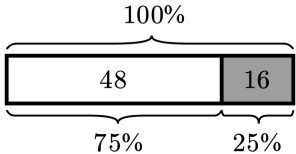
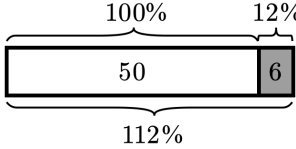


**Percentages  
Student Guide**

**Math 6 Unit 6 Accelerated  
Part 2**

## Glossary

Term	Definition
<b>percent decrease</b>	<p>A percent decrease tells us how much a quantity went down, expressed as a percentage of the starting amount.</p> <p>For example, a store had 64 hats in stock on Friday. They had 48 hats left on Saturday. The amount of hats went down by 16.</p> <p>This was a 25% decrease because 16 is 25% of 64.</p> 
<b>percent increase</b>	<p>A percent increase tells us how much a quantity went up, expressed as a percentage of the starting amount.</p> <p>For example, Elena had \$50 on Monday. She helped a neighbor, so she had \$56 on Tuesday. The amount went up by \$6.</p> <p>This was a 12% increase because 6 is 12% of 50. <math>\frac{6}{50} = 0.12 = 12\%</math>.</p> 
<b>percent error</b>	<p>Percent error is a way to describe error, expressed as a percentage of the correct or desired amount.</p> <p>For example, a box is supposed to have 150 folders in it. Clare counts only 147 folders in the box.</p> <p>This is an error of 3 folders. The percent error is 2% because 3 is 2% of 150. <math>\frac{3}{150} = 0.02 = 2\%</math>.</p>
<b>repeating decimal</b>	<p>A repeating decimal has digits that repeat in the same pattern over and over. The repeating digits are marked with a line above them. If the repeating digits are all zeroes, we call the decimal terminating.</p> <p>For example, the decimal representation of <math>\frac{1}{3}</math> is <math>0.\overline{3}</math>, which means 0.33333....</p> <p>The decimal representation of <math>\frac{25}{22}</math> is <math>1.\overline{136}</math>, which means 1.1363636....</p>

<b>terminating decimal</b>	<p>A terminating decimal has a finite number of non-zero digits after the decimal point.</p> <p>The decimal representation of <math>\frac{2}{25}</math> is 0.08.</p>
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### Unit 4 Summary

Prior Learning	Math 7, Unit 4	Future Learning
Grades 3–5 <ul style="list-style-type: none"> <li>Fraction operations</li> </ul> Math 6 <ul style="list-style-type: none"> <li>Equivalent ratios</li> <li>Unit rates</li> </ul> Math 7, Unit 2 <ul style="list-style-type: none"> <li>Proportional relationships</li> </ul>	<ul style="list-style-type: none"> <li>Percentages as proportional relationships</li> <li>Applying percentages</li> </ul>	Math 7, Unit 6 <ul style="list-style-type: none"> <li>Solving equations</li> </ul> High School <ul style="list-style-type: none"> <li>Exponential functions</li> </ul>

### Percentages as Proportional Relationships

This unit continues the study of proportional relationships, now incorporating fractional quantities and percentages.

A 4-by-6 photograph can be scaled and printed to be many different sizes.

In this example, each value in the second column is  $\frac{3}{2}$  times the length of the value in the first column.

Height (in.)	Width (in.)
4	6
$1\frac{1}{2}$	$2\frac{1}{4}$
5	$7\frac{1}{2}$

Increasing or decreasing an original amount by a percentage is another example of a proportional relationship. The original amount is always represented by 100% or 1.

Three runners training for a race agree that they will each run 10% further next week than they ran this week.

Each value in the second column is 10% greater than the value in the first column. The constant of proportionality is 1.10.

This is an example of a **percentage increase**.

