Exponent and Scientific Notation
Student Guide

Math 7 Unit 8 Accelerated

Glossary

Term	Definition
base (of an exponent)	When numbers are written using exponents, the base is the larger bottom value, whereas the exponent is the smaller superscript to the top right of the base. The base number represents the factor being raised to a certain power (represented by the exponent value). For example, in the number 5^3, the base is 5 and the exponent is 3.
exponent	The value that a number or expression is raised to. When this value is a positive integer, it tells us how many times the number or expression is multiplied by itself.
power of ten	A number written as a power of ten means that it is in the form 10^n, where n represents the exponent of 10 needed to remain equivalent. For example, $10,000$ written as a power of ten is 10^4, since $10,000 = 10^4$.
scientific notation	Scientific notation is a way to write very large or very small numbers. We write these numbers by multiplying a number greater than or equal to 1, but less than 10 by a power of 10. For example, the number $425,000,000$ in scientific notation is 4.25×10^8. The number 0.0000000000783 in scientific notation is 7.83×10^{-11}.

Unit 7 Summary

Prior Learning	**Math 8, Unit 7**	**Future Learning**
Math 6 ● Operations with whole number exponents (e.g., using $V = s^3$)	● Exponent properties ● Scientific notation	Math 8, Unit 8 ● Square and cube roots High School ● Exponential and polynomial functions ● Rational exponents

Exponent Properties

$$8^6 = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$$

Base = 8

Exponent = 6

Power = 8^6

Exponents are a way of keeping track of how many times a number has been repeatedly multiplied.

Using our understanding of repeated multiplication, we can figure out several properties of exponents.

$$10^3 \cdot 10^4 = (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10$$

$$10^3 \cdot 10^4 = 10^7$$

Another way to multiply powers with the same base is to add their exponents together.

$$\left(7^2\right)^3 = (7 \cdot 7) \cdot (7 \cdot 7) \cdot (7 \cdot 7)$$

$$\left(7^2\right)^3 = 7^6$$

Another way to express powers of powers can be found by multiplying the exponents together.

$$10^{-3} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{10^3}$$

$$8^0 = 1$$

Negative exponents and zero exponents extend from the properties of positive exponents.

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Scientific Notation

The United States mint has made over $500,000,000,000$ pennies.

A single carbon atom weighs $0.00000000000000000000002$ grams.

The distance from Earth to the moon is $240,000$ miles.

$$500,000,000,000 = 5 \cdot 10^{11}$$

$$0.00000000000000000000002 = 2 \cdot 10^{-23}$$

Another way to write very large and very small numbers is as multiples of powers of 10.

Writing numbers in this way helps avoid errors since it would be easy to accidentally add or take away a zero when writing out the decimal.

$240\,000 = 2 \cdot 10^{5} + 4 \cdot 10^{4}$ or $24 \cdot 10^{4}$

Scientific notation: $2.4 \cdot 10^{5}$

Scientific notation is one specific way to write numbers.

Numbers in scientific notation are written as a number between 1 and 10 multiplied by a power of 10.

$$2.4 \cdot 10^{5} < 1.5 \cdot 10^{6}$$

It is more efficient to compare numbers when they are both written in this form.

Try This at Home

Exponent Properties

1.1 Carlos and Amara were trying to understand the expression $3^4 \cdot 3^5$. Amara said, "Since we are multiplying, we will get 3^{20}." Carlos said, "But I don't think you can get 3 twenty times by multiplying everything together." Do you agree with either of them?

1.2 Next, Carlos and Amara were thinking about the expression $(3^4)^5$. Amara said, "Okay, this one will be 3^{20} because you will have five groups of four 3s." Carlos said, "I agree it will be 3^{20}, but it's because there will be four groups of five 3s." Do you agree with either of them?

Scientific Notation

This table shows the top speeds of different vehicles.

Vehicle	Speed (kilometers per hour)
Sports car	$4.15 \cdot 10^2$
Apollo command and service module (Mother ship of the Apollo spacecraft)	$3.99 \cdot 10^4$
Jet boat	$5.1 \cdot 10^2$
Autonomous drone	$2.1 \cdot 10^4$

2.1 Order the vehicles from fastest to slowest.

2.2 The top speed of a rocket sled is $10,326$ kilometers per hour. Is this faster or slower than the autonomous drone?

2.3 Estimate how many times as fast the Apollo command and service module is as the sports car.

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Solutions:

1.1 Carlos is correct. Rewriting $3^4 \cdot 3^5$ to show all the factors looks like:

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

We can see that there are a total of 3s multiplied together nine times. This helps us understand what's going on when we use the rule to write $3^4 \cdot 3^5 = 3^{4+5} = 3^9$.

1.2 This time, Amara is correct. When we look at $(3^4)^5$, the outside exponent of 5 tells us that there are five factors of 3^4 being multiplied together. So $(3^4)^5 = 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4$. We could write this out the long way as:

$$(3^4)^5 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3).$$

This helps us understand what's going on when we use the rule to write $(3^4)^5 = 3^{4 \cdot 5} = 3^{20}$.

2.1 The order from fastest to slowest is:

- Apollo CSM
- Autonomous drone
- Jet boat
- Sports car

Since all of these values are in scientific notation, we can look at the power of 10 to compare. Both the speeds of the Apollo CSM and the autonomous drone have the highest power of ten , 10^4, so they are the fastest. The Apollo CSM is faster than the drone because 3.99 is greater than 2.1. Similarly, the jet boat is faster than the sports car because 5.1 is greater than 4.15, even if their speeds both have the same power of ten, 10^2 .

