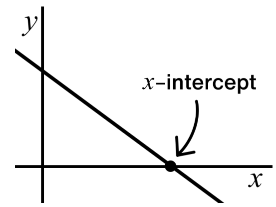
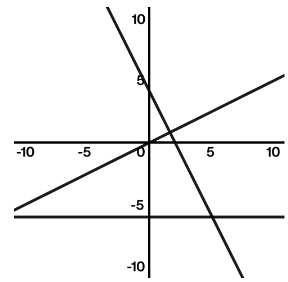
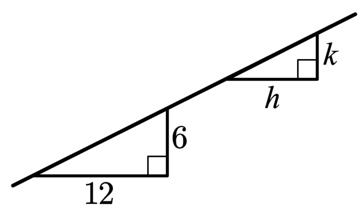
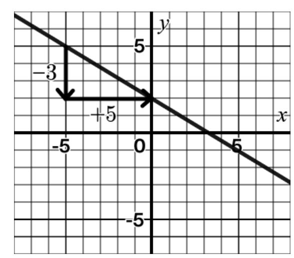
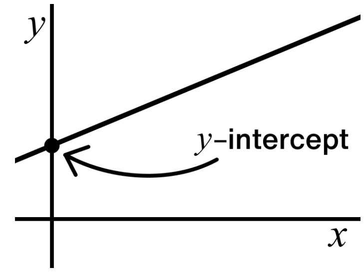
Linear Relationships and Systems of Linear Equations Student Guide

Math 7 Unit 4 Accelerated
Part 1

Glossary

Term	Definition
horizontal intercept (x-intercept)	The horizontal intercept, sometimes called the x–intercept, is the point where the graph of a line crosses the horizontal axis or when $y = 0$. 
linear relationship	A relationship between two quantities is linear if there is a constant rate of change. The relationship is called *linear* because its graph is a line. 
slope (rate of change)	Slope is a number that describes the direction and steepness of a line. To calculate the slope, divide the vertical distance between any two points on the line by the horizontal distance between those points. For example, the slope of the first line is $\frac{k}{h} = \frac{6}{12} = \frac{1}{2}$.  Slope represents the amount that y changes when x increases by 1. That's why the slope of a line is sometimes called a rate of change.  The slope of the second line is $-\frac{3}{5}$.
solution	A solution to an equation is a value or set of values that makes the equation true. The solution to the equation $4x + 3 = 23$ is $x = 5$ because $4(5) + 3 = 23$. One solution to the equation $4x + 3y = 24$ is $(6,\ 0)$ because $4(6) + 3(0) = 24$.

Unit 8.3, Student Goals and Glossary

vertical intercept (y-intercept)	The vertical intercept, sometimes called the y–intercept, is the point where the graph of a line crosses the vertical axis or when $x = 0$.	

Unit 3 Summary

Prior Learning

Grade 6
- Calculating unit rates

Grade 7
- Exploring proportional relationships

Grade 8, Unit 2
- Calculating slope

Grade 8, Unit 3

- Revisit proportional relationships
- Represent linear relationships
- Linear equations and points

Future Learning

Grade 8
- Solve systems of linear equations.
- Analyze linear functions and piecewise linear functions.

High School
- Quadratic and exponential functions
- Calculate average rate of change.

Proportionality Revisited

Here is an example of a proportional relationship between the amount of carpet bought and its cost. We can identify the **constant of proportionality** or **slope** (1.5) in every representation.

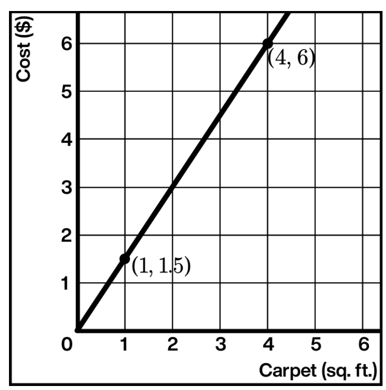
Table

Carpets (sq. ft.)	Cost (dollars)
0	0
1	1.50
4	6

Equation

$$y = 1.5x$$

Graph



Representing Linear Relationships

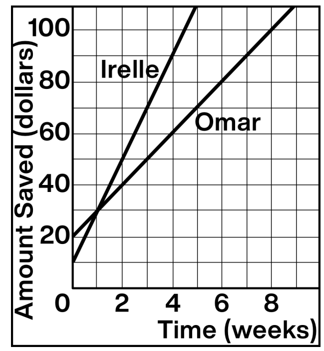
A relationship between two quantities is called a **linear relationship** if its graph is a line.

For example, Irelle and Omar save some of the money they earned.

Let w represent the number of weeks passed.
Let s represent the amount saved.

Equation for Irelle's savings: $s = 20w + 10$

Equation for Omar's savings: $s = 10w + 20$



Another example is measuring the amount of money on a public transit fare card over time.

