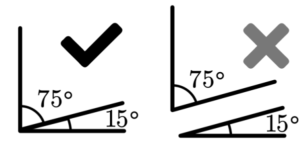
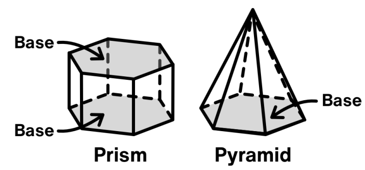
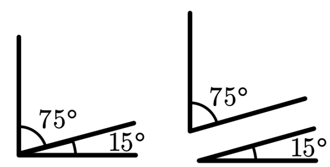
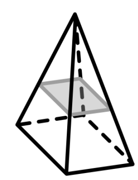
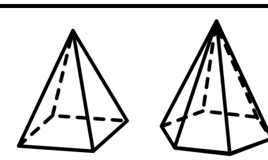
Volume and Surface Area Student Guide

Math 7 Unit 7 Accelerated

Glossary

Term	Definition
adjacent angles	Adjacent angles share a side and a vertex. In the diagram on the left, the $75°$ angle and the $15°$ angle are adjacent. In the diagram on the right, they are not adjacent. 
base	A prism has two identical bases that are parallel. A pyramid has one base. A prism or pyramid is named for the shape of its base. 
complementary angles	Complementary angles have measures that add up to 90 degrees. For example, a $75°$ angle and a $15°$ angle are complementary. 
cross section	A cross section is the new face you see when you slice through a three-dimensional figure. For example, if you slice a rectangular pyramid parallel to the base, you get a smaller rectangle as the cross section. 
identical copy	An identical copy of a figure is one that has the same shape and size.
prism	A prism is a solid that has two bases that are identical copies. The bases are connected by rectangles or parallelograms.  Triangular Prism, Hexagonal Prism
pyramid	A pyramid is a solid in which the base is a polygon. All of the other faces are triangles that meet at a single vertex.  Rectangular Pyramid, Hexagonal Pyramid

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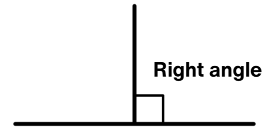
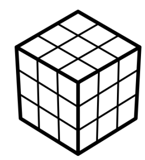
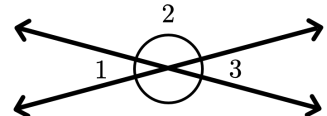
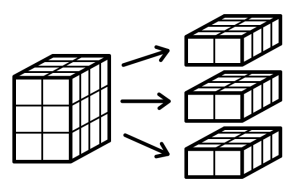
Unit 7.7, Student Goals and Glossary

right angle	A right angle is half of a straight angle. It measures $90°$.	
straight angle	A straight angle forms a straight line. It measures $180°$.	
supplementary angles	Supplementary angles have measures that add up to 180 degrees. For example, a $165°$ angle and a $15°$ angle are supplementary.	
surface area	The surface area of a polyhedron is the number of square units that covers all the faces of the polyhedron, without any gaps or overlaps. For example, if the six faces of a cube each have an area of 9 square centimeters, then the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.	
vertical angles	Vertical angles are angles opposite each other where two lines cross. Their angle measures are equal. Angles 1 and 3 are a pair of vertical angles. Another pair is angles 2 and 4.	
volume	Volume is the number of cubic units that fill a three-dimensional region without any gaps or overlaps. For example, the volume of this rectangular prism is 24 cubic units, because it is composed of 3 layers that are each 8 cubic units.	

Glossary

Term	Definition
cone	A cone is a three-dimensional figure that tapers from a circular base to a point.
cylinder	A cylinder is a three-dimensional figure like a prism, but with bases that are circles.
dependent variable	A dependent variable is a variable representing the output of a function.
function	A function is a rule that assigns exactly one output to each possible input.
independent variable	An independent variable is a variable representing the input of a function.
radius	A radius is a line segment that goes from the center of a circle to any point on the circle. A radius can go in any direction. Every radius of a circle is the same length. We also use the word *radius* to mean the length of this segment.
sphere	A sphere is a three-dimensional figure in which all cross-sections in every direction are circles.
volume	The volume is the number of cubic units that fill a three-dimensional region without any gaps or overlaps.

Unit 7 Summary

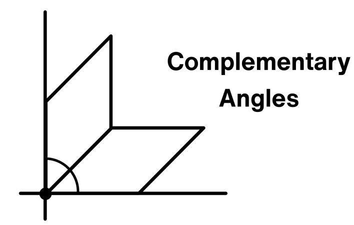
Prior Learning	Math 7, Unit 7	Future Learning
Math 6 • Area and surface area • Volume of rectangular prisms Math 7 • Solving equations • Properties of circles	• Angle relationships • Building and drawing triangles with given conditions • Volume and surface area of non-rectangular prisms	Math 8, Units 1 and 5 • Congruence • Volume of cylinders, cones, and spheres High School • Triangle congruence theorems

Angle Relationships

We can use common angle relationships to determine unknown angles in diagrams.

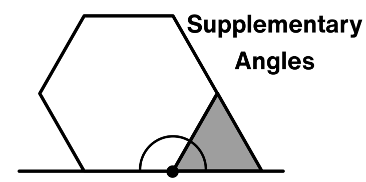
If two angles add to $90°$, they are *complementary angles*.

In the diagram, each marked angle must be $45°$ because $2(45) = 90$.



If two angles add to $180°$, they are *supplementary angles*.

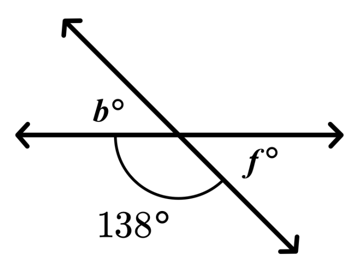
If one angle of the triangle is $60°$, the larger marked angle must be $120°$ because $60 + 120 = 180$.



We can write equations based on angle relationships.

For example, $f + 138 = 180$ because they are supplementary angles.

It is also true that $b + 138 = 180$, so b and f are equal.



Angles b and f are called *vertical angles*, angles that are opposite each other where two lines cross.

The measures of vertical angles are always equal.

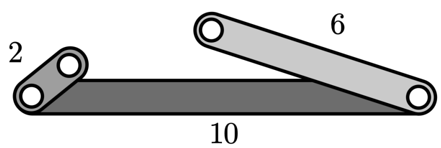
Drawing Triangles

The second part of the unit is all about drawing polygons based on descriptions. How many triangles are possible to draw based on given information?

Sometimes it is not possible to draw any triangle.

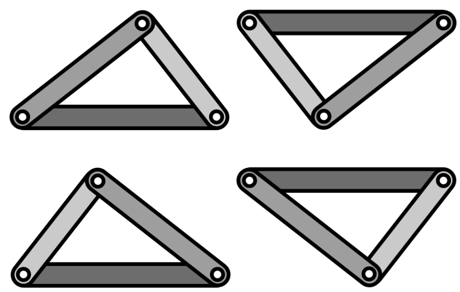
The two shortest sides are not long enough to form a triangle.

They would need to be longer than the third side to connect.



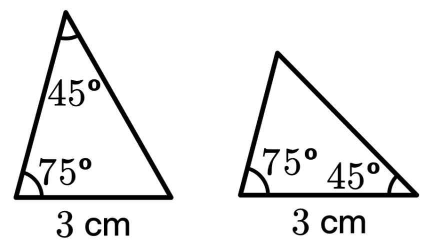
Sometimes it is only possible to draw one triangle.

