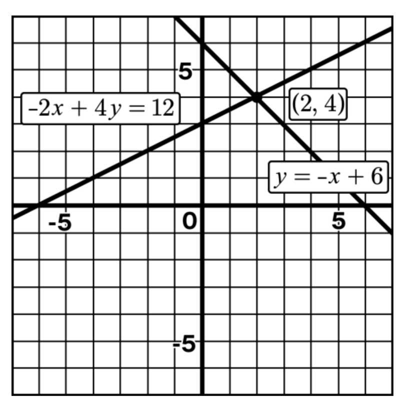
Linear Relationships and Systems of Linear Equations Student Guide

Math 7 Unit 4 Accelerated Part 2

Glossary

Term	Definition
constant term	In an expression like $5x + 2$, the number 2 is called the constant term because it doesn't change when x changes. • In the expression $7x + 9$, 9 is the constant term. • In the expression $5x + (-8)$, -8 is the constant term. • In the expression $12 - 4x$, 12 is the constant term.
infinitely many solutions	An equation has infinitely many solutions if it is true for any value of the variable. For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x. A system of equations has infinitely many solutions if the equations in the system are equivalent.
solution	A solution to an equation is a value or set of values that makes the equation true. The solution to the equation $4x + 3 = 23$ is $x = 5$ because $4(5) + 3 = 23$. One solution to the equation $4x + 3y = 24$ is $(6, 0)$ because $4(6) + 3(0) = 24$.
solution to a system of equations	A solution to a system of equations is a set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the intersection point. For example, $(2, 4)$ is the solution to this system of equations, and the intersection point on the graph. • $y = -x + 6$ • $-2x + 4y = 12$ 

system of equations	A system of equations is two or more equations that represent the constraints on a shared set of variables. For example, these equations make up a system of equations: • $x + y = -2$ • $x - y = 12$
term	A term is a part of an expression. It can be a single number, a variable, or a number and a variable multiplied together. For example, the expression $5x + 18$ has two terms. The first term is $5x$ and the second term is 18.

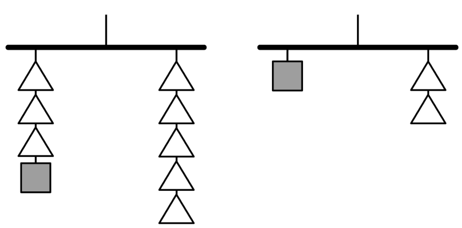
Unit 4 Summary

Prior Learning	**Grade 8, Unit 4**	**Future Learning**
Grade 8, Unit 3 • Writing linear equations, such as $y = 2x + 3$ • Graphing equations • Slope and y-intercept	• Solve equations in one variable, such as $3x + 20 = 7x$. • Solve systems of two linear equations using graphs and symbols.	Unit 5 • Use equations to describe functions. High School • Solve nonlinear equations. • Solve systems of more than two equations.

Solving Linear Equations

Solving an equation means finding all values that make the equation true.

$x = 2$ is a solution to the equation $3x = 6$ because $3(2) = 6$.



$3x+5=5x$ $\quad$ $5=2x$

A true equation is like a balanced hanger—if you perform the same operations to both sides, the hanger remains balanced.

The equations $3x + 5 = 5x$ and $5 = 2x$ are equivalent because we subtracted $3x$ (removed three triangles) from both sides.

When an equation requires several operations in order to determine a solution, we write each equation on its own line.

Here we use the **distributive property**: Add $6x$, subtract 2, and divide by 11 to both sides of the equation to determine a solution.

$$
\begin{aligned}
2(-3x+4) &= 5x+2 \\
-6x+8 &= 5x+2 \\
8 &= 11x+2 \\
6 &= 11x \\
\frac{6}{11} &= x
\end{aligned}
$$

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Unit 8.4, Family Resource

Systems of Linear Equations

A system of equations is a set of two (or more) equations where the variables represent the same values.

