Pythagorean Theorem and Irrational Numbers Student Guide

Math 7 Unit 9 Accelerated

Glossary

Term	Definition
cube root	The cube root of a number n is the number whose cube is n. It is also the edge length of a cube with a volume of n. We write the cube root of n as $\sqrt[3]{n}$. The cube root of 64 is 4 because 4^3 is 64. $\sqrt[3]{64}$ is also the edge length of a cube that has a volume of 64.
hypotenuse	The hypotenuse is the side of a right triangle that is opposite the right angle. It is the longest side of a right triangle.
irrational number	Irrational numbers are numbers that are not rational; they cannot be written as a fraction of two integers. For example, 2 is a rational number because it can be written as $\frac{2}{1}$, whereas π or $\sqrt{3}$ are irrational because they cannot be written as a fraction of two integers.
legs	The legs of a right triangle are the sides that make the right angle. They are the two sides that are not the hypotenuse.
Pythagorean theorem	The Pythagorean theorem describes the relationship between the side lengths of right triangles. The square of the hypotenuse is equal to the sum of the squares of the legs. This is written as $a^2 + b^2 = c^2$.
rational number	Rational numbers are numbers that can be written as a fraction of two integers. Some examples of rational numbers are: $\frac{1}{3}$, $\frac{-7}{4}$, 0, 0.2, -5, and $\sqrt{9}$.
square root	The square root of a positive number n is the positive number whose square is n. It is also the the side length of a square whose area is n. We write the square root of n as $\sqrt{n}$. The square root of 16 is 4 because 4^2 is 16. $\sqrt{16}$ is also the side length of a square that has an area of 16.

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Unit 8 Summary

Prior Learning	Math 8, Unit 8	Future Learning
Math 6 ● Areas of parallelograms and triangles Math 7 ● Operations with rational numbers ● Converting fractions to decimals using long division	● Square roots and cube roots ● The Pythagorean theorem ● Rational and irrational numbers	High School ● Trigonometry ● Rational exponents ● Square root and cube root functions ● Complex numbers

Square Roots and Cube Roots

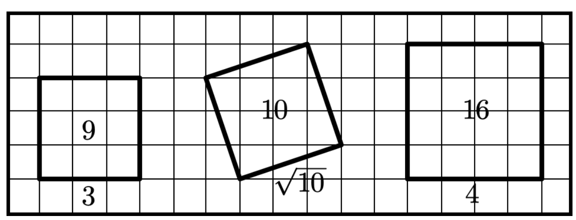
We call the length of the side of a square whose area is a square units $\sqrt{a}$

(pronounced “the square root of a”).

$\sqrt{9} = 3$ because $3^2 = 9$.

$\sqrt{16} = 4$ because $4^2 = 16$.

$\sqrt{10}$ is between 3 and 4 because 10 is between 9 and 16.



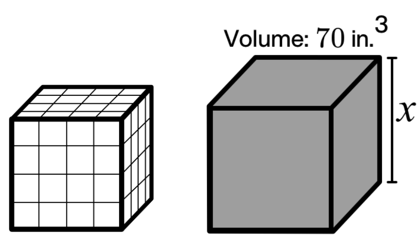
We call the length of the edge of a cube whose volume is a cubic units $\sqrt[3]{a}$

(pronounced “the cube root of a”).

$\sqrt[3]{64} = 4$ because $4^3 = 64$.

$\sqrt[3]{70} > 4$ because $\sqrt[3]{70} > \sqrt[3]{64} = 4$.

$\sqrt[3]{70} \approx 4.12$ because $(4.12)^3 \approx 69.93 \approx 70$.



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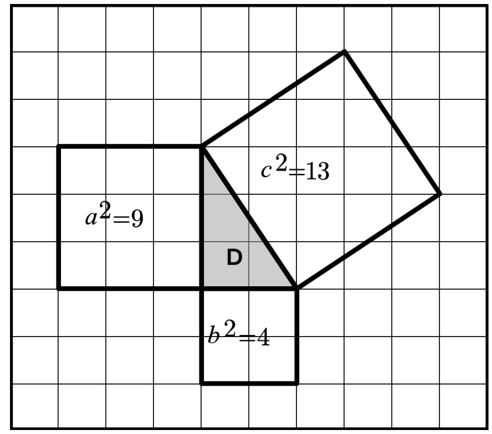
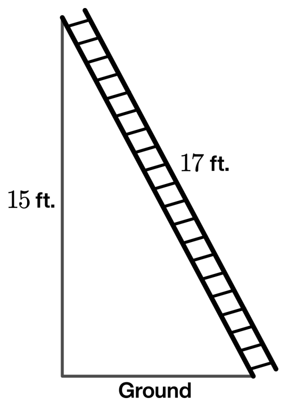
Pythagorean Theorem

In triangle D, the square of the hypotenuse is equal to the sum of the squares of the legs.

This relationship is true for all **right triangles**.

We can describe this relationship as $a^2 + b^2 = c^2$, where a and b are the lengths of the legs, and c is the length of the hypotenuse of a right triangle.





What can the Pythagorean theorem be used for?

- Deciding if a triangle is a right triangle.
- Calculating one side length of a right triangle if we know the other two side lengths.

Rational and Irrational Numbers

Rational numbers are numbers that can be written as a fraction of two integers. We call numbers that cannot be written this way irrational numbers.

Irrational Number

Definition	Facts/Characteristics
A number that cannot be written as a fraction of two integers.	**Their decimal representations are neither terminating nor repeating.**
Examples	**Non-Examples**
$\sqrt{7}$ π $5(\sqrt[3]{15})$	$\sqrt{9}$ $\frac{3}{4}$ -5.34 $-5.\overline{34}$

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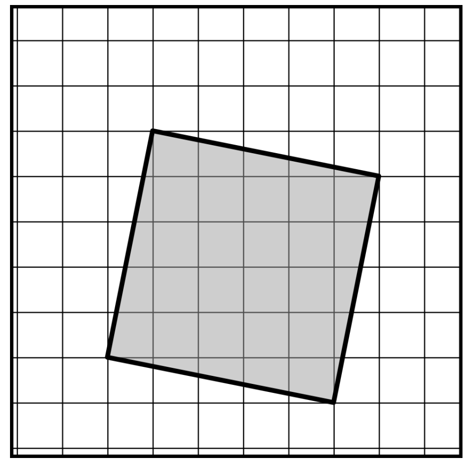
Try This at Home

Square Roots and Cube Roots

1.1 If each grid square represents 1 square unit, what is the area of this titled square?

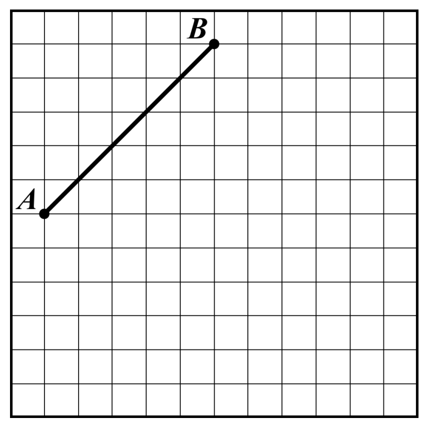


1.2 What is the side length of this tilted square?

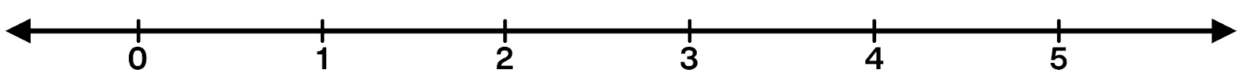
2. Draw a square so that segment AB is along one side of the square.

Exact length of AB:_________

Approximate length of AB:______________



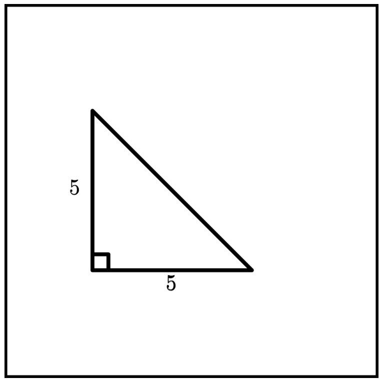
3. Plot the following numbers on the number line below: $\sqrt{27}$,$\sqrt[3]{27}$,$\sqrt[3]{5}$, $\sqrt{5}$



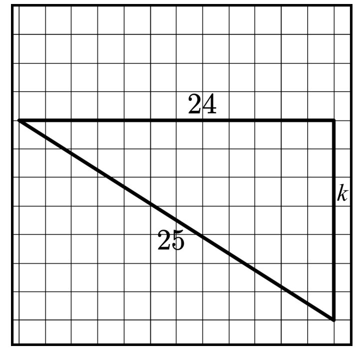
Pythagorean Theorem

4.1 Label the hypotenuse of this triangle with the letter c.

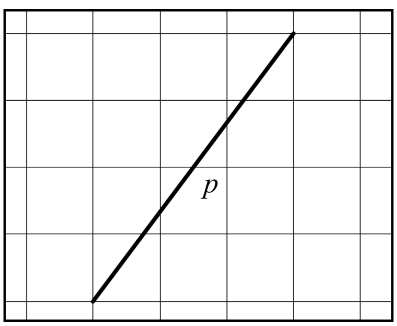
Then determine its length.