All of the triangles with side lengths of 3 units, 4 units, and 5 units are *identical copies.*



Sometimes it is possible to draw more than one triangle.

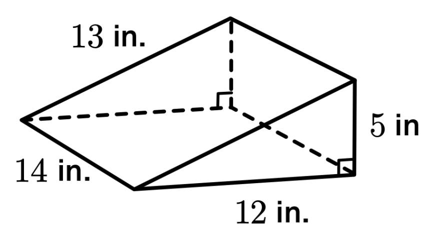
One side length of 3 cm, one $75°$ angle, and one $45°$ angle could describe two triangles that are not identical copies.



Solid Geometry

There are two features we often measure in a three-dimensional object: its *volume* (how much space is inside the object) and its *surface area* (the amount of material needed to cover the object).

A *prism* is a solid that has two *bases* that are identical. In this prism, the bases are right triangles.



Volume: We can calculate the volume of any prism by multiplying the area of the base by the height.

Volume = Area of Base • Height

Volume = $\frac{1}{2}(5 \cdot 12) \cdot 14 = 30 \cdot 14 = 420$ cubic inches

Surface area: This is the sum of the area of each face.

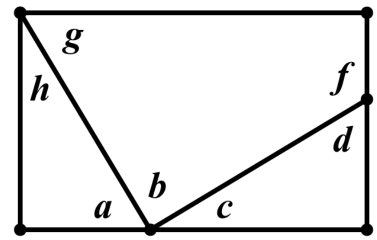
This prism has two triangular faces and three rectangular faces.

Surface Area = $30 + 30 + 70 + 168 + 182 = 480$ square inches

Try This at Home

Angle Relationships

Here is a rectangle.



1.1 List two angles that are **complementary.**

1.2 List two angles that are **supplementary.**

1.3 If angle h is $31°$, determine the measure of angle g. Label it on the diagram.

1.4 If angle f is $121°$, determine the measure of angle d. Label it on the diagram.

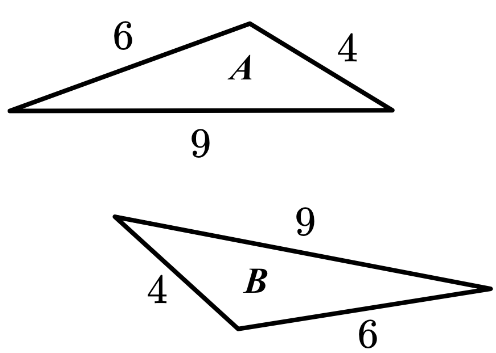
1.5 If the measure of angle b is $90°$, are angles a and c complementary? Explain your thinking.

Drawing Triangles

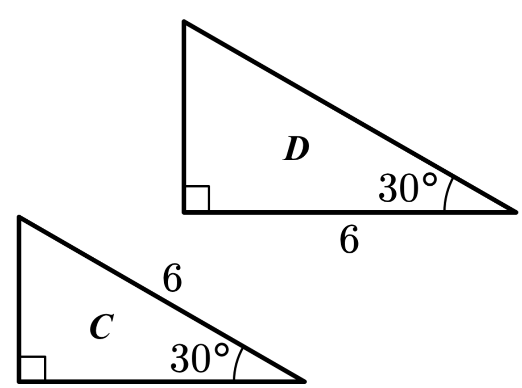
2. How many nonidentical triangles can be made using the side lengths 5 cm, 15 cm, and 25 cm? Explain your thinking.

For each pair of triangles, explain what is similar and different about the triangles. Then determine whether or not the triangles are identical copies.

3.1

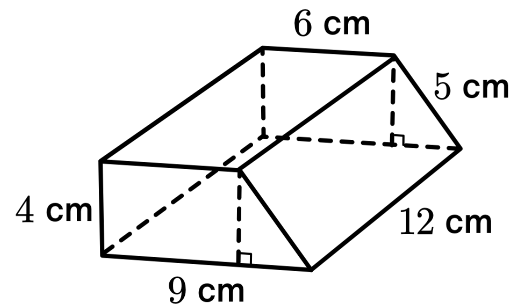


3.2



Solid Geometry

Here is a prism.



4.1 Shade one base of the prism.

4.2 Calculate the volume of this prism. Organize your calculations so that others can follow them.

4.3 How many faces does the prism have?

4.4 Calculate the surface area of this prism. Organize your calculations so that others can follow them.

4.5 If this were a box and you wanted to know how much cardboard you would need to build it, what would be more useful information: volume or surface area? Explain your thinking.

Unit 7.7, Family Resource

Solutions:

1.1 Angles g and h

1.2 Angles d and f

1.3 $g = 59°$ $(31 + 59 = 90)$

1.4 $d = 59°$ $(121 + 59 = 180)$

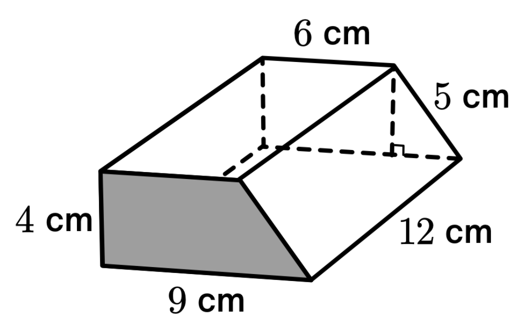
1.5 Yes. *Explanations vary.* The sum of the measures of angles a, b, and c is $180°$. If the measure of angle b is $90°$, then the measures of the other two angles must add up to $90°$, which means they are complementary angles.

2. None. *Explanations vary.* In order to connect and make a triangle, the two shortest sides need to be longer than the third side. $5 + 15 < 25$, so the sides are too short to create a triangle.

3.1 These are identical triangles. They are the same shape and size, even though one triangle is turned in a different direction.

3.2 These are not identical triangles. They are the same shape, but not the same size. Both triangles are facing the same direction and both have two equal sides. The equal sides in triangle D are 6 units long. The equal sides in triangle C are less than 6 units long.

4.1 See figure.



4.2 Base Area = Area of Rectangle + Area of Triangle

$A = 4 \cdot 6 + \frac{1}{2} \cdot 4 \cdot 3 = 24 + 6 = 30$ square cm

Volume = Base Area • Height

$V = 30 \cdot 12 = 360$ cubic cm

4.3 6 faces. 2 bases and 4 other faces.

4.4 *Strategies vary.*

- Surface Area $= 30 + 30 + 108 + 60 + 72 + 48 = 348$ square units
- Surface Area $= 2(30) + 12(9 + 5 + 6 + 4) = 60 + 12(24) = 348$ square units

4.5 Surface area. *Explanations vary.* Surface area is the number of square units that covers all the faces of the object, without any gaps or overlaps. Volume is more about the amount of space that fills up an object.