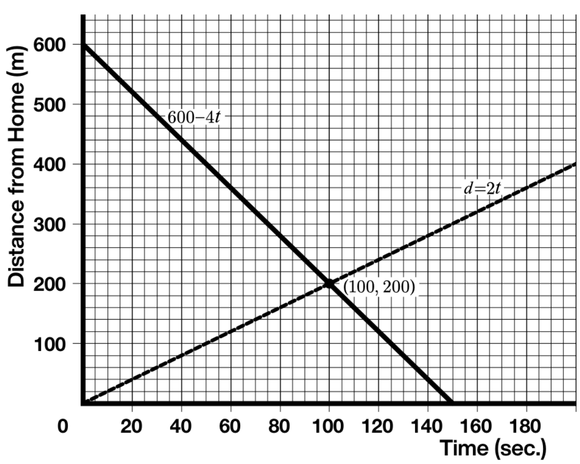
Solving a system of equations means finding values for the variables that make both equations true.

$$y = 2x + 5$$

$$y = 3x + 1$$

Here is an example of a situation where systems of equations are useful:

Yona is running home from school at 4 meters per second. Her brother Haruto is walking to school from home at 2 meters per second. They leave at the same time and their school is 600 meters from their home. When will Yona and Haruto pass each other? How far will they be from home?



If you write an equation for each child's distance from home, the two equations form a system:

Yona: $d = 600 - 4t$

Haruto: $d = 2t$

The solution to the system is the point where the lines cross on the graph.

The question asks when the distances will be equal, so you can set these expressions equal to each other and solve for the time.

$$600 - 4t = 2t$$
$$600 = 6t$$
$$100 = t$$

Once you know the time, use the equations to find the children's distances at that time.

Yona and Haruto pass each other when 100 seconds have passed. They are 200 meters from home.

$$d = 600 - 4(100)$$
$$d = 600 - 400$$
$$d = 200$$

$$d = 2(100)$$
$$d = 200$$

Try This at Home

Solving Linear Equations

1. Solve this equation: $3(3 - 3x) = 2(x + 3) - 30$

2. Elena and Noah work on the equation $\frac{1}{2}(x + 4) = -10 + 2x$ together.

   Here is their work:

   Do you agree with their solutions? Explain or show your reasoning.

Elena:

$$\frac{1}{2}(x+4) = -10+2x$$
$$(x+4) = -20+2x$$
$$x+24 = 2x$$
$$24 = x$$
$$x = 24$$

Noah:

$$\frac{1}{2}(x+4) = -10+2x$$
$$x+4 = -20+2x$$
$$-3x+4 = -20$$
$$-3x = -24$$
$$x = -8$$

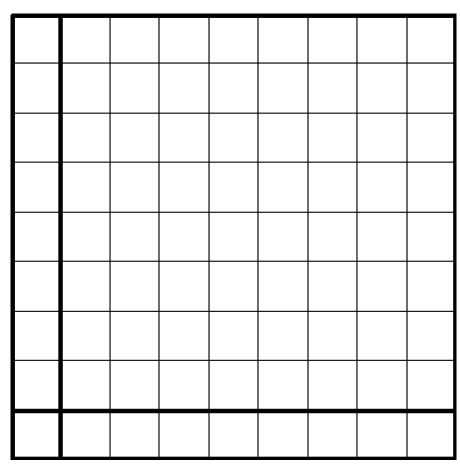
Systems of Linear Equations

Tiam and Maneli are biking in the same direction on the same path, but they start at different times. Tiam is riding at a constant speed of 18 miles per hour. Maneli started riding at a constant speed of 12 miles per hour a quarter of an hour (15 minutes) before Tiam started.