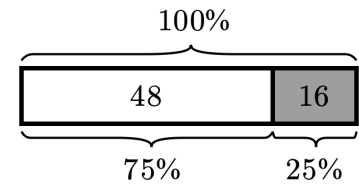
Miles Ran This Week	Miles to Run Next Week
5	5.5
11	12.1
6.5	7.15

## Unit 7.4, Family Resource

Here is an example of a **percentage decrease**.

The computer club had 64 students. Then, they lost 16 students.

This is a 25% decrease because  $\frac{16}{64} = 0.25$ .



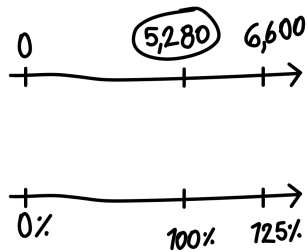
The club now has 48 students, which is 75% of the starting amount:

$$0.75 \cdot 64 = 48.$$

Sometimes problems require us to work backwards. The population of Boom Town has increased by 25% since last year. The population is now 6 600. What was the population last year?

We can use a variety of representations to solve the problem:

### Double Number Line



### Table

OLD	NEW
100%	125%
5,280 (circled)	6,600

Arrows indicate a multiplier of  $\times 1.25$  from the OLD row to the NEW row.

### Equation

$$6,600 = 1.25 \cdot b$$

$$\frac{6,600}{1.25} = \frac{1.25 \cdot b}{1.25}$$

$$5,280 = b$$

## Applying Percentages to Solve Problems

Percentages are useful in a variety of real-world situations.

A customer buys an item that costs \$20. The customer has an 18% off coupon, and then pays a sales tax of 7.5%.

82% of the bill remains after the 18% off coupon, and 82% of \$20 is  $20 \cdot 0.82 = 16.40$ .

For the total after tax, you can calculate  $16.40 \cdot 1.075 = 17.63$ .

The customer will pay a total of \$17.63.

Original Cost	\$20.00
18% Off Coupon	\$ 0.00
Subtotal	\$ 16.40
7.5% Tax	\$ 1.23
<hr/>	
Total	\$17.63

We can also use **percent change** to analyze statistics about the larger society in which we live.

## Try This at Home

### Percentages as Proportional Relationships

A supermarket offers some food by the pound. A customer orders  $1\frac{1}{2}$  pounds of potato salad for \$9 and  $1\frac{3}{4}$  pounds of coleslaw for \$11.20.

- 1.1 How much would 5 pounds of potato salad cost?
- 1.2 Which food is more expensive per pound?
2. A car dealership pays \$8350 for a car. They sell it for 17% more than they paid. How much does the dealership sell the car for?
3. On Tuesday, the high temperature was 54° Fahrenheit. This was 10% lower than the high temperature on Monday. What was the high temperature on Monday?

### Applying Percentages to Solve Problems

4. A restaurant bill before tip was \$18.75. If you paid \$22, what percent tip did you leave for the server?

The price tag on a backpack is \$34.20.

- 5.1 The store has a 15% off sale. What is the new price of the backpack?
- 5.2 The sales tax in this city is 5%. How much would a customer pay after the sale and the tax?

### Solutions:

- 1.1 \$30. One approach is to divide the cost by the weight to find the cost per pound.

$$9 \div 1\frac{1}{2} = 6 \text{ dollars per pound. 5 pounds at that rate is } \$30.$$

- 1.2 Coleslaw is more expensive. One approach is to divide each cost by each weight.

**Potato salad:**  $9 \div 1\frac{1}{2} = 6$  dollars per pound

**Coleslaw:**  $11.20 \div 1\frac{3}{4} = 6.40$  per pound

2. \$9 769.50. One approach is to multiply  $8350 \cdot 1.17 = 9769.5$ .

3. 60°. One approach is to write and solve an equation, where 90% of some number is 54°:

$$0.9x = 54 \rightarrow x = \frac{54}{0.9} = 60.$$

4. About 17.3%. One approach is write and solve an equation, where 18.75 multiplied by an unknown number is 22.  $18.75x = 22 \rightarrow x = \frac{22}{18.75} = 1.17333\dots$  The 1 that comes before the decimal represents the original 100%, while the rest of the decimal number is the growth. When written as a rounded percent, .17333 is 17.3%.

