Functions
Student Guide

Math 7 Unit 5 Accelerated

Glossary

Term	Definition
cone	A three-dimensional figure that tapers from a circular base to a point.
cylinder	A three-dimensional figure that has two parallel circular bases connected by a curved surface.
decreasing	A function, or segment of a function, is decreasing if the values of the output go down when the values of the input go up.
dependent variable (output)	The value of a dependent variable is based on the value of another variable or set of variables. It is sometimes called the output. The dependent variable is typically on the vertical axis of a graph and the right-hand column of a table.
function	A function is a relationship that assigns exactly one output to each possible input.
increasing	A function, or segment of a function, is increasing if the values of the output go up when the values of the input go up.
independent variable (input)	The value of an independent variable is not based on the value of any other variable. It is sometimes called the input. The independent variable is typically on the horizontal axis of a graph and the left-hand column of a table.
linear	A function is linear when its graph is a straight, non-vertical line. A vertical line is not a function because it has an input with different outputs.
linear function	A linear function can be defined by an equation in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept.
non-linear	A function is non-linear when its graph is not a straight line.
radius	A line segment that connects the center of a circle with a point on the circle. The term radius can also refer to the length of this segment.

sphere	A three-dimensional figure in which all cross-sections in every direction are circles.
volume	Volume is the number of cubic units that fill a three-dimensional region without any gaps or overlaps.

Unit 5 Summary

Prior Learning	Math 8, Unit 5	Future Learning
Math 7 ● Volume of prisms ● Area of circles Math 8, Unit 3 ● Linear relationships	● Introduction to functions ● Representing and interpreting functions ● Volume of cylinders, cones, and spheres	High School ● Function notation ● Cross–sections and volumes in context

Defining Functions

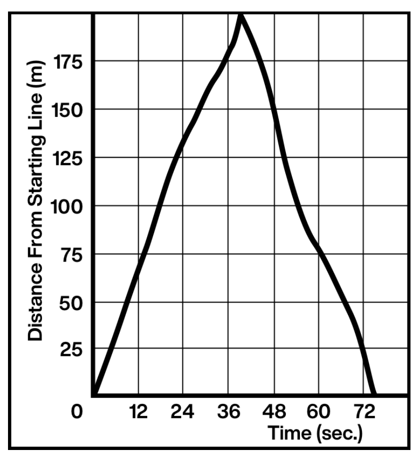
A function is a rule that assigns exactly one output to each possible input.

Examples	Non-Examples
Input: Name **Output:** First letter of that name (e.g., Sneha → S) **Input**: Any number **Output:** Three more than the input (e.g., $7 \rightarrow 10$)	**Input:** Letter **Output:** A name beginning with that letter (e.g., S → Sora) **Input:** Digit **Output:** A number whose last digit is the input (e.g., $7 \rightarrow 207$)

Here are some more **examples** of functions:

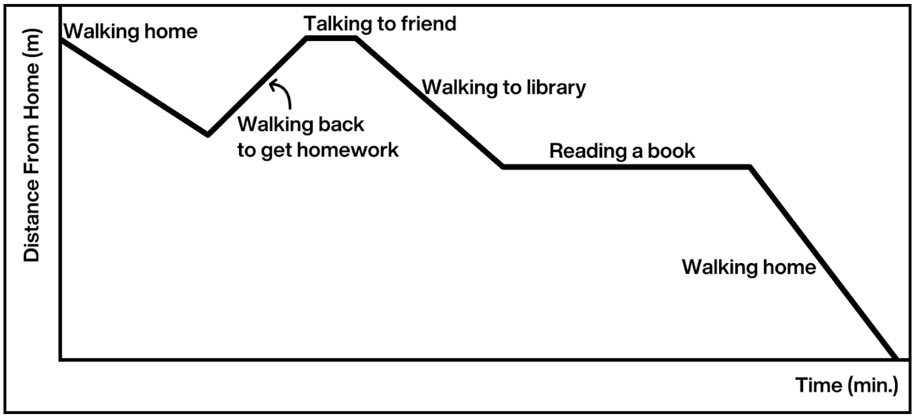
$$y = 4 - 3x$$

Input	Output
-2	4π
-1	1π
0	0
1	1π
2	4π



Representing and Interpreting Functions

A function can represent a story. Here is one example:



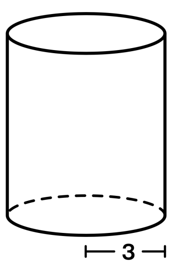
Independent Variable:
Time (min.)