4.2 Calculate the length of k.

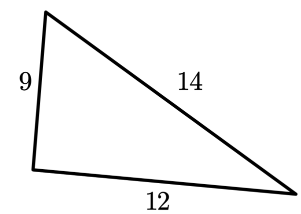


4.3 How long is line segment p?



4.4 Is this a right triangle?

Why or why not?



Rational and Irrational Numbers

5. Write each rational number as a decimal. $\frac{3}{5}, \frac{6}{11}, \frac{17}{6}$.

6.1 Write some examples of rational numbers. Try to include examples of numbers that are rational but that someone might think are irrational.

6.2 Write some examples of irrational numbers.

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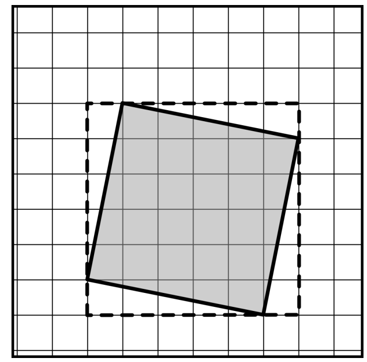
Solutions:

1.1 The area of the square is 26 square units.

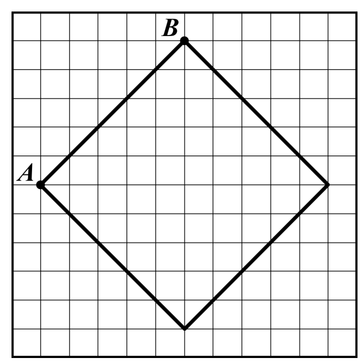


One way to find the area of a tilted square is to enclose the square in a larger square whose area you do know. The side length of this square is 6. Its area is $6 \cdot 6 = 36$ square units.

To find the area of the tilted square, subtract out the areas of the four triangles between the larger square and the original ($4 \cdot \frac{1}{2} \cdot 1 \cdot 5 = 10$ square units).

1.2 The side length of the square is $\sqrt{26}$ units because the square root of the area is the side length of a square.

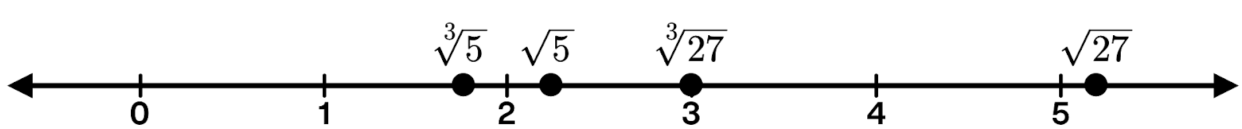
2. **Exact length of AB (as a square root):** $\sqrt{50}$ units



Area of the large square: $10^2 = 100$ square units
Area of the triangles: $4 \cdot \frac{1}{2} \cdot 5 \cdot 5 = 50$ square units
Area of the tilted square: $100 - 50 = 50$ square units
Side length of the tilted square: $\sqrt{50}$ units

Approximate length of AB: $\sqrt{50}$ is between 7 and 8 because 50 is between 49 or 7^2 and 64 or 8^2.

3. Plot the following numbers on the number line below: $\sqrt{27}, \sqrt[3]{27}, \sqrt[3]{5}, \sqrt{5}$



4.1 The length of the hypotenuse is $\sqrt{50}$ units.

$a^2 + b^2 = c^2$

$(5)^2 + (5)^2 = c^2$

$25 + 25 = c^2$

$50 = c^2$

$c = \sqrt{50}$

4.2 The length of k is 7 units.

$a^2 + b^2 = c^2$

$(k)^2 + (24)^2 = 25^2$

$k^2 + 576 = 625$

$k^2 = 49$

$k = 7$

4.3 Line segment p is 5 units long.

$a^2 + b^2 = c^2$

$(3)^2 + (4)^2 = p^2$

$9 + 16 = p^2$

$25 = p^2$

$p = 5$

4.4 This is **not** a right triangle because the Pythagorean theorem is not true.

$9^2 + 12^2 \neq 14^2$

$81 + 144 \neq 196$

$225 \neq 196$

If the hypotenuse were 15, the triangle would be a right triangle.

5.

$\frac{3}{5}$

$\frac{3}{5} = \frac{6}{10} = 0.6$

$\frac{6}{11}$

$$\begin{array}{r|l} & 0.5454\ldots \\ 11 & 6.00000 \\ & -55 \\ & \ \ 50 \\ & \ -44 \\ & \ \ \ 60 \\ & \ \ -55 \\ & \ \ \ \ 50 \\ & \ \ \ -44 \\ & \ \ \ \ \ 60 \end{array}$$

$\frac{17}{6}$

$$\begin{array}{r|l} & 2.8333\ldots \\ 6 & 17.00000 \\ & 12 \\ & \ \ 50 \\ & \ -48 \\ & \ \ \ 20 \\ & \ \ -18 \\ & \ \ \ \ 20 \\ & \ \ \ -18 \\ & \ \ \ \ \ 20 \end{array}$$

6.1 *Responses vary.* Some examples: $\frac{3}{5}, 0.16, \frac{\sqrt{16}}{\sqrt{100}}, \sqrt[3]{8}, 7, .1\overline{66}$

6.2 *Responses vary.* Some examples: $\frac{\sqrt{3}}{5}, \sqrt{8}, \sqrt[3]{16}, 7\pi, 16 \cdot \sqrt{7}$

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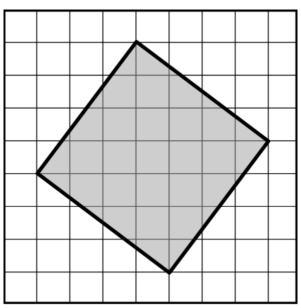
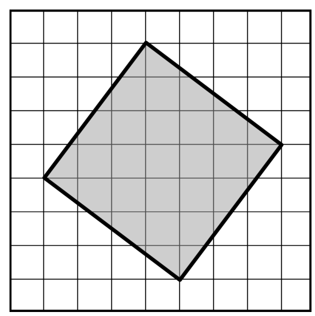
Unit 8.8, Lesson 1: Notes Name ___________________________________

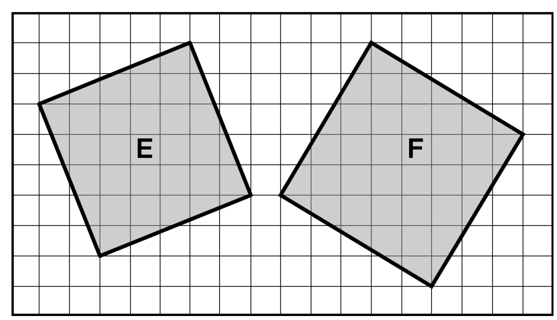
Learning Goal(s):

Sometimes we want to find the area of a square, but we don't know the side length. When this is true, we can use strategies such as "decompose and rearrange" and "surround and subtract."

Decompose and Rearrange	**Surround and Subtract**
	

Use any strategy to calculate the area of each square.

Square E Square F



Which of these squares must have a side length that is greater than 5 but less than 6? _________
Explain how you know.

Summary Question
If you don't know the side length of a square, how can you find its area?

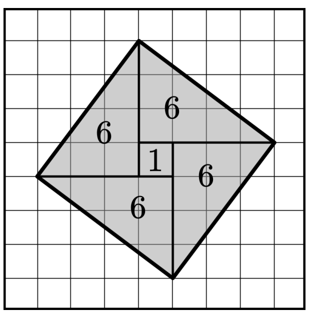
Unit 8.8, Lesson 1: Notes Name ___________________________________

Learning Goal(s):

- **I can calculate the area of a triangle.**
- **I can calculate the area of a tilted square on a grid by using methods like “decompose and rearrange” and “surround and subtract.”**

Sometimes we want to find the area of a square, but we don’t know the side length. When this is true, we can use strategies such as “decompose and rearrange” and “surround and subtract.”

