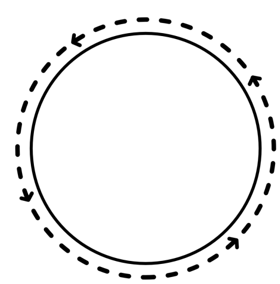
Proportional Relationships
Student Guide

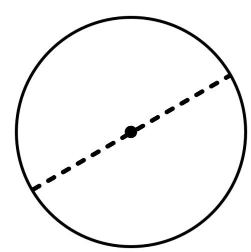
Math 6 Unit 5 Accelerated
Part 2

Glossary

<table>
<tr><th>Term</th><th>Definition</th></tr>
<tr><td>circle</td><td>A shape made out of all the points that are the same distance from a center.</td></tr>
<tr><td>circumference</td><td>The circumference of a circle is the distance around the circle. If you imagine the circle as a piece of string, it is the length of the string.

If the circle has a diameter d, then the circumference is $C = \pi d$.

The circumference of a circle with a radius of 5 cm is $C = \pi \cdot 2 \cdot 5 = 10\pi$ cm, or about 31.416 cm.
</td></tr>
<tr><td>diameter</td><td>A diameter is a line segment that goes from one edge of a circle to the other and passes through the center.

Every diameter of a circle is the same length.
</td></tr>
</table>

Unit 3 Summary

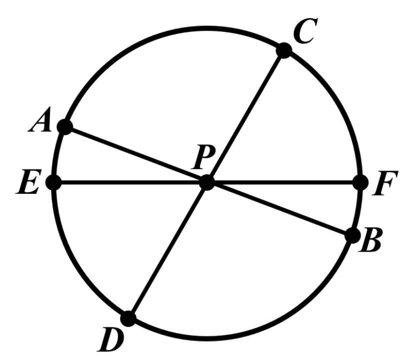
Prior Learning	**Math 7, Unit 3**	**Future Learning**
Math 6 ● Area of triangles and quadrilaterals ● Evaluating formulas Math 7 ● Proportional relationships	● Circumference of a circle ● Area of a circle	Math 8, Unit 5 ● Volume of cylinders, cones, and spheres

Circumference of a Circle

Circles are shapes made up of all the points that are the same distance away from a center.

Here are some common measurements of a circle.

- The **radius** goes from the center to the edge of a circle.
- The **diameter** goes from one edge of a circle to the other and passes through the center.
- The **circumference** is the distance around the circle.



There is a proportional relationship between the diameter and circumference of a circle.

The constant of proportionality of this relationship is π (pronounced "pie").

Common approximations for π are 3.14, $\frac{22}{7}$, and 3.14159, but none of these are exactly π.

The relationship between the diameter and circumference of a circle is exactly $C = \pi d$.

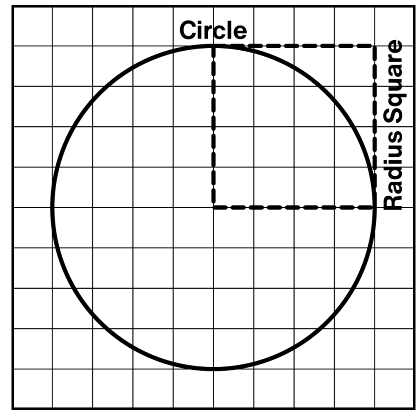
If AP is 5 inches, then AB is $2 \cdot 5 = 10$ inches.

The circumference is $C = \pi(10) = 10\pi$ inches, or about 31.4 inches.

Area of a Circle

We can estimate the area of a circle using radius squares.

A little more than 3 radius squares cover any circle, so this circle's area would be a little more than $3 \cdot 4^2 = 48$ square units.



The relationship between the radius and area of a circle is exactly $A = \pi r^2$.
The area of the circle above is $\pi(4)^2 = 16\pi \approx 50.27$ square units.

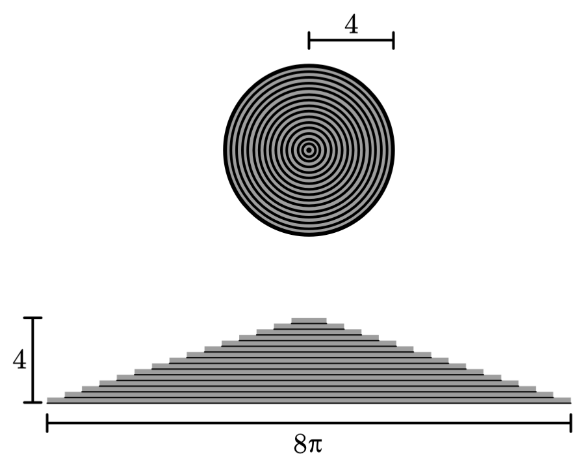
We can prove that this formula is correct by cutting a circle into rings and rearranging the rings into a triangle.

