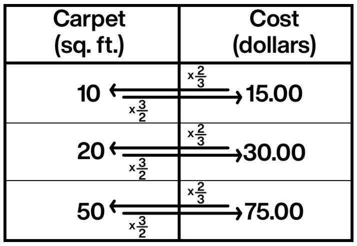
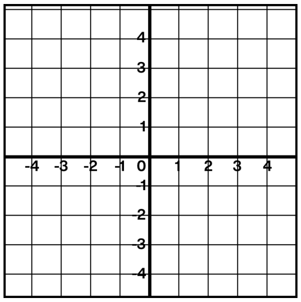
Proportional Relationships
Student Guide

Math 6 Unit 5 Accelerated
Part 1

Glossary

<table>
<tr><th>Term</th><th>Definition</th></tr>
<tr><td>constant of proportionality</td><td>A constant of proportionality is a number that the value of one quantity is multiplied by to get the value of the other quantity in a proportional relationship.
In this table, one constant of proportionality is 1.5.
The cost is $\$1.50$ for every square foot of carpet.

 </td></tr>
<tr><td>coordinate plane</td><td>The coordinate plane is a grid of two perpendicular axes that intersect at a point called the origin, or $(0,\ 0)$.

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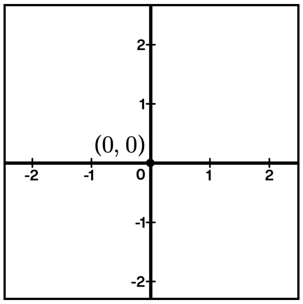
<tr><td>equivalent ratios</td><td>Two ratios are equivalent if you can multiply each number in the first ratio by the same factor to get the numbers in the second ratio.
For example, 10 square feet of carpet costs $\$15$. If you buy 20 square feet of carpet, it would cost $\$30$, because $10: 15$ and $20: 30$ are equivalent ratios.
<table><tr><th>Carpet (sq. ft.)</th><th>Cost (dollars)</th></tr><tr><td>10</td><td>15.00</td></tr><tr><td>20</td><td>30.00</td></tr><tr><td>50</td><td>75.00</td></tr></table></td></tr>
<tr><td>origin</td><td>The origin is the point $(0,\ 0)$ in the coordinate plane.
This is where the horizontal axis and the vertical axis cross.

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<table>
<tr><td>reciprocal</td><td>Two factors whose product is 1 are called reciprocals.

In this example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \cdot \frac{2}{3} = 1$.</td></tr>
<tr><td>proportional relationship</td><td>A proportional relationship is a set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

<table>
<tr><th>Carpet (sq. ft.)</th><th>Cost (dollars)</th></tr>
<tr><td>10 $\xrightarrow{\times 1.5}$</td><td>15.00</td></tr>
<tr><td>20 $\xrightarrow{\times 1.5}$</td><td>30.00</td></tr>
<tr><td>50 $\xrightarrow{\times 1.5}$</td><td>75.00</td></tr>
</table>
</td></tr>
</table>

Unit 2 Summary

Prior Learning	**Math 7, Unit 2**	**Future Learning**
Grades 3–5 - Fraction operations - Graphing coordinates Math 6 - Equivalent ratios - Unit rates Math 7, Unit 1 - Scale factor	- Proportional relationships (in tables, equations, and graphs)	Math 7, Unit 4 - Proportional relationships and percentages Math 8 - Slope - Linear relationships

Proportional Relationships in Tables

Carpets are sold at a price per square foot, so the ratios for amount of carpet to cost are all equal.

$\frac{\$15}{10\, sq.ft.}=\frac{\$30}{20\, sq.ft.}=\frac{\$75}{50\, sq.ft}=\$1.5$ per square foot

This is called a **proportional relationship.**

Carpet (sq. ft.)	**Cost (dollars)**
10	**15.00**
20	**30.00**
50	**75.00**

In this relationship, every square foot of carpet costs $\$1.50$.