Unit 5 Summary

Prior Learning	Math 8, Unit 5	Future Learning
Math 7 • Volume of prisms • Area of circles Math 8, Unit 3 • Linear relationships	• Introduction to functions • Representing and interpreting functions • Volume of cylinders, cones, and spheres	High School • Function notation • Cross–sections and volumes in context

Defining Functions

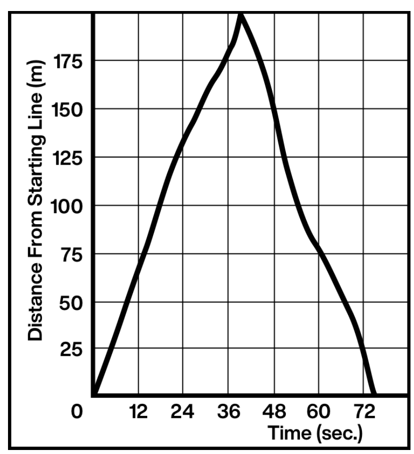
A function is a rule that assigns exactly one output to each possible input.

Examples	Non-Examples
Input: Name **Output:** First letter of that name (e.g., Sneha → S) **Input**: Any number **Output:** Three more than the input (e.g., $7 \rightarrow 10$)	**Input:** Letter **Output:** A name beginning with that letter (e.g., S → Sora) **Input:** Digit **Output:** A number whose last digit is the input (e.g., $7 \rightarrow 207$)

Here are some more **examples** of functions:

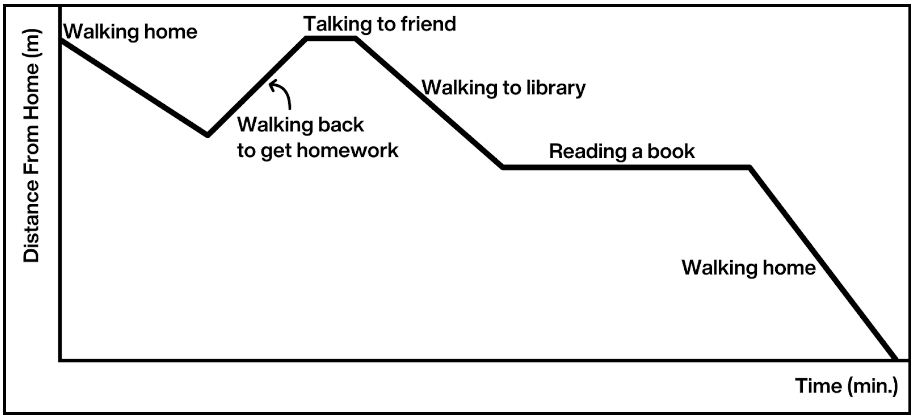
$$y = 4 - 3x$$

Input	Output
-2	4π
-1	1π
0	0
1	1π
2	4π



Representing and Interpreting Functions

A function can represent a story. Here is one example:



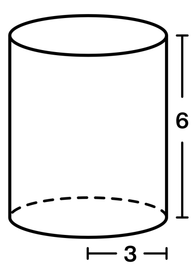
Independent Variable:
Time (min.)

Dependent Variable:
Distance From Home (m)

Volume of Cylinders, Cones, and Spheres

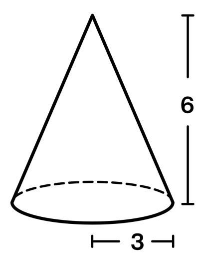
Volume is the number of cubic units that fill a 3-D region without any gaps or overlaps.

Cylinder



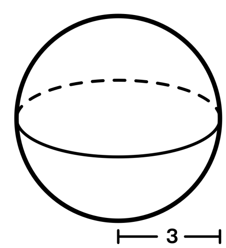
$$V = B \cdot h$$
$$= \pi r^2 \cdot h$$
$$= \pi \cdot (3)^2 \cdot (6)$$
$$= 9 \cdot 6 \cdot \pi$$
$= 54\pi$ cubic units

Cone



$$V = \frac{1}{3} B \cdot h$$
$$= \frac{1}{3} \pi r^2 \cdot h$$
$$= \frac{1}{3} \pi \cdot (3)^2 \cdot (6)$$
$$= \frac{1}{3} \cdot 9 \cdot 6 \cdot \pi$$
$= 18\pi$ cubic units

Sphere



$$V = \frac{4}{3} \pi r^2 \cdot r$$
$$= \frac{4}{3} \pi r^3$$
$$= \frac{4}{3} \pi \cdot (3)^3$$
$$= \frac{4}{3} \cdot 27 \cdot \pi$$
$= 36\pi$ cubic units

Try This at Home

Defining Functions

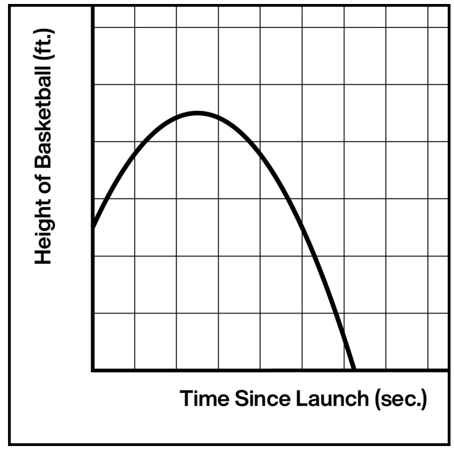
1.1 This table represents the total amount of data used compared to how many phone calls were made in a month.

# of Phone Calls	**Total Data Used** (GB)
10	4.3
19	6.2
35	7.5
10	8.3

a. Name the independent variable (input) and dependent variable (output).

b. Decide whether the situation represents a function or not. Explain your thinking.

1.2 This graph represents the height of a basketball over time.



a. Name the independent variable (input) and dependent variable (output).

b. Decide whether the situation represents a function or not. Explain your thinking.

1.3 Brown rice costs $\$2$ per pound. Beans cost $\$1.60$ per pound. Jamar has $\$10$ to spend to make a large meal of beans and rice for a potluck dinner. The amount of brown rice he can buy, r, is related to the amount of beans he can buy, b.

a. Name the independent variable (input) and dependent variable (output).

b. Decide whether the situation represents a function or not. Explain your thinking.

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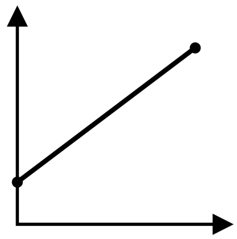
Unit 8.5, Family Resource

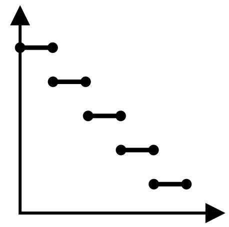
Representing and Interpreting Functions

Match each of the following situations with a graph (you can use a graph multiple times). Name the independent and dependent variables.

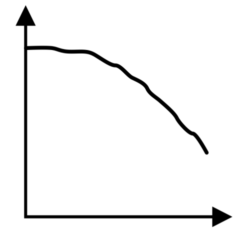
A.



B.



C.



2.1 Daeja takes a handful of popcorn out of the bag every 5 minutes.

2.2 A plant grows the same amount every week.

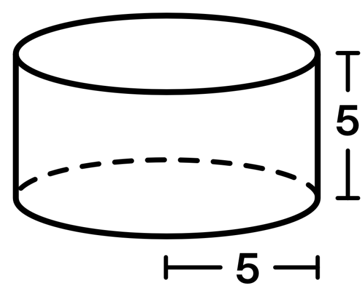
2.3 The day started very warm, but then it slowly got colder.

2.4 A cylindrical glass sits on a counter.
The more water you pour in, the higher the water level is.

3. Write an equation in the form $y = mx + b$ that could represent the plant's growth. Explain what each number means in terms of the situation.