The height of the triangle is the radius of the circle.

The base of the triangle is its circumference.

The area of the triangle is:

$$\begin{aligned} A &= \frac{1}{2} \cdot b \cdot h \\ &= \frac{1}{2} \cdot 8\pi \cdot 4 \\ &= 16\pi \text{ square units.} \end{aligned}$$

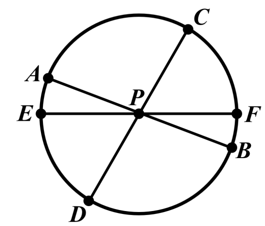


Try This at Home

Circumference of a Circle

1.1 AP is a radius of this circle. List every other radius.

1.2 EF is a diameter of this circle. List every other diameter.

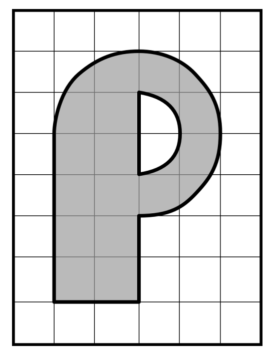


A candle has a diameter of 12 centimeters.

2.1 What is the distance from the edge of the candle to the wick (at the center)?

2.2 Would a ribbon 40 centimeters long wrap around the candle? Explain your thinking.

3. Determine the total perimeter of this figure.



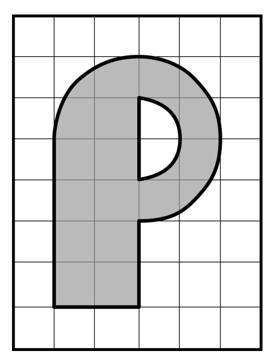
Area of a Circle

A rectangular wooden board, 20 inches wide and 40 inches long, has a circular hole cut out of it.

4.1 If the diameter of the circle is 6 inches, what is the area of the circular hole?

4.2 What is the area of the board after the circle is removed?

5. Determine the total shaded area of this figure.



Unit 7.3, Family Resource

Solutions:

1.1 BP, CP, DP, EP, FP

1.2 AB, CD

2.1 6 centimeters. This would be the radius of the circle, which is half of the diameter.

2.2 Yes.

Explanations vary. The distance around the candle is its circumference, which would be $C = \pi(12) = 12\pi \approx 37.7$ centimeters. This means a 40-centimeter ribbon would wrap around.

3. $4\pi + 10$ units

The perimeter of the outside of the shape is $\frac{3}{4} \cdot \pi \cdot 4 = 3\pi$ units plus 8 units for the straight edges. The perimeter of the inside of the shape is 2 units plus $\frac{1}{2} \cdot \pi \cdot 2 = \pi$ units. $(3\pi + 8) + (\pi + 2) = 4\pi + 10$ units.

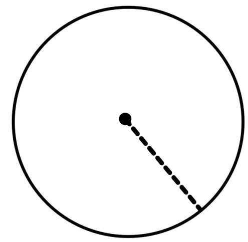
4.1 $\pi(3^2) = 9\pi \approx 28.3$ square inches

4.2 $800 - 9\pi \approx 771.7$ square inches

5. $2.5\pi + 8$ square units

The area of the large shape is $\frac{3}{4} \cdot \pi \cdot (2^2) = 3\pi$ square units for the part of a circle plus $2 \cdot 4 = 8$ square units for the area of the rectangle. The area of the hole is $\frac{1}{2} \cdot \pi \cdot (1^2) = 0.5\pi$ square units. $(3\pi + 8) - (0.5\pi) = 2.5\pi + 8$ square units.