This number 1.5 is called a **constant of proportionality.**

Carpet (sq. ft.)	**Cost (dollars)**
10 $\xrightarrow{x\ 1.5}$	**15.00**
20 $\xrightarrow{x\ 1.5}$	**30.00**
50 $\xrightarrow{x\ 1.5}$	**75.00**

Another constant of proportionality in this example is $\frac{2}{3}$.

You get $\frac{2}{3}$ of a square foot of carpet for every dollar spent.

Carpet (sq. ft.)	**Cost (dollars)**
10 $\xleftarrow{x\frac{2}{3}}$	**15.00**
20 $\xleftarrow{x\frac{2}{3}}$	**30.00**
50 $\xleftarrow{x\frac{2}{3}}$	**75.00**

Unit 7.2, Family Resource

Proportional Relationships in Equations

The cost of carpet is 1.5 times the number of square feet.
We can represent this relationship with the equation:

$y = 1.5x$	x represents the number of carpet bought. y represents the cost of the carpet, in dollars.

In general, the equation for a proportional relationship looks like:

$y = kx$	x and y represent the two related quantities. k represents the constant of proportionality.

Proportional Relationships in Graphs

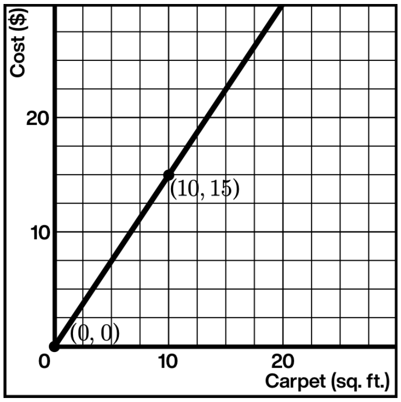
Graphs of proportional relationships:

- Lie on a line.
- Include the point $(0, 0)$, called the origin.

If you buy 10 square feet of carpet, it costs $\$15$.

If you buy 0 square feet of carpet, it costs $\$0$.

These are represented by the points $(10,\ 15)$ and $(0,\ 0)$.



Using Proportional Relationships

We can identify the constant of proportionality (1.5) in every representation.

Description

Each square foot of carpet costs $\$1.50$.

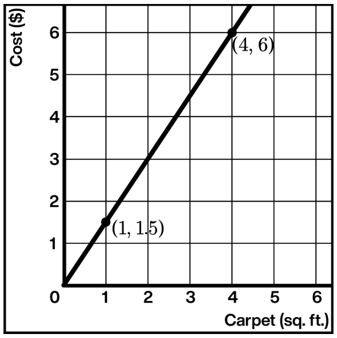
Table

Carpets (sq. ft.)	Cost (dollars)
0	0
1	1.50
4	6

Equation

$y = 1.5x$

Graph



$$\frac{6}{4} = 1.5$$

☐ $c = 6.28r$

Proportional Relationships in Graphs

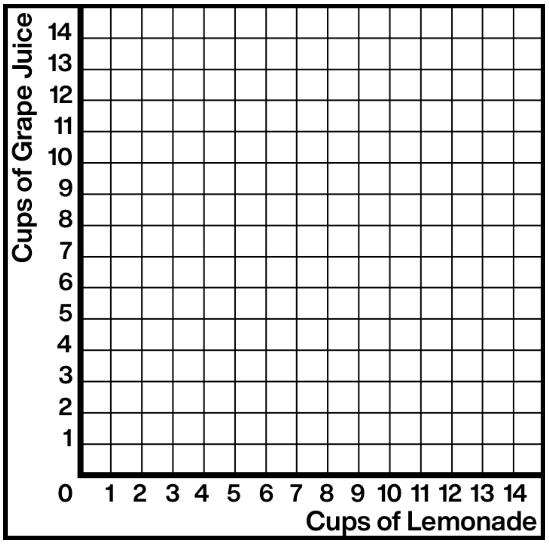
Here is a brief recipe for Grape-Ade:

For every 6 cups of lemonade, mix in 3 cups of grape juice.