Dependent Variable:
Distance From Home (m)

Volume of Cylinders, Cones, and Spheres

Volume is the number of cubic units that fill a 3-D region without any gaps or overlaps.

Cylinder



$V = B \cdot h$

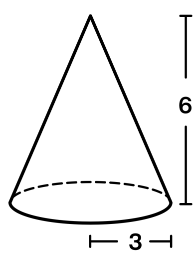
$= \pi r^2 \cdot h$

$= \pi \cdot (3)^2 \cdot (6)$

$= 9 \cdot 6 \cdot \pi$

$= 54\pi$ cubic units

Cone



$V = \frac{1}{3} B \cdot h$

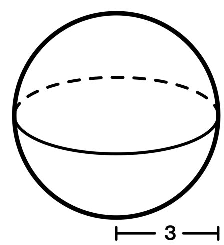
$= \frac{1}{3} \pi r^2 \cdot h$

$= \frac{1}{3} \pi \cdot (3)^2 \cdot (6)$

$= \frac{1}{3} \cdot 9 \cdot 6 \cdot \pi$

$= 18\pi$ cubic units

Sphere



$V = \frac{4}{3} \pi r^2 \cdot r$

$= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \pi \cdot (3)^3$

$= \frac{4}{3} \cdot 27 \cdot \pi$

$= 36\pi$ cubic units

Try This at Home

Defining Functions

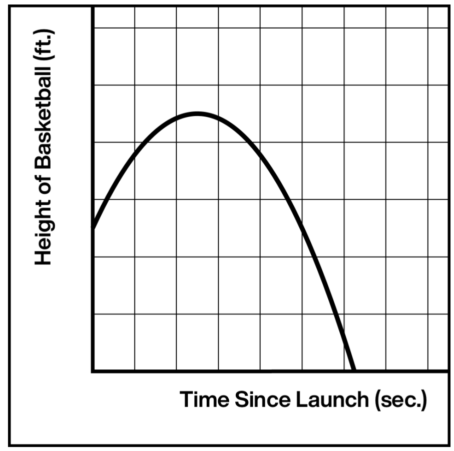
1.1 This table represents the total amount of data used compared to how many phone calls were made in a month.

# of Phone Calls	**Total Data Used** (GB)
10	4.3
19	6.2
35	7.5
10	8.3

a. Name the independent variable (input) and dependent variable (output).

b. Decide whether the situation represents a function or not. Explain your thinking.

1.2 This graph represents the height of a basketball over time.



a. Name the independent variable (input) and dependent variable (output).

b. Decide whether the situation represents a function or not. Explain your thinking.

1.3 Brown rice costs $\$2$ per pound. Beans cost $\$1.60$ per pound. Jamar has $\$10$ to spend to make a large meal of beans and rice for a potluck dinner. The amount of brown rice he can buy, r, is related to the amount of beans he can buy, b.

a. Name the independent variable (input) and dependent variable (output).

b. Decide whether the situation represents a function or not. Explain your thinking.

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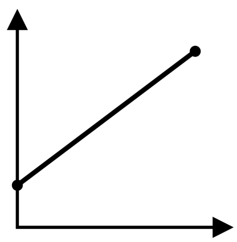
Unit 8.5, Family Resource

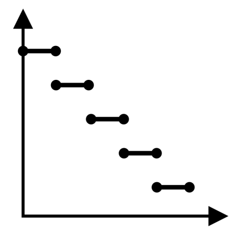
Representing and Interpreting Functions

Match each of the following situations with a graph (you can use a graph multiple times). Name the independent and dependent variables.