Unit 7.3, Student Goals and Glossary

pi	Pi is a number that represents the constant of proportionality between the diameter and circumference of any circle. The symbol for pi is π. Some common approximations for π are $\frac{22}{7}$, 3.14, and 3.14159.
radius	A line segment that goes from the center to the edge of a circle. Every radius of a circle is the same length. 

desmos

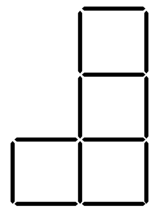
Unit 7.3, Lesson 1: Notes

Name ________________________________

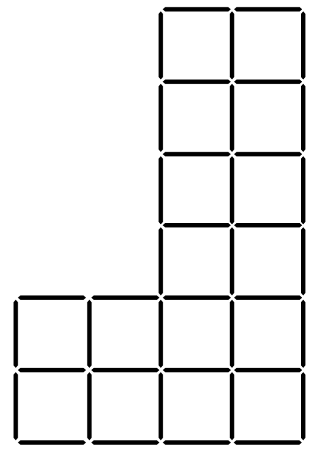
My Notes

1. Here is a shape with a side length of 2 toothpicks. Sketch a scaled copy of this shape with a side length of 4 toothpicks.

Bottom Side Length: 2



Bottom Side Length: 4



2. Complete the table with the number of toothpicks needed to build the perimeter and interior of each shape.

Side Length	Perimeter	Interior
2	10	3
4	20	22

3. Explain which relationships are proportional: side length and perimeter, side length and interior toothpicks, both, or neither.

Only the relationship between side length and perimeter is proportional. *Explanations vary.* There is the same constant of proportionality from side length to perimeter because $2 \cdot 5 = 10$ and $4 \cdot 5 = 20$.

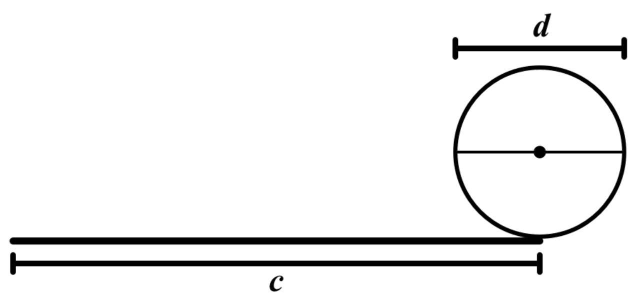
Summary

- ☐ I can explain whether or not the relationship between a side length or a diagonal of a shape and its perimeter is proportional.
- ☐ I can use proportional relationships to figure out missing side lengths, diagonals, and perimeters.

Name ____________________________________

My Notes

1. Describe the relationship between the diameter of a circle, d, and its **circumference**, C. ***Responses vary.***



The relationship between the diameter of a circle and its circumference is proportional. You can write the formula $C = \pi d$ to represent the relationship.

2. List some things you know about π. ***Responses vary.***

- **It is the constant of proportionality between the diameter of a circle and its circumference.**
- **It cannot be written as an exact decimal.**
- **It is close to but not exactly 3.14 and $\frac{22}{7}$.**

3. Complete the table with measurements for each object.

Object	**Radius (cm)**	**Diameter (cm)**	**Circumference (cm)**
Coaster	5	10	10π
Ring	1.2	2.4	2.4π
Hoop	$\frac{150}{2\pi} \approx 23.87$	$\frac{150}{\pi} \approx 47.75$	150

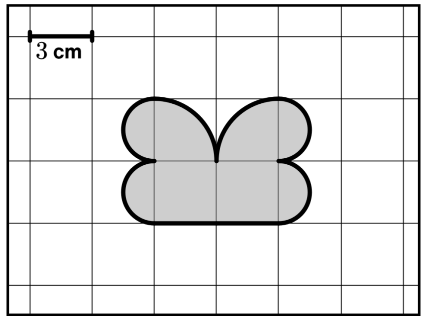
Summary

- ☐ I can describe the relationship between the radius, diameter, and circumference of a circle.
- ☐ Given the radius, diameter, or circumference of a circle, I can calculate the other two measurements.

Name ______________________

My Notes

1. Irene calculated the perimeter of the shape below as $9\pi + 6$ centimeters. Explain how you know she is correct.



Responses vary.

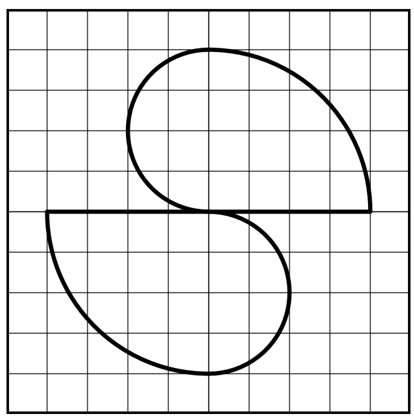
The total perimeter is equal to the perimeter of 2 quarter circles, 4 semicircles, and 2 straight edges.

