describing the same quantity. $x + x + x$ is equivalent to $3x$ because they both describe three copies of an unknown number, x.
exponent	Exponents describe repeated multiplication. For example, $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$. 2^4 is called "2 to the power of 4" or "2 to the fourth." In 2^4, 2 is called the base and 4 is called the exponent. 
independent variable	The independent variable is the variable in a relationship that is the cause. For example, if we are exploring the distance a boat can travel in different amounts of time, the independent variable is the amount of time, t. The independent variable is typically on the horizontal axis of a graph and the left-hand column of a table. 

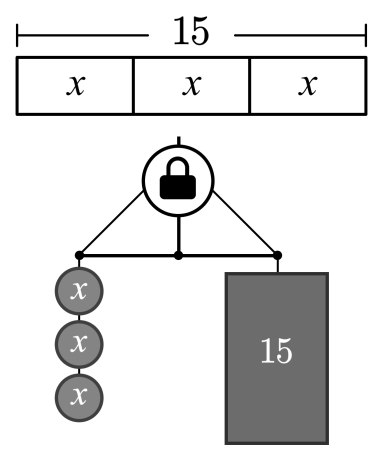
Unit 6 Summary

Prior Learning	Math 6, Unit 6	Future Learning
Grades 1–5 • Basic operations ($+, -, \times, \div$) • Operations with grouping symbols • Graphing positive numbers • Powers of 10 Math 6 • Dividing fractions (Unit 4) • Decimal operations (Unit 5)	• Solving equations • Equivalent expressions • Expressions involving exponents • Introduction to representing relationships	Math 6, Unit 7 • Graphing with positive and negative numbers Math 7 • Proportional relationships • Solving more complex equations • Factoring and expanding expressions

Solving Equations

A *solution* is a value of a variable that makes an equation true.

Tape diagrams and hangers can help us make sense of equations.
Here is a tape diagram and a hanger that show the equation $3x = 15$.



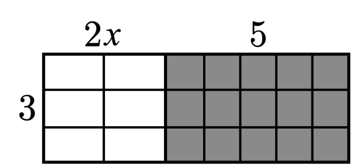
Solving an equation is the process of determining a solution.
In the equation $3x = 15$, the solution is $x = 5$ because $3(5) = 15$.

Replacing x with 5 in the hanger will keep the hanger balanced.

Equivalent Expressions

Equivalent expressions are different ways of describing the same quantity.
$x + x + x$ is equivalent to $3x$ because they both describe three copies of an unknown number, x.

The area of this rectangle can be written in two different ways.



$3(2x + 5)$
the length times width

$6x + 15$
the sum of two smaller areas

$$3(2x + 5) = 3(2x) + 3(5) = 6x + 15$$

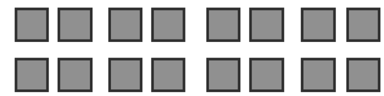
This is an example of the *distributive property.*

Expressions Involving Exponents

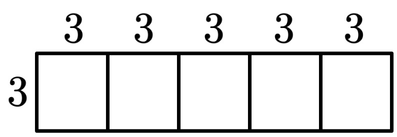
Exponents are a way to describe repeated multiplication.

2^4 is called “2 to the power of 4” or “2 to the fourth”.

In 2^4, 2 is called the base and 4 is called the exponent.



$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$



$$5 \cdot 3^2 = 5 \cdot 9 = 45$$

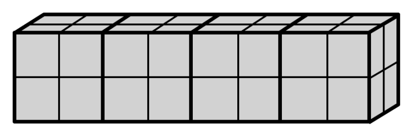
Diagrams can help make sense of expressions that involve exponents and other operations.

For example, $5 \cdot 3^2$ can describe 5 copies of a 3–by–3 square.

Exponents can also appear in expressions with variables.

What is the value of $4x^3$ when $x = 2$?

$$4(2)^3 = 4(2 \cdot 2 \cdot 2) = 4(8) = 32$$



32 unit cubes

Introduction to Representing Relationships

Math can help make sense of the relationship between two different quantities or variables.

Tables, equations, and graphs can each show the same relationship in different ways.