The steepness of this line (called the **slope**) is

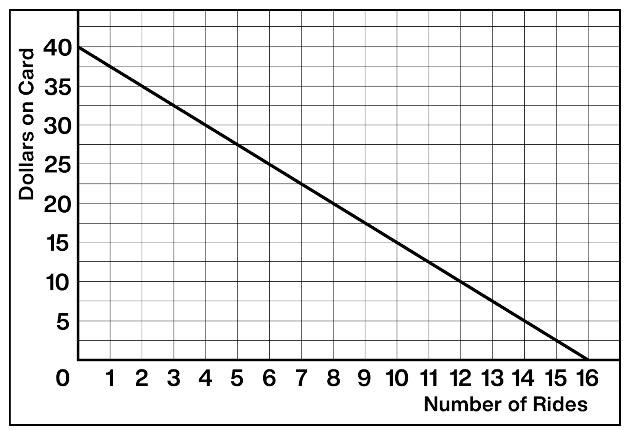
$$\frac{vertical\ change}{horizontal\ change} = \frac{-40}{16} = -2.5.$$

The **vertical intercept** (y-intercept) of this line is $(0, 40)$, which means the card started out with $\$40$ on it.

The **horizontal intercept** (x-intercept) of this line is

$(16, 0)$, which means there is no money left on the card after 16 rides.

One equation for this relationship is $y = -2.5x + 40$, where x represents the number of rides you take and y represents the money left on the card.



Linear Equations and Points

In general, the slope-intercept form of a linear equation looks like:

$y = mx + b$	x and y represent the two related quantities. m represents the slope of the graphed line. b represents the y-intercept of the line.

A *solution* to an equation is a value (or values) that makes the equation true.

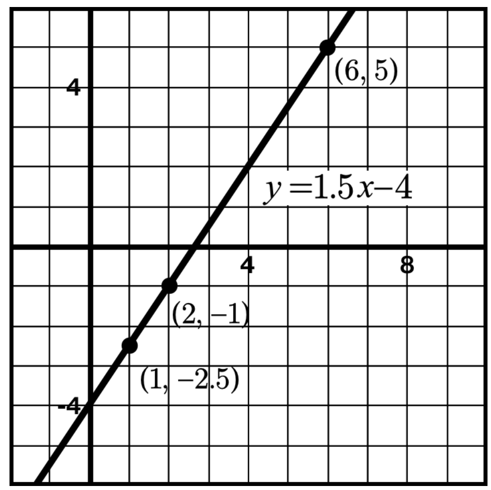
The graph of an equation is all of the ordered pairs that make the equation true.

The point $(2, -1)$ is on the line $y = 1.5x - 4$.

We can show that the point is a solution to the equation by substituting 2 and –1 for x and y.

$$y = 1.5x - 4$$
$$-1 = 1.5(2) - 4$$
$$-1 = 3 - 4$$
$$-1 = -1 \checkmark$$

Another form for a linear equation is **standard form**.
An equation for this line in standard form is $3x - 2y = 8$.



Try This At Home

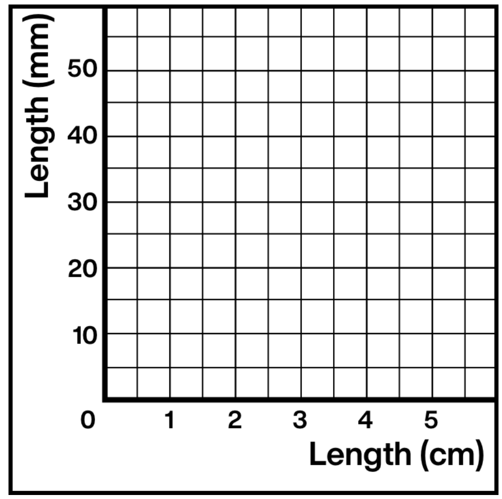
Proportionality

This table shows some lengths measured in centimeters and the equivalent lengths in millimeters.

1.1 Complete the table.

Length (cm)	Length (mm)
1	10
2.5	
4	
	55

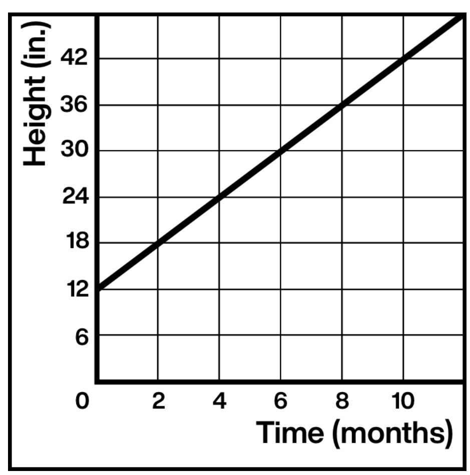
1.2 Sketch a graph of the relationship between centimeters and millimeters.



1.3 How would the graph look different if the y-axis were scaled by 10s instead of by 5s?

Representing Linear Relationships

This graph shows the height, in inches, of a bamboo plant each month after it was planted.



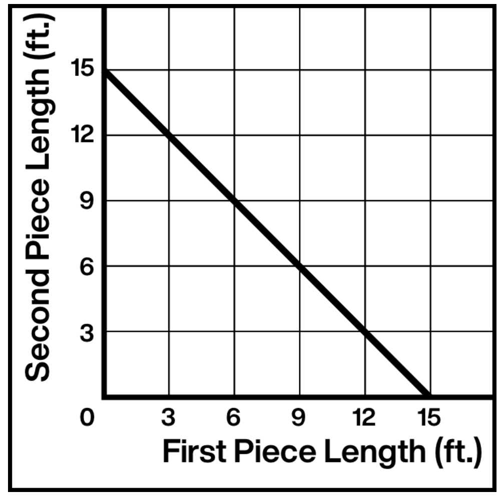
2.1 What is the slope of this line? What does that value mean in this context?