8. Create a graph that represents the relationship between the amounts of lemonade and the amounts grape juice in different-sized batches of Grape-Ade.

9. Choose one point on your graph. Explain what that point means in a sentence.

10. What is a constant of proportionality for this relationship? Circle where you see the constant of proportionality in the graph.



Using Proportional Relationships

11. Describe a proportional relationship between quantities that you might encounter in your life.

12. What is a constant of proportionality in the relationship from Problem 4?

What does this number mean?

Try This at Home

Proportional Relationships in Tables

Here is a brief recipe for pineapple soda:

For every 5 cups of soda water, mix in 2 cups of pineapple juice.

1. Create a table that shows at least three possible combinations of soda water and pineapple juice to make pineapple soda.

2. How much pineapple juice would you mix with 20 cups of soda water?

3. How much soda water would you mix with 20 cups of pineapple juice?

4. What is one constant of proportionality for this situation?

Proportional Relationships in Equations

5. Write an equation that represents the relationship in the recipe above, using s for cups of soda water and p for cups of pineapple juice.

6. Write a second equation that represents this relationship.

7. Select all the equations that represent a proportional relationship:

   ☐ $K = C + 283$

   ☐ $m = \frac{1}{4}j$

   ☐ $V = s^3$

   ☐ $h = \frac{14}{x}$

Unit 7.2, Family Resource

Solutions:

1. *Responses vary.*

Cups of Soda Water	Cups of Pineapple Juice
5	2
10	4
2.5	1

2. 8 cups of pineapple juice would be needed for 20 cups of soda water. One way to think about this is that you need 4 times the recipe because $5 \cdot 4 = 20$, and so $2 \cdot 4 = 8$ cups. Another way to think about it is that there are $\frac{2}{5}$, or 0.4, cups of pineapple juice per cup of soda water. Therefore, you would need $\frac{2}{5} \cdot 20 = 8$ cups of pineapple juice.

3. 50 cups of soda water would be needed for 20 cups of pineapple juice. There are $\frac{5}{2}$, or 2.5, cups of soda water per cup of pineapple juice. $\frac{5}{2} \cdot 20 = 50$ cups of soda water.

4. Both $\frac{2}{5}$ and $\frac{5}{2}$ are constants of proportionality for this situation.

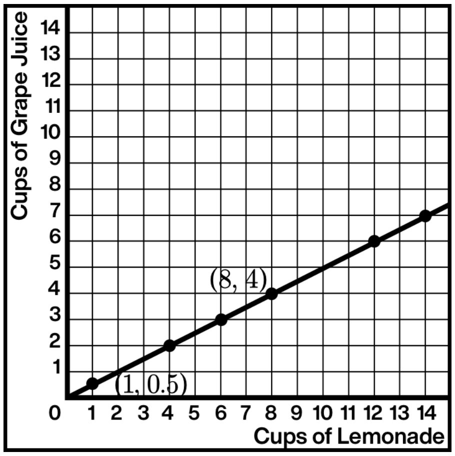
5. Two equations that represent this situation are $p = 0.4s$ and $s = 2.5p$, where s represents the number of cups of soda water and p represents the number of cups of pineapple juice used.

6. See above.

7. ✓ $m = \frac{1}{4}j$

   ✓ $c = 6.28r$

8.



9. The point $(8, 4)$ means that you can make Grape-Ade using 8 cups of lemonade and 4 cups of grape juice.

10. The constant of proportionality is 0.5 or $\frac{1}{2}$. You can see this as the second coordinate of the point $(1, 0.5)$, or in the simplified ratio $\frac{4}{8}=\frac{1}{2}$.

11. *Responses vary.*
    - Miles driven on a new tank vs. gallons of gas used
    - Number of toy cars purchased vs. cost
    - Amount of flour used in cookies vs. number of cookies baked

12. *Responses vary.* The meaning of the constant of proportionality often involves “per” or “for every.”

Unit 7.2, Lesson 2: Notes

Name ______________________________

My Notes

1. What does it mean for two quantities to be in a **proportional relationship**?

   In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity.