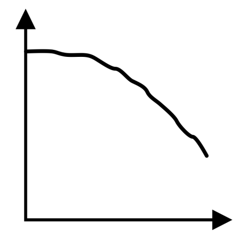
A.



B.



C.



2.1 Daeja takes a handful of popcorn out of the bag every 5 minutes.

2.2 A plant grows the same amount every week.

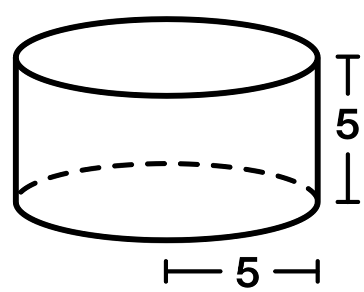
2.3 The day started very warm, but then it slowly got colder.

2.4 A cylindrical glass sits on a counter.
The more water you pour in, the higher the water level is.

3. Write an equation in the form $y = mx + b$ that could represent the plant's growth. Explain what each number means in terms of the situation.

Volume of Cylinders, Cones, and Spheres

This cylinder has a height and radius of 5 cm.
Express your answers in terms of π .



4.1 What is the diameter of the base?

4.2 What is the area of the base?

4.3 What is the volume of the cylinder?

4.4 What would the volume of a cone with the same height and radius be?

4.5 What would the height be if the volume of the cylinder remained the same, but the radius doubled?

Unit 8.5, Family Resource

Solutions:

1.1a The *independent variable* represents the input of a function. The *dependent variable* represents the output of a function. Here, the independent variable is the number of phone calls; the dependent variable is the total data used.

1.1b This relationship is **not** a function because the number of calls does not uniquely determine the amount of data. For example, 10 phone calls results in both 4.3 GB and 8.3 GB of data.

1.2a By convention, the *independent variable* is represented on the horizontal axis and the *dependent variable* on the vertical axis. The independent variable in this situation is the time since launch. The dependent variable is the height of the basketball.

1.2b This relationship is a function because there is exactly one height for each time.

1.3a It is possible for either variable to be the independent variable. In this case, we are wondering about how much rice can be bought, so the independent variable is the amount of brown rice purchased. The dependent variable is the amount of beans purchased.

1.3b This relationship is a function because for every amount of beans, there is only one possible amount of rice Lin can buy if he wants to spend exactly $\$10$.

2.1 Graph B, Independent variable = Time (minutes), Dependent variable = Amount of popcorn left in bag

2.2 Graph A, Independent variable = Time (weeks), Dependent variable = Height of the plant

2.3 Graph C, Independent variable = Time (hours), Dependent variable = Temperature outside

2.4 Graph A, Independent variable = Volume of water poured in the glass, Dependent variable = Height of water in the glass

3. The equations vary. An example equation is $y = 2x + 5$, where 5 represents the height of the plant when you start measuring and 2 represents the number of inches the plant grows every week.

4.1 10 cm. The diameter is twice the length of the radius, and $2(5) = 10$.

4.2 25π cm^2. The area of a circle is π times the radius squared, or $(5)^2 \cdot \pi$.

4.3 125π cm^3. The volume is the area of the base times the height. The area of the base here is 25π, so the volume is 125π cm^3 since $25\pi \cdot 5 = 125\pi$.

4.4 $\frac{125\pi}{3}$ cm^3. The volume of a cone is one-third the volume of the corresponding cylinder.