Volume of Cylinders, Cones, and Spheres

This cylinder has a height and radius of 5 cm.
Express your answers in terms of π .



4.1 What is the diameter of the base?

4.2 What is the area of the base?

4.3 What is the volume of the cylinder?

4.4 What would the volume of a cone with the same height and radius be?

4.5 What would the height be if the volume of the cylinder remained the same, but the radius doubled?

Solutions:

1.1a The *independent variable* represents the input of a function. The *dependent variable* represents the output of a function. Here, the independent variable is the number of phone calls; the dependent variable is the total data used.

1.1b This relationship is **not** a function because the number of calls does not uniquely determine the amount of data. For example, 10 phone calls results in both 4.3 GB and 8.3 GB of data.

1.2a By convention, the *independent variable* is represented on the horizontal axis and the *dependent variable* on the vertical axis. The independent variable in this situation is the time since launch. The dependent variable is the height of the basketball.

1.2b This relationship is a function because there is exactly one height for each time.

1.3a It is possible for either variable to be the independent variable. In this case, we are wondering about how much rice can be bought, so the independent variable is the amount of brown rice purchased. The dependent variable is the amount of beans purchased.

1.3b This relationship is a function because for every amount of beans, there is only one possible amount of rice Lin can buy if he wants to spend exactly $\$10$.

2.1 Graph B, Independent variable = Time (minutes), Dependent variable = Amount of popcorn left in bag

2.2 Graph A, Independent variable = Time (weeks), Dependent variable = Height of the plant

2.3 Graph C, Independent variable = Time (hours), Dependent variable = Temperature outside

2.4 Graph A, Independent variable = Volume of water poured in the glass, Dependent variable = Height of water in the glass

3. The equations vary. An example equation is $y = 2x + 5$, where 5 represents the height of the plant when you start measuring and 2 represents the number of inches the plant grows every week.

4.1 10 cm. The diameter is twice the length of the radius, and $2(5) = 10$.

4.2 25π cm^2. The area of a circle is π times the radius squared, or $(5)^2 \cdot \pi$.

4.3 125π cm^3. The volume is the area of the base times the height. The area of the base here is 25π, so the volume is 125π cm^3 since $25\pi \cdot 5 = 125\pi$.

4.4 $\frac{125\pi}{3}$ cm^3. The volume of a cone is one-third the volume of the corresponding cylinder.

4.5 1.25 cm. If the radius doubled, then it would be 10 cm. There are various methods for finding the height. One method is to organize each quantity using a table. A sample table is shown below.
Radius (cm): 10
Base area (sq. cm):100π
Height (cm): $\frac{125\pi}{100\pi} = 1.25$
Cylinder volume (cubic cm): 125π

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Unit 7.7, Lesson 9: Notes Name ______________________

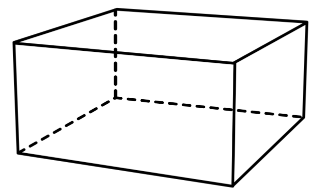
My Notes

1. Explain in your own words what a *cross section* is.

Here is a rectangular prism.

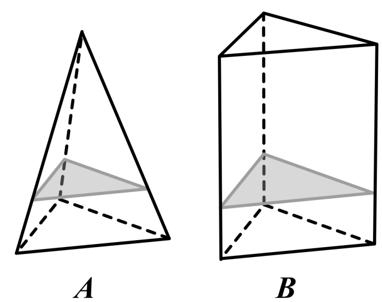
2. Select **all** the possible cross sections of this prism.

- ☐ Triangle
- ☐ Rectangle
- ☐ Pentagon
- ☐ Hexagon
- ☐ Octagon



Here is a triangular pyramid and a triangular prism.

3.1 If you cut both the pyramid and the prism parallel to their bases, how would the cross sections be **similar**?



3.2 How would they be **different**?

Summary

- ☐ I can describe cross sections of a solid.
- ☐ I can compare and contrast cross sections of prisms and pyramids.

Name ____________________________________

My Notes

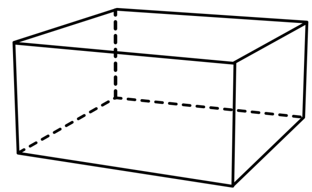
1. Explain in your own words what a *cross section* is.

   ***Responses vary.* A cross section is a shape you see when you slice through a three-dimensional object.**

Here is a rectangular prism.

2. Select **all** the possible cross sections of this prism.

   - ✓ Triangle
   - ✓ Rectangle
   - ✓ Pentagon
   - ✓ Hexagon
   - ☐ Octagon

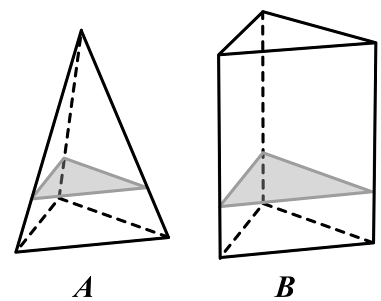


Here is a triangular pyramid and a triangular prism.

3.1 If you cut both the pyramid and the prism parallel to their bases, how would the cross sections be **similar**?

Responses vary.

- **They are both triangles.**
- **They are both the same shape as the base.**
- **They both are scaled copies of the base.**



3.2 How would they be **different**? ***Responses vary.***

- **The cross section of the prism is the same size as its base. The cross section of the pyramid is smaller than its base.**
- **The cross section of the pyramid is almost always smaller than the cross section of the prism.**

Summary

☐ I can describe cross sections of a solid.

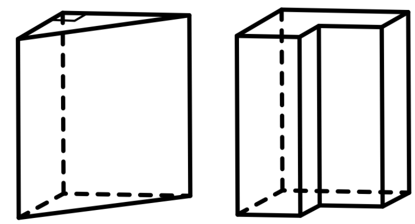
☐ I can compare and contrast cross sections of prisms and pyramids.

desmos

Unit 7.7, Lesson 10: Notes Name ______________________________

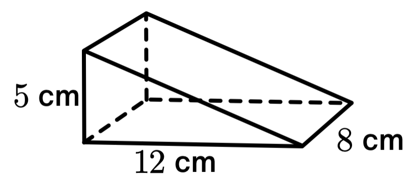
My Notes

1. Describe a strategy for calculating the volume of a prism.



2.1 Shade in a base of this prism.

2.2 Calculate the volume.
Show all of your calculations.



2.3 Sketch and label a **rectangular** prism with the same volume.

Summary

- ☐ I can explain how the volume of a prism is related to the area of its base and its height.
- ☐ I can calculate the volume of rectangular and triangular prisms.

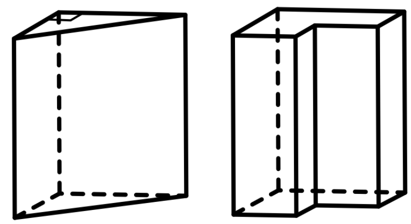
Name ________________________________

My Notes

1. Describe a strategy for calculating the *volume of a prism.*

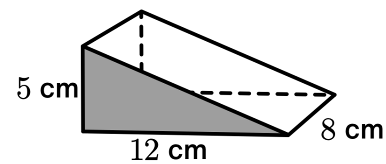
Responses vary.