2.2 The drone is faster than the rocket sled. One approach is to convert the rocket sled's speed into scientific notation. $10,326$ is equivalent to $1032.6 \cdot 10$, which is equivalent to $103.26 \cdot 10 \cdot 10$. By continuing that process, we can determine that the rocket sled's speed is $1.0326 \cdot 10^4$. The drone's speed is $2.1 \cdot 10^4$ kilometers per hour and 2.1 is greater than 1.0326, so the drone must be faster.

2.3 To compare the speeds of the Apollo CSM and the sports car, we can try to find the missing number in this equation: $? \cdot 4.15 \cdot 10^2 = 3.99 \cdot 10^4$. To find the missing value (?), we need to compute $\frac{3.99 \cdot 10^4}{4.15 \cdot 10^2}$. Since we are estimating, we can simplify the calculation to $\frac{4 \cdot 10^4}{4 \cdot 10^2}$.

Using properties of exponents and our understanding of fractions, we can conclude that $\frac{4 \cdot 10^4}{4 \cdot 10^2} = 1 \cdot 10^2$, so the Apollo CSM is about 100 times as fast as the sports car!

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Unit 8.7, Lesson 1: Notes Name ________________________________

Learning Goal(s):

Exponents make it easy to show repeated multiplication. It is easier to write 2^6 than to write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. Imagine writing 2^{100} using multiplication!

For each expression below, write an equivalent expression that uses exponents:

A. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$	B. $5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 \cdot 8 \cdot 5$	C. $10 \cdot 10 \cdot 10 + 10 \cdot 10$

Consider this situation: Each day, the number of grains of rice you have triples. On day one, you have 3 grains of rice. On day two, you have 9 grains of rice.

- On what day will you have 243 grains of rice?

- On what day will you have 3^{13} grains of rice?

- How many grains of rice will you have *two days after* you have 3^{13} grains of rice?

Summary Question

When is it useful to express a number or expression with exponents?

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Unit 8.7, Lesson 1: Notes Name ______________________________

Learning Goal(s):
• **I can use exponents to describe repeated multiplication.** • **I can explain the meaning of an expression with an exponent.**

Exponents make it easy to show repeated multiplication. It is easier to write 2^6 than to write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. Imagine writing 2^{100} using multiplication!

For each expression below, write an equivalent expression that uses exponents:

A. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$	B. $5 \cdot 8 \cdot 5 \cdot 8 \cdot 5 \cdot 8 \cdot 5$	C. $10 \cdot 10 \cdot 10 + 10 \cdot 10$
7^5	$5^4 \cdot 8^3$ **or equivalent**	$10^3 + 10^2$ **or equivalent**

Consider this situation: Each day, the number of grains of rice you have triples. On day one, you have 3 grains of rice. On day two, you have 9 grains of rice.

- On what day will you have 243 grains of rice?

Day five. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$**, so you will have** 243 **grains of rice on day five.**

- On what day will you have 3^{13} grains of rice?

Day thirteen. Day one had 3^1 **grains of rice. Day two had** 3^2 **grains of rice. Continuing the pattern, the day that will have** 3^{13} **grains of rice will be day thirteen.**

- How many grains of rice will you have *two days after* you have 3^{13} grains of rice?

3^{15} **grains of rice. This is** 9 **times more than** 3^{13}**.**

Summary Question
When is it useful to express a number or expression with exponents?

Responses vary. **Exponents are especially useful when we have an expression with a lot of repeated multiplication.**

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Unit 8.7, Lesson 2: Notes Name ________________________________

Learning Goal(s):

Sometimes writing an expression in an equivalent way can help us compare it to other expressions. The fact that exponents represent repeated multiplication can help us write equivalent expressions.

Decide if Expression 1 is equivalent to Expression 2 for each pair. Consider "expanding" each expression, as shown in Pair A.

	Expression 1	Expression 2	Equivalent?
Pair A	$\left(12^2\right)^3$ $(12 \cdot 12)(12 \cdot 12)(12 \cdot 12)$	$12^4 \cdot 12^2$ $(12 \cdot 12 \cdot 12 \cdot 12)(12 \cdot 12)$	YES NO
Pair B	$7^3 \cdot 2^3$	$(7 \cdot 2)^3$	YES NO
Pair C	$16^3 + 16^2 + 16$	16^6	YES NO
Pair D	15^6	$(5 \cdot 3 \cdot 3 \cdot 5)^4$	YES NO

Summary Question

Show or explain why $6^5 \cdot 6^3$ is equivalent to $\left(6^4\right)^2$. Then write another expression that is equivalent to both of them.

Learning Goal(s):
- **I can describe what it means for two expressions with exponents to be equivalent.**
- **I can create equivalent expressions with exponents.**

Sometimes writing an expression in an equivalent way can help us compare it to other expressions. The fact that exponents represent repeated multiplication can help us write equivalent expressions.

Decide if Expression 1 is equivalent to Expression 2 for each pair. Consider "expanding" each expression, as shown in Pair A.

	Expression 1	Expression 2	Equivalent?
Pair A	$\left(12^2\right)^3$ $(12 \cdot 12)(12 \cdot 12)(12 \cdot 12)$	$12^4 \cdot 12^2$ $(12 \cdot 12 \cdot 12 \cdot 12)(12 \cdot 12)$	**YES** NO
Pair B	$7^3 \cdot 2^3$ $7 \cdot 7 \cdot 7 \cdot 2 \cdot 2 \cdot 2$	$(7 \cdot 2)^3$ $(7 \cdot 2)\cdot(7 \cdot 2)\cdot(7 \cdot 2)$	**YES** NO
Pair C	$16^3 + 16^2 + 16$ $16 \cdot 16 \cdot 16 + 16 \cdot 16 + 16$	16^6 $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$	YES **NO**
Pair D	15^6 $15 \cdot 15 \cdot 15 \cdot 15 \cdot 15 \cdot 15$	$(5 \cdot 3 \cdot 3 \cdot 5)^4$ $(15 \cdot 15)(15 \cdot 15)(15 \cdot 15)(15$	YES **NO**

Summary Question

Show or explain why $6^5 \cdot 6^3$ is equivalent to $\left(6^4\right)^2$. Then write another expression that is equivalent to both of them.