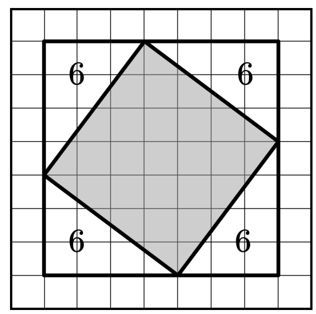
Decompose and Rearrange



Area =
4 **triangles +** 1 **square**

$4 \cdot 6 + 1 = 25$ **square units**

Surround and Subtract



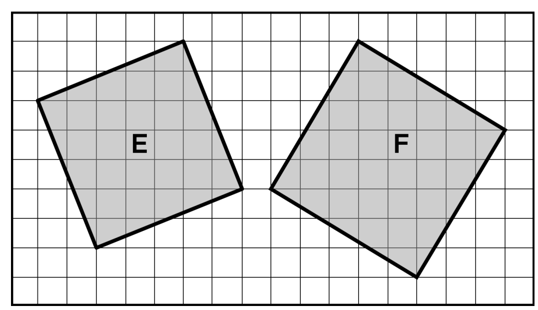
Area =
square - 4 **triangles**

$7 \cdot 7 - 4 \cdot 6 = 25$
square units

Use any strategy to calculate the area of each square.

Square E
29 **square units**

Square F
34 **square units**



Which of these squares must have a side length that is greater than 5 but less than 6? **Both**
Explain how you know.

A square with a side length of 5 has an area of 25 square units, and a square with a side length of 6 has an area of 36 square units. Squares E and F both have areas between 25 and 36 square units, so they both have side lengths greater than 5 and less than 6.

Summary Question
If you don’t know the side length of a square, how can you find its area?

I can enclose the square in a larger square whose area I do know. Then I can subtract out the areas of the four triangles that are between the larger square and the original square. This gives me the area of the original square.

Unit 8.8, Lesson 2: Notes Name ______________________________

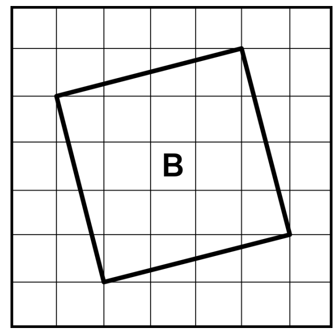
Learning Goal(s):

Sometimes we want to know the side length of a square whose length is not countable using a grid. When this is true, we can take the square root of the area in order to find the side length.

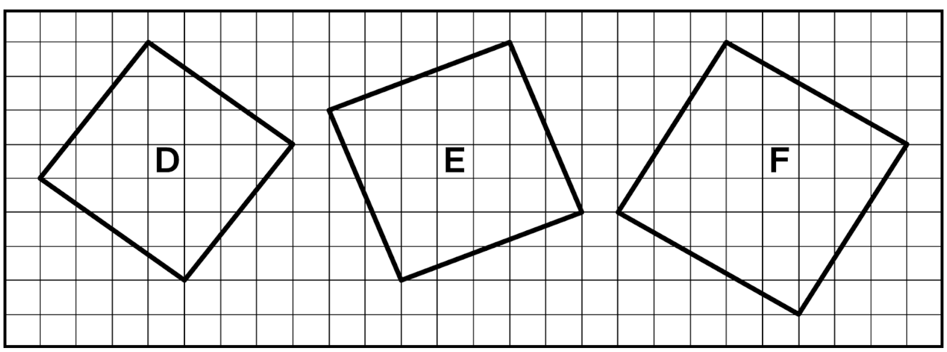
Square B has an area of 17.

We say the side length of a square with an area of 17 units is $\sqrt{17}$ units.

This means that $(\quad)^2 =$ ______.



Find each missing value.



Square	Side Length of Square (units)	Area of Square (square units)
D		25
E	$\sqrt{29}$	
F		

Summary Question

Explain the meaning of $(\sqrt{9})^2 = 9$ using squares and side lengths.

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Unit 8.8, Lesson 2: Notes Name ______________________________

Learning Goal(s):

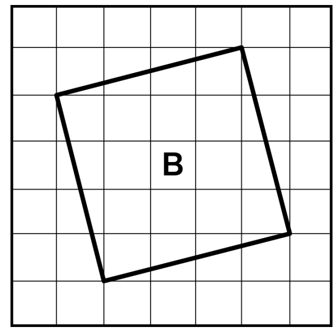
- **I can explain the meaning of square roots in terms of side length and area of a square.**
- **I can write the side length or the area of a square using square root notation (like $\sqrt{3}$).**

Sometimes we want to know the side length of a square whose length is not countable using a grid. When this is true, we can take the square root of the area in order to find the side length.

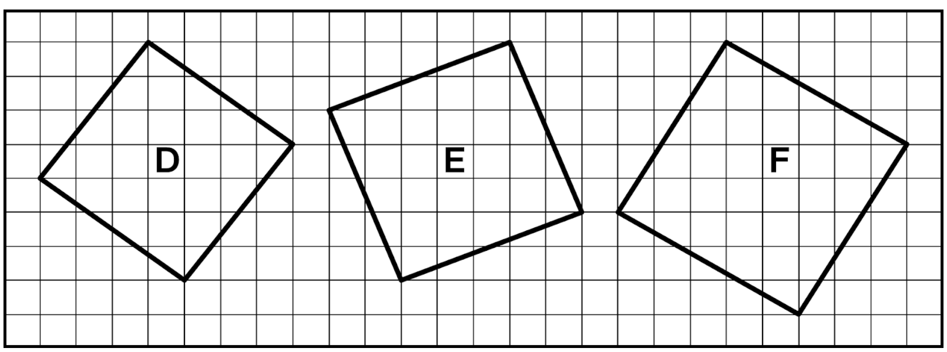
Square B has an area of 17.

We say the side length of a square with an area of 17 units is $\sqrt{17}$ units.

This means that $\left(\sqrt{17}\right)^2 = 17$.



Find each missing value.



Square	Side Length of Square (units)	Area of Square (square units)
D	$\sqrt{25}$	25
E	$\sqrt{29}$	29
F	$\sqrt{34}$	34

Summary Question

Explain the meaning of $\left(\sqrt{9}\right)^2 = 9$ using squares and side lengths.

A square with an area of 9 square units has a side length of $\sqrt{9}$ or 3. This makes sense because $3 \cdot 3 = 9$.

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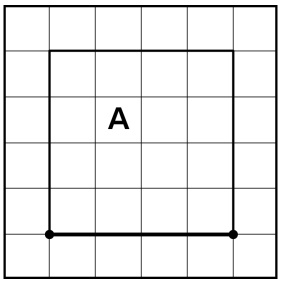
Unit 8.8, Lesson 3: Notes Name ___________________________________

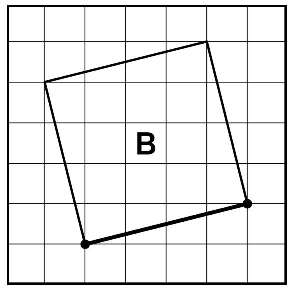
Learning Goal(s):

Determine the side length of each square. Use a square root if the value is not exact.



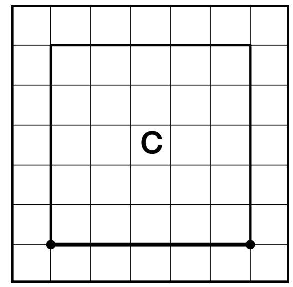
Area: 16 square units

Side length:________



Area: 17 square units

Side length:________



Area: 25 square units

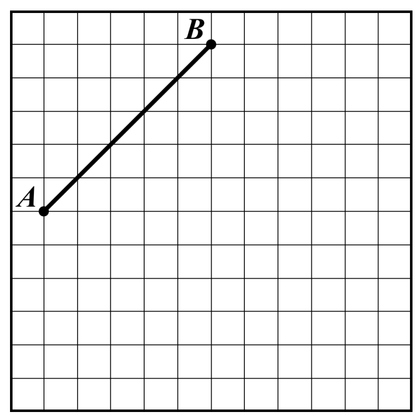
Side length:_________

Square B has a side length of ______ units. In order to approximate numbers like ______, we can find two integer values that the number lies between. Square B has an area between ______ and ______ square units, so its side length must be between ______ and ______ units.

Draw a square so that segment AB is along one side of the square.

Exact length of AB (as a square root):_____________

Approximate length of AB :______________



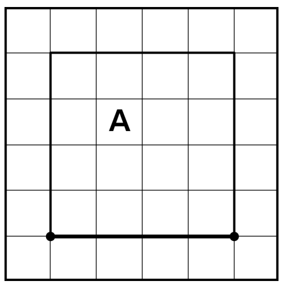
Summary Question

What two integers does $\sqrt{60}$ lie between? Explain how you know. Then use a calculating device to approximate $\sqrt{60}$ as closely as possible.

Unit 8.8, Lesson 3: Notes Name ____________________________________

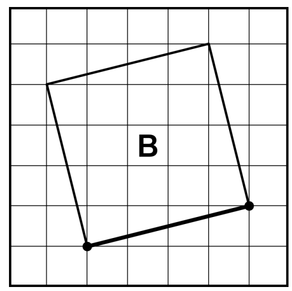
Learning Goal(s):
- **I can approximate a square root as a decimal.**

Determine the side length of each square. Use a square root if the value is not exact.