Here is an example:

n = the number of quarters in my pocket

v = the value of my quarters (in cents)

Description

Every quarter in my pocket is worth 25 cents.

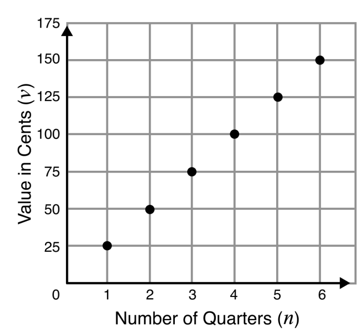
Table

n	v
1	25
2	50
3	75

Equation

$$v = 25n$$

Graph



Try This at Home

Solving Equations

1.1 Determine the solution to each equation. Draw a diagram if it helps you with your thinking.

$x + 2 = 11$ $2x = 11$ $x - 11 = 2$

Matias bought 2 plants, which cost $\$11$ total. x represents the cost of each plant.

1.2 Which of the equations above represents this situation? Explain how you know.

1.3 Explain what the solution to the equation means in this situation.

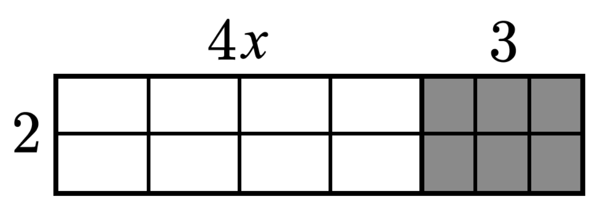
Equivalent Expressions

2. At Kai's pizza shop, they charge $\$4$ for delivery on top of the cost of the pizza.
How much would the total charge be if the cost of the pizza was:

$\$15$? $\$24$? d dollars?

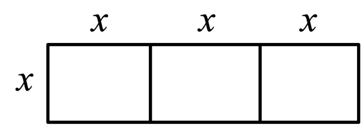
3. Select all the expressions that describe the area of this rectangle.

- ☐ $2(4x + 3)$
- ☐ $2(4x + 6)$
- ☐ $8x + 6$
- ☐ $(4x + 3) + (4x + 3)$
- ☐ $6x + 5$



Expressions Involving Exponents

4.1 Which expression represents the diagram on the right?



- ☐ $3 + x^2$
- ☐ $(3 + x)^2$
- ☐ $3x^2$

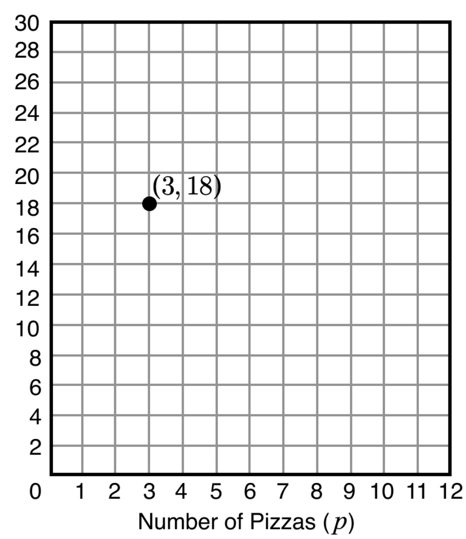
4.2 Determine the value of each expression when $x = 4$.

5. What is $2(4)^3$? Explain how you know.

Introduction to Representing Relationships

Kai uses 6 mushrooms on every large Super Mushroom Pizza. They are wondering about the relationship between the number of pizzas made, p, and the number of mushrooms they use, m.

6.1 They started making a graph of the relationship. What does the point $(3, 18)$ mean in Kai's situation?



6.2 Add at least three more points to Kai's graph. Use a table if it helps you with your thinking.

p	m

6.3 Write an equation to represent the relationship between p and m.

Solutions:

1.1 $x = 9$ $x = 5.5$ (or equivalent) $x = 13$

1.2 $2x = 11$

Explanations vary. Since each plant costs the same amount, we are doubling the cost of one plant, which we can show with $2x$.

1.3 *Responses vary.* This means that each plant that Matias bought cost $\$5.50$.