- 5.1 \$29.07. One approach is to calculate  $34.20 \cdot 0.85$ , which is 29.07.

- 5.2 \$30.52. One approach is to multiply the answer from the previous problem, 29.07, by 1.05.

**My Notes**

Kwasi is making banana bread.

1. He only has a  $\frac{1}{4}$  cup measuring scoop. How many scoops of sugar and flour does he need?

**Kwasi's Recipe**

**Number of servings: 6**

- 2 lb. of bananas
- $\frac{1}{2}$  cup of butter
- $\frac{3}{4}$  cup of sugar
- $2\frac{1}{2}$  cups of flour
- 1 tsp. of baking soda

2. A person Kwasi is planning to share his banana bread with wants to know how much sugar there is per serving in his recipe. What should Kwasi tell them?
3. Kwasi wants to make a larger loaf to serve 10 people. How much of each ingredient will he need?

**Summary**

I can use the constant of proportionality to solve problems that involve fractions.



**My Notes**

Kwasi is making banana bread.

1. He only has a  $\frac{1}{4}$  cup measuring scoop. How many scoops of sugar and flour does he need?

**3 scoops of sugar**

**10 scoops of flour**

2. One person Kwasi is planning to share his banana bread with wants to know how much sugar there is per serving in his recipe. What should Kwasi tell them?

**There is  $\frac{1}{8}$  of a cup of sugar in each serving.**

3. Kwasi wants to make a larger loaf to serve 10 people. How much of each ingredient will he need?
- $3\frac{1}{3}$  lb. of bananas
  - $\frac{5}{6}$  cup of butter
  - $1\frac{1}{4}$  cups of sugar
  - $4\frac{1}{6}$  cups of flour
  - $1\frac{2}{3}$  tsp. of baking soda

**Kwasi's Recipe**

**Number of servings: 6**

- 2 lb. of bananas
- $\frac{1}{2}$  cup of butter
- $\frac{3}{4}$  cups of sugar
- $2\frac{1}{2}$  cups of flour
- 1 tsp. of baking soda

**Summary**

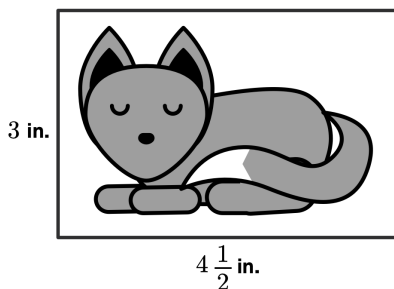
I can use the constant of proportionality to solve problems that involve fractions.

**My Notes**

StuckStickers makes pins in addition to stickers.

1. Here is Cho's design and thinking for a pin that is  $\frac{3}{4}$  inches wide.

Cho's Design

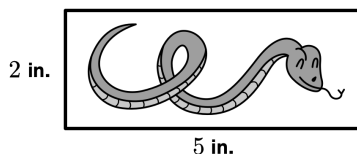


Cho's Thinking

Height (in.)		Width (in.)
3	$\cdot 1\frac{1}{2}$	$4\frac{1}{2}$
	$\div 1\frac{1}{2}$	$\frac{3}{4}$

Explain how Cho figured out how tall the pins should be.

2. Hamza wants to create pins with his design as well. Hamza's pins will be  $\frac{4}{5}$  inches tall. What will the width of his pin be?



**Summary**

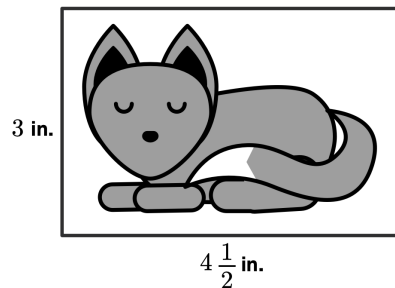
I can use a table to determine an unknown value in a proportional relationship.