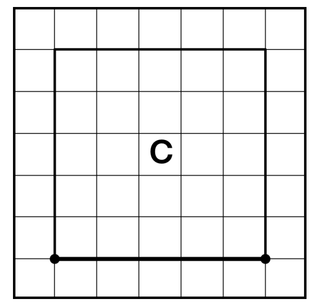
Area: 16 square units

Side length: 4 **units**



Area: 17 square units

Side length: $\sqrt{17}$ **units**



Area: 25 square units

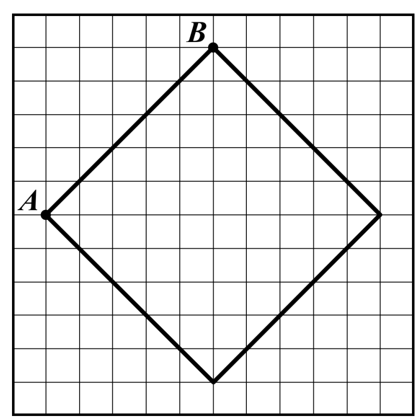
Side length: 5 **units**

Square B has a side length of $\sqrt{17}$ units. In order to approximate numbers like $\sqrt{17}$, we can find two integer values that the number lies between. Square B has an area between 16 and 25 square units, so its side length must be between 4 and 5 units.

Draw a square so that segment AB is along one side of the square.

Exact length of AB (as a square root): $\sqrt{50}$ **units**

Approximate length of AB: **Between** 7 **and** 8 **units**



Summary Question

What two integers does $\sqrt{60}$ lie between? Explain how you know. Then use a calculating device to approximate $\sqrt{60}$ as closely as possible.

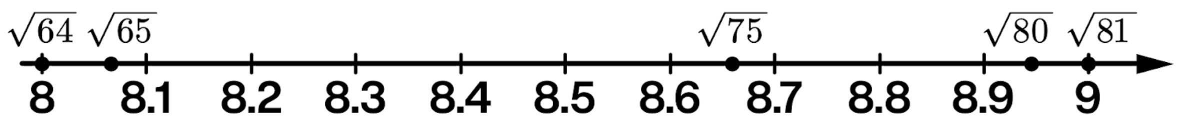
$\sqrt{60}$ **is larger than** 7 **because** $\sqrt{60}$ **is larger than** $\sqrt{49}$**, and** $\sqrt{49} = 7$. **Similarly** $\sqrt{60}$ **is smaller than** 8 **because** $\sqrt{60}$ **is less than** $\sqrt{64}$**, and** $\sqrt{64} = 8$**.**

Unit 8.8, Lesson 4: Notes Name ______________________________

Learning Goal(s):

We can approximate the values of square roots by looking for whole numbers nearby.

- $\sqrt{65}$ is a little more than _____ , because $\sqrt{65}$ is a little more than $\sqrt{64}$ = ______.
- $\sqrt{80}$ is a little less than ______ , because $\sqrt{80}$ is a little less than $\sqrt{81}$ = ______.
- $\sqrt{75}$ is between ______ and ______ , because 75 is between 64 and 81 .
- $\sqrt{75}$ is approximately ______. We can check this by calculating _______________.

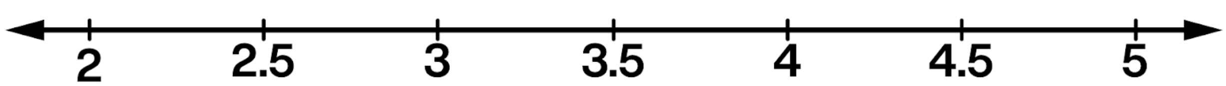


Under each description, write the square root(s) that lie between the integers described.

- $\sqrt{6}$
- $\sqrt{12}$
- $\sqrt{24}$
- x when $x^2 = 8$

Between 2 and 3	**Between 4 and 5**

Add each number above to the number line below.



Summary Question

Where would $\sqrt{17}$ belong on the number line above? Explain how you know.

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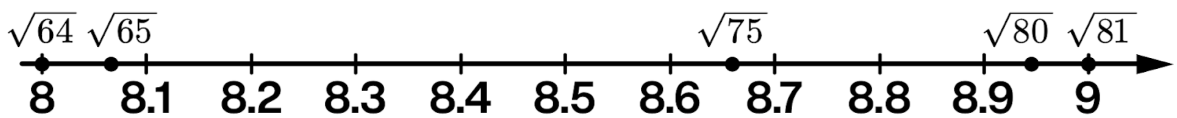
Unit 8.8, Lesson 4: Notes Name ______________________

Learning Goal(s):

- **I can plot square roots on a number line.**
- **I can identify the two whole numbers a square root is between and explain why.**

We can approximate the values of square roots by looking for whole numbers nearby.

- $\sqrt{65}$ is a little more than 8, because $\sqrt{65}$ is a little more than $\sqrt{64} = 8$.
- $\sqrt{80}$ is a little less than 9, because $\sqrt{80}$ is a little less than $\sqrt{81} = 9$.
- $\sqrt{75}$ is between 8 and 9, because 75 is between 64 and 81.
- $\sqrt{75}$ is approximately 8.67. We can check this by calculating $8.67^2 = 75.1689$, **which is close to** 75.

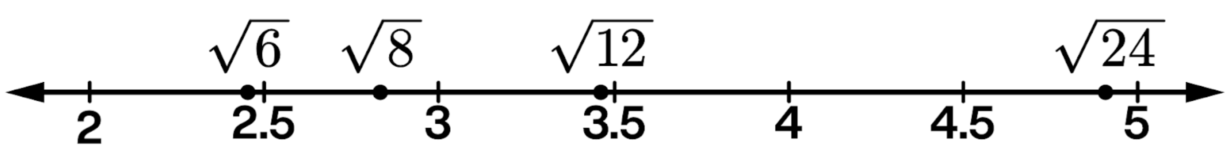


Under each description, write the square root(s) that lie between the integers described.

- $\sqrt{6}$
- $\sqrt{12}$
- $\sqrt{24}$
- x when $x^2 = 8$

Between 2 and 3	**Between 4 and 5**
$\sqrt{6}$ x **when** $x^2 = 8$	$\sqrt{24}$

Add each number above to the number line below.



Summary Question

Where would $\sqrt{17}$ belong on the number line above? Explain how you know.

$\sqrt{17}$ **is between** 4 **and** 5 **because** $\sqrt{17}$ **is larger than** $\sqrt{16} = 4$ **and smaller than** $\sqrt{25} = 5$. **It is much closer to** 4 **because** $\sqrt{17}$ **is much closer to** $\sqrt{16}$ **than it is to** $\sqrt{25}$.

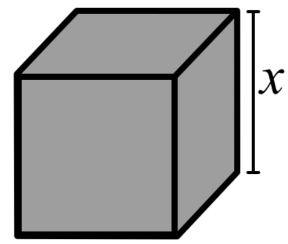
desmos

Unit 8.8, Lesson 5: Notes Name ______________________________

Learning Goal(s):

Sometimes we are interested in the edge length of a cube instead of the side length of a square.

The number $\sqrt[3]{17}$, read as "cube root of 17," is the edge length of a cube that has a volume of 17.



We can approximate the value of a cube root in a similar way to approximating a square root:

$\sqrt[3]{17}$ is more than ______ , because $\sqrt[3]{17}$ is more than $\sqrt[3]{8}$ = ______ .

$\sqrt[3]{17}$ is less than ______ , because $\sqrt[3]{17}$ is less than $\sqrt[3]{27}$ = ______ .

$\sqrt[3]{17}$ is approximately ______ , because $(2.57)^3 = 16.9746$.

Find each missing value without using a calculator.

Exact Edge Length of Cube (units)	Approximate Edge Length of Cube (units)	Volume of Cube (cubic units)
	Between _____ and _____	60
$\sqrt[3]{4}$	Between _____ and _____	
	Between _____ and _____	25

Summary Question

Approximate the value of x when $x^3 = 81$. Explain your thinking.

desmos

Unit 8.8, Lesson 5: Notes Name ______________________

Learning Goal(s):

- **I can explain the meaning of a cube root, like $\sqrt[3]{35}$, in terms of its edge length and volume.**
- **I can identify the two whole numbers a cube root is between and explain why.**

Sometimes we are interested in the edge length of a cube instead of the side length of a square.

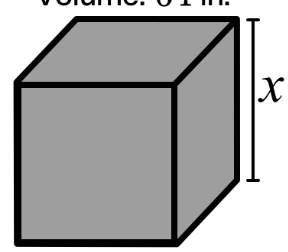
The number $\sqrt[3]{17}$, read as " cube root of 17," is the edge length of a cube that has a volume of 17.