2. $\$19$ $\$28$ $d + 4$

3. ✓ $2(4x + 3)$
✓ $8x + 6$
✓ $(4x + 3) + (4x + 3)$

4.1 ✓ $3x^2$

4.2 $3 + (4)^2 = 3 + 16 = 19$
$(3 + 4)^2 = (7)^2 = (7 \cdot 7) = 49$
$3(4)^2 = 3(4 \cdot 4) = 3(16) = 48$

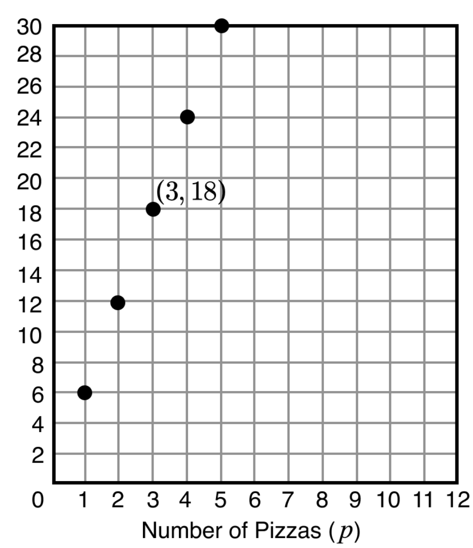
5. $2(4)^3 = 128.$

Explanations vary. Any number to the power of 3 is $\# \cdot \# \cdot \#$, so $4^3 = 4 \cdot 4 \cdot 4 = 64$. The 2 in front means that there are two of the 64s and $64 \cdot 2 = 128$.

6.1 *Responses vary.* $(3, 18)$ means that when Kai makes 3 pizzas, they use 18 mushrooms.

6.2 *Responses vary.* See the graph to the right.

p	m
1	6
2	12
4	24

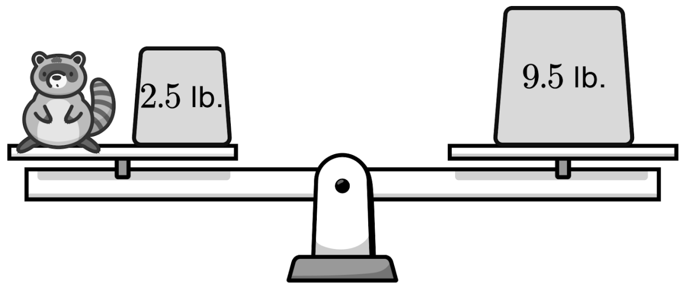


6.3 $m = 6p$

Name ____________________________________

My Notes

This raccoon and 2.5 pounds balance with a 9.5 lb. weight.

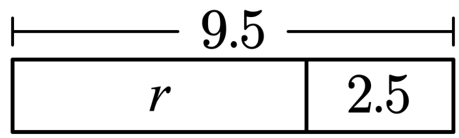


1.1 Nekeisha wrote $r + 2.5 = 9.5$ to represent the situation. How is the equation like balancing the raccoon and weights?

Each side of the equation represents a side of the see-saw. The raccoon's weight plus 2.5 pounds is on the left, so the left side of the equation is $r + 2.5$. The right side of the equation and the see-saw are both 9.5 lbs.

1.2 Nekeisha also drew a tape diagram to help determine the weight of the racoon.

Explain how this tape diagram is like the equation.



The tape diagram is like the left side of the equation because it adds r and 2.5.
The total width of the tape diagram is like the right side of the equation.

1.3 How much does the racoon weigh?
Use the equation or tape diagram if it helps your thinking.

7 pounds

Summary

- [ ] I can make connections between tape diagrams and equations.
- [ ] I can use reasoning and tape diagrams to figure out unknown values.

desmos

Unit 6.6, Lesson 2: Notes Name ______________________

My Notes

Here is a situation along with an equation that represents it.

Kiandra sold 40 hats and made $\$320$. The hats cost h dollars each.

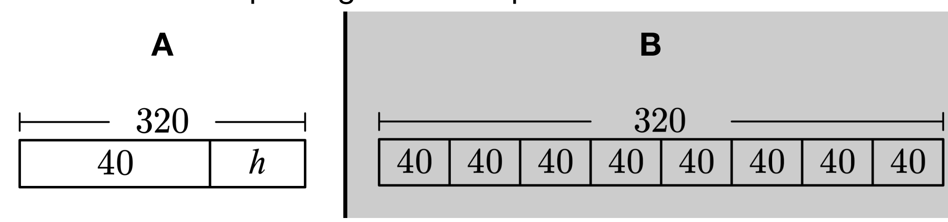
Equation	Solution	Meaning of the Solution
$40h = 320$	$h = 8$	The hats cost $\$8$ each.