2.2 At what point does the line intersect the y-axis? What does that value mean in this context?

2.3 Write an equation showing the relationship between the two variables. Use x for the time in months and y for the height in inches.

Linear Equations and Points

A length of ribbon is cut into two pieces. The graph shows the length of the second piece, y, for each possible length of the first piece, x.



3.1 How long is the original ribbon?

Explain how you know.

3.2 What is the slope of the line?

What does it represent?

3.3 List two possible pairs of lengths for the two pieces and explain what they mean.

3.4 Write an equation for the relationship between the length of the first piece (x) and the length of the second piece (y).

Amplify Desmos Math

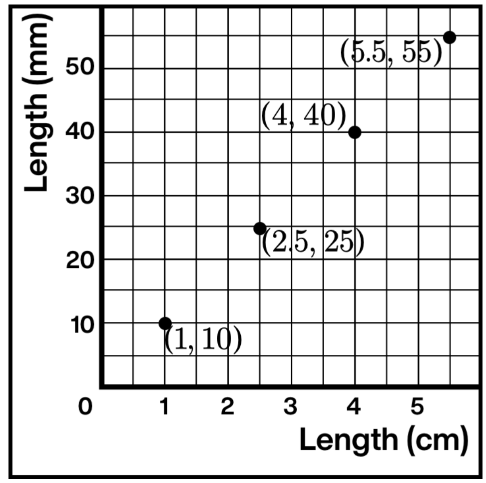
Unit 8.3, Family Resource

Solutions:

1.1

Length (cm)	Length (mm)
1	10
2.5	25
4	40
5.5	55

1.2



1.3 The graph would look less steep because each point would be twice as close to the x-axis.

2.1 3. Every month that passes, the bamboo plant grows an additional 3 inches.

2.2 $(0, 12)$. This bamboo plant was planted when it was 12 inches tall.

2.3 $y = 3x + 12$

3.1 15 feet. When the first piece is 0 feet long, the second is 15 feet long, so that is the length of the ribbon.

3.2 -1. For each length the first piece increases by, the second piece must decrease by the same length. For example, if we want the first piece to be 1 foot longer, then the second piece must be 1 foot shorter.

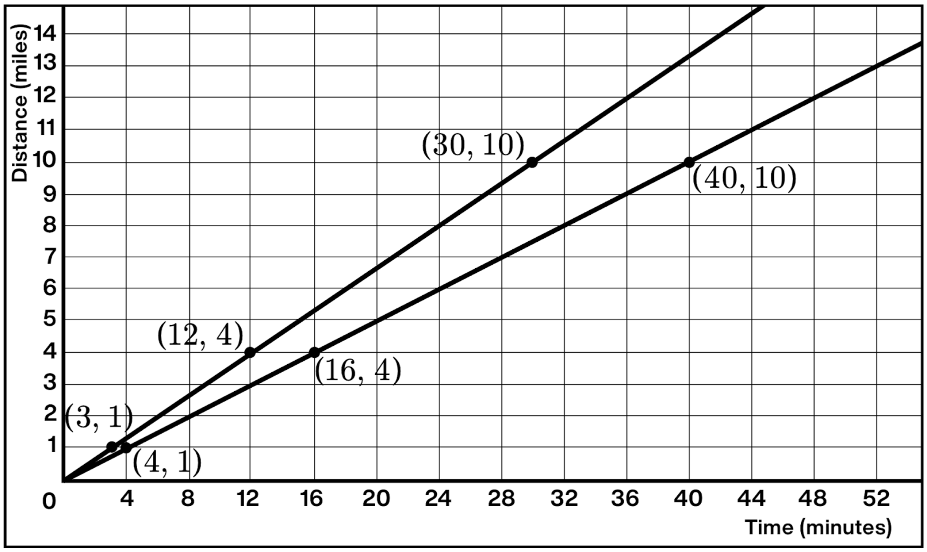
3.3 *Responses vary*. Two possible pairs: $(14.5, 0.5)$, which means the first piece is 14.5 feet long, so the second piece is only a half foot long. $(7.5, 7.5)$, which means each piece is 7.5 feet long, so the original ribbon was cut in half.

3.4 $x + y = 15$

Unit 8.3, Lesson 1: Notes Name ___________________________

Learning Goal(s):
- **I can graph a proportional relationship from a story.**
- **I can use the constant of proportionality to compare the pace of different turtles.**

Here are the graphs showing Jasmine and Sothy's distance on a long bike ride. Jasmine rides 4 miles every 16 minutes, and Sothy rides 4 miles every 12 minutes.