2. Complete the tables so that one table shows a proportional relationship and the other does not. ***Responses vary.***

Proportional Relationship

x	y
2	8
6	24
1	4

Not a Proportional Relationship

x	y
2	8
6	16
4	4

3. Show (or explain) how you know that the table on the left represents a proportional relationship.

   ***Responses vary*. For each row, I multiplied the x-value by 4 to get the corresponding y-value.**

Summary

☐ I can identify patterns in tables that represent proportional relationships.

☐ I can use a table to calculate unknown quantities in a proportional relationship.

Unit 7.2, Lesson 3: Notes

Name ____________________

My Notes

1. What is a **constant of proportionality**? Give an example.

   In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity. This number is called the *constant of proportionality*. For example, since feet and inches are in a proportional relationship, I can multiply any number of feet by a constant of proportionality (12) to find the number of inches.

2. An 8-ounce glass of apple juice contains 26 grams of sugar. Complete the table to determine the amount of sugar in different sizes of apple juice.

Apple Juice		
	Volume (oz.)	Sugar (grams)
Glass	8	26
Bottle	12	39
Carton	32	104
Jug	128	416

3. What is the constant of proportionality in this relationship? What does it tell us about the situation?

   3.25. **There are 3.25 grams of sugar for each ounce of apple juice.**

Summary

☐ I can determine the constant of proportionality from a table and explain what it means.

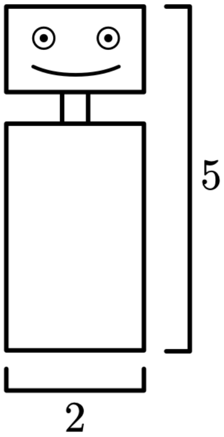
☐ I can use the constant of proportionality to calculate unknown information in a table.

☐ I can justify whether a table represents a proportional relationship or not.

Unit 7.2, Lesson 4: Notes Name ______________________________

My Notes

The table shows measurements for three robots. The relationship between width and height is proportional.



1. Complete the table.

Robot Width in inches (w)	**Robot Height in inches (h)**
2	5
6	15
11	27.5

2. Write instructions explaining how to calculate the height of the robot given any robot width.

   ***Responses vary*. If you know the width of the robot, you can multiply it by the constant of proportionality, 2.5, to find the height of the robot.**

3. Write an equation that relates the robot height, h, to the robot width, w.

   $h = 2.5w$ **(or equivalent)**

Summary

☐ I can explain where to find the constant of proportionality as a value in a table.

☐ I can write equations to represent proportional relationships.

Unit 7.2, Lesson 6: Notes

Name ____________________________

My Notes

It took Jayden 6 minutes to fill a bathtub with 24 gallons of water from a faucet that was flowing at a steady rate.

Time in Minutes (t)	Gallons of Water (w)
0	0
2	8
4	16
6	24

1. What are the two constants of proportionality for this situation?

   $4, \frac{1}{4}$

   How are they related?

   ***Responses vary*. The constants of proportionality are reciprocals of each other.**

2. What does each constant of proportionality tell you about this situation?

   ***Responses vary.* 4 tells me that 4 gallons are added to the bathtub each minute. $\frac{1}{4}$ tells me that it takes $\frac{1}{4}$ of a minute to fill the bathtub with 1 gallon of water.**

3. Write two equations that relate w and t in this situation.

   $w = 4t, \ t = \frac{1}{4} w$

Summary

☐ I can explain what *reciprocal* means and how it is related to constants of proportionality.

☐ I can write two equations for the same proportional relationship.

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Unit 7.2, Lesson 7: Notes

Name ____________________

My Notes

x	y
0	3
2	7
12	27
3.5	10

1. Use the equation $y = 2x + 3$ to complete the table.

2. Does the equation represent a proportional relationship?

   Explain.

   No.