1.1 What is the *variable* in the equation? ____ h ____

What does the variable represent in this situation?

h **represents the cost of each hat.**

1.2 Circle the tape diagram that represents this situation.



1.3 Determine the *solution* to the equation.

$h = 8$

1.4 Explain what the solution means in this situation.

Each hat costs $\$8$ **.**

Summary

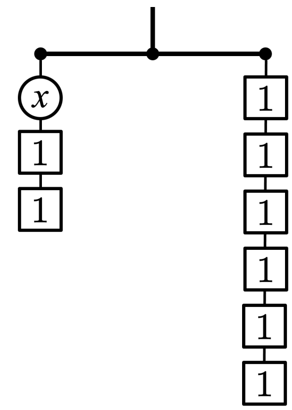
☐ I can make connections between tape diagrams, equations, and situations.

☐ I know what the terms *variable* and *solution* mean when solving equations.

Unit 6.6, Lesson 3: Notes Name ____________________

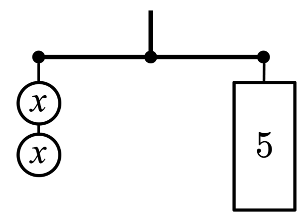
My Notes

1. What value of x balances this hanger?



$x = 4$

2.1 Which equation represents this hanger?



A. $2x = 5$

B. $x + 2 = 5$

C. $5 \cdot 2 = x$

D. $x + 5 = 2$

2.2 Determine the value of x that balances this hanger

$x = \frac{5}{2}$ **or** $x = 2.5$

Summary

☐ I can make connections between balanced hangers and true equations.

☐ I can use balanced hangers to solve equations.

Unit 6.6, Lesson 4: Notes

Name ____________________

My Notes

1. Daeja and Juana solved this equation: $6 = \frac{1}{2}s$.

   Daeja: The solution is $s = 12$.
   Juana: The solution is $s = 3$.

   Who is correct? **Daeja is correct.**

   ***Explanations vary.* When I substitute 12 in for s, I get $6 = \frac{1}{2} \cdot 12$, which is true.**

Determine the solution to each equation.

Draw a hanger or a tape diagram if it helps you with your thinking.

2.1 $y + 1.8 = 14.7$

$y = 12.9$

2.2 $1.8 = 3t$

$0.6 = t$

Summary

☐ I can solve equations that include whole numbers, decimals, and fractions.

Name ______________________________

My Notes

1. You must be 3 feet tall to ride a roller coaster.

   Mauricio is $2\frac{1}{4}$ feet tall.

   Which equation represents the number of feet Mauricio must grow, f, in order to ride the roller coaster?

   A. $3 + 2\frac{1}{4} = f$

   B. $2\frac{1}{4} + f = 3$

   C. $3 + f = 2\frac{1}{4}$

   D. $2\frac{1}{4}f = 3$

Here is an equation: $0.5 \cdot 32 = x$.

2.1 Write a situation to match this equation. ***Responses vary.***

A shirt's original price is $\$32$. It is on sale for 50% of the original price. The new price of the shirt is x dollars.

2.2 Solve this equation.

$x = 16$

2.3 Explain what the solution represents in your situation.

Responses vary. **The new price of the shirt is $\$16$.**

Summary

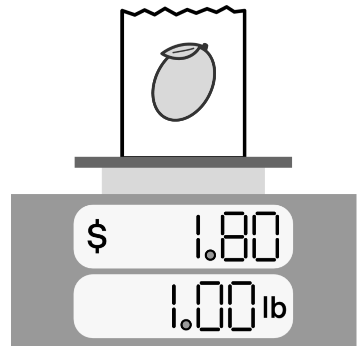
- ☐ I can write a situation to represent an equation.
- ☐ I can explain what the solution to an equation means in a situation.