3.1 Write equations to represent the relationship between time and distance biked for each person.

3.2 Graph both equations on the same set of axes.

3.3 Use the equations and/or the graph to find the time and distance that Tiam and Maneli meet.



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Solutions:

1. There are many possible ways to solve this equation, all with a correct solution of $x = 3$.

   Here is one example:

$$
\begin{aligned}
3(3 - 3x) &= 2(x + 3) - 30 \\
9 - 9x &= 2x + 6 - 30 \\
9 - 9x &= 2x - 24 \\
-9x &= 2x - 33 \\
-11x &= -33 \\
x &= 3
\end{aligned}
$$

2. No, they both have errors in their solutions.
   - Elena multiplied both sides of the equation by 2 in her first step. She did not multiply the $2x$ by the 2. The second line should be $(x + 4) = -20 + 4x$.
   - We can check Elena's solution by replacing x with 24 in the original equation to see if the equation is true. Since 14 is not equal to 38, Elena's solution is not correct.

$$
\begin{aligned}
\tfrac{1}{2}(x+4) &= -10+2x \\
\tfrac{1}{2}(24+4) &= -10+2(24) \\
\tfrac{1}{2}(28) &= -10+48 \\
14 &= 38
\end{aligned}
$$

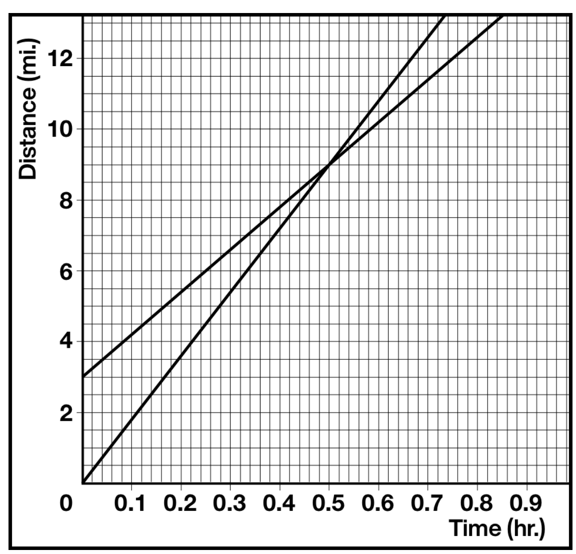
   - Noah divided both sides in his last step. He wrote -8 as the quotient on the right hand side instead of 8. $\frac{-24}{-3} = 8$. His last line should be $x = 8$.
   - We can also check Noah's solution by replacing x with -8 in the original equation to see if the equation is true. Noah's solution is not correct.

3.1 **Tiam:** $d = 18t$

Maneli: $d = 12\left(t + \frac{1}{4}\right)$ or $d = 12t + 3$

3.2 See the graph on the right.

3.3 Using the graph, Tiam and Maneli are at the same time and distance when their graphs cross, which is 0.5 hours since Tiam started riding (0.75 hours since Maneli started riding). They meet after having biked 9 miles.



Using equations, you can set the two expressions equal to each other and write the equation $18t = 12\left(t + \frac{1}{4}\right)$.

One way to solve this equation is shown on the right.

First, use the distributive property to rewrite the right-hand side of the equation.

Then, subtract $12t$ from both sides.

Finally, divide both sides of the equation by 6.

$$18t = 12\left(t + \frac{1}{4}\right)$$
$$6t = 3$$
$$t = \frac{1}{2}$$

After a half hour, Tiam and Maneli have each ridden 9 miles. Both strategies—using the graph and the equations—give the same result.

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Unit 8.4, Lesson 9: Notes Name ______________________