We can approximate the value of a cube root in a similar way to approximating a square root:

$\sqrt[3]{17}$ is more than 2, because $\sqrt[3]{17}$ is more than $\sqrt[3]{8} = 2$.

$\sqrt[3]{17}$ is less than 3, because $\sqrt[3]{17}$ is less than $\sqrt[3]{27} = 3$.

$\sqrt[3]{17}$ is approximately 2.57, because $(2.57)^3 = 16.9746$.



Find each missing value without using a calculator.

Exact Edge Length of Cube (units)	**Approximate Edge Length of Cube (units)**	**Volume of Cube (cubic units)**
$\sqrt[3]{60}$	Between 3 and 4	60
$\sqrt[3]{4}$	Between 1 and 2	4
$\sqrt[3]{25}$	Between 2 and 3	25

Summary Question

Approximate the value of x when $x^3 = 81$. Explain your thinking.

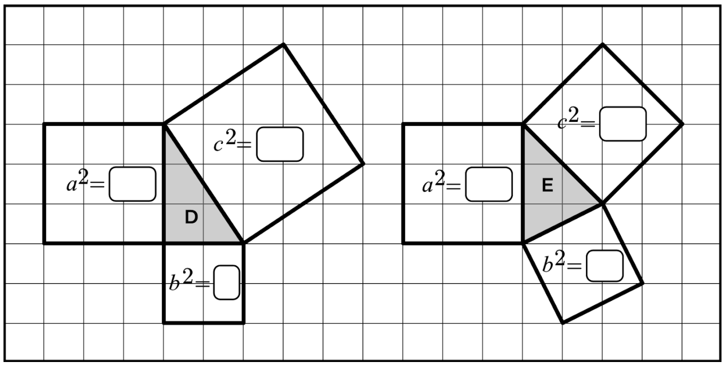
x **is equal to** $\sqrt[3]{81}$**, which is between** 4 **and** 5**. We know this is true because** $\sqrt[3]{64} = 4$**, and** $\sqrt[3]{125} = 5$**. In particular,** $x \approx 4.327$ **since** $4.327^3 = 81.014$**.**

desmos

Unit 8.8, Lesson 6: Notes Name ______________________________

Learning Goal(s):

Find the missing values. Record what you notice and wonder.



I notice . . .

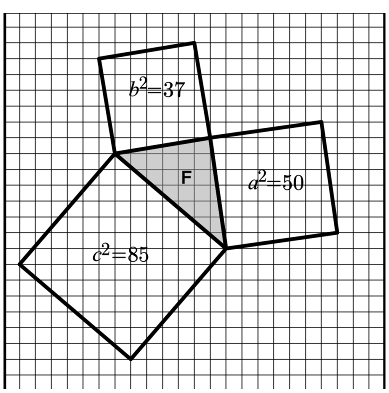
I wonder . . .

In Triangle D, the square of the hypotenuse is equal to the sum of the squares of the legs.

This relationship is true for **all** right triangles. It is often known as the **Pythagorean theorem.**

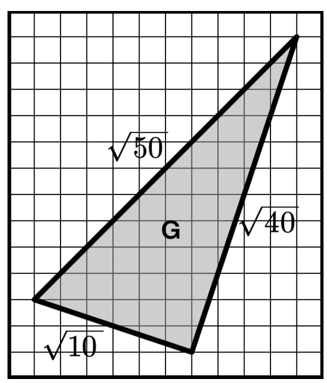
Another way to describe this relationship is $a^2 + b^2 = c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse of a right triangle.

Decide if the Pythagorean theorem is true for each triangle. Show your thinking.



Yes / No

Your thinking:



Yes / No

Your thinking:

Summary Question

What does the Pythagorean theorem tell us about the side lengths of a right triangle?

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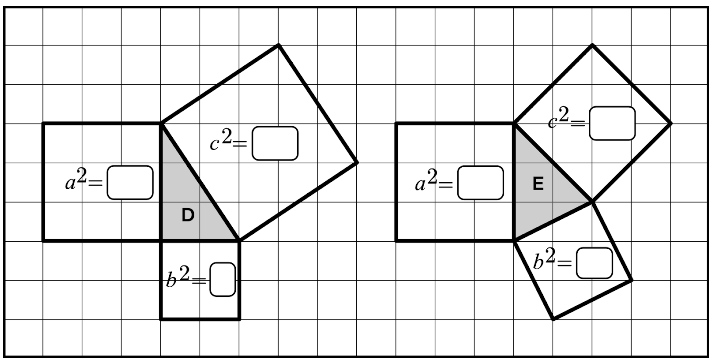
Unit 8.8, Lesson 6: Notes Name ____________________

Learning Goal(s):

- **I can explain what the Pythagorean theorem says.**

Find the missing values. Record what you notice and wonder.



I noticed . . .

Responses vary.

I noticed that a^2 is equal in both diagrams.

I noticed the two small squares equaled the large square for Triangle D.

I wonder . . .

Responses vary.

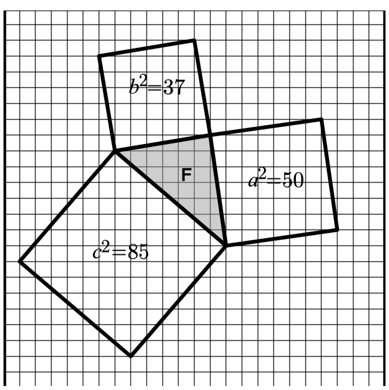
How are the sides of the triangles related?

In Triangle D, the square of the hypotenuse is equal to the sum of the squares of the legs.

This relationship is true for **all** right triangles. It is often known as the **Pythagorean theorem.**

Another way to describe this relationship is $a^2 + b^2 = c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse of a right triangle.

Decide if the Pythagorean theorem is true for each triangle. Show your thinking.



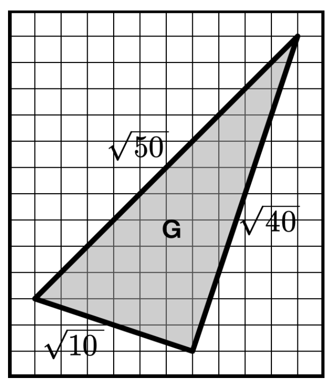
Yes / **No**

Your thinking:

Responses vary.

$37 + 50 \neq 85$ **so**

$a^2 + b^2 \neq c^2$



Yes / No

Your thinking:

Responses vary.

$(\sqrt{10})^2 + (\sqrt{40})^2$

$= 10 + 40$

$= 50$

$= (\sqrt{50})^2$

Summary Question

What does the Pythagorean theorem tell us about the side lengths of a right triangle?

Responses vary. **The Pythagorean theorem tells us that the sum of the squares of the shorter side lengths is equal to the square of the longest side length (the hypotenuse).**

Unit 8.8, Lesson 7: Notes Name ______________________________

Learning Goal(s):

We observed that $a^2 + b^2 = c^2$ is true for many right triangles with legs of a and b. How do we know this relationship is **always** true? Proofs help us know when a relationship will always be true.

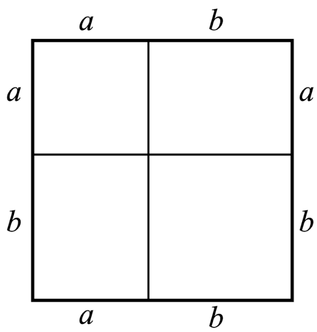


Figure G

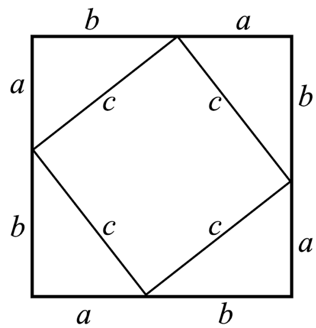


Figure H

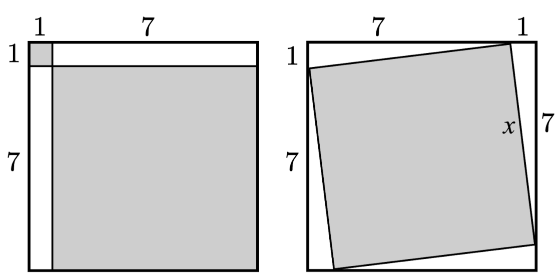
Habib wrote the following proof of the Pythagorean theorem based on the diagram:

$$a^2 + b^2 + ab + ab = c^2 + 4 \cdot \frac{1}{2}\, ab$$

$$a^2 + b^2 + 2ab = c^2 + 2ab$$

$$a^2 + b^2 = c^2$$

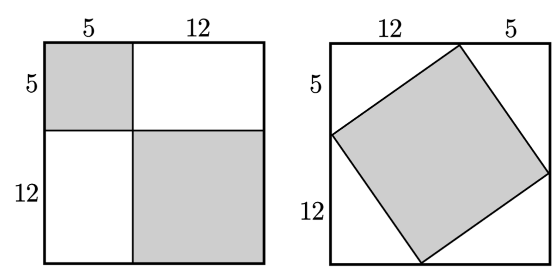
Describe Habib's strategy for proving the Pythagorean theorem. Use the diagrams if that helps to support your thinking.