4.5 1.25 cm. If the radius doubled, then it would be 10 cm. There are various methods for finding the height. One method is to organize each quantity using a table. A sample table is shown below.
Radius (cm): 10
Base area (sq. cm):100π
Height (cm): $\frac{125\pi}{100\pi} = 1.25$
Cylinder volume (cubic cm): 125π

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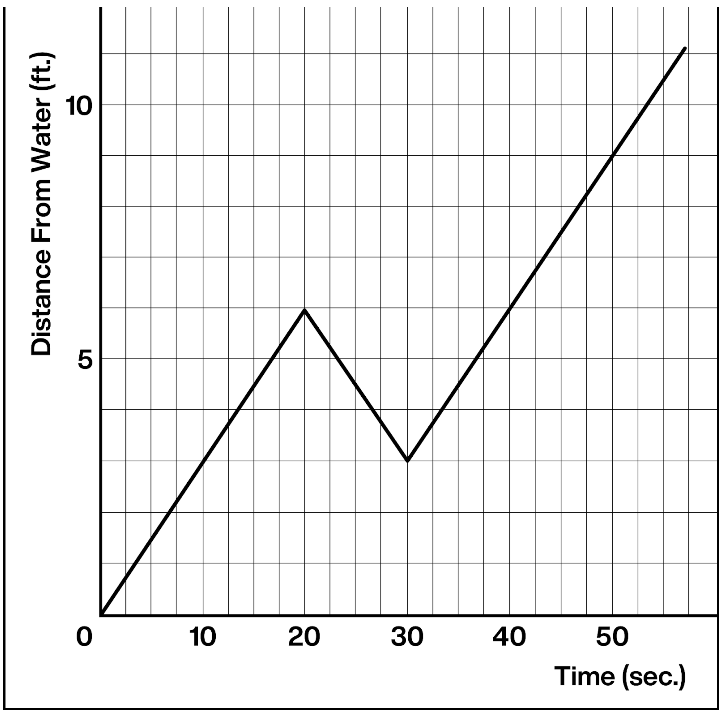
Unit 8.5, Lesson 1: Notes Name ______________________________

Making Sense of Graphs

Learning Goal(s): • **I can make connections between scenarios and the graphs that represent them.**

Here is the graph of a turtle's journey.

	What story does the graph tell about the turtle's journey? **The turtle started at the water and traveled away from the water for 20 seconds. Then it walked backwards towards the water. At 30 seconds, the turtle began to walk away from the water again.**
What is the turtle's distance from the water after 40 seconds? **After 40 seconds, the turtle is 6 feet away from the water.**	When is the turtle's distance from the water 3 feet? **The turtle's distance from the water is 3 feet at both 10 seconds and 30 seconds.**

Summary Question

How does a point on a graph represent part of a story? Give at least one example.

A point on a graph can represent part of a story because it represents a point in time. For example, in the story of the turtle, the point $(10, 3)$ says that the turtle is 3 feet from the water after traveling for 10 seconds.

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Unit 8.5, Lesson 2: Notes Name ______________________________

Introduction to Functions

Learning Goal(s):

- **I can write rules when I know input-output pairs.**
- **I know that a function is a rule with exactly one output for each allowable input.**
- **I can identify rules that do and do not represent functions.**

Function

Definition	Facts/Characteristics
A function is a rule that assigns exactly one output to each possible input.	**In a table, you will never see one input with multiple different outputs.**
Examples	**Non-Examples**
Name → First Letter (e.g. Sarah → S) **Number → 7 (e.g., 5→ 7)**	**First Letter → Name (e.g., S→ Sarah)** **Digit → Number starting with that digit (e.g., 7→ 704)**

For each rule, decide if the rule represents a function or not. Explain your thinking.

Possible Inputs: Any person Rule: Output the month the person was born in. Function? **Yes** No **Each person is only born in one month, so each possible input has only one output.**	Possible Inputs: Any month Rule: Output a person born in that month. Function? Yes **No** **Each month has many people born in that month, so each input has many possible outputs.**

Summary Question

Why might it be useful to know whether a rule is a function?

Knowing a rule is a function can be useful because functions are predictable in a specific way. You can know that for any input-output pair, there are no other possible outputs for that input.