Name ____________________

My Notes

1. Mangos cost $\$1.80$ per pound. Complete the table.

Mangos (lb.)	**Total Cost ($)**
1	1.80
2	3.60
5	9.00
10	18.00
p	$1.80p$



2.1 Adnan paid x dollars for a pizza and an extra $\$10.00$ to have it delivered. Write an expression for the total cost.

$x + 10$

2.2 Explain how each part of your expression relates to the situation.

Responses vary. **x is the cost of the pizza and 10 is the cost of delivery.**

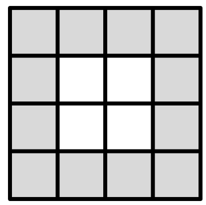
Summary

☐ I can write an expression with a variable to represent a situation.

My Notes

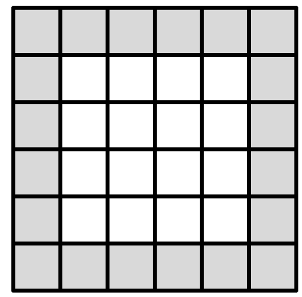
How many gray tiles are used to make the border for each square?

1.1 2-by-2



12 **tiles**

1.2 4-by-4



20 **tiles**

1.3 10-by-10

44 **tiles**

1.4 n-by-n

$4n + 4$ **tiles**
(or equivalent)

2. Show or explain how you know that $2n + 2$ and $2(n + 1)$ are equivalent.

Responses vary. **One way to think about** $2(n + 1)$ **is two groups of** $n + 1$**, which is the same as** $2n + 2$**.**

Summary

☐ I can explain what it means for two expressions to be equivalent.

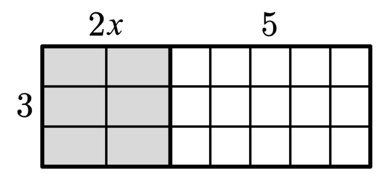
☐ I can justify whether two expressions are equivalent.

desmos

Unit 6.6, Lesson 8: Notes Name ______________________________

My Notes

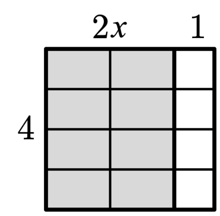
1. Write two equivalent expressions that could be used to represent the area of this rectangle.



Expression 1	**Expression 2**
$3(2x + 5)$	$6x + 15$

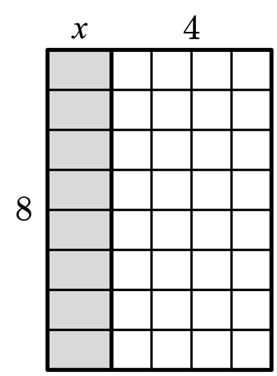
2.1 Write an expression that is equivalent to $8x + 4$. Draw a rectangle if it helps you with your thinking.

$4(2x + 1)$ **(or equivalent)**



2.2 Show or explain how you know that $8x + 4$ and $8(x + 4)$ are **not** equivalent.

Responses vary. $8x + 4$ **and** $8(x + 4)$ **are not equivalent because** $8(x + 4)$ **is equivalent to** $8x + 32$**.**



Summary

☐ I can use an area model to write equivalent expressions.

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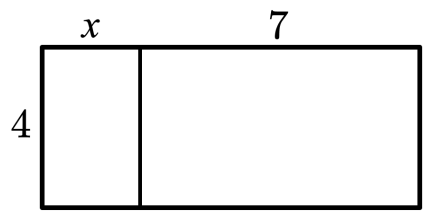
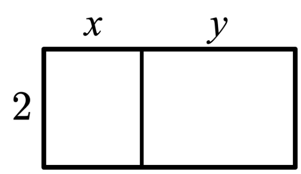
Unit 6.6, Lesson 9: Notes Name ______________________________