Find the value of x.

Summary Question

Show how you can see the equation $5^2 + 12^2 = 13^2$ in the figures on the right. Explain how this relates to the Pythagorean theorem.



Unit 8.8, Lesson 7: Notes Name ____________________

Learning Goal(s):

- **I can explain why the Pythagorean theorem is true for every right triangle.**
- **I can use the Pythagorean theorem to find unknown side lengths in right triangles.**

We observed that $a^2 + b^2 = c^2$ is true for many right triangles with legs of a and b. How do we know this relationship is **always** true? Proofs help us know when a relationship will always be true.

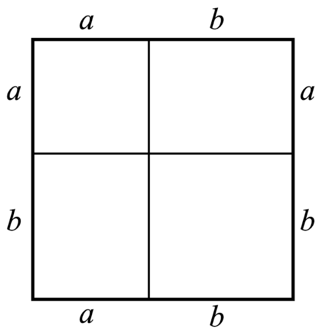


Figure G

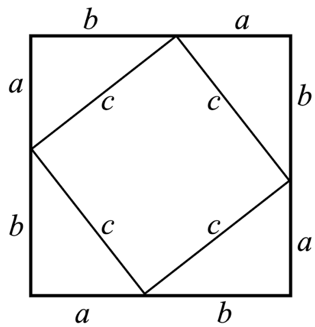


Figure H

Habib wrote the following proof of the Pythagorean theorem based on the diagram:

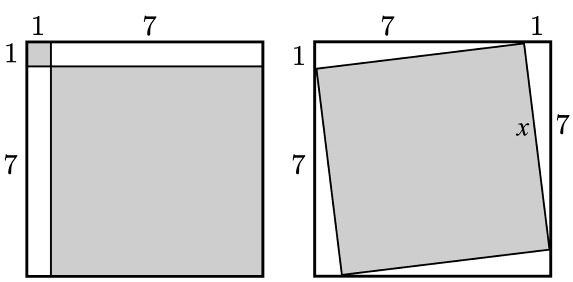
$$a^2 + b^2 + ab + ab = c^2 + 4 \cdot \frac{1}{2}ab$$

$$a^2 + b^2 + 2ab = c^2 + 2ab$$

$$a^2 + b^2 = c^2$$

Describe Habib's strategy to prove the Pythagorean theorem. Use the diagrams if that helps to support your thinking.

Habib found the total area for Figure G and for Figure H by adding up the areas of the individual parts. Since figures G and H have the same total area, Habib set the areas equal. Then, they simplified and subtracted $2ab$ from each side to get $a^2 + b^2 = c^2$.



Find the value of x.

$$7^2 + 1^2 = x^2$$

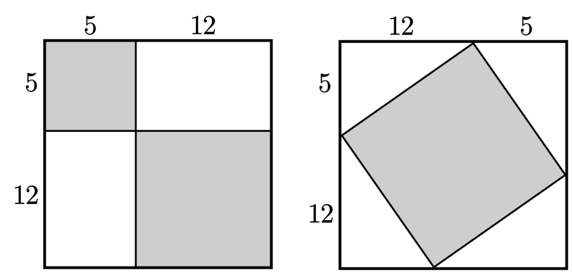
$$49 + 1 = x^2$$

$$50 = x^2$$

$$x = \sqrt{50}$$

Summary Question

Show how you can see the equation $5^2 + 12^2 = 13^2$ in the figures on the right. Explain how this relates to the Pythagorean theorem.



In the left figure, I see $5^2 + 12^2$ as the total area of the two shaded squares. The Pythagorean theorem says that the area of the shaded square in the right figure will be equal to the sum of the shaded squares in the left figure, so its area is 169 square units, or 13^2 square units.

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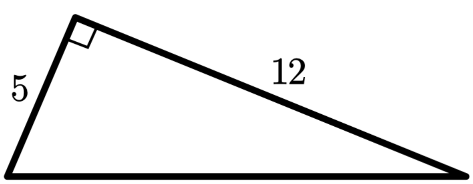
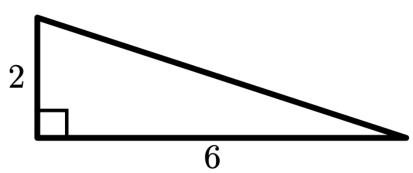
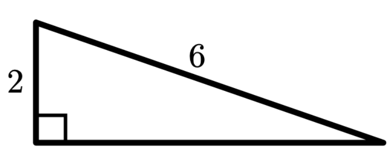
Unit 8.8, Lesson 8: Notes Name ___________________________________

Learning Goal(s):

Sometimes we know the length of two sides of a right triangle and want to find the third. In this situation, we can use the Pythagorean theorem.

Highlight the hypotenuse of each triangle. Then find the length of the missing side of the triangle.

Triangle	Missing Side Length
	
	
	

Summary Question

How can you use the Pythagorean theorem to find an unknown side length in a right triangle?

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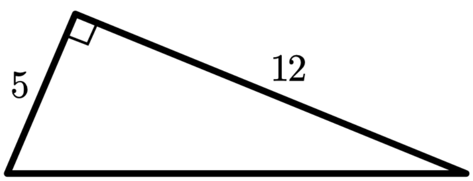
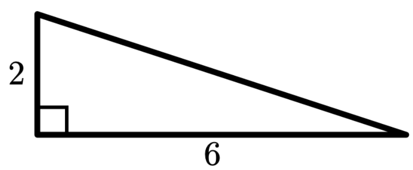
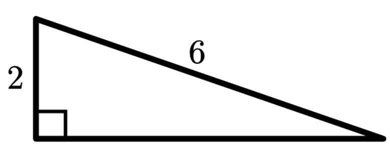
Unit 8.8, Lesson 8: Notes Name ______________________________

Learning Goal(s):
- **I can identify which side is the hypotenuse and which sides are the legs in a right triangle.**
- **I can use the Pythagorean theorem to find unknown side lengths in right triangles.**

Sometimes we know the length of two sides of a right triangle and want to find the third. In this situation, we can use the Pythagorean theorem.

Highlight the hypotenuse of each triangle. Then find the length of the missing side of the triangle.

Triangle	**Missing Side Length**
	$5^2 + 12^2 = c^2$ $25 + 144 = c^2$ $169 = c^2$ $c = 13$
	$2^2 + 6^2 = c^2$ $4 + 36 = c^2$ $40 = c^2$ $c = \sqrt{40}$
	$2^2 + b^2 = 6^2$ $4 + b^2 = 36$ $b^2 = 32$ $b = \sqrt{32}$

Summary Question
How can you use the Pythagorean theorem to find an unknown side length in a right triangle?

You can enter the side lengths you know into the equation and solve for the missing length. Remember that you will need to take a square root at some point!

Unit 8.8, Lesson 9: Notes Name ______________________________

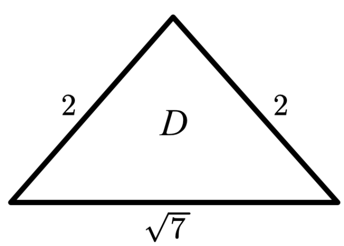
Learning Goal(s):

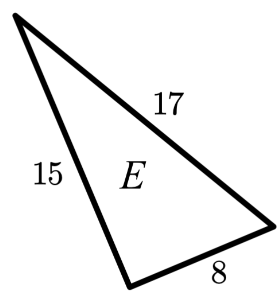
Sometimes it's hard to tell if a triangle is a right triangle just by looking. In this situation, we can use what is called the converse of the Pythagorean theorem to help us decide.

If ____ , the triangle is a right triangle.

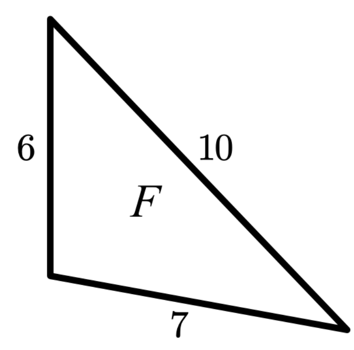
If ____, the triangle is not a right triangle.

Use the converse of the Pythagorean theorem to decide which of the following are right triangles.





Change **one** of the values to make triangle F into a right triangle.



Summary Question

Explain how to tell if a triangle is a right triangle using its side lengths.