**My Notes**

StuckStickers makes pins in addition to stickers.

1. Here is Cho’s design and thinking for a pin that is  $\frac{3}{4}$  inches wide.

Cho’s Design



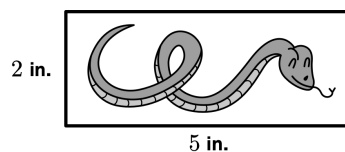
Cho’s Thinking

Height (in.)		Width (in.)
3	$\cdot 1\frac{1}{2}$ →	$4\frac{1}{2}$
	← $\div 1\frac{1}{2}$	$\frac{3}{4}$

Explain how Cho figured out how tall their pins should be.

**Responses vary.** Cho made a table with the height and width. Then, they figured out what the constant of proportionality is between height and width ( $1\frac{1}{2}$ ). Since they knew the width, they needed to divide by  $1\frac{1}{2}$  to figure out the height.

2. Hamza wants to create pins with his design as well. Hamza’s pins will be  $\frac{4}{5}$  inches tall. What will the width of his pin be?



$$2 \cdot \frac{5}{2} = 5 \text{ inches}$$

$$\frac{4}{5} \cdot \frac{5}{2} = 2 \text{ inches}$$

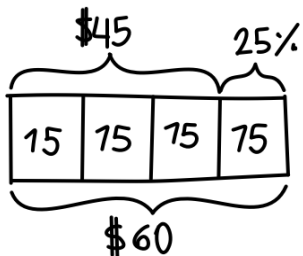
His pin will be 2 inches wide.

**Summary**

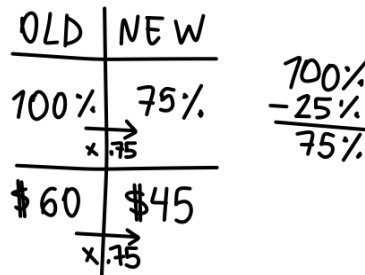
I can use a table to determine an unknown value in a proportional relationship.

**My Notes**

Here are two different representations Pablo used to figure out the new price of a pair of headphones after using a coupon.



TAPE DIAGRAM



TABLE

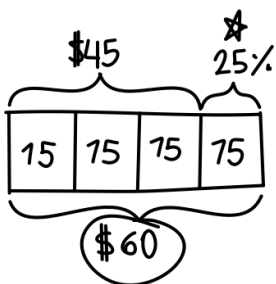
1. What was the original price of the headphones?  
Circle where you see it in each representation.
2. What is the **percent increase** or **percent decrease**? Star where you see it in each representation.
3. Choose one representation and explain how Pablo used it to figure out the new price of the headphones.

**Summary**

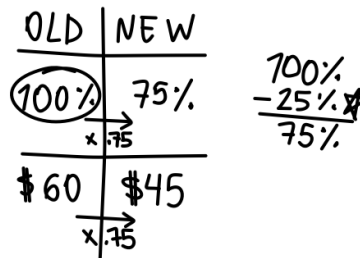
- I can use tape diagrams and tables to represent adding or subtracting a percentage from 100% .
- I can determine the new amount if I know the original amount and the percent change.

**My Notes**

Here are two different representations Pablo used to figure out the new price of a pair of headphones after using a coupon.



TAPE DIAGRAM



TABLE

1. What was the original price of the headphones?  
Circle where you see it in each representation.  
  
\$60
2. What is the **percent increase** or **percent decrease**? Star where you see it in each representation.  
  
25% **decrease**
3. Choose one representation and explain how Pablo used it to figure out the new price of the headphones. **Responses vary.**
  - **Tape Diagram:** Pablo broke the tape diagram into four sections that were each 25%. Then, he figured out that each 25% represented \$15. Since the coupon was 25% off, he counted the rest as the new price, which was  $3 \cdot 15 = 45$  dollars.
  - **Table:** Pablo figured out that a coupon for 25% off means that there is 75% left. He used the constant of proportionality 0.75 to figure out that the new price was  $0.75 \cdot 60 = 45$  dollars.