Which graph goes with which rider? **The line that includes the points $(3,\ 1)$ and $(30,\ 10)$ represents Sothy.** **The line that includes the points $(4,\ 1)$ and $(40,\ 10)$ represents Jasmine.**	Who rides faster? **Sothy rides faster.**
Jasmine and Sothy start a bike trip at the same time. How far have they traveled after 24 minutes? **Jasmine traveled 6 miles and Sothy traveled 8 miles.**	How long will it take each of them to reach the end of the 12-mile bike path? **Jasmine will take 48 minutes and Sothy will take 36 minutes.**

Summary Question

How can you graph a proportional relationship from a story?

To graph a proportional relationship from a story, look for how one measurement is related to another measurement. Then you can graph that point and draw a line that includes both $(0,\ 0)$ and the graphed point.

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Unit 8.3, Lesson 2: Notes Name ______________________________

Learning Goal(s):
- **I can graph a proportional relationship from an equation.**
- **I can tell when two graphs are of the same proportional relationship even if the scales are different.**

Rate of Change

Definition

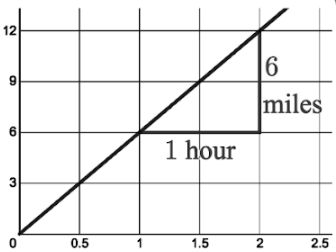
The amount y changes when x increases by 1.

Facts/Characteristics

The rate of change in a linear relationship is also the slope of its graph.

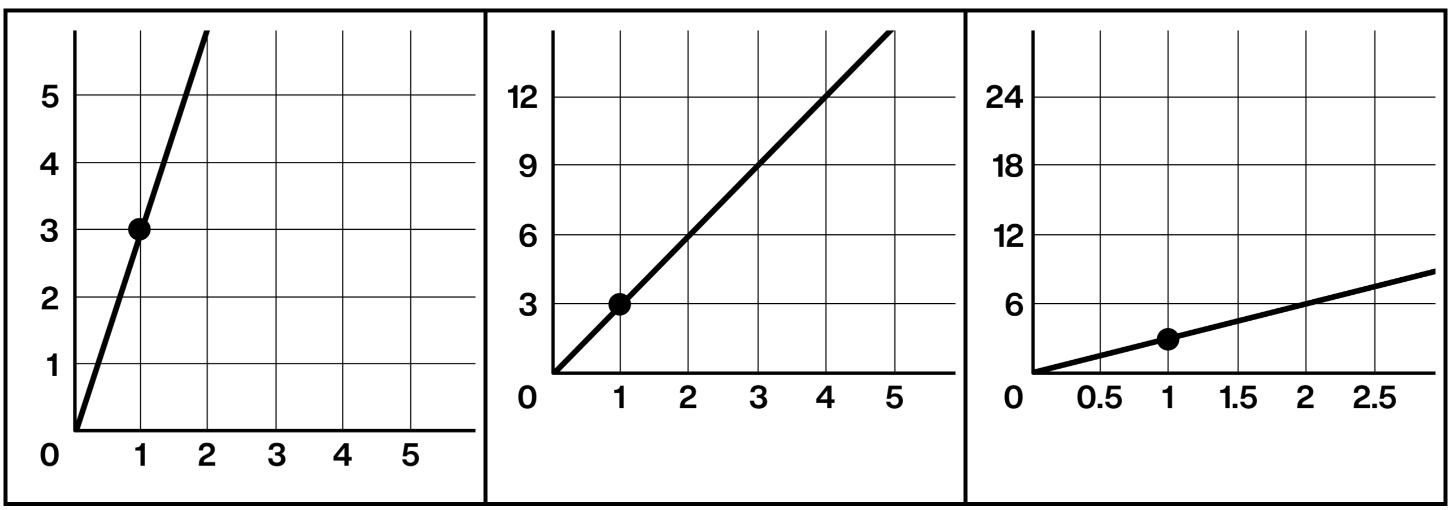
Examples

I bike 6 miles in 1 hour.



Non-Examples

Sketch the graph of the proportional relationship $y = 3x$ by scaling the axes three different ways.



Summary Question

How can you tell when two graphs have the same proportional relationship?

When the rate of change is the same for different graphs, then the graphs represent the same proportional relationship.

Unit 8.3, Lesson 3: Notes Name ____________________________

Learning Goal(s):
- **I can compare proportional relationships represented in different ways.**

When two proportional relationships are represented in different ways, we compare them by finding a common piece of information.

Terrance's earnings are represented by the equation $y = 14.5x$, where y is the amount of money he earns, in dollars, for working x hours.

The table shows some information about Jaylin's pay.

Time Worked (hours)	Earnings (dollars)
7	92.75
4.5	59.63
37	490.25

How much does Terrance get paid per hour?

$\$14.50$ **per hour**

How much does Jaylin get paid per hour?

$\$13.25$ **per hour**

After 20 hours, how much more does the person who gets paid a higher rate have?

Terrance is paid a higher rate than Jaylin. Terrance earns $\$1.25$ more per hour than Jaylin, which means that after 20 hours of work, Terrance has $\$25$ more than Jaylin.

Summary Question

How can you determine the rate of change of a proportional relationship from . . .

. . . a table? . . . a graph? . . . an equation?

From a table: Divide the y-value by the corresponding x-value. You should get the same value regardless of which pair of values is selected.