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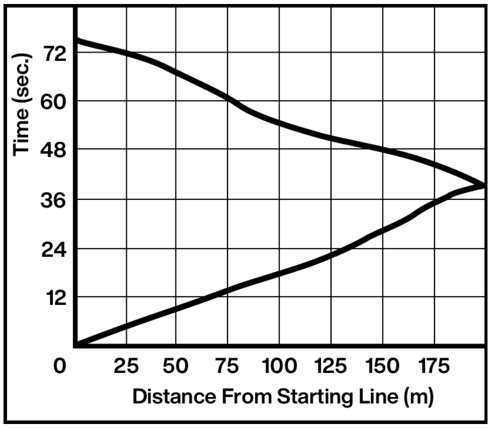
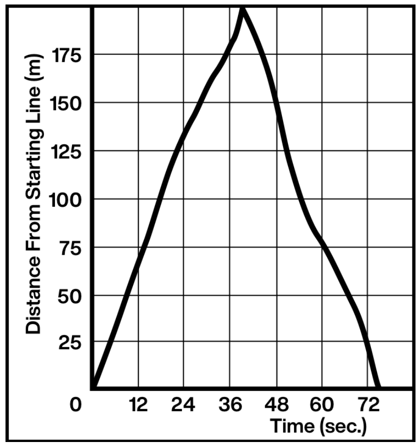
Unit 8.5, Lesson 3: Notes Name ________________________________

Graphs of Functions and Non-Functions

Learning Goal(s): • **I can explain why a graph does or does not represent a function.** • **I can use precise language to describe functions (e.g., “is a function of” or “determines”).**

Ariana is running once around the track. The graphs below show the relationship between her time and her distance from the starting point.





Estimate when Ariana was 100 meters from her starting point. **After about 18 seconds and 54 seconds.**	Estimate how far Ariana was from the starting line after 60 seconds. **About 75 meters from the starting line.**
Is time a function of Ariana's distance from the starting point? Explain how you know. **No. Time is not a function of distance from the starting point because there are some distances (like 100 meters) that correspond with more than one time (18 and 54 seconds).**	Is Ariana’s distance from the starting point a function of time? Explain how you know. **Yes. Distance from the starting point is a function of time because every time corresponds to only one distance.**

Summary Question

What is something you won’t see on the graph of a function?

On the graph of a function, you won’t see any x-values that corresponds to multiple y-values. For example, you would never see both the points $(0, 7)$ and $(0, 5)$ on the graph of a function because then one input would have two outputs.

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Unit 8.5, Lesson 4: Notes Name ___________________________

Functions and Equations

Learning Goal(s):

- **I can represent a function with an equation.**
- **I can name the independent and dependent variables for a function.**

In each situation, complete the table with a possible *independent variable* or *dependent variable.*

Question or Equation	**Independent Variable**	**Dependent Variable**
How many pickles can I make?	The number of cucumbers	The number of pickles
How much does my ice cream cost if I get different amounts of toppings?	**Number of toppings**	Cost of my ice cream cone
How does hours of sleep affect performance on tests?	**Hours of sleep**	**Test score**
$y = 3x + 5$	x	y

What is the *independent variable*? How is it represented on a graph? ***Responses vary.***

The independent variable is the input in a function. It is what is affecting the other variable. The independent variable is represented on the horizontal or x-axis of a graph.

What is the *dependent variable*? How is it represented on a graph? ***Responses vary.***

The dependent variable is the output in a function. It is what is affected by the other variable. The dependent variable is represented on the vertical or y-axis on a graph.

Brown rice costs $\$2$ per pound and beans cost $\$1.60$ per pound. Rudra has $\$10$ to spend on these items. The amount of brown rice, r, is related to the amount of beans, b, Rudra can buy.

Rudra wrote the equation $r = \frac{10-1.60b}{2}$. What is the dependent variable? How do you know?

The dependent variable is r (the amount of rice Rudra can buy). The equation is solved for r, so it is easiest to calculate the amount of rice needed based on the amount of beans Rudra buys.

Summary Question

How does the choice of independent and dependent variables affect the equation of a function?