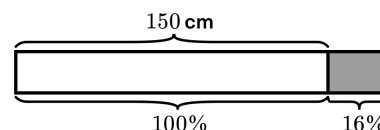
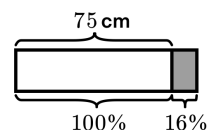
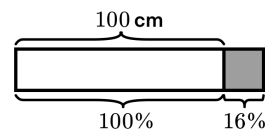
**Summary**

- I can use tape diagrams and tables to represent adding or subtracting a percentage from 100%.
- I can determine the new amount if I know the original amount and the percent change.

**My Notes**

1. Each rectangle is 16% longer than the original. Complete the table with the length of each new rectangle.

Original Rectangle Length (cm)	New Rectangle Length (cm)
100	
75	
150	



2. Write at least two different equations that represent the relationship between the length of the original rectangle,  $b$ , and the length of new rectangle,  $c$ .
3. Write at least one equation for the relationship between the length of an original rectangle and the length of a new rectangle that is 16% **shorter**.

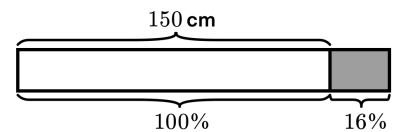
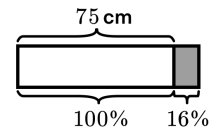
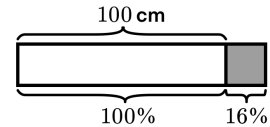
**Summary**

I can write an equation to represent adding or subtracting a percentage from 100% .

**My Notes**

1. Each rectangle is 16% longer than the original. Complete the table with the length of each new rectangle.

Original Rectangle Length (cm)	New Rectangle Length (cm)
100	116
75	87
150	174



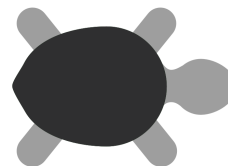
2. Write at least two different equations that represent the relationship between the length of the original rectangle,  $b$ , and the length of new rectangle,  $c$ . **Responses vary.**
- $c = 1.16b$
  - $c = (1 + 0.16)b$
  - $c = 1b + 0.16 \cdot b$
3. Write at least one equation for the relationship between the length of an original rectangle and the length of a new rectangle that is 16% shorter. **Responses vary.**
- $c = 0.84b$
  - $c = (1 - 0.16)b$
  - $c = 1b - 0.16 \cdot b$

**Summary**

I can write an equation to represent adding or subtracting a percentage from 100% .

**My Notes**

At a turtle sanctuary, the number of nesting turtles decreased by 20% compared to last year.



This year, there are 180 nesting turtles.

1. Create each representation to show how many nesting turtles were at the sanctuary last year.

**Double Number Line**

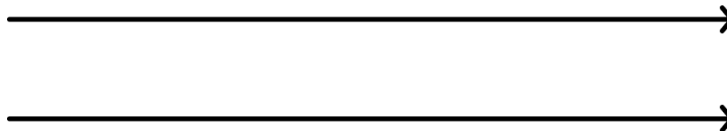


Table	Equation
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2. How many nesting turtles were at the sanctuary last year?

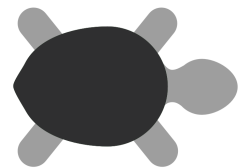
**Summary**

- I can use double number lines to represent adding or subtracting a percentage from 100%.
- I can determine the original amount if I know the new amount and the percent change.



**My Notes**

At a turtle sanctuary, the number of nesting turtles decreased by 20% compared to last year.



This year, there are 180 nesting turtles.