   ***Explanations vary*. The constant of proportionality is not the same for each row.**

[✓ $\frac{x}{2} = y$]

$y = 2x + 1$

[✓ $y = 1.5x$]

3. Circle the equations that represent a proportional relationship.

4. How can you tell if an equation represents a proportional relationship?

   ***Responses vary*. Proportional relationships have equations of the form $y = kx$.**

Summary

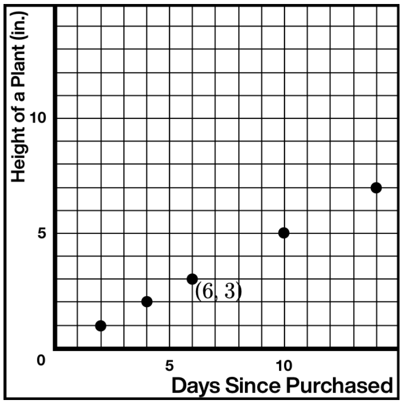
☐ I can explain why a relationship is proportional or not by looking at the equation.

Unit 7.2, Lesson 8: Notes

Name ______________________________

My Notes

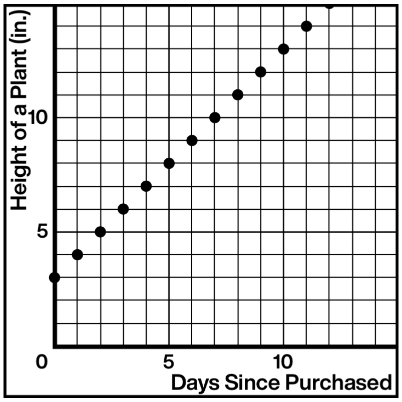
A plant's height is proportional to the number of days since it was purchased. On Day 6, it was 3 inches tall.



1. Add more points to the graph to represent the plant's height on other days.

2. Should the origin, $(0, 0)$, be included in this relationship? Why or why not?

   Yes. *Responses vary*. In proportional relationships, the y-value is equal to the x-value times the constant of proportionality, k. This means that the y-value when x is 0 is equal to $0 \cdot k$, which is always equal to 0.



3. This graph shows information about a different plant. Does this represent a proportional relationship? Why or why not?

   No. *Responses vary*. The graph doesn't go through the origin, so the relationship isn't proportional.

Summary

☐ I can explain what a proportional relationship looks like when represented with a graph.

☐ I can justify if a graph represents a proportional relationship or not.

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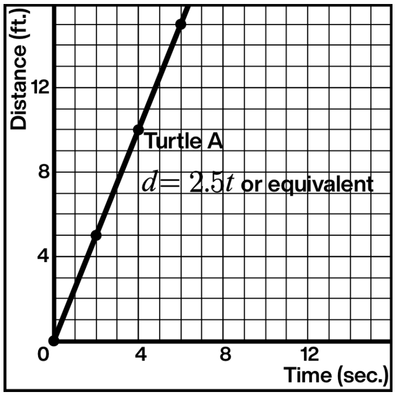
Unit 7.2, Lesson 10: Notes Name ________________________________

My Notes

Two turtles went for a walk. They each walked a distance, d, after t seconds.

1. Turtle A's walk is represented in the graph. Write an equation for this relationship.



$d = 2.5t$ **or equivalent**

2. Turtle B's walk is represented by the equation $d = 2t$.

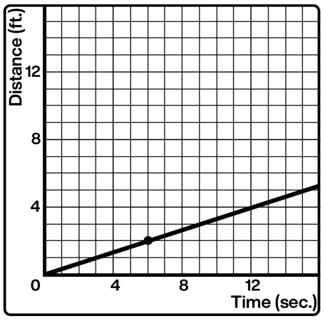
Which turtle walked faster? **Turtle A.**

Explain how you know.

Explanations vary. **Turtle A walked 2.5 feet per second, while Turtle B walked 2 feet per second.**

3. Explain how you know the equation $d = \frac{1}{3}t$ matches the graph.

***Responses vary*. The graph goes through the points $(0, 0)$ and $(6, 2)$, so a constant of proportionality is $\frac{2}{6}$ or $\frac{1}{3}$, which is the same as the constant of proportionality in the equation.**



$$d = \frac{1}{3}t$$

Summary

☐ I can write an equation of a proportional relationship from a point on a graph.