Total perimeter:

$$2\left(\frac{1}{4} \cdot 6 \cdot \pi\right) + 4\left(\frac{1}{2} \cdot 3 \cdot \pi\right) + 2(3)$$

= $3\pi + 6\pi + 6 = 9\pi + 6$ **cm**

2. Calculate the perimeter of the shape below. Show all of your thinking.



Total perimeter:
2 **quarter circles**
$+\ 2$ **semicircles**
$+\ 2$ **straight edges**

Total perimeter:

$$2\left(\frac{1}{4} \cdot 8 \cdot \pi\right)$$

$$+\ 2\left(\frac{1}{2} \cdot 4 \cdot \pi\right)$$

$$+\ 2(4)$$

Total perimeter = $4\pi + 4\pi + 8 = 8\pi + 8$ **units**

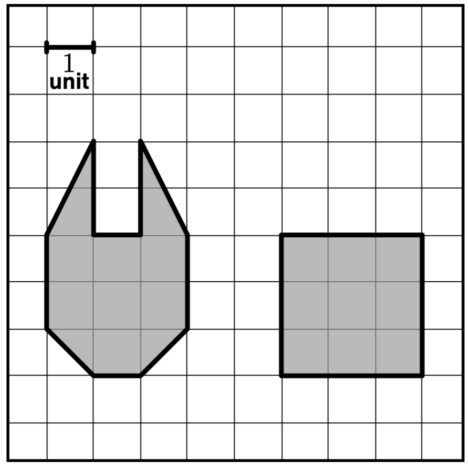
Summary

☐ I can calculate the perimeter of a complex shape that includes parts of circles.

☐ I can write perimeter as an expression that includes π, such as $20\pi + 50$.

My Notes

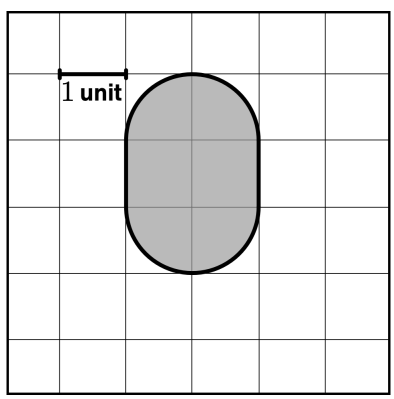
1. Tiara says these two figures have the same area. Is Tiara correct? Explain and show your thinking.



No. *Explanations vary.*

The two triangles on top of the left shape can be rearranged to make a 2-by-1 rectangle. The two triangles on the bottom can be rearranged to make a 1-by-1 square. This makes the total area of the left shape $2+6+1+1=10$ square units, 1 square unit more than the area of the square.

2. Do you think the area of this shape is more than 4 square units, less than 4 square units, or exactly 4 square units? Explain your thinking.



More than 4 square units. *Explanations vary.*

Each rounded section takes up more than half of the unit square, so the total area is more than $2+4(0.5)$, or 4 square units.

Summary

☐ I can determine the area of a complex shape using a variety of strategies.

☐ I can estimate the area of a shape with curved edges.

desmos

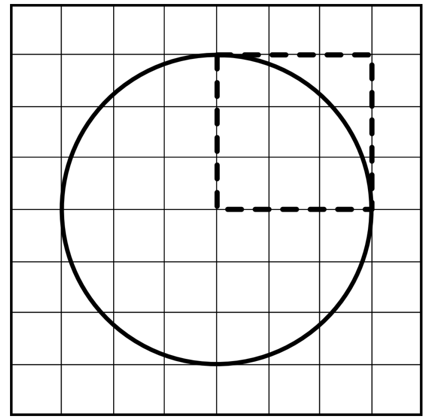
Unit 7.3, Lesson 6: Notes Name ______________________

My Notes

1. Draw a radius square for this circle.

What is the area of the radius square?