My Notes

1. Complete the table.

Area Model	Product	Sum
	$4(x + 7)$	$4x + 28$
	$2(x + y)$	$2x + 2y$

2.1 The expressions $2(m + 8)$ and $2m + 16$ are equivalent.
Write an expression that is equivalent to $2(m - 8)$.

$2m - 16$ **(or equivalent)**

2.2 The expressions $3p - 18$ and $3(p - 6)$ are equivalent.
Write an expression that is equivalent to $18 - 3p$.

$3(6 - p)$ **(or equivalent)**

Summary

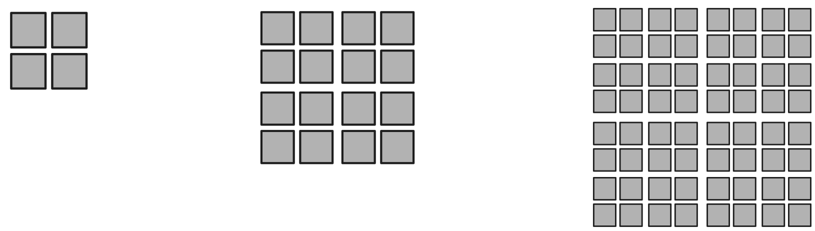
☐ I can write equivalent expressions, including expressions that have subtraction.

Unit 6.6, Lesson 10: Notes Name ______________________________

My Notes

The number of squares in each images represents a power of 4.

4^1 4^2 4^3



1. Explain how you could figure out the value of 4^4.

   Responses vary. **Each step has 4 times as many squares as the step before. There are 64 squares in 4^3 . If this is multiplied by 4, then $64 \cdot 4 = 256$ squares, so $4^4 = 256$.**

2. Complete the table.

With Exponent	Without Exponent
3^5	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$(\frac{1}{2})^4$	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
$(0.6)^3$	$0.6 \cdot 0.6 \cdot 0.6$

3. Select **all** the expressions that are equal to 81.

   ☐ 1^{81} ✓ 81^1 ✓ 3^4 ☐ 2^9 ✓ $3^3 \cdot 3$

Summary

☐ I can explain what an expression with an exponent means (e.g., 3^5).

☐ I can decide whether two expressions that include exponents are equivalent.

My Notes

Here are two figures.

Figure A

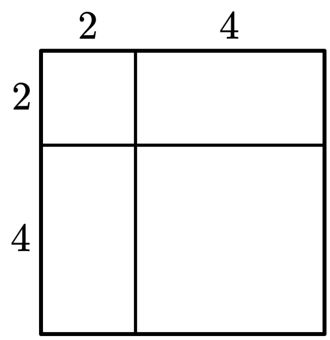
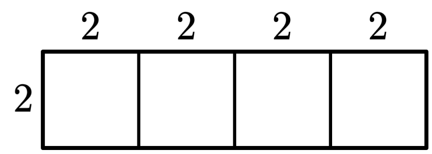


Figure B



1. Match each figure with an expression that describes its area. You will have one expression left over.

$(4 \cdot 2)^2$	$4 \cdot 2^2$	$(2 + 4)^2$
Figure _____	Figure __**B**__	Figure __**A**__

Calculate the value of each expression.

2.1 $(4 \cdot 2)^2$	2.2 $4 \cdot 2^2$	2.3 $(2 + 4)^2$
64	16	36

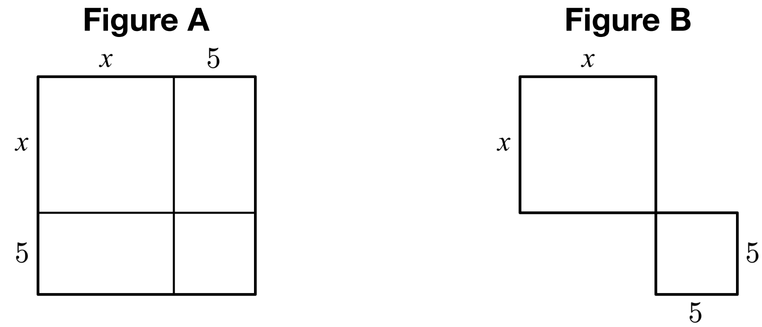
Summary

☐ I can determine the value of an expression that has an exponent and addition, subtraction, multiplication, or division.

 Name ____________________

My Notes

Here are two figures. They are not drawn to scale.