1. Create each representation to show how many nesting turtles were at the sanctuary last year.

**Double Number Line**

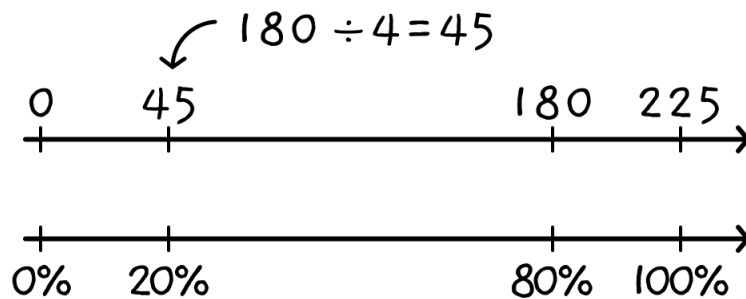


Table	Equation						
<table border="1" style="margin: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Last Year</th> <th style="padding: 5px;">This Year</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; padding: 5px;">100%</td> <td style="text-align: center; padding: 5px;">80%</td> </tr> <tr> <td style="text-align: center; padding: 5px;">180</td> <td style="text-align: center; padding: 5px;">225</td> </tr> </tbody> </table>	Last Year	This Year	100%	80%	180	225	$180 = 0.8 \cdot x$ $225 = x$
Last Year	This Year						
100%	80%						
180	225						

2. How many nesting turtles were at the sanctuary last year?

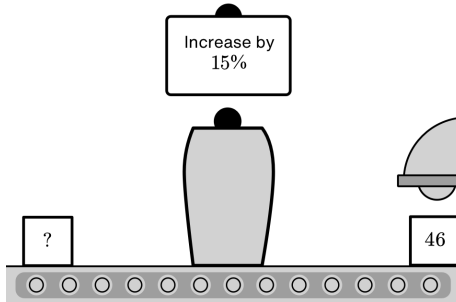
**There were 225 nesting turtles last year.**

**Summary**

- I can use double number lines to represent adding or subtracting a percentage from 100%.
- I can determine the original amount if I know the new amount and the percent change.

**My Notes**

1. A number went into this machine and 46 came out.  
What number went in? Explain your strategy.



2. 50 went into a different machine and 46.5 came out. What percent increase or decrease did this machine use?
3. What are some important things to remember about figuring out the original value given the new value and a percent increase or decrease?

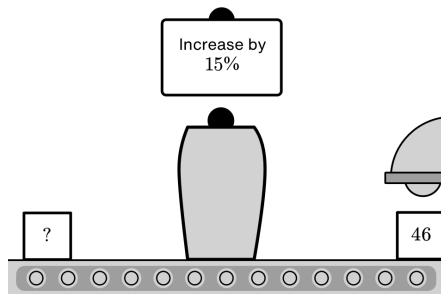
**Summary**

I can determine the original amount if I know the new amount and the percent change for one-step and multistep problems.

**My Notes**

1. A number went into this machine and 46 came out.

What number went in? Explain your strategy.



**Explanations vary.**

**Since this machine increases every input by 15%, every number that goes in gets multiplied by 1.15. In order to figure out what number went in, I worked backwards and divided 46 by 1.15 and got 40.**

2. 50 went into a different machine and 46.5 came out. What percent increase or decrease did this machine use?

**The number decreased by 7%.  $\frac{3.5}{50} = 0.07$ .**

3. What are some important things to remember about figuring out the original value given the new value and a percent increase or decrease? **Responses vary.**

- **Figure out the constant of proportionality using the percent increase or decrease first.**
- **Undoing an increase of 10% is not the same as decreasing by 10%.**
- **You can divide by the constant of proportionality in order to work backwards.**

**Summary**

I can determine the original amount if I know the new amount and the percent change for one-step and multistep problems.

**My Notes**

1. What are **sales tax** and **tip**?

2. Use this receipt to figure out the total amount this customer paid for their \$20 meal after an 18% off coupon and 7.5% sales tax.

Original Cost	\$20.00
18% Off Coupon	\$ 0.00
Subtotal	\$ 0.00
7.5% Tax	\$ 0.00
<hr/>	
Total	\$?.??