Responses vary. $6^5 \cdot 6^3$ **is eight factors of** 6**. The expression** $\left(6^4\right)^2$ **is also eight factors of** 6**.**

Another equivalent expression is: $\left(6^2\right)^4$**.**

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Unit 8.7, Lesson 3: Notes Name ______________________

Learning Goal(s):

Sometimes, we want to investigate whether two expressions are equivalent. In those instances, it can be helpful to convert between exponents and repeated multiplication.

For each pair, decide if Expression 1 is equivalent to Expression 2.

	Expression 1	**Expression 2**	**Equivalent?**
Pair A	$(5^5)^2$	$5^4 \cdot 5^3$	YES NO
Pair B	$4^3 \cdot 2^5$	8^8	YES NO
Pair C	$15^3 \cdot 2^3$	$(5 \cdot 2)^3 \cdot 3^3$	YES NO

Decide whether each expression below is equivalent to 10^6. For any that are not, change the expression so that it *is* equivalent to 10^6.

A. $10 \cdot 10^3 \cdot 10^2$	B. 100^5	C. $10^3 + 10^3$	D. $(10^2)^3$

Summary Question

What are some important things to remember when determining whether expressions with exponents are equivalent?

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Unit 8.7, Lesson 3: Notes Name ____________________

Learning Goal(s):
- **I can explain why two expressions with exponents are equivalent.**

Sometimes, we want to investigate whether two expressions are equivalent. In those instances, it can be helpful to convert between exponents and repeated multiplication.

For each pair, decide if Expression 1 is equivalent to Expression 2.

	Expression 1	**Expression 2**	**Equivalent?**
Pair A	$(5^5)^2$ $(5^5)\cdot(5^5)$	$5^4\cdot 5^3$ $(5\cdot 5\cdot 5\cdot 5)\cdot(5\cdot 5\cdot 5)$	YES **NO**
Pair B	$4^3\cdot 2^5$ $(4\cdot 4\cdot 4)\cdot(2\cdot 2\cdot 2\cdot 2\cdot 2)$ $(4\cdot 2)(4\cdot 2)(4\cdot 2)\cdot 2\cdot 2$	8^8 $8\cdot 8\cdot 8\cdot 8\cdot 8\cdot 8\cdot 8\cdot 8$	YES **NO**
Pair C	$15^3\cdot 2^3$ 30^3	$(5\cdot 2)^3\cdot 3^3$ 30^3	**YES** NO

Decide whether each expression below is equivalent to 10^6. For any that are not, change the expression so that it *is* equivalent to 10^6.

A. $10\cdot 10^3\cdot 10^2$ **Equivalent**	B. 100^5 **NOT equivalent** **Equivalent:** 100^3	C. 10^3+10^3 **NOT equivalent** **Equivalent:** $10^3\cdot 10^3$	D. $(10^2)^3$ **Equivalent**

Summary Question
What are some important things to remember when determining whether expressions with exponents are equivalent?

***Responses vary*. It's important to remember that I can rewrite a number in different ways. For instance, sometimes it's helpful to write out all the factors of a number to see how it compares to a different number.**

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Unit 8.7, Lesson 4: Notes Name ______________________________

Learning Goal(s):

Expressions that have a single base and a single exponent (like 7^3) are sometimes preferable to expressions with more parts because they can help us easily compare numbers to each other.

For each expression below, fill in the blanks. The first row has been done for you.

Expression	Expanded Expression	Single Power
$\left(12^2\right)^3$	$(12 \cdot 12)(12 \cdot 12)(12 \cdot 12)$	12^6
A. $\frac{6^5 \cdot 6^2}{6^4}$		
B. $7^3 \cdot 2^3$		
C. $\frac{\left(3^3\right)^2}{3^4}$		
D. $\frac{9^2 \cdot 3^5}{3^3}$		

Which of the four above expressions (A, B, C, or D) is greatest? Explain your reasoning.

Summary Question

Describe a strategy for rewriting an expression like $\frac{\left(6^{30}\right)^3}{6^{40}}$ as a single power.

Learning Goal(s):
- **I can divide expressions with exponents that have the same base.**
- **I can rewrite expressions with positive exponents as a single power.**

Expressions that have a single base and single exponent (like 7^3) are sometimes preferable to expressions with more parts because they can help us easily compare numbers to each other.

For each expression below, fill in the blanks. The first row has been done for you.

Expression	Expanded Expression	Single Power
$(12^2)^3$	$(12 \cdot 12)(12 \cdot 12)(12 \cdot 12)$	12^6
A. $\frac{6^5 \cdot 6^2}{6^4}$	$\frac{(6 \cdot 6 \cdot 6 \cdot 6 \cdot 6) \cdot (6 \cdot 6)}{6 \cdot 6 \cdot 6 \cdot 6}$	6^3
B. $7^3 \cdot 2^3$	$7 \cdot 7 \cdot 7 \cdot 2 \cdot 2 \cdot 2 \rightarrow 7 \cdot 2 \cdot 7 \cdot 2 \cdot 7 \cdot 2$	14^3
C. $\frac{(3^3)^2}{3^4}$	$\frac{(3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3 \cdot 3}$	3^2
D. $\frac{9^2 \cdot 3^5}{3^3}$	$\frac{(9 \cdot 9) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3} \rightarrow \frac{(3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)}{3 \cdot 3 \cdot 3}$	3^6

Which of the four above expressions (A, B, C, or D) is greatest? Explain your reasoning.