- **Calculate the area of the base.**
- **Multiply that area by the height (the distance between the bases).**



2.1 Shade in a base of this prism.

2.2 Calculate the volume.
Show all of your calculations.



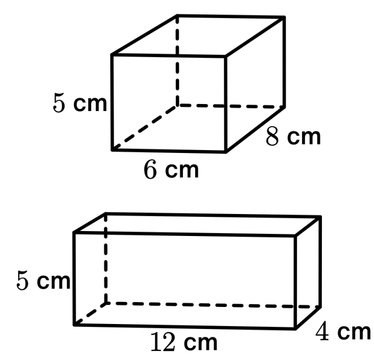
Volume = Base Area · Height

$$V = \frac{1}{2}(5 \cdot 12) \cdot 8$$

$V = 30 \cdot 8 = 240$ **cubic cm**

2.3 Sketch and label a **rectangular** prism with the same volume.

Responses vary.



Summary

- ☐ I can explain how the volume of a prism is related to the area of its base and its height.
- ☐ I can calculate the volume of rectangular and triangular prisms.

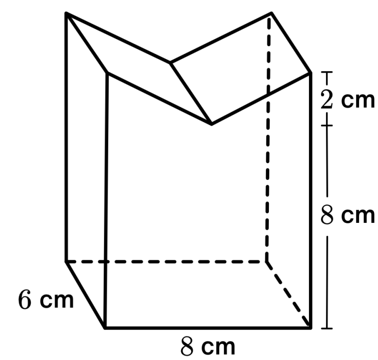
desmos

Unit 7.7, Lesson 11: Notes

Name ______________________________

My Notes

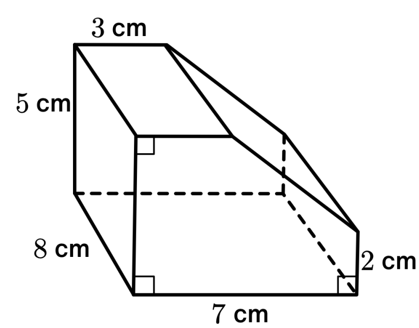
1.1 Sketch the base of this prism and label its dimensions.



1.2 What is the area of the base? Explain or show your reasoning.

1.3 What is the volume of the prism?

2. Use any strategy to calculate the volume of this prism. Show all of your thinking.



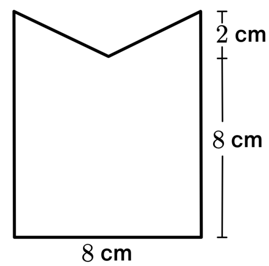
Summary

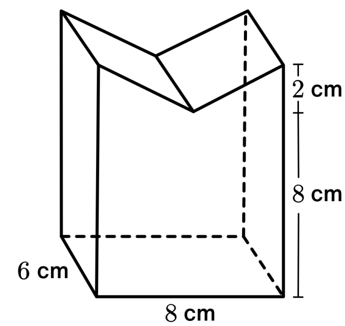
☐ I can calculate the volume of more complicated prisms.

Unit 7.7, Lesson 11: Notes Name ____________________

My Notes

1.1 Sketch the base of this prism and label its dimensions.





1.2 What is the area of the base? Explain or show your reasoning.

The base is made up of a square and two triangles.
The area of the square is $8 \cdot 8 = 64$ **square cm.**

The area of each triangle is $\frac{1}{2}(2 \cdot 4) = 4$ **square cm.**

In total, the area is $64 + 2(4) = 72$ **square cm.**

1.3 What is the volume of the prism?

Volume = Base Area · Height

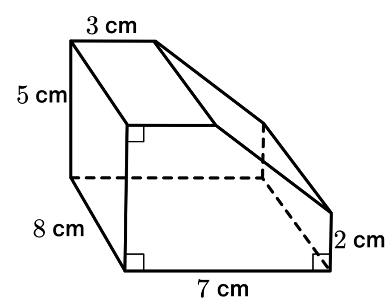
$V = 72 \cdot 6$
$V = 432$ **cubic cm**

2. Use any strategy to calculate the volume of this prism. Show all of your thinking.

Volume = Base Area · Height

Area = Rectangle – Triangle

$V = (7 \cdot 5 - 0.5 \cdot 4 \cdot 3)(8$
$V = (35 - 6)(8)$
$V = (29)(8)$
$V = 232$ **cubic cm**



Summary

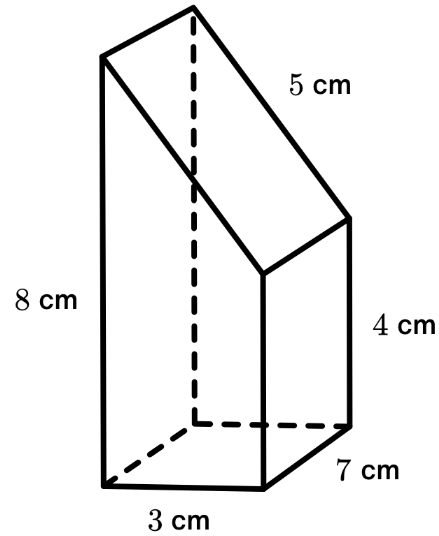
☐ I can calculate the volume of more complicated prisms.

desmos

Unit 7.7, Lesson 12: Notes Name ______________________________

My Notes

Here is a prism.



1.1 How many faces does this prism have?

1.2 Sketch and label one of the bases.

1.3 Calculate the surface area of your prism.

1.4 Explain a strategy for calculating the surface area of this prism.

Summary

- ☐ I can calculate the surface area of a prism.
- ☐ I can compare and contrast different strategies for calculating surface area.

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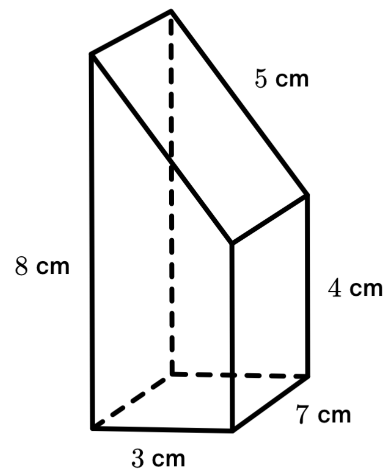
Unit 7.7, Lesson 12: Notes Name ______________________

My Notes

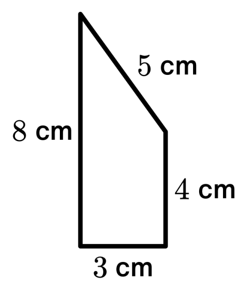
Here is a prism.



1.1 How many faces does this prism have?

6 **faces**

1.2 Sketch and label one of the bases.



1.3 Calculate the surface area of your prism. ***Strategies vary.***

- **SA** = $18 + 18 + 28 + 35 + 56 + 21 = 176$ **square units**
- **SA** = $2(18) + 1(28 + 35 + 56 + 21) = 176$ **square units**
- **SA** = $2(18) + 7(4 + 5 + 8 + 3) = 176$ **square units**

1.4 Explain a strategy for calculating the surface area of this prism. ***Responses vary.***

- **Calculate the area of each face and add them together. Make sure you include two copies of the base.**
- **Calculate the area of each shape and then multiply by how many of that shape there are.**
- **Calculate the area of the big rectangle that wraps aroun the shape and add that to the area of the bases.**

Summary

☐ I can calculate the surface area of a prism.