3. Which would result in the greatest total amount?

- Tax first, then coupon.
- Coupon first, then tax.
- They are the same.
- Not enough information.

Explain your thinking.

**Summary**

I can solve multistep problems about sales tax and tip.

My Notes

1. What are **sales tax** and **tip**? *Responses vary.*
- **Sales tax is a fee (an amount of money) paid to the government, usually a percentage of the price of the item. Different states charge different percentages. Additionally, some local governments, like counties and cities, also charge a sales tax.**
  - **Tip is an amount that you add onto a bill to pay the waiter or other people who help you at a store or a restaurant. Tips at a restaurant are usually between 10% and 20% .**

2. Use this receipt to figure out the total amount this customer paid for their \$20 meal after an 18% off coupon and 7.5% sales tax.

**Coupon:**

$$\$20.00 \cdot 0.82 = \$16.40$$

**Tax:**

$$\$16.40 \cdot 1.075 = \$17.63$$

**This customer paid \$17.63 for their meal.**

Original Cost	\$20.00
18% Off Coupon	\$ 0.00
Subtotal	\$ 0.00
7.5% Tax	\$ 0.00
<hr/>	
Total	\$?.??

3. Which would result in the greatest total amount?
- Tax first, then coupon.
  - Coupon first, then tax.
  - They are the same.
  - Not enough information.

Explain your thinking.

***Explanations vary. You are multiplying by two numbers, and you can multiply in either order and get the same answer.***

Summary

I can solve multistep problems about sales tax and tip.

**My Notes**

Adrian is a 25-year-old who plays in a band and works 30 hours per week as a server. He makes minimum wage, which is \$5.45 per hour in his town. Adrian also collects tips. The average tip he receives is 15% of the bill. The typical bill is \$25 per table, and he serves 70 tables in an average week.

1. How much money does Adrian make in a typical week?

Imagine that the average tip Adrian receives is 20% instead of 15%.

- 2.1 How much money would he make now?
- 2.2 By what percent would his pay increase?

---

**Summary**

I can use proportional relationships and percent change to analyze an issue in society.

**My Notes**

Adrian is a 25-year-old who plays in a band and works 30 hours per week as a server. He makes minimum wage, which is \$5.45 per hour in his town. Adrian also collects tips. The average tip he receives is 15% of the bill. The typical bill is \$25 per table, and he serves 70 tables in an average week.

1. How much money does Adrian make in a typical week?

$$30 \cdot 5.45 + (70 \cdot 25 \cdot 0.15) = \$426$$

Imagine that the average tip Adrian receives is 20% instead of 15%.

- 2.1 How much money would he make now?

$$30 \cdot 5.45 + (70 \cdot 25 \cdot 0.20) = \$513.50$$

- 2.2 By what percent would his pay increase?

**\$513.50 is about a 20.5% increase compared to \$426.**

---

**Summary**

I can use proportional relationships and percent change to analyze an issue in society.





**My Notes**

Between 2017 and 2018, the city of San Francisco raised its minimum wage from \$14.00 to \$15.00 .

- 1.1 What is the percent increase?  $\sim 7.14\%$
- 1.2 Write an equation for the relationship between the minimum wage in 2017,  $x$ , and the minimum wage in 2018,  $y$  .

$$y = 1.0714x$$

- 1.3 If the percent increase stayed constant, how much should minimum wage be in San Francisco in 2020?  
Show or explain your thinking.

Year	Minimum Wage in SF
2018	\$15.00
2019	$\$15.00 \cdot 1.0714 = \$16.07$
2020	$\$16.07 \cdot 1.0714 = \$17.22$

- 2. Explain to a family member how the cost of college has changed over time compared to minimum wage. What is important for them to know? **Responses vary.**
  - **The cost of college was more affordable in 1990 than it is now.**
  - **The cost of college now is more money than a person can afford if they work a minimum wage job for 20 hours a week or even more.**

**Summary**

I can write equations to represent the cost of college over time.