B is greatest of the four. *Responses vary*. A cannot be the greatest expression because $6^3 < 14^3$. C cannot be the greatest expression because $3^2 < 3^6$. I know I can rewrite 3^6 as 9^3, which helps me see that $9^3 < 14^3$.

Summary Question

Describe a strategy for rewriting an expression like $\frac{(6^{30})^3}{6^{40}}$ as a single power.

***Responses vary*. If the numbers are small, I like to write out every factor of an expression. Then I reduce factors that are in both the numerator and denominator. If the numbers are large, like in this expression, I *imagine* writing out every factor. E.g., 30 factors of 6 three times will make 90 factors of 6.**

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Unit 8.7, Lesson 5: Notes Name ____________________

Learning Goal(s):

Our concept of "exponents as repeated multiplication" is less helpful when the exponent is zero or a negative number. Patterns can help us discover what zero or negative numbers mean as exponents.

Powers of 8		
8^3	$1 \cdot 8 \cdot 8 \cdot 8$	512
8^2	$1 \cdot 8 \cdot 8$	64
8^1	$1 \cdot 8$	8
8^0	1	1
8^{-1}	$1 \div 8$	$\frac{1}{8}$
8^{-2}	$1 \div 8 \div 8$	$\frac{1}{8^2}$ or $\frac{1}{64}$
8^{-3}	$1 \div 8 \div 8 \div 8$	$\frac{1}{8^3}$ or $\frac{1}{512}$

Examine the **Powers of 8** table. How do the numbers change as you look *down* the table from 8^3 to 8^2 to 8^1?

Based on the patterns in the table, what is another way to represent 8^{-5}?

Why does it make sense that $8^0 = 1$?

Write each expression as a single power:

A. $\frac{7^4 \cdot 7^{-2}}{7^{12}}$

B. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

C. $\frac{2^{-4}}{\left(2^{-5}\right)^2}$

Summary Questions

What is the relationship between 10^5 and 10^{-5}?

What is the value of $10^5 \cdot 10^{-5}$?

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Unit 8.7, Lesson 5: Notes Name ____________________

Learning Goal(s):
- **I can explain what it means for a number to be raised to a zero or a negative exponent.**
- **I can determine if two expressions with positive, zero, and negative exponents are equivalent.**

Our concept of "exponents as repeated multiplication" is less helpful when the exponent is zero or a negative number. Patterns can help us discover what zero or negative numbers mean as exponents.

Powers of 8		
8^3	$1 \cdot 8 \cdot 8 \cdot 8$	512
8^2	$1 \cdot 8 \cdot 8$	64
8^1	$1 \cdot 8$	8
8^0	1	1
8^{-1}	$1 \div 8$	$\frac{1}{8}$
8^{-2}	$1 \div 8 \div 8$	$\frac{1}{8^2}$ or $\frac{1}{64}$
8^{-3}	$1 \div 8 \div 8 \div 8$	$\frac{1}{8^3}$ or $\frac{1}{512}$

Examine the **Powers of 8** table. How do the numbers change as you look *down* the table from 8^3 to 8^2 to 8^1?

As we go down the table, each number is one-eighth the previous number.

Based on the patterns in the table, what is another way to represent 8^{-5}?

$\frac{1}{8^5}$ **(or equivalent)**

Why does it make sense that $8^0 = 1$?

***Responses vary*. If we continue the pattern of dividing by 8, then the row below $8^1 = 8$ should say $8^0 = 1$.**

Write each expression as a single power:

A. $\frac{7^4 \cdot 7^{-2}}{7^{12}}$

7^{-10}

B. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

5^{-3} **or** $\left(\frac{1}{5}\right)^3$

C. $\frac{2^{-4}}{\left(2^{-5}\right)^2}$

2^6

Summary Questions

What is the relationship between 10^5 and 10^{-5}?

***Responses vary.* The two numbers are reciprocals. 10^5 is 1 multiplied by 10 five times, while 10^{-5} is 1 divided by 10 five times.**

What is the value of $10^5 \cdot 10^{-5}$?

$10^5 \cdot 10^{-5}$ is the same as $10^5 \cdot \frac{1}{10^5}$, which is $\frac{10^5}{10^5}$, which is 1.

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Unit 8.7, Lesson 6: Notes

Name ________________________________

Learning Goal(s):

Patterns emerge when we rewrite expressions with exponents. We can generalize these patterns into exponent rules.

Fill in the blanks. Then write why the rule makes sense.

Symbolic Rule	Example	Why It Makes Sense
$x^m \cdot x^n = x^{m+n}$	$8^5 \cdot 8^2 = 8^7$	Both sides of the equal sign have seven factors of 8.
$\left(x^m\right)^n = \left(x^n\right)^m = x^{m \cdot n}$	$\left(11^2\right)^3 = \left(11^3\right)^2 = 11^6$	
	$6^3 \cdot 5^3 = 30^3$	
$\frac{x^m}{x^n} = x^{m-n}$		
$x^{-n} = \frac{1}{x^n}$		
	$188^0 = 1$	

Summary Question

Explain why $15^{10} \cdot 2^{13}$ is equivalent to $30^{10} \cdot 2^3$.

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Unit 8.7, Lesson 6: Notes Name ____________________

Learning Goal(s):
- **I can explain and use rules for properties of exponents.**

Patterns emerge when we rewrite expressions with exponents. We can generalize these patterns into widely-applicable exponent rules.

Fill in the blanks. Then write why the rule makes sense.