☐ I can compare and contrast different strategies for calculating surface area.

desmos

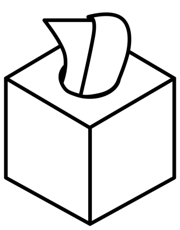
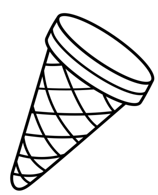
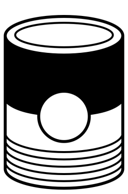
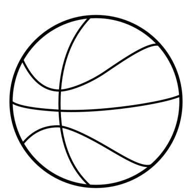
Unit 8.5, Lesson 10: Notes Name ______________________________

Exploring Volume

Learning Goal(s):

Definition	Facts/Characteristics
Volume	
Examples	Non-Examples

For each household object, name the 3-D solid it most resembles and a fact you learned today.

 Name: Fact:	 Name: Fact:
 Name: Fact:	 Name: Fact:

Summary Question
How would you describe volume to a 3rd grader?

desmos

Unit 8.5, Lesson 10: Notes Name ____________________

Exploring Volume

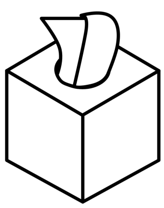
Learning Goal(s):

- **I recognize the following 3-D shapes: cylinder cone, rectangular prism, and sphere.**
- **I can estimate the volumes of different solids.**

Volume

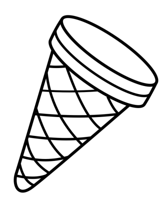
Definition	Facts/Characteristics
The volume is the number of cubic units that fill a three-dimensional region without any gaps or overlaps.	**It is a 3-D measurement (e.g., cm^3)**
Examples	**Non-Examples**
A pitcher holds 5 gallons of water. **500 beans inside a jar.**	**The amount of wrapping paper on a present.** **Number of sides of a cube.**

For each household object, name the 3-D solid it most resembles and a fact you learned today.



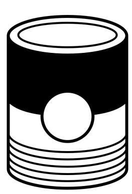
Name: **Cube**

Fact: **Has the largest volume for specific dimensions.**



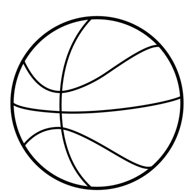
Name: **Cone**

Fact: **Base shape is the same as a cylinder but comes to a point.**



Name: **Cylinder**

Fact: **Base shape is a circle.**



Name: **Sphere**

Fact: **Only uses one measurement (radius) for all three dimensions.**

Summary Question

How would you describe volume to a 3rd grader?

Volume is the amount of space that a shape takes up. For example, with a cake, the volume is the amount of cake, not the amount of icing on top of the cake.

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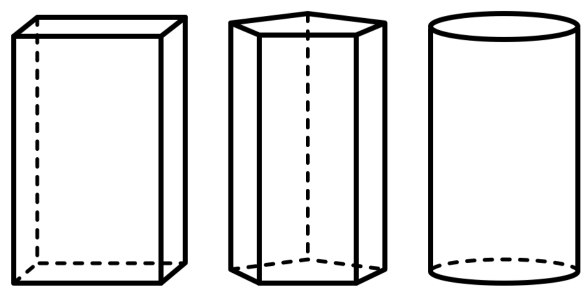
Unit 8.5, Lesson 11: Notes Name ______________________

The Volume of a Cylinder

Learning Goal(s):

Here is the formula for the volume of a prism.

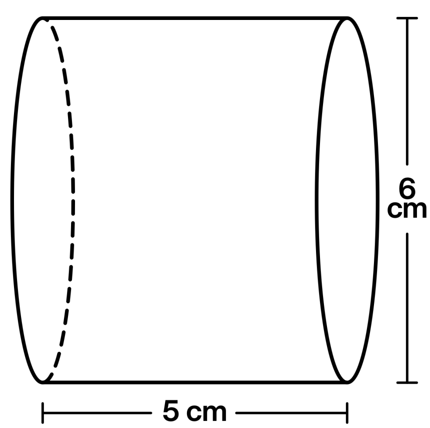
$V = Bh$



Explain what each of the variables mean.

Use these figures if it helps you explain your thinking.

Find the volume of the cylinder (exactly or rounded to the nearest tenth).



Summary Question

How is finding the volume of a cylinder like finding the volume of a prism?

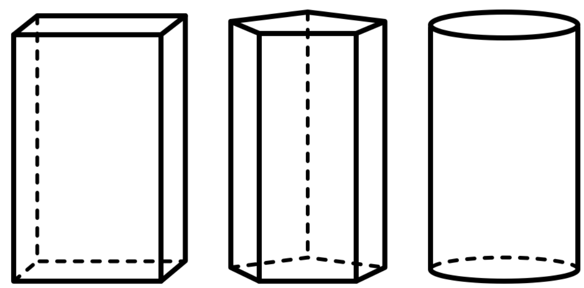
Unit 8.5, Lesson 11: Notes Name ____________________

The Volume of a Cylinder

Learning Goal(s):
- **I can explain the parts of the formula for the volume of a cylinder.**
- **I can calculate the volume of a cylinder.**

Here is the formula for the volume of a prism.

$V = Bh$

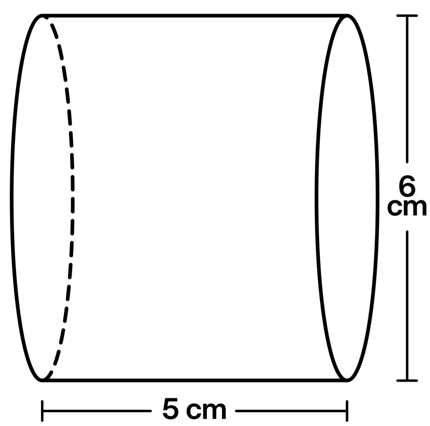


Explain what each of the variables mean.

Use these figures if it helps you explain your thinking.

V represents the volume of a prism, which is found by multiplying the area of the base of the prism, B, by the height of the prism, h.

Find the volume of the cylinder (exactly or rounded to the nearest tenth).



The base has an area of 9π cm^2 (since $\pi \cdot 3^2 = 9\pi$).
The volume is 45π cm^3 (since $9\pi \cdot 5 = 45\pi$).

Using 3.14 as an approximation for π, the volume of the cylinder is approximately 141.3 cm^3.

Summary Question
How is finding the volume of a cylinder like finding the volume of a prism?

Finding the volume of a cylinder is like finding the volume of a prism because they both involve finding the area of the base shape and multiplying that by the height.

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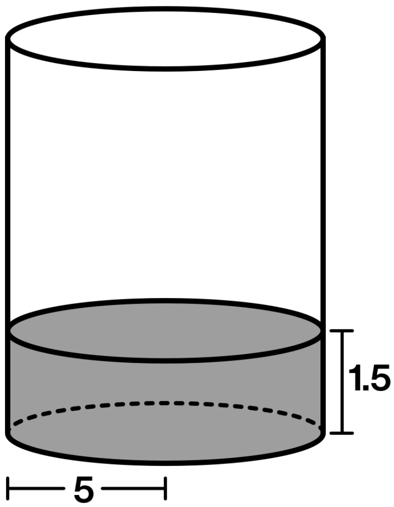
Unit 8.5, Lesson 12: Notes Name ____________________

Scaling Cylinders Using Functions

Learning Goal(s):

Imagine a water tank that is shaped like a cylinder.