From a graph: Look at how much the y-value changes when the x-value increases by 1.

From an equation: The rate of change is the amount by which y changes when x increases by 1. For a proportional relationship, this can be found by setting $x = 1$ and solving for y.

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Unit 8.3, Lesson 4: Notes Name ______________________________

Learning Goal(s):

- **I can find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship.**

Linear Relationship

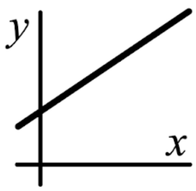
Definition

When one quantity changes by a certain amount, the other quantity always changes by a set amount.

Facts/Characteristics

In a linear relationship, one quantity has a constant rate of change with respect to the other. The relationship is called linear because its graph is a line.

Examples

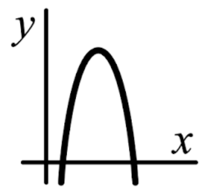


$y=2x+10$

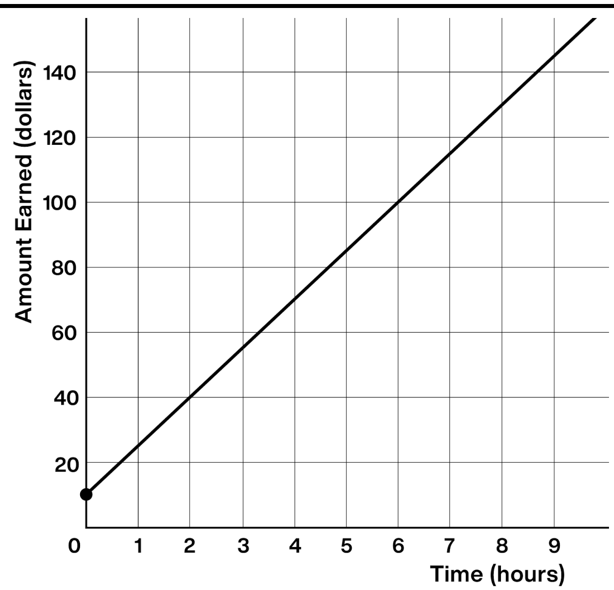
For each cup added, the total height increases 2 inches.

Non-Examples

$y=|x|+10$



Aniyah starts babysitting. She charges $\$10$ for traveling to and from the job, and $\$15$ per hour. Here is a graph of Aniyah's earnings based on how long she works.



Show how to figure out the rate of change using the graph.

Looking at the points $(2,\ 40)$ and $(6,\ 100)$, we can see that when Aniyah's hours increase by $6-2=4$, her earnings increased by $100-40=60$ dollars.

So the rate of change is $\frac{60}{4}$ ($\$15$ per hour).

Summary Question

How can you find the rate of change of a linear relationship?

The rate of change can be found by figuring out the slope of the line representing the relationship.

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Unit 8.3, Lesson 5: Notes Name ____________________

Learning Goal(s):
- **I can interpret the vertical intercept of a graph of a real-world situation.**
- **I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.**
- **I can use patterns to write a linear equation to represent a situation.**

Vertical Intercept

Definition

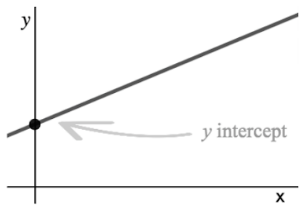
The vertical intercept is the point where the graph of a line crosses the vertical axis.

Facts/Characteristics

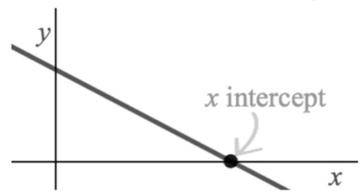
This is the y value when $x=0$.

Proportional relationships have a vertical intercept of 0.

Examples

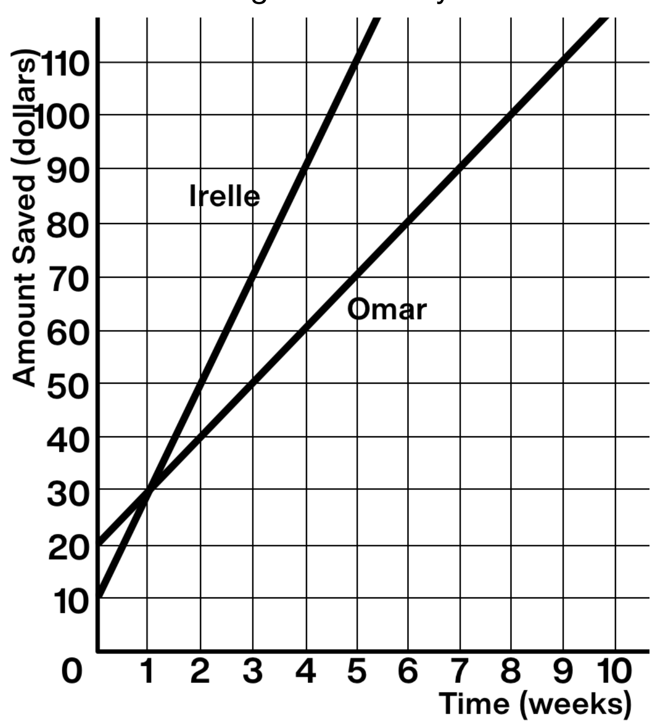


Non-Examples



Omar and Irelle decide to save some of the money they earn to use during the school year.