Depending on the choice of variables, the format of the equation can change. Equations can be very useful when they are solved for the dependent variable (e.g., $r =$ when trying to answer questions about how much rice Rudra can buy).

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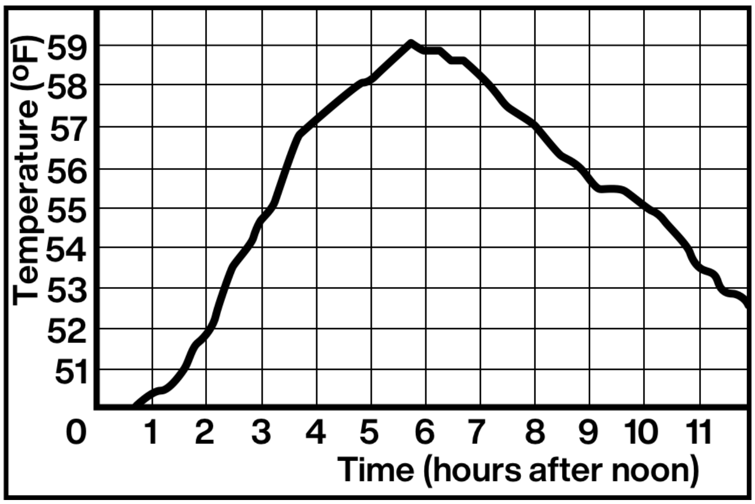
Unit 8.5, Lesson 5: Notes Name ______________________________

Interpreting Graphs of Functions

Learning Goal(s):
- **I can explain the story told by the graph of a function.**
- **I can find and interpret points on the graph of a function.**
- **I can determine whether a function is increasing or decreasing based on whether its rate of change is positive or negative.**

This graph shows the temperature between noon and midnight on one day.



Tell the story of the temperature on this day.

The temperature starts cold (around 50° F) and then begins to get warmer throughout the day. The temperature is hottest between 5 p.m. and 6 p.m. Then the temperature decreases at a slower rate than it increased earlier in the day.

Did the temperature change more between 1 p.m. and 3 p.m. or between 7 p.m. and 9 p.m.? Explain your thinking.

The temperature changed more between 1 p.m. and 3 p.m. (about 4.5°F) than between 7 p.m. and 9 p.m. (about 2.5°F).

Was it warmer at 3 p.m. or 9 p.m.?

The temperature is higher at 9 p.m. than at 3 p.m.

Summary Question

How can you tell from a graph whether a function is increasing or decreasing?

In a graph, if the line is going up as the x-values get larger, then the function is increasing. If the line is going down as the x-values get larger, then the function is decreasing.