If you triple the height of the water, will you triple the volume inside the container?

Yes No

Explain your thinking.

If you triple the radius of the water tank, will you triple the volume inside the container?

Yes No

Explain your thinking.

What are all of the ways you could change the water or the tank so that its volume is 4 times its current amount?

Summary Question

Why is the relationship between radius and volume non-linear?

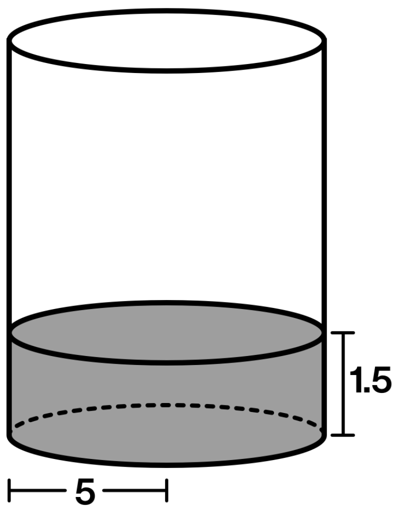
desmos

Unit 8.5, Lesson 12: Notes Name ___________________

Scaling Cylinders Using Functions

Learning Goal(s): • **I can analyze the relationship between the height or radius of a cylinder and its volume.** • **I can explain why the relationship between height and volume is linear but the relationship between radius and volume is not.**

Imagine a water tank that is shaped like a cylinder.

	If you triple the height of the water, will you triple the volume inside the container? **Yes** No Explain your thinking. **The relationship between the height and the volume of a cylinder is linear, so if you multiply the height by a scale factor, the volume will change by the same factor.**
	If you triple the radius of the water tank, will you triple the volume inside the container? Yes **No** Explain your thinking. **The relationship between the radius and the volume of a cylinder is not linear, so if you multiply the height by a scale factor, the volume will not change by the same scale factor.**
What are all of the ways you could change the water or the tank so that its volume is 4 times its current amount? **You could multiply the height of the water by 4 or double the radius of the tank.**	

Summary Question

Why is the relationship between radius and volume non-linear?

The radius of a cylinder represents 2 dimensions of the object (because the area of the circle involves r^2), so the relationship to volume is not linear. The height only represents 1 dimension of the object, so its relationship to volume is linear.

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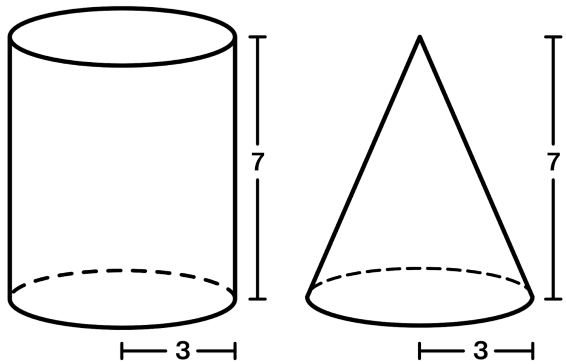
Unit 8.5, Lesson 13: Notes Name ____________________

Volumes of Cones

Learning Goal(s):



Find the volume of the cylinder above.	Find the volume of the cone above.
Sketch a cone. Label the diameter 8 units and the height 5 units.	Find the volume of the cone whose diameter is 8 units and height is 5 units.

Summary Question

How would you explain the relationship between the volume of a cone and the volume of a cylinder to a 3rd grader?

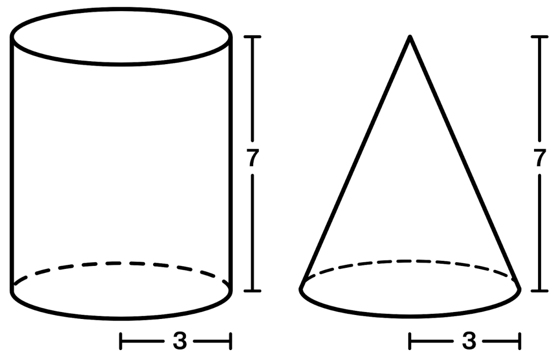
desmos

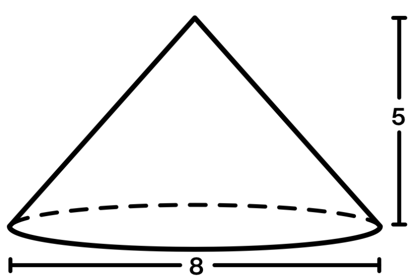
Unit 8.5, Lesson 13: Notes Name ____________________

Volumes of Cones

Learning Goal(s):

- **I can explain the relationship between the volume of a cone and the volume of a cylinder.**
- **I can use the formula for the volume of a cone.**



Find the volume of the cylinder above. **The volume of the cylinder is** 63π **cubic units. The area of the base is** $(3)^2 \cdot \pi = 9\pi$ **square units,** $9\pi \cdot 7 = 63\pi$**.**	Find the volume of the cone above. **The volume of the cone is** 21π **cubic units, which is** $\frac{1}{3}$ **the volume of a cylinder with the same radius and height.** $\frac{1}{3} \cdot 63\pi = 21\pi$**.**
Sketch a cone. Label the diameter 8 units and the height 5 units. 	Find the volume of the cone whose diameter is 8 units and height is 5 units. **The volume of the cone is** $\frac{80}{3}\pi$ **cubic units. This is** $\frac{1}{3}$ **the volume of a cylinder with the same radius and height. The volume of this cylinder is** $(4)^2 \cdot \pi \cdot 5 = 80\pi$**, so** $\frac{1}{3} \cdot 80\pi = \frac{80}{3}\pi$**.**

Summary Question

How would you explain the relationship between the volume of a cone and the volume of a cylinder to a 3rd grader?

If you filled a cone with water, you could pour 3 **cones full of water into** 1 **cylinder.**

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Unit 8.5, Lesson 14: Notes Name ________________________________

Finding Cylinder and Cone Dimensions

Learning Goal(s):

Use the space below to find the missing dimensions of each object (rounded to the nearest tenth). Use 3.14 as an approximation for π. Circle your answers.

1. A cylinder has a radius of 4 cm and a volume of 80π cm^3. What is the height of the cylinder?
2. A cylinder with a volume of 405 in.^{3} has a diameter of 10 in. What is its height?
3. A cone with a volume of 135π in.^{3} has a height of 5 in. What must the radius of the cone be?

Summary Question

How are the strategies for finding the missing dimensions of a cone and a cylinder . . .

. . . similar? . . . different?

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Unit 8.5, Lesson 14: Notes Name ____________________________________

Finding Cylinder and Cone Dimensions

Learning Goal(s):
- **I can find missing information about a cylinder or cone if I know its volume and other information.**
- **I can compare and contrast strategies for finding information about a cone or cylinder.**

Use the space below to find the missing dimensions of each object (rounded to the nearest tenth). Use 3.14 as an approximation for π. Circle your answers.