☐ I can compare related proportional relationships based on their graphs.

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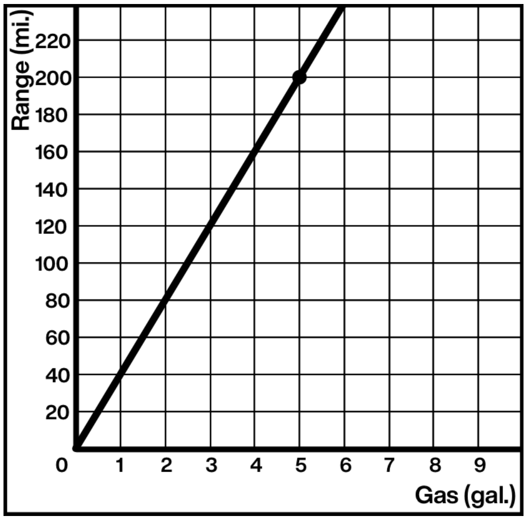
Unit 7.2, Lesson 9: Notes Name ______________________________

My Notes

The graph shows how far a car travels using any amount of gas.



1. Determine the constant of proportionality for the relationship between gallons of gas and miles.

   40

2. What does the constant of proportionality say about the car?

   ***Responses vary*. The constant of proportionality tells us the gas mileage of the car, or how far it can travel using 1 gallon of gas.**

3. In general, how can you use a graph to find the constant of proportionality for a proportional relationship?

   - **Determine the y-value when the x-value is 1.**
   - **Determine the number you need to multiply the x-value by to find the y-value. For example, each y-coordinate in the example is 40 times the x-coordinate.**

Summary

☐ I can interpret points on the graph of a proportional relationship.

☐ I can identify the constant of proportionality from a graph of a proportional relationship.

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Unit 7.2, Lesson 11: Notes

Name ______________________________

My Notes

1. Create a table, an equation, and a graph of this proportional relationship.

Description

A bakery uses 7 scoops of chocolate for every 2 cups of milk to make chocolate milk.

The constant of proportionality is the scoops of chocolate for each cup of milk, $\frac{7}{2}$.

Table

Cups of Milk (c)	Scoops of Chocolate (s)
2	7
4	14
1	3.5

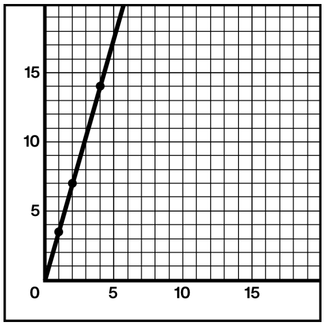
Multiply each value in the first column by 3.5 to get the values in the second column.

Equation

$$s = 3.5c$$

The constant of proportionality is k in the equation $s = kc$, so here it is 3.5.

Graph



2. Circle or show where you can see the constant of proportionality in each representation.

Summary

☐ I can create four different representations of a proportional relationship (description, table, graph, equation).

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Unit 7.2, Lesson 12: Notes

Name ____________________

My Notes

Information about the fuel usage of two cars is shown below.

Car A	**Car B**
20 **gallon tank**	12 **gallon tank**
24.9 **miles per gallon**	552 **miles per tank**

1. Which vehicle can go farther on 1 gallon of gas?

   ***Responses vary*. Car A can go 24.9 miles on 1 gallon of gas. Car B can go $552 \div 12 = 46$ miles on 1 gallon of gas. Therefore, Car B can go farther on 1 gallon of gas.**

2. Which vehicle can go farther on a full tank of gas?

   ***Responses vary*. Car A can go $20 \cdot 24.9 = 498$ miles on a tank of gas. Car B can go 552 miles on a tank of gas. Therefore, Car B can go farther on a full tank of gas.**

Summary

☐ I can model a real-world situation by deciding what information is important and making assumptions.

☐ I can use proportional relationships to answer a question about a real-world situation.