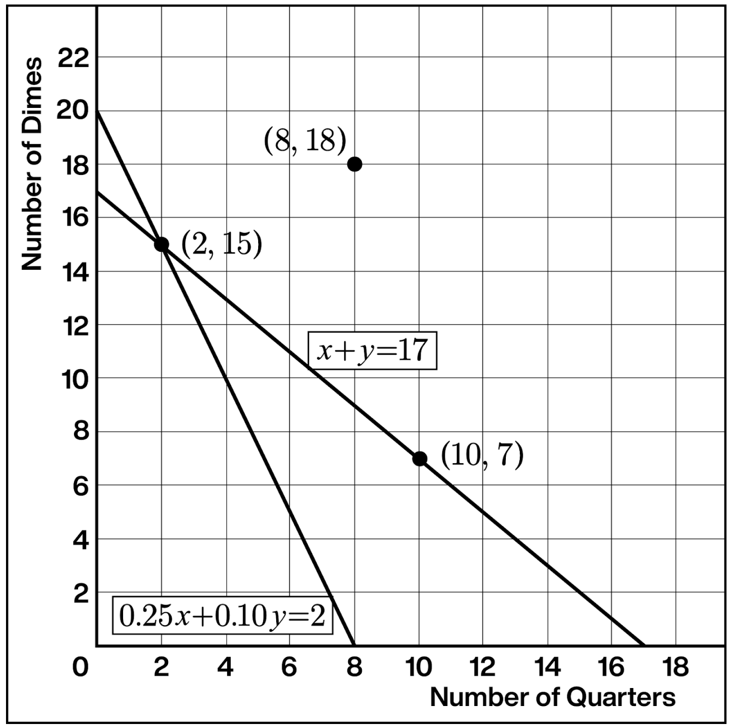
Learning Goal(s):

- **I can identify and interpret points that satisfy two relationships at the same time using graphs.**

Values of x and y that make an equation **true** correspond to points (x, y) on the graph. For example, if we have x number of quarters and y number of dimes, and the total cost is $\$2.00$, then we can write an equation like this to represent the relationship between x and y: $0.25x + 0.10y = 2$.

Since 2 quarters is $\$0.50$ and 15 dimes is $\$1.50$, we know that $x = 2$, $y = 15$ is a **solution** to the equation, and the point $(2, 15)$ is a point on the graph. The line shown is the graph of the equation.

We also know that the quarters and dimes together total 17 coins. That means that: $x + y = 17$



1. Label the graph of each equation on the coordinate plane.
2. Pick another point on the coordinate plane and explain what it means in context:
   Responses vary. **The point $(10, 7)$ means there are 10 quarters and 7 dimes totaling 17 coins.**

In general, if we have two lines in the coordinate plane:

- The coordinates of a point that is on both lines make **both** equations true.
- The coordinates of a point on only one line make **one** equation true.
- The coordinates of a point on neither line make **neither** equation true.

Summary Question

If you are given two linear relationships, how can you determine x- and y-values that will make both relationships true?

Responses vary. **By looking at the graph of the two linear relationships, the ordered pair that falls on both lines will make both relationships true.**

Unit 8.4, Lesson 10: Notes Name ______________________________

Learning Goal(s):
- **I can use graphs to find an ordered pair that two real-world situations have in common.**