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Unit 8.8, Lesson 9: Notes Name ____________________

Learning Goal(s):

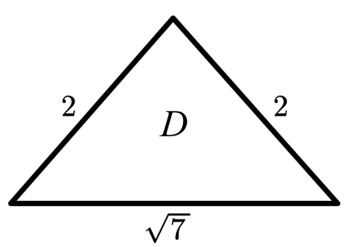
- **I can explain why it is true that if the side lengths of a triangle satisfy the equation $a^2 + b^2 = c^2$, then it must be a right triangle.**
- **I can determine whether a triangle is a right triangle if I know its side lengths.**

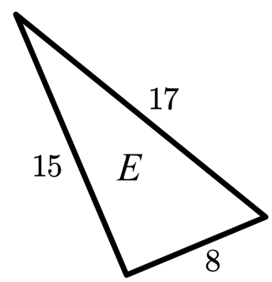
Sometimes it's hard to tell if a triangle is a right triangle just by looking. In this situation, we can use what is called the converse of the Pythagorean theorem to help us decide.

If $a^2 + b^2 = c^2$, the triangle is a right triangle.

If $a^2 + b^2 \neq c^2$, the triangle is not a right triangle.

Use the converse of the Pythagorean theorem to decide which of the following are right triangles.





$2^2 + 2^2 \neq (\sqrt{7})^2$, **so** D **is not a right triangle.**

$8^2 + 15^2 = 17^2$, **so** E **is a right triangle.**

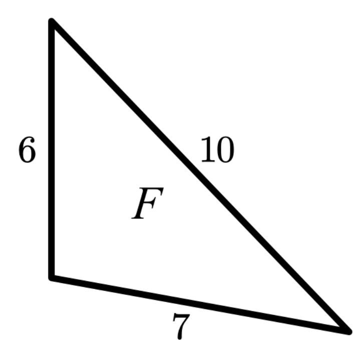
Change **one** of the values to make Triangle F into a right triangle.

Responses vary.

Change 7 **to** 8**, then** $6^2 + 8^2 = 10^2$ **and** F **is a right triangle.**

Change 6 **to** $\sqrt{51}$**, then** $(\sqrt{51})^2 + 7^2 = 10^2$ **.**

Change 10 **to** $\sqrt{85}$**, then** $6^2 + 7^2 = (\sqrt{85})^2$**.**



Summary Question

Explain how to tell if a triangle is a right triangle using its side lengths.

Responses vary.

Square each of the shorter side lengths and add them together. If that is the same as the hypotenuse squared, then the triangle is a right triangle.

desmos

Unit 8.8, Lesson 10: Notes Name ____________________________

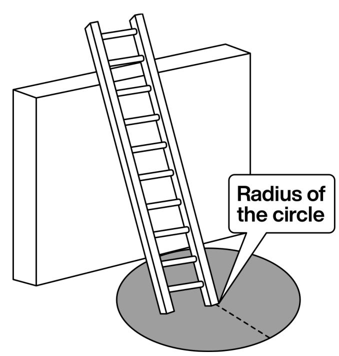
Learning Goal(s):

Name some situations in your world that might involve right triangles.

A 17-foot ladder is leaning against a wall. The ladder can reach a window 15 feet up the wall. How far from the wall should the base of the ladder be so that it reaches the window?

Draw a picture of the situation.	Write your answer to the question. Show all of your thinking.

To avoid accidents, the fire department wants to create a circular no-walk zone under the ladder with a radius that is the distance between the ladder and the wall. What is the area of the no-walk zone?



Summary Question

What are some things that are important to remember when using the Pythagorean theorem?

desmos

Unit 8.8, Lesson 10: Notes Name ____________________

Learning Goal(s):

- **I can use the Pythagorean theorem to solve problems.**

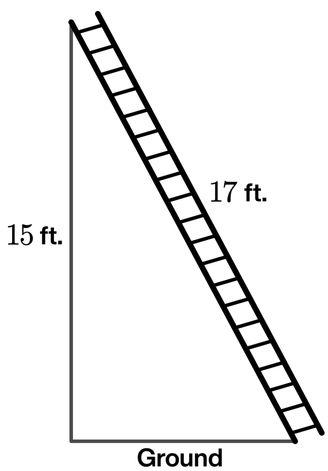
Name some situations in your world that might involve right triangles.

Responses vary.

- **Shadows**
- **Roofs**
- **Staircases & other architecture**
- **Bridges & electricity towers**
- **Sails on ships**
- **U.K. flag (the Union Jack)**

A 17-foot ladder is leaning against a wall. The ladder can reach a window 15 feet up the wall. How far from the wall should the base of the ladder be so that it reaches the window?

Draw a picture of the situation.



Write your answer to the question. Show all of your thinking.

Let d represent the distance from the wall to the ladder.

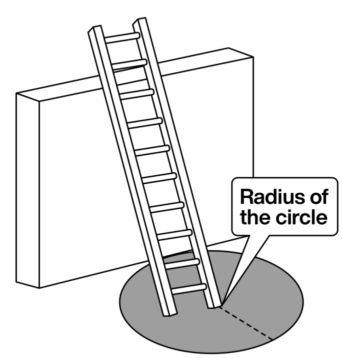
$$d^2 + 15^2 = 17^2$$
$$d^2 + 225 = 289$$
$$d^2 = 64$$

In this situation, d must be 8.
Place the ladder 8 feet from the wall.

To avoid accidents, the fire department wants to create a circular no-walk zone under the ladder with a radius that is the distance between the ladder and the wall. What is the area of the no-walk zone?

Area of a circle $= \pi \cdot r^2$

Area of no-walk zone $= \pi \cdot (8)^2 = 64\pi \approx 201$ **square feet**



Summary Question

What are some things that are important to remember when using the Pythagorean theorem?

Responses vary. **You need to use the squares of the side lengths rather than the lengths themselves. Remember which lengths are the legs (vs. the hypotenuse) of the right triangle.**

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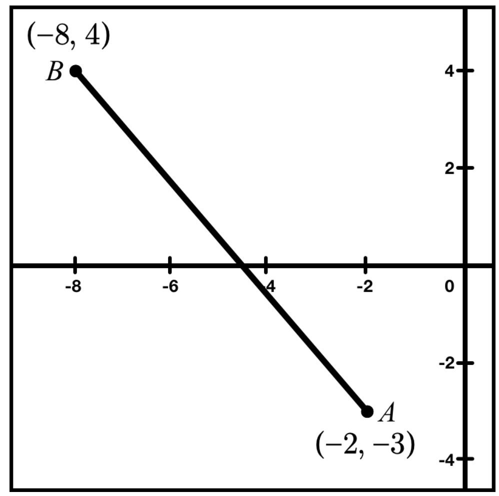
Unit 8.8, Lesson 11: Notes Name ______________________

Learning Goal(s):

Sometimes we want to find the distance between two points that are not easily countable on a grid.

Draw a right triangle whose hypotenuse is $\overline{AB}$.

Use the tools you have to calculate the length of $\overline{AB}$.

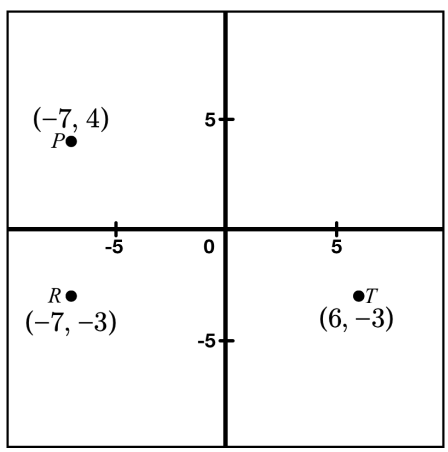


Calculate the distances between each pair of points on the graph.

$\overline{PR}$ = ______ units

$\overline{RT}$ = ______ units

$\overline{PT}$ = ______ units



Ready for more?

Plot a point that is exactly $\sqrt{29}$ units away from point R.

Summary Question

How is using the Pythagorean theorem on a grid similar to or different from using it on a triangle?

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Unit 8.8, Lesson 11: Notes Name ______________________

Learning Goal(s):

- **I can calculate the distance between two points in the coordinate plane.**
- **I can calculate the length of a diagonal line segment in the coordinate plane.**

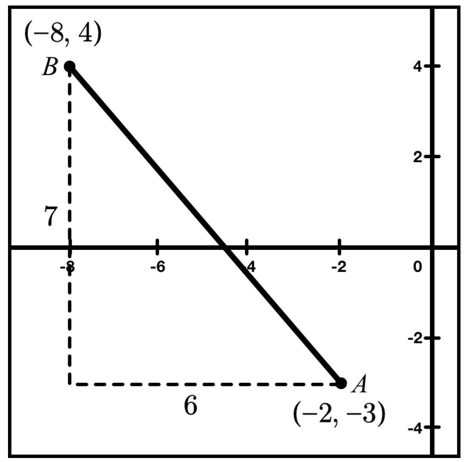
Sometimes we want to find the distance between two points that are not easily countable on a grid.

Draw a right triangle whose hypotenuse is $\overline{AB}$.