Symbolic Rule	Example	Why It Makes Sense
$x^m \cdot x^n = x^{m+n}$	$8^5 \cdot 8^2 = 8^7$	Both sides of the equal sign have seven factors of 8.
$\left(x^m\right)^n = \left(x^n\right)^m = x^{m \cdot n}$	$\left(11^2\right)^3 = \left(11^3\right)^2 = 11^6$	***Responses vary*. Three groups of two factors of 11 makes six factors of 11. Two groups of three factors of 11 also makes six factors of 11.**
$a^x \cdot b^x = (a \cdot b)^x$	$6^3 \cdot 5^3 = 30^3$	***Responses vary*. I can rearrange the 6s and 5s to group them as 30s, and then count them.**
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{10^9}{10^7} = 10^2$	***Responses vary*. Seven 10s in the denominator and seven 10s in the numerator can reduce to 1, leaving only *two* factors of 10.**
$x^{-n} = \frac{1}{x^n}$	$4^{-3} = \frac{1}{4^3}$	***Responses vary*. If positive exponents signify repeated multiplication, negative exponents signify repeated division.**
$x^0 = 1$	$188^0 = 1$	***Responses vary*. I know that $\frac{x^3}{x^3} = x^0$ from a different rule and also $\frac{x^3}{x^3} = 1$. Therefore, $x^0 = 1$.**

Summary Question

Explain why $15^{10} \cdot 2^{13}$ is equivalent to $30^{10} \cdot 2^3$.

$15^{10} \cdot 2^{13}$ can be rewritten as $15^{10} \cdot 2^{10} \cdot 2^3$. I also know that ten factors of 15 multiplied by ten factors of 2 is equivalent to ten factors of 30.

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Unit 8.7, Lesson 7: Notes Name ______________________________

Learning Goal(s):

The United States Mint has made over $500,000,000,000$ pennies. Exactly how many pennies is that? One way to make sense of that number is by considering how many thousands, millions, or billions of pennies that is. Another way of making sense is to rewrite it using powers of 10.

Number	In Billions	In Millions	In Thousands	Rewrite as a Multiple of a Power of 10
$500,000,000,000$	______ billion (10^9)			
$500,000,000,000$		______ million (10^6)		
$500,000,000,000$			______ thousand (10^3)	

Write two different expressions that represent the weight of the object using a power of ten.

Object and Weight	Expression #1	Expression #2
Bus: $7,810$ **kg**	$781 \cdot 10^1$	
Ship: $4,850,000$**kg**		
Cell Phone: 0.13 **kg**		

Summary Question

What does it mean to write a number using a single multiple of a power of 10?

Unit 8.7, Lesson 7: Notes Name ______________________________

Learning Goal(s):

- **I can represent large and small numbers as multiples of powers of 10.**

The United States Mint has made over $500,000,000,000$ pennies. Exactly how many pennies is that? One way to make sense of that number is by considering how many thousands, millions, or billions of pennies that is. Another way of making sense is to rewrite it using powers of 10.

Number	**In Billions**	**In Millions**	**In Thousands**	**Rewrite as a Multiple of a Power of 10**
$500,000,000,000$	500 billion (10^9)			$500 \cdot 10^9$
$500,000,000,000$		$500,000$ million (10^6)		$500,000 \cdot 10^6$
$500,000,000,000$			$500,000,000$ thousand (10^3)	$500,000,000 \cdot 10^3$

Write two different expressions that represent the weight of the object using a power of ten.

Object and Weight	**Expression #1**	**Expression #2**
Bus: $7,810$ **kg**	$781 \cdot 10^1$	$7.81 \cdot 10^3$
Ship: $4,850,000$ **kg**	$485 \cdot 10^4$	$4.85 \cdot 10^6$
Cell Phone: 0.13 **kg**	$13 \cdot 10^{-2}$	$1.3 \cdot 10^{-1}$

Summary Question

What does it mean to write a number using a single multiple of a power of 10?

Responses vary. **It means that I multiply a first factor by a second factor that is 10 raised to a power. When I multiply the first and second factor, I will get the original number.**

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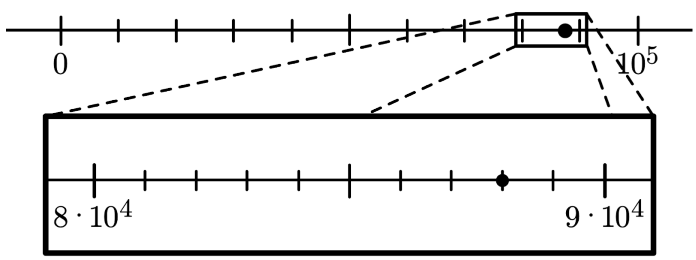
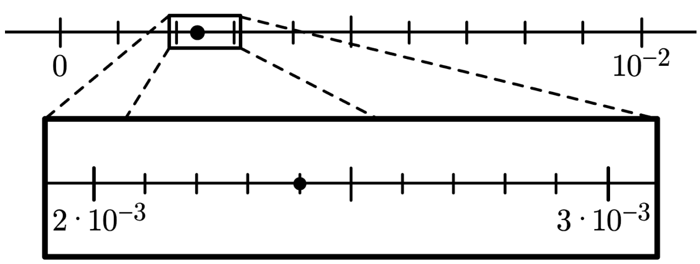
Unit 8.7, Lesson 8: Notes Name ______________________________

Learning Goal(s):

For each example below, write the number shown on the number line diagram.