Here are graphs of how much money they will save after 10 weeks if they each follow their plans.



How much money does Omar have to start? $\$20$	How much money does Irelle have to start? $\$10$
How much money does Omar plan to save per week? $\$10$ **per week**	How much money does Irelle plan to save per week? $\$20$ **per week**

Summary Question

How can you find the vertical intercept and the slope from a graph?

To find the vertical intercept, look at the value where the graph intersects the y -axis.

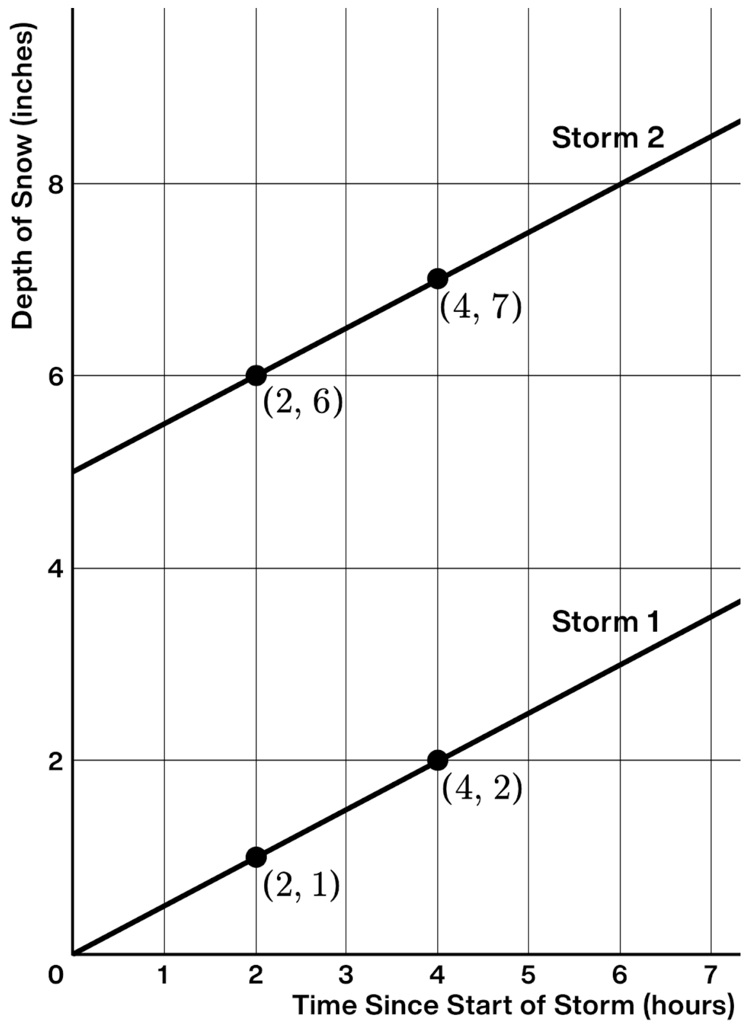
To find the slope, look at how much y increases when x increases by 1 .

Unit 8.3, Lesson 6: Notes Name ____________________

Learning Goal(s):

- **I can explain where to find the slope and the vertical intercept in both an equation and its graph.**
- **I can write equations of lines using $y = mx + b$.**

Snow fell at the same rate for two separate snow storms. During the storms, Raven measured the depth of snow on the ground for each hour.



The depth of snow on the ground for Storm 1 is a **proportional** relationship because there were 0 inches of snow on the ground at the start of the storm.

What is the equation representing Storm 1?

$$y = \frac{1}{2}x$$

The depth of snow on the ground for Storm 2 is a **linear** relationship because there were 5 inches of snow on the ground at the start of the storm.

What is the equation representing Storm 2?

$$y = \frac{1}{2}x + 5$$

Summary Question

How do you use a graph to write the equation of a line using $y = mx + b$?

In the equation of a line $y = mx + b$, m represents the rate of change and the slope of the graph, and b is the vertical intercept of the line.

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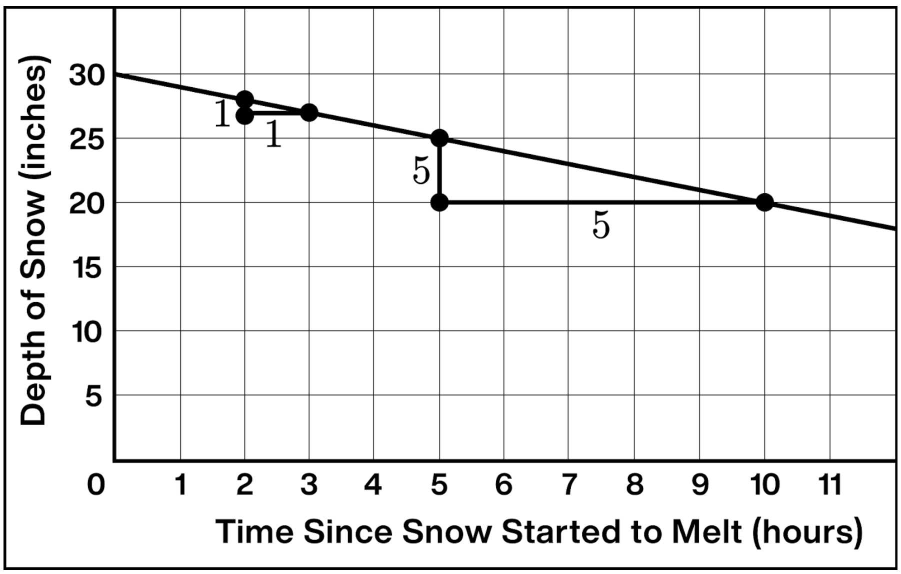
Unit 8.3, Lesson 7: Notes Name ______________________