Use the tools you have to calculate the length of $\overline{AB}$.

$$\overline{AB} = \sqrt{6^2 + 7^2} = \sqrt{36 + 49} = \sqrt{85}$$

$\overline{AB}$ is $\sqrt{85}$ units long.

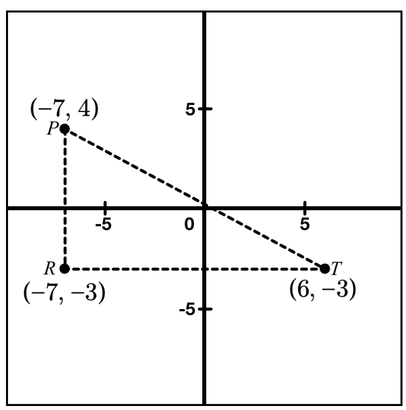


Calculate the distances between each pair of points on the graph.

$\overline{PR}$ = 7 units

$\overline{RT}$ = 13 units

$\overline{PT}$ =
$\sqrt{7^2 + 13^2} = \sqrt{49 + 169} = \sqrt{218}$
units



Ready for more?

Plot a point that is exactly $\sqrt{29}$ units away from point R.

Responses vary.
One correct response is $(-5, 2)$ because the distance between the x-coordinates of $(-5, 2)$ and $R = (-7, -3)$ is 2 and the distance between the y-coordinates is 5.
$\sqrt{5^2 + 2^2} = \sqrt{29}$.

Summary Question

How is using the Pythagorean theorem on a grid similar to or different from using it on a triangle?

On a grid and on a triangle, you use the same equation to find the missing sides. On a grid, the distance is often not given, and you need to figure it out using the coordinates you know.

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Unit 8.8, Lesson 12: Notes Name ____________________

Learning Goal(s):

Sometimes it's helpful to rewrite fractions as decimals. Can you think of times this might be true?

Decimals can either terminate (stop) or continue infinitely. When the decimal repeats indefinitely, we draw a line over the repeating digits.

Expand $0.56\overline{73}$ = ______________________________…

Describe how Kwame calculated that $\frac{2}{11} = .\overline{18}$ in your own words.

$$
\begin{array}{r}
0.1818...\\
11\overline{)\,2.00000}\\
\underline{-11}\\
90\\
\underline{-88}\\
20\\
\underline{-11}\\
90\\
\underline{-88}\\
20
\end{array}
$$

Use any strategy to write each fraction as a decimal. Decide whether it is terminating or repeating.

$\frac{3}{8}$	$\frac{3}{11}$	$\frac{98}{6}$
Terminating or repeating?	Terminating or repeating?	Terminating or repeating?

Summary Question

What are some clues you can use to predict if a fraction will be a terminating or a repeating decimal?

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Unit 8.8, Lesson 12: Notes Name ______________________________

Learning Goal(s):

- **I can write a fraction as either a repeating or a terminating decimal.**

Sometimes it's helpful to rewrite fractions as decimals. Can you think of times this might be true?

Decimals can either terminate (stop) or continue infinitely. When the decimal repeats indefinitely, we draw a line over the repeating digits.

Expand $0.56\overline{73} = 0.5673737373737373737373737373...$

Describe how Kwame calculated that $\frac{2}{11} = .\overline{18}$ in your own words.

Kwame divided 2 by 11 in order to find the decimal. He used long division and added trailing zeros when needed.

$$\begin{array}{r} 0.1818... \\ 11 \overline{)\,2.00000} \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \end{array}$$

Use any strategy to write each fraction as a decimal. Decide whether it is terminating or repeating.

$\frac{3}{8} = 0.375$

$$\begin{array}{r} 0.375 \\ 8 \overline{)\,3.000} \\ -24 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Terminating or repeating?

$\frac{3}{11} = 0.\overline{27}$

$$\begin{array}{r} 0.2727... \\ 11 \overline{)\,3.00000} \\ -22 \\ \hline 80 \\ -77 \\ \hline 30 \\ -22 \\ \hline 80 \\ -77 \\ \hline 30 \end{array}$$

Terminating or **repeating**?

$\frac{98}{6} = 16.\overline{3}$

$$\begin{array}{r} 16.333... \\ 6 \overline{)\,98.00} \\ 6 \\ \hline 38 \\ -36 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 20 \end{array}$$

Terminating or **repeating**?

Summary Question

What are some clues you can use to predict if a fraction will be a terminating or a repeating decimal?

Responses vary. **If the denominator of the fraction is a power of 2 (like 8), then the fraction terminates. If the denominator of the fraction is a multiple of 3 (like 6), then the fraction repeats.**

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Unit 8.8, Lesson 13: Notes

Name ______________________________

Learning Goal(s):

Some decimals terminate, while others repeat. However, **all** terminating and repeating decimals can be written as fractions. Look at the example below to see what we mean.

Describe each step of Adhira's process for converting $4.\overline{85}$ to $\frac{481}{99}$.

$x = 4.\overline{85}$

1.	1. $100x = 485.\overline{85}$
2.	2. $-x = -4.\overline{85}$
3.	3. $99x = 481$
4.	4. $x = \frac{481}{99}$

Use any strategy to write each decimal as a fraction.

$5.\overline{37}$	$5.\overline{3}$	$0.3\overline{7}$

Summary Question

What question(s) do you have about converting repeating decimals into fractions? (You can also record a question you imagine someone else having about this topic.)

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Unit 8.8, Lesson 13: Notes Name ______________________________

Learning Goal(s):

- **I can write a repeating decimal as a fraction.**
- **I understand that every number has a decimal expansion.**

Some decimals terminate, while others repeat. However, **all** terminating and repeating decimals can be written as fractions. Look at the example below to see what we mean.

Describe each step of Adhira's process for converting $4.\overline{85}$ to $\frac{481}{99}$.

$x = 4.\overline{85}$

1. **Multiply both sides by 100. This keeps the decimal part of the number the same as it was.** — $100x = 485.\overline{85}$
2. **Multiply x by -1. The decimal part of $100x$ and $-x$ are equal.** — $-x = -4.\overline{85}$
3. **Combine $-x$ and $100x$. This is now equal to a whole number.** — $99x = 481$
4. **Divide both sides by 99. This is the value of $1x$.** — $x = \frac{481}{99}$

Use any strategy to write each decimal as a fraction.

$5.\overline{37}$

$100x = 537.\overline{37}$

$-x = -5.\overline{37}$

$99x = 532$

$x = \frac{532}{99}$ **(or equivalent)**

$5.\overline{3}$

$10x = 53.\overline{3}$

$-x = -5.\overline{3}$

$9x = 48$

$x = \frac{48}{9}$ **(or equivalent)**

$0.3\overline{7}$

$100x = 37.\overline{7}$

$-10x = -3.\overline{7}$

$90x = 34$

$x = \frac{34}{90}$ **(or equivalent)**

Summary Question

What question(s) do you have about converting repeating decimals into fractions? (You can also record a question you imagine someone else having about this topic.)

Responses vary.

- **How do you decide what to multiply by x?**
- **Why do you multiply x by a negative number in Step 2?**

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Unit 8.8, Lesson 14: Notes Name ______________________________

Learning Goal(s):

Rational numbers are numbers that can be written as a fraction of two integers. What if a number cannot be written as a fraction of two integers? We call this type of number an irrational number.

Definition	Facts/Characteristics
Irrational Number	
Examples	Non-Examples

Write each number as a rational number. If it is impossible, write "irrational."

0.16	$\frac{\sqrt{16}}{\sqrt{100}}$	$\sqrt{8}$	x when $x^3 = 64$	$\sqrt[3]{16}$

Summary Question

What does it mean when someone says that $\sqrt{3}$ is irrational?

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Unit 8.8, Lesson 14: Notes Name ____________________

Learning Goal(s):

- **I know what a rational number is and can give an example.**
- **I know what an irrational number is and can give an example.**

Rational numbers are numbers that can be written as a fraction of two integers. What if a number cannot be written as a fraction of two integers? We call this type of number an irrational number.

Irrational Number

Definition	Facts/Characteristics
A number that cannot be written as a fraction of two integers.	**Their decimal representations are neither terminating nor repeating.**
Examples	**Non-Examples**
$\sqrt{7}$ π $5(\sqrt[3]{15})$	$\sqrt{9}$ $\frac{3}{4}$ -5.34 $-5.\overline{34}$

Write each number as a rational number. If this is impossible, write "irrational."

0.16	$\frac{\sqrt{16}}{\sqrt{100}}$	$\sqrt{8}$	x when $x^3 = 64$	$\sqrt[3]{16}$
$\frac{16}{100}$ **(or equivalent)**	$\frac{4}{10}$ **(or equivalent)**	**Irrational**	4	**Irrational**

Summary Question

What does it mean when someone says that $\sqrt{3}$ is irrational?

This means that $\sqrt{3}$ cannot be written as a fraction of whole numbers or as a repeating or terminating decimal.