1.1 Match each figure with an expression that describes its area. You will have one expression left over.

$x + 5^2$	$(x + 5)^2$	$x^2 + 5^2$
Figure _____	Figure **A**	Figure **B**

1.2 Explain why $(x + 5)^2$ and $x + 5^2$ are not equivalent.

Responses vary. $(x + 5)^2$ **and** $x + 5^2$ **are not equivalent because they represent different diagrams. When** $x = 1$**,** $(x + 5)^2$ **is** $(1 + 5)^2 = 36$ **and** $x + 5^2$ **is** $1 + 5^2 = 26$**.**

Calculate the value of each expression when $x = 2$.

2.1 $x + 3^3$	2.2 $(x + 1)^4$	2.3 $5x^3$
$2 + 3^3$	$(2 + 1)^4$	$5(2)^3$
$2 + 27$	3^4	$5 \cdot 8$
29	81	40

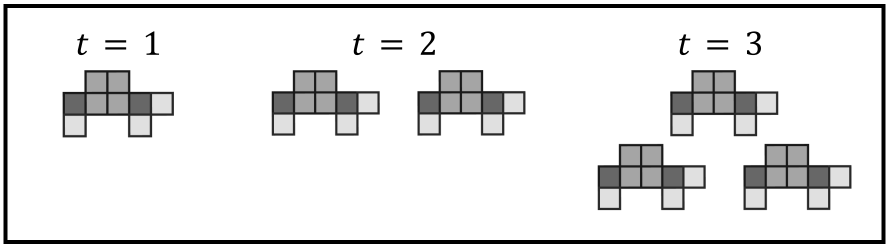
Summary

☐ I can determine the value of an expression that has a variable, an exponent, and addition, subtraction, multiplication, or division for a specific value of the variable.

Answer Key

My Notes

Here is a pattern of turtles.



The *independent variable* is t, the number of turtles.

1.1 Explain what an *independent variable* is. ***Responses vary.***

The independent variable is the variable in a relationship that you can control.

1.2 Explain what a *dependent variable* is. Give one example.

Responses vary. **The dependent variable is the variable that changes as a result of the independent variable. An example is the total area of the turtles.**

Adah made a table to represent the relationship between the number of turtles, t, and the total area, a.

t	a
1	9
2	18
3	27

2.1 What is the dependent variable?

Total area, a

2.2 Which equation represents this relationship?

$t = 9a$ ✓ $a = 9t$ $a = t + 9$

Explanations vary.

If you look at the numbers in the table you see that the area is equal to nine times the number of turtles.

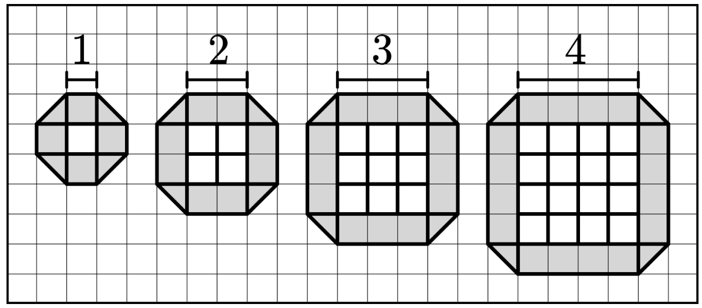
Summary

☐ I understand what the independent and dependent variables are in a relationship.

☐ I can use a table or an equation to represent a relationship.

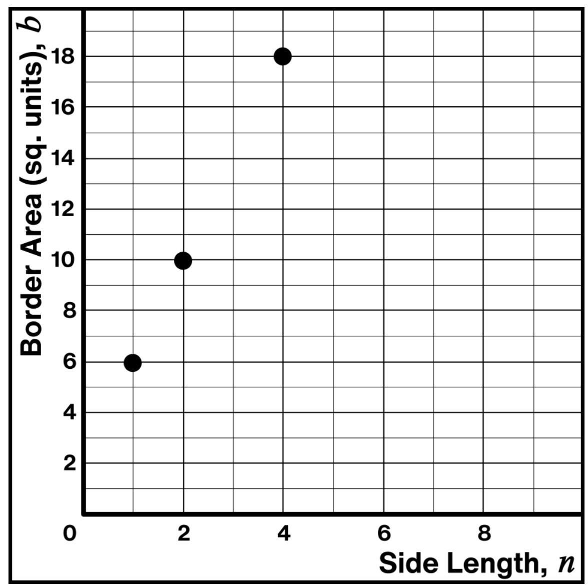
My Notes

Kanna is exploring the relationship between the side length, n, and the total area of the border, b.