Learning Goal(s):

- **I can give an example of a situation that would have a negative slope when graphed.**
- **I can look at a graph and tell if the slope is positive or negative and explain how I know.**

The snow on the ground was 30 inches deep. On a warm day, the snow began to melt. The graph shows the relationship between the time since the snow started to melt and the depth of the snow.



The slope of this graph is -1 since the rate of change is -1 inches of snow per **hour**.

This means that the depth of snow **decreased** at a rate of 1 inch per hour.

The vertical intercept is 30 **inches**.

This means that the snow was 30 inches deep when the time since snow started to melt was 0 hours.

Summary Task

Give an example of a different situation that would have a negative slope when graphed. Explain how you know the slope would be negative.

The amount of cereal in my bowl as I eat the cereal. This would have a negative slope because as time increases, the cereal in my bowl decreases.

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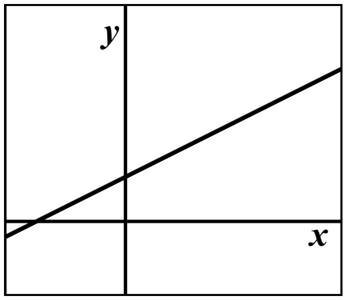
Unit 8.3, Lesson 8: Notes Name ______________________________

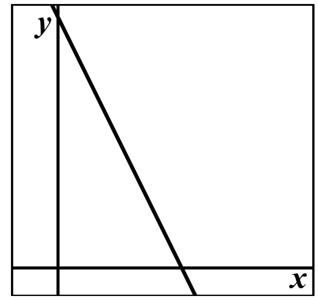
Learning Goal(s):

- **I can calculate positive and negative slopes given two points on the line.**
- **I can describe a line precisely enough that another student can draw it.**

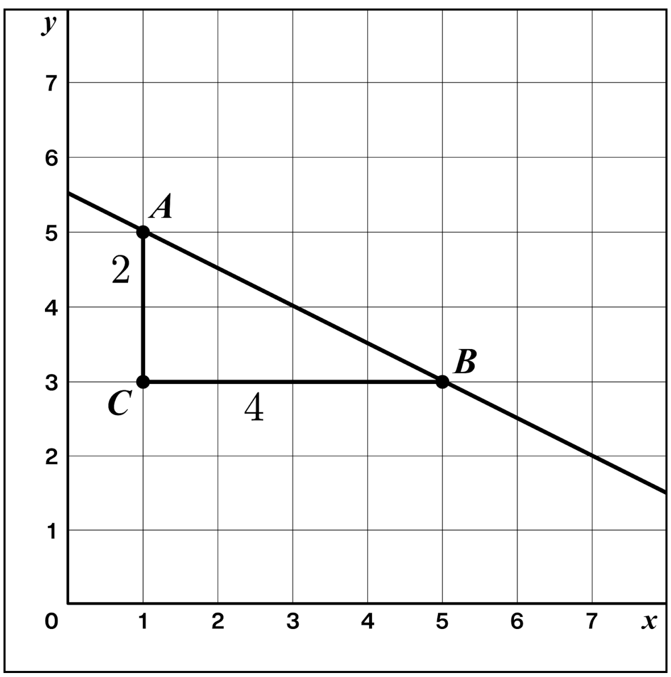
From left to right, if the graph increases, then the slope is **positive.**



From left to right, if the graph decreases, then the slope is **negative** .



Now we know two different ways to find the slope of a line:



1. We learned earlier that one way to find the slope of a line is by drawing a slope triangle.

Using the slope triangle shown here, the slope of the line is:

$$-\frac{2}{4} = -\frac{1}{2}$$

2. We can also compute the slope of this line using just two points.

Using the points $A = (1, 5)$ and $B = (5, 3)$, the slope of the line is:

$$\frac{3-5}{5-1} = \frac{-2}{4} = -\frac{1}{2} \text{ or } \frac{5-3}{1-5} = \frac{2}{-4} = -\frac{1}{2}$$

Summary Question

How can you calculate slope using two points on a line?

To calculate the slope of any two coordinates, find the vertical change by subtracting the y-coordinates and find the horizontal change by subtracting the x-coordinates (in the same order). Then divide the vertical change by the horizontal change.