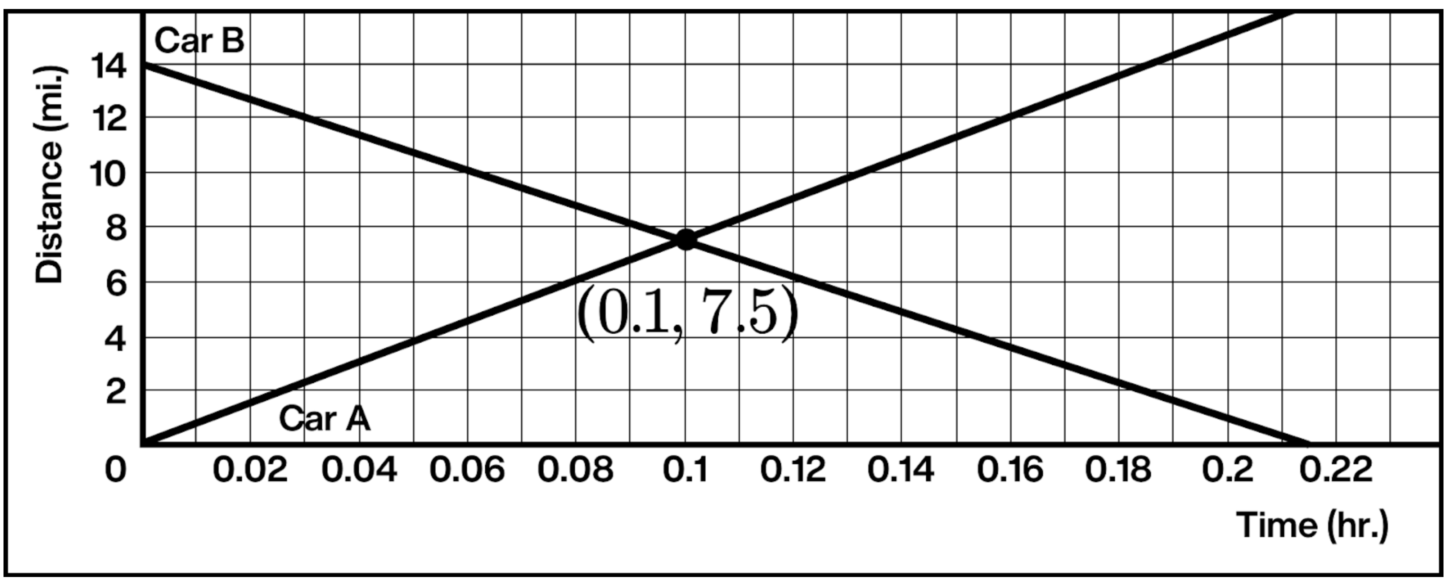
The solutions to an equation correspond to **points** on its graph. For example, if Car A is traveling 75 miles per hour and passes a rest area when $t = 0$, then the distance in miles it has traveled from the rest area after t hours can be represented by the equation $d = 75t$.

1. What is one point that will be on this graph? How do you know?

   Responses vary. **The point $(2,\ 150)$ will be on the graph of this equation because $150 = 75 \cdot 2$. 2 hours after passing the rest area, the car has traveled 150 miles.**

If you have **two** equations, you can ask whether there is an ordered pair that is a solution to both equations simultaneously.

For example, if Car B is traveling towards the rest area and its distance from the rest area is $d = 14 - 65t$, we can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is yes, then the solution will correspond to a point **that is on both lines**.



Looking at the coordinates of the intersection point, we see that Car A and Car B will both be 7.5 miles from the rest area after 0.1 hours.

Summary Question

How can you tell when two linear relationships will be the same?

You can determine when two linear relationships will be the same by graphing both on the same coordinate plane and finding their point of intersection.

Unit 8.4, Lesson 11: Notes

Name ____________________________________

Learning Goal(s):

- **I can explain the solution to a system of equations in a real-world context.**
- **I can explain what a system of equations is.**
- **I can make graphs to find an ordered pair that two real-world situations have in common.**

System of Equations

Definition

A system of equations is a set of two or more equations.

Facts/Characteristics

Each equation contains two or more variables. We want to find values for the variables that make all the equations true.

Examples

These equations make up a system of equations:

$$\begin{cases} x+y=-2 \\ x-y=12 \end{cases}$$

The solution to this system is $x=5$ and $y=-7$ because when these values are substituted for x and y, each equation is true: $5+(-7)=-2$ and $5-(-7)=12$