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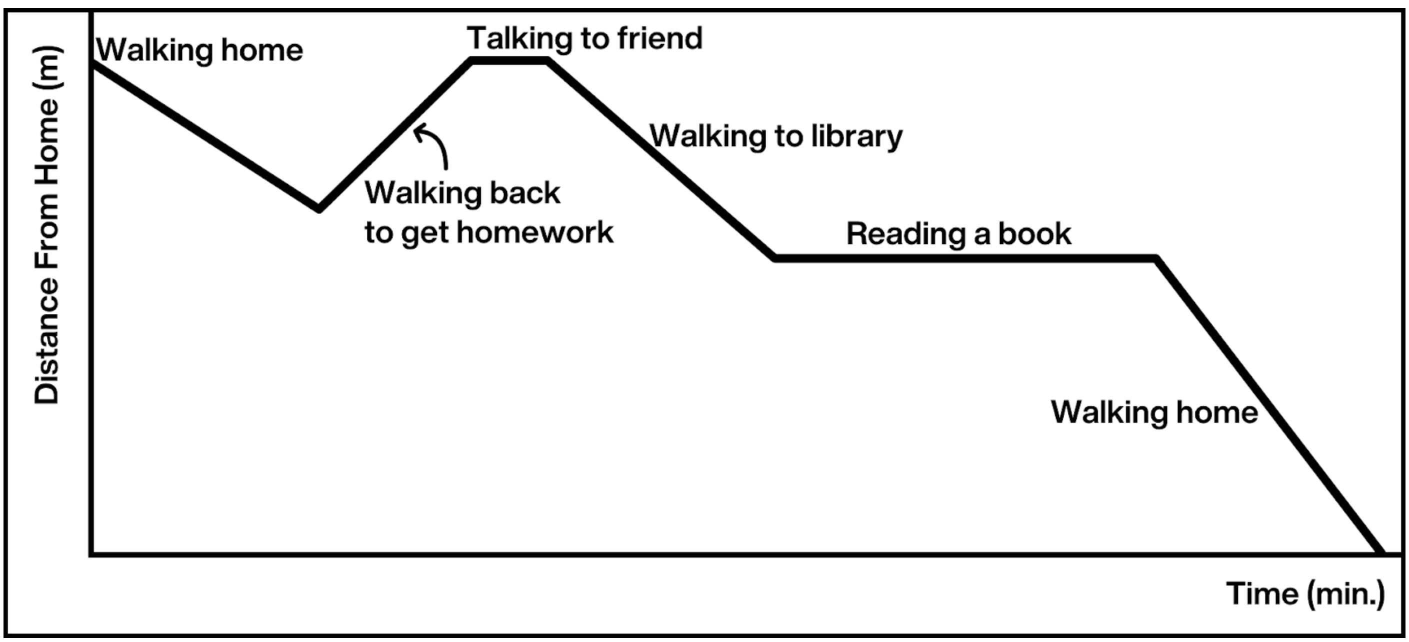
Unit 8.5, Lesson 6: Notes Name ______________________________

Creating Graphs of Functions

Learning Goal(s): • **I can draw the graph of a function that represents a real-world situation.** • **I can explain that graphs can appear different depending on the variables chosen.**

Elena starts to walk home from school. She turns around and goes back to school because she left something in her locker. At school, she runs into a friend who invites her to the library to do homework. She goes to the library, reads a book, then heads home to do her chores.

Label both axes so that the graph accurately represents the situation.	Label each segment with what is happening in the story during that time. (E.g., in the first segment, she is **walking home from school**).



Summary Question

What is important to pay attention to when drawing the graph of a function from a story?

Make sure to pay attention to the quantity being measured. For example, when creating a graph between "height" and time, ask yourself "the height of what?" Also, pay attention to the units! If you measure in feet, but the graph asks for meters, your graph will have the right shape, but will be the wrong size. It'll be squished.

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Unit 8.5, Lesson 7: Notes Name ______________________

Comparing Representations of Functions

Learning Goal(s):
- **I can explain the strengths and weaknesses of different representations.**
- **I can compare inputs and outputs of functions that are represented in different ways.**

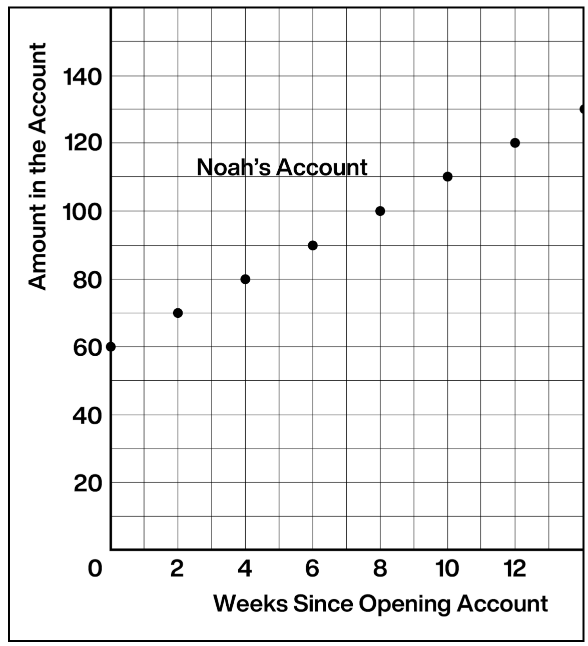
Elena opened an account on the same day as Noah. The amount of money, E, in Elena's account is given by the function $E = 8w + 70$, where w is the number of weeks since the account was opened. The graph below shows some data about the amount of money in Noah's account.