I can solve problems about the cost of college over time.

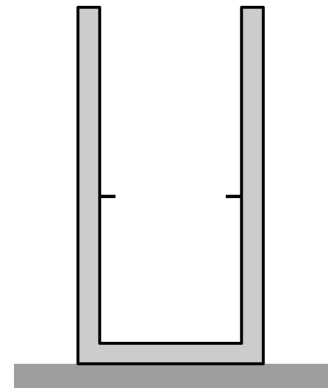
**My Notes**

1. What is **percent error**? Create your own example.

2. Diamond is making a bookshelf with shelves that are supposed to be 17.6 centimeters long. Complete the table with the percent error of each shelf that Diamond builds.

Shelf Width (cm)	Percent Error
17.1	
18.25	
16.5	

Desired shelf length:  
17.6 cm



3. The acceptable percent error is 5% for a shelf to fit. Will all of the shelves fit? Why or why not?

**Summary**

I can explain what percent error is and how to calculate it.

I can decide whether a value is within an acceptable percent error.

**My Notes**

1. What is **percent error**? Create your own example.

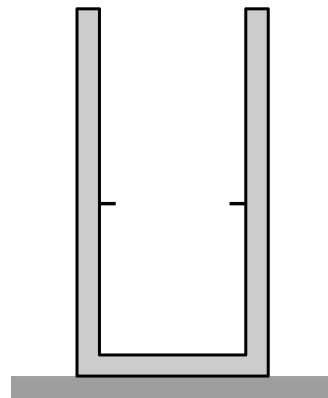
**Responses vary.** Percent error is how far away the value you have is from the value you wanted, written as a percent.

**Example:** I wanted to grow my hair exactly 12 inches long. Right now, my hair is only 11.5 inches. The percent error is  $\frac{0.5}{12} \approx 0.042$ , which is 4.2%.

2. Diamond is making a bookshelf with shelves that are supposed to be 17.6 centimeters long. Complete the table with the percent error of each shelf that Diamond builds.

Shelf Width (cm)	Percent Error
17.1	$\frac{0.5}{17.6} \approx 2.8\%$
18.25	$\frac{0.65}{17.6} \approx 3.7\%$
16.5	$\frac{1.1}{17.6} = 6.25\%$

Desired shelf length:  
17.6 cm



3. The acceptable percent error is 5% for a shelf to fit. Will all of the shelves fit? Why or why not?

**No. Explanations vary.** The smallest shelf will be too short because it has a percent error of 6.25%.

**Summary**

- I can explain what percent error is and how to calculate it.
- I can decide whether a value is within an acceptable percent error.



**My Notes**

Here is information about the wage gap.

In 1963, when the Equal Pay Act was passed, women were paid 41% less than what men were paid on average, which was about \$5 978 per year. By 2004, women were paid \$29 900 per year on average, which is about 23% less than what men were paid.

*Source: National Organization for Women*

1. Write at least two questions that you could figure out using this information and whose answer is not already given.

**Responses vary.**

- How much were women paid on average in 1963?
- How much were men paid on average in 2004?

2. Answer one of the questions that you asked.

**Responses vary based on the questions asked.**

- Women were paid  $\$5\,978 \cdot 0.59 \approx \$3\,527$  on average, which is  $\$5\,978 - \$3\,527 = \$2\,451$  less than men were paid in the same year.
- Men were paid  $\frac{29\,900}{0.77} \approx \$38\,831$  on average, which is  $\$38\,831 - \$29\,900 = \$8\,931$  more than women were paid in the same year.

3. What are some characteristics of a good question you could ask using a set of information?

**Responses vary.**

- You can answer it with only the information you know.
- It is interesting or provides insight into the situation.
- The answer isn't already available in the information you have.

**Summary**

- I can write a question about a real-world situation that involves percent increase or decrease.
- I can use what I know to answer questions about the world we live in.