<table>
<tr><td rowspan="2"></td><td>Write the number shown on the number line diagram.</td></tr>
<tr><td>What is another way to write this number?</td></tr>
<tr><td rowspan="2"></td><td>Write the number shown on the number line diagram.</td></tr>
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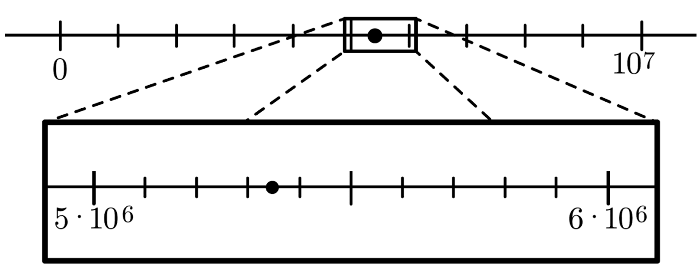
<tr><td rowspan="2"></td><td>Write the number shown on the number line diagram.</td></tr>
<tr><td>What is another way to write this number?</td></tr>
</table>

Summary Question

When a number is given as a multiple of a power of 10, what is a strategy for writing an equivalent number?

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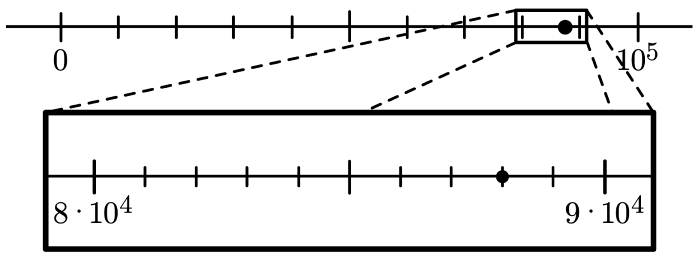
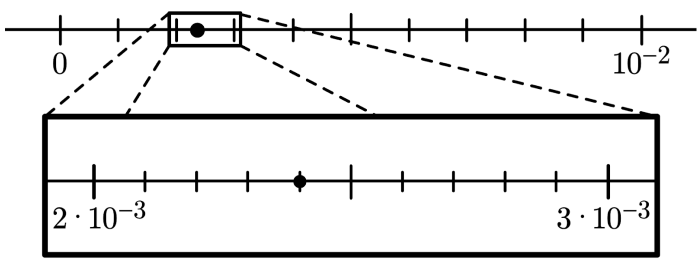
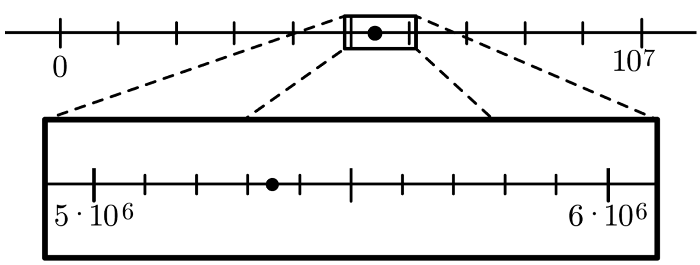
Unit 8.7, Lesson 8: Notes Name ____________________________

Learning Goal(s): • **I can represent large and small numbers as multiples of powers of 10 using number lines.**

For each example below, write the number shown on the number line diagram.

	Write the number shown on the number line diagram. $8.8 \cdot 10^4$ What is another way to write this number? $88,000$ **or equivalent**
	Write the number shown on the number line diagram. $2.4 \cdot 10^{-3}$ What is another way to write this number? $\frac{24}{10,000}$ **or equivalent**
	Write the number shown on the number line diagram. $5.35 \cdot 10^6$ What is another way to write this number? $5,350,000$ **or equivalent**

Summary Question

When a number is given as a multiple of a power of 10, what is a strategy for writing an equivalent number?

***Responses vary.* You can use repeated multiplication to write out the expression, and then multiply all of the factors to find the number in standard form.**

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Unit 8.7, Lesson 9: Notes Name ______________________

Learning Goal(s):

Powers of 10 and exponent rules can be helpful for making calculations with large or small numbers. The table below shows the number of people in the United States in 2014 and how much total oil they used for energy.

	Estimated Amount	**Write Using a Power of 10**
Population of United States in 2014	$300{,}000{,}000$ people	
Total Oil Used	$2{,}000{,}000{,}000{,}000$ kilograms	

Approximately how many kilograms of oil did the average person in the United States use in 2014?

The table shows the total number of creatures as well as the approximate masses of each creature.

Creature	**Total**	**Mass of One Individual (kg)**
Humans	$7.5 \cdot 10^9$	$6 \cdot 10^1$
Ants	$5 \cdot 10^{16}$	$3 \cdot 10^{-6}$

Which is more massive: the total mass of all humans or the total mass of all the ants? About how many times more massive is it?

Summary Question

If you have two very large numbers, how can you tell which is larger?

 Name ______________________

Learning Goal(s):

- **I can apply what I learned about powers of 10 to answer questions about real-world situations.**

Powers of 10 and exponent rules can be helpful for making calculations with large or small numbers. The table below shows the number of people in the United States in 2014 and how much total oil they used for energy.

	Estimated Amount	**Write Using a Power of 10**
Population of United States in 2014	$300,000,000$ people	$3 \cdot 10^8$
Total Oil Used	$2,000,000,000,000$ kilograms	$2 \cdot 10^{12}$

Approximately how many kilograms of oil did the average person in the United States use in 2014?

$\frac{2 \cdot 10^{12}}{3 \cdot 10^8} = \frac{2}{3} \cdot 10^4 \approx 0.66 \cdot 10^4$**,** **which is about $6,600$ kilograms of oil per person on average.**

The table shows the total number of creatures as well as the approximate masses of each creature.

Creature	**Total**	**Mass of One Individual (kg)**
Humans	$7.5 \cdot 10^9$	$6 \cdot 10^1$
Ants	$5 \cdot 10^{16}$	$3 \cdot 10^{-6}$

Which is more massive: the total mass of all humans or the total mass of all the ants? About how many times more massive is it?