1. Use Kanna's table to create a graph of the relationship. Be sure to label each axis with what it represents.

n	b
1	6
2	10
4	18



2. If the graph were larger, it would include the point $(6, 26)$. Describe what this point means in the situation.

   The point $(6, 26)$ means that when the side length is 6 units, the area of the border is 26 square units.

Summary

☐ I can represent relationships using tables and graphs.

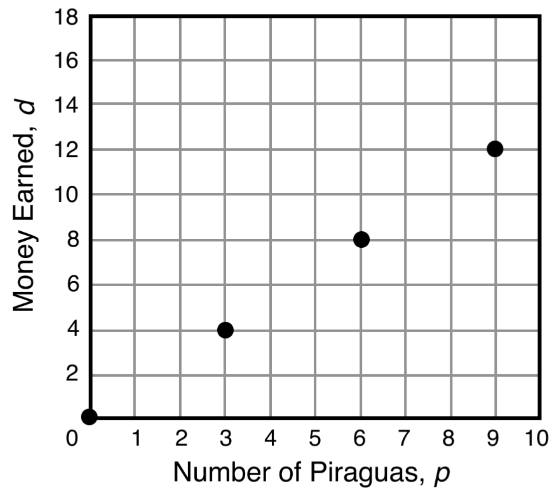
desmos

Unit 6.6, Lesson 15: Notes

Name ______________________

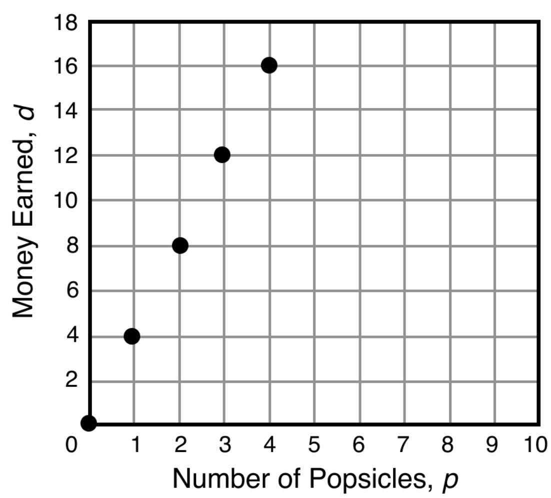
My Notes

1. Create a table that represents this graph.



p	d
0	0
3	4
6	8
9	12

2. Which equation represents this graph?



$p = 4d$

$d = 4 + p$

✓ $d = 4p$

Explain how you know.

Explanations vary. **If you substitute 1 in for p in the equation, you get $d = 4$. This means that 1 popsicle earns $\$4$, which matches with the point $(1, 4)$ in the graph.**

Summary

☐ I can connect tables, graphs, and equations that represent the same relationship.

Unit 6.6, Lesson 16: Notes Name ____________________

My Notes

In 2021, one regular-fare subway ride costs $\$2.75$ in New York City.

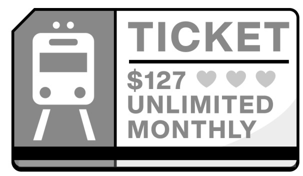
1.1 Write an equation to represent the relationship between total cost, t, and number of rides, r.

$$t = 2.75r$$

1.2 Use the equation to determine how much 15 rides would cost.

$t = 2.75(15) = 41.25$ **So it would cost** $\$41.25$ **for** 15 **rides.**

An unlimited monthly pass costs $\$127$.



2.1 Describe things to consider when buying an unlimited monthly pass.

You might consider how many times in a month you ride the subway.

2.2 Explain when it would be a good deal to buy the unlimited monthly pass.

It would be a good idea to buy the monthly pass when the price of all your single ride tickets is more than $\$127$**. You can use the equation to find out how many rides that would be.** $127 = 2.75r$ **when** $r = 46.2$**. This means that after** 46 **rides you would begin to save money with the monthly pass.**

Summary

☐ I can use tables, graphs, and equations to analyze an issue in society.