Who started out with more money in their account? Explain how you know.

Elena starts out with more money. When $w = 0$, Elena has $\$70$, while Noah has $\$60$.

Who is saving money at a faster rate? Explain how you know.

Elena is saving money at a faster rate. She adds $\$8$ to her account each week. Noah adds $\$10$ every 2 weeks, or $\$5$ per week.



Write one question that might be easier to answer using the equation than using the graph.

If the trend continues this way, how much money will be in the account after 20 weeks?

Write one question that might be easier to answer using the graph than using the equation.

Is the amount of money in the account increasing or decreasing over time?

Summary Question: What are the strengths of using . . .

. . . a table? . . . a graph? . . . an equation?

One strength of a table is that it is easy to pick out specific input-output pairs. Graphs make it easier to see the big picture, like trends and comparative rates of change. Equations are useful for finding *any* input-output pair.

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Unit 8.5, Lesson 9: Notes Name ____________________

Modeling With Piecewise Linear Functions

Learning Goal(s):

- **I can create graphs of nonlinear functions with pieces of linear functions.**
- **I can calculate and interpret rates of change in context.**

Deiondre gave their dog a bath in a bathtub. This graph shows the volume of water in the tub, in gallons, as a function of time, in minutes.

Why do you think this function is called a piecewise linear function?

This function is made up of three different pieces of linear functions.

At what rate did the water in the tub fill up? Explain how you know.

The water in the tub filled up at a rate of 4 gallons every minute. In the first 3 minutes, the amount of water went from 0 gallons to 12 gallons. $\frac{12}{3} = 4.$

At what rate did the water in the tub drain? Explain how you know.

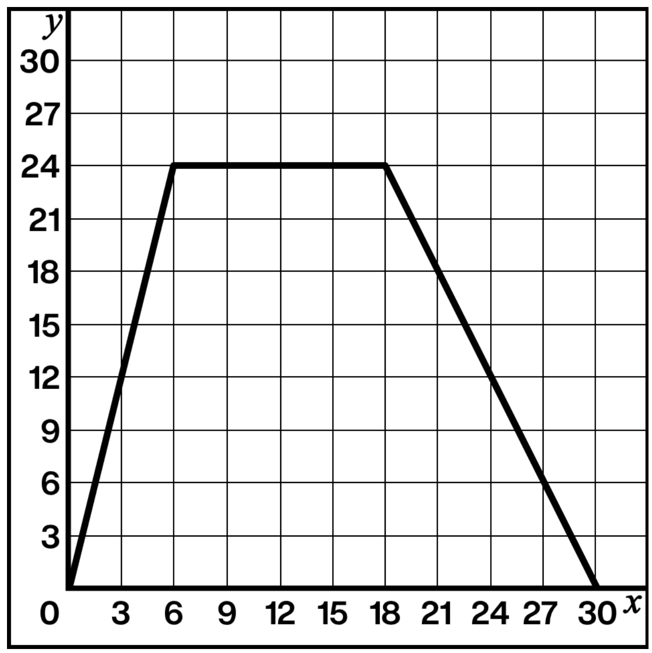
The tub drained at a rate of 2 gallons every minute. In the last 3 minutes, the amount of water went from 6 gallons to 0 gallons. $\frac{6}{3} = 2.$

Write an equation in the form $y = mx + b$ for any linear segment of this function.

1st: $y = 4x$

2nd: $y = 24$

3rd: $y = -2x + 60$



x **represents the time (min.).**

y **represents the water in the bath (gal.).**

Summary Question

How would you describe a piecewise linear function to someone who has never seen one?

A piecewise linear function is made up of some number of linear segments connected to make a continuous function.