Total Mass of Humans:

$(7.5 \cdot 10^9)(6 \cdot 10^1)$
$= 45 \cdot 10^{10}$ **kilograms**

Total Mass of Ants:

$(5 \cdot 10^{16})(3 \cdot 10^{-6})$
$= 15 \cdot 10^{10}$ **kilograms**

How Many Times More Massive?

$\frac{45 \cdot 10^{10}}{15 \cdot 10^{10}} \approx 3$

The total mass of all humans is $45 \cdot 10^{10}$ kilograms. That's 3 times as much as the total mass of all the ants, which is $15 \cdot 10^{10}$ kilograms.

Summary Question

If you have two very large numbers, how can you tell which is larger?

Responses vary. **If you rewrite the numbers using the same power of 10, the number with the larger first factor is larger.**

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Unit 8.7, Lesson 10: Notes Name ______________________________

Learning Goal(s):

Definition	Facts/Characteristics
Scientific Notation	
Examples	Non-Examples

Write each number using scientific notation, or say if it is already written using scientific notation.

Number	**Scientific Notation**
$540,000$	
0.003	
$6.8 \cdot 10^9$	
$12 \cdot 10^{-2}$	
$97 \cdot 10^5$	

Summary Question

What is important to pay attention to when writing a number in scientific notation?

Unit 8.7, Lesson 10: Notes Name ______________________________

Learning Goal(s):

- **I can tell whether or not a number is written in scientific notation.**
- **I can rewrite a large or small number using scientific notation.**

Scientific Notation

Definition	Facts/Characteristics
Scientific notation is a way to write numbers (often very large or very small ones). We write them as a number between 1 and 10 multiplied by a power of 10.	- Scientific notation can help us compare very large or very small numbers. - The power of 10 has to be an integer.
Examples	**Non-Examples**
$5.23 \cdot 10^8$ $8.7 \cdot 10^{12}$ $3 \cdot 10^2$	$48\,200$ $36 \cdot 10^5$ $0.781 \cdot 10^3$

Write each number using scientific notation, or say if it is already written using scientific notation.

Number	Scientific Notation
$540{,}000$	$5.4 \cdot 10^5$
0.003	$3 \cdot 10^{-3}$
$6.8 \cdot 10^9$	**Already in scientific notation**
$12 \cdot 10^{-2}$	$1.2 \cdot 10^{-1}$
$97 \cdot 10^5$	$9.7 \cdot 10^6$

Summary Question

What is important to pay attention to when writing a number in scientific notation?

Responses vary. **Make sure the first factor is greater than or equal to 1 and less than 10. The second factor should be an integer power of 10.**

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Unit 8.7, Lesson 11: Notes Name ______________________________

Learning Goal(s):

Comparing the relative sizes of very large or very small numbers is easier with scientific notation. The table shows the total numbers of humans and ants.

	Approximate Number	Scientific Notation
Humans	$7,500,000,000$	
Ants	$50,000,000,000,000,00($	

About how many ants are there for every human?

Ants weigh about $3 \cdot 10^{-6}$ kilograms each. Humans weigh about $6.2 \cdot 10^{1}$ kilograms each. About how many ants weigh the same as one human?

There are about $3.9 \cdot 10^{7}$ residents in California. The average Californian uses about 180 gallons of water per day. About how many gallons of water total do Californians use in a day?

Summary Question

Describe a strategy you used to divide two numbers given in scientific notation.

Learning Goal(s):

- **I can use scientific notation and estimation to compare very large or very small numbers.**
- **I can multiply and divide numbers given in scientific notation.**

Comparing the relative sizes of very large or very small numbers is easier with scientific notation. The table shows the total number of humans and ants.

	Approximate Number	Scientific Notation
Humans	7 500 000 000	$7.5 \cdot 10^9$
Ants	50 000 000 000 000 000	$5 \cdot 10^{16}$

About how many ants are there for every human?

$\frac{5 \cdot 10^{16}}{7.5 \cdot 10^9} \approx 0.67 \cdot 10^7$**, which is about** 6.7 **million ants per human.**

Ants weigh about $3 \cdot 10^{-6}$ kilograms each. Humans weigh about $6.2 \cdot 10^1$ kilograms each. About how many ants weigh the same as one human?

$\frac{6.2 \cdot 10^1}{3 \cdot 10^{-6}} \approx 2 \cdot 10^7$**, so about** 20 **million ants weigh the same as one human.**

There are about $3.9 \cdot 10^7$ residents in California. The average Californian uses about 180 gallons of water per day. About how many gallons of water total do Californians use in a day?

$\left(3.9 \cdot 10^7\right)\left(1.8 \cdot 10^2\right) \approx 8 \cdot 10^9$**, so Californians use about** 8 **billion gallons of water in a day.**

Summary Question

Describe a strategy you used to divide two numbers given in scientific notation.

Responses vary. **I rounded each of the first factors and divided them. I also divided the powers of** 10 **using exponent properties. My final answer is those two numbers multiplied together.**

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Unit 8.7, Lesson 11: Notes

Name ________________________________

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Unit 8.7, Lesson 12: Notes Name ____________________

Learning Goal(s):

The table below shows the diameters for objects in our solar system.

Object	Diameter (km)
Sun	$1.392 \cdot 10^6$
Mars	$6.785 \cdot 10^3$
Jupiter	$1.428 \cdot 10^5$
Neptune	$4.95 \cdot 10^4$
Saturn	$1.2 \cdot 10^5$

If we place Mars and Neptune next to each other, are they wider than Saturn?

First, add the diameters of Mars and Neptune:

$$6.785 \cdot 10^3 + 4.95 \cdot 10^4$$

To add these numbers, we can either rewrite them as multiples of 10^3 or as multiples of 10^4.