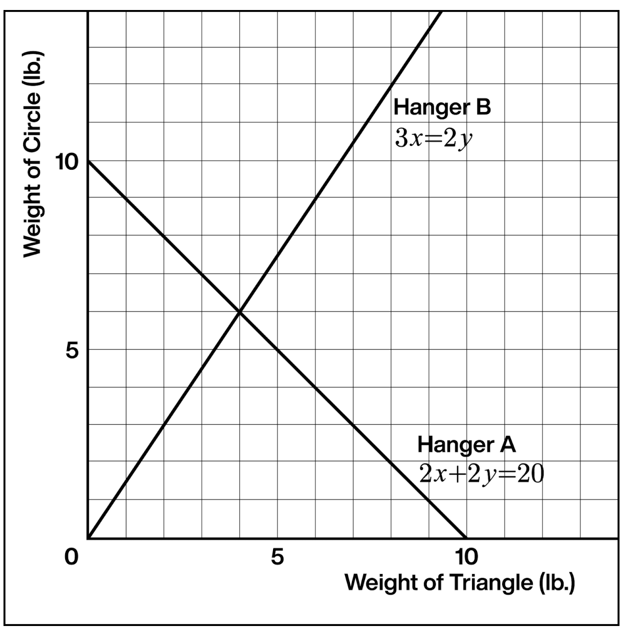
Non-Examples

10

$3x+15$

$y=4x+7$

The system of equations below represents the weights of two balanced hangers.



What is the solution to the system of equations?

$(4,\ 6)$

What does the solution tell us about the hangers?

The hanger will balance when the triangles weigh 4 lb. and the circles weigh 6 lb.

Summary Question

What does it mean to solve a system of equations?

Solving a system of equations means finding the values of the variables that make each equation true.

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Unit 8.4, Lesson 12: Notes Name ______________________________

Learning Goal(s): • **I can solve systems of equations using algebra.**

Sometimes it is easier to solve a system of equations without having to graph the equations and look for an **intersection point.**

In general, whenever we are solving a system of equations we know that we are looking for a pair of (x, y) values that makes both equations **true**. In particular, we know that the value for y will be the **same** in both equations.

If we have a system like this:

$$y = 2x + 6$$
$$y = -3x - 4$$

we know the y **-value** of the solution is the same in both equations, so we can write the following:

$$2x + 6 = -3x - 4$$

and we can solve this equation for x:

$$\begin{aligned} 2x + 6 &= -3x - 4 \\ 5x + 6 &= -4 \\ 5x &= -10 \\ x &= -2 \end{aligned}$$

Solving for x is only half of what we are looking for; we know the value for x, but we need the corresponding value for y.

Since both equations have the same y-value, we can use either equation to find the y-value:

$$y = 2(-2) + 6$$
$$y =- 3(-2) - 4$$

In both cases, we find that $y = 2$. So the solution to the system is $(-2, 2)$.

We can verify this by graphing both equations in the coordinate plane.

Summary Question

What are the first steps you can take when solving the system of equations?

$$y = 2x$$
$$y = -3x + 10$$

Since I know the y-value is the same in both equations, I can write $2x = -3x + 10$. Then I can use balancing moves to get all of the variables on one side of the equation.

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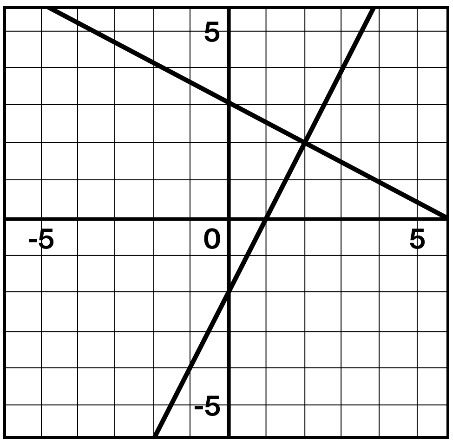
Unit 8.4, Lesson 13: Notes Name ________________________________

Learning Goal(s):

- **I can determine whether a system of equations has no solutions, one solution, or infinitely many solutions.**

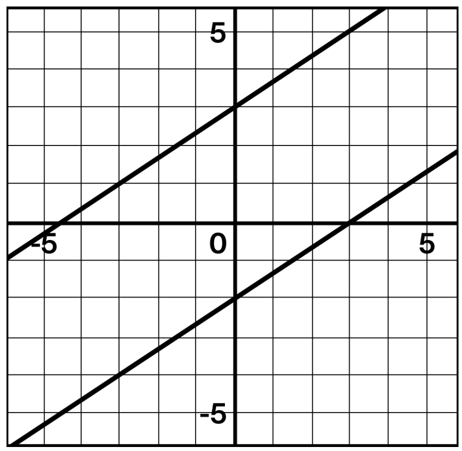
The x- and y-values that make both equations true are known as the **solution** to a system of equations. Depending on the equations, a system can have **no solutions**, **one solution**, or **infinitely many** solutions.



$$y = 2x - 2$$
$$y = -\frac{1}{2}x + 3$$

If the two lines of a system intersect at a point, there is **one** solution.