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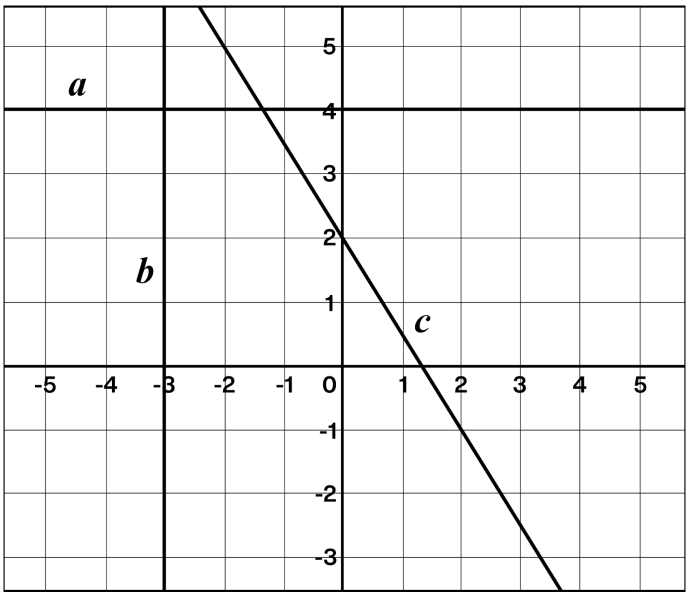
Unit 8.3, Lesson 9: Notes Name ________________________________

Learning Goal(s):

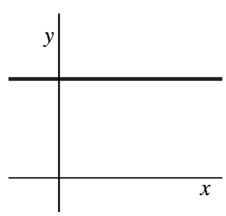
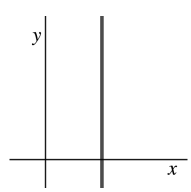
- **I can write equations of lines that have a positive or negative slope.**
- **I can write equations of vertical and horizontal lines.**



Here are three lines on a coordinate grid.

Write an equation for each line.

Line	Equation
a	$y = 4$
b	$x = -3$
c	$y = -\frac{3}{2}x + 2$

	Description	Graph	Slope	Equation Example
Horizontal Lines	**Lines where the y-value does not change, while the x-value changes.**		**Since the y-value does not change, the slope is 0.**	$y = 10$
Vertical Lines	**Lines where the x-value does not change, while the y-value changes.**		**Since the x-value does not change, the slope is undefined.**	$x = 2$

Summary Question

Write an example of an equation for a . . .

. . . horizontal line.

Responses vary.

$y = 3$

. . . vertical line.

Responses vary.

$x = -8$

. . . line with a negative slope.

Responses vary.

$y = -2x + 6$

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Unit 8.3, Lesson 10: Notes Name ____________________

Learning Goal(s):

- **I know that the graph of an equation is a visual representation of all the solutions to the equation.**
- **I understand what the solution to an equation with two variables is.**

Solution to an Equation With Two Variables

Definition	Facts/Characteristics
Solutions are any pair of values that can be substituted in for the variables to make the equation true.	**When graphed, solutions to an equation with two variables are points that are on the line representing the equation.**

Examples	Non-Examples
One solution to the equation $4x + 3y = 24$ **is** $(6, 0)$ **because** $4(6) + 3(0) = 24$**.**	**The point** $(10, 0)$ **is not a solution to the equation** $4x + 3y = 24$ **because** $4(10) + 3(0) \neq 24$**.**

Here are some facts about solutions in two variables:

1. A solution to a linear equation is a pair of values that makes the equation **true**.
2. Solutions can be found by **substituting** a value for one of the variables and solving the equation for the other.
3. The set of all the solutions to a linear equation can be shown in the coordinate plane and is called the **graph** of the equation.
4. The graph of a linear equation is **a line** .
5. Any points in the coordinate plane that **do not** lie on the graph of the linear equation are **not solutions** to the equation.
6. The number of solutions might be limited in a real-world situation even though the equation has **infinite** solutions.

Summary Question

How can you find solutions to linear equations? How do you know when you've found a solution?

Solutions can be found by substituting a value for one of the variables and solving the equation for the other. You know you've found a solution when the point lies on the graph of the equation.

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Unit 8.3, Lesson 11: Notes Name ______________________

Learning Goal(s):

- **I can find solutions to linear equations given either the x - value or the y -value.**
- **I can write linear equations to reason about real-world situations.**

No matter the form of a linear equation, we can always find solutions to the equation by starting with one value and then solving for the other value.

Let's think about the linear equation $2x - 4y = 12$.

Find the y -intercept by making $x = 0$.	Find the x -intercept by making $y = 0$.
$2x - 4y = 12$ $2(0) - 4y = 12$ $0 - 4y = 12$ $-4y = 12$ $y = -3$	$2x - 4y = 12$ $2x - 4(0) = 12$ $2x - 0 = 12$ $2x = 12$ $x = 6$

Based on your work above, what are the coordinates of two points on the line $2x - 4y = 12$?

The points $(0, -3)$ **and** $(6, 0)$ **lie on the graph of the line** $2x - 4y = 12$.

Summary Question

Once you have identified one solution to your equation, what are some ways you can find others?

You can find solutions to linear equations by:

- **Graphing the line and finding the coordinates of additional points on the line.**
- **Substituting a value for one of the variables and solving the equation for the other.**