Method 1: Rewrite each number as a multiple of 10^3.

Method 2: Rewrite each number as a multiple of 10^4.

If we place Jupiter and Neptune next to each other, are they wider than the Sun?

About how much wider is Jupiter than Neptune?

Summary Question

What are some important things to remember when adding numbers written in scientific notation?

Unit 8.7, Lesson 12: Notes Name ____________________________________

Learning Goal(s):

- **I can add and subtract numbers given in scientific notation.**

The table below shows the diameters for objects in our solar system.

Object	Diameter (km)
Sun	$1.392 \cdot 10^6$
Mars	$6.785 \cdot 10^3$
Jupiter	$1.428 \cdot 10^5$
Neptune	$4.95 \cdot 10^4$
Saturn	$1.2 \cdot 10^5$

If we place Mars and Neptune next to each other, are they wider than Saturn?

First, add the diameters of Mars and Neptune:

$$6.785 \cdot 10^3 + 4.95 \cdot 10^4$$

To add these numbers, we can either rewrite them as multiples of 10^3 or as multiples of 10^4.

Method 1: Rewrite each number as a multiple of 10^3.

$6.785 \cdot 10^3 + 49.5 \cdot 10^3 = 56.285 \cdot 10^3$ **or about** $5.6 \cdot 10^4$ **km, which is not wider than Saturn.**

Method 2: Rewrite each number as a multiple of 10^4.

$0.6785 \cdot 10^4 + 4.95 \cdot 10^4 = 5.6285 \cdot 10^4$ **km, which is not wider than Saturn.**

If we place Jupiter and Neptune next to each other, are they wider than the Sun?

$1.428 \cdot 10^5 + 4.95 \cdot 10^4$

$= 14.28 \cdot 10^4 + 4.95 \cdot 10^4$**, which is about** $19 \cdot 10^4$ **or** $1.9 \cdot 10^5$ **km. They are not wider than the Sun.**

About how much wider is Jupiter than Neptune?

$1.428 \cdot 10^5 - 4.95 \cdot 10^4$

$\approx 14 \cdot 10^4 - 5 \cdot 10^4$

$= 9 \cdot 10^4$

Jupiter is about $90,000$ **km wider than Neptune.**

Summary Question

What are some important things to remember when adding numbers written in scientific notation?

Responses vary. **Make sure the powers of** 10 **are the same. To rewrite a number with a different power of ten, multiply the first factor by** 10 **to make the exponent smaller by** 1**, or divide the first factor by** 10 **to make the exponent larger by** 1**.**

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Unit 8.7, Lesson 13: Notes Name ______________________________

Learning Goal(s):

Use the table to answer questions about different life forms on our planet.

Creature	Number	Mass of One Individual (kg)
Humans	$7.5 \cdot 10^{9}$	$6.2 \cdot 10^{1}$
Sheep	$1.75 \cdot 10^{9}$	$6 \cdot 10^{1}$
Chickens	$2.4 \cdot 10^{10}$	$2 \cdot 10^{0}$
Antarctic Krill	$7.8 \cdot 10^{14}$	$4.86 \cdot 10^{-4}$
Bacteria	$5 \cdot 10^{30}$	$1 \cdot 10^{-12}$

Which is larger: the total mass of all humans or of all the Antarctic krill?

How can you tell which creature has the greatest total mass?

About how many more chickens are there than sheep?

Summary Question

What are some important things to remember about adding, subtracting, multiplying, and dividing numbers written in scientific notation?

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Unit 8.7, Lesson 13: Notes Name ______________________________

Learning Goal(s):

- **I can use scientific notation to compare different quantities and answer questions about real-world situations.**

Use the table to answer questions about different life forms on our planet.

Creature	Number	Mass of One Individual (kg)
Humans	$7.5 \cdot 10^9$	$6.2 \cdot 10^1$
Sheep	$1.75 \cdot 10^9$	$6 \cdot 10^1$
Chickens	$2.4 \cdot 10^{10}$	$2 \cdot 10^0$
Antarctic Krill	$7.8 \cdot 10^{14}$	$4.86 \cdot 10^{-4}$
Bacteria	$5 \cdot 10^{30}$	$1 \cdot 10^{-12}$

Which is larger: the total mass of all humans or of all the antarctic krill?

Humans:

$\left(7.5 \cdot 10^9\right)\left(6.2 \cdot 10^1\right) \approx 45 \cdot 10^{10}$ **kg**

Antarctic krill:

$\left(7.8 \cdot 10^{14}\right)\left(4.86 \cdot 10^{-4}\right) \approx 40 \cdot 10^{10}$ **kg**

The total mass of all humans is larger.

How can you tell which creature has the greatest total mass?

***Responses vary.* Bacteria has the greatest total mass because multiplying the number of bacteria by the mass of one bacteria will give me 10^{18}, which is larger than any of the other products.**

About how many more chickens are there than sheep?

Responses vary. $2.4 \cdot 10^{10} - 1.75 \cdot 10^9$ **can be rewritten as** $2.4 \cdot 10 \cdot 10^9 - 1.75 \cdot 10^9$ **or** $24 \cdot 10^9 - 1.75 \cdot 10^9$**.**

There are $22.25 \cdot 10^9$ **more chickens than sheep, which is about about** 22 **billion.**

Summary Question

What are some important things to remember about adding, subtracting, multiplying, and dividing numbers written in scientific notation?

***Responses vary.* When adding or subtracting numbers written in scientific notation, you have to have the same exponent for the powers of** 10**. When multiplying or dividing, you can multiply or divide the first factors, and then multiply or divide the powers of** 10**.**