1. A cylinder has a radius of 4 cm and a volume of 80π cm^3. What is the height of the cylinder?

Radius	**Base Area**	**Height**	**Cylinder Volume**
4 cm	16π **cm**	$\frac{80\pi}{16\pi} = 5$ **cm**	80π cm^3

2. A cylinder with a volume of 405 in.^{3} has a diameter of 10 in. What is its height?

Diameter	**Radius**	**Base Area**	**Height**	**Cylinder Volume**
10 **in.**	$\frac{10}{2} = 5$ **in.**	25π **in.^{2}**	$\frac{405}{25\pi} \approx 5.2$ in.	405 in.^{3}

3. A cone with a volume of 135π in.^{3} has a height of 5 in. What must the radius of the cone be?

Radius	**Base Area**	**Height**	**Cylinder Volume**	**Cone Volume**
9 **in.**	$\frac{405\pi}{5} = 81\pi$ **in.**	5 in.	$135\pi \cdot 3 = 405\pi$ **in.^{3}**	135π in.^{3}

Summary Question

How are the strategies for finding the missing dimensions of a cone and a cylinder . . .

. . . similar? . . . different?

In order to find the missing dimensions of both figures, work backwards from the volume. The strategies are different because, in a cone, the area of the base multiplied by the height is divided by 3 to find the total volume. Therefore, it is important to consider the 3 when working backwards.

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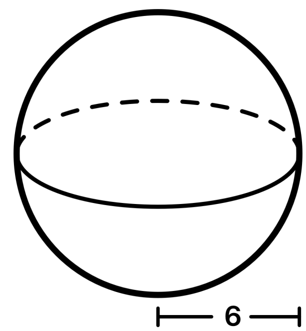
Unit 8.5, Lesson 15: Notes Name ______________________________

Volumes of Spheres

Learning Goal(s):

Darryl, Na'ilah, and Maia calculated the volume of the sphere on the right. Each of them made an error in their calculations. Identify their errors and explain what they might have been thinking.



Darryl: $V = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(36) = 48\pi$ in.^{3}	Maia: $V = \frac{4}{3}\pi(6)^3 = \frac{4}{3}(18.84)^3 \approx 8196$ in.^{3}
Na'ilah: $V = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(18) = 24\pi$ in.^{3}	Find the volume of the sphere.

Summary Question

What advice would you give a student to help them find the volume of a sphere?

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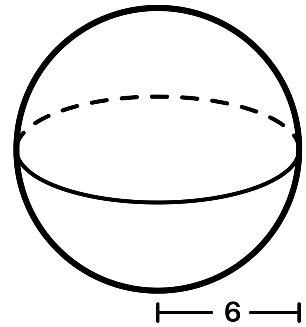
Unit 8.5, Lesson 15: Notes Name ______________________

Volumes of Spheres

Learning Goal(s): • **I can compare the volumes of a cone, a cylinder, a hemisphere, and a sphere.** • **I can find the volume of a sphere when I know the radius or the diameter.**

Darryl, Na'ilah, and Maia calculated the volume of the sphere on the right. Each of them made an error in their calculations. Identify the error and explain what they might have been thinking.



Darryl: $V = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(36) = 48\pi$ in.^{3} **Darryl found the value of** $(6)^2 = 6 \cdot 6$ **instead of** $(6)^3 = 6 \cdot 6 \cdot 6$**.**	Maia: $V = \frac{4}{3}\pi(6)^3 = \frac{4}{3}(18.84)^3 \approx 8196$ in.^{3} **Maia multiplied the value for** π **by** 6 **before using the exponent** $(\ )^3$**.**
Na'ilah: $V = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(18) = 24\pi$ in.^{3} **Na'ilah found the value of** $6 \cdot 3$**, not** $(6)^3$**.**	Find the volume of the sphere. **The volume of the sphere is** 288π **in.^{3}.** $(6)^3 \cdot \pi = 216\pi$ **and** $\frac{4}{3} \cdot 216\pi = 288\pi$**.**

Summary Question

What advice would you give a student to help them find the volume of a sphere?

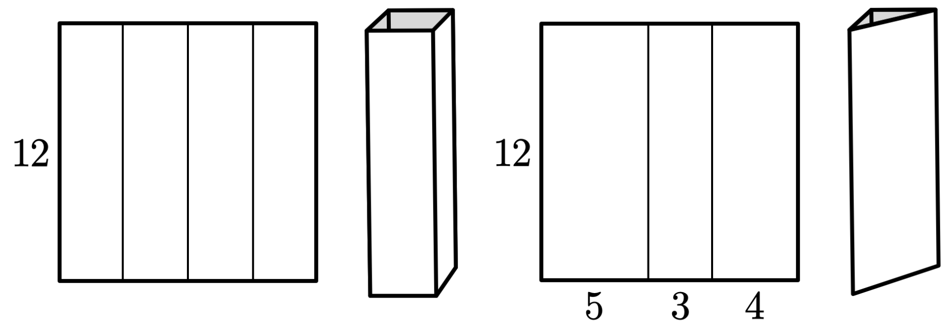
Make sure that you cube the radius first.
What it means to cube a number is to multiply the number by itself three times.

Unit 7.7, Lesson 13: Notes

Name ______________________________

My Notes

Imani is folding origami paper to make pencil holders for their room. They fold two holders: a square prism and a right triangular prism.



1. Which container holds more pencils? Calculate the amount each container can hold to support your claim.

2. Imani added a bottom and a top to each container. Which container do you think uses more paper? Calculate the amount of paper each container uses to support your claim.

Summary

- ☐ I can decide whether volume or surface area is more useful to answer a question about a situation.
- ☐ I can answer a question about a real-world situation using my knowledge of surface area and volume.

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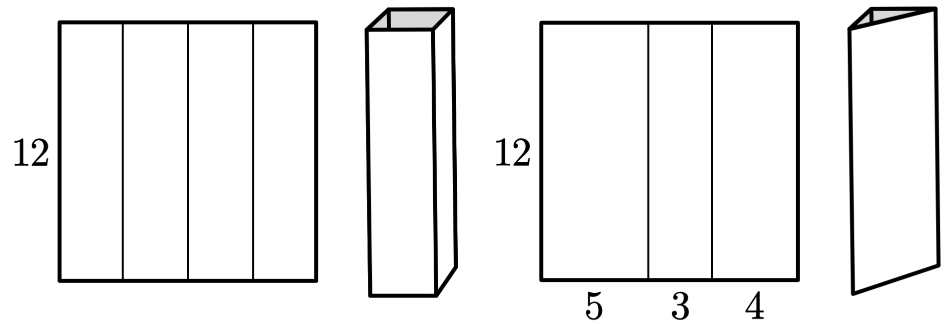
Unit 7.7, Lesson 13: Notes Name ____________________

My Notes

Imani is folding origami paper to make pencil holders for their room. They fold two holders: a square prism and a right triangular prism.



1. Which container holds more pencils? Calculate the amount each container can hold to support your claim.
   The square prism.

   Explanations vary. **The base of the square pencil holder is 3-by-3 inches, so the area of its base is 9 square inches. The area of the base of the triangular pencil holder is $0.5 \cdot 4 \cdot 3 = 6$ square inches. Since both pencil holders are the same height, the square pencil holder has a larger volume and can hold more pencils.**

2. Imani added a bottom and a top to each container. Which container do you think uses more paper? Calculate the amount of paper each container uses to support your claim.
   The square prism.

   Explanations vary. **The amount of paper used for the side faces of each prism are the same since they are different ways of folding the same size paper. This means that the only difference is the area of the base. We already know the area of the base of the square is larger than the triangle, so it also must use more paper.**

Summary

- ☐ I can decide whether volume or surface area is more useful to answer a question about a situation.
- ☐ I can answer a question about a real-world situation using my knowledge of surface area and volume.