9 **square units**



2. Estimate the area of this circle using radius squares.

The area is about 3 **times the area of the radius square, so the area of the circle is a little more than** $3 \cdot 9 = 27$ **square units.**

3. What is the formula for the relationship between the radius of a circle and its area?

$A = \pi \cdot r^2$ **or Area =** $\pi \cdot$ **(radius)^2**

4. Use the formula to calculate the exact area of the circle.

$A = \pi \cdot r^2$

$A = \pi \cdot 3^2 = 9\pi \approx 28.27$ **square units**

Summary

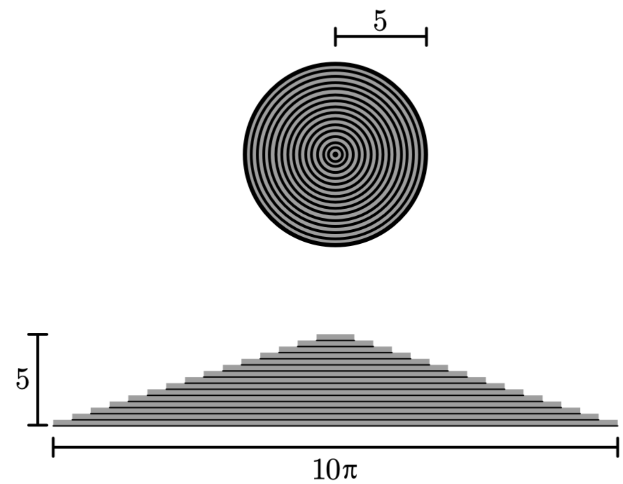
☐ I can describe the relationship between the radius of any circle and its area.

☐ I can calculate the area of a circle.

Unit 7.3, Lesson 7: Notes Name ______________________________

My Notes

Here is a circle cut into rings and unrolled into a triangle shape.



1. Calculate the area of the circle.

   $A = \pi \cdot r^2 = \pi \cdot 5^2 = 25\pi$ **square units**

2. Label the base and the height of the triangle.

   The base is the same length as the circumference of the circle, or 10π units.

   The height is equal to the radius, or 5 units.

3. Calculate the area of the triangle. How is it related to the area of the circle?

   $A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 10\pi \cdot 5 = 25\pi$

   The area of the triangle is equal to the area of the circle!

Summary

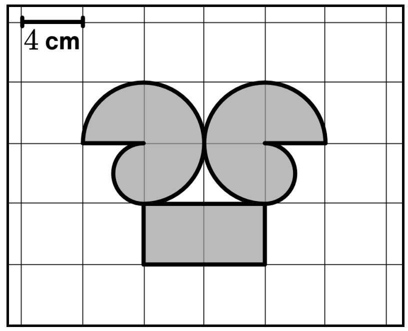
- ☐ I can explain whether the relationship between the radius and area of a circle is proportional or not.
- ☐ I can explain the formula of a circle's area by rearranging the circle into a triangle of the same area.

Unit 7.3, Lesson 8: Notes

Name ______________________________

My Notes

1. Amari calculated the area of the shape below as $28\pi + 32$ square centimeters. Explain how you know they are correct.



Responses vary.

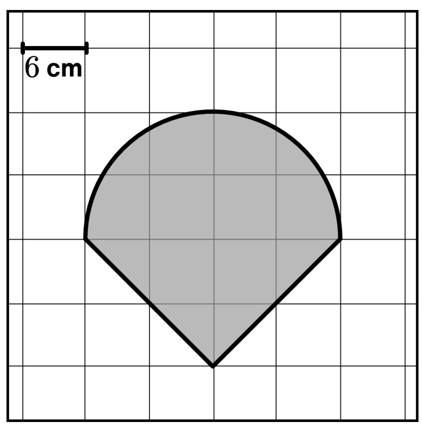
The total area is equal to the area of 6 quarter circles, 2 semicircles, and 2 squares.

Total area:

$$6\left(\frac{1}{4} \cdot 4^2 \cdot \pi\right) + 2\left(\frac{1}{2} \cdot 2^2 \cdot \pi\right) + 2(4 \cdot 4)$$

Area: $24\pi + 4\pi + 32 = 28\pi + 32$ **square centimeters**

2. Determine the area of the shape below.



Total area:
semicircle + triangle

Total area:

$$\frac{1}{2} \cdot 12^2 \cdot \pi + \frac{1}{2} \cdot 12 \cdot 24$$

Total area:
$72\pi + 144$ **square units**

Summary

- [ ] I can calculate the area of a complex shape that includes parts of circles.
- [ ] I can write area as an expression that includes π, such as $20\pi + 50$.