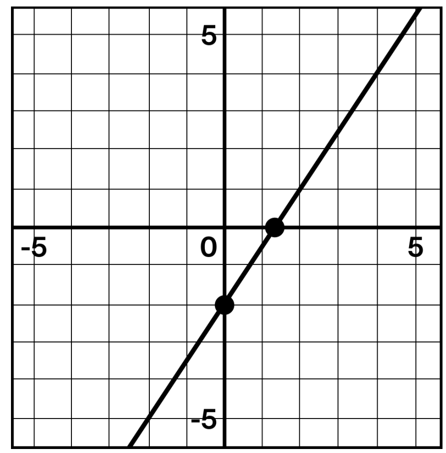
If the two lines have **different** slopes, there is one solution.



$$y = \frac{2}{3}x - 2$$
$$y = \frac{2}{3}x + 3$$

If the two lines of a system **do not** intersect at a point, there are **no** solutions.

If the two lines have **the same** slope and different y-intercepts, there are no solutions.



$$y = 1.5x - 2$$
$$y = \frac{3}{2}x - 2$$

If the two equations have the **same** slope and the **same** y-intercept, the system has **infinite** solutions.

Summary Question

How can you tell from the structure of the equations if a system has no solutions, one solution, or infinite solutions?

If the two equations have different slopes, there is one solution. If the two equations have the same slope but different y-intercepts, there are no solutions. If the two equations have the same slope and the same y-intercept, there are infinitely many solutions.

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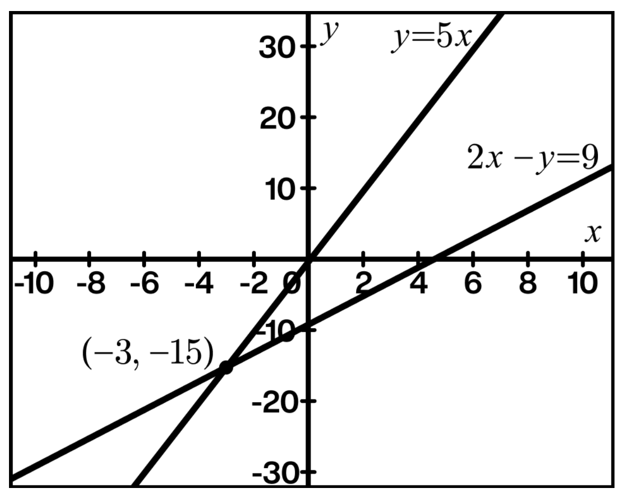
Unit 8.4, Lesson 14: Notes Name ___________________

Learning Goal(s): • **I can solve systems of equations using a variety of strategies.**

When we have a system of linear equations where one of the equations is of the form $y = [stuff]$ or $x = [stuff]$, we can solve it algebraically by using **substitution**. The basic idea is to replace a variable with an equivalent **expression**.	$\begin{cases} y=5x \\ 2x-y=9 \end{cases}$
Since we know that $y = 5x$, we can substitute ***5x*** for y in $2x - y = 9$.	$2x - (\; 5x \;) = 9$
And then solve the equation for x.	$-3x = 9$ $x = -3$
We can find y using either equation. Let's use the first one:	$y = 5x$ $y = 5 \cdot -3$ $y = -15$
The x- and y-values that make both equations true are known as the **solution** to the system.	Solution $(-3, \; -15)$

We can check this by looking at the graphs of the equations in the system:

They intersect at $(-3, -15)$.



Summary Question
Describe one strategy you can use for solving a system of equations algebraically.

We can use substitution to solve a system of equations. In one equation, when a variable is equal to an expression, we can replace that variable with